Solid State Coherent Quantum Dot System for Quantum Computing and Quantum Transmission

Hideaki Matsueda*

***Abstract* — Recent progress in nano-meter structure and measurements is going to provide us the freedom to harness fast and seemingly weak correlations between atoms in an ensemble, manifesting them at macroscopic level. So we propose a prospective model of a solid state integrated circuit for quantum computation, based on our coherence retentive dynamic dipole-dipole interaction theory. In this model, each qubit is a block consisting of ensemble of quantum dots, and the energetics of which suggests stability up to room temperature. Moreover, quantum permutation circuits that can transfer a qubit state to any position in the quantum circuit and also rearrange the ordering of qubits are given, together with the constituent qubit swap (Π_4) gate, as well as a quantum controlled controlled not (CCN) gate of the block structure.**

1 Introduction

Nanoscopic effects such as dipole-dipole interaction are becoming to be considered as the principal device mechanisms, due to the recent developments in nanostructure fabrication and precision measurements. The theory shows that the inclusion of the dipole-dipole interaction in the starting Hamiltonian is decisive to describe phenomena among ensemble of dipoles [1]. We have been investigating the consequences of the dynamic dipole-dipole resonance interaction among electric dipoles each confined three-dimensionally, i.e. in a quantum dot [1][2].

A quantum controlled not (CN) gate and a quantum controlled controlled not (CCN) gate have been proposed, employing pairs of blocks of quantum dot ensemble or a pair of molecules with a gigantic dipole moment [2]. Furthermore, in our model, if the pair of blocks are situated close enough to significantly induce the inter-block quantum resonance, it is expected to constitute a realistic solid state qubit swap (Π_4) gate. This gate is useful to construct a programable mechanism to transfer a quantum state to an arbitrary position in a quantum circuit, and also to construct other circuits that perform perfect shuffling and bit reversal permutation [3].

In this paper, we propose an integrated solid state computing system constructed out of blocks of quantum dot array (qubits), computing row of the blocks, sequential collection of the rows, coherent light source array for input of initial states, and an array of light source and detector for the readout. The CN gate and the Π_4 gate are included in the rows of this system.

*Department of Information Science, Kochi University, 2-5-1 Akebono-cho, Kochi 780-8520, Japan. E-mail: matsueda@is.kochi-u.ac.jp, Tel: +81-88-844-8334, Fax: +81-88-844-8345.

2 Stability Energetics of the Dipole-Dipole Blocks

2.1 Dipole-Dipole Energy

An array of dipole moments $\mathbf{p}_m$ arranged at positions $\mathbf{R}_m$ imposes a resultant Electric field $\mathbf{E}(n)$ on a site at $\mathbf{R}_n$ as given below.

$$\begin{aligned}\mathbf{E}(n) &= -\frac{1}{4\pi\epsilon}\sum_m\left[\frac{\mathbf{p}_m}{\mid \mathbf{R}_n-\mathbf{R}_m\mid^3}\right.\\ &\qquad\left.-3\,\frac{\mathbf{p}_m\cdot(\mathbf{R}_n-\mathbf{R}_m)\ (\mathbf{R}_n-\mathbf{R}_m)}{\mid \mathbf{R}_n-\mathbf{R}_m\mid^5}\right] \quad (1)\end{aligned}$$

in SI unit. Then the average energy Q of the total dipole-dipole interaction in a block per activated single quantum dot may be given as

$$Q = -\frac{1}{N}\sum_n \mathbf{p}_n\cdot\mathbf{E}(n) \quad (2)$$

where N is the total number of activated quantum dots in the block.

In an ensemble of quantum dots only part of which is excited, the resonance dipole-dipole interaction allows the excited sites (excitons) to travel among the ensemble keeping the total energy constant, without any aid of external field, i.e. energy is transferred between quantum dots through the dynamic dipole-dipole resonance coupling.

Then, as is explained in ref.[1], it is natural to postulate that there is correlation in phase and direction, among the dipoles in the dots being excited and the other dipoles in the dots being de-excited, or that they are coherant each other conserving the parity. Thus, the overall coherence among the ensemble is expected to be maintained starting from the original coherent ensemble of excitations. So the process may well be called dynamic dipole-dipole resonance induction.

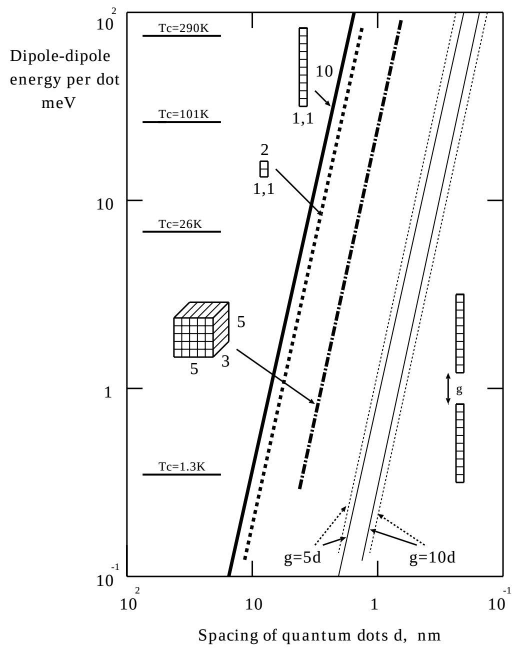

Figure 1: Dipole-dipole interaction energy per quantum dot as the function of the spacing (d) of quantum dots, and with inter-block separation (g) as the parameter.

The dipole-dipole energy of eq.(2) is plotted in Fig.1 as a function of the separation d of quantum dots, for the array having identical dipole moments all directed along z direction in blocks of dimensions (x, y, z): (1, 1, 10), (1, 1, 2) and (5, 3, 5). The dipole moment is assumed to be, as an example, that of GaAs of which technology is quite advanced [2].

The field of eq.(1) could be thought of as the molecular (or Weiss) field in ferromagnetic materials, and equivalent Curie Temperature T_c is estimated as indicated in Fig.1.

2.2 Ensemble Stability

It is seen from Fig.1 that the block having the minimum dimension in x-y plane has the largest dipole-dipole energy, ex. (1, 1, 10) block which is an one dimensional array with 10 activated quantum dots along the z direction. In this case, the parallel (ferromagnetic type) phase is stable even at room temperature if the quantum dot spacing is less than 1.7nm, and it is stable at 77K and 4K if the spacing is below 2.6nm and 7.1nm respectively.

In Fig.1, inter-block dipole-dipole interaction energy is also plotted as a function of the dot spacing d, for the inter-block separation g which is 5 times and 10 times of the dot spacing ($g = 5d$, $10d$), in the cases of tandem arrangements of (1,1,2) block pair and (1,1,10) pair.

Furthermore, the coupling between the dipole moments of the block and dipole moments in the environment, should be minimized by surrounding the device by vacuum or materials of very small dielectric constants. Numerical estimation similar to the above shows that vacuum spacing of 500μm is sufficient to suppress the environmental dipole-dipole coupling below 10^{-11} meV per dot in a block of 10 dots, assuming as the environment an array of 10 dipoles each moment of which is 100 times larger than GaAs. Furthermore, the spontaneous emission rate should also be considerably controlled, by some cavity designed to have a photonic band gap wide enough to cover the wavelength range of importance.

The dipole-dipole interacting ensemble is expected to have resistance not only against bit error, but also against phase error, on the basis of statistical considerations as presented before [2].

3 Quantum Computing Integration

3.1 Quantum Computing Architecture

It is expected to construct a quantum computing system out of the blocks consisting of quantum dots, corresponding to the smaller boxes (shorter rectangles) in sections A to D in Fig.2. Each block works as a bit (qubit) for computation. Every row in Fig.2 executes a bank of operations step by step [2]. In each row, a collective Coulombic interaction between the blocks, together with a photonic pulse, is expected to create the conditional relation between the necessary qubits, resulting in the entanglements for the quantum computing.

The first row (S, having longer rectangles) in Fig.2 represents an array of coherent light sources needed for the initial preparation of superposition, as well as for some inputs of numbers. Moreover, the excitation energy of the different blocks are assumed to be sufficiently different from each other, to be well distinguished and selectively addressed by the lights or photons of different wavelengths.

The last row (R, having longer rectangles) in Fig.2 represents an array of probing light sources and the photodetectors, for the final readout of the computed results.

3.2 Quantum Computing Process

The overall process of the quantum computation may be understood as comprising four steps as demonstrated in Fig.2 and ref.[2]. Computation proceeds as the downward sequential execution of the elemental row operation, which is the unitary (or quasiunitary) evolution driven by the propagation of the excited states into the fields below initially kept at the ground states. A consecutive activation of the row of blocks is assumed, employing methods of clock signal and biased band gap, as explained in the next section for the case of a CCN gate [2].

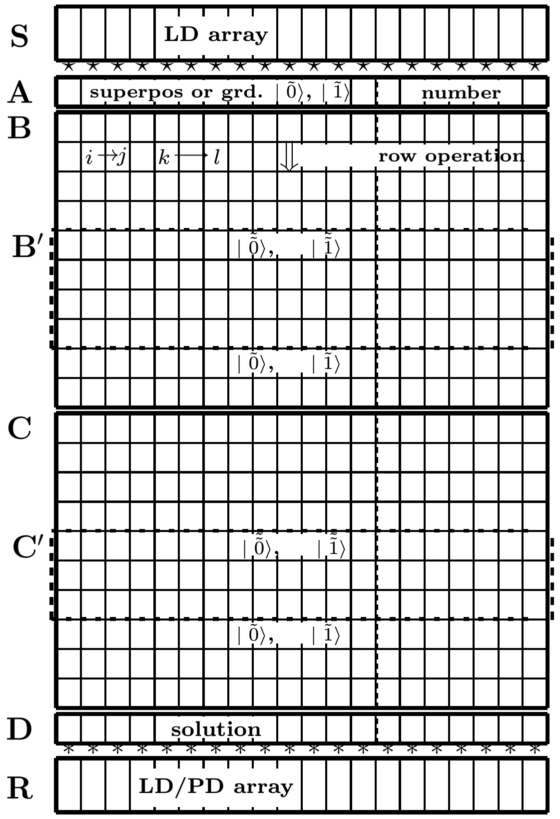

Figure 2: A solid state qunatum computer; (S) light sources, (A) input port, (B) first computing field, (C) second computing field, (D) output port, and (R) probing and detecting elements.

Each elemental row operation includes numbers of unit operations among the different bits, such as CN, CR (controlled rotation), and CPS (controlled phase shift) as indicated by $i \to j$ and $k \to l$ (i, k control bits, j, l target bits). The Π_4 operation is also included where it is necessary in the row. Every unit operation in a row is executed just once in parallel, row by row. The temporal nature of this pairwise logical relation may be visualized by a diagram in a state space, as demonstrated elsewhere [2].

4 Cotrolled Controlled Not Gate

As a simple example, a detailed structure of a CCN gate is given in Fig.3, which consists of two CN, three CR, and two CPS gates, beeing an essential structure to execute Boolean algebra in an universal quantum computing systems, as detailed in ref.[2].

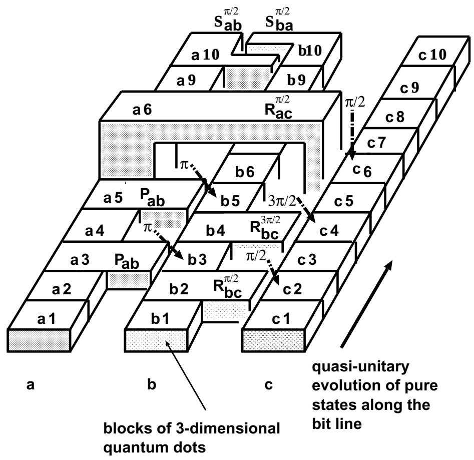

Figure 3: A solid block CCN gate, combining two CN gates (P_{ab}), two $\frac{\pi}{2}$ controlled rotators ($R_{bc}^{\pi/2}$ and $R_{ac}^{\pi/2}$), one $\frac{3\pi}{2}$ controlled rotator ($R_{bc}^{3\pi/2}$), and a pair of $\frac{\pi}{2}$ controlled phase shifters ($S_{ab}^{\frac{\pi}{2}}$ and $S_{ba}^{\frac{\pi}{2}}$). Arrows with π, $\frac{\pi}{2}$, and $\frac{3\pi}{2}$ denote the respective photonic pulses.

It is helpful if the depth and/or width of the quantum dots could be implemented to cause a small but sequential decrease of the energy gap, starting from the input port toward the output port, block by block along each bit line. Excitons generating the dipole moments may be transferred in one-way fashion through this biased path, bacause there exists some dissipative energy loss which deprives the excitons of their energy necessary to climb back the line. Application of microwave pulses with proper energy may work as clock signals. These methods are expected to facilitate the directional quasi-unitary evolution of the whole system that is needed and sufficient for the quantum computation. The spatio-temporal evolution of the quantum entanglements in the CCN gate is demonstrated in ref.[2].

5 Qubit Swap gate and Circuits

A Π_4 gate may be implemented by a couple of blocks (or molecules) facing each other and a negative electromagnetic $\frac{\pi}{2}$ pulse to activate the dynamic dipole-dipole interaction between the blocks, as sketched in Fig.4. The couple of blocks is constantly irradiated by an electromagnetic beam which is polarized along the couple $(\mathsf{a} - \mathsf{b})$ as an example, and which with the electric field from a dipole nonlinearly increases the dielectric constant

ϵ of the blocks. The negative $\frac{\pi}{2}$ pulse[1] means stopping of this irradiation, ceasing the nonlinear increase of the dielectric constant ϵ in eq.(1), in a manner analogous to the linear electrooptic (Pockels) effect.

The quantum resonance between the two blocks cause a Rabi type oscilation between (a) the state where the left block a is excited and (b) the state right block b is excited. The state (a) corresponds to logical state $|1\ 0\rangle$, and state (b) to $|0\ 1\rangle$. In cases where the both sites a and b have same condition, i.e. $|0\ 0\rangle$ and $|1\ 1\rangle$, there is no Rabi type oscillation to swap the states.

It is expected from eq.(1) that the strength of the dynamic dipole-dipole interaction could be adjusted, to facilitate Rabi type oscillation of practical period only under the negative electromagnetic pulse. Then, the swapping between the different states $|1\ 0\rangle$ and $|0\ 1\rangle$ should be accomplished by the negative $\frac{\pi}{2}$ pulse. The frequency (photon energy) of the irradiation or the negative $\frac{\pi}{2}$ pulse may be tuned to the most effective value for the modulation of the dielectric constant ϵ, in order to start and stop the inter-block dynamic dipole-dipole resonance interaction. At the same time, the frequency should be sufficiently different from that of (π or $\frac{\pi}{2}$) optical pulses for other gates such as CN or CCN, to avoid interference.

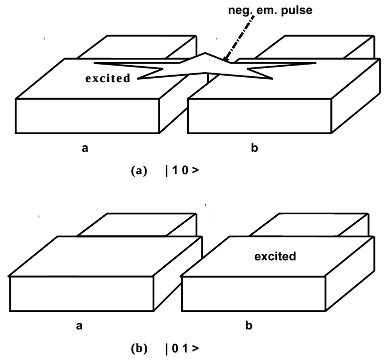

Figure 4: A qubit swap (Π_4) gate activated by a negative electromagnetic $\frac{\pi}{2}$ pulse. (a) before the pulse, bit a is excited representing $|10\rangle$, (b) after the pulse, bit b is excited representing $|01\rangle$.

The Π_4 gate is useful to construct a mechanism to transfer a quantum state to an arbitrary position in a quantum circuit, and also circuits to perform bit reversal permutation shown in Fig.5, and perfect shuffling. These devices work on electronic excitations, in a manner similar to a directional coupler for optical signals, and are programmable to the extent of the freedom in the externally applied negative electromagnetic pulses.

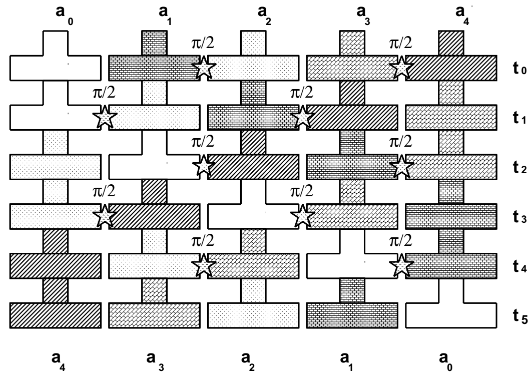

Figure 5: Conceputual scheme of a bit reversal permutation circuit with an odd number of bits, consisting of the qubit swap (Π_4) gates activated by negative electromagnetic $\frac{\pi}{2}$ pulses; $|a_0\ a_1\ a_2\ a_3\ a_4\rangle \longrightarrow |a_4\ a_3\ a_2\ a_1\ a_0\rangle$.

6 Concluding Remarks

A prospective model of a solid state integrated circuit for quantum computation is given, on the basis of our theory of coherence retentive dynamic dipole-dipole resonance interaction. In this model, each qubit is a block consisting of ensemble of quantum dots, or molecules with gigantic dipole moments.

A quantum CCN gate and a Π_4 (qubit swap) gate are proposed as the essential elements of the solid state quantum computing system employing the qubit blocks, assuming some adequate nonlinear electrooptic effect for on and off of the bit swapping.

References

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[2] Matsueda H., Chaos, Solitons, and Fractals, **10** (10), (1999) 1737-1748; Lecture Notes in Computer Science **1509** (Springer, Berlin, 1999) 468-477; *Proc. The European Conference on Circuit Theory and Design (ECCTD'97)*, Budapest (30 Aug. - 3 Sept., 1997), 265-270; Int. J. Circ. Theor. Appl., **29** (2001); Matsueda H, Cohen DW., Int. J. Theor. Phys., **38** (2), (1999) 701-711.

[3] Fijany A., Williams C. P., Lecture Notes in Computer Science, **1509** (Springer 1999) 10-33.

[1]This could be a positive normal pulse, if we could choose the combination of the material and the wavelength.