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A Study of Sensitivity for a Class of Matrix-Derived Log-Domain Ladder Filters

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Abstract – This paper reports the results of a study of sensitivity properties for a class of recently proposed matrix-derived log-domain ladder filters. Formulae for calculating individual gain and phase sensitivities are given. The ladder topologies feature an additional network for realising floating capacitors normally associated with transmission zeros. Despite this, low sensitivity in the passband is preserved.

1. Introduction

A formal method for synthesising log-domain ladder filters with finite transmission-zeros has been recently proposed[1]. Designs with different topologies can be derived from a single passive ladder prototype. They feature an additional network for simulating floating capacitors, necessitated by the non-linear internal node voltages to which the capacitors are connected. It is therefore important to ascertain that these introduced elements do not degrade the low sensitivity property of the ladder filters. Synthesis procedures and sensitivity formulae for these filters will be derived.

2. Sensitivity Analysis

A nodal admittance equation can be written for a passive ladder network, Fig. 1, as

$$\mathbf{C}\dot{\mathbf{V}} + \mathbf{\Gamma}\int\mathbf{V} + \mathbf{G}\mathbf{V} = \mathbf{U}Vin \quad (1)$$

where $\mathbf{C}$, $\mathbf{\Gamma}$ and $\mathbf{G}$ are coefficient matrices representing capacitances, inverse inductances and conductances respectively[1]. $\mathbf{U}$ is a column vector of input connections. Factorising $\mathbf{\Gamma}$ as

$$\mathbf{\Gamma} = \mathbf{\Gamma_L}\mathbf{\Gamma_R} \quad (2)$$

and introducing an intermediate variable vector $\mathbf{X}$, defined by

$$\mathbf{X} = \mathbf{\Gamma_R}\int\mathbf{V} \quad (3)$$

(1) can be expressed as

$$\mathbf{C}\dot{\mathbf{V}} + \mathbf{\Gamma_L}\mathbf{X} + \mathbf{G}\mathbf{V} = \mathbf{U}Vin \quad (4)$$

where different combinations of $\mathbf{\Gamma_L}$ and $\mathbf{\Gamma_R}$(either direct or LU decompositions[1]) result in different topologies.

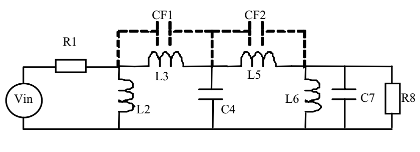

Fig. 1. A typical 6th order passive ladder prototype

The diagonal and off-diagonal elements in the $\mathbf{C}$-matrix constitute grounded and floating capacitors respectively. They can be separated by splitting $\mathbf{C}$ into $\mathbf{C}_{diag}+\mathbf{C}_{off}$. Equation (4) becomes

$$\mathbf{C}_{diag}\dot{\mathbf{V}} = \mathbf{U}Vin - \mathbf{\Gamma_L}\mathbf{X} - \mathbf{G}\mathbf{V} - \mathbf{C}_{off}\dot{\mathbf{V}} \quad (5)$$

Transform $\dot{\mathbf{V}}$ on the RHS of (5) into a non-derivative vector by introducing a current vector $\mathbf{I}$ defined by

$$\dot{\mathbf{V}} = \mathbf{C}_{diag}^{-1}\mathbf{I} \quad (6)$$

Substituting (6) into (5) yields

$$\mathbf{C}_{diag}\dot{\mathbf{V}} = \mathbf{U}Vin - \mathbf{\Gamma_L}\mathbf{X} - \mathbf{G}\mathbf{V} - \mathbf{C}_{off}\mathbf{C}_{diag}^{-1}\mathbf{I} \quad (7)$$

Now define

$$\boldsymbol{\alpha} = \mathbf{C}_{off}\mathbf{C}_{diag}^{-1} \quad (8)$$

From (6), the new $\mathbf{I}$ variable is given by

$$\mathbf{I} = \mathbf{C}_{diag}\dot{\mathbf{V}} \quad (9)$$

Terms resulting from this equation will be distinguished from those of (7) by subscript I. The final system is now described by equations (7), (9) and (3) as

$$\mathbf{C}_{diag}\dot{\mathbf{V}} = \mathbf{U}Vin - \mathbf{\Gamma_L}\mathbf{X} - \mathbf{G}\mathbf{V} - \boldsymbol{\alpha}\mathbf{I} \quad (10a)$$

$$\mathbf{I_{DI}}\mathbf{I} = \mathbf{U_I}Vin - \mathbf{\Gamma_{LI}}\mathbf{X} - \mathbf{G_I}\mathbf{V} - \boldsymbol{\alpha}_{\mathbf{I}}\mathbf{I} \quad (10b)$$

$$\mathbf{I_D}\dot{\mathbf{X}} = \mathbf{\Gamma_R}\mathbf{V} \quad (10c)$$

where $\mathbf{I_D}$ is an identity matrix. It is expedient to remain in the linear current domain(as opposed to the log voltage domain) to exploit the many state-space tools available. Prior to the sensitivity analysis, we have to write the matrix equations in augmented form in order to access the intermediate variables(i.e. X_i). There are various ways to do this. For conciseness, consider a simple 3rd order $\mathbf{\Gamma_L\Gamma_R}$=$\mathbf{LU}$ lowpass design with its signal-flowgraph(SFG) shown in Fig. 2.

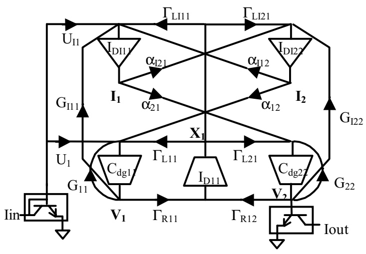

Fig. 2 SFG of 3rd order $\mathbf{\Gamma_L\Gamma_R}$=$\mathbf{LU}$ log-domain lowpass ladder filter

Define the augmented variable vector as $Y=(V_1\ V_2\ I_1\ I_2\ X_1)^T$, then the resulting augmented matrices become

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$$C_{diag}(s)=\begin{bmatrix} \overset{\mathbf{C}_{\text{diag}}}{sC_{diag_{11}}} & 0 & 0 & 0 & 0\\ 0 & sC_{diag_{22}} & 0 & 0 & 0\\ 0 & 0 & \overset{\mathbf{I}_{\mathbf{DI}}}{I_{DI_{11}}} & 0 & 0\\ 0 & 0 & 0 & I_{DI_{22}} & 0\\ 0 & 0 & 0 & 0 & \overset{\mathbf{I}_{\mathbf{D}}}{I_{D_1}} \end{bmatrix}\quad G=\begin{bmatrix} \overset{\mathbf{G}}{-G_{11}} & 0 & 0 & 0 & 0\\ 0 & -G_{22} & 0 & 0 & 0\\ \overset{\mathbf{G}_{\mathbf{I}}}{-G_{I_{11}}} & 0 & 0 & 0 & 0\\ 0 & -G_{I_{22}} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Gamma=\begin{bmatrix} 0 & 0 & 0 & 0 & \overset{\mathbf{\Gamma}_{\mathbf{L}}}{\Gamma_{L_{11}}}\\ 0 & 0 & 0 & 0 & \Gamma_{L_{21}}\\ 0 & 0 & 0 & 0 & \overset{\mathbf{\Gamma}_{\mathbf{LI}}}{\Gamma_{LI_{11}}}\\ 0 & 0 & 0 & 0 & \Gamma_{LI_{21}}\\ \underset{\mathbf{\Gamma}_{\mathbf{R}}}{-\Gamma_{R_{11}}} & -\Gamma_{R_{12}} & 0 & 0 & 0 \end{bmatrix}\quad \alpha=\begin{bmatrix} 0 & 0 & 0 & \overset{\boldsymbol{\alpha}}{-\alpha_{12}} & 0\\ 0 & 0 & -\alpha_{21} & 0 & 0\\ 0 & 0 & 0 & \overset{\boldsymbol{\alpha}_{\mathbf{I}}}{-\alpha_{I12}} & 0\\ 0 & 0 & -\alpha_{I21} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\quad U=\begin{bmatrix} \overset{\mathbf{U}}{U_1}\\ 0\\ \overset{\mathbf{U}_{\mathbf{I}}}{U_{I_1}}\\ 0\\ 0 \end{bmatrix} \tag{11}$$

From these, the system transfer-functions from the input to the internal nodes are given by

$$H_i(s)=T_i\left(C_{diag}(s)-\Gamma-G-\alpha\right)^{-1}U \tag{12}$$

where the overall filter output is selected by T_2=[0 1 0 0 0]. The forward transfer-functions from each variable node to the output are given by the vector

$$\begin{aligned} N(s)&=\left[T_2\left(C_{diag}(s)-\Gamma-G-\alpha\right)^{-1}\right]^T\\ &=\left[N_1(s)\;\; N_2(s)\;\; .\,.\,.\;\; N_5(s)\right]^T \end{aligned} \tag{13}$$

It is conveniently designated N(s) as it is also a vector of the noise transfer-functions.

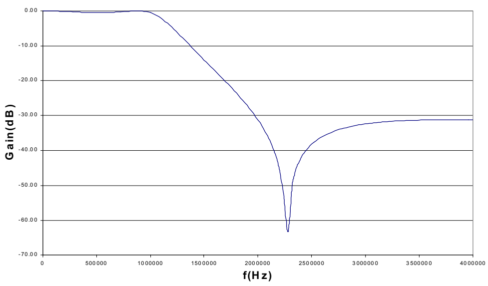

Fig. 3 Gain transfer-function(H_2) of the 3rd order $\mathbf{\Gamma_L\Gamma_R}$= **LU** log-domain filter

It can be shown[2] that

$$\frac{\partial H(s)}{\partial A_{ij}}=N_i(s)H_j(s) \tag{14}$$

The classical sensitivity function is defined as

$$S_{A_{ij}}^{H(s)}\triangleq\frac{\partial H(s)}{\partial A_{ij}}\frac{A_{ij}}{H(s)} \tag{15}$$

Therefore, substituting (14) into (15) yields

$$S_{A_{ij}}^{H(s)}=N_i(s)H_j(s)A_{ij}H_2(s)^{-1} \tag{16}$$

The normalised magnitude and phase sensitivities are given respectively by

$$S_{A_{ij}}^{|H(s)|}=\mathrm{Re}\left\{S_{A_{ij}}^{H(s)}\right\}=\mathrm{Re}\left\{N_i(s)H_j(s)A_{ij}H_2(s)^{-1}\right\} \tag{17a}$$

$$S_{A_{ij}}^{\phi}=\mathrm{Im}\left\{S_{A_{ij}}^{H(s)}\right\}=\mathrm{Im}\left\{N_i(s)H_j(s)A_{ij}H_2(s)^{-1}\right\} \tag{17b}$$

For instance, (17a) indicates that the resultant relative (per unit) change in $H(s)$ is S times the relative change in parameter A_{ij}, i.e.

$$\frac{\Delta|H|}{|H|}=S_{A_{ij}}^{|H|}\frac{\Delta A_{ij}}{A_{ij}} \tag{18}$$

Figs. 4-7 show plots of the normalised magnitude deviations in dB resulting from a 1% change(i.e. $\Delta A_{ij}/A_{ij}$= 0.01) in the matrix elements.

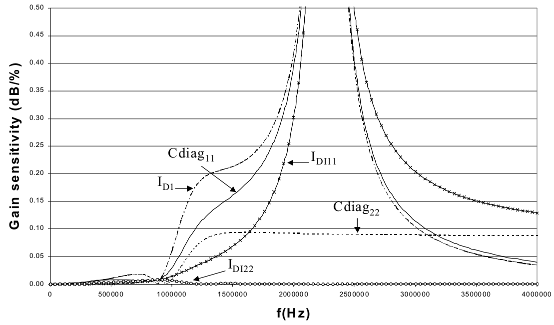

Fig. 4 Gain sensitivity of integrating capacitors($\mathbf{C}_{\text{diag}}$, $\mathbf{I}_{\mathbf{D}}$) and I-variable currents($\mathbf{I}_{\mathbf{DI}}$)

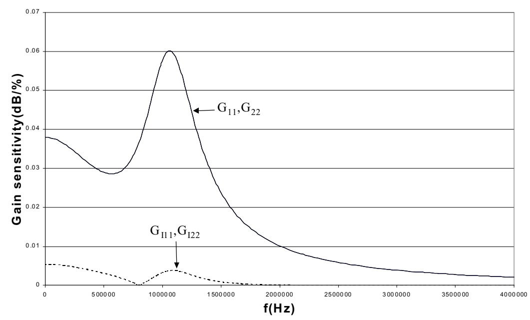

Fig. 5 Gain sensitivity of conductance(**G**) elements

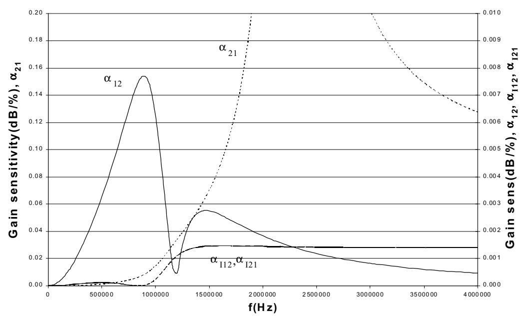

Fig. 6 Gain sensitivity of α-elements

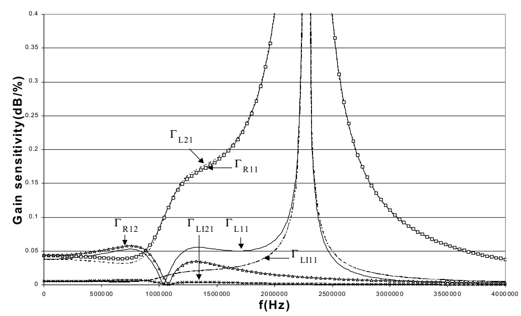

Fig. 7 Gain sensitivity of inverse-inductance($\mathbf{\Gamma}$) elements

Physically, $\mathbf{C}_{diag}$ and $\mathbf{I_D}$ are implemented by grounded capacitors while all others are implemented by current sources[1]. Therefore, these sensitivites relate directly to capacitor and transistor matching ratios respectively.

The sensitivity or absolute deviation in the passband is low, including those introduced for simulating the floating capacitors(α and elements with I-subscript). Preservation of the low sensitivity property thus qualifies these designs as true ladder simulations. Using (17a,b), the sensitivity of any matrix element can be plotted, which is an indispensable aid to the design process.

For an indicative assessment of the sensitivity of bandpass structures, we select the 6th order $\mathbf{\Gamma_L \Gamma_R = LU}$ elliptic bandpass design[1]. The frequency response and SFG are given in Figs. 8 and 9 respectively.

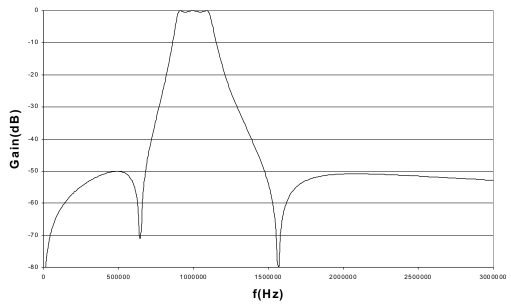

Fig. 8 Gain transfer-function of the 6th order $\mathbf{\Gamma_L \Gamma_R = LU}$ log-domain bandpass filter

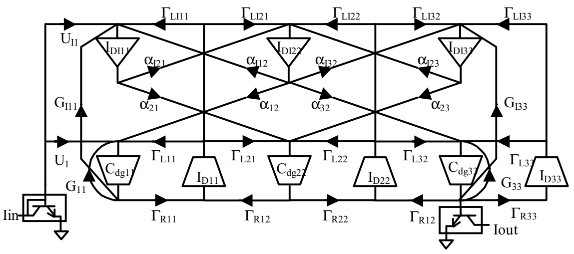

Fig. 9 $\mathbf{\Gamma_L \Gamma_R = LU}$ 6th order elliptic bandpass structure

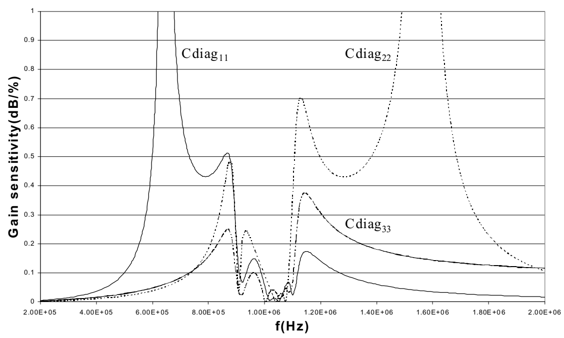

Fig. 10 Gain sensitivity of integrating capacitors($\mathbf{C_{diag}}$)

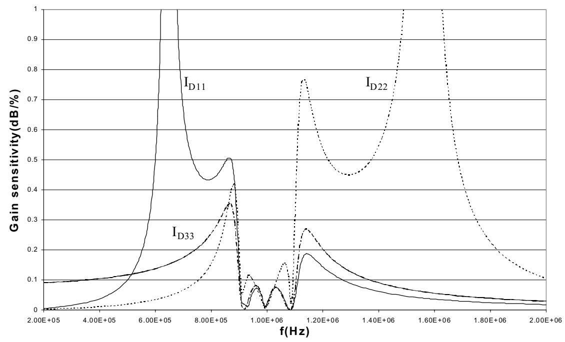

Fig. 11 Gain sensitivity of integrating capacitors($\mathbf{I_D}$)

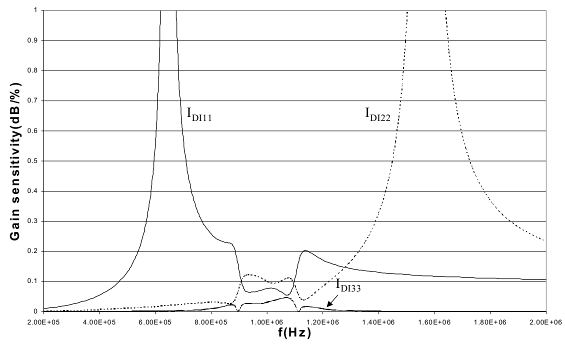

Fig. 12 Gain sensitivity of **I-**variable currents($\mathbf{I_{DI}}$)

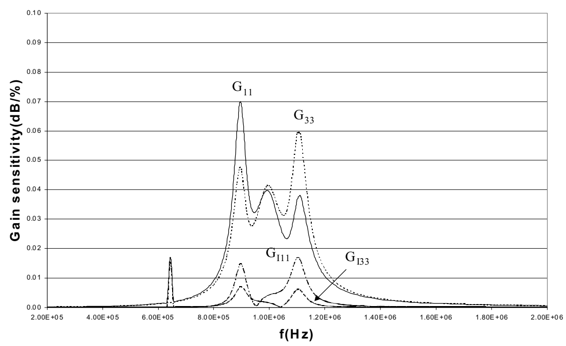

Fig. 13 Gain sensitivity of conductance($\mathbf{G}$,$\mathbf{G_I}$) elements

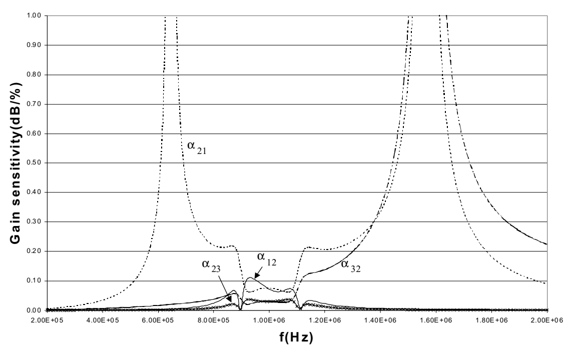

Fig. 14 Gain sensitivity of α-elements

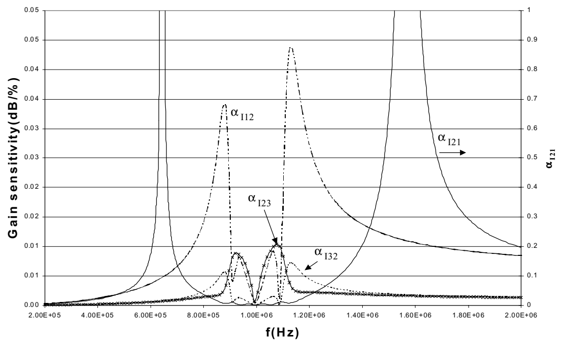

Fig. 15 Gain sensitivity of α_I-elements

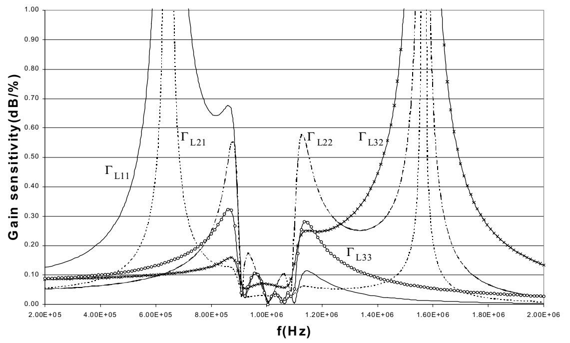

Fig. 16 Gain sensitivity of Γ_L-elements

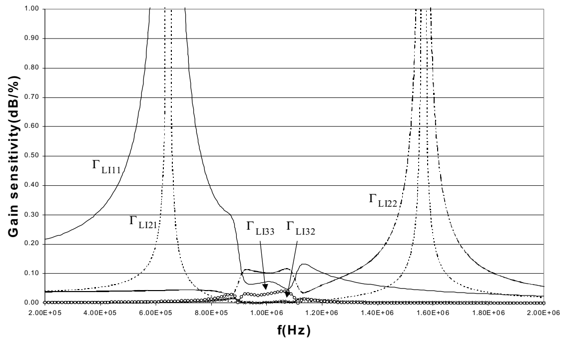

Fig. 17 Gain sensitivity of Γ_{LI}-elements

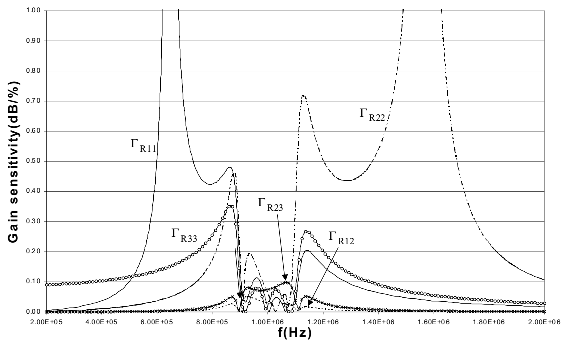

Fig. 18 Gain sensitivity of Γ_R-elements

From Figs. 10-18, the bandpass structure also maintains low sensitivity over the passband including those introduced for simulating the floating capacitors(α and elements with I-subscript).

To compare and assess the sensitivity of different designs, a suitable sum-sensitivity measure could be

$$S_{total}^{|H(s)|} = \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\left| S_{C_{diag}(s)_{ij}}^{|H(s)|} \right| + \left| S_{G_{ij}}^{|H(s)|} \right| + \left| S_{\Gamma_{ij}}^{|H(s)|} \right| + \left| S_{\alpha_{ij}}^{|H(s)|} \right| \right)$$

where N is the order of the augmented matrices. This was applied to 6^{th} order elliptic bandpass log-domain filters[1] derived by the four different matrix-decompositions of $\mathbf{\Gamma_L \Gamma_R}$= **LU**, **UL**, $\mathbf{I\Gamma}$ and $\mathbf{\Gamma I}$, where **L** and **U** are lower, upper triangular matrices respectively and **I** is an identity matrix. The sum-sensitivities in the passband are plotted in Fig. 19. The $\mathbf{\Gamma_L \Gamma_R}$= $\mathbf{I\Gamma}$ topology has low, even sensitivity throughout the passband while the other designs have various degrees of asymmetric skew.

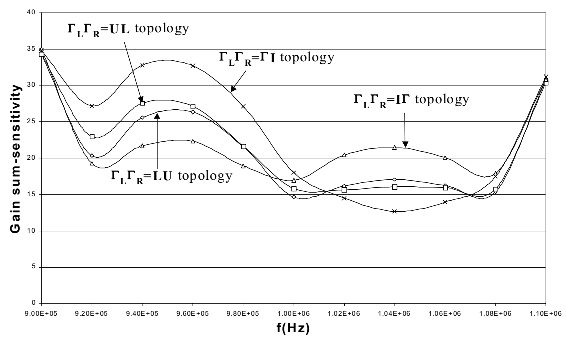

Fig. 19 Passband sum-sensitivity for different 6^{th} order elliptic bandpass log-domain filter topologies

3. Conclusions

Formulas for analysing and comparing the sensitivities of the new log-domain ladder filters have been developed. Low passband sensitivity has been confirmed for both lowpass and bandpass designs despite their unconventional topologies and additional elements.

Acknowledgements

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References

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