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Capacity of Hybrid Open-loop and Closed-loop MIMO with Channel Uncertainty at Transmitter

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ABSTRACT OF MASTER'S THESIS

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Multiple-input and multiple-output (MIMO) technology has attracted attention in wireless communications, since it offers significant increases in data throughput and link range without additional bandwidth or transmit power. Open-loop transmit diversity techniques such as space-time coding can be used to achieve a large portion of the available capacity. However these codes are designed under the assumption that the transmitter has no knowledge about the channel. In this thesis, we consider the case when the transmitter has partial but not perfect knowledge about the channel. In addition, it is investigated how to improve a predetermined code so that the channel imperfection is taken into account. Imperfect knowledge of channel is modeled by using a statistical method to describe the correlation between the assumed and the true channel. Ergodic mutual information is chosen as the performance criterion. The resulting optimization problem can be solved numerically. In addition, an efficient approximation method is used for the special case of independently fading channel coefficients. This simple formula can be utilized for evaluating average capacity for correlated vector Rician channel. Simulation results demonstrate significant gain over conventional methods in a scenario with non-perfect channel knowledge. The accuracy of the capacity approximation method is also validated through the numerical results.

Keywords: MIMO system, transmit diversity, feedback system, Rician channel capacity.
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Monen lähetin- ja vastaanotinantennin tekniikat (multiple-input multiple-output, MIMO) ovat olleet laajan kiinnostuksen kohteena langattomasa tietoliikenteessä. Ne lupaavat huomattavaa lisäystä tiedonsiirtokapasiteettiin ja -kantamaan ilman että joudutaan lisäämään kaistanleveyttä tai lähetystehoa. Avoimen kontrollisilmukan lähetysdiversiteettimenetelmillä, kuteen esimerkiksi ns. tila-aika koodeilla, voidaan saavuttaa suurin osa kanavan kapasiteetistä. Nämä koodit on suunniteltu olettaen, että lähettäjällä ei ole etukäteistietoa kanavasta. Tässä diplomityössä tutkitaan tapausta, jossa lähettäjällä on osittaista, mutta ei täydellistä etukäteistietoa kanvasta. Työssä tarkastellaan myös menetelmiä parantaa ennalta annettua koodia, kun otetaan huomioon kanavatiedon epäteydellisyys. Kanavatiedon epäteäydellisyyttä mallinnetaan tilastollisesti, käytettävissä olevan tiedon ja täsmällsen tiedon korrelaatiomatriisin kautta. Suorituskykymittarina käytetään ergodista keskinäisinformaatiota, mikä johtaa siihen, että ratkaistava optimointiongelma edellyttää numeerista ratkaisemista. Työssä kehitetään tehokas analyyttinen aproksimaatio, jota voidaan käyttää kun kanavakertoimet häipyvät riippumattomasti. Tuloksena olevaa yksinkertaista yhtälöä voidaan käyttää laskemaan korreloituneen Rice:n jakauman mukaan häipyvän vektorikanavan keskimääräinen kapasiteetti. Simulaatiotulokset osoittavat huomattava suorituskyvyn parantumista, kun verrataan tavanomaisiin lähetysmenetelmiin. Ehdotetun aproksimaation tarkkuus todennetaan vertaamalla numeerisiin tuloksiin.

Avainsanat:	MIMO-järjestelmä, lähetysdiversiteetti, takaisinkytkentä Rice:n kanavan kapasiteetti
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List of Abbreviations

AWGN	Addictive White Gaussian Noise
BS	Base Station
CL	Closed-loop
CSI	Channel State Information
FDD	Frequency Division Duplexing
i.i.d	Independent Identically Distributed
MIMO	Multiple-input and Multiple-output
MMSE	Minimum Mean Square Error
MRC	Maximal Ratio Combining
MRT	Maximal Ratio Transmission
MSE	Mean Square Error
OL	Open-loop
PDF	Probability Density Function
SNR	Signal to Noise Ratio
STBC	Space Time Block Code
TAS	Transmit Antenna Selection
UMTS	Universal Mobile Telecommunications System
ZF	Zero Forcing

List of Symbols

$(\cdot)^*$	Complex Conjugate Operation
$(\cdot)^\dagger$	Conjugate Transpose Operation
$\lvert \cdot \rvert$	Absolute Value or Determinant of Matrix
$\lVert \cdot \rVert$	Euclidean Norm for Vectors or Frobenius Norm for Matrices
$\Re[\cdot]$	Real Part Operation
$\Im[\cdot]$	Imaginary Part Operation
$E[\cdot]$	Expected Value
$\mathbf{I}_N$	N Dimensional Identity Matrix
$\lambda_{max}(\mathbf{X})$	Largest Eigenvalue of $\mathbf{X}$
$\mathbf{X}^{1/2}$	Hermitian Square Root of $\mathbf{X}$
$\chi^2(l)$	Chi-square Distribution With l Degree of Freedom
$\mathbb{C}^N$	N Dimensional Complex Space

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Chapter 1

Introduction to Transmit Diversity

The purpose of this chapter is to give a short introduction to the most promising transmit diversity techniques. Our focus is on space-time diversity methods. Transmit diversity methods can be divided into two categories: open-loop (OL) transmit diversity (mainly space-time block coding) and closed-loop (CL) transmit diversity. After the brief explanations on above mentioned transmit diversity techniques, we will unfold the research area of hybrid open-loop and closed-loop methods. The Chapter ends with an outline of the structure of this thesis.

1.1 Open-loop Transmit Diversity: Space-Time Block Coding

Multiple-input multiple-output (MIMO) systems have great potential to improve the performance of wireless systems. Compared to wire-line systems, the inferior capacity of wireless cellular systems is caused by several different physical constraints like co-channel and adjacent channel interference, channel propagation loss, and flat or multi-path fading. Multi-antenna transmission and reception techniques are currently seen as one of the most promising approaches for significantly increasing the coverage, capacity and spectral efficiency of wireless systems.

Traditionally, multi-antenna techniques have mainly been considered in the downlink direction of cellular systems, where the base station (BS) are the transmitters. The reason for this is that deploying multiple antennas in the user terminal is not straightforward due to cost, complexity of signal processing, and power consumption. Diversity reception in BS is a mature technology and it has been successfully applied in base stations to increase cell coverage. However, receive diversity in BS alone does not provide full access to the promised capacity gains. In order to better exploit the capacity promised by MIMO information theory, transmit diversity techniques has to be used in addition to receive diversity techniques.

One of the first forms of transmit diversity was antenna hopping. In a system using antenna hopping, two or more transmit antennas are used interchangeably to achieve diversity effect. However, antenna hopping is a suboptimal way to utilize transmit antennas. A more systematic transmission technique that can use multiple antennas is so called space-time block coding (STBC). Space-time coding finds its applications in cellular communications as well as in wireless local area networks. Some of the work on space-time coding focuses on explicitly improving the system performance in terms of error probability analysis and other research capitalizes on the promises of information theory to use transmit antennas for increasing the throughput. Generally speaking, the design of space-time codes amounts to finding a constellation of matrices that satisfy certain optimality criteria. In particular, the construction of space-time coding scheme is to a large extent a trade-off between three conflicting goals of maintaining a simple decoding, optimizing the error performance and maximizing the information rate.

One of the first space-time codes is due to Alamouti [1], which is a code designed for two transmit antennas ($n_T = 2$). For reliable reception, only one receive antenna is needed ($n_R = 1$). In the first time interval, two complex symbols x_1 and x_2 are transmitted simultaneously from the first and second antenna. During the second interval the complex symbols $-x_2^*$ and x_1^* are transmitted from first and second antenna. Therefore, the encoding matrix is:

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}. \tag{1.1}$$

To see the optimality of this scheme in a flat fading channel, let us consider the received signal:

$$\mathbf{y} = \frac{1}{\sqrt{2}}\mathbf{X}\mathbf{h} + \mathbf{n}, \tag{1.2}$$

where $\mathbf{h}$ is the channel vector $\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$, $\mathbf{n}$ is the noise vector $\begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$. $\mathbf{X}$ is normalized by $\frac{1}{\sqrt{2}}$ so that the transmit power is kept the same as the closed-loop case in section 1.2. Conjugating the received signal during the second symbol period, the received signal may be written in terms of an equivalent signal model as

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \mathcal{H} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}, \tag{1.3}$$

where the equivalent channel matrix is

$$\mathcal{H} = \frac{1}{\sqrt{2}} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}. \tag{1.4}$$

Now the space-time matched filtering of (1.1) proceeds simply by applying the Hermitian conjugate of the equivalent channel matrix on the received signal (1.3)

$$\mathcal{H}^\dagger \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \frac{1}{2}(|h_1|^2 + |h_2|^2) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \mathcal{H}^\dagger \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}. \tag{1.5}$$

Assuming an i.i.d fading scenario with h_i, $\hat{h}_i$ and noise n_i distributed as $\mathcal{CN}(0,1)$, the signal to noise ratio (SNR) after detection is

$$SNR = \frac{1}{2}\frac{E[|x|^2]}{E[|n|^2]}(|h_1|^2 + |h_2|^2) = \frac{1}{2}\gamma(|h_1|^2 + |h_2|^2), \tag{1.6}$$

γ is SNR before detection, therefore $\frac{|h_1|^2+|h_2|^2}{2}$ is often defined as processing gain or coding gain (SNR gain) of Alamouti code as

$$SNR_{gain} = \frac{|h_1|^2 + |h_2|^2}{2}. \tag{1.7}$$

Notice that the matched filter gave a result where both symbols have been transmitted at half power over both channels, and have been maximum ratio combined (MRC) at the receiver. This is a consequence of the fact that the equivalent channel matrix is proportional to a unitary matrix. Compared to MRC at the receiver, with two receive antennas, there is a 3 dB loss due to splitting the transmit power into two between the antennas. Finally we claim that with one receive antenna, Alamouti code is an optimal linear *open-loop* transmit diversity scheme, assuming that the receiver has sufficient channel state information (CSI). It provides full diversity, with linear matched filter detection and it reaches channel capacity [27].

Generalizing Alamouti code to more than two transmit antennas is a non-trivial task. The underlying characteristic of (1.1), explaining its excellent performance, was found to be the unitary of the code matrix. For Alamouti code, this is simply expressed as

$$\mathbf{X}\mathbf{X}^{\dagger} = (|x_1|^2 + |x_2|^2)\mathbf{I_2}, \tag{1.8}$$

where $\mathbf{I_2}$ is the two-dimensional identity matrix. Orthogonality/unitary leads to the theory of orthogonal designs, for real or complex modulation symbols. These have a simple linear detection scheme with optimal MRC performance. Based on the principle of orthogonality/unitary, the problem of designing rate 1, full diversity space-time block code was solved in [2]. In [2], it is also proved that complex orthogonal designs for rate 1 exist only when $n_t = 2$.

A new class quasi-orthogonal codes was proposed in [13] [14]. The original quasi-orthogonal space-time block code accomplishes full rate transmission, but unlike orthogonal space-time code, it does not have full spatial diversity. The 4 transmit antennas, rate 1 quasi-orthogonal code proposed in [14] is the so-called ABBA code

$$\mathbf{X_{ABBA}} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix}. \tag{1.9}$$

Here, one has two copies of the 2×2 Alamouti block code with symbols x_1, x_2 on the block diagonal, and two copies of Alamouti code with symbols x_3, x_4 on the block anti-diagonal, i.e. in the form

$$\begin{bmatrix} A & B \\ B & A \end{bmatrix} \tag{1.10}$$

where

$$\mathbf{A} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, \tag{1.11}$$

$$\mathbf{B} = \begin{bmatrix} x_3 & x_4 \\ -x_4^* & x_3^* \end{bmatrix}. \tag{1.12}$$

This scheme is thus called “ABBA”. Using the equivalent channel representation the received signal over 4 consecutive time slots is

$$\mathbf{y} = \mathbf{H} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \mathbf{n}, \tag{1.13}$$

where H is given by

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ h_4^* & -h_3^* & h_2^* & -h_1^* \end{bmatrix}. \quad (1.14)$$

For such space-time code, the receiver can be decomposed in 2 stages: the space-time matched filtering operation, and the decoding part. Indeed, applying the matched filter results in

$$\begin{aligned} \mathbf{z} &= \mathbf{H}^\dagger \mathbf{y} && (1.15) \\ &= \mathbf{H}^\dagger \mathbf{H} \mathbf{x} + \mathbf{H}^\dagger \mathbf{n} && (1.16) \\ &= \begin{bmatrix} \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & \beta \\ \beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{bmatrix} + \mathbf{x} + \mathbf{H}^\dagger \mathbf{n}. && (1.17) \end{aligned}$$

Each diagonal element α of the detection matrix is the sum of the 4 square magnitudes of the channel coefficients h_i (full diversity order):

$$\alpha = \sum_{i=1}^{4} |h_i|^2, \quad (1.18)$$

whereas each interference term is equal to

$$\beta = 2\Re[h_1^* h_3 + h_2^* h_4], \quad (1.19)$$

where $\Re[x]$ denotes the real part of the complex x. Then, an additional decoding stage is required in order to retrieve the input signal, and its complexity is reduced since it amounts to solve separately 2 subsystems involving $[x_1, x_3]$ on one hand, and $[x_2, x_4]$ on the other hand. Different strategies can be applied for this final step, from maximum likelihood to ZF or MMSE block equalization. Of course, these non-orthogonal schemes can then be extended for more transmit antennas, in which case the number of interference terms to be canceled increases.

1.2 Closed-loop Transmit Diversity

Open-loop transmit diversity methods discussed in previous section are designed to operate without channel state information at the transmitter. In contrast, closed-loop concepts exploit channel state information that is provided to the transmitter using closed-loop signalling. The channel state information can be used to weight the signals transmitted from the BS antennas. The weighting should be such that the signals arrive co-phased in the receiver. Constructive signal combining increases the received signal power, therefore SNR gain and link capacity will also increase.

The signal model considered in closed-loop transmit diversity for two transmit antennas and one receive antennas is

$$y = \mathbf{hw}x + n, \tag{1.20}$$

where $\mathbf{h}$ represents the complex channel $[h_1, h_2]$ and $\mathbf{w}$ is complex weighting factors $[w_1, w_2]$ which is used to weight the channel coefficients, so that the received signals combine coherently in the receiver. Again, in order to keep total transmission power constant, it is required that $\|\mathbf{w}\|^2 = 1$. The transmitted scalar symbol is x.

In order to detect the transmitted symbol, the receiver forms the following test statistics

$$\begin{aligned} z &= \mathbf{w}^\dagger \mathbf{h}^\dagger y & (1.21) \\ &= \mathbf{w}^\dagger \mathbf{h}^\dagger (\mathbf{hw}x + n) & (1.22) \\ &= |\mathbf{hw}|^2 x + \mathbf{w}^\dagger \mathbf{h}^\dagger n. & (1.23) \end{aligned}$$

From equation (1.10) the instantaneous SNR at the receiver can be calculated as

$$SNR = \frac{E[|x|^2](|\mathbf{hw}|^2)^2}{E[|n|^2]|\mathbf{w}^\dagger \mathbf{h}^\dagger|^2} = \gamma |\mathbf{hw}|^2, \tag{1.24}$$

where γ is SNR at the transmitter, therefore $|\mathbf{hw}|^2$ could be defined as the coding gain of this 2×1 closed-loop transmit diversity scheme.

The optimum feedback weight calculation problem is to find out a weighting vector $\mathbf{w}$ which maximizes the SNR in equation (1.24), subject to the power constraint $\|\mathbf{w}\|^2 = 1$ as

$$\mathbf{w}_{optimal} = \arg \max_{s.t. \|\mathbf{w}\|^2 = 1} |\mathbf{hw}|^2, \tag{1.25}$$

this optimization problem can be solved by the Cauchy-Schwarz inequality and the solution is given as

$$w_1 = \frac{h_1^*}{\sqrt{|h_1|^2 + |h_2|^2}}, \quad w_2 = \frac{h_2^*}{\sqrt{|h_1|^2 + |h_2|^2}}. \tag{1.26}$$

Plugging the above optimal weight values into (1.24), we obtain the corresponding instantaneous SNR as

$$SNR = \gamma(|h_1|^2 + |h_2|^2), \tag{1.27}$$

the SNR gain is therefore

$$SNR_{gain} = |h_1|^2 + |h_2|^2. \tag{1.28}$$

This closed-loop transmission technique is often considered as transmit beamforming or maximal ratio transmission (MRT) system, which is different from maximal ratio combining (MRC) system in that the latter is the so-called receive diversity method, the signals are coherently combined at the receiver in order to achieve diversity effect. By comparing the coding gain of Alamouti code in equation (1.6) and coding gain of the 2×1 closed-loop transmit method in equation (1.26), we can conclude that relative to a system that uses optimal transmit beamforming, the Alamouti code provides the same diversity order, but has a 3 dB loss in post-detected SNR value. In other words, being able to have complete channel state information gives us a 3 dB gain in SNR over the Alamounti code, which does not require any CSI at transmitter.

In the case of matrix channel $\mathbf{H}$, where $\mathbf{H}$ is the $n_r \times n_t$ channel gain matrix, we have

$$\mathbf{y} = \mathbf{H}\mathbf{w}x + \mathbf{n}. \tag{1.29}$$

The optimum feedback weight calculation problem is now

$$\mathbf{w}_{optimal} = \arg \max_{s.t. \|\mathbf{w}\|^2=1} \|\mathbf{H}\mathbf{w}\|^2. \tag{1.30}$$

To solve this maximization problem note that [22]

$$\frac{\|\mathbf{H}\mathbf{w}\|^2}{\|\mathbf{w}\|^2} = \frac{\mathbf{w}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{w}}{\mathbf{w}^\dagger\mathbf{w}} \leq \lambda_{max}(\mathbf{H}^\dagger\mathbf{H}), \tag{1.31}$$

with equality if $\mathbf{w}$ is proportional to the eigenvector of $\mathbf{H}^\dagger\mathbf{H}$ that corresponds to the largest eigenvalue. Hence, the $\mathbf{w}_{optimal}$ that solves (1.30) is the eigenvector of $\mathbf{H}^\dagger\mathbf{H}$ that corresponds to the largest eigenvalue and is normalized such that $\|\mathbf{w}\|^2 = 1$. The resulting SNR gain is

$$SNR_{gain} = \lambda_{max}(\mathbf{H}^\dagger\mathbf{H}). \tag{1.32}$$

The above optimal performance requires complete knowledge of the optimal beamforming vector. Unfortunately, in system where the forward and reverse channels are not reciprocal, this requires coarsely quantizing the channel or beamforming vector to accommodate the limited bandwidth of the feedback channel. In [15] the problem of quantized beamforming for i.i.d channel is considered. To support the limitations of the feedback channel, they assume the use of a codebook of possible beamforming vectors known to both the transmitter and receiver. The codebook is restricted to have fixed cardinality N and is designed off-line. The receiver is assumed to convey the best beamforming vector from the codebook over the channel. A primary contribution of [15] is to provide a constructive method for designing a quantized beamforming codebook. Using the distribution of the optimal unquantized beamforming vector, the codebook design problem is equivalent to the problem of packing one-dimensional subspaces known as Grassmannian line packing. These codebooks depend on the number of transmit antennas and the size of the codebook but are independent of the number of receive antennas. The optimal codebook (maximized SNR gain for a given number of feedback bits) should minimize the maximized correlation between a pair of codebook vectors as

$$\mathbf{w}_{optimal} = \arg \max_{s.t.\mathbf{w} \in \mathbb{C}^{n_t \times N}} \min_{1 \le i \le j \le N} |\mathbf{w_i}^\dagger \mathbf{w_j}|, \tag{1.33}$$

where $\mathbf{w_j}$ is viewed as coordinates of a point on surface of a hypersphere with unit radius centered at origin, the point defines a straight line in a complex space $\mathbb{C}^{n_t}$.

The two lines defined by $\mathbf{w_i}$ and $\mathbf{w_j}$ have a distance defined as

$$d(\mathbf{w_i}, \mathbf{w_j}) = \sin\theta_{ij} = \sqrt{1 - |\mathbf{w_i}^\dagger \mathbf{w_j}|}, \tag{1.34}$$

which is known as chordal distance. Chordal distance refers to a distance between two points on the sphere as Euclidian distance of the chord joining them. The codebook design is equivalent to

$$\mathbf{w}_{optimal} = \arg \max_{s.t.\mathbf{w} \in \mathbb{C}^{n_t \times N}} \min_{1 \le i \le j \le N} d(\mathbf{w_i}, \mathbf{w_j}). \tag{1.35}$$

This is the same as Grassmannian line packing problem, which does not have a closed-form solution, the optimal codebook are found by simulations. For $n_t = 2$ and $N = 4$ an optimum Grassmannian codebook consists of the four vectors

$$\mathbf{w} = \begin{bmatrix} -0.1612 - 0.7348i & -0.0787 - 0.3192i & -0.2399 + 0.5985i & -0.9541 \\ -0.5135 - 0.4128i & -0.2506 + 0.9106i & -0.7641 - 0.0212i & 0.2996 \end{bmatrix}. \tag{1.36}$$

The above codebook is optimal because correlation between codewords are the same and equal to $1/\sqrt{3}$.

There are other quantization schemes which are not optimal but are easy to implement in practice. For example, order and co-phase algorithms quantize amplitude and phase independently. In WCDMA, the receiver selects the optimum quantized weights from a set of predetermined weights. In closed loop 'mode 2' from UMTS specification, 8 phases and 2 distinct amplitudes ($\sqrt{0.8}$ and $\sqrt{0.2}$) are used as codebook.

1.3 Hybrid Open-loop and Closed-loop Methods

We have already showed that there is a significant performance gap between open-loop and closed-loop transmit diversity schemes. However, the advantage of closed-loop diversity method comes at price that the transmitter has to have channel state information, which in a real FDD system requires that CSI is fed back to the transmitting side. Therefore, a natural question to ask here is whether it is possible to narrow the performance gap of STBC and beamforming by exploiting channel state information to the open-loop system especially to the STBC system? The answer to this question is yes. In fact, tuning antenna weights based on partial or full CSI on top of STBC has been exploited by many researchers and variety of feedback techniques have been appeared in the literatures. For example in [24] the signal model

$$\mathbf{y} = \frac{1}{\sqrt{2}}\mathbf{XWh} + \mathbf{n}, \tag{1.37}$$

is used where $\mathbf{W}$ is a diagonal weighting matrix on top of Alamouti code $\mathbf{X}$ as

$$\mathbf{W} = \begin{bmatrix} w & 0 \\ 0 & 2 - w \end{bmatrix} \tag{1.38}$$

Here w has to be a function of channel itself as $w = \frac{2|h_1|}{|h_1|+|h_2|}$. Thus w is the fed back partial CSI. Notice that, here we only utilize channel amplitude for feedback information, phase feedback can is not used in this simple diagonal weighting feedback. With unquantized (complete) channel amplitude information, this straightforward diagonal weighting method could achieve a 1.3 dB gain over Alamouti's open-loop STBC transmit diversity method.

Similarly in [25] it was suggested to use one bit amplitude feedback to improve the performance of the Alamouti code. One bit is used to choose one of the following two weighting matrices

$$\mathbf{W_1} = \begin{bmatrix} a^2 & 0 \\ 0 & 1-a^2 \end{bmatrix}^{1/2}, \quad \mathbf{W_2} = \begin{bmatrix} 1-a^2 & 0 \\ 0 & a^2 \end{bmatrix}^{1/2}, \tag{1.39}$$

for some constant a that satisfy $0 < a < 1$, where $a^2 = 0.5$ corresponds to unweighted Alamouti code and $(\cdot)^{1/2}$ denotes the Hermitian square root. This scheme is different from the first one in that the quantized feedback codebook idea is used. In the first case, channel amplitude remain unquantized, however in the second case we need only 1 bit feedback overhead to specify which feedback weighting matrix is used. This scheme therefore could be easily implemented in practice and it is a good tradeoff between feedback complexity and system performance. Based on this scheme the error performance is analyzed for the general case of finite quantized feedback weighting matrix, where the transmitter pre-multiply the STBC matrix $\mathbf{X}$ with a weighting matrix $\mathbf{W}$ taken from a finite matrix constellation $\Omega = \{\mathbf{W_1}, ..., \mathbf{W_K}\}$.

Besides the above diagonal weighting schemes, there are several other methods reported in the literature which try to narrow the performance gap between STBC and beamforming. Among them, transmit antenna selection (TAS) with space-time coding (TAS/STBC) has been investigated by many researchers [3] [4] [5]. Although these schemes are straightforward and relatively easy to implement in practice, the performance loss of these TAS/STBC schemes are non-trivial. For example, in [4] it is proved that the TAS/STBC method achieves a full diversity order asymptotically, but the lose in coding gain is non-negligible.

The feedback information could also be utilized to orthogonalize the class of quasi-orthogonal space-time block code. In [18] it was shown that by exploiting channel state information at the transmitter, it is possible to obtain simultaneously all the advantage of having a rate 1 orthogonal block coding scheme with full diversity order for more than 2 transmit antennas. Associating complex feedback weights $[w_1, w_2, w_3, w_4]$ with each of the transmit antennas, the interference part in ABBA code becomes

$$\beta = 2\Re[h_1^* h_3 w_1^* w_3 + h_2^* h_4 w_2^* w_4]. \tag{1.40}$$

If the weights are selected so that $\beta = 0$, the orthogonality of the code is restored by canceling all the self-interference β. Several sets of transmit weights could easily be found to solve the above equation. One of them consists in choosing 4 transmit weights, with unit modulus, as follows

$$w_1 = 1, \tag{1.41}$$
$$w_2 = 1, \tag{1.42}$$
$$w_3 = \exp(i[\angle(h_1 h_3^*) + \pi/2]), \tag{1.43}$$
$$w_3 = \exp(i[\angle(h_2 h_4^*) + \pi/2]). \tag{1.44}$$

This set of transmit weights defines a new space-time coding scheme providing CSI-based Orthogonal Transmit Diversity. In a similar way, other rate 1 orthogonal block codes with full diversity can be obtained by exploiting transmit CSI from non-orthogonal rate 1 codes when a real or imaginary interference term needs to be canceled to restore the orthogonality of the detection scheme.

The above discussed space-time code gives the best achievable transmit diversity advantage, compensating for fading. However, it has also to be compared to other approaches with multiple transmit antennas, that use CSI at the transmitter. The most classical solution in this area consists in performing maximum ratio transmission (MRT). In MRT, not only the spatial diversity is exploited, but beamforming enables to get additional array gain. However, MRT has 2 flaws. First, its extension to multiple receive antennas requires to compute the eigenvector corresponding to the highest eigenvalue of the channel matrix, which is demanding complexity-wise. Second, it necessitates a full complex weight per emitting antenna (gain and phase). In the above case, only 2 phases computed from the knowledge of the channel coefficients are required at the emitter for 4 transmit antennas. Therefore, in the quasi-orthogonal case, the tradeoff of using these feedback information becomes more complicated than the orthogonal case. The feedback information could be utilized to orthogonalize the code or to do beamforming at the transmitter in order to increase system performance. The optimal tradeoff between these factors remains an open problem for a given quality of channel feedback.

In [16], a new family of full-rate space-time block codes were proposed using a single parameter feedback for communication over Rayleigh fading channels for 3 and 4 transmit antennas. The proposed rate-one codes achieve full diversity, and the performance is similar to maximum receiver ratio combining. The decoding complexity of these codes are only linear even while performing maximum-likelihood decoding. The partial channel information is a real phase parameter (θ) that is a function of all the channel gains, and has a simple closed-form expression for 3 and 4 transmit antennas. This feedback information enables to derive channel orthogonal designs starting from quasi-orthogonal space-time block code. The feedback complexity is significantly lower than conventional closed-loop transmit beamforming. The proposed code for 3 transmit antennas is

$$\mathbf{g_1} = \begin{bmatrix} x_1 & x_2 & e^{j\theta}x_3 \\ x_2^* & -x_1^* & e^{j\theta}x_4^* \\ x_3 & -x_4 & -e^{j\theta}x_1 \\ x_4^* & x_3^* & -e^{j\theta}x_2^* \end{bmatrix}, \tag{1.45}$$

$$\mathbf{g_2} = \begin{bmatrix} x_1 & x_2 & e^{j\theta}x_3 \\ -x_2^* & -x_1^* & -e^{j\theta}x_4^* \\ x_3^* & x_4^* & -e^{j\theta}x_1^* \\ -x_4 & x_3 & e^{j\theta}x_2 \end{bmatrix}. \tag{1.46}$$

And, for the 4 transmit antennas case, the proposed channel orthogonalization space-time block codes are

$$\mathbf{g_3} = \begin{bmatrix} x_1 & x_2 & e^{j\theta}x_3 & e^{j\theta}x_4 \\ x_2^* & -x_1^* & e^{j\theta}x_4^* & -e^{j\theta}x_3^* \\ x_3 & -x_4 & -e^{j\theta}x_1 & e^{j\theta}x_2 \\ x_4^* & x_3^* & -e^{j\theta}x_2^* & -e^{j\theta}x_1^* \end{bmatrix}, \tag{1.47}$$

$$\mathbf{g_4} = \begin{bmatrix} x_1 & x_2 & e^{j\theta}x_3 & e^{j\theta}x_4 \\ -x_2^* & x_1^* & -e^{j\theta}x_4^* & e^{j\theta}x_3^* \\ x_3^* & x_4^* & -e^{j\theta}x_1^* & -e^{j\theta}x_2^* \\ -x_4 & x_3 & e^{j\theta}x_2 & -e^{j\theta}x_1 \end{bmatrix}. \tag{1.48}$$

The transmitter has to be informed about the angle θ (which is a function of the channel realizations) in order to orthogonalize the code.

The concept of combining closed-loop and open-loop schemes is considered in [20] [21] [26]. In these papers the performance criterion used is to minimize codeword error probability. A predetermined orthogonal space-time block code is linearly transformed in order to adapt the code to the available side information. The performance criterion used cannot be solved in a closed-form fashion, therefore numerical optimization is used in order to obtain the optimal weighting matrix $\mathbf{w}$. The proposed beamformer is capable to combine the benefits of conventional beamforming with those given by orthogonal space-time block coding. Generalizations to a quasi-orthogonal scheme is considered in [19], where a quasi-orthogonal code is considered to be the predetermined space-time code. In both cases, the modeling of channel feedback is done by considering the correlation between the true channel and the channel information available.

So far we have seen that open-loop transmit diversity (STBC) and closed-loop transmit diversity schemes use different signal models to represent their transmit architectures. For open-loop systems, in order to achieve diversity, the transmitted symbols must be encoded over time. This requires that the transmit antennas emit different symbols at the same time instant. However, closed-loop transmit diversity systems transmit the same symbol in a time from different antennas, which is usually called scalar coding. The diversity gain of the closed-loop scheme comes at the ability to weight this scalar symbol before transmission so that the received signals from different path will combine coherently at the receiver, achieving both diversity gain and coding gain. Notice that the weighting coefficients are associated with each transmit antenna and the weights must get new values according to different channel realizations. This same idea could also be used for open-loop systems. In fact the above discussed diagonal weighting methods are indeed applying the same spirit to space-time codes. Diagonal weights is nothing but associating the diagonal elements as antenna weights. However, we claim that these weighting schemes for space-time coding system are not optimal in SNR gain sense than beamforming system even when we have complete channel state information. The reason is that STBC is developed on the sole purpose to achieve diversity gain without CSI. It rely on different permutations of transmit symbols in time and space (antennas) to have diversity gain, therefore STBC have its inherent encoding structure, which is different from closed-loop system where the same scalar value is transmitted by different antennas. This fixed encoding structure will prevent STBC systems to obtain the same coding gain as beamforming systems.

This thesis is organized as follows. In Chapter 2 the transmission model and the model for channel state information are introduced, followed by which, derivations of the capacity optimization problem are given. Simplifications of both the CSI model and the maximization problem are unfolded in Chapter 3. We also give an approximation method to effectively solve the capacity optimization problem in Chapter 3 as well. In Chapter 4, numerical results are given to illustrate the significant gains compared with conventional space-time coding as well as beamforming. The accuracy of the approximation method is also illustrated there and some interpretations of performance curves are given as well. Finally, concluding marks and possible future research directions are discussed in Chapter 5.

Chapter 2

Problem Formulation

After the brief explaining of the research problem in Chapter 1, we will begin to unfold the formulations of our problem in detail in this chapter. First of all, the adopted signal model, which could essentially combine the benefits of open-loop and closed-loop transmit diversity is introduced. Next the modeling of imperfect CSI at transmitter is described, as well as the chosen performance criterion, maximizing Ergodic mutual information. Merits of choosing this criterion are explained. Finally, we formulate the capacity optimization problem to be solved in the next chapter.

2.1 Signal Model

In order to exploit the full degrees of freedom in linear STBC, we adopt a transmit signal model, which will allow us to combine the benefits of an open-loop system and a closed-loop system in a manner that maximizes the Ergodic mutual information according to the channel feedback quality. We could therefore optimally narrow the performance gap between Alamouti scheme and MRT system by the degree of accuracy of channel feedback. The accuracy of the channel state information is modeled by using a statistical approach as a simple correlation coefficient.

The signal model for open-loop transmit diversity is

$$\mathbf{y} = \mathbf{X}\mathbf{H} + \mathbf{n}, \tag{2.1}$$

where $\mathbf{X}$ is space-time block codes having $T \times n_T$ dimensions. T is the number of symbol intervals used to encode $\mathbf{X}$. And $\mathbf{H}$ represents $n_T \times n_R$ dimensions of complex channel coefficients between transmit and receive antennas. For this model there is no channel state information at the transmitter. If we consider now some CSI available at the transmitter, the modified signal model exists in the literature is

$$\mathbf{y} = \mathbf{XWH} + \mathbf{n}, \tag{2.2}$$

the matrix $\mathbf{W}$ is diagonal weighting matrix, having each of the diagonal element w_i as transmit antenna weight representing CSI at transmitter. We could also consider $\mathbf{W}$ matrix as precoding matrix which is used to tune antenna weights according to channel condition before transmission.

The signal model for closed-loop transmit diversity scheme is,

$$\mathbf{y} = \mathbf{Hw}x + n, \tag{2.3}$$

here, the difference is that only a scalar value x is transmitted in a time, no symbol coding over time mechanism is exploited. In order to combine the benefits of transmit beamforming and space-time block coding, the signal model adopted here is

$$\mathbf{y} = \mathbf{XPWh} + \mathbf{n}. \tag{2.4}$$

We concentrate on two transmit antennas, and use a predetermined space-time code $\mathbf{X} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$. The transmission is adapted to the available channel state information by means of a linear transformation $\mathbf{PW}$. $\mathbf{P}$ is diagonal beam weighting matrix $\mathbf{P} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$, $\mathbf{W}$ is unitary beam forming matrix $\mathbf{W} = \begin{bmatrix} w_1 & w_2 \\ -w_2^* & w_1^* \end{bmatrix}$ with optimal weighting entries [23] as $w_i = \frac{h_i^*}{\|\mathbf{h}\|}$, $i = 1, 2$. In order to keep total transmission power constant, it is required that $P_1^2 + P_2^2 = 1$ and $\|\mathbf{w}\|^2 = 1$.

Straightforward calculation reveals that the SNR gain of this signal model (2.4) when using a 2×1 system is

$$SNR_{gain} = |h_1|^2(P_1^2|w_1|^2 + P_2^2|w_2|^2) + |h_2|^2(P_1^2|w_2|^2 + P_2^2|w_1|^2) + 2\Re[w_1 w_2^* h_1 h_2^*](P_1^2 - P_2^2). \tag{2.5}$$

In order to give some insights on the above expression, let us discuss to two extreme cases regarding different values of beam power allocation matrix $\mathbf{P}$:

- When $P_1 = P_2 = 1/\sqrt{2}$, the signal model in (2.4) becomes

$$\begin{aligned}\mathbf{y} &= \mathbf{XPWh} + \mathbf{n} && (2.6)\\ &= \frac{1}{\sqrt{2}}\mathbf{XWh} + \mathbf{n} && (2.7)\\ &= \mathcal{H}\begin{bmatrix} x_1 \\ x_2^* \end{bmatrix} + \mathbf{n}, && (2.8)\end{aligned}$$

where $\mathcal{H}$ is the equivalent channel calculated by rearranging symbols so that the transmitted signal vector $[x_1, x_2]$ is factored out. Finally, the detection process is done by multiplying the received signal vector with Hermitian conjugate of the equivalent channel $\mathcal{H}$, which could be interpreted as the effective channel where the signal vector $\mathbf{x}$ is propagating. The detected symbol vector $\mathbf{z}$ is therefore

$$\begin{aligned}\mathbf{z} &= \mathcal{H}^\dagger \mathbf{y} && (2.9)\\ &= \mathcal{H}^\dagger\mathcal{H}\begin{bmatrix} x_1 \\ x_2^* \end{bmatrix} + \mathbf{n}' && (2.10)\\ &= \frac{1}{2}(|h_1|^2 + |h_2|^2)\begin{bmatrix} x_1 \\ x_2^* \end{bmatrix} + \mathbf{n}', && (2.11)\end{aligned}$$

it can be easily checked that for the equivalent channel $\mathcal{H}$ in this case, $\mathcal{H}^{\mathbf{H}}\mathcal{H} = \frac{1}{2}(|h_1|^2 + |h_2|^2)$ always holds. Clearly from the above derivations we can see that both symbols x_1 and x_2 are shielded by the diversity $\|\mathbf{h}\|^2$. The diversity order is the same as for the Alamouti code. Inserting $P_1 = P_2 = 1/\sqrt{2}$ into (2.5), the SNR gain becomes

$$\begin{aligned}SNR_{gain} &= \frac{|h_1|^2}{2}(|w_1|^2 + |w_2|^2) + \frac{|h_2|^2}{2}(|w_2|^2 + |w_1|^2) && (2.12)\\ &= \frac{1}{2}(|h_1|^2 + |h_2|^2), && (2.13)\end{aligned}$$

which is the same as equation (1.7) for the Alamouti code.

- When $P_1 = 1 \quad P_2 = 0$, the signal model in (2.4) becomes

$$\begin{aligned}\mathbf{y} &= \mathbf{XPWh} + \mathbf{n} && (2.14)\\ &= \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} w_1 & w_2 \\ -w_2^* & w_1^* \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \mathbf{n} && (2.15)\\ &= \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \|\mathbf{h}\| \\ 0 \end{bmatrix} + \mathbf{n} && (2.16)\\ &= \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}\begin{bmatrix} \|\mathbf{h}\| \\ 0 \end{bmatrix} + \mathbf{n} && (2.17)\\ &= \begin{bmatrix} x_1\|\mathbf{h}\| \\ -x_2^*\|\mathbf{h}\| \end{bmatrix} + \mathbf{n}, && (2.18)\end{aligned}$$

which tells us that the diversity gain is maintained to be two. Notice that, from equation (2.15) to (2.16), complete channel state information is assumed (i.e. $w_i = \frac{h_i^*}{\|\mathbf{h}\|}$, $i = 1, 2$). Then it becomes clear that the signal model gives MRT scheme in this case. When plugging $P_1 = 1$ and $P_2 = 0$ into the SNR gain equation (2.5), we see that

$$\begin{aligned} SNR_{gain} &= |h_1|^2|w_1|^2 + |h_2|^2|w_2|^2 + 2\Re[w_1 w_2^* h_1 h_2^*] && (2.19)\\ &= |h_1 w_1 + h_2 w_2|^2 && (2.20)\\ &= |h_1|^2 + |h_2|^2, && (2.21) \end{aligned}$$

which corresponds to the case of SNR gain in maximal ratio transmission (1.28). We see again the well known fact that a $n_t = 2, n_r = 1$ MRT system, having complete knowledge about channel, has twice the coding gain over the Alamouti code while maintains the same diversity order.

Therefore, the considered signal model has the ability to combine both open-loop and closed-loop transmit diversity schemes given different values of diagonal beam weighting matrix $\mathbf{P}$. Notice that, this diagonal weighting matrix $\mathbf{P}$ is different from the weighing matrix in equations (1.38) (1.39). In these cases the weighting coefficients are associated with each transmit antennas whereas the element in $\mathbf{P}$ matrix is associated with beam of the transmission signal model.

2.2 Modeling Side Information at the Transmitter

In the previous section we have showed that by choosing different values for the matrix $\mathbf{P}$, it is possible for the signal model in (2.4) to be reformed into Alamouti transmission scheme or MRT scheme. We have seen that by using $P_1 = 1$ and $P_2 = 0$, it is possible to achieve maximal coding gain and diversity gain that a 2×1 system could possibly provide. However the above discussion is based on the assumption that complete channel state information is available at the transmitter, conditioned on which $P_1 = 1$ and $P_2 = 0$ will become the optimal choice for the beam weighting matrix $\mathbf{P}$. We have also seen that by choosing $P_1 = P_2 = 1/\sqrt{2}$ the signal model contracts into the Alamouti transmission scheme, where no channel state information is required at all. In summary, when there is no CSI a $P_1 = P_2 = 1/\sqrt{2}$ solution should be used, and the solution is optimal indeed for the open-loop case. On the other hand, complete CSI will lead to the solution $P_1 = 1$, $P_2 = 0$, which is optimal in the closed-loop case. A question to ask here is what will be beam weighting matrix $\mathbf{P}$ when imperfect CSI is available at the transmitter? Naturally we would guess that as the degree of accuracy in the channel state information decreases from the complete CSI case, P_1 will likely to move from $P_1 = 1$ to $P_1 = 1/\sqrt{2}$ and at the same time P_2 will move from $P_2 = 0$ to $P_1 = 1/\sqrt{2}$. Finally, when the CSI goes to a completely un-trustable state we would expect to see the solution $P_1 = P_2 = 1/\sqrt{2}$.

Before we could explicitly model the degree of accuracy of the available CSI, we have to understand where the transmitter's uncertainty about the channel comes from. We use $\hat{\mathbf{h}}$ to denote the imperfect CSI or side information available in the transmitter. The true channel state information is represented as $\mathbf{h}$. The differences of $\mathbf{h} - \hat{\mathbf{h}}$ may arise due to:

- Channel estimation error in the receiver.
- Channel variations during the feedback delay in the reverse channel (from receiver to transmitter).
- Feedback weight quantization error.
- Errors induced by the feedback channel.

For modeling channel estimation error, the following model exists in the literature [6] [7] [33]:

$$\hat{\mathbf{h}} = \sqrt{1-\epsilon^2}\mathbf{h} + \epsilon\mathbf{v}, \tag{2.22}$$

where the channel estimation error $\mathbf{v}$ is a complex Gaussian random variable independent of $\mathbf{h}$ having zero-mean and unit variance and $\epsilon \in [0, 1]$ is a measure of the accuracy of the channel estimation. The value $\epsilon = 0$ indicates that there is no estimation error. The mean square error (MSE) between of the channel estimation is given by

$$MSE = E[|\hat{h}_i - h_i|^2] = 2(1 - \sqrt{1-\epsilon^2}). \tag{2.23}$$

Channel delay modeling is motivated by the Jakes' model [8], which describes the variations of the channel due to movement of the mobile receiver as a function of time. In this model, the channel coefficients are samples of a stationary

Gaussian process with an autocorrelation function proportional to $J_0(2\pi f_m \tau)$, where J_0 is the zero-order Bessel function of the first kind, τ denoted the time lag and f_m is the maximum Doppler frequency. Hence, the outdated channel estimates available at the transmitter are correlated with the current channel and the amount of such correlation is determined by the time it takes to feed back the estimates.

It is generally not an easy task to statistically model the feedback weights quantization error. The reason is that quantization error is algorithm dependent, depending on different quantization schemes, we will have different models for the error performance. In [29], modeling of phase only quantization scheme is given in a closed form expression for a equal-gain combing system. For detailed analysis of variance other quantization algorithms, see reference [23].

Finally, for modeling of error induced by feedback channel, it is also required to have channel model from information theory perspective. In [9], a discrete memoryless multilevel channel is used to model the imperfection of the feedback channel.

We now consider the imperfect CSI case, as we have seen that the beam weighting matrix $\mathbf{P}$ shall change the value according to the quality of channel feedback information. Loosely speaking, $\mathbf{P}$ should be at least a function of the degree of accuracy in CSI and the transmitting power. Therefore, for a given CSI quality and transmission power, we would expect to find a optimal $\mathbf{P}$ such that a performance criterion is optimized. The performance criterions could be mutual information, bit/block error rate, SNR gain, or error performance.

In this thesis, we choose Ergodic mutual information as the performance criterion to be maximized over. The maximization is done with respect to the beam weighting matrix $\mathbf{P}$ for a given degree of accuracy of the CSI. To this end, the beamforming matrix $\mathbf{W}$ is now a function of the imperfect CSI $\hat{\mathbf{h}}$ as $\mathbf{W}(\hat{\mathbf{h}})$, where each entry is given by $w_i = \frac{\hat{h_i^*}}{\|\hat{\mathbf{h}}\|}$, $i = 1, 2$. We are using beamforming matrix $\mathbf{W}(\hat{\mathbf{h}})$ as if it has correct value as $\mathbf{W}(\mathbf{h})$ in the full CSI case. Therefore, we could define the 'weighting error' as $\mathbf{W}(\hat{\mathbf{h}}) - \mathbf{W}(\mathbf{h})$. Assuming that $\hat{\mathbf{h}}$ and $\mathbf{h}$ are jointly complex Gaussian, the statistics of the side information and its relation to the true channel are completely described by the vector of means $\mathbf{m}_{\hat{\mathbf{h}}}$, the covariance matrix $\mathbf{R}_{\hat{\mathbf{h}}\hat{\mathbf{h}}}$ and the cross-covariance matrix $\mathbf{R}_{\hat{\mathbf{h}}\mathbf{h}}$ [26]. In view of the Jakes model, the joint Gaussian assumption is reasonable since the side information and the true channel are samples of the same Gaussian random process. Clearly, the quality of side information is closely related to the degree of correlation with the true channel, as represented by the cross-covariance matrix. Note that this modeling of imperfect CSI can not be applied to model feedback quantization error. Because quantization error should reasonably be assumed to be uniformly distributed within the quantization level.

Since the true channel and side information are jointly complex Gaussian, the probability density function (pdf) of the true channel, conditioned on the imperfect CSI, is also a complex Gaussian distributed, which is completely described by the conditional mean vector $\mathbf{m}_{\mathbf{h}|\hat{\mathbf{h}}}$ and the conditional covariance matrix $\mathbf{R}_{\mathbf{h}\mathbf{h}|\hat{\mathbf{h}}}$ as [26] [28]

$$p(\mathbf{h}|\hat{\mathbf{h}}) = \frac{e^{-(\mathbf{h}-\mathbf{m}_{\mathbf{h}|\hat{\mathbf{h}}})^* R^{-1}_{\mathbf{h}\mathbf{h}|\hat{\mathbf{h}}}(\mathbf{h}-\mathbf{m}_{\mathbf{h}|\hat{\mathbf{h}}})}}{\pi^{n_r n_t}\det(R_{\mathbf{h}\mathbf{h}|\hat{\mathbf{h}}})}. \tag{2.24}$$

All cross-covariance and covariance matrices are assumed to be constant and invertible. From this it follows that the conditional mean of $\mathbf{h}$ is given by [28] is

$$\mathbf{m}_{\mathbf{h}|\hat{\mathbf{h}}} = E[\mathbf{h}|\hat{\mathbf{h}}] = \mathbf{m}_{\mathbf{h}} + \mathbf{R}_{\mathbf{h}\hat{\mathbf{h}}}\mathbf{R}^{-1}_{\hat{\mathbf{h}}\hat{\mathbf{h}}}(\hat{\mathbf{h}} - \mathbf{m}_{\hat{\mathbf{h}}}), \tag{2.25}$$

and the conditional covariance of $\mathbf{h}$ is [28]

$$\mathbf{R}_{\mathbf{h}\mathbf{h}|\hat{\mathbf{h}}} = E[(\mathbf{h} - \mathbf{m}_{\mathbf{h}|\hat{\mathbf{h}}})(\mathbf{h} - \mathbf{m}_{\mathbf{h}|\hat{\mathbf{h}}})^*|\hat{\mathbf{h}}] \tag{2.26}$$

$$= \mathbf{R}_{\mathbf{h}\mathbf{h}} - \mathbf{R}_{\mathbf{h}\hat{\mathbf{h}}}\mathbf{R}^{-1}_{\hat{\mathbf{h}}\hat{\mathbf{h}}}\mathbf{R}^*_{\mathbf{h}\hat{\mathbf{h}}}. \tag{2.27}$$

Notice that $\mathbf{m}_{\mathbf{h}|\hat{\mathbf{h}}}$ is the minimum mean-square estimate (MMSE) of $\mathbf{h}$ based on $\hat{\mathbf{h}}$ with $\mathbf{R}_{\mathbf{h}\mathbf{h}|\hat{\mathbf{h}}}$ being the corresponding error covariance matrix. Since $\mathbf{R}_{\mathbf{h}\mathbf{h}|\hat{\mathbf{h}}}$ describes the remaining uncertainty when the imperfect channel state information is known, it should be apparent that, loosely speaking, high quality side information corresponds to small $\mathbf{R}_{\mathbf{h}\mathbf{h}|\hat{\mathbf{h}}}$ if measured in a suitable norm, where a large $\mathbf{R}_{\mathbf{h}\mathbf{h}|\hat{\mathbf{h}}}$ corresponds to side information of low quality. Formally, perfect side information and no side information are defined as [26]

- Perfect side information$\Leftrightarrow$ $\|\mathbf{R}_{\mathbf{h}\mathbf{h}|\hat{\mathbf{h}}}\| \rightarrow 0$.
- No side information$\Leftrightarrow$$\|\mathbf{R}_{\mathbf{h}\mathbf{h}|\hat{\mathbf{h}}}\|^{-1} \rightarrow 0$.

2.3 Performance Criterion: Ergodic Mutual Information

In this section, we will derive a performance criterion for the optimization problem, which takes the available side information into account. Before we could say anything on this performance criterion, let us first make two terminologies clear, namely the channel capacity and the capacity achieved by space-time codes (mutual information). In information theory, channel capacity is the tightest upper bound on the amount of information that can be reliably transmitted over a communication channel. The channel capacity of a discrete memoryless channel is

$$C = \max_{p(x)} I(X;Y), \tag{2.28}$$

where the maximum is taken over all the possible input distributions $p(x)$. According to the channel coding theorem, when the transmission rate R is smaller that channel capacity C, it is possible to send information with an arbitrarily low probability of error, and when the transmission rate is larger than channel capacity, the probability of error is bounded away from zero. The channel coding theorem proves the existence of good codes (codes with small probability of error) for long block length, but the code obtained is very difficult to decode. In other words, the channel coding theorem does not provide a practical coding scheme but proves the the existence of good codes.

One possibility to achieve large portion of the channel capacity promised by information theory while maintaining the decoding complexity is to use space-time codes. Data is encoded using a space-time block code and the encoded data is split into n streams which are simultaneously transmitted using n transmit antennas. The received signal at each receive is a linear superposition of the n transmitted signals perturbed by noise. Maximum likelihood decoding is achieved in a simple way thought decoding of signals transmitted from different antennas rather than joint detection. This uses the orthogonal structure of the space-time block code and gives a maximum-likelihood decoding algorithm which is based only on linear processing at the receiver [2]. In [2], it is shown that for complex constellations and for the specific cases of two, three and four transmit antennas, these diversity schemes are improved to provide, respectively all, 3/4 and 3/4 of the maximum possible transmission rate promised by information theory.

In summary, space-time codes have elegant mathematical solution for providing full diversity over coherent, flat-fading channel and they require extremely simple encoding and decoding. Although these codes provide full diversity at low computational costs, it is shown in [10] that the space-time block codes incur a loss in capacity because they convert the matrix channel into a scalar addictive white gaussian noise (AWGN) channel whose capacity is (at least sometimes) smaller than the true channel capacity. In [10], the loss in capacity is quantified as a function of channel rank, code rate and number of receive antennas. Space-time block code could achieve channel capacity for a channel with rank one but is sub-optimal for a channel with rank greater than one. The rank one channel occurs, for example, when there is only one receive antenna. Therefore, the 2×1 Almounti code achieves channel capacity and is called capacity optimal conditioned on that no knowledge of CSI at the transmitter is provided. It should be noted that the maximal ratio transmission scheme achieves channel capacity for the 2×1 closed-loop case when complete channel knowledge is available. We will show in a while how to achieve the available channel capacity when there is incomplete channel knowledge at the transmitter.

The capacity achieved by a linear STBC code can be written as [27]

$$C = \log\det(\mathbf{I}_{n_r} + \frac{\gamma}{n_t}\mathcal{H}\mathcal{H}^\dagger), \tag{2.29}$$

where γ is average SNR or the transmit power and n_t, n_r is the number of transmit and receive antennas respectively. $\mathcal{H}$ is the equivalent channel, which in our case could be calculated from the signal model in equation (2.4) as

$$\mathbf{y} = \mathbf{XPWh} + \mathbf{n} = \mathcal{H}\begin{bmatrix} x_1 \\ x_2^* \end{bmatrix} + \mathbf{n}. \tag{2.30}$$

In our problem the capacity is a function of the SNR γ, the true channel $\mathbf{h}$, the beam power allocation $\mathbf{P}$ and the imperfect channel knowledge $\hat{\mathbf{h}}$ as

$$C(\gamma, \mathbf{h}, \mathbf{P}|\hat{\mathbf{h}}) = \log\det(\mathbf{I}_{n_r} + \frac{\gamma}{n_t}\mathcal{H}\mathcal{H}^\dagger). \tag{2.31}$$

Recall that the power allocation matrix $\mathbf{P}$ and the beamforming matrix $\mathbf{W}$ depend on the imperfect channel knowledge $\hat{\mathbf{h}}$. In the transmitter, we could only obtain the imperfect CSI $\hat{\mathbf{h}}$ which is correlated to an given degree with the true channel vector $\mathbf{h}$. Assuming that the distribution of $\mathbf{h}$ given $\hat{\mathbf{h}}$ is specified as (2.24), we can average out the true but unknown channel to obtain

$$C(\gamma, \mathbf{P}|\hat{\mathbf{h}}) = \int C(\gamma, \mathbf{h}, \mathbf{P}|\hat{\mathbf{h}}) p(\mathbf{h}|\hat{\mathbf{h}}) d\mathbf{h}. \quad (2.32)$$

The STBC capacity is now a function of imperfect CSI, the SNR and the diagonal beam power allocation matrix $\mathbf{P}$. Therefore, for a given SNR value γ and a given quality of channel state information, we would like to find the optimal power allocation $\mathbf{P}$ matrix such that the capacity in the left hand side of equation (2.32) is maximized

$$\mathbf{P}_{optimal} = \arg \max_{s.t. P_1^2 + P_2^2 = 1} C(\gamma, \mathbf{P}|\hat{\mathbf{h}}). \quad (2.33)$$

In order to keep total transmission power constant, we have to impose the power constraint $P_1^2 + P_2^2 = 1$. Note that the problem formulation assumes that we know the statistics of $\mathbf{h}$ and $\hat{\mathbf{h}}$, and the degree of correlation between these two variables. This information is used to select the optimum $\mathbf{P}$ matrix.

From the results in the last section, we could already have two special cases of the optimal solutions $\mathbf{P}$. When complete channel information is known, the optimal solution is $\mathbf{P_{opt}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and when no CSI is available at the transmitter the optimal solution to maximize capacity is $\mathbf{P_{opt}} = \begin{bmatrix} 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \end{bmatrix}$. Here we could consider using the SNR gain as the performance criterion as well. This is problematic, however. If the expected SNR gain is maximized, the optimal solution will always be the beamforming solution with $\mathbf{P_{opt}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, irrespective the quality of the CSI. Intuitively the reason is that when maximizing the expected SNR gain the diversity gain is not seen at all. Therefore, using SNR gain as a performance criterion is insufficient as compared with capacity criterion. However, we have to keep in mind that when using the capacity criterion, we assume that we have capacity-achieving outer codes such as turbo codes concatenated with the space-time block coding system. Optimization of expected mutual information as a performance criterion has been proposed in [29], where a sufficient condition for maximizing the expected mutual information is given.

Chapter 3

Simplification and Approximation

In the previous chapter we have presented the capacity optimization problem in equation (2.33). However, this optimization does not permit a closed form solution easily. Therefore, in this chapter we will try to reformulate this optimization problem so that a tight approximation method is applicable. Firstly, a simplified fading scenario is proposed where practical assumptions are made. Secondly, we utilize a unitary transformation to remove channel dependence of the imperfect CSI. Finally, a approximation method is used to give an analytical solution to this optimization problem.

3.1 Simplified Fading Scenario

In the simplified fading scenario it is assumed that the antennas at both the transmitter and receiver are spaced sufficiently far apart so that the fading is independent. A rich scattering environment with non-line-of-sight condition is also assumed. It is reasonable to model the true channel coefficient h_i as independent and identically distributed (i.i.d) zero-mean complex Gaussians. Let σ_h^2 denote the variance of each individual channel coefficient. The coefficients of the channel estimates $\hat{h}_i$ are modeled in the same way with variance $\sigma_{\hat{h}}^2$. Each estimated channel coefficient $\hat{h}_i$ is assumed to be correlated with the corresponding true channel coefficient h_i and uncorrelated with all others. In order to describe the degree of correlation, introduce the normalized correlation coefficient ρ as

$$\rho = E[h_i \hat{h}_i^*]/\sigma_{\hat{h}}\sigma_h. \tag{3.1}$$

Thus, assuming h_i and $\hat{h}_i$ are jointly complex Gaussian, the distribution of the true channel and the side information is completely characterized by the covariance matrices

$$\mathbf{R}_{\mathbf{hh}} = \sigma_h^2 \mathbf{I}_{n_r n_t}, \quad \mathbf{R}_{\mathbf{h}\hat{\mathbf{h}}} = \sigma_h \sigma_{\hat{h}} \rho \mathbf{I}_{n_r n_t}, \quad \mathbf{R}_{\hat{\mathbf{h}}\hat{\mathbf{h}}} = \sigma_{\hat{h}}^2 \mathbf{I}_{n_r n_t}, \tag{3.2}$$

and the mean vectors $\mathbf{m}_{\mathbf{h}} = \mathbf{m}_{\hat{\mathbf{h}}} = \mathbf{0}$. Straightforward calculations using equations (2.25) and (2.26) show that this model leads to a conditional channel distribution described by

$$\mathbf{m}_{\mathbf{h}|\hat{\mathbf{h}}} = \frac{\sigma_h}{\sigma_{\hat{h}}} \rho \hat{\mathbf{h}}, \quad \mathbf{R}_{\mathbf{hh}|\hat{\mathbf{h}}} = \sigma_h^2 (1-\rho^2) \mathbf{I}_{n_r n_t}. \tag{3.3}$$

Upon these simplifying assumptions, the conditional probability function in equation (2.24) can be simplified to

$$p(\mathbf{h}|\hat{\mathbf{h}}) = \frac{e^{-\frac{\|\mathbf{h}-\rho\hat{\mathbf{h}}\|^2}{1-\rho^2}}}{\pi^2 (1-\rho^2)^2}, \tag{3.4}$$

which is the distribution of the true channel conditioned on the estimated channel vector and the correlation is specified by the factor ρ in our case of a 2×1 system for the simplified fading scenario.

The normalized correlation coefficient $\rho = E[h_i \hat{h}_i^*]$ has an apparent interpretation as the channel feedback quality. It is used to measure the accuracy of the imperfect CSI. Perfect channel knowledge now corresponds to $\rho \to 1$ where equation (3.4) becomes a δ-function. Therefore the integration problem in equation (2.32) simplifies to a replacement of imperfect CSI $\hat{\mathbf{h}}$ with full CSI $\mathbf{h}$, which implies fully correctness of the channel state information obtained. On the other hand, no channel information corresponds to $\rho \to 0$ where the imperfect CSI $\hat{\mathbf{h}}$ is weighted by zero, meaning the CSI is not at all reliable.

3.2 Removing Channel Dependence by Unitary Transform

We now utilize a unitary transformation on the right hand side of equation (2.32), which will greatly simplify the optimization problem in equation (2.33) and lead us to some interesting observations.

In the signal model adopted in the previous chapter

$$\mathbf{y} = \mathbf{XPWh} + \mathbf{n}, \tag{3.5}$$

the unitary beamforming matrix is defined as

$$\mathbf{W} = \frac{1}{\|\hat{\mathbf{h}}\|}\begin{bmatrix} \hat{h}_1^* & \hat{h}_2^* \\ -\hat{h}_2 & \hat{h}_1 \end{bmatrix}. \tag{3.6}$$

This matrix has the property that

$$\mathbf{W}\hat{\mathbf{h}} = \begin{bmatrix} \|\hat{\mathbf{h}}\| \\ 0 \end{bmatrix}. \tag{3.7}$$

We redefine the complete channel knowledge $\mathbf{h}$ as

$$\mathbf{h} = \rho\hat{\mathbf{h}} + \mathbf{W}^{\dagger}\tilde{\mathbf{h}}, \tag{3.8}$$

where $\tilde{\mathbf{h}}$ is a new channel variable. Under this change of variable, the probability function of $\mathbf{h}$ conditioned on $\hat{\mathbf{h}}$ is now a function of $\tilde{\mathbf{h}}$ as

$$p(\mathbf{h}|\hat{\mathbf{h}}) = \frac{e^{-\frac{\|\mathbf{h}-\rho\hat{\mathbf{h}}\|^2}{1-\rho^2}}}{\pi^2(1-\rho^2)^2} \tag{3.9}$$

$$= \frac{e^{-\frac{\|\mathbf{W}^{\dagger}\tilde{\mathbf{h}}\|^2}{1-\rho^2}}}{\pi^2(1-\rho^2)^2} \tag{3.10}$$

$$= \frac{e^{-\frac{\|\tilde{\mathbf{h}}\|^2}{1-\rho^2}}}{\pi^2(1-\rho^2)^2} = p(\tilde{\mathbf{h}}). \tag{3.11}$$

The step from equation (3.10) to (3.11) is often considered as the definition of a unitary transformation. This same definition leads to the fact that the Jacobian of the change of variable in equation (3.8) is one, as

$$d\mathbf{h} = d(\rho\hat{\mathbf{h}}) + d(\mathbf{W}^{\dagger}\tilde{\mathbf{h}}) \tag{3.12}$$

$$= \det(\mathbf{W}^{\dagger})d\tilde{\mathbf{h}} \tag{3.13}$$

$$= d\tilde{\mathbf{h}}. \tag{3.14}$$

What happens in the transformation (3.8) is that a 'Rician like' [1] fading variable $\mathbf{h}$ is separated into its specular $\rho\hat{\mathbf{h}}$ and fading components $\mathbf{W}^{\mathbf{H}}\tilde{\mathbf{h}}$. In our case, we know the specular component, which represents the CSI at transmitter, and we average over the fading component. Now we apply the same

[1] The adopted signal model is operating on Rayleigh channel, however the CSI modeling makes it look like a Rician channel problem.

change of variable rule (3.8) to the capacity expression for STBC in equation (2.31), and after some simplifications we get

$$C = \log\det(\mathbf{I}_{n_r} + \frac{\gamma}{n_t}\mathcal{H}\mathcal{H}^*) \quad (3.15)$$

$$= \log(1 + \frac{\gamma}{n_t}A) \quad (3.16)$$

$$= \log[1 + \gamma(P_1^2\|\hat{\mathbf{h}}\|^2\rho^2 + 2\Re\{\tilde{h_1}\}\|\hat{\mathbf{h}}\|\rho + P_1^2|\tilde{h_1}|^2 + P_2^2|\tilde{h_2}|^2)], \quad (3.17)$$

where $A = |h_1|^2(P_1^2|w_1|^2 + P_2^2|w_2|^2) + |h_2|^2(P_1^2|w_2|^2 + P_2^2|w_1|^2) + 2\Re[w_1 w_2^* h_1 h_2^*]$ $(P_1^2 - P_2^2)$ and the capacity C is now a function of the new channel variable, $C = C(\gamma, \tilde{\mathbf{h}}, \mathbf{P}|\hat{\mathbf{h}})$. The capacity integral in (2.32) can now be expressed in terms of the transformed conditional pdf in (3.11) and the transformed capacity expression in (3.17). Notice that, after this change of variable the only dependence left on $\hat{\mathbf{h}}$ in the capacity integral is through $\|\hat{\mathbf{h}}\|$, the dependence on other degrees of freedom in $\hat{\mathbf{h}}$ vanishes. Specifically we cannot see any angular dependence of the estimated channel $\hat{\mathbf{h}}$. To prepare for the approximation method in the next section, we move the variable $\Re\{\tilde{h_1}\}$ in the capacity expression (3.17) into the pdf expression in (3.11). For this, we redefine again the channel variable as $\breve{\mathbf{h}}$ we get the capacity expression as

$$C = \log[1 + \gamma(P_1^2|\breve{h}_1|^2 + P_2^2|\breve{h}_2|^2)], \quad (3.18)$$

and the average capacity over the corresponding distribution is

$$E(C) = \int \log[1 + \gamma(P_1^2|\breve{h}_1|^2 + P_2^2|\breve{h}_2|^2)]\frac{e^{-\frac{\|\breve{\mathbf{h}} - \mathbf{m}_{\breve{\mathbf{h}}}\|^2}{1-\rho^2}}}{\pi^2(1-\rho^2)^2}d\breve{\mathbf{h}}, \quad (3.19)$$

where $m_{\breve{h}}$ is mean of new channel variable $\breve{\mathbf{h}}$. [2]

Now it becomes more clear that the new channel variables $\breve{h}_1$ and $\breve{h}_2$ are distributed according to a Rician distribution. The capacity integration in equation (3.19) is therefore a problem of evaluating mean (Ergodic) capacity in a Rician environment. However, notice that the channel variables $\breve{\mathbf{h}}$ are weighted by P_1^2 and P_1^2 respectively. In other words, the channel random variable have different variance, therefore the covariance matrix of channel variable $\mathbf{R}_{\breve{\mathbf{h}}}$ is not a identity matrix $\mathbf{I}_2$ but a diagonal matrix, with the diagonal elements proportional to the weighting P_1^2 and P_1^2. [3]

[2] Obviously, only $\Re\{\breve{h}_1\}$ has non-zero mean $\rho\|\hat{\mathbf{h}}\|$.

[3] The non-diagonal elements of the covariance matrix $\mathbf{R}_{\breve{\mathbf{h}}}$ are zero, since there is no cross-correlation between random channel variables.

3.3 Capacity Approximation

Recall that our problem is to maximize the capacity with respect to the beamforming power allocation matrix $\mathbf{P}$ as

$$\mathbf{P}_{optimal} = \arg \max_{s.t. P_1^2+P_2^2=1} \int \log[1+\gamma(P_1^2|\breve{h}_1|^2 + P_2^2|\breve{h}_2|^2)] \frac{e^{-\frac{\|\breve{\mathbf{h}}-\mathbf{m}_{\breve{\mathbf{h}}}\|^2}{1-\rho^2}}}{\pi^2(1-\rho^2)^2} d\breve{\mathbf{h}} \quad (3.20)$$

Due to the power constraint $P_1^2 + P_2^2 = 1$ in this optimization problem, we only have one parameter P_1 or P_2 to optimize over, given the SNR γ, feedback quality ρ and the estimated channel norm $\|\hat{\mathbf{h}}\|$. In order to solve this problem, we could do numerical optimization by using certain non-linear optimization algorithms.

From the development of the previous section we saw that our problem in (3.20) involves the evaluation of the capacity in a correlated Rician fading channel, where the correlation is specified by a diagonal covariance matrix $\mathbf{R}_{\breve{\mathbf{h}}}$. However, to the best of our knowledge, the closed-form capacity expression for correlated Rician fading channel is not available. Therefore in order to ease the numerical burden to optimize the four fold integration [4] in equation (3.20), we therefore utilize a approximation method outlined in [30]. The approximation method discussed in [30] includes the the following two steps. Firstly, since the channel variables $|\breve{h}_1|$ and $|\breve{h}_2|$ are Rician distributed, the random variables $|\breve{h}_1|^2$ and $|\breve{h}_2|^2$ are non-central chi-square distributed. The random variable inside the log function in (3.20) can be seen as a weighted sum of chi-square distributed variable. It is a common practice in statistics and engineering [11] to approximate a weighted sum of non-central chi-square variables by a single central one with different degree of freedom and a proper scaling factor as

$$1 + \gamma(P_1^2|\breve{h}_1|^2 + P_2^2|\breve{h}_2|^2) \approx \alpha\chi^2(l). \quad (3.21)$$

Here l denotes the number degrees of freedom of the chi-square distribution and the parameters α and l should be chosen that both sides of (3.21) will have the same first two moments (i.e. mean and variance). The pdf of the non-central chi-square distribution is given as [32]

$$p_Y(y) = \frac{1}{2\sigma^2}(\frac{y}{s^2})^{(n-2)/4} e^{-(s^2+y)/2\sigma^2} I_{n/2-1}(\sqrt{y}\frac{s}{\sigma^2}), \quad y \geq 0, \quad (3.22)$$

[4] Two complex channels have four degree of freedom.

where Y is defined as

$$Y = \sum_{i=1}^{n} X_i^2, \tag{3.23}$$

and by definition the non-centrality parameter s^2 is defined as

$$s^2 = \sum_{i=1}^{n} m_i^2. \tag{3.24}$$

When $s^2 = 0$ the pdf (3.22) contracts to the central chi-square distribution. $I_\alpha(x)$ is the α:th-order modified Bessel function of the first kind, which may be represented by infinite series as

$$I_\alpha(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{\alpha+2k}}{k!\Gamma(\alpha+k+1)} \tag{3.25}$$

Finally, we state that the first two moments of a noncentral chi-square distributed variable are

$$E(Y) = n\sigma^2 + s^2 \tag{3.26}$$
$$E(Y^2) = 2n\sigma^4 + 4\sigma^2 s^2 + (n\sigma^2 + s^2)^2 \tag{3.27}$$
$$\sigma_y^2 = 2n\sigma^4 + 4\sigma^2 s^2. \tag{3.28}$$

Therefore, now we could do the mean and variance fits from equation (3.21) according to equations (3.26) (3.27) (3.28) as

Mean fits:

$$1 + \gamma[P(1-\rho^2 + \|\hat{\mathbf{h}}\|^2\rho^2) + (1-P)(1-\rho^2)] = \alpha l \tag{3.29}$$

Variance fits:

$$\gamma^2[P^2((1-\rho^2)^2 + 2(1-\rho^2)\|\hat{\mathbf{h}}\|^2\rho^2) + (1-P)^2(1-\rho^2)^2] = 2\alpha^2 l, \tag{3.30}$$

where $P = P_1^2$. From the above two equations, the parameter α and l could be calculated as simple functions of γ, P, ρ and $\|\hat{\mathbf{h}}\|^2$. The capacity can now be calculated due to a Lemma by Porteous [31], which can be stated as follows

Lemma 1 : If $u \sim \chi^2(k)$,then

$$E(\ln u) = \ln k - \frac{1}{k} - \frac{1}{3k^2} + \frac{2}{15k^4} + o(k^{-6}). \tag{3.31}$$

The essence of the above Lemma is to use asymptotic expansion in k [31] and ignore the high order terms $o(k^{-6})$. Therefore the error incurred by this approximation is bounded by the last term $\frac{2}{15k^4}$.

Combining the chi-square approximation in equation (3.21) with the Porteous Lemma, we are now in a position to determine the average channel capacity in equation (3.19). To this end, inserting the parameter α and l obtained from (3.29) and (3.30) into equation (3.21), and use Lemma 1 to obtain the capacity in (3.19) as

$$E[C] \approx \log_2(\alpha l) - c, \tag{3.32}$$

where c is defined by

$$c = \frac{1}{\ln 2}(\frac{1}{l} + \frac{1}{3l^2} - \frac{2}{15l^4}). \tag{3.33}$$

Notice that, from (3.29) αl is actually the approximated mean of the random variable on the left hand side of (3.21). Therefore, the term $\log_2(\alpha l)$ in equation (3.32) is the Jensen bound [12] of the average capacity $E[C]$. Equation (3.32) reveals that if $\log_2(\alpha l)$ is used to approximate $E[C]$, a correction term must be added. This correction term is determined by c.

Since we have integrated out the unknown true channel, the optimization problem becomes trivial in the sense that equation (3.32) becomes a combination of polynomials and simply functions in γ, P, ρ and $\|\hat{\mathbf{h}}\|^2$. Thus the approximation method avoids numerical integration, which makes it computational efficient and affordable in practice.

Chapter 4

Numerical Results and Discussion

In order to examine the accuracy of the approximation method discussed in the previous chapter and to examine the performance of the proposed beamformer, numerical simulations are done for several cases in this chapter. In addition we will discuss some of the properties of the proposed transmission method. Finally general interpretations will be given from the performance curves as well.

4.1 Comparison with Beamforming and Space-Time Coding

Our proposed beamformer is compared with the Alamouti code corresponding to no CSI at transmitter ($\rho = 0$) and to maximal ratio transmission corresponding to complete CSI at the transmitter ($\rho = 1$). For all the examined cases, the simplified fading scenario in Chapter 3.1 is assumed. We assume perfect knowledge of σ_h^2, $\sigma_{\hat{h}}^2$, ρ and the noise variance σ^2. The variance of the estimated and true channel coefficients are set at $\sigma_h^2 = \sigma_{\hat{h}}^2 = 1$. The channel is constant during the transmission of a codeword and independently fading from from one codeword to another. The average transmission power is denoted by γ.

We first study the accuracy of the approximation method in equation (3.32). We use numerical optimization from equation (3.20) as our benchmark to compare our approximation with. The channel quality parameter is set to $\rho = 0.8$. The Ergodic mutual information as a function of SNR for various transmission methods is depicted in Fig. 4.1. As can be seen from Fig. 4.1, the

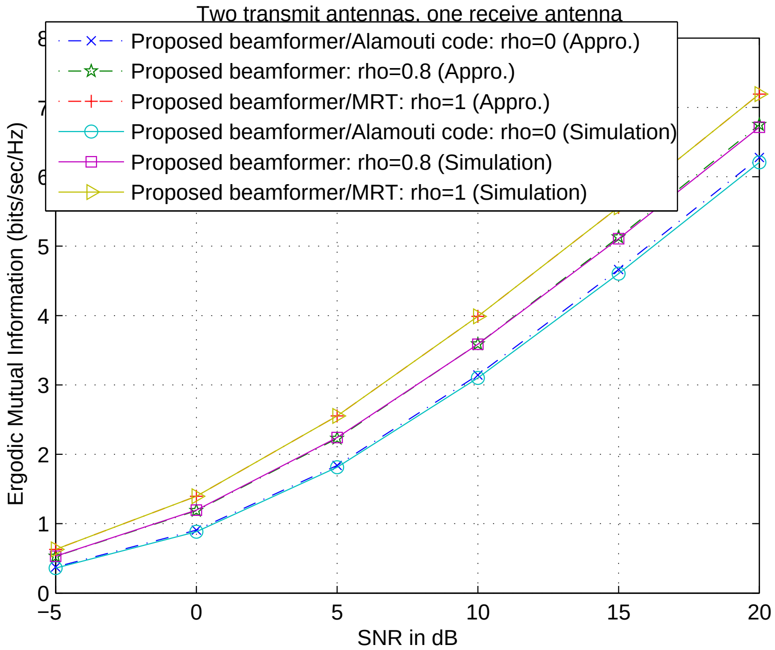

Figure 4.1: Proposed beamformer when $\rho = 0.8$.

capacity of the proposed beamformer is larger than the Alamouti code for all SNR values. However, the capacity is smaller than the MRT case, as expected. Therefore, the proposed beamformer combines the advantages of both open-loop and closed-loop transmit diversity. Note that the two curves for Alamouti code and maximal ratio transmission also correspond to the capacity of our proposed beamformer in the case of $\rho = 0$ and $\rho = 1$, respectively. This is also the same with the results outlined in Chapter 2.1 and 2.3, where $\mathbf{P} = \begin{bmatrix} 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \end{bmatrix}$ and $\mathbf{P} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ correspond to the optimal solutions when $\rho = 0$ and $\rho = 1$, irrespective of channel realization. The capacity values are very well given by the approximation. There is almost complete agreement between numerically integrated and approximated capacity.

In the second case, we study how well the capacity approximation formula (3.32) will give us the power allocation value, P. We also discuss the sensitivity of the capacity on the value of P. Fig. 4.2 shows a case when the feedback quality is relatively low ($\rho = 0.4$). The upper side of this picture depicts the average power allocation, P, (averaged over multiple channel realizations) with respect to the considered SNR range. We can see that the approximation error could be as large as 10% in this case. However, in the lower side of Fig. 4.2,

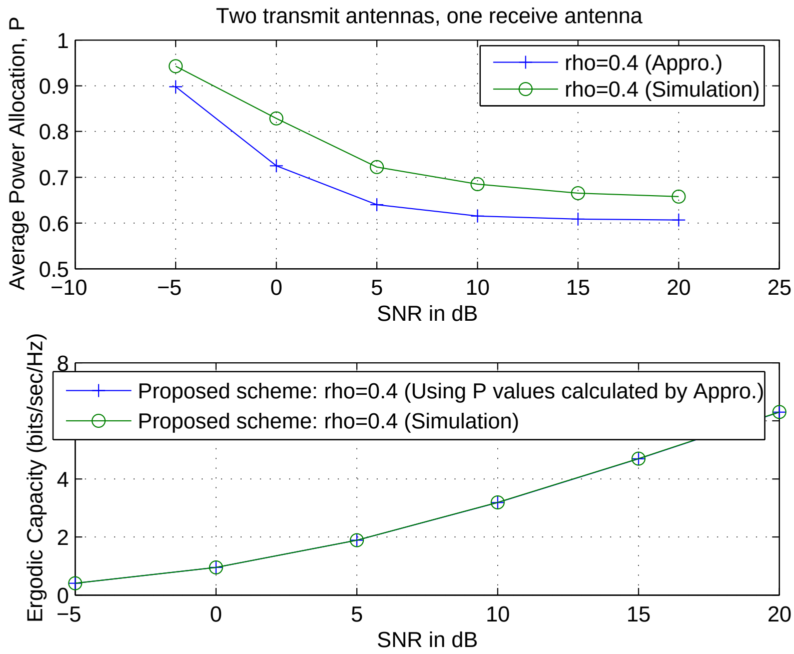

Figure 4.2: The insensitivity of capacity on the value P when ρ is small.

we see that the difference in power allocation P will result a negligible performance loss in capacity domain, when ρ is quite small.

In order to further understand the sensitivity of our transmission scheme, we show the capacity differences when there are errors or deviations from the optimal P in Fig. 4.3. This time we set a condition where the channel feedback quality is relatively high ($\rho = 0.8$). In this case, we can see clearly the capacity loss due to deviations from $P_{optimal}$: when there are 30% error on P, the capacity loss can be 0.5 bits/sec/Hz and 10% deviations could still be tolerable even when ρ is as large as 0.8 in this case, which also validates our use of the approximation method.

Finally in Fig. 4.4, we illustrate the impact of channel feedback quality ρ on the choice of optimal power allocation, P. As seen from Fig. 4.4, when ρ is large ($\rho = 0.9$ and $\rho = 0.7$) the optimal power allocation is almost 1, reflecting the fact that MRT is always optimal in order to maximize capacity in these cases. However, when the feedback quality begins to decrease, the MRT scheme is no longer optimal. We see that when $\rho = 0.3$ the optimal P is approaching 0.5, corresponding to the transmission scheme of the Alamouti

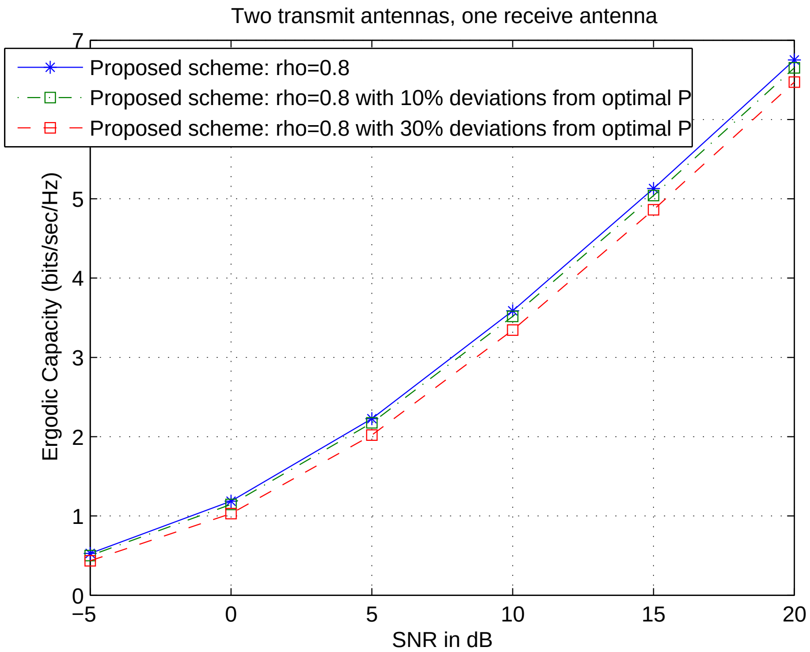

Figure 4.3: Sensitivity analysis when ρ is large.

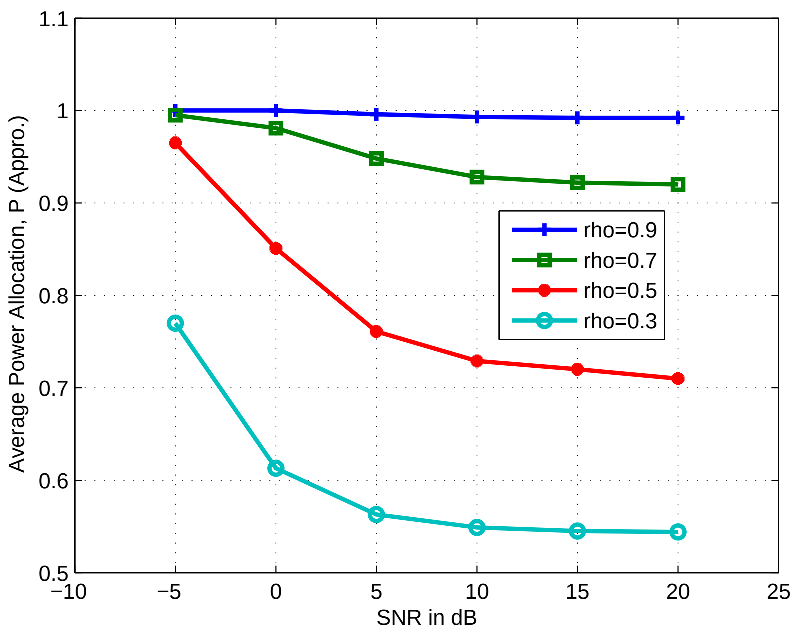

Figure 4.4: Dependence of P on SNR.

code as SNR increases. The general picture is that as the SNR increases the power allocation P will decrease.

4.2 General Interpretations

From the numerical results in last section, we could see that our proposed beamformer could combine the open-loop (Alamouti code) and closed-loop (MRT) transmit diversity as the special cases of the optimal solutions from the capacity optimization problem (3.20). Therefore the proposed beamformer could benefit from open-loop and closed-loop methods according to the channel feedback quality as indicated by ρ. Using Ergodic mutual information as the performance criterion, we have achieved the optimal combing of open-loop and closed-loop in capacity optimal sense, in other words, we have achieved our goal to narrow the performance gap between Alamouti and MRT scheme optimally based on the different ρ values.

Although our choice of ρ in Fig. 4.1 is as large as 0.8, there is still substantial capacity difference compared with the capacity that a 2×1 system could possibly offer. This is so because parameter ρ is crucial to the system performance, only when ρ is as large as 0.99 the capacity difference can be neglected and when $\rho = 0.4$ the capacity is almost the same as the open-loop system. Intuitively this means that when the feedback error is large, the proposed beamformer can not utilize these coarse information efficiently to increase system performance. In fact the similar effect can be seen in [26], where the performance criterion is to minimize the block error rate.

If we compare the performance of Fig. 4.2 with that of Fig. 4.3, we could notice that the degree of sensitivity of P in capacity depends on ρ. In other words, conditional on different ρ values, the importance of power allocation is also different. When $\rho = 0.4$ the 10% error on $P_{optimal}$ will result in a negligible capacity loss: below 0.01 bits/sec/Hz. However, 10% error for $\rho = 0.8$ case the capacity loss is more prominent: around 0.1 bits/sec/Hz. This is due to the fact that as the channel feedback quality ρ increases, the accuracy of power allocation P becomes more important. Intuitively speaking, when ρ is small the maximal mutual information that could be achieved by the proposed beamformer is comparable with the capacity for open-loop case. We have potentially less mutual information to lose than in the case when ρ is large, where the power allocation becomes more important.

Finally, the impact of SNR on P for different ρ values could be seen from Fig. 4.4. Clearly the average power allocation will always decrease with increasing SNR. Therefore, we can conclude that the MRT scheme might be preferred when SNR is low and as SNR increases the open-loop system might be preferred [29]. When the SNR increases, the proposed beamformer try to maximize the diversity gain since the high SNR now already provides high coding gain. The optimal power allocation P also dependents on ρ. As we can see that when $\rho = 0.9$ the beamformer will always choose values in the vicinity of $P_1 = 1$, $P_2 = 0$, implying the optimality of MRT in this case. But when $\rho = 0.3$, the proposed beamformer is already approaching to the open-loop case ($\rho \to 0.5$) when SNR is above $0dB$.

Chapter 5

Conclusion and Future Work

Following the illustrations from last chapter, in this chapter we are going to conclude the main results achieved in this thesis. And most importantly, some possible future research directions based on this thesis are discussed as well.

5.1 Concluding Remarks

In order to communicate efficiently over a wireless channel, one should exploit spatial diversity arising from independent fading propagation paths. This has led to the development of efficient open-loop and closed-loop transmit diversity solutions with the idea of enhancing capacity by means of taking advantage of the spatial diversity available in the channel. Our research problem of optimal combining open-loop and closed-loop diversity is introduced in Chapter 1. We adopted a transmission method which is used to optimize a linear transformation of the predetermined space-time code. We consider the case when the transmitter has only partial knowledge about the channel. The modeling of imperfect channel knowledge is shown Chapter 2, where the capacity optimization problem is unfolded as well. The resulting optimization problem could be solved numerically. In addition, an efficient approximation method is used for the special case of independently fading channel coefficients in Chapter 3. This simple formula can be utilized for evaluating average capacity for correlated vector Ricean channel. Simulation results in Chapter 4 demonstrate significant gain over conventional methods in a scenario with non-perfect channel knowledge. The accuracy of the capacity approximation method is validated through these numerical results.

5.2 Possible Future Work

As an important research area, many interesting aspects of the design of combining open-loop and closed-loop antenna system remain to be explored. We close this thesis by summarizing just two of the more obvious issues that warrant further investigation.

- Firstly, this thesis has not considered the cases of more than two antennas at the transmitter. It would be worth exploring the possibility of a general signal model/transmission architecture to effectively combine beamforming and orthogonal space-time block code or even to include the case of non-orthogonal codes. A scheme considering a 4×1 system with non-orthogonal code is in preparation, where the predetermined space-time code is chosen to be the code in [23].

- Secondly, further research work may consider explicitly modeling feedback quality ρ. We could model ρ as $\rho(N)$, where N is number of feedback bits [29]. Therefore, given N bits of side information, the transmitter could follow a vector quantization based approach to determine a locally (single user) optimal transmission strategy. Alternatively, ρ could be considered as a a decreasing function of SNR as $\rho(SNR)$, which is more reasonable for practical applications because the channel estimation usually becomes more accurate as the SNR increases.

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