

The Implementation and Development of a Time-Domain Model of Dispersive Transmission Line

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Abstract — In the time domain, the accurate and efficient simulation of lossy and dispersive transmission lines is still a problem.

Therefore, a time-domain model of the dispersive transmission line based on series approximation and recursive integration was implemented into APLAC. In addition, model was developed to take into account the initial state.

The model was tested carefully and results obtained were accurate enough compared with results based on the frequency-domain model. The recursive integration made the calculation efficient.

The new model improves the transmission line simulation capability of APLAC.

1 Introduction

The problem of transient simulation of lossy transmission lines has recently received more attention because operating speeds in high-speed digital electronics are increasing. In that case, the conventional lumped models are no longer adequate to describe the transmission line behavior of electronic interconnects. However, in the time domain, the accurate and efficient simulation of transmission lines is still a problem because lossy and dispersive transmission lines are traditionally modeled and analyzed in the frequency domain as opposed to nonlinear devices or time-dependent components which must be characterized in the time domain.

One solution to transmission line simulation problem is the model developed by Valtonen [3]. In the model, the computation is kept as exact as required. The characteristic admittance and propagation function of a transmission line in the time domain were obtained exactly with inverse Laplace transforms. The power series approximation of the modified Bessel functions allows calculating convolution integrals recursively and the amount of computation is proportional to the number of time steps.

The model has been implemented into APLAC circuit simulator and developed in such a way that it takes for the first time, the non-zero initial state into account contrary to many other models which assume a zero initial state. The relevant simulation

results are also presented. The detailed derivation of the whole model and a comparison of other transmission line simulation approaches are available in reference [2].

2 Transmission line model

2.1 Frequency-domain model for transmission line

The model equations for the transmission line in the frequency domain can be formulated as follows

$$\begin{cases} I_1(s) = Y(s)U_1(s) - 2I^-(s)e^{-\gamma(s)\ell} \\ I_2(s) = Y(s)U_2(s) - 2I^+(s)e^{-\gamma(s)\ell} \\ J_1(s) = \frac{1}{2}[Y(s)U_2(s) + I_2(s)]e^{-\gamma(s)\ell} \\ J_2(s) = \frac{1}{2}[Y(s)U_1(s) + I_1(s)]e^{-\gamma(s)\ell}, \end{cases} \quad (1)$$

where the characteristic admittance

$$Y(s) = \sqrt{\frac{sC + G}{sL + R}} = Y_0 \sqrt{\frac{s + b}{s + a}}, \quad (2)$$

where $Y_0 = \sqrt{C/L}$, $a = R/L$, and $b = G/C$. The propagation coefficient becomes $\gamma(s) = \frac{\tau}{\ell} \sqrt{(s + a)(s + b)}$, where $\tau = \ell\sqrt{LC}$ is the delay of the line. $I^+(s)$ and $I^-(s)$ are the incident and reflected current waves propagating along the line, respectively. Fig. 1 presents the model corresponding to these equations.

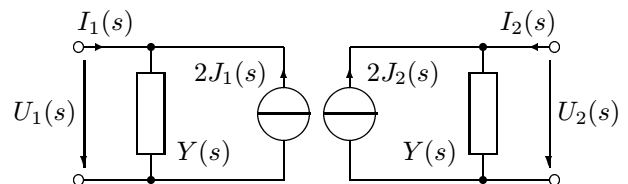


Figure 1: Transmission line model in the frequency domain

According to eqs. (1), the expressions for the propagating current waves are

$$\begin{cases} I^-(s) = \frac{1}{2}[Y(s)U_2(s) + I_2(s)] \\ I^+(s) = \frac{1}{2}[Y(s)U_1(s) + I_1(s)]. \end{cases} \quad (3)$$

2.2 Model for lossy and dispersive transmission line in time domain

First, the frequency-domain model based on eqs. (1) has to be transferred into the time domain.

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The model equations become

$$\begin{cases} i_1(t) = \int_{-\infty}^t y(t-x)u_1(x)dx - 2j_1(t) \\ j_1(t) = \int_{-\infty}^t \kappa(t-x)i^-(x)dx \\ i_2(t) = \int_{-\infty}^t y(t-x)u_2(x)dx - 2j_2(t) \\ j_2(t) = \int_{-\infty}^t \kappa(t-x)i^+(x)dx, \end{cases} \quad (4)$$

where

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}, \quad \kappa(t) = \mathcal{L}^{-1}\{e^{-\gamma(s)l}\} \quad (5)$$

The integrals can be divided into two parts

$$\int_{-\infty}^t f(x,t)dx = \int_{-\infty}^0 f(x,t)dx + \int_0^t f(x,t)dx,$$

where the first part specifies the initial state and the latter determines the transient phenomenon itself. Fig. 2 shows the model of the transmission line in the time domain.

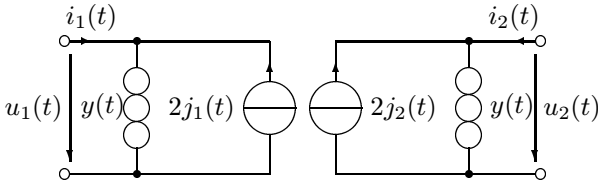


Figure 2: Transmission line model in the time domain with convolution conductance

2.2.1 Model for convolution conductance

The inverse Laplace transform for the convolution conductance is [1]:

$$y(t) = Y_0 [\delta(t) + \nu e^{-\mu t} \{l_0(\nu t) + l_1(\nu t)\}]. \quad (6)$$

Here, $\delta(t)$ is the Dirac delta function, l_0 and l_1 are the modified Bessel functions and $\mu = \frac{b+a}{2}$, $\nu = \frac{b-a}{2}$. Now, assuming a zero initial state the integral (4) can be written as $i(t) = Y_0 u(t) + \hat{i}(t)$

$$\hat{i}(t) = \nu Y_0 e^{-\mu t} \int_0^t \{l_0[\nu(t-x)] + l_1[\nu(t-x)]\} e^{\mu x} u(x) dx. \quad (7)$$

To construct the recursion, the modified Bessel functions are approximated with polynomial series

$$l_0(x) + l_1(x) \approx \sum_{k=0}^M C_k x^k, \quad (8)$$

where M is the order of approximation. Applying this approximation yields

$$\hat{i}(t) = \sum_{k=0}^M \hat{i}_k(t) \quad (9)$$

$$\hat{i}_k(t) = C_k \nu^{k+1} Y_0 e^{-\mu t} \int_0^t (t-x)^k e^{\mu x} u(x) dx. \quad (10)$$

Time derivatives yield the recursion

$$\frac{d\hat{i}_0}{dt} = -\mu \hat{i}_0 + C_0 \nu Y_0 u \quad (11)$$

$$\frac{d\hat{i}_k}{dt} = -\mu \hat{i}_k + \nu k \frac{C_k}{C_{k-1}} \hat{i}_{k-1}, \quad k > 0. \quad (12)$$

Now, the numerical integration

$$x(t_n) = B_0 \frac{dx(t_n)}{dt} + \sum_{m=1}^N \left[A_m x(t_{n-m}) + B_m \frac{dx(t_{n-m})}{dt} \right] \quad (13)$$

gives

$$\hat{i}_0(t_n) = \sum_{m=0}^N [a_m \hat{i}_0(t_{n-m}) + b_m \nu Y_0 u(t_{n-m})] \quad (14)$$

$$\hat{i}_k(t_n) = \sum_{m=0}^N \left[a_m \hat{i}_k(t_{n-m}) + b_m k \nu \frac{C_k}{C_{k-1}} \hat{i}_{k-1}(t_{n-m}) \right], \quad (15)$$

where $k > 0$, $a_0 = 0$,

$$a_m = \frac{A_m - \mu B_m}{1 + \mu B_0}, \quad \text{and} \quad b_m = \frac{B_m}{1 + \mu B_0}. \quad (16)$$

By considering an unknown voltage $u(t_n)$ as a variable, $\hat{i}_k(t)$ can be formulated as $\hat{i}_k(t_n) = G_k u(t_n) + J_k(t_n)$. By utilizing the recursion, the following expressions can be introduced

$$G_0 = b_0 \nu Y_0 \quad (1)$$

$$G_k = b_0 \nu k \frac{C_k}{C_{k-1}} G_{k-1}, \quad k > 0 \quad (1)$$

$$J_k(t_n) = G_k \sum_{m=1}^N \left(a_m + \frac{b_m}{b_0} \right) u(t_{n-m}) \quad (1)$$

$$+ \sum_{m=0}^N \left[a_m J_k(t_{n-m}) + \frac{b_m}{b_0} \frac{G_k}{G_{k-1}} J_{k-1}(t_{n-m}) \right]$$

$$J_{-1}(t_n) = 0.$$

Fig. 3 depicts the model for convolution conductance, where

$$\mathbf{G} = Y_0 + \sum_{k=0}^M G_k \quad J_G(t_n) = \sum_{k=0}^M J_k(t_n) \quad (20)$$

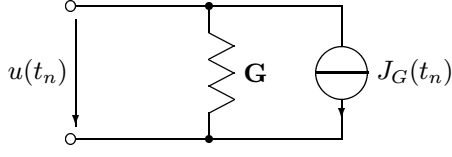


Figure 3: Model for the convolution conductance.

2.2.2 Model for current source with memory

The other part of the transmission line model is a current source. Now, the inverse Laplace transform (5) gives [1]

$$\kappa(t) = e^{-\mu\tau} \delta(t-\tau) + \frac{\nu\tau e^{-\mu t}}{\sqrt{t^2 - \tau^2}} \text{l}_1\left(\nu\sqrt{t^2 - \tau^2}\right) \varepsilon(t-\tau), \quad (21)$$

where $\varepsilon(t)$ is a unit step function.

The integral of interest from eq. (4) is

$$j(t) = \int_0^t \kappa(t-x) i(x) dx = j_a(t) + j_b(t), \quad (22)$$

where $j_a(t) = e^{-\mu\tau} i(t-\tau)$ and

$$j_b(t) = \nu\tau e^{-\mu t} \int_0^{t-\tau} \frac{\text{l}_1\left(\nu\sqrt{(t-x)^2 - \tau^2}\right)}{\sqrt{(t-x)^2 - \tau^2}} e^{\mu x} i(x) dx. \quad (23)$$

Approximating the modified Bessel function l_1 , applying the binomial expansion and differentiating a couple of times [2] yields finally

$$j(t) = j_a(t) + \sum_{k=0}^{M''} \sum_{p=0}^k j_{k,p}(t) \quad (24)$$

$$j_{k,p}(t_n) = \sum_{m=1}^N a_m j_{k,p}(t_{n-m}) \quad (25)$$

$$+ D_{k,p} \tau e^{-\mu\tau} \sum_{m=0}^N b_m i(t_{n-m} - \tau)$$

$$+ \sum_{m=0}^N b_m \hat{j}_{k,p}(t_{n-m}) \quad (26)$$

$$\begin{aligned} \hat{j}_{k,p}(t_n) &= \sum_{m=1}^N a_m \hat{j}_{k,p}(t_{n-m}) \\ &+ 2p D_{k,p} e^{-\mu\tau} \sum_{m=0}^N b_m i(t_{n-m} - \tau) \\ &- \frac{2(2p-1)(k-p+1)}{\tau^2} \sum_{m=0}^N b_m j_{k,p-1}(t_{n-m}), \end{aligned} \quad (27)$$

where the current term $j_{k,-1} = 0$ and

$$D_{k,p} = (-1)^{k+p} \binom{k}{p} C_{2k+1} \nu^{2k+2} \tau^{2k}. \quad (28)$$

2.2.3 Summary of the model

The total value of current source \mathbf{J} includes both the actual current source and, in addition, the current terms from convolution conductance J_G :

$$\mathbf{J}_1(t_n) = J_{G1}(t_n) - 2j_1(t_n) \quad (29)$$

$$\mathbf{J}_2(t_n) = J_{G2}(t_n) - 2j_2(t_n). \quad (30)$$

Fig. (4) presents the complete model.

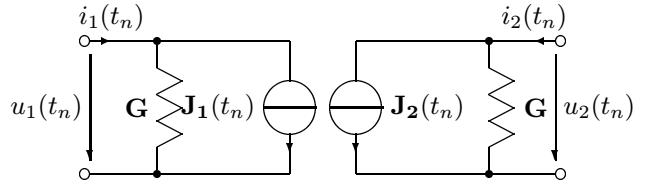


Figure 4: Transmission line model in the time domain.

According to fig. 4, $i_1(t_n) = \mathbf{G}u_1(t) + \mathbf{J}_1(t)$ and thus on the basis of eqs. (3) $i^+(t) = \mathbf{G}u_1(t) + J_{G1}(t) - j_1(t)$, and similarly $i^-(t) = \mathbf{G}u_2(t) + J_{G2}(t) - j_2(t)$.

When the simulation time $t < \tau$, the DC values of propagating currents i^+ and i^- are used. Otherwise, the values of $i^+(t-\tau)$ and $i^-(t-\tau)$ have to be interpolated.

3 Initial state of transient analysis

The transient sources have to be initialized to a steady state that is equivalent to the DC state. After carrying out the DC analysis, the currents i_1 and i_2 are calculated on the basis of y-parameters

$$\begin{cases} i_1 = y_{11}u_1 + y_{12}u_2 \\ i_2 = y_{12}u_1 + y_{11}u_2. \end{cases} \quad (31)$$

Due to continuity, these currents have to be the same in transient analysis when $t = 0$.

The expressions for the propagating currents can be written

$$\begin{cases} i^+ = \frac{1}{2} (i_1 + \mathbf{G}u_1 + J_{G1}) \\ i^- = \frac{1}{2} (i_2 + \mathbf{G}u_2 + J_{G2}), \end{cases} \quad (32)$$

where the terms $\mathbf{G}u + J_G$ arise from the convolution conductance. First, the values for G_k are determined. Subsequently, the values for J_G can be calculated by demanding $J_k(t_n) = J_k(t_{n-m})$ for all m and k in formula (19). Now, $\mathbf{G}u + J_G$ is known

and the values for i^+ and i^- can be determined. Note that calculating the values for i_1 and i_2 from the time domain model would require the complete value of the current source which, again, depends on the values of i^+ and i^- . Therefore, DC values for i_1 and i_2 were used.

As the values of $i^+(0)$ and $i^-(0)$ are now known, $\hat{j}_{k,p}$ and $j_{k,p}$ can be determined as with J_k by demanding $j_{k,p}(t_n) = j_{k,p}(t_{n-m})$ and $\hat{j}_{k,p}(t_n) = \hat{j}_{k,p}(t_{n-m})$ for all indices m , k , and p in formulae (25) and (27). Finally, the "old" values of J_k , $j_{k,p}$, and $\hat{j}_{k,p}$ are initialized.

4 Simulation examples

Subsequently, some substantial simulation examples are presented to verify the operation of the model. Fig. 5 shows the circuit used in these simulations.

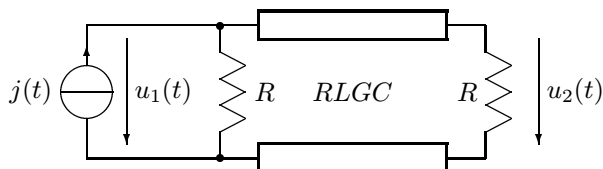


Figure 5: Circuit used in simulation examples.

The operation of the model is compared with harmonic balance (HB) results. HB solves the steady state of a nonlinear circuit in the frequency domain where analyzing dispersive transmission lines is straightforward.

The values of the distributed parameters used in these simulations are $R=0.35 \Omega/\text{m}$, $L=265 \text{ nH}/\text{m}$, $C=94.3 \text{ pF}/\text{m}$, and $G=0 \text{ S}$.

4.1 Simulation with sinusoidal excitation

Now, the current source $j(t)$ is composed of several sinusoidal signals at harmonic frequencies and also includes a distinct DC component. Fig. 6 displays the voltage at the end of the line.

The error between transient and HB analysis results is estimated by calculating the squared error normalized with the squared value in every simulation point. As the frequency increases, error always decreases below 0.1 %.

4.2 Simulation with pulse excitation

The simulation of a pulse is one of the most interesting, because as a wideband signal, a pulse shows distinctly the effect of dispersion. Fig. 7 displays the voltage at the end of the line.

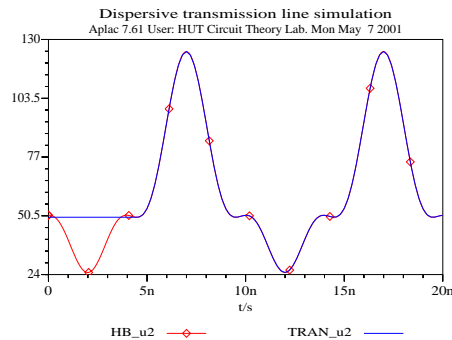


Figure 6: Simulation of $u_2(t)$ with sinusoidal excitation

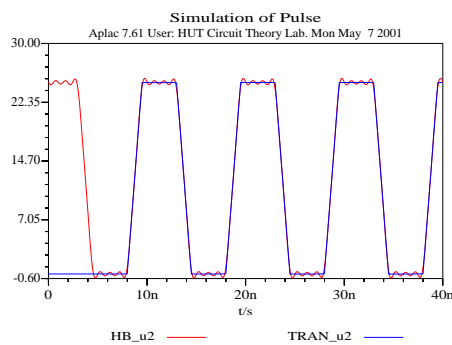


Figure 7: Simulation of $u_2(t)$ with pulse excitation

Now, the HB results contain additional oscillation due to the limited number of harmonic frequencies used. However, the model seems to be capable of simulating this kind of wideband signals, too.

5 Conclusions

The new transmission line model has been implemented into the circuit simulator APLAC. The dispersive transmission lines can now be simulated in the time domain, which is not possible with traditional transmission line models. This new model can be extended to include the skin effect.

References

- [1] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, Dover Publications, 1965.
- [2] A. Lehtovuori, *Time-domain model of dispersive transmission line*, Master's Thesis, Helsinki University of Technology, 2000. (www.aplac.hut.fi/publications/dt-anu/)
- [3] M. Valtonen, *Computer-Aided Analysis of Mixed Lumped and Distributed Circuits in Time Domain*, Proc. of the ISCAS 1978, pp. 762-766, 1978.