

# Analysis of SI Circuits in MAPLE Program

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**Abstract** — This paper is a presentation of symbolic analysis of switched currents (SI) circuits by software for analysis of switched capacitors (SC) circuits – SCSYRUP. It is a special library of functions for MAPLE software which allows the symbolic analysis of switched circuits. An example of an SI circuit analysis by means of this library is presented in this paper.

## 1 Introduction

Circuit symbolic analysis is very useful instrument especially for circuit design and optimization. Such an analysis is frequently combined with a suitable mathematical software (MATHEMATICA, MAPLE, ...) which enables further processing of symbolic results, unlike [1, 2]. In [3] we have published an efficient tool serving for symbolic analysis of SC circuits - SCSYRUP, in the form of the library of functions under MAPLE. The library facilitates symbolic and numeric analysis of idealized SC circuits directly in MAPLE since it is programmed in MAPLE macro language [3, 4]. The advantage of the library is significantly supported by MAPLE ability to effectively process symbolic mathematical expressions.

Good experience gained by SCSYRUP application led the authors to extend library ability and made it more universal. Modified algorithm determined to the switched-current circuit analysis was created for this purpose. The circuit description is based on modified nodal-charge equations [5, 6] making possible to include resistive elements. The simple transformation of charges into currents is the main goal of the developed procedure. This leads to the correct evaluation of nodal voltages in the case of SI circuit.

In contrast to more complicated simulation software [7, 8] solving non-ideal switched SC or SI circuits, the extended version of SCSYRUP is limited to the analysis of idealized circuits. But support of MAPLE symbolic mathematical engine gives possibility to extend computations and to include a set of additional conditions, which can simulate non-ideal circuit behaviour. The symbolic form of results obtained is kept in this case.

## 2 Method description

The algorithm used is based on switched-current (SI) basic cell description using SC circuit modified capacitance matrix. Let us consider basic configuration of dynamic current-mirror shown in Fig.1.

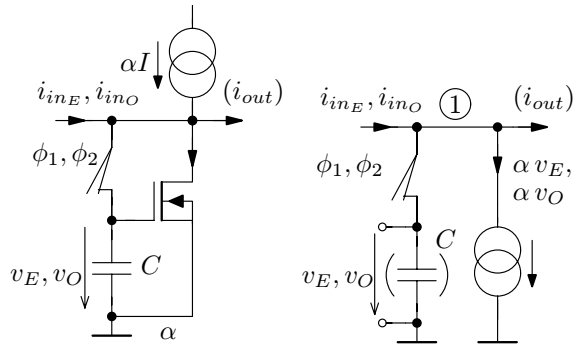


Figure 1: Basic cell of SI circuit

To accomplish the starting conditions of the charge-voltage description, the SI cell must be modelled by voltage controlled charge source with transfer constant  $\alpha_Q$  (instead of current source with transconductance  $\alpha$ ) and memory capacitor  $C$  and ideal switches. The resultant capacitance matrix of circuit in Fig.1 switched in even phase ( $\phi_1$ ) can be written in the following form (1).

$$[\mathbf{Q}] = \begin{bmatrix} C + \alpha_Q & 0 & -\frac{C}{\sqrt{z}} \\ 0 & 0 & \alpha_Q \\ -\frac{C}{\sqrt{z}} & 0 & C \end{bmatrix} \times [\mathbf{V}] \quad (1)$$

The charge transfer from even to odd phase is than:

$$T_Q = \frac{Q_{outO}}{Q_{inE}} = -\frac{\alpha_Q \sqrt{z}}{zC + z\alpha_Q - C} \quad (2)$$

As evident, the transfer function  $T_Q$  contains additional terms, corresponding "parasitic" changes of memory capacitor charge. This effect can be eliminated in idealized circuit description by minimizing capacitance  $C$ . When  $C \rightarrow 0$ , the Eq.(2) limits into the correct known formula (3).

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$$T_Q = -\frac{1}{\sqrt{z}} \quad (3)$$

In fact, the described procedure corresponds to the charge  $\rightarrow$  current transformation in the circuit description (in other words, "charge is divided by time"). In this case, the "starting" description of VCCS by voltage controlled charge source can be turned back ( $\alpha_Q \rightarrow \alpha$ ) and original nodal voltage - charge description changes into voltage - current equations. Note that presented transformation does not change the numeric value of VCCS gain (transconductance  $\alpha$ ).

It is important to say, the procedure of capacitance zeroing should be performed as the last step of transfer evaluation to avoid the complication in description of phase-to-phase energy transfer. The symbolic or special case of semisymbolical analysis is necessary with respect to correct simulation result. This fact limits the described method of memory capacitor zeroing. The mentioned problem can be solved by a new element "T" for the circuit description of SI basic cell. Its matrix representation for even and odd phase switching has the following expression in SCSYRUP library:

| T 1 1 1 $\alpha$ |                           |       | T 1 1 2 $\alpha$ |       |                           |
|------------------|---------------------------|-------|------------------|-------|---------------------------|
|                  | $u_s$                     | $u_l$ |                  | $u_s$ | $u_l$                     |
| $i_{in_s}$       | $\alpha$                  | 0     | $i_{in_s}$       | 0     | $\frac{\alpha}{\sqrt{z}}$ |
| $i_{in_l}$       | $\frac{\alpha}{\sqrt{z}}$ | 0     | $i_{in_l}$       | 0     | $\alpha$                  |

Now the currents are used instead of charges – it is a case of modified node voltages method applied for circuit switched in two phases. In our case the circuit contains only one non-grounded node. It means the matrix has only  $2 \times 2$  dimension. The memory effect is here described by current source controlled by voltage in even and odd phase *with non zero transfer (transconductance) from one phase to the other* as can be seen from the above mentioned matrix form.

Presented procedure leads to the simple and easy description of SI structures and their effective analysis in both symbolic or numerical form.

### 3 The SCSYRUP library

Original version of the SCSYRUP library destined for SC circuit analysis was published in [3, 4]. New version of the library enables to describe SI circuit by following elements:

- "T" element which represents basic memory cell of SI circuits, see Fig. 1,
- switch in even or odd phase,
- all type of controlled sources which can be used for example for modelling output conductance of memory cell<sup>1</sup> (VCCS), in circuits with switched transconductances [9] (VCCS), for current measuring in individual circuit branches (CCVS, CCCS), ...

Following properties of SI circuit can be analysed using above mentioned elements:

- current transfers<sup>2</sup>
- voltage transfers<sup>3</sup> in case of suitably adapted circuit,
- mixed transfers<sup>3</sup>, resp. transfer immittance,
- two-port parameters of SI circuits as is demonstrated on the following example.

The outputs of SCSYRUP functions can be processed by standard MAPLE commands as is shown in [3].

### 4 Conclusion

The algorithm of SI circuit symbolic analysis used in SCSYRUP library was successfully tested on the design of SI-filters and other tasks. Modified SCSYRUP library now represents an universal tool for symbolic and numerical analysis of discrete-time idealized circuits in SC and SI implementation.

The presentation of the algorithms for SI circuits analysis using the SCSYRUP library is original contribution of the authors to this problem.

### 5 An Example of an SI circuit analysis

In the following example of the current mode circuit the applicability of presented method is demonstrated. It is implementation of a simple serial resonant circuit (see Fig 2) by technique of the switched transconductances [9].

Procedure of the circuit description and its analysis is evident from the following listing of MAPLE program. Precise syntax of commands and computing abilities of SCSYRUP library are published in [4] or in library help.

The listing of MAPLE program follows.

<sup>1</sup>In this case circuit losses are modelled. Switching is always idealized – circuit without transient process.

<sup>2</sup>of circuit in different phases

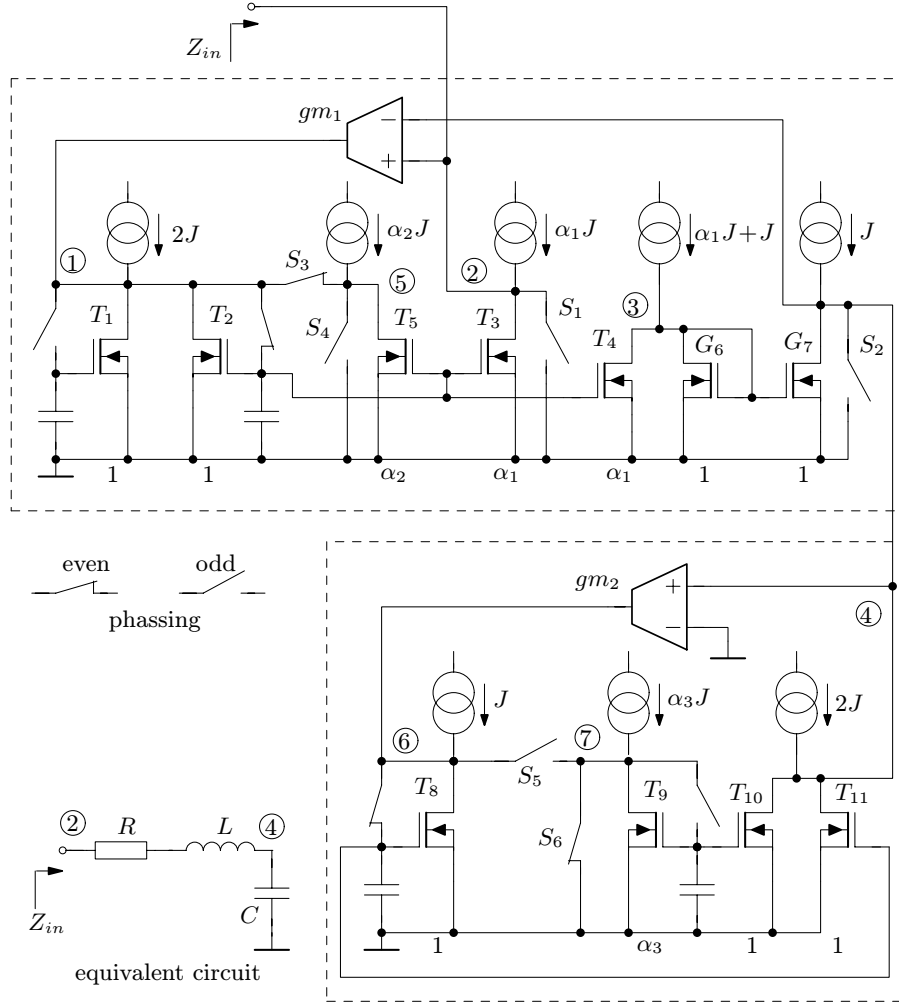


Figure 2: Realization of RLC circuit by technique of the switch transconductances

Activation of the library

```
> with(scsyup):
```

Description of the circuit

```
> RLC_c:=TEXT('resonant circuit',
> # Current cell
> # Tx in out phase gain
> 'T1 1 1 2 1',
> 'T2 1 1 1 1',
> 'T3 1 2 1 alpha1',
> # Switch
> # Sx node1 node2 phase
> 'S3 5 1 1',
> 'S4 5 0 2',
> # VCCS
> # Gx in+ in- out+ out- gain
> 'G6 3 0 3 0 1',
```

```
> 'G7 3 0 4 0 1',
```

```
> 'G1 2 4 0 1 gm1',
```

```
:
```

```
> '.end':
```

Computation of the circuit input impedance in node 2 in even phase.

```
> Zinz:=TransferN(RLC_c,2,2,EE,QQ,VV);
```

$$Zinz := ((\alpha_3 gm_2 \alpha_2 + \alpha_3 gm_2 + \alpha_1 gm_1) z^2 + (-2 \alpha_3 gm_2 - \alpha_3 gm_2 \alpha_2) z + \alpha_3 gm_2) / (z \alpha_1 gm_1 \alpha_3 gm_2 (z - 1))$$

Transformation of the input impedance from  $z$  domain to  $s$  domain by means of inverse backward Euler transformation.

```
> Zins:=expand(simplify
(subs(z=1/(1-s),Zinz)));
```

$$Z_{ins} := \frac{\alpha 2}{\alpha 1 \, gm1} + \frac{1}{s \, gm2 \, \alpha 3} + \frac{s}{\alpha 1 \, gm1}$$

Computation of the voltage transfer function from node 2 to node 4 both in even phase (EE).

> Tz:=TransferN(obvod03,2,4,EE,VV,VV);

$$Tz := \alpha 1 \, gm1 \, z^2 / ((\alpha 3 \, gm2 \, \alpha 2 + \alpha 3 \, gm2 + \alpha 1 \, gm1) \, z^2 + (-2 \, \alpha 3 \, gm2 - \alpha 3 \, gm2 \, \alpha 2) \, z + \alpha 3 \, gm2)$$

Transformation of the transfer function from  $z$  domain to  $s$  domain by means of inverse backward Euler transformation.

> Ts:=expand(simplify  
(subs(z=1/(1-s),Teez)));

$$Ts := \frac{\alpha 1 \, gm1}{\alpha 3 \, gm2 \, \alpha 2 \, s + \alpha 1 \, gm1 + \alpha 3 \, gm2 \, s^2}$$

Computation of the four-port chain parameters from "RL part" of the circuit. The matrix is only  $2 \times 2$  because transfer in odd phase is zero.

> Arl:=FourportN(RL\_c,2,4,A);

$$Arl := \begin{bmatrix} \frac{(\alpha 3 \, gm2 \, \alpha 2 + \alpha 3 \, gm2 + \alpha 1 \, gm1) \, z^2 + (-2 \, \alpha 3 \, gm2 - \alpha 3 \, gm2 \, \alpha 2) \, z + \alpha 3 \, gm2}{\alpha 1 \, gm1 \, z^2} & \frac{\alpha 2 \, z + z - 1}{z \, \alpha 1 \, gm1} \\ \frac{\alpha 3 \, gm2 \, z - \alpha 3 \, gm2}{z} & 1 \end{bmatrix}$$

## References

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- [7] Biolek D., Zaplatílek K.: *Frequency Domain Analysis of Switched Networks by Generalized Transfer Functions*, Proceedings of the ECCTD'93 Davos, Vol. 2, pp. 957-962, Switzerland 1993.
- [8] Bičák J.: *Analysis of Switched-Current Filters*, Radioelektronika '96, Technical University, Brno 1996, p. 50-53, ISBN 80-214-0617-8.
- [9] Toumazou C., Hughes J.B. and Battersby N.C.: *SWITCHED-CURRENTS an analogue technique for digital technology*, Peter Peregrinus Ltd., London 1993.

$$Arl := \begin{bmatrix} 1 & \frac{\alpha 2 \, z + z - 1}{z \, \alpha 1 \, gm1} \\ 0 & 1 \end{bmatrix}$$

Computation of the chain parameters from "C part" of the circuit. Matrix must be again only  $2 \times 2$ .

> Ac:=FourportN(C\_c,4,4,A);

$$Ac := \begin{bmatrix} 1 & 0 \\ \frac{\alpha 3 \, gm2 \, (z - 1)}{z} & 1 \end{bmatrix}$$

Computation of the four-port chain parameters of the whole circuit.

> Arlc:=FourportN(obvod03,2,4,A);

The same result can be obtained by multiplying  $Arl \times Ac$ . Dividing a circuit to smaller part and their analysis is computationally less intensive. In case of presented circuit the voltage transfer function can be also obtained by this way

$$Tz = \frac{1}{Arlc_{11}}.$$