Synthesis of broadband multiport phase commutators and uncouplers

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***Abstract* - Methods of synthesis, mathematical models and parameters of broadband multiport phase commutators and uncouplers are considered in this paper. These circuits are used for a solution of an optimal broadband matching problem for a complex *n*-port load. Structure parameters and a synthesis of these networks are based on eigenvalues and eigenvectors of multiport scattering matrices on *jw*-axis of the complex frequency plane.**

1 Introduction

A design of an optimum multiport equalizer to match an arbitrary multiport load is one of the classic problems in the circuit theory [1,2,]. An application of broadband *n*-port ***phase commutators*** and ***uncouplers*** may carry out a solution of this problem [3,4,5].

The paper presents mathematical models and structures of the multiport broadband phase commutators and the uncouplers for a given resistive or complex *n*-port load. It is used resistive and complex normalized scattering matrices for a network connection [4,5,6].

The phase commutator is such network who provides a distribution of output signals with the same amplitudes and different phase of its (Fig.1,a). With the change of an excitation input of the phase commutator we have different distributions of output signal phases.

The uncoupler is named a multiport network providing a total ***diagonal*** scattering matrix of the cascade connection of the uncoupler and the whole multiport complex load (Fig.1,b).

A synthesis theory of the phase commutators and the uncouplers is based on power parameters coupled with eigenvalues and eigenvectors of the multiport matrices. Formulas for a computing of these parameters are presented in the paper.

Obtained results may be used for the design of the multiport networks of the different structure [7].

2 Scattering matrices and parameters of phase commutators and uncouplers

Consider a base theory of the phase commutators and the uncouplers with use of a complex normalized scattering matrix on the imaginary axis of the *p*-plane.

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The complex normalized scattering matrix $\mathbf{S}_N$ of a double-side *n*-order network *N* (Fig.1,a) is provided by block relations:

$$\mathbf{S}_N = \begin{bmatrix} \mathbf{S}_{\alpha\alpha} & \mathbf{S}_{\alpha\beta} \\ \mathbf{S}_{\beta\alpha} & \mathbf{S}_{\beta\beta} \end{bmatrix}. \qquad (1)$$

For the *reciprocal* coupling network *N*:

$$\mathbf{S}_{\alpha\alpha} = \mathbf{S}_{\alpha\alpha t}\ , \quad \mathbf{S}_{\beta\beta} = \mathbf{S}_{\beta\beta t}\ , \quad \mathbf{S}_{\alpha\beta} = \mathbf{S}_{\beta\alpha t} \qquad (2)$$

and for the *lossless* network matrix $\mathbf{S}_N$ is ***unitary***:

$$\mathbf{S}_N \mathbf{S}_N^{+} = \mathbf{1}, \qquad (3)$$

superscript (+) denotes the *hermit conjugate* matrix.

For the cascade connection of multiport networks (Fig.1,b) a total scattering matrix is given by [5,6]:

$$\mathbf{S} = \mathbf{S}_{\alpha\alpha} + \mathbf{S}_{\alpha\beta}\mathbf{S}_L(\mathbf{1} - \mathbf{S}_{\beta\beta}\mathbf{S}_L)^{-1}\mathbf{S}_{\beta\alpha}\,, \qquad (4)$$

where $\mathbf{S}_L$ - scattering matrix of the multiport load.

It is proved that a total normalized average power absorbed by the whole multiport network (Fig.1,b) for the arbitrary excitation vector is given by ***Rayleigh ratio*** and limited by the minimum and the maximum eigenvalues of a dissipation matrix $\mathbf{D}$ [2,5,6]:

$$d_{\min} \le P / P_{\max} = \mathbf{a}_\alpha^{+}\mathbf{D}\,\mathbf{a}_\alpha / \mathbf{a}_\alpha^{+}\mathbf{a}_\alpha \le d_{\max}\,, \qquad (5)$$

where $\mathbf{D} = \mathbf{1} - \mathbf{S}^{+}\mathbf{S}$ - ***dissipation*** matrix of the whole network; $\mathbf{a}_\alpha = 0.5\mathbf{R}_\alpha^{0.5}\mathbf{E}_\alpha$ - excitation vector; $\mathbf{R}_\alpha$ - real parts of the diagonal impedance matrix $\mathbf{Z}_\alpha$; d_i - *real eigenvalues* of the matrix **D.**

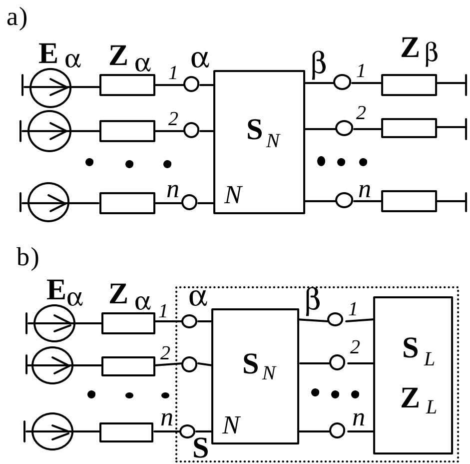


Figure 1: Phase commutator (a) and uncoupler for a multiport complex load (b).

The matrix $\mathbf{D}$ is ***hermetian*** and *unitary similar* to the diagonal *positive real* matrix of its *eigenvalues* d_i:

$$\mathbf{D}=\mathbf{V}\{d_i\}\,\mathbf{V}^+,\ \ d_i{=}d_i^*,\ \ \mathbf{V}\,\mathbf{V}^+=\mathbf{1}, \qquad (6)$$

where $\mathbf{V}$- a complex *unitary* matrix of the *eigenvectors* of the matrix $\mathbf{D}$.

In general case the scattering matrix of the multiport load has a singular expansion [2]:

$$\mathbf{S}_L=\mathbf{V}_L\{s_i\}\mathbf{W}_L\ ,\ \ \mathbf{V}_L\mathbf{V}_L^+{=}\mathbf{1}\,,\ \ \mathbf{W}_L\,\mathbf{W}_L^+{=}\mathbf{1}\,, \qquad (7)$$

where s_{Li} - singular values of the matrix $\mathbf{S}_L$;$\mathbf{V}_L$, $\mathbf{W}_L$ - complex *unitary* matrices of eigenvectors of matrices $\mathbf{S}_L\mathbf{S}_L^+$ and $\mathbf{S}_L^+\mathbf{S}_L$ correspondingly. Notice that for a *normal* scattering matrix ($\mathbf{S}_L\mathbf{S}_L^+{=}\,\mathbf{S}_L^+\mathbf{S}_L$) we have:

$$\mathbf{S}_L=\mathbf{V}_L\{s_i\}\mathbf{V}_L^+\ ,\ \ \mathbf{V}_L\mathbf{V}_L^+{=}\mathbf{1}\,. \qquad (8)$$

For the *lossless* coupling network N in (Fig.1,b):

$$d_i=1-|\,s_i\,|^2\,. \qquad (9)$$

Hence the ***optimization*** and the multiport ***matching*** problem comes to *maximization* (*minimization*) of the eigenvalues (singular values) of the dissipation matrix $\mathbf{D}$ (matrix $\mathbf{S}_L$) at the frequency band [5,6].

The double-side multiport network N (Fig.1) is named the ***uncoupled-matched network*** that has zero diagonal blocks of the scattering matrix:

$$\mathbf{S}_{\alpha\alpha}=\mathbf{S}_{\beta\beta}=\mathbf{0}\,. \qquad (10)$$

In general the ***phase commutator*** is the *uncoupled-matched network* and provides a transmission between inputs and outputs only with various distributions of output signal phases with a change of excitation input.

The synthesis methods of the phase commutators by use of two-port hybrids are described in [4,7].

For the *cascade connection* of the uncoupled-matched network and multiport load (Fig.1,b) from (4) and (10) we have:

$$\mathbf{S}=\mathbf{S}_{\alpha\beta}\,\mathbf{S}_L\,\mathbf{S}_{\beta\alpha}\,. \qquad (11)$$

Further, if these *transmission blocks* of scattering matrix of *uncoupled-matched network* are equal to *hermit conjugate* eigenvectors matrices of the load scattering matrix:

$$\mathbf{S}_{\alpha\beta}{=}\,\mathbf{V}_L^+\,,\ \ \mathbf{S}_{\beta\alpha}=\mathbf{W}_L^+\,, \qquad (12)$$

then ***total scattering matrix*** of this cascade connection from (11) is ***diagonal***:

$$\mathbf{S}=\{s_i\}=\mathbf{V}_L^+\,\mathbf{S}_L\mathbf{W}_L^+\ . \qquad (13)$$

It means that the ***all input ports*** of cascade connection (Fig.1,b) are mutually ***uncoupled***; this uncoupled-matched network is named by **uncoupler** for given n-port load. Notice that in general case this uncoupler is *nonreciprocal* because $\mathbf{S}_{\alpha\beta}\neq\mathbf{S}_{\beta\alpha t}$.

normal load scattering matrix $\mathbf{S}_L$	real orthogonal eigenvectors $\mathbf{T}$ of $\mathbf{S}_L$
nonreciprocal uncoupler	*reciprocal* uncoupler
$\mathbf{S}_N=\begin{bmatrix}\mathbf{0} & \mathbf{V}^+\\ \mathbf{V} & \mathbf{0}\end{bmatrix}$ $\mathbf{S}_{\beta\alpha}=\mathbf{V},\ \mathbf{V}\mathbf{V}^+=\mathbf{1}$	$\mathbf{S}_N=\begin{bmatrix}\mathbf{0} & \mathbf{T}_t\\ \mathbf{T} & \mathbf{0}\end{bmatrix}$ $\mathbf{S}_{\beta\alpha}=\mathbf{V}=\mathbf{T},\mathbf{T}\mathbf{T}_t{=}\,\mathbf{1}$

Table 1: Scattering matrices of the uncouplers.

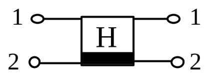
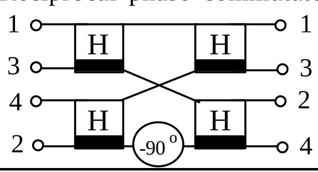
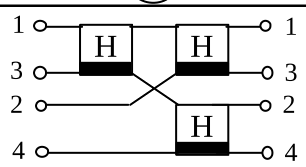
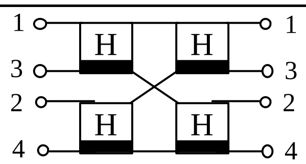
m	n	Transmission matrix	Circuit	Uncoupler	Phase commutator
1	2	$\mathbf{T}=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$		reciprocal	reciprocal
2	4	$\mathbf{V}=\frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & j & -1 & -j\\ 1 & -1 & 1 & -1\\ 1 & -j & -1 & j\end{bmatrix}$	Reciprocal phase commutator 	*non-reciprocal*	reciprocal
3	4	$\mathbf{T}=\frac{1}{2}\begin{bmatrix}1 & \sqrt{2} & 1 & 0\\ 1 & 0 & -1 & \sqrt{2}\\ 1 & -\sqrt{2} & 1 & 0\\ 1 & 0 & -1 & -\sqrt{2}\end{bmatrix}$		reciprocal	——
4	4	$\mathbf{T}=\frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\\ 1 & -1 & -1 & 1\end{bmatrix}$		reciprocal	reciprocal

Table 2: Matrices and structures of the uncouplers and the phase commutators (m is a number of the circuit).

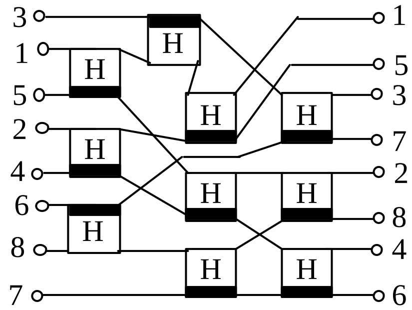
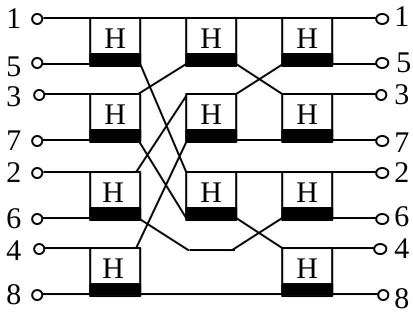
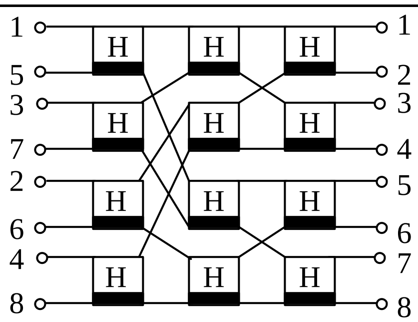
m	n	Transmission matrix	Circuit	Uncoupler	Phase commutator
5	8	$\mathbf{V}=\frac{1}{\sqrt{8}}\begin{bmatrix}1&1&1&1&1&1&1&1\\1&\alpha&j&-\alpha^{-1}&-1&-\alpha&-j&\alpha^{-1}\\1&j&-1&-j&1&j&-1&-j\\1&-\alpha^{-1}&-j&\alpha&-1&\alpha^{-1}&j&-\alpha\\1&-1&1&-1&1&-1&1&-1\\1&-\alpha&j&\alpha^{-1}&-1&\alpha&-j&-\alpha^{-1}\\1&-j&-1&j&1&-j&-1&j\\1&\alpha^{-1}&-j&-\alpha&-1&-\alpha^{-1}&j&\alpha\end{bmatrix}$	Hybrids, phase shifters, circulators $\alpha=\exp\ (j\pi/4)$	*non-reciprocal*	reciprocal
6	8	$\mathbf{T}=\frac{1}{\sqrt{8}}\begin{bmatrix}1&\sqrt{2}&\sqrt{2}&\sqrt{2}&1&0&0&0\\1&1&0&-1&-1&1&\sqrt{2}&1\\1&0&-\sqrt{2}&0&1&-\sqrt{2}&0&\sqrt{2}\\1&-1&0&1&-1&1&-\sqrt{2}&1\\1&-\sqrt{2}&\sqrt{2}&-\sqrt{2}&1&0&0&0\\1&-1&0&1&-1&-1&\sqrt{2}&-1\\1&0&-\sqrt{2}&0&1&\sqrt{2}&0&-\sqrt{2}\\1&1&0&-1&-1&-1&-\sqrt{2}&-1\end{bmatrix}$		reciprocal	——
7	8	$\mathbf{T}=\frac{1}{\sqrt{8}}\begin{bmatrix}1&1&1&1&1&1&1&1\\1&\sqrt{2}&1&0&-1&-\sqrt{2}&-1&0\\1&1&-1&-1&1&1&-1&-1\\1&0&-1&\sqrt{2}&-1&0&1&-\sqrt{2}\\1&-1&1&-1&1&-1&1&-1\\1&-\sqrt{2}&1&0&-1&\sqrt{2}&-1&0\\1&-1&-1&1&1&-1&-1&1\\1&0&-1&-\sqrt{2}&-1&0&1&\sqrt{2}\end{bmatrix}$		reciprocal	——
8	8	$\mathbf{T}=\frac{1}{\sqrt{8}}\begin{bmatrix}1&1&1&1&1&1&1&1\\1&-1&1&-1&1&-1&1&-1\\1&1&-1&-1&1&1&-1&-1\\1&-1&-1&1&1&-1&-1&1\\1&1&1&1&-1&-1&-1&-1\\1&-1&1&-1&-1&1&-1&1\\1&1&-1&-1&-1&-1&1&1\\1&-1&-1&1&-1&1&1&-1\end{bmatrix}$		——	reciprocal

Table 2: Matrices and structures of the uncouplers and the phase commutators (cont.).

For particular cases of a ***normal*** load scattering matrix $\mathbf{S}_L$ and the *real orthogonal* eigenvectors $\mathbf{T}$ a general structure and the transmission blocks of the uncoupler scattering matrix are shown in table 1. It is proved that the uncoupler is *reciprocal* for *real orthogonal* eigenvectors of $\mathbf{S}_L$ only [4,7].

3 Phase commutators and uncouplers

Consider a synthesis of the phase commutators for resistive loads (Fig.1,a) and the uncouplers for a multi-port load with ***ring symmetry*** in given frequency band (Fig.1,b).The corresponding transmission matrices and results of the synthesis of these networks are shown in table 2. For *ring symmetry* scattering matrices for 2- and 4- orders of the multiport load have next forms:

$$\mathbf{S}_{L2}=\begin{bmatrix}a&b\\b&a\end{bmatrix},\qquad \mathbf{S}_{L4}=\begin{bmatrix}a&b&c&b\\b&a&b&c\\c&b&a&b\\b&c&b&a\end{bmatrix}, \qquad (14)$$

Impedance and dissipation matrices have the same kind. The eigenvectors of these matrices are shown in 1 and 2 lines of the table 2. The eigenvalues of matrices (14):

$$n=2:\quad s_{1,2}=a\pm b\ , \qquad (15)$$

$$n=4:\quad s_{1,3}=a\pm 2b+c\ ,\ s_2=s_4=a-c\ . \qquad (16)$$

For the 8-order multiport *ring* complex load the scattering matrix and corresponding eigenvalues are:

$$\boldsymbol{S}_{L8} = \begin{bmatrix} a & b & c & d & e & d & c & b \\ b & a & b & c & d & e & d & c \\ c & b & a & b & c & d & e & d \\ d & c & b & a & b & c & d & e \\ e & d & c & b & a & b & c & d \\ d & e & d & c & b & a & b & c \\ c & d & e & d & c & b & a & b \\ b & c & d & e & d & c & b & a \end{bmatrix}, \qquad (17)$$

$$s_{1,5} = a \pm 2b + 2c \pm 2d + e, \; s_3 = s_7 = a - 2c + e, \; s_{2,4} = s_{8,6} = a \pm b\sqrt{2} \mp d\sqrt{2} - e. \qquad (18)$$

The eigenvectors of the impedance and scattering matrices are shown in line 5 of the table 2.

For $\boldsymbol{n = 2}$ we have a simple symmetrical load, then the both the phase commutator and the uncoupler are *reciprocal* and its are usual hybrids (m = 1 from table 2). Principle of the work of the hybrid is presented in (Fig.2). This two-port phase commutator realize equal and opposite phases of the output signals with switching of the inputs. Input parameters of this uncoupler with the symmetrical load are described by (14), (15). Constructions and frequency characteristics of the hybrid on transmission lines are presented in [4,5].

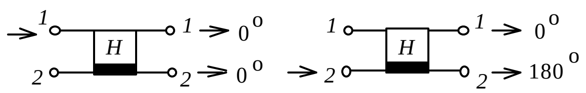


Figure 2: Principle of the work of the hybrid.

For $\boldsymbol{n = 4}$ and $\boldsymbol{n = 8}$ the matrices (14), (17) have *double* eigenvalues (16), (18); it means that there are ***infinite number*** of schemes realized by any linear combination of the eigenvectors corresponding to the multiple eigenvalues. Part of these is shown in table 2.

For $\boldsymbol{n = 4}$ three networks are represented. **First** of these has ***complex*** unitary transmission matrix that describes *nonreciprocal* uncoupler and reciprocal phase commutator. The uncoupler may be realized by use of hybrids, phase shifters and circulators. In the same case *reciprocal* phase commutator is a known *Butler matrix* and realizes a *quadrature phasing* of output signals with switching of it's inputs with phase shift $\Delta\varphi = (k-1)\pi/2$, where k is number of excitation input. Thus *Butler matrix* is not uncoupler for this ring load.

The **second** and the **third** *real orthogonal* matrices for $\boldsymbol{n = 4}$ are designed by summation and subtraction of the eigenvectors corresponding to eigenvalues 2 and 4 in (16). In this case networks 3 and 4 from table 2 are *reciprocal* uncouplers and circuit 4 is *reciprocal* phase commutator. But scheme 3 is not commutator because transmission matrix consists of zero elements and not all inputs connected to all outputs.

All uncoupler (m = 2, 3, 4) have insulated inputs for *ring* load with parameters described by (14), (16).

For $\boldsymbol{n = 8}$ four matrices are shown in table 2. **First** of these (m = 5, table 2) represents a complex unitary transmission matrix corresponding to *nonreciprocal* uncoupler for the ring complex load and *reciprocal* phase commutator with output phases: $\Delta\varphi = (k-1)\pi/4$.

The **next** networks (6 and 7 from the table 2) are designed by the summation and subtraction of the eigenvectors corresponding to *double* eigenvalues 2 and 8, 3 and 7, 4 and 6 in (18). In this case we obtain *real orthogonal* transmission matrices too. These devices are *reciprocal* uncouplers for the given ring load but its are not phase commutators because the zero elements exist in the transmission matrix.

At last **fourth** network (m=8) is a *reciprocal* phase commutator with *real orthogonal* matrix but it is not uncoupler because last *four columns* of the transmission matrix are not eigenvectors of the matrix (17).

All uncoupler (5 - 8) have insulated inputs for the given ring load with the parameters described by (17), (18). All reciprocal uncouplers and phase commutators (6 - 8) may be realized by use of the hybrids [4, 7].

4 Conclusions

The presented theory based on the eigenvalues and eigenvectors of the n-port matrices, methods of synthesis and the structures of the broadband multiport phase commutators and uncouplers may by use for solution different circuit problems, including an optimization of the power signal transmitting in multiport structures.

Acknowledgments

This work has been supported by the Rector's Project W/WE/5/01 in the Technical University of Bialystok.

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