Description of Symmetrical Lossless Two-ports in Two Kinds of Elements for the Design of Microwave Communication Systems in MMIC Realization

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Abstract - **In this paper, a semi-analytic method is presented to describe symmetrical lossless networks with two-kinds of elements, namely, distributed and lumped elements in cascade connection. In terms of the independently chosen parameters, element values of the lossless symmetrical networks are given by explicit formulas up to nine elements. It is expected that the results presented in this paper will find immediate usage in performance assessment, design and simulation of analog/digital wireless communication systems put on MMICs or VLSI chips.**

1 Introduction

Lossless two ports are considered as the major building blocks of antenna systems, microwave amplifiers, analog/digital interface circuits and models for interconnects of high speed, high frequency analog/digital communications systems layed out on the same MMIC and VLSI chips. Furthermore, many special applications demand the utilisation of symmetrical, lossless two-ports in two kinds of elements. For example, design of microwave amplifiers or antenna matching networks may require symmetrical lossless two-ports constructed by excessive number of elements to assure the sharp roll-off on the performance characteristics and to facilitate the production of the same value elements employing the MMIC or VLSI technology. Another example is the model of a naturally symmetrical interconnects utilising the mixed element lossless two-port structures. Therefore in this work, we propose a semi analytic method to describe symmetrical-lossless two-ports in two kinds of elements, namely lumped and distributed elements.

Mixed element design problems has been extensively investigated but it has not yet been thoroughly elaborated [1][2]. Nevertheless, over the last couple years, novel techniques have been proposed to construct the lossless two-ports with mixed lumped and distributed elements for restricted class of topologies and related practical designs have been introduced in [3-10]. In the new approaches, lossless two-ports constructed with two kinds of elements, are described in terms of the real normalised Bounded-Real Strictly Hurwitz Scattering Parameters (BR-SHSP). In a similar manner, in this work, symmetrical lossless two-ports realised with mixed element structures are also described in terms of BR-SHSP in two variables up to nine elements symmetrical ladder forms. Element values of the lossless-two ports are obtained in terms of some preselected independent set of parameters.

2 Scattering Description of Symmetrical Lossless Two-Ports in Two-kinds of Elements

Symmetrical lossless two-ports, constructed with mixed-lumped and distributed elements can be described in terms two variable scattering matrix $S = S(p,\lambda)$ or transfer scattering matrix $T = T(p,\lambda)$. Here, $p = \sigma + j\omega$ is the conventional complex frequency variable associated with lumped elements and $\lambda = tan(p\tau) = \Sigma + j\Omega$ is the Richard variable, associated with the equal length transmission lines. In the classical literature, equal length or commensurate transmission lines are also known as unit elements.

Using Belevitch representation, the scattering and the transfer scattering parameters of a symmetrical lossless two-port are given by,

$$S_{11} = \frac{h(p,\lambda)}{g(p,\lambda)} \qquad S_{12} = \sigma\frac{f(-p,-\lambda)}{g(p,\lambda)}$$

$$S_{21} = \frac{f(p,\lambda)}{g(p,\lambda)} \qquad S_{22} = -\sigma\frac{h(-p,-\lambda)}{g(p,\lambda)} \tag{1}$$

and

$$T_{11} = \sigma\frac{g(-p,-\lambda)}{f(p,\lambda)}; \qquad T_{12} = \frac{h(p,\lambda)}{f(p,\lambda)}$$

$$T_{21} = \sigma\frac{h(-p,-\lambda)}{f(p,\lambda)} \qquad T_{22} = \frac{g(p,\lambda)}{f(p,\lambda)}. \tag{2}$$

where $\sigma = \pm 1$ and the two variable real polynomials $g(p,\lambda)$ and $h(p,\lambda)$ can be expressed in the coefficient form as

$$g(p,\lambda) = \sum_{i=0}^{n_p}\sum_{j=0}^{n_\lambda} g_{ij}p^i\lambda^j \;, h(p,\lambda) = \sum_{i=0}^{n_p}\sum_{j=0}^{n_\lambda} h_{ij}p^i\lambda^j \tag{3a}$$

If the symmetrical structure consists of only series inductor and shunt capacitor type of lumped elements connected with unit element (UE) then,

$$f(p,\lambda) = \left(1-\lambda^2\right)^{n_\lambda/2} \tag{3b}$$

In the above formulation n_p stands for the total number of lumped circuit elements, n_λ designates the total number of unit elements. Alternative expressions are given in the matrix forms as follows.

$$g(p,\lambda)=[P]^T\Lambda_G[\lambda];\quad h(p,\lambda)=[P]^T\Lambda_H[\lambda]$$

where $[P]^T=[1\ \ p\ \ p^2\ \cdots\ p^n]$ and $[\lambda]=[1\ \ \lambda\ \ \lambda^2\ \cdots\ \lambda^n]^T$

In this representation, Λ_H and Λ_G are called connectivity matrices and they are formed by the corresponding real polynomial coefficients of (3a). For symmetrical structures, $S_{11}(p,\lambda)=S_{22}(p,\lambda)$. Therefore, the numerator polynomial $h(p,\lambda)$ must be odd in p and even in λ or vice versa. Since the symmetrical two-port is lossless then,

$$S(p,\lambda)S^T(-p,-\lambda)=I \tag{4a}$$

where I is the unity matrix. Employing the Belevitch form of (1) and the losslessness condition of (4a) we have,

$$g(p,\lambda)g(-p,-\lambda)=h(p,\lambda)h(-p,-\lambda)+f(p,\lambda)f(-p,-\lambda) \tag{4b}$$

Cascade connection of each circuit elements of the structures given in Fig.1, result in the complete scattering parameters in (p,λ). In order to carry out algebraic manipulations, one needs to derive transfer scattering matrices for each single lumped element (L,C) and unit element. Then, for each circuit configuration transfer scattering matrices of the elements are multiplied to obtained the transfer scattering parameter of the complete structure which in turn yields the scattering matrix of the composite structure in two variable (p,λ).

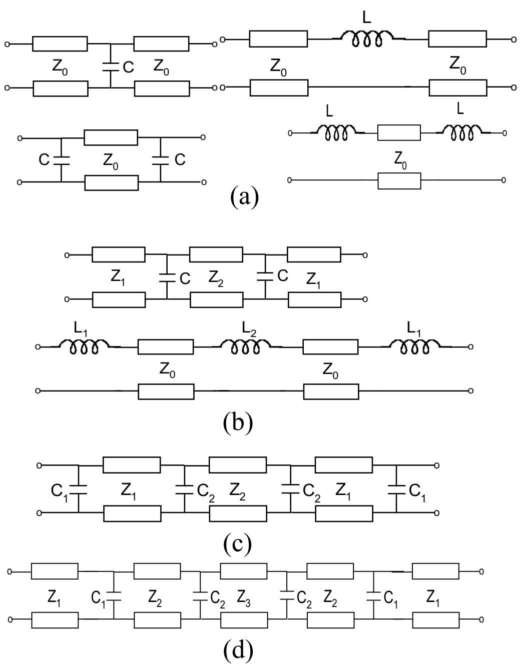

Figure 1: Low-pass Based Symmetrical Lossless two-ports with lumped-distributed elements. a) 3 elements b)5 elements c) 7 elements d)9 elements

Thus, the connectivity matrices can be obtained for 3,5,7 and 9 element circuit topologies as in [10]. As far as the description of the symmetrical lossless two-port is concerned, some of the entrees of the Λ_H matrix are chosen as independent parameters and rest of the other entrees of Λ_H and Λ_G matrices are determined in terms of these parameters employing the losslessness condition of (4). Then, we can make the following major "Statement".

The Main Statement: Description of lossless two-ports constructed with mixed lumped and distributed elements

Any lossless two-port constructed with mixed lumped and distributed elements, can be described in terms of some selected independent entrees of the connectivity matrix Λ_H such that the total number of independent parameters is equal to the total number of unique circuit element of the two-port.

Verification of the above statement can be found in [3-6]. Based on the main statement, complete entrees of the connectivity matrices Λ_H and Λ_G and the element values of the Low Pass based, Symmetrical Lossless two ports constructed with two kind of elements (LPSL) are obtained in terms of these independent parameters. For example, referring to Fig. 1b of [UE – C – UE – C – UE] configuration, we have 5 distinct elements. However, total number of unknowns is three: Z_1, C and Z_2. In this case, entrees h_{10}, h_{21}, and h_{23} of the connectivity matrix Λ_H are selected as independent parameters such that $h_{10}<0$, $h_{21}<0$ and $h_{23}>0$. Then, rest of the other entrees of the matrices Λ_H and Λ_G are determined in terms of these parameters. Finally, the unknown element values are computed from the equations given below.

$$h_{10}=-\mathrm{C};\quad h_{21}=-(1/2)\,\mathrm{Z}_2\,\mathrm{C}^2;\quad h_{23}=(1/2)\,\mathrm{Z}_1^2\,\mathrm{Z}_2\,\mathrm{C}^2$$

Hence, the symmetrical lossless two-port of Fig.1b is fully described in terms of independently selected parameters h_{10}, h_{21}, and h_{23}. Employing the similar approach, the coefficient relations for 3, 5 and 7 elements symmetrical structures of Fig.1 are obtain as shown in Table 1. In the course of algebraic manipulations, we employed the symbolic computation toolbox of Matlab 5.1. For each circuit component, transfer scattering matrices were defined symbolically and they were multiplied in the given order of Fig.1 to end up with the coefficient relations of Table 1 [10]. The results can also be extended to higher number of elements [10]. Due to the space limitations details are omitted here. Complete results will be presented at the conference.

Numerical properties of the connectivity matrices are quite interesting. Here, we just present a statement, which greatly facilitates the selection of the independent parameters.

Statement 2: Property of the connectivity matrix $\Lambda_H = [h_{ij}]$

Symmetrical two-ports constructed with low pass LC *lumped ladder sub sections connected with unit elements yield the following generic form of the connectivity matrix* $\Lambda_H = [h_{ij}]$

$$\Lambda_H = \begin{bmatrix} 0 & h_{01} & 0 & 2 & h_{0n_\lambda} \\ h_{10} & 0 & h_{12} & 2 & h_{1n_\lambda} \\ 0 & h_{21} & 0 & 3 & h_{2n_\lambda} \\ h_{30} & 0 & h_{32} & 3 & h_{3n_\lambda} \\ 0 & h_{41} & 0 & 3 & h_{4n_\lambda} \\ 0 & 0 & h_{52} & 3 & h_{5n_\lambda} \\ 4 & 4 & 4 & 6 & 4 \\ 0 & 0 & 0 & 3 & h_{n_p n_\lambda} \end{bmatrix} \qquad (5)$$

As it is seen from (5), alternating entrees of Λ_H is zero. It is found that this property greatly facilitates the selection of independent parameters to fully describe the symmetrical lossless two-ports under consideration. Verification of this statement is straightforward and can be found in [10].

In the next section we present an example where a symmetrical matching network is designed with mixed lumped and distributed elements for the solution of a double matching problem.

3 Example

We wish to solve the double matching problem for the given complex generator and the load terminations, depicted in Fig.2 [10]. Here, it is desired to design a symmetrical equaliser of order 5 as described in Table 1. The transducer power gain (TPG) of the doubly terminated structure is expressed in terms of the independent parameters h_{10}, h_{21}, h_{23} and it is optimised employing the Levenberg-Marquardt technique over the normalised frequency band $0 \le \omega \le 1$. Hence, the following results are obtained.

$$h(p,\lambda) = p^T \begin{bmatrix} 0 & 2.5451 & 0 \\ -1.3297 & 0 & -0.9141 \\ 0 & -5.1059 & 0 \\ 0 & 0 & -0.6132 \end{bmatrix} \lambda$$

$$g(p,\lambda) = p^T \begin{bmatrix} 1 & 3.2369 & 1 \\ 1.3297 & 7.6883 & 1.6565 \\ 0 & 5.1059 & 0.9542 \\ 0 & 0 & 0.6132 \end{bmatrix} \lambda$$

In terms of the independent coefficients h_{10}, h_{21}, h_{23} the element values are obtained as $Z_0 = (1/2)h_{01} + (1/2)\sqrt{h_{01}^2 + 4}$, $C_1 = (h_{10} - h_{12})/(1 + Z_0^2) - h_{10}$ $C_2 = -2(h_{10} - h_{12})/(1 + Z_0^2)$ The resulting 5 element circuit realization and the performance of the matched structure are shown in Fig. 2 and Fig. 3 respectively.

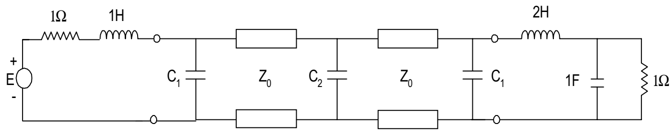

Figure 2: Double matching problem (Z_0=2.8910, C_1=1.2852, C_2=0.0888, τ=0.4 fixed)

It should be noted that the configuration shown in Fig.2 provides a satisfactory roll-off at the stop band and it is very easy to implement as MMIC.

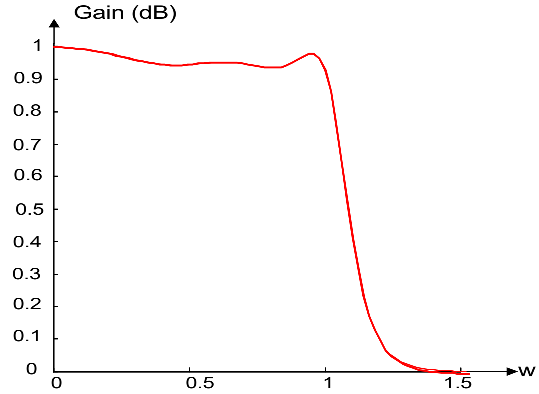

Figure 3. The performance of the matched structure

4 Summary

In this paper, a semi-analytic procedure is introduced to describe symmetrical lossless two-ports formed with low pass, lumped ladder subsections connected with unit elements. The lossless two-port is described by means of its Bounded real Scattering Parameters in two variable (p,λ) in the Belevitch form. Using the real coefficients of $h(p,\lambda)$ and $g(p,\lambda)$ connectivity matrices Λ_H and Λ_G have been defined. It has been stated that the complete scattering parameters of the lossless two-port under consideration can be described in terms of the independently selected set of entrees of the connectivity matrix Λ_H.

Explicit element values are given by means of the selected independent parameters for 3, 5 and 7 elements symmetrical structures. An example is included to design a symmetrical lossless matching network with mixed lumped and distributed elements for a double matching problem. It is expected that results introduced in this work will find immediate applications for the design and the simulation of the miniaturised communication systems manufactured on MMIC.

Topology: [C]-[UE]-[C]

$h_{01}=\frac{Z_0}{2}-\frac{1}{2Z_0}$ $h_{10}=-C$ $h_{21}=-\frac{1}{2}Z_0C^2$ $h_{00}=0$,

$h_{11}=0$, $h_{20}=0$ $g_{01}=\frac{Z_0}{2}+\frac{1}{2Z_0}$, $g_{00}=1$, $g_{20}=0$,

$g_{10}=C$, $g_{11}=CZ_0$, $g_{21}=(1/2)Z_0C^2$

Independent parameters: $h_{10}<0$ $h_{21}<0$

Topology: [C_1]-[UE]-[C_2]-[UE]-[C_1]

$h_{00}=0$, $h_{01}=-\frac{1}{Z_0}+Z_0$, $h_{02}=0$, $h_{10}=-C_1-(1/2)C_2$

$h_{11}=0$, $h_{12}=-C_1+(1/2)C_2Z_0^2$ $h_{20}=0$,

$h_{21}=-C_1^2Z_0-C_1C_2Z_0$ $h_{22}=0$, $h_{30}=0$, $h_{31}=0$,

$h_{32}=-(1/2)C_1^2C_2Z_0^2$ $g_{00}=1$, $g_{01}=(1/Z_0)+Z_0$,

$g_{02}=1$, $g_{10}=C_1+(1/2)C_2$, $g_{11}=2C_1Z_0+C_2Z_0$,

$g_{12}=C_1+(1/2)C_2Z_0^2$ $g_{20}=0$, $g_{21}=C_1^2Z_0+C_1C_2Z_0$

$g_{22}=C_1C_2Z_0^2$, $g_{30}=0$, $g_{32}=0$ $g_{32}=(1/2)C_1^2C_2Z_0^2$

Independent parameters: h_{01}, h_{12} and $h_{10}<0$

Topology: [C_1]-[UE_1]-[C_2]-[UE_2]-[C_2]-[UE_1]-[C_1]

$h_{00}=0$ $h_{01}=Z_1+\frac{1}{2}Z_2-\frac{1}{Z_1}-\frac{1}{2Z_2}$ $h_{02}=0$ $h_{11}=0$

$h_{03}=\frac{1}{2}\frac{Z_1^2}{Z_2}-\frac{1}{2}\frac{Z_2}{Z_1^2}$ $h_{10}=-C_1-C_2$ $h_{13}=0$ $h_{20}=0$

$h_{12}=-C_2\frac{Z_2}{Z_1}+C_2Z_1Z_2+C_2Z_1^2-C_1\frac{Z_1}{Z_2}-C_1-C_1\frac{Z_2}{Z_1}$

$h_{21}=-2C_1C_2Z_1-C_1C_2Z_2-(1/2)C_2^2Z_2-C_1^2Z_1-(1/2)C_1^2$

$h_{22}=0$ $h_{23}=-C_1C_2Z_2+(1/2)C_2^2Z_1^2Z_2-(1/2)C_1^2\frac{Z_1^2}{Z_2}$

$h_{30}=0$ $h_{31}=0$ $h_{32}=-C_1^2C_2Z_1Z_2-C_1C_2^2Z_1Z_2-C_1^2C_2Z_1^2$

$h_{33}=0$ $h_{40}=0$ $h_{41}=0$ $h_{42}=0$ $h_{43}=-(1/2)C_1^2C_2^2Z_1^2Z_2$

$g_{00}=1$ $g_{01}=Z_1+\frac{1}{2}Z_2+\frac{1}{Z_1}+\frac{1}{2Z_2}$ $g_{02}=1+\frac{Z_2}{Z_1}+\frac{Z_1}{Z_2}$

$g_{03}=\frac{1}{2}\frac{Z_1^2}{Z_2}+\frac{1}{2}\frac{Z_2}{Z_1^2}$ $g_{10}=C_1+C_2$ $g_{13}=C_2Z_2+C_1\frac{Z_1^2}{Z_2}$

$g_{11}=2C_1Z_1+C_1Z_2+C_2Z_2+2C_2Z_1$

$g_{12}=C_2\frac{Z_2}{Z_1}+C_2Z_1Z_2+C_2Z_1^2+C_1\frac{Z_1}{Z_2}+C_1+C_1\frac{Z_2}{Z_1}$

$g_{21}=2C_1C_2Z_1+C_1C_2Z_2+\frac{1}{2}C_2^2Z_2+C_1^2Z_1+\frac{1}{2}C_1^2Z_2$

$g_{22}=2C_1C_2Z_1Z_2+C_2^2Z_1Z_2+2C_1C_2Z_1^2$

$g_{23}=C_1C_2Z_2+(1/2)C_2^2Z_1^2Z_2+(1/2)C_1^2(Z_1^2/Z_2)$

$g_{32}=C_1^2C_2Z_1Z_2+C_1C_2^2Z_1Z_2+C_1^2C_2Z_1^2$

$g_{33}=C_1C_2^2Z_1^2Z_2$ $g_{43}=(1/2)C_1^2C_2^2Z_1^2Z_2$

Independent parameters: h_{01}, $h_{10}<0$ h_{03} and $h_{43}<0$

Table 1. Coefficient relations for symmetrical networks

References

[1] Youla D. C. "Synthesis of n-ports Containing Lumped and Distributed Elements". *Proc. Symp Generalized Networks*, pages 289-343 Polytechnic Press, New York, 1966.

[2] Fettweis A. "On the Scattering Matrix and the Scattering Transfer Matrix of Multi-dimensional Lossless Two-ports". *Archiv Electr. Übertragung*, vol. 36, pages 374-381, Sept. 1982.

[3] Aksen A. "Design of lossless two ports with mixed, lumped and distributed elements for Broadband Matching". *Ph.D.thesis*, Ruhr University, Bochum, 1994.

[4] Yarman B.S.and Aksen A. "An Integrated Design Tool to Construct Lossless Matching Networks with Mixed Lumped and Distributed Elements for Matching Problems". *IEEE Trans. Circuits and Systems*, vol.39, Sept. 1992, pages 713-723.

[5] Aksen A. and Yarman B.S. "A Semi–analytical Procedure to Describe Lossless Two-ports with Mixed Lumped and Distributed Elements". *IEEE Trans. Circuit and Systems*, vol.5-6, 1994, pages 205-208.

[6] Aksen A. Yarman B. S. "Cascade Synthesis of Two-variable Lossless Two-port Networks of Mixed Lumped Elements and Transmission Lines: A Semi-analytic Procedure". *NDS-98, The First International Workshop on Multidimensional Systems*, Poland, July 12-14, 1998, pages 35-37

[7] Sertbas A., Aksen A. and Yarman B.S. "Construction of Analog RF Circuits with Lumped and Distributed Components for High Speed/high Frequency Mobile Communication MMICs". *ECCTD'99*, Stresa-Italy, vol.2, 1999, pages 1123-26.

[8] Yarman B.S and Carlin H.J. "A Simplified Real Frequency Technique Applied to Broadband Multistage Microwave Amplifiers", *IEEE Trans. Mic. Theo. and Tech.*, vol.30, Dec.1982, pages 2216-2222.

[9] Carlin H. J. and Yarman B. S. "The Double Matching Problem: Analytic and Real Frequency Solutions" *IEEE Trans. Circuits and Systems,* vol. 30, pages 15-28, Jan. 1983.

[10] Çimen E. G., 'Design of Lossless Two-Ports with Lumped and Mixed Elements Using Parametric Method', *Ph.D. thesis*, Istanbul University, 2000.