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Performance of Relaying Protocols

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In wireless systems, cooperative diversity and relaying can exploit the benefit of spatial diversity and combat heavy pathloss without requiring multiple antennas at the receivers and transmitters. For practical networks, the use of relays is motivated by the need for simple, inexpensive terminals with limited power and a single antenna.

The motivation for this thesis is to study and propose practical relaying protocols that can reduce the power consumption and ameliorate the performance with minimum additional complexity. Based on a dual-hop communication model, we exploit two upper bounds for the end-to-end SNR. These bounds further inspire us to propose new relaying protocols for wireless communication systems. We examine the case of a single user and relay under Rayleigh and Nakagami-m fading conditions. Based on the general upper bound, a new protocol is introduced: Clipped gain. This protocol makes it possible to save the transmit power by stopping the transmission when the quality of the first hop leads to an outage.

We consider also user selection and user scheduling for dual-hop communication with multiple users and relays over a Rayleigh fading channel. We introduce new scheduling protocols based on one-bit feedback information. To the best of our knowledge, most of the available literature uses full channel state information to perform user selection and user scheduling. Interestingly, our protocols based on one bit feedback greatly improve the system performance while adding less additional complexity.

To carry out rigorous comparison, close-form expressions are derived and analytical results used to assess the outage probability performance.

**Keywords:** Cooperative diversity, multiple antennas, relay, dual-hop, relaying protocols, user selection, user scheduling, feedback.
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<tr>
<td>AF</td>
<td>Amplify-and-forward</td>
</tr>
<tr>
<td>AoF</td>
<td>Amount of fading</td>
</tr>
<tr>
<td>AP</td>
<td>Access point</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive white Gaussian noise</td>
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<tr>
<td>BC</td>
<td>Broadcast mode</td>
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<td>BEP</td>
<td>Bit error probability</td>
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<td>BER</td>
<td>Bit error rate</td>
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<td>BS</td>
<td>Base station</td>
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<tr>
<td>CDF</td>
<td>Cumulative distribution function</td>
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<td>CDMA</td>
<td>Code division multiple access</td>
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<td>CG</td>
<td>Clipped gain</td>
</tr>
<tr>
<td>CS</td>
<td>Centralized scheduling</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel state information</td>
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<tr>
<td>CT</td>
<td>Continued transmission</td>
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<tr>
<td>DF</td>
<td>Decode-and-forward</td>
</tr>
<tr>
<td>DL</td>
<td>Downlink</td>
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<tr>
<td>DS</td>
<td>Distributed scheduling</td>
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<td>DSS</td>
<td>Distributed scheduling simplified</td>
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<tr>
<td>EF</td>
<td>Estimate-and-forward</td>
</tr>
<tr>
<td>FDD</td>
<td>Frequency-division duplexing</td>
</tr>
<tr>
<td>FG</td>
<td>Fixed gain</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>IT</td>
<td>Interrupted transmission</td>
</tr>
<tr>
<td>LDGM</td>
<td>Low-density generator matrix</td>
</tr>
<tr>
<td>LDPC</td>
<td>Low-density parity-check</td>
</tr>
<tr>
<td>MA</td>
<td>Multiple-access mode</td>
</tr>
<tr>
<td>MGF</td>
<td>Moment generating function</td>
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<tr>
<td>MIMO</td>
<td>Multiple-input multiple-output</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability density function</td>
</tr>
<tr>
<td>RF</td>
<td>Radio frequency</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to noise ratio</td>
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<tr>
<td>TES</td>
<td>Selection threshold equaled to the outage threshold</td>
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<tr>
<td>TIS</td>
<td>Selection threshold inferior to the outage threshold</td>
</tr>
<tr>
<td>TSS</td>
<td>Selection threshold superior to the outage threshold</td>
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<tr>
<td>VG</td>
<td>Variable gain</td>
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<tr>
<td>UL</td>
<td>Uplink</td>
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<td>Unlimited gain</td>
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Chapter 1

Introduction

In wireless systems, cooperation diversity and relaying techniques have attracted considerable interest, since increasing the data rate and coverage at low cost is a central issue in next-generation communication systems. Relaying is an attractive solution that can exploit the benefit of spatial diversity and combat heavy pathloss without requiring multiple antennas at the receivers and transmitters. To incorporate efficiently the use of relays into wireless systems, practical protocols deploying low-complexity and energy-efficient relays constitute an interesting area of research.

1.1 Motivation of the thesis

For practical networks, the use of relays is motivated by the need for simple, inexpensive terminals with limited power and a single antenna. In this thesis, one aim is investigate the performance of relaying protocols and develop new algorithms for cooperative communication using such relays.

To improve the system performance, the channel state information (CSI) can be exploited. Taking advantage of the CSI in relay system offers numerous opportunities to dramatically improve the performance. Obviously, in a relay system represented by a dual-hop channel model, the knowledge about first-and second-hop CSI may be exploited. The first-hop channel state information is available at the relay. We propose in this thesis one relay gain protocol for a single-user and single relay network model, Clipped Gain, that takes advantage of this information to save transmit power.

To obtain the channel state information at the transmitter, feedback may be employed. Digital feedback consists of sending information bits about the channel condition available at the receiver to the transmitter. Feedback can be used in
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a single-user and single-relay setting to enhance the rate and reliability between
the source and the destination, whereas the use of feedback in a multiuser and
multirelay system helps to achieve beamforming and relay selection to improve the
performance. However, exploiting feedback in a practical wireless relay system is
challenging as there may be several channels to be estimated and the feedback
information may eventually have to traverse multiple paths. Moreover, the number
of feedback bits is limited. In this thesis, the focus is on developing and studying
new relay and user scheduling algorithms that exploit one-bit feedback. Even this
limited information about the channel allows good system performance improvement
while keeping added complexity low. By contrast, user selection and user scheduling
to date have mostly employed full channel state information and thus disregard the
feedback and complexity requirements of getting such CSI.

We develop various cooperative diversity protocols for multiuser and multirelay
network model based on one-bit feedback. Those efficient power-saving protocols set
transmission thresholds on the first- and second hop SNR to avoid the cases when
the SNRs of the links are too poor. Yielding already large performance improvement
with low added complexity to the relay, those protocols are well-suited for practical
networks such as cellular relay network.

1.2 Scope of the thesis

The goal of this thesis is to study and propose new practical relaying protocols that
can reduce the power consumption and ameliorate the performance with minimum
additional complexity. To carry out more rigorous comparison, analytical results
are used. Namely, we derive close-form expressions to assess the outage probability
performance of the new protocols.

Based on a dual-hop communication model, we exploit two upper bounds for the
end-to-end SNR. We exploit a general upper bound that applies for all amplify-
and-forward protocols and also an upper bound for wireless systems that use a
theoretical unlimited gain. These bounds will further inspire the proposal of new
relaying protocols for wireless communication systems.

Based on the general upper bound, we introduce a new protocol, clipped gain,
for single-user and single-relay network. This protocol makes it possible to save the
transmit power by stopping the transmission when the quality of the first hop leads
to an outage. We examine the outage performance of the CG protocol over Rayleigh
and Nakagami channels and further compare this performance to existing practical
protocols.
CHAPTER 1. INTRODUCTION

We consider also user selection and user scheduling in a multiuser and multirelay network setting over a Rayleigh fading channel. We use a system divided into partitions to introduce new scheduling protocols based on one-bit feedback information. First, we propose scheduling schemes that set a transmission threshold on the first-hop SNR. Thus, we introduce several schemes that select one dual-hop link for transmission among the links having a favorable first-hop SNR. We assess the performance of those schemes in terms of outage probability and draw conclusions on the optimum threshold value.

Then, the study is extended to the case where not only the quality of the first hop but the quality of both links of the dual-hop transmission is taken into account. We introduce new Distributed Scheduling schemes (DS and DSS) and Centralized Scheduling scheme (CS) based on one-bit feedback information on the first- and second link SNR and one-bit feedback information on the end-to-end SNR, respectively.

1.3 Structure of the thesis

This thesis is divided into two main parts. The first part, consisting of Chapters 2 and 3, provides a background review of the relevant literature on the topic of study. The second part consists of Chapters 4, 5, 6, and 7 comprises the contribution of this thesis.

Chapter 2 provides an overview of the fundamental relay communication concept and the wireless environment.

Chapter 3 presents a review of the use of feedback to improve the system performance. This chapter also provides a deeper review of the literature on relay protocols.

Chapter 4 sets the system model used in this thesis and introduces the new relaying protocols.

Chapter 5 provides the performance closed-form expressions and the comparison results between the new CG protocol and the existing relay protocols.

Chapter 6 provides the performance closed-form expressions and the results analysis of the new scheduling protocols for a multiuser and multirelay network.

Chapter 7 summarizes the contributions of the thesis and highlights some suggestions for further research.
Chapter 2

Relaying in wireless communication

This chapter provides a review of the concepts used throughout our study. First, an overview of relaying concepts is given. Then, channel fading models and performance measures used in this thesis are briefly presented. We summarize also the substantial background that contributes to the setup of our study.

By definition, relaying in wireless communication involves a terminal acting as a relay between the source and destination. The relay assists the source in transmitting to the destination.

The classical relay channel [56] is composed by three terminals, one terminal serving as source communicates with a destination terminal via a “relay” that receives, processes and re-transmits the signal of interest. The relay can be in some cases a terminal in the network that does not have any information to receive or to transmit; in more recent research work however, the relay is considered to be a transmitting and/or receiving terminal that cooperates by serving as relay for one another.

2.1 Three terminal relay channel

The transmission of information over a communication channel between three terminals was originally introduced by van der Meulen [56]. Basically, in the case of a three terminals relay channel, the source node communicates with the destination through two paths as depicted in Figure 2.1: the direct path and relay transmission path. The relay transmission path is composed of the backward channel and the forward channel where the backward channel is the channel between the transmitter and the relay and the forward channel is the channel between the relay and the
CHAPTER 2. RELAYING IN WIRELESS COMMUNICATION

receiver.

![Three terminals relay channel diagram](image)

Figure 2.1: Three terminals relay channel

To take into account certain implementation constraints, the terminals are considered to employ half duplex transmission [33, 32], i.e. terminals do not receive and transmit at the same time. Current limitation in radio implementation is due to insufficient isolation between transmit and receive circuitry and severe attenuation over wireless transmission resulting in a drowning of the signal at the receiver input (the transmit signal can be 150 dB above the received signal in the case of a single antenna terminal). Thus, we assume half duplex transmission. To ensure half duplex operation, transmitted and received signals can be separated in time or in frequency, or orthogonal signals can be used. In the case of time division duplexing for instance, each channel is divided into orthogonal subchannels (time slots) and communication takes place over two time slots, thereby different time slots are allocated for transmission and reception.

2.2 Relaying methods

This section outlines different strategies employed by relaying terminals, including fixed relaying methods such as amplify-and-forward, decode-and-forward, and estimate-and-forward relaying and adaptive relaying methods that adapt based on channel quality measurements between terminals. The principle of the coded cooperative relaying method is also briefly reviewed.

2.2.1 Fixed relaying methods

For those methods, the channel quality measurement such as the SNR measurement of the backward channel is not exploited by the relay and the latter receives and
processes the signal in an invariable fashion. The difference between those relaying methods is in the way the relay processes the signal before forwarding.

- **Amplify-and-forward**

  For *amplify-and-forward* relaying methods [44], the relay simply amplifies its received signal by a factor $\beta$ before forwarding the signal to the destination terminal.

  As the signal received at the relay is degraded due to fading of the backward coefficient and the noise at the relay input, the signal forwarded by the relay and consequently the signal received at the destination terminal contains an amplified degradation in addition to the amplified desired signal.

  In the literature, different types of *amplify-and-forward* relays have been studied. Those include Variable Gain (VG) relay [18], Unlimited Gain (UG) relay, and Fixed Gain (FG) relay. The aforementioned relaying methods are characterized with a different amplifying gain $\beta$. The VG and UG relaying methods assume a perfect knowledge at the relay of the backward fading coefficients as the amplifying gain use this information to invert the effect of the backward channel.

  Note that the fading coefficient of the channel will not be referred to in the following as the channel state information (CSI); the CSI will only denote the channel quality measurements such as the SNR of the channel links $^1$. Thus, the fixed relaying methods differ from the adaptive relaying methods (see Section 2.2.2) as those latter use the channel quality measurements or CSI to select a suitable cooperative or noncooperative mode.

- **Decode-and-forward**

  While using the *decode-and-forward* methods [33], the relay fully decodes the message from the source before forwarding the information to the destination. One can notice that in this case there is no amplification of backward degradation but possible decoding error.

- **Estimate-and-forward**

  Those relaying methods are used only when the information from the direct link is available. The relay does not fully decode the received signal from the source. It encodes and quantizes a version of the received signal and forwards

---

$^1$This information about the channel quality has also been referred to as the channel quality information (CQI)
this extracted information to the destination. The forwarded information may contain some estimation error. Therefore, this information is used as side information by the destination while decoding the direct link information.

Note that the demodulate-and-forward relaying method is a form of estimate-and-forward method in which the relay decodes only a fraction of its received signal from the source.

Obviously, the decode-and-forward and estimate-and-forward methods perform better in many cases. However the amplify-and-forward methods put less burden on the relay and are practical when the complexity is an important issue. Furthermore, the performances of the decode-and-forward and estimate-and-forward methods are limited by the quality of the backward channel [33] and in some cases, for instance in an interference relay network [47], the relays cannot decode the input signal reliably.

2.2.2 Adaptive relaying methods

If the information on the backward channel quality is available, the relaying methods can be adaptive meaning that the relay can adjusts its behavior according to the conditions of the source-relay channel. If the backward channel is poor the relay does not gives any benefit for the cases where the signal is forwarded using fixed relaying methods; since the channel quality information or CSI is available to the relay, it can adapt its relaying method. The following relaying methods have been proposed in [33].

• Selection relaying Since the fading coefficients are known to the appropriate receiver, the SNR of the backward channel can be measured to high accuracy by the relay terminal; thus, the relay can adapt its transmission according to the realized channel states. If the measured SNR falls below a certain threshold, the source shall use repetition or more powerful codes to continue to transmit to the destination. If the measured SNR is above the threshold, then the relay forward the received signal from the source using a fixed relaying method i.e. either amplify-and-forward or decode-and-forward method.

• Incremental relaying Incremental relaying protocols use limited feedback from the destination terminal (for instance a single bit indicating the success or failure of the direct link transmission). Based on this information, the relay can either forwards its received signal from the source or stays idle. The relay transmits only if the destination has sent a feedback indicating failure of the
direct link; thus incremental relaying can improve spectral efficiency over fixed and selection relaying by repeating only when it is necessary.

Note that selection and incremental relaying method combined have paved the way to several other adaptive relaying methods that will be briefly reviewed in the following chapter. Those methods combined with power control constraint can lead to important power saving.

2.2.3 Coded relaying methods

The original scheme involving the coded relaying method has been proposed by Hunter and Nosratinia. The coded cooperation method integrates cooperation into channel coding and requires that both the source and the relay transmit a part of the codeword [25]. In this early work, the relay has to decode the transmitted information from the source and then forwards a part of the codeword if the decoding is successful. Hunter and Nosratinia proposed that the decoding attempt’s success can be determined by checking the cyclic redundancy check codes so that no feedback is required between the terminals. This method can maintain the system performance and rate at the cost of added complexity at the receiver.

In more recent work on coded cooperation, the schemes are based on the use of decode-and-forward relays as well as demodulate-and-forward relays (a form of estimate-and-forward relaying method) [9, 10] and dirty paper coding cooperation between transmitters [27, 38]. In this section, we are going to describe these three coded cooperation. However, it should be noted that the methods described in the following have been selected for illustrative purpose and are not the only works available on coded cooperation.

- Using a Decode-and-forward relay

The first work on coded cooperation method using the decode-and-forward relay was proposed by Hunter and Nosratinia [25, 26]. Each user data is encoded into two segments N1 and N2. In their data transmission period, each cooperative user is transmitting first their own codeword segment N1 and the segment N2 of their partner if the decoding is successful. The success of the decoding attempt is checked by CRC code. If the user failed to decode his/her partner transmission, he/she then transmits his/her own remaining codeword segment N2. Thus, the coded operation enable cooperation in favorable cases only with no feedback between users [39] but obviously at the cost of added complexity at the receiver.
Using a Demodulate-and-forward relay

In [15, 9, 10], the authors propose cooperation schemes that use coded cooperation based on decode-and-forward relay and repeat-accumulate codes.

This demodulate-and-forward cooperative scheme was first introduced in [15]. In this scheme, the demodulate-and-forward relay decodes only a fraction of the received signal. Note that the demodulate-and-forward relaying method is a form of the estimate-and-forward relaying method. Further, those bits provided by this partial demodulation form the information bits which will be used to generate the new codeword to be transmitted to the destination by the relay. The codeword results from concatenating the information bits and the punctured parity bits formed by the repeat-accumulate code. The repeat-accumulate code is a class of turbo like codes which can be described as follow: The information bits are first repeated $q$ times, then randomly interleaved before being fed into a truncated rate-1 recursive convolutional encoder with transfer function $\frac{1}{1+D}$ [15]. The encoder output then forms the parity bits. The rate of the repeat-accumulate code can be changed by puncturing the parity bits. Hence, the relay can adjust the code rate to allow a constant transmission power.

The demodulate-and-forward cooperative scheme based on repeat-accumulate code was first design for sensor networks.

Using an Estimate-and-forward relay

In [8], the authors focus on code design for both the decode-and-forward and estimate-and-forward relays. We summarize here as an illustrative example the use of low-density parity-check (LDPC) code as a component of the estimate-and-forward relaying method only. The channel state information (CSI) of all channels involved is assumed to be perfectly known at transmitters and receivers.

In this relaying method, the relay forwards a quantized estimate of its received signal. The communication between the source terminal and the destination terminal is done in half duplex mode: first, during the broadcast mode (BC), the source sends its information to the relay and the destination, the relay stays idle. Then, during the multiple-access mode (MA), both the source and the relay transmit to the destination (the relay sends its estimated information from the message it has received in BC mode and the source sends new information).
The information to be sent by the source is separated into two parts \((w,v)\). In BC mode, the source encodes the first part \(w\) using LDPC code to generate a codeword \(X_{BC}\). Corrupted versions of \(X_{BC}\) are received at the relay and the destination as respectively \(V_{BC}\) and \(Y_{BC}\). Neither the relay nor the destination tries to decode the received signal. The relay generates the estimate \(\hat{V}_{BC}\) from the received signal \(V_{BC}\). On the other hand, the destination cannot decode its received signal \(Y_{BC}\) yet and therefore stores \(Y_{BC}\) for further decoding. Then, in MA mode, the relay encodes the estimate signal \(\hat{V}_{BC}\) using the LDPC code to yield a codeword \(W_{MA}\) and transmits this new codeword over the relay-destination channel. In MA mode, the source also transmits. It sends a codeword \(X_{MA}\) containing the second part of the information, \(v\). The source transmission utilizes the remaining capacity of the multiple access channel formed by the source and the relay acting as two transmitters and the destination acting as receiver. At the end of the MA mode, the destination first decodes the codeword \(W_{MA}\) and \(X_{MA}\) and then decodes the codeword \(Y_{BC}\) using the information carried by \(W_{MA}\) as side information.

- Using Dirty paper coding

N. Jindal and A. Goldsmith, and then Ng and A. Goldsmith examine a cooperation scheme using two transmitters and dirty paper coding [27, 38]. The studied communication system is composed of two transmitters and two receivers. In this strategy, the two transmitters first exchange their intended messages to the destinations and then jointly encode both messages using dirty paper coding. Therein, the transmitters form a two-antenna broadcast channel.

In these papers, the authors assume that there are small distances between source and destination nodes and the results indicate that cooperation is beneficial only if there is a good inter-transmitter channel.

Some cooperative schemes that use coding and the decode-and-forward relay include distributed turbo code [60], distributed convolutional coding [54], and dynamic decode-and-forward [5]. Even though these schemes provide excellent performance, like many of the scheme available, the relay only assists the communication between the source and the destination node if it had successfully decoded its received message from the source.

The coded cooperative scheme using the demodulate-and-forward relaying protocol has also been developed for parity check code [11] and low-density generator
matrix (LDGM) code [17]. Meanwhile, both the decoding and the estimation of the source information bits at the relay increase the hardware complexity at the relay.

2.3 Wireless channel

In this section, we describe the channel impairments that affect the wireless transmissions and provide the fairly general mathematical model that is used in the sequel.

2.3.1 Multipath propagation: large scale and small scale fading

In wireless mobile communication systems, a signal travels from the transmitter to the receiver over multiple paths. Multipath arises because the propagated signal is reflected, diffracted, and scattered by the objects presented in the channel environment.

- Reflection occurs when a propagating signal falls on a surface with a dimension much larger than the signal wavelength $\lambda$.
- Diffraction occurs when the electromagnetic waves encounter an impenetrable obstacle. Secondary waves are then formed and diffracted field can even reach a shadowed receiver.
- Scattering occurs when the objects in the channel environment causes the reflected energy to spread out in all directions.

If the radio transmission propagated in an ideal free space, perfectly uniform and nonabsorbing, the attenuation of RF energy between the transmitter and the receiver will behave according to an inverse square law: the received power expressed in terms of transmitted power will be attenuated only by the pathloss factor. The large scale fading is due to prominent terrain contours that shadow the receiver. It represents the average signal power attenuation or pathloss resulting from radio propagation over a large area [53]. The small scale fading superimpose on the large scale fading. The small scale fading or multipath fading is due to the presence of many objects in the environment that induce a fluctuation in the receiver signal’s amplitude, phase, and angle of arrival. Thus, the receiver observes multiple path and time delayed versions of the transmitted signal and the received signal can be characterized by the large- and small scale fading. Furthermore, the receive signal is corrupted by additive noise and interference at the radio receiver input.
The receive signal \( r(t) \) is generally written in terms of a convolution between the transmitted signal \( x(t) \) and the impulse response of the channel \( h(t) \):

\[
r(t) = x * h(t) + n(t)
\]

(2.1)

Where \( n(t) \) is the noise contribution at the receiver input.

A mobile radio signal can be written in terms of two components \( e(t) \) and \( h_0(t) \) [36]:

\[
r(t) = e(t) \cdot h_0(t) \cdot x(t) + n(t)
\]

(2.2)

Where \( e(t) \) is the large scale fading log-normally distributed and \( h_0 \) is the small scale fading.

With the relative motion of the transmitters, receivers and scattering objects, the multipath fading manifests itself in a time-spreading and time-variant phenomenon. For signal dispersion, the fading degradation can be categorized as frequency-flat or frequency selective. Similarly, the time-variant degradation can be categorized as fast- or slow fading. A nice analysis of those fading degradation has been written by B. Sklar [53]. In this thesis, however, these considerations are omitted and the relaying protocols will be examined only for flat fading and time invariant channel in the sequel. Our protocols can be extended to the case where the channel exhibits a frequency selective and time variant behavior but the improvement measured will be less conclusive as other forms of diversity are available; the examination of such cases is beyond the scope of this thesis.

2.3.2 Statistical models of fading channel

This section presents the statistical model used to characterize the channel effects.

Rayleigh fading channel

The small scale fading can be modeled with the Rayleigh fading model given that the envelope of the receive signal is distributed according to a Rayleigh probability density function (PDF) (2.3). This fading model applies to a fairly common situation in mobile radio environment when all multiple reflective waves are received from the surrounding and there is no line-of-sight component. The Rayleigh PDF is expressed as:
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\[ P(a \mid \sigma) = \begin{cases} \frac{a}{\sigma^2} e^{-\frac{a^2}{2\sigma^2}} & \text{for } a \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.3) \]

Where \( a \) is the envelope of the received signal and \( 2\sigma^2 \) is the mean power of the multipath signal.

The power of the received signal and therefore the SNR are exponentially distributed. The exponential PDF can be expressed as follow:

\[ P(\gamma) = \frac{1}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}} \quad (2.4) \]

Where \( \overline{\gamma} \) is the average of \( \gamma \) over all the channel realizations.

Rayleigh fading is a reasonable statistical model for signal propagation over a highly built-up environment. When there are many objects on the path that scatter the radio signal and no line-of-sight propagation, the central limit theorem stipulate that the channel impulse response can be modeled by a Gaussian process and the Rayleigh distribution is therefore an appropriate assumption for the envelope of the response of the channel.

Rice fading channel

When the wireless channel path is composed of two components: the strong line of sight component and an additive Rayleigh component, the received signal envelope can be statistically described by the Rice distribution:

\[ p(a \mid \nu, \sigma) = \frac{a}{\sigma^2} \exp \left( -\frac{(a^2 + \nu^2)}{2\sigma^2} \right) I_0 \left( \frac{a\nu}{\sigma^2} \right) \quad (2.5) \]

Where \( a \) is the envelope of the received signal, \( 2\sigma^2 \) is the mean power of the multipath signal, and \( I_0(z) \) is the modified Bessel function of the first kind with order zero. The distribution was derived by Rice. When \( \nu = 0 \), the distribution reduces to a Rayleigh distribution. The Rice factor \( K = \frac{\nu^2}{\sigma^2} \) is the relation between line-of-sight (non-fading) and fading components [58].

Nakagami fading channel

Nakagami fading is a more general statistical model. Based on the empirical results for short ionospheric propagation and on the Rayleigh and Rician model for a small scale fading environment, the \( m \)-Nakagami model describes large scale experiments
on rapid fading in high frequency long distance propagation. The envelope follows the Nakagami pdf given by:

\[ p(a | m, \sigma) = \frac{2}{\Gamma(m)} \left( \frac{m}{2\sigma^2} \right)^m a^{2m-1} e^{-\frac{ma^2}{2\sigma^2}} \]  

(2.6)

Where is the envelope of the received signal under Nakagami fading, \( \Gamma(\cdot) \) is the gamma function \([21, eq. 8.310.1]\), \( 2\sigma^2 \) is the mean power of the signal, and \( m \) is the factor describing the severity of fading; \( m \geq \frac{1}{2} \).

If the envelope of the radio signal is Nakagami distributed, then its corresponding power is Gamma distributed.

\[ P(\gamma) = \frac{m^m}{\gamma^m \Gamma(m)} \gamma^{m-1} e^{-\frac{m\gamma}{\gamma}} \]  

(2.7)

Where \( \Gamma(\cdot) \) is the gamma function, \( \gamma \) is the average of \( \gamma \), and \( m \) is the factor describing the severity of fading which verifies \( m \geq \frac{1}{2} \). The Nakagami distribution reduces to Rayleigh distribution for the special case where the factor \( m \) is equal to one.

2.4 Fading channel performance

In principle, there are several methods to evaluate the performance limit over fading channel that can be used. Most generally, these measures will debrief the tradeoffs between achievable channel rate and the distortion on the signals.

2.4.1 Ergodic fading process

In the cases where the fading is an ergodic process, the performance can be examined in term of the well-known Shannon capacity or capacity region. The capacity region represents the largest set of transmission rates that can be reliably communicated over the channel, in the sense of asymptotically negligible error probability with long codewords and unconstrained decoding complexity. The Shannon capacity for the simplest cases of the relay channel with additive white Gaussian noise has been given in [13]. Cover and El Gamar [12] have focused on certain non-faded relay channel, developing the channel capacity for degraded channel and reversely degraded channel, and the lower bound for general relay channel. The degraded channel is characterized by a degraded signal at the destination when compared to the signal received at the relay; on the other hand, the reversely degraded channel is defined such that the signal at the relay is worse than the signal at the destination.
The more general case including various relay channel models of the Gaussian multiple access channel with cooperative diversity has been studied by Laneman [32]. He has developed an outer bound on the capacity region for the Gaussian multiple access channel with cooperative diversity and then compared this obtained outer bound with the sets of achievable rate for non-cooperative transmission and decode-and-forward transmission.

### 2.4.2 Non-Ergodic fading process

When the fading process is non-ergodic, for instance, when fading varies slowly or delay constrains limit the coding interval to a finite number of channel realizations preventing the fading process from revealing its ergodic structure, the Shannon capacity can be arbitrary small or equal to zero and is not a pertinent performance measure [32].

Laneman has evaluated the performance of several cooperative transmission protocols in non-ergodic fading environments in term of outage probability. He has examined the direct transmission, cooperative transmission based on amplify-and-forward\(^2\), and decode-and-forward transmission. Based on equivalence for outage probability for large SNR for those protocols, he has shown that the cooperative diversity protocol gives better result than direct transmission, for instance by using the amplify-and-forward protocol, the outage probability is reduced greatly while compared to the direct transmission. He has also demonstrated that the large SNR performance of adaptive decode-and-forward transmission is identical to that of amplify-and-forward transmission for large SNR due to the limits of decode-and-forward protocol on performance because it requires full decoding at the relay.

Thus far, in the following parts, we limit our study to the case of a non-ergodic fading environment. We will examine and compare the performance of diverse cooperation protocols in terms of outage probability [52] by deriving closed-form expressions and carrying out simulations.

### 2.4.3 System performance measures

In this section, a brief description of the performance measure used in this thesis is given.

\(^2\)Actually, the amplify-and-forward protocol preferred to here is the Variable Gain protocol that will be described in the upcoming chapter
Outage Probabilities for Relay Fading channel

When the fading process is non-ergodic, since the realized channel signal to noise ratio does not support any fixed rate, no reliable transmission can be guaranteed for any fixed and non-zero rate. Meanwhile, for one given fixed rate, the probability that this rate is achievable over the fading channel can be examined.

In a noise limited system, the outage probability is defined as the probability that the instantaneous error probability exceeds a specified value or equivalently, the probability that the SNR falls below the threshold $\gamma_{th}$. The threshold $\gamma_{th}$ is a predetermined protection ratio that may depend on the type of modulation employed and the type of application supported; when the equivalent end-to-end SNR of the system is above this ratio, the quality of service is considered to be satisfactory. The outage probability denoted by $P_{out}$ can be obtained by integrating the probability density function of the SNR from zero to $\gamma_{th}$, i.e. by evaluating the cumulative distribution function of the SNR at $\gamma = \gamma_{th}$ \cite{52}. The outage probability is hence:

$$P_{out} = \int_{0}^{\gamma_{th}} p_{\gamma}(\gamma) d\gamma$$  \hspace{1cm} (2.8)

From the outage probability, one can obtain the PDF by deriving the outage probability with respect to SNR $\gamma$. One expression of the moment generating function can be derived from both outage probability and PDF expression.

Moment generating function

By definition, the moment generating function of a random variable $\gamma$ is the Laplace transform of its PDF, namely:

$$M_\gamma(s) = \int_{0}^{\infty} p_{\gamma}(\gamma)e^{-s\gamma} d\gamma$$  \hspace{1cm} (2.9)

In our case of study, the random variable $\gamma$ represents the SNR of the system.

As the PDF is the derivative of the outage probability with respect to SNR and $P_{out}(0) = 0$, the Laplace transform of the outage probability can be expressed in terms of the Laplace transform of the PDF as follows:

$$L[P_{\gamma}(s)] = \frac{L[p_{\gamma}(s)]}{s}$$  \hspace{1cm} (2.10)

Therefore, the MGF can be derived from the PDF expression as well as the outage probability expression.

The expression of the MGF may be used for the system performance evaluation of
digital modulation over fading channel using the MGF-based approach (see Section 2.4.3 Average Bit Error Probability).

**Average signal-to-Noise ratio**

The SNR measured at the output of the receiver can usually serve as an indicator of the overall fidelity of the system. In order to avoid fading impairment, the average SNR can be taken for the performance measurement purposes. The average is taken over the probability distribution of the fading.

\[ \bar{\gamma} = \int_0^\infty \gamma p_\gamma(\gamma) \, d\gamma \]  \hspace{1cm} (2.11)

The average SNR can also be yield by taking the first derivative of MGF expression (2.9) with respect to \( s \) and evaluating the result at \( s = 0 \):

\[ \bar{\gamma} = \frac{dM_\gamma(s)}{ds} \bigg|_{s=0} \]  \hspace{1cm} (2.12)

**Average bit error probability**

Average bit error probability is one of the most revealing performance criterions about the system behavior. The average Bit error rate for a wide variety of M-ary modulations obtained by the MGF-based approach for the performance evaluation of digital modulation over fading channels can be found in [52]. For our interest, we will also use some well-know result on the MGF-based approach to evaluate the BEP for some modulations and to validate the closed-form expression of the MGF.

### 2.5 Network architecture

The network architecture can be classified into two broad classes called infrastructure and ad-hoc networks as in wireless local area network terminology. Those classes of network are defined in the following sections.

#### 2.5.1 Infrastructure network

In an infrastructure network, the mobile terminals are connected to an access point which is, in turn, connected to the backbone network. The terminals do not communicate directly with each other; instead, they communicate though their assigned access point. Another distinctive feature of the infrastructure network is the power and the processing asymmetry between access point and other radio terminals: the
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Base station is high-powered and usually stationary and the radio terminals are low powered and mobile. An example of such network is the current cellular network where the access point is the base station.

2.5.2 Ad-hoc network

In the ad-hoc network, the radio terminals have more symmetric power and the terminals do not need to communicate via a centralized access point; this is the main difference between an ad-hoc network and an infrastructure network. The terminals communicate mainly with the nearby terminals in multihop transmission. The advantages of the ad-hoc networks are the fast and ad-hoc deployment and their robustness to the loss of terminals. Another advantage is due to the multihop transmission that can combat path loss and limit interference in the network.

2.5.3 Integrating cooperation in the network architecture

The multihop transmission is an effective method to combat pathloss and fading. This notion is traditionally studied in ad-hoc networks. However, since the introduction of relaying, fixed infrastructure can also take advantage of multihop communication. Relaying is foreseen as a solution to reduce infrastructure cost. The ambitious throughput and coverage requirements of future communication systems demandes fundamental infrastructure enhancement [43]. In a cellular network for instance, the brute force solution consisting of increasing the density of base stations in cellular networks is costly; therefore, relaying is a promising architecture upgrade. If the density of relays in the network is moderately high, a user terminal has good chances to be closer to a relay than to the base station. Therefore, the propagation loss is larger from the BS to the user than from the relay to the user. As a result, relaying can potentially solve the coverage problem for a high data rate. Infrastructure network can integrate fixed relays or mobile relays.

The fixed relays can be used in a cellular network for instance. Those relays are radio terminals that differ from the BS by being less powered and have a smaller coverage [39]. Additionally, relays are not connected to the backhaul. Instead, the data received wirelessly from the BS is stored and then forwarded to the user terminal, and reversely.

The cellular networks and more generally wireless networks can also use mobile relays. One way to incorporate mobile relaying in the wireless network is to allow single-antenna mobile terminal in the multiuser environment to share their antennas and generate a virtual multiple-antenna transmitter. This method is called
cooperative communication. In this scheme, the broadcast nature of the wireless medium and its ability to achieve diversity through independent channels are exploited. Namely, in the cooperative diversity schemes, the transmitted signal is received by multiple cooperating terminals and usefully forwarded.

This form of relaying that can be applied for both ad-hoc and infrastructure-based network. In a cooperative communication system, each user is assumed to transmit data for other users (cooperation) as well as his/her own data. Figure 2.2 illustrates the cooperative communication scheme. Each terminal has only one antenna and cannot individually achieve diversity. However, cooperative terminals can receive and process the information for each another. Then, the cooperative terminal forwards some version of this received information along with its own data to the destination. Further, the destination terminal combines the information received from both the direct-when available- and relay paths [33, 50, 32, 39]. By performing this combining, the redundant information hereby obtained, allows to achieve diversity, coding gain, or SNR gain. Alternatively, with multiple antennas formed by the source terminal and the cooperative terminals, it is also possible to transmit several parallel data streams thereby obtaining multiplexing gain, i.e. an increase in rate [63, 8]. The following chapter gives a more detailed literature review on the topic.

Figure 2.2: Cooperative communication: two terminals communicating with the same destination

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3 Cooperative relaying can involve fixed relay node as well. However, in this dissertation, cooperative relaying will refer wireless communication via mobile relay.  
4 The icons used in the figure resemble base stations or handsets uniquely for graphical representation convenience; the idea of cooperative communication is however general and can be also applied for ad-hoc networks and wireless sensor networks.
Chapter 3

Background and literature

This chapter provides a literature survey of the existing works on relaying protocols that involve power control, feedback, and relay or user scheduling in cooperative communication.

As mentioned in the preceding chapter, the basic idea of cooperative communication is that the single-antenna terminals can share their antennas to obtain a benefit similar to a multiple-antenna system. Mobile wireless channels suffer from the effects of fading (see Section 2.3); and creating diversity by transmitting independent copies of the signal can mitigate those deteriorations. Thus, one way to obtain different independent copies of the signal that each single-antenna terminal alone cannot generate is to allow terminals to cooperate with each other. Cooperating terminals receive, process, and forward the signal from different locations. As a result, multiple faded versions of the signal is then available at the receiver.

The idea behind cooperative communication has first been introduced by Cover and El Gamal. Their work treated the case of certain non-faded relay channel with additive white Gaussian noise. They developed the lower bound on capacity for three different random coding schemes: facilitation, cooperation and observation \(^1\). The facilitation scheme implies that the relay helps the source by inducing as little interference as possible. However, the other schemes cooperation and observation are more involved. In the cooperation scheme, the relay fully decodes the source message and retransmits some information about that signal to the destination, and the destination suitably combines the received signals to achieve a higher rate than the direct transmission. In the observation scheme, the relay encodes a quantized version of its received signal and the destination combines information about the

\(^1\)The names facilitation and cooperation were introduced by the authors. However, the name of the last scheme, observation, has been given by Laneman in his PhD dissertation
relay’s received signal with its own to form a better estimation of the source message.

Obviously, those schemes have inspired the future works on cooperative systems. However, recent research works on cooperative communication have taken a different emphasis: relaying schemes that exploit diversity and enhance rate and reliability are intensively studied and the relays now may act as a source as well as a cooperative terminal.

In the following, first, the use of feedback in wireless communication is briefly reviewed. We focus in particular on the case of cooperative communication. Indeed, the use of some knowledge on the channel condition (channel state information CSI) provided by the feedback links may allow relay systems to take advantage of power control or scheduling to maximize the benefit of cooperation. Then, we summarize the existing research results on single-user and single-relay system and on multiuser and multirelay system. Those results include investigation on the fundamental system performance, power control, and relay and user scheduling.

3.1 Feedback in wireless communication

The channel quality knowledge at the transmitter can bring many benefits and yield large performance improvement. Feedback is a solution to allow the transmitter to obtain the information about the channel condition. Digital feedback, or more commonly referred to as limited feedback [37], consists of sending information bits about the channel conditions available at the receiver (e.g., through training) to the transmitter.

The use of feedback in communication systems has been first mentioned by Shannon in [51]. Since then, the application of feedback in wireless communication systems has been intensely studied for various communication models combining one or several of the following characteristics: single-user, multiuser, single-antenna and multiple-antenna.

In those systems, the feedback bits convey some form of knowledge of the wireless channel condition (channel state information CSI). Numerous research works have shown that few feedback bits about the channel condition can already allow near optimal channel adaptation ([37] and references therein).

3.1.1 Single-user systems

In single-user systems, only one source and one destination is considered. This is the basic case of study in wireless network and it has been widely investigated. In
both single- and multiple-antenna systems, feedback information provided by the receiver to the transmitter can bring significant benefit.

- **Single-antenna system**
  In single-antenna systems, the receiver can obtain information about the wireless channel through techniques such as training. This information is then fed into a quantizer to get a number of feedback bits to be sent to the transmitter in the overhead. This feedback information is exploited to obtain performance enhancement associated with rate adaptation and adaptive coded modulation.

- **Multiple-antenna system**
  In the multiple-antenna systems such as MIMO system, limited feedback can offer beamforming and interference mitigation capabilities.

- **Relay system**
  Feedback can be used in a relay system to enhance the rate and reliability between the source and the destination terminal. Generally, feedback information is provided for destination-relay and relay-source links. However, feedback information on the direct link’s condition (destination-source feedback) is unusual as the channel has often poor fading condition (see Figure: 3.1).

![Figure 3.1: Block diagram of a relay system with feedback links](image)

Exploiting CSI knowledge in a single-user and single-relay system may offer diversity [23] and power control [4, 3].

In the sequel, full CSI knowledge and limited CSI knowledge to achieve diversity and/or power control for single-user and single-relay system will be reviewed further.
3.1.2 Multiuser systems

Feedback in a multiuser system allows adaptation of the transmitted signal across multiple users. This additional degree of freedom comes at the cost of an increasing amount of information to be treated proportional to the number of users, i.e. the number of channels in the system.

- Single-antenna system

In multiuser systems, by sharing the spectrum or scheduling the transmission, a larger rate or better reliability can be obtained. In a cellular framework with multiuser and single-antenna terminals for instance, the BS needs some knowledge about the channels’ SNR condition to perform user scheduling.

In [30], the BS uses perfect CSI information to schedule the transmission to the users with the largest received SNR. Therein, multiuser diversity gain is achieved and the throughput is maximized.

The perfect CSI information assumption about all the wireless channels required an infinite and perfect feedback from all the users. The practical case of limited feedback has also been investigated and the information from the users about the channels’ SNR quality can also be limited to a subset of users, e.g. users having the SNR exceeding a predetermined threshold [19, 20]. In [19, 20], the authors propose an user selection scheme in which the users decide locally based on their received SNR either or not whether he/she should report his/her link SNR to the scheduler (i.e. if his/her SNR exceeds the threshold). The scheduler picks for transmission the user with the highest SNR among the users who have sent the feedback to the BS (or access point). In the case of scheduling outage where no SNR has been reported, two solutions have been proposed: the scheduler can randomly choose one user or assume that the previous best user remain optimal. These method are quite bandwidth optimal [20].

Limited feedback in a multiuser single-antenna system may also be used to vary the rate and power to minimize outage [45] or to allow users to compete for access to the spectrum [2].

- Multiple-antenna system

In a multiuser multiple-antennas system, limited feedback finds its use in suppressing the inter-user interference and scheduling. One nice overview of the limited feedback benefit for multiuser multiple-antennas systems can be found in [37].
In cellular systems that commonly use frequency-division duplexing (FDD), CSI is critical to enable the base station (BS) to precode the signal to allow the suppression of the inter-user interference in downlink (DL).

And like for a single-antenna system, in the case of a multiuser with multiple-antennas, limited feedback can be used for user selection. Indeed, if the number of users that actively communicate with the BS (or AP) exceeds the number of receiving antennas at the latter, a subset of users that will be allowed to transmit or receive the communication should be set. By designing selection rules, multiple-antenna systems can extract an important capacity gain. The selection design rules are a tradeoff between selecting the users with high SNR to obtain diversity gain and selecting users with an orthogonal channel to maximize the multiplexing gain.

- Relay system

Exploiting feedback in a practical wireless relay network is challenging as there may be several channels to be estimated and the feedback information may eventually have to traverse multiple paths.

Feedback information in a multiuser relay system can help to achieve beamforming [59, 62] and relay selection to improve the performance. Optimal relay beamforming with unlimited feedback has been studied in [59]. Although the requirements for an unlimited amount of feedback synchronization among relay are impractical, the results provide a performance upper bound for practical beamforming schemes with limited feedback. Meanwhile, in [62], the authors investigate the case of beamforming with limited feedback for a multiuser relay system and show that this scheme needs significantly more feedback than relay selection schemes. Moreover, beamforming schemes require synchronization among all relay terminals. Therefore, the authors have concluded that selection relaying is more attractive.

In the following, we will concentrate on reviewing more in detail the cases of single-user and single-relay, and multiuser and multirelay systems. For the multiuser system, we will consider only the use of feedback for user and relay scheduling and selection. In [62], additional information and comparison between beamforming with limited feedback and user selection for multiuser relay system can be found.
3.2 Single-user and single-relay systems

Single-user and single-relay systems have been intensively studied. The protocols proposed for those systems include simple protocols with fixed relaying methods as well as more involved protocols with adaptive relaying methods. We summarize here a few selected works that gives some fundamental results on the performance of single-user and single-relay protocols.

First, we recall some results and conclusions of previous studies on protocols using fixed relaying methods, more specifically protocols with the *amplify-and-forward* relaying method. Then, we describe two protocols that exploit CSI knowledge to offer diversity and power control.

### 3.2.1 Fixed relaying method

Performance of single-user and single-relay systems using fixed relaying methods such as FG, UG, and VG have been studied in terms of outage probability, average BER, and AoF over Rayleigh and Nakagami fading.

In [41, 40, 18], the authors have studied the outage probability performance of two-hop systems employing FG, UG, and VG relay respectively, over a Rayleigh fading channel; and in [42, 55], they studied those relay protocols over a Nakagami fading channel.

The performance of the systems employing VG relays over a Rayleigh fading channel was first studied in [34], then in [18] and [24]. In [18], the outage probability expression for a two-hop communication through one VG relay has been derived. The authors further use this expression to yield the average outage probability of a scheme where M terminals relay the communication between the source and the destination. The multiuser scheme considered use the VG protocol and the communication propagate through two-hop links over Rayleigh fading channel. In [24], the author give the BER performance result for two-hop systems using VG relay over a Rayleigh fading channel.

In addition, [40, 42] present the UG protocol as a benchmark protocol that gives tight bounds on the performances of the VG protocol for Rayleigh and Nakagami channels, respectively. In both works, the performance of a two-hop communication system with UG relay is compared to that of a regenerative system. UG relay is a hypothetical relay able to invert the channel regardless of its amplitude; this relay protocol is not applicable in practice because when the fading coefficient is too small, the relay gain is supposed to become infinitely large to compensate the fading effect. Meanwhile, the performance of the UG protocol can be used as a benchmark for the
practical VG protocol [40, 42]. In [40], the authors derive and apply the closed-form expressions of the PDF, MGF, and CDF of the harmonic mean of two exponential independent variables to the case of UG relay communication. The expression of the MGF and the outage probability thereby obtained are then used to carry out the performance comparison with a regenerative relay system. Numerical results show that the regenerative relay system outperforms the UG relay system at low average SNR; however, the two systems have similar performance at high average SNR. In [42], the study leads to the same observation while comparing the outage probability and BER performance measures of UG relay system to those of the DF relay system over Nakagami channel. In addition, the author also show in [42] that the use of relay decrease the amount of fading.

The performance of VG relay systems is also compared to that of FG relay systems in [41, 55]. In [41], the authors first study a general fixed gain relay and derive the closed-form expressions of outage probability, PDF, and MGF for systems with this general relay protocol. Then, they propose a specific fixed gain protocol, semi-blind relay, and apply the closed-form formulas to the case of this relay gain. The semi-blind relay gain requires the statistical knowledge of the first hop average fading power; its expression is obtained by taking the expectation of the VG relay gain expression over the first-hop distribution of fading. Thus, the definition of the semi-blind relay gain has been set without normalizing the transmit power at the relay output. As a consequence, the use of this relay protocol in cooperative communication context (good first-hop channel condition) may result in amplification saturation. Moreover, the performance comparison with VG protocol may not be fair as the output power of the semi-blind relay can vary in time. In [41], the outage performance of a two-hop system with semi-blind FG relay is compared to that of a system with VG relay, the exact performance of which was given in [18]. The performance comparison with a system using VG relay in terms of average BER is also illustrated for a DPSK constellation. The BER performance results used for VG relay were derived in [24].

In [55], the authors study the systems employing the VG and FG protocols over a Nakagami fading channel. The FG protocol is this paper is a general FG protocol depending on a positive constant C. No specific value for C has been suggested; therein, the relay transmit power is not normalized and has not been taken into account for this performance comparison. The performance of VG and FG protocols are compared in terms of outage probability, average BER, and AoF over a Nakagami fading channel. The given numerical results show that for low SNR region, FG relay systems slightly outperform the VG relay systems and for medium-to-large
SNR region, VG relay systems outperform the FG relay systems. However, the authors argue that FG relays may serve as an efficient solution for practical systems regarding their lower complexity nature.

3.2.2 Power control and adaptive relaying methods

CSI when available can be exploited to offer diversity and power saving. Following single-user and single-relay protocols propose adaptive relaying schemes in [4, 3, 23] using limited feedback to take advantage of the measured CSI.

- Optimal power control with perfect CSI

In [3, 4], the author proposes an optimal power control scheme at the source and the relay based on perfect CSI knowledge and using VG relay over a Rayleigh fading channel.

In this scheme, the network channel state, defined by the 3-tuple of fading coefficients from the direct, first-, and second-link is assumed to be perfectly measured at the destination. For a given channel state, the minimum sum power (including relay and source transmission power) that removes outage is found. This sum power is then compared to a cut-off region $s$ defined to meet the long term power constraint: if the minimum sum power is greater than $s$ then the transmission is shut off and the system is considered to be in outage. The parameter $s$ is given by numerical computation proposed in [3]. Therein, for every given channel state, the minimum transmit power to avoid outage is used or the transmission is stopped and outage is minimized while satisfying the power constraint.

The optimal power control policy is not an applicable scheme but it can serve as a lower bound on the outage probability of any limited feedback power control scheme.

- Power control with limited feedback

In [4], a power control scheme with limited feedback has also been proposed. For $M$ feedback bits, the space of the possible channel states (including first-, second-, and direct link CSI) is quantized into $L = 2^M$ regions. Each region is associated with a power 3-tuple from a power control codebook. When the measured channel state falls into the region $q$, the index of the associated power duplet is transmitted to both the source and the relay through a noiseless feedback link, and upon its reception, the source power takes the value $P_{sq}$.
and the relay power takes the value $P_{rq}$ corresponding to the selected power duplet.

Unlike in the case of optimal power control with perfect CSI, the source and the relay in this scheme are considered to have individual power constraint. The codebook has been designed to minimize the outage probability while meeting the power constraints. The assigned power duplet $P_q$ corresponding to the region $R_q$ is the minimum required to guarantee zero outage for any possible channel state in the region. The lower boundary of each region $R_q$ is expressed in terms of the corresponding power duplet $P_q$, and any point above this boundary need at the maximum $P_q$ power level to transmit outage-free.

To solve the regions $R_q$ and the associated power level $P_q$, a suboptimal method similar to that in [28] for multiple-antenna systems with limited feedback has been proposed. The power control levels and the region boundaries are found in an iterative fashion in order to make the method amenable to a large number of feedback bits. The method is as follow: first, equal total power is allocated to every region. Then, the power level of the L region is solved along with its lower boundary. Iteratively, the region L-1 and so on can be found until the power level $P_1$ is reached.

The general power control scheme with limited feedback is also applied in [4] to the case of one bit feedback and an approximation at high SNR to the outage probability has been developed.

- Power control with one-bit feedback

In [23], the authors focus on a relay-assisted two-hop system over a Rayleigh fading channel and study the case of both amplify-and-forward relay and decode-and-forward relay. The studied communication system is a modified two-hop wireless communication system that had a reliable feedback channel from the receiver to the transmitter. The binary feedback from the receiver informs the transmitter about the current state of the channel. There is a feedback link between both the destination and the relay and the relay and the source. The feedback bit indicates whether or not the channel SNR is above a predetermined threshold $\tau$ (respectively $\tau_1$ and $\tau_2$ for first- and second link channel). Based on this information, the transmitter then decides to transmit or to cease the transmission. The feedback thereby allows preventing the transmission when the quality of the channel is bad and hence enhances the performance of the system. The proposed scheme uses a relay gain similar
to that of the unlimited gain relay protocol. However, this system will not encounter the problem of saturation as the gain has a floor defined by the transmitting threshold which prevents the fading amplitude of the first link of being too small. The numerical results in [23] shows that the feedback greatly enhances the system performance. The advantage of the feedback scheme with DF relay over the feedback scheme with an AF relay is clearer than when DF and AF schemes without feedback are compared. However, an AF binary feedback scheme can outperform a DF scheme with no feedback. Meanwhile, the results derived in [23] are those of a hypothetically perfect case as the outage probability closed form expression have been derived with the assumption that the first- and second- hop SNRs will never fall below the predetermined protection thresholds. This assumption is actually available only in the case of multiuser and multirelay with infinite amount of source and relay so that there will be always a source-relay and a relay-destination link with corresponding SNR above the thresholds $\tau_1, \tau_2$. Therefore, the results from [23] may be seen as a lower bound for a multiuser and multirelay binary feedback scheme with transmission thresholds.

Obviously, power control in the case of single-user and single-relay communication can offer great gain in terms of diversity and power saving at the cost of a few bits of limited feedback. The schemes summarized above are just a sample of the research results on adaptive relaying methods, other methods may involve coding gain for instance such as the hybrid scheme in [3], in which the adaptive protocol chooses between the DF and EF protocols to obtain a larger achievable rate at each time slot depending on the position of the relay between the source and the destination. Furthermore, in a multiuser and multirelay system, proper scheduling of the transmitter can also yield a great performance benefit as seen in the above relay assisted communication with binary feedback.

### 3.3 Multiuser and multirelay systems

Because a wireless radio network comprises several radio terminals willing to transmit at each time instant and several potential relays, relay and source selection and power allocation motivate various schemes. The selection schemes are an effective alternative to methods using several relays for transmission and coherent addition at the destination [7] for exploiting multiuser diversity [30]. The current methods proposed to achieve the coherent combination of multiple faded version of the trans-
mitted signal such as orthogonal transmission and maximal-ratio-combiner [50, 46], RAKE receiver for CDMA systems [33, 46], pre-coding [50], and distributed space-time codes [35], result in bandwidth expansion problems or require exact channel knowledge and synchronization. Thus, scheduling and selection diversity schemes have recently been considered as a method to achieve full diversity without sacrificing network capacity or power efficiency for multiuser and multirelay network.

User and/or relay scheduling strategies have been proposed for cellular networks [57, 29, 49, 22] as well as ad-hoc and mesh networks [9, 6, 10]. Both mobile relays and fixed relay networks have been studied [29, 7, 35, 6, 10] and [49], and the scheduling schemes involve from the simple AF relay protocols to the demodulate-and-forward and DF relay protocols to exploit cooperation diversity. Power allocation schemes have also been considered for a single source-destination pair with a multiple relays network setting [61]. In [61], the authors study both the case where the all the relays participate in forwarding the communication (all participate power allocation scheme) and the case where only one relay is selected for forwarding (relay selection power allocation scheme).

In the the following, we first summarize the all participate power allocation and the relay selection power control scheme scheme in [61], then we describe scheduling schemes based on three different relaying protocols: demodulate-and-forward relay with coded cooperation relaying method, DF relay, and AF relay. The first scheduling scheme has been studied in a sensor network, the second for a static mesh network, and the last scheme was proposed for a cellular network.

### 3.3.1 Power allocation for single source and multirelay network

In [61], the system under consideration has one source node communicating with one destination node through $m$ relays and the relay protocol employed in this scheme is the VG protocol. The all participate power allocation scheme divides the transmit power among the source and the relays to maximize the channel capacity and thereby minimize the system outage probability. This optimization problem is modeled with both sum and individual power constraints. Since instantaneous throughput is maximized for every channel realization with power constraint on all the transmitting nodes, the minimum outage is achieved.

Meanwhile, this scheme has high requirements in terms of computation and feedback: the scheduler has to compute the optimal power for every relay and that for every channel realization; then, it has to notify all the relays of their assigned transmit power values using the feedback channel. In practice, this all participate
optimal power allocation scheme may not be feasible as the feedback channel to the relays may have limited capacity.

3.3.2 Relay selection with power allocation for single source and multirelay network

In [61], the authors also propose a relay selection scheme with power allocation for a single source and multirelay network. As for the all participate power allocation scheme, this relay selection scheme still requires perfect CSI at the scheduler and feedback channel.

The relay selection is based on the maximization of the instantaneous SNR. In the case of the tree-terminal cooperative communication, it has been shown that maximizing the instantaneous SNR gives an optimal transmission power ratio for the relay and source transmit power. Hence, to perform selection, this optimal power ratio and the corresponding maximized instantaneous SNR are computed. Then, the relay that has the highest maximized SNR is selected for the communication. Finally, the scheduler uses the feedback channel to notify the selected relay and the source of their assigned transmit power.

This selection scheme with power allocation requires perfect CSI at the destination and accurate feedback to both the source and the relay. Moreover, the performance results are only presented for medium to high SNR region and are based on simulations.

3.3.3 Relay selection based on demodulate-and-forward relay and Coded cooperation method

In [16, 9, 10], the authors propose relay selection schemes that use demodulate-and-forward relay and coded cooperation. In those schemes, the selected relay chooses only a fraction of the received bit from the source and demodulates them. The result of the demodulation serves then as information bits to form the new relay codeword. The repeat-accumulate code is used to form the parity bits from the information bits, and the relay transmitted symbols are obtained by concatenating the information bits with the punctured parity bits.

The Repeat-accumulate code used in those schemes is a turbo-like code introduced in [14] and can be described as follow: The information bits are first repeated $q$ times, then interleaved before being fed into a truncated rate-1 recursive convolutional encoder with transfer function: $\frac{1}{1+D}$. The decoding of Repeat-accumulate code can be perform using the sum-product algorithm [31].
The relay selection scheme in [9] is done assuming perfect channel state information at all the receiving nodes: not only the destination has full CSI from direct, first-and second transmission link but also the relay need to have the perfect knowledge about the second link CSI. The mutual information is then calculated for every potential relay and the relay having the largest value is chosen to assist the communication. In [10], the proposed selection scheme assume the CSI knowledge from the first and second links, and the best relay is selected based on the calculation of the Bhattacharyya parameter. Indeed, the relay selection uses the fact that the union bound can provide upper bounds on the maximum likelihood bit error rate (BER) and frame error rate (FER) for convolution codes, and that its evaluation requires only the Bhattacharyya parameter and the weight enumerator. Using the results from coding theory applied to demodulate-and-forward relaying, the Bhattacharyya parameter is used to characterize the relays channels (the associated BER improves when the value of the parameter decreases). In the selection scheme, the best relay is chosen based on the computed value of the Bhattacharyya parameter.

3.3.4 Relay selection based on DF relay method

In [6, 7], the authors propose relay selection scheme based on the DF relay protocol. Relay selection has been considered [7] for both one source-destination pair [6, 7] and multiple source-destination pairs. In the case of one source node communicating with one destination node, one relay is selected out of a pool of potential relay based on its relay-destination channel instantaneous power. The transmission is done in two time slots as follows: in the first time slot, the source sends its information. The destination and the m-1 potential relays attempt decode this information. Each potential relay node decides whether it has decoded the information correctly and if it does, it declares itself part of the decoding set D(s) of eligible relay for transmission. The selected relay is the one having the best instantaneous relay-destination channel from the decoding set D(s). In the second time slot, the selected relay forwards the decoded information to the destination. Finally, the destination combines the information from the source and the relay using maximal ratio combining.

The case of multiple source-destination pairs has been proposed for a static mesh network of access points. In a multiple-source and destination network model, the relay is considered to be able to transmit to several sources. The transmission is done in two phases: in the first phase, each source sends its data using full power, and in the second phase, each relay divides its power evenly between the sources it is assisting to forward the data to the destination. In the distributed relay assignment
scheme, the most practical scheme among the selection schemes proposed, each relay is chosen independently for each of source-destination pairs. Every source is assumed to transmit to a different destination. The selected relay for a source-destination pair is the relay with the highest instantaneous relay-destination channel SNR.

The relay selection scheme for a multiple-source and destination network model in [7] is based on the SNR of the relay-destination link. It has been shown to give a better performance result than the distributed space-time code scheme in [35]. However, as in [35], the selection scheme requires feedback information at the relay nodes and furthermore, the relay needs to perform the decision of relaying for different source-destination pairs.

### 3.3.5 Relay selection based on AF relay method

In [49, 29], scheduling schemes for cellular network with multiuser are proposed. In [49], the authors consider a cellular network with a fixed infrastructure and introduce a simplified channel model to yield compact result for performance formulas. In [29], the network model involve mobile relays and study the outage probability of two general scheduling policies in downlink over Rayleigh fading channels.

In [49], the fixed infrastructure relays lead to the assumption of stationary links between the BS and the relays. Thus, only the link between the users and the relays are taken to be Rayleigh fading, the links between the relays and the BS are modeled as an AWGN channel. Using this model, the authors derive closed-form expressions for DL and UL transmission performances with transmission and reception turns granted based on maximum SNR scheduling. In this scheme, the CSI of all the links (equivalent end-to-end SNR) are assumed to be known at the BS to perform scheduling. The performance analysis and comparison has been carried for both VG and FG relay protocols. Due to the asymmetric channel model, the system performance analysis differs for UL and DL transmission. The numerical result in [49] indicates that while FG and VG relay have equally good performance for DL, for UL on the other hand, the VG protocol should be preferred.

In [29], the authors present the DL performance of a distributed scheduling (DS) scheme and a centralized scheduling (CS) scheme for an infrastructure network with multiuser and multirelays. UG, the studied relay protocol, is taken as an approximation to the practical VG relay protocol. The channel model is a Rayleigh flat fading channel and the relays are assumed to be mobile so that the channels between the BS and relays and relays and users are time varying.

The scheduling policies are based on a system model divided into partitions where
the number of partition is equal to the number of relay plus one [57, 29]. Every partition consists of one transmitter and a number of receivers as shown in Figure 3.2. Thus the BS partition contains the BS and the relays and the relay partitions contain one given relay and the users assigned to that relay.

![Multiuser and multirelay system model with partitions](image)

Figure 3.2: Multiuser and multirelay system model with partitions

The schemes, designed for DL transmission are as follow:

- **Centralized scheduling**
  
The CS scheme is based on the perfect knowledge of the end-to-end equivalent SNR of all the two-hop links assumed to be available at the BS. Based on this information, the BS schedules the two-hop link which has the maximum $\gamma_{eq}$ value. In this work, the authors also consider the case where the relays are intelligent enough to select the user with the highest relay-user SNR by themselves. In this case, the performance of the system remains the same with a reduced required feedback amount at the BS but the users still need to send their measured CSI to the relay of their partition.

- **Distributed scheduling**
  
The DS scheme is strictly capacity-suboptimal compared to the CS scheme but reduce the amount of required feedback. Every transmitter performs scheduling in their own partition based on the SNR measurements. More specifically,
the BS selects the relay with the highest BS-relay SNR and the selected relay simultaneously selects the user with the highest relay-user SNR in its partition.

The scheduling policies considered in [29, 49] are both based on maximum SNR scheduling and thus require the perfect CSI knowledge and feedback at the BS. Besides, the schemes presented in [29] involve the benchmark UG protocol which is not applicable in practice and thus the derived performance results for DL are only an upper bound of the performance of the practical VG relays. Thus, reducing the amount of feedback and using practical scheme could be an object of future work on multiuser and multirelay scheduling. In the next chapter, we propose new scheduling scheme for infrastructure network that uses limited feedback bits and examine their outage probability performance.
Chapter 4

System model

This chapter sets the channel model for the dual-hop communication used in our study. In this model, the source is communicating with the destination through a relay terminal. This relay terminal is considered to be mobile; those mobile relay terminals can be encountered for instance in a wireless communication system where the users’ terminals are used to relay the signals for one another. We do not consider the direct link as it is assumed that a relay is used when the direct link is in deep fade (see figure 4.1) such as in wireless communication systems where users are far from the BS or where there is an obstruction on the direct link. Note that the users close to the BS, or more generally the users that suffer only from a moderate or small path-loss and shadowing, would communicate via a single hop link.

![Figure 4.1: Three terminals relay channel without direct path](image)

Based on this dual-hop communication model, an expression of the end-to-end SNR is derived and this expression is used in the upcoming chapters to evaluate the outage probability of relaying protocols.

We derive also two upper bounds for the end-to-end SNR expression: a general upper bound by neglecting a positive term’s contribution and an upper bound for end-to-end expression that uses the unlimited gain (UG) given below in this chapter.
CHAPTER 4. SYSTEM MODEL

We will focus on wireless communication systems for the case of a single-user and single-relay but also for the more general case of multiusers and multirelays.

In section 4.2, we describe several relaying protocols for single-user and single-relay communication and introduce a new protocol, clipped gain, that allows to save the transmit power by stopping the transmission when the first link condition would have lead to an outage.

We examine also the dual-hop communication with multiuser and multirelay in the corresponding Section 4.2. To introduce the new scheduling policies, a system divided into partitions similar to the system model in [57, 29] is considered. Based on this system model, we introduce several scheduling protocols based on one-bit feedback information. Exploiting the upper bounds mentioned before, a transmission threshold can be set on the first- and second link SNR avoiding a waste of transmit power when the links’ SNRs are too poor. First, we focus on the first-hop SNR and introduce scheduling schemes that set a transmission threshold on the first-hop SNR. Thus, we introduce several schemes that select one dual-hop link for transmission among the links having the SNRs above the predefined threshold.

Then, scheduling is extended to the case where not only the quality of the first hop link but the quality of both links of the dual-hop transmission is taken into account. We describe the new Distributed Scheduling (DS) and the new Centralized Scheduling (CS) policies based on respectively one-bit feedback information on the first- and second link SNR and one-bit feedback information on the end-to-end SNR. This one-bit information renders the comparison result between the links’ SNR and a predefined threshold in both DS and CS cases and allows the scheduler to perform a random choice among the favorable links for transmission.

4.1 Channel model for dual-hop link

This model depicts the three-terminal communication channel in which the source is communicating with the destination through a relay terminal. The direct link is not considered here. The amplify-and-forward relay amplifies its input signal by a factor $\beta$ which is defined by the relaying protocol used.

The dual-hop link channel model is depicted in Figure 4.2. The signal generated by the source terminal is affected by the pathloss, shadowing and flat fading of the first hop link before reaching the relay terminal. The effect of pathloss and shadowing is given by the constant average channel energy factor $E_1$ and the effect of flat fading is given by the random variable $h_1$. Note that the constant value of $E_1$ in our model results from, in practice, the assumption of a perfect long-term power.
control. The backward coefficient of the channel is then $\sqrt{E_1}h_1$. Additionally, an additive white Gaussian noise with variance $\sigma_1$ is added to the received signal at the relay. We assume that the source transmits a signal $x(t)$ with a unit average power. The received signal at the relay terminal $R$ can be written as:

$$r(t) = \sqrt{E_1}h_1x(t) + n_1,$$  (4.1)

The signal is amplified at the relay $R$ by a gain factor $\beta$ before being forwarded to the destination. The transmit signal at the relay is:

$$t(t) = \beta \cdot r(t),$$  (4.2)

Using a second hop link channel model similar to the one of the first hop link, the forward channel coefficient between the relay and the destination is $\sqrt{E_2}h_2$ and the additional Gaussian noise variance at the destination receiver is $\sigma_2$. The received signal at the destination can be then written as:

$$y(t) = \sqrt{E_2}h_2t(t) + n_2,$$  (4.3)

Using the expressions (4.1), (4.2), and (4.3), the end-to-end SNR can be written as:

$$\gamma_{eq} = \frac{E_1E_2 | h_1\beta h_2 |^2}{E_2 | \beta h_2 |^2 \sigma_1^2 + \sigma_2^2}.$$  (4.4)

We use the subscript 1 (resp. 2) to denote parameters, statistics, and random variables associated with the first (resp. second) link of a two-hop communication model. Thus, the average SNR of the first and second hop can be defined as: $\gamma_1 = \overline{\gamma_1}|h_1|^2$ and $\gamma_2 = \overline{\gamma_2}|h_2|^2$ where $\overline{\gamma_1} = \frac{E_1}{\sigma_1^2}$ and $\overline{\gamma_2} = \frac{E_2}{\sigma_2^2}$ are respectively the first
and second link average SNR.

**General Upper bound for the end-to-end SNR**

Considering the above expression for the equivalent end-to-end SNR, an upper bound can be derived by neglecting the contribution of the power of the noise at the destination. As a result, the equivalent end-to-end SNR is bounded from above by the first-hop SNR:

\[
\gamma_{eq} = \frac{E_1 E_2 |h_1 \beta h_2|^2}{\beta h_2^2 \sigma_1^2 + \sigma_2^2} \leq \frac{E_1 E_2 |h_1 \beta h_2|^2}{\beta h_2^2 \sigma_1^2} = \gamma_1
\]

From this inequality, we can notice an important implication: whenever the first-hop SNR \( \gamma_1 \) is inferior to the outage threshold \( \gamma_{th} \), the equivalent end-to-end SNR is inferior to \( \gamma_{th} \) (i.e. the system is in outage). Therein, by setting a transmission condition on the first-hop SNR, the transmission in the case where the outage event is certain may be avoided. The transmission threshold on the first-hop SNR can be used in the case of a single source and a single-relay as well as in the case of a scheduled transmission with multiuser and multirelay.

### 4.2 Dual-hop communication with single-user and single-relay

In this section, we consider a dual-hop communication system with one source communicating with the destination through one relay. In a wireless communication system, this corresponds to the case where there is a single user and a single relay.

#### 4.2.1 Single-user and single-relay network model

In this dual-hop relaying communication model, there are a single source terminal and a single-relay terminal. We consider that the direct communication link is obstructed or in deep fade and, therefore, the communication between the source terminal and the destination terminal is possible only via the dual-hop link as depicted in Figure 4.3. The channel model described in Section 4.1 can be applied.

In the wireless communication system, the source terminal may be the user or the BS depending on, respectively, if the UL or the DL communication is considered.
4.2.2 Variable gain protocol

The VG protocol is a practical protocol that equalizes the fluctuations of the backward (or first-hop) channel based on the knowledge of the first-hop instantaneous fading amplitude. By limiting the transmit power at the relay to unity, the relay gain of the VG protocol is [33, 48]:

$$\beta_{VG} = \frac{1}{\sqrt{E_1 | h_1 |^2 + \sigma_1^2}},$$  \hspace{1cm} (4.6)

Using the VG relay gain, the end-to-end SNR (4.4) can be written as:

$$\gamma_{eq}^{VG} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \gamma_1 + 1},$$ \hspace{1cm} (4.7)

4.2.3 Unlimited gain protocol

The unlimited relay gain is a hypothetical gain that is able to invert the channel regardless of its magnitude. Indeed, the expression of this gain is a tight upper bound of the expression of the variable gain: the expression of the unlimited gain can be seen as a variable gain expression in which the additive noise power at the relay has been neglected. The relay gain of UG protocol is:

$$\beta_{UG} = \frac{1}{\sqrt{E_1 | h_1 |^2}},$$ \hspace{1cm} (4.8)

As a result, when the fading coefficient value of the first hop drops close to zero as encountered under Rayleigh fading condition, an infinite relay gain and therefore an infinite transmit power is required. This protocol is not applicable in practice; however, as argued in [40, 48], its performance may serve as benchmark for practical protocols.
Indeed, using the UG relay gain, the end-to-end SNR (4.4) can be written as:

\[ \gamma_{eq}^{UG} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \gamma_1} \]  

(4.9)

The expression of the end-to-end SNR obtain using UG protocol (4.9) is a tight upper bound for the expression of the end-to-end SNR using VG protocol (4.7) [40]. As a result, the performance of the UG protocol in terms of outage probability and bit error rate is a lower bound to those of the VG protocol.

4.2.4 Fixed gain protocol

The relay forwards the received signal from the source with a fixed gain which does not depend on the fading amplitude of the backward channel. The fading amplitude of the first-hop channel is considered unavailable at the relay; though, the relay uses only the average channel energy, a statistical knowledge about the backward channel that varies slowly (relative to the fading coefficient) and as such can be obtained though estimation or adaptation without continuous monitoring of the channel [41].

By limiting the transmit power at the relay to unity, the relay gain of the FG protocol is [48]:

\[ \beta^{FG} = \frac{1}{\sqrt{E_1 + \sigma^2_1}} \]  

(4.10)

Using the FG relay gain, the end-to-end SNR (4.4) can be written as:

\[ \gamma_{eq}^{FG} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \gamma_1 + 1} \]  

(4.11)

4.2.5 Clipped gain protocol

In this section, we introduce a new relaying protocol based on the fact that the equivalent SNR is always limited by the source-relay SNR. When \( \gamma_1 < \gamma_{th} \) then \( \gamma_{eq} < \gamma_{th} \) and the destination will be in outage whatever the relay-destination SNR is.

Recalling that unlimited gain protocol is an ideal case that does not take into account the noise at the relay needing an infinite gain and transmitted power (see section 4.2.3). Therefore, the unlimited gain is not an applicable protocol when the first hop is in deep fade.

To yield the clipped gain protocol, we pick only good channel conditions between the source and the relay. Let us assume that the relay knows the threshold value SNRth of the receiver. This is a reasonable assumption because the threshold is a
system parameter. The gain could be set then to zero when $\gamma_1 \leq \gamma_{th}$, avoiding loss in performance whenever the outage is certain. The resulting clipped gain will be a practical protocol that can help to save transmit power:

$$\beta_{CG}^2 = \begin{cases} 0, & \gamma_1 < \gamma_{th} \\ \frac{G}{E_1|h_1|^2}, & \gamma_1 \geq \gamma_{th} \end{cases} \quad (4.12)$$

Thus,

$$\gamma_{eq}^{CG} = \begin{cases} 0, & \gamma_1 < \gamma_{th} \\ \frac{G\gamma_1 \gamma_2}{\gamma_1 + G\gamma_2}, & \gamma_1 \geq \gamma_{th} \end{cases} \quad (4.13)$$

Where $G$ is the normalization factor that set the transmit power to unity, $\varepsilon[|t|^2] = \varepsilon[E_1|h_1\beta|^2] + \varepsilon[\sigma_1^2/\beta^2] = 1$. The factor $G$ can be evaluated for a Rayleigh fading channel (A.1.2) and a Nakagami fading channel (A.1.1) respectively. For the Rayleigh fading channel, the normalization factor is:

$$G = \frac{1}{e^{-\frac{\gamma_{th}}{\gamma_1}} + \frac{1}{\gamma_1}E_1(\frac{\gamma_{th}}{\gamma_1})} \quad (4.14)$$

For the Nakagami fading channel, the normalization factor is:

$$G_N = \frac{\Gamma[m_1]}{\left(\frac{m_1}{\gamma_1} \Gamma\left(\left(m_1 - 1, \frac{\gamma_{th}m_1}{\gamma_1}\right) + \Gamma\left(m_1, \frac{\gamma_{th}m_1}{\gamma_1}\right)\right)\right)} \quad (4.15)$$

One can verify that the normalization factor over the Nakagami fading channel $G_N$ reduces to the normalization factor over the Rayleigh fading channel $G$ for $m_1 = 1$.

The advantages of the clipped gain protocol independently of the fading model can be observed from the relay gain expression (4.12): by integrating the factor $G$ (Rayleigh fading case) or $G_N$ (Nakagami fading case) in the relay gain and setting this gain to zero when the SNR of the first link is below the threshold SNR, we obtain a constant average relay transmit power and a finite relay gain yielding an applicable protocol for the relay channel. Thus, the power normalization factor allows the saving of transmit power when the condition of the first link is poor and to use this power when the SNR of the first link is above $\gamma_{th}$.

The CG protocol can be seen as a power control scheme in which the transmit power at the relay is not constant: the power level is either zero or $P$ depending on the realized CSI of the source-relay link. In [4], the one-bit feedback power
control scheme also proposed a non constant transmit power for the relay and the source depending on the channel states measured at the destination. In this case, the channel states for the first-, second-, and direct link are assumed to be perfectly known at the destination. The channel state is then mapped to an element of a predesigned codebook containing two power-duplets (source and relay transmit power). Upon reception of the index sent through a noiseless feedback link from the destination, the source and the relay transmit power take value from the selected power-duplet. In the case of CG gain, only the first-link SNR is measured at the relay, and if we assume that the relay is able to perform the comparison between the measured SNR and the transmission threshold, the decision of transmitting or switching off transmission can be taken at the relay and there will be no feedback bit needed from the destination.

4.3 Dual-hop communication with multiuser and multirelay

In this section, we describe the multiuser and multirelay network model and propose new scheduling protocols for such a network.

In the precedent section, we have proposed the CG protocol, a power control scheme with a transmission threshold based on the general upper bound on the equivalent SNR. Meanwhile, the CG protocol has been consider only for a single-user and single-relay setting, and for a more realistic network model, multiuser and multirelay nodes should be taken into account. Thus, we consider in the sequel an infrastructure network with multiuser and multirelay and propose new selection schemes for this setting.

Selection cooperation has been shown to be an efficient method to achieve full diversity without sacrificing network capacity or power efficiency. In our scheduling protocols, we select a dual-hop for transmission such that its first-hop or/and second hop SNR satisfy the selection criterion based on the upper bounds on the equivalent end-to-end SNR (Section 4.1 and Section 4.3.2).

For dissertation convenience, we use the cellular network notation such as user and BS, we stress however that those scheduling protocols can be applied for other general infrastructure networks.
4.3.1 Multiuser and multirelay network model

We focus on wireless dual-hop system model with multiple users and multiple amplify-and-forward relays.

The direct links between the users and the BS are not considered here as we assume that they are subject to obstruction or deep fade.

In this system model, similarly to the system model in [57], there are M relays and N users divided into M+1 partitions as shown in Figure 4.4. The BS partition, $P_{BS}$, includes the BS and the relays in the network; there are M relay partitions ($P_{Ri}$) and each of them consists of one relay and the users that communicate through that relay. Assuming that users are uniformly distributed in the network, every relay partition contains $N/M$ user terminals on average.

The users close to the BS are not considered here as they suffer from only a moderate or small path loss and can use one hop communication. Assuming that the relays are mobile, the channels between the relays and the BS and the channels between the relays and the users are time varying. We assume that both first- and
second hop are Rayleigh flat fading channels.

For a fair scheduling, the M relays are assumed to be at approximately the same
distance to the BS and the channels between the BS and to experience similar
fading. Similar assumptions are also made for the users within a relay partition.
Thus, within a partition, it is assumed that all links are subject to independent and
identically distributed (i.i.d) fading and the dual-hop channel model in Section 4.1
applies.

Multiple access is time division multiple access; we further consider half-duplex
relaying that requires two independent time slots for transmission with the assump-
tion that the destination terminal listen to only the second hop channel.

4.3.2 Upper bound using unlimited gain protocol

In this section, an additional upper bound to the equivalent end-to-end SNR of a
dual-hop communication is derived. For this derivation, the relay is taken to be
an UG gain relay. This upper bound will be used subsequently in the scheduling
scheme for a multiuser and multirelay network.

Similarly as in the derivation of the upper bound in Section 4.1, as the term
$E_2 \left| \beta h_2 \right|^2 \sigma_1^2$ in the expression of $\gamma_{eq}$ (4.4) is positive, the equivalent end-to-end
SNR can be upper bounded by:

$$\gamma_{eq} = \frac{E_1 E_2 \left| h_1 \beta h_2 \right|^2}{E_2 \left| \beta h_2 \right|^2 \sigma_1^2 + \sigma_2^2} \leq \frac{E_1 E_2 \left| h_1 \beta h_2 \right|^2}{\sigma_2^2}$$

Recalling the unlimited gain expression:

$$\beta_{UG} = \frac{1}{E_1 \left| h_1 \right|}$$

By using the unlimited gain amplify-and-forward protocol, the latter upper bound
in (4.4) will simplify to the signal-to-noise ratio of the relay to destination link such
that:

$$\gamma_{eq} \leq \frac{E_1 E_2 \left| h_1 \beta h_2 \right|^2}{\sigma_2^2} = \gamma_2$$

This upper bound along with the general upper bound (4.5) can be used to perform
scheduling in the multiuser and multirelay network. By setting lower bounds $\tau_1$
and $\tau_2$ on, respectively, the SNR of the first hop and the SNR of the second hop
for scheduling, the performance of the system can be improved. Such scheduling
schemes are considered in the next section.
4.3.3 Multiuser and multirelay scheduling

Using the system model with partition introduced in Section 4.3.1, we present new scheduling schemes for an infrastructure network with multiuser and multirelay. The first schemes, $\gamma_1$-selection scheduling, are based on the general upper bound on the equivalent end-to-end SNR (Section 4.1). Those schemes set a transmission threshold on the first-hop SNR and thus select one dual-hop link for transmission among the links having a favorable first-hop SNR.

Then, we use both the general and the UG upper bound on the equivalent end-to-end SNR (Section 4.1 and Section 4.3.2) to set selection thresholds for scheduling in our schemes. We introduce $\gamma_1$, $\gamma_2$-selection scheduling schemes in which the quality of both first- and second link of the dual-hop transmission is taken into account. We introduce new Distributed Scheduling (DS) and Centralized Scheduling (CS) policies based on one-bit information at the scheduler on the first- and second link SNR and one-bit information on the end-to-end SNR, respectively. In both DS and CS cases, this one-bit information renders the comparison result between the links’ SNR and a predefined threshold and allows the scheduler to perform a random choice among the links having a satisfactory SNR for transmission.

$\gamma_1$-selection scheduling

In this section, we introduce new scheduling schemes in which the selection of the dual-hop link for transmission is based on the values of the first-hop SNRs. Indeed, as the equivalent end-to-end SNR of the dual-hop relay channel has been shown to be always limited by the source-relay (or first-link) SNR, a proper selection of this source-relay link can improve the outage performance of the transmission. The scheduling principle of the following schemes is to choose preferably a dual-hop transmission in which the source-relay link SNR is above the selection threshold $\tau$. Thus, for both UL and DL transmission, information about the SNR of the source-relay links is needed at the scheduler to perform selection. Meanwhile, unlike the previously proposed scheduling schemes in [29, 49], the BS does not have the exact SNR measurements. Indeed, the only information needed at the scheduler is whether the first link SNR is above or below the selection threshold.

Assuming that the SNR and the comparison to the selection threshold can be computed at the relay, the one-bit CSI can be obtained at the BS as follow:

- In UL transmission, the CSI information needed for scheduling is the information about the quality of the link between the users and the relays compared to the selection threshold. The SNR of the user-relay links can be measured
and compared to the threshold at the relays from the pilot signals sent by the
users (source terminals in UL transmission case). Then, for every user, the
relay sends one bit information (i.e. 0 or 1) to the BS notifying if the SNR of
the corresponding user-relay link is above or below the threshold.

- In the DL transmission, the CSI information needed is on the quality of the
  links between the BS and the relays. In this case, a pilot signal is sent by
  the BS to every relay. As in UL transmission case, the SNR of the first links
  should be computed and compared to the selection threshold at the relays.
  Then, the relays send one bit feedback indicating whether the BS-relay SNR
  is above the selection threshold or not.

Based on the information at the BS about the SNR condition of the first link
(the first links denote the BS-relay links and user-relay links in DL and UL respec-
tively), the $\gamma_1$-selection scheduling schemes follow the same selection principle: the
BS schedules whenever possible the transmission to a dual-hop that has the first-link
SNR above the selection threshold, i.e. the transmission turn is given randomly to
one dual-hop from the pool that satisfy this condition. Thus, in DL transmission
(illustrated in Figure 4.5), the scheduler makes a random choice of one relay among
the relays that satisfy the condition of having the BS-relay SNR above $\tau$. Then, the
relay receiving the transmission from the BS forwards the signal to one user chosen
according to a round-robin order among those served by it (see algorithm 1). In
UL transmission (illustrated in Figure 4.6), the first links of the dual-hop channel
correspond to the links between the users and their relays and the second links corre-
spond to the links between the relays and the BS. Therefore, the scheduling policy
is as follows: the scheduler first makes a round-robin choice of one relay thereby
fixing the second link; then, the chosen relay sends (to the scheduler), for every
user in its partition, one bit information that indicates whether the corresponding
user-relay link SNR is below or above $\tau$. Based on this knowledge, one user served
by the selected relay is randomly chosen among those with user-relay SNR superior
to the threshold. In the case where there is no user in the chosen relay’s partition
that has a first link SNR above the threshold, the scheduler repeats the selection
process with another relay (see Algorithm 2).

We propose and study three scheduling schemes based on the $\gamma_1$ selection thresh-
old $\tau$. All the schemes follow the scheduling policies for UL and DL mentioned
above with the use of the threshold $\tau$ for selecting the first link such that its SNR is
superior or equal to $\tau$. The scheduling schemes differ in the threshold value $\tau$ and
the selection policy taken when there is no first link that satisfies the criterion.
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![Diagram of system model with user terminals and relays]

Figure 4.5: $\gamma_1$ selection scheduling in DL
Figure 4.6: \( \gamma_1 \) selection scheduling in UL
In the $\gamma_1$ selection scheduling, the values of the predefined thresholds are divided into three cases that characterize our studied schemes:

- Selection threshold is equal to the outage threshold (TES): $\tau = \gamma_{th}$
- Selection threshold is inferior to the outage threshold (TIS): $\tau < \gamma_{th}$
- Selection threshold is superior to the outage threshold (TIS): $\tau > \gamma_{th}$

When there is no dual-hop link that satisfies the selection criterion, i.e., when all the first hop links’ SNR are below the threshold, two different solutions may be considered depending on the value assigned to the transmission threshold $\tau$. The scheduler may choose randomly one link among all the possibilities and schedule the transmission for this link (continued transmission CT) or the scheduler may choose no link for transmission and, therefore, the system is kept idle during this time slot (interrupted transmission IT).

The TIS, TES, and TSS algorithms are as follow:

**Algorithm 1**  
**TIS:** $\tau < \gamma_{th}$, **TES:** $\tau = \gamma_{th}$, and **TSS-IT** $\tau > \gamma_{th}$ in DL  
$\mathcal{R}$ is the ensemble of all the relays in the network.  
$\mathcal{R}_\tau = \{ R_i \in \mathcal{R} : \gamma_{BS-R_i} > \tau \}$

- if $\mathcal{R}_\tau \neq \emptyset$
  1. Choose randomly one relay $\overline{R} \in \mathcal{R}_\tau$, $\gamma_1 := \gamma_{BS-\overline{R}}$
  2. Round-robin scheduling of one user $\overline{U}$ in the relay $\overline{R}$’s partition: $P_{\overline{R}}$  
  BS transmits to the scheduled user

- if $\mathcal{R}_\tau = \emptyset$
  IT: No relay scheduled and stop transmitting.

**Algorithm 2**  
**TIS:** $\tau < \gamma_{th}$, **TES:** $\tau = \gamma_{th}$, and **TSS-IT** $\tau > \gamma_{th}$ in UL  
$\mathcal{R}$ is the ensemble of all the relays in the network.

1. Round-robin scheduling of one relay $\overline{R} \in \mathcal{R}$

2. $\mathcal{U}_\tau = \{ U_i \in P_{\overline{R}} : \gamma_{U_i-\overline{R}} > \tau \}$
- if $U_{\tau} \neq \emptyset$
  Choose randomly one user $\bar{U} \in U_{\tau}$, $\gamma_1 := \gamma_{\bar{U} - \bar{R}}$
  Schedule the transmission to the chosen user.

- if $U_{\tau} = \emptyset$
  Start again from step 1 if there is still at least one relay from $R$ that have not been selected
  Otherwise: IT No user is chosen for transmitting

Algorithm 3 \textit{TSS-CT} $\tau > \gamma_{th}$ in DL
\( R \) is the ensemble of all the relays in the network.
\( R_{\tau} = \{ R_i \in R : \gamma_{BS-Ri} > \tau \} \)

- if $R_{\tau} \neq \emptyset$ same step 1 and 2 than in algorithm 1
- if $R_{\tau} = \emptyset$
  CT:
  1. choose randomly one relay $R$ among the $M$ relays
  2. make a round-robin choice of one user among the users in the relay $R$ ’s partition $P_R$

Algorithm 4 \textit{TSS-CT} $\tau > \gamma_{th}$ in UL
\( R \) is the ensemble of all the relays in the network.

1. Round-robin scheduling of one relay $\bar{R} \in R$

2. $U_{\tau} = \{ U_i \in P_{\bar{R}} : \gamma_{U_i-\bar{R}} > \tau \}$

- if $U_{\tau} \neq \emptyset$
  same instructions than in algorithm 2
- if $U_{\tau} = \emptyset$
  Start again from step 1 if there is still relay from $R$ that have not been selected
  Otherwise: CT: choose randomly one user in $P_{\bar{R}}$ then schedule the transmission to the chosen user.
For the $\gamma_1$-selection scheduling schemes, i.e. TIS, TES, and TSS, the end-to-end signal-to-noise ratio $\gamma_{eq}$ can be expressed as:

$$
\gamma_{eq} = \begin{cases} 
    0 & \gamma_1 \leq \tau \\
    \frac{G\gamma_1\gamma_2}{\gamma_1 + G\gamma_2} & \gamma_1 > \tau 
\end{cases}
$$

(4.18)

Where $\gamma_1$ is the selected first-hop SNR, i.e. $\gamma_{BS-R}$ in DL (see Algorithm 1 and 3) and $\gamma_{T-R}$ in UL (see algorithm 2 and 4).

The predefined threshold, that set a floor value to the selected SNR of the first hop $\gamma_1$, may prevent the system from scheduling the transmission for the cases where the value of the equivalent end-to-end SNR is certainly leading to an outage event. This is especially straightforward observation in the case where the threshold $\tau$ is taken to be equal to the outage threshold or inferior to it (respectively TES and TIS protocols) because any first link SNR inferior to that threshold will lead to an equivalent SNR inferior to the outage threshold. Note that in the case TES and TIS protocols, if there is no first link with a SNR greater than $\tau$, no link is selected and the radio terminals remain idle (see Algorithms 1 and 2). TIS and TES schemes hence help the system to avoid transmission when the channel condition leads to certain outage. Therefore, it allows power saving and performance enhancement.

In addition to investigating the performance of TIS and TES schemes, we consider also the TSS scheme with $\tau > \gamma_{th}$; as the chosen $\gamma_1$, despite satisfying the selection criterion of being superior to the outage threshold, may still lead to an outage of the overall transmission and that for any second link SNR condition. Indeed, according to the general upper bound (Section 4.1), we have $\gamma_{eq} < \gamma_1$; additionally, TES selection gives $\gamma_{th} < \gamma_1$ but does not guarantee that $\gamma_{th} < \gamma_{eq}$. Therefore, the TSS protocol where the threshold is superior to the outage threshold $\gamma_{th}$ is envisaged as it discards some supplementary cases where the selected first hop SNR might not be high enough to ensure an equivalent SNR superior to the outage threshold. Both continued transmission (CT) and interrupted transmission (IT) are considered for the TSS protocol. IT and CT policies intervene in the case where there is no first link SNR above the threshold $\tau$. Note that CT is studied for TSS protocol as the scheduling of one randomly chosen dual-hop link that does not satisfy the selection criterion $\gamma_1 > \tau$ may still improve the outage performance of the system; for instance, $\gamma_1$ may be in the interval $[\gamma_{th} : \tau]$ (i.e. $\gamma_{th} < \gamma_1 < \tau$) for which we cannot conclude that its selection will result in an outage of the system.
In Chapter 6, we develop the closed-form expressions for the $\gamma_1$-selection protocols and compare their performance.

**$\gamma_1$, $\gamma_2$-selection scheduling schemes**

In this chapter, we consider two generalized scheduling policies: *centralized scheduling* and *distributed scheduling*.

In the CS policy, the knowledge about all the dual-hop links in the system is collected at the BS. The BS schedules one dual-hop, i.e. one relay and one user, based on this centralized information. We present in the following one scheme that uses this policy: the CS scheduling scheme.

In the DS policy, the feedback about the forward channel condition is collected within every partition and is available at the corresponding transmitter. Then, scheduling is performed within the transmitters’ partitions. We present two scheduling schemes that use this policy: the DS scheme and the DS Simplified scheme (DSS).

- **DS** For every transmission, one bit of feedback from every potential receiver in one partition is available to the transmitter. In the relays’ partitions, the feedback bit from the user to the relay indicates whether the SNR of the relay-user link is above the selection threshold $\tau_1 = \gamma_{th}$. In the BS partition, the feedback bit from the relay to the BS indicates whether the relay-BS SNR is above the selection threshold $\tau_2 = \gamma_{th}$ and whether there are any users in that relay’s partition that satisfies the previous selection criterion: $\gamma_{relay-user} > \gamma_{th}$. Note that the selection thresholds $\tau_1$ and $\tau_2$ are taken to be equal to the outage threshold.

The feedback bits are determined as follows. First, every user terminal measures its $\gamma_{relay-user}$ (second-hop SNR) and compares this measure to $\tau_2$. The comparison result is set to 1 if the SNR is above the threshold and 0 otherwise. The users send those comparison results under the form of one bit feedback to their assigned relay. The relay selects randomly one user from the pool of users having sent 1 if this pool is not empty, otherwise no user is selected.

On the other hand, the relays also measure their $\gamma_{BS-relay}$ (first-hop SNR) and compare the measured SNRs to the threshold $\tau_1$. The comparison result is set to 1 if the SNR is above the threshold and 0 otherwise. If there has been a selected user, the relay sends its comparison result to the BS. Otherwise, in the case where no user has been selected, the relay sends a 0 feedback to the BS.
The scheduling scheme selects the user and the relay within the partition as shown in Algorithm 5 and Figure 4.7. First, every relay selects if possible one user in its partition that has $\gamma_{\text{relay-user}}$ above $\gamma_{\text{th}}$. Then, the BS selects one relay that has both selected user in its partition and a $\gamma_{\text{BS-relay}}$ above $\gamma_{\text{th}}$. If there is no relay and user selected, the transmission is stopped. Not transmitting in those cases is the best option. Indeed, the choice of a dual-hop that does not fulfill the selection criterion for one of the two links leads to an outage event according to the general upper bound and UG upper bound.

**Algorithm 5** DS in DL

$\mathcal{R}$ is the ensemble of all the relays in the network.

For every $R_j \in \mathcal{R}$:

$\mathcal{U}_{\gamma_{\text{eq}},R_j} = \{ U_i \in P_{R_j} : \gamma_{R_j-U_i} > \gamma_{\text{th}} \}$

- if $\mathcal{U}_{\gamma_{\text{eq}},R_j} \neq \emptyset$ Choose randomly one user $U_{R_j} \in \mathcal{U}_{\gamma_{\text{eq}},R_j}$
- if $\mathcal{U}_{\gamma_{\text{eq}},R_j} = \emptyset$

No user is selected in $P_{R_j}$.

$\mathcal{R}_{\gamma_{\text{eq}}} = \{ R_i \in \mathcal{R} : \gamma_{\text{BS-relay}} > \gamma_{\text{th}}, \mathcal{U}_{\gamma_{\text{eq}},R_i} \neq \emptyset \}$

- if $\mathcal{R}_{\gamma_{\text{eq}}} \neq \emptyset$ Choose randomly one relay $R \in \mathcal{R}_{\gamma_{\text{eq}}}$, $\gamma_{\text{BS-R}} := \gamma_1$

$U := U_{R}, \gamma_{U-U} := \gamma_2$

The transmission is scheduled for the selected relay and the selected user.

- if $\mathcal{R}_{\gamma_{\text{eq}}} = \emptyset$

No relay is selected, stop transmitting.
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(a) User with relay-user SNR > threshold

(b) Base station BS with relay-user SNR > threshold and at least one user in its partition with relay-user SNR > threshold

(c) Selected user

Figure 4.7: DS scheduling scheme
• DSS

In this simplified DS scheme, the receiver selection is still performed within a transmitter’s partition based on the feedback from all the receivers in this partition. As in DS, the users send one-bit feedback to their assigned relay and the relays send their feedback to the BS. However, unlike in the DS scheme, the feedback from the relays to the BS only results from the comparison between the BS-relay links SNR and the threshold (i.e. 1 if $\gamma_{BS-relay} > \gamma_{th}$, 0 otherwise). The BS selects randomly one relay that has $\gamma_{BS-relay} > \gamma_{th}$ without knowing if there is one user in the selected relay’s partition that satisfies the selection criterion. The selected relay then selects randomly one user in its partition with $\gamma_{relay-user} > \gamma_{th}$ if such user exist, otherwise, the transmission is stopped.

**Algorithm 6 DSS in DL**

$\mathcal{R}$ is the ensemble of all the relays in the network.

$\mathcal{R}_{\gamma_{th}} = \{ R_i \in \mathcal{R} : \gamma_{BS-Ri} > \gamma_{th} \}$

- if $\mathcal{R}_{\gamma_{th}} \neq \emptyset$
  1. Choose randomly one relay $R \in \mathcal{R}_{\gamma_{th}}$, $\gamma_1 := \gamma_{BS-R}$
  2. $\mathcal{U}_{\gamma_{th}} = \{ U_i \in \mathcal{P}_R : \gamma_{U_i-R} > \gamma_{th} \}$
     * if $\mathcal{U}_{\gamma_{th}} \neq \emptyset$ Choose randomly one user $U \in \mathcal{U}_{\gamma_{th}}$, $\gamma_2 := \gamma_{U-R}$
       Schedule the transmission to the selected user.
     * if $\mathcal{U}_{\gamma_{th}} = \emptyset$ No user is selected, stop transmitting.
- if $\mathcal{R}_{\gamma_{th}} = \emptyset$
  No relay scheduled and stop transmitting.
Figure 4.8: DSS scheduling scheme
• CS

For every transmission, this scheduling scheme assumes that the information whether the equivalent end-to-end SNR is above a predefined threshold \( \tau_{eq} \) is available at the BS for all links. The equivalent SNR of the dual-hop link is measured and compared to the threshold by the user terminal. Then the user sends the comparison result in one bit (i.e. 1 or 0) to the BS. Based on this feedback information from every user, the scheduler choose randomly one of the dual-hop link with an SNR above the threshold. When there is no dual-hop link that fulfills this selection criterion, the system ceases transmitting. Stopping the transmission in such cases seems to be a natural decision as selecting any dual-hop link for transmitting will certainly lead to an outage.

The CS protocol can be written as:

\[
\text{Algorithm 7} \quad \text{CS in DL} \\
\mathcal{D}_{\gamma_{th}} = \{ (R_i, U_i) : \gamma_{eq,R_i,U_i} = \frac{\gamma_{BS-R_i,\gamma_{R_i-U_i}}}{\gamma_{BS-R_i}+\gamma_{R_i-U_i}} > \gamma_{th} \} \\
- \text{if } \mathcal{D}_{\gamma_{th}} \neq \emptyset \\
1. \text{Choose randomly one dual-hop link characterized by } (R, U), \gamma_{eq} := \gamma_{eq,R,U} \\
2. \text{Schedule the transmission to the selected user } U \text{ via the selected relay } R. \\
- \text{if } \mathcal{D}_{\gamma_{th}} = \emptyset \\
\text{No relay-user pair is scheduled and the transmission is interrupted.}
\]

One can notice that CS is capacity-optimal but requires one-bit feedback for every user at the BS.

In Chapter 6, we derive the outage probability closed-form expressions for the \( \gamma_1, \gamma_2 \)-scheduling schemes and assess their performance.
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Figure 4.9: CS scheduling scheme
Chapter 5

Performance of single-user and single-relay protocols

In this chapter we compare the performance of the single-user and single-relay protocols introduced in Chapter 4. We recall for this purpose the outage probability expressions of the VG, UG, and FG relay systems undergoing Rayleigh fading and Nakagami fading. Those closed-form expressions have been derived in, respectively, [18, 40, 41] and [55, 42, 55]. Furthermore, we also introduce the outage probability closed-form expressions of the CG protocol for Rayleigh and Nakagami fading channel and compare the performance of our scheme to that of the existing protocols mentioned above.

5.1 Performance over Rayleigh fading channel

In this section, we first recall the outage probability expressions of the VG, UG, FG, and DF (decode-and-forward) protocols over a Rayleigh fading channel. Then, we develop the outage probability expression for the CG protocol over Rayleigh fading channel and compare its performance to that of the existing protocol under the same fading conditions. The close-form expressions for VG, UG, FG, and DF have been derived in [18, 40, 41, 40], respectively.

To evaluate the closed-form outage probability expression of the CG protocol, we use the results derived in Appendix B and take into account the absence of transmission mode of the CG protocol (when $\gamma_1 < \gamma_{th}$, see chapter 4) as an outage occurrence.
5.1.1 Outage probability of the variable gain protocol

The following close-form expression for VG outage probability has been derived in [41]:

$$P_{vG}^{out}(\gamma_{th}) = 1 - e^{-\gamma_{th}^2} \left( \frac{1}{\gamma_1 + \frac{1}{\gamma_2}} \right) \cdot \left( 2 \sqrt{\frac{(1 + \gamma_{th})\gamma_{th}}{\gamma_1 \gamma_2}} K_1 \left( 2 \sqrt{\frac{(1 + \gamma_{th})\gamma_{th}}{\gamma_1 \gamma_2}} \right) \right)$$ (5.1)

where $K(.)$ is the first-order modified Bessel function of the second kind defined in [21] Eq. (9.6.22).

5.1.2 Outage probability of the unlimited gain protocol

The outage probability of the UG protocol was given in [40] as a tight bound to the performance of the VG protocol:

$$P_{out}^{UG}(\gamma_{th}) = 1 - e^{-\gamma_{th}^2} \left( \frac{1}{\gamma_1 + \frac{1}{\gamma_2}} \right) \cdot \left( 2 \sqrt{\frac{2\gamma_{th}}{\gamma_1 \gamma_2}} K_1 \left( \frac{2\gamma_{th}}{\sqrt{\gamma_1 \gamma_2}} \right) \right)$$ (5.2)

where $K(.)$ is the first-order modified Bessel function of the second kind defined in [21] Eq. (9.6.22).

5.1.3 Outage probability of the fixed gain protocol

In [41], the authors have derived the outage probability close-form expression for a general fixed gain relay. Originally, this expression has been applied to a semi-blind relay gain protocol. However, this fixed gain protocol will not be used in the sequel as it does not normalize the relay transmit power and cannot provide a fair comparison with the other protocols.

Hence, the following FG outage probability close-form expression is obtained using the FG gain Eq. (4.10) that leads to a constant relay transmit power and the close-form expression for a general fixed gain relay in [41]:

$$P_{out}^{FG}(\gamma_{th}) = 1 - e^{-\gamma_{th}^2} \left( \frac{1}{\gamma_1 + \frac{1}{\gamma_2}} \right) \cdot \left( 2 \sqrt{\frac{(1 + \gamma_1)\gamma_{th}}{\gamma_1 \gamma_2}} K_1 \left( 2 \sqrt{\frac{(1 + \gamma_1)\gamma_{th}}{\gamma_1 \gamma_2}} \right) \right)$$ (5.3)

where $K(.)$ is the first-order modified Bessel function of the second kind defined in [21] Eq. (9.6.22).
5.1.4 Outage probability of decode-and-forward protocol

The outage probability for the decode-and-forward relay gain protocol over a Rayleigh fading channel has been derived in [40]. The system is considered to be in outage if either one of the link is in outage.

\[ P_{\text{out}}^{DF}(\gamma_{\text{th}}) = 1 - e^{-\gamma_{\text{th}} \left( \frac{1}{\gamma_1} + \frac{1}{\gamma_2} \right)}, \]  

(5.4)

5.1.5 Outage probability of the clipped gain protocol

The outage probability of a general two-hop relay channel can be written in terms of conditional probabilities as follows:

\[ P_{\text{out}}(\gamma_{\text{th}}) = P[\gamma_{\text{eq}} < \gamma_{\text{th}} \mid \gamma_1 < \gamma_{\text{th}}] \cdot P[\gamma_1 < \gamma_{\text{th}}] + \]  
\[ P[\gamma_{\text{eq}} < \gamma_{\text{th}} \mid \gamma_1 \geq \gamma_{\text{th}}] \cdot P[\gamma_1 \geq \gamma_{\text{th}}], \]  

(5.5)

Where \( P[\gamma_{\text{eq}} < \gamma_{\text{th}} \mid \gamma_1 < \gamma_{\text{th}}] \) and \( P[\gamma_{\text{eq}} < \gamma_{\text{th}} \mid \gamma_1 \geq \gamma_{\text{th}}] \) are the outage probability given that the first-hop SNR is below / above the outage threshold and \( P[\gamma_1 < \gamma_{\text{th}}] \) and \( P[\gamma_1 \geq \gamma_{\text{th}}] \) are the probability of having the first-hop SNR below / above the outage threshold.

Using the general upper bound (Equation 4.5) derived in Chapter 4, one can notice that if the source relay SNR is inferior to the threshold SNR, the outage will happen. Thereby, (5.5) gives:

\[ P_{\text{out}}(\gamma_{\text{th}}) = P[\gamma_1 < \gamma_{\text{th}}] + P[\gamma_{\text{eq}} < \gamma_{\text{th}} \mid \gamma_1 \geq \gamma_{\text{th}}] \cdot P[\gamma_1 \geq \gamma_{\text{th}}], \]  

(5.6)

The precedent expression of the outage probability in terms of conditional probability for a general two-hop communication leads to the following expression for clipped gain relay:

\[ P_{\text{out}}^{CG}(\gamma_{\text{th}}) = P[\gamma_1 < \gamma_{\text{th}}] + P[\gamma_{\text{eq}} < \gamma_{\text{th}} \mid \gamma_1 \geq \gamma_{\text{th}}] \cdot \frac{G\gamma_1\gamma_2}{\gamma_1 + G\gamma_2} < \gamma_{\text{th}} \cdot P[\gamma_1 \geq \gamma_{\text{th}}], \]  

(5.7)

where, \( G \) is the relay transmit power normalization factor over a Rayleigh fading channel (see Equation 4.14) which has been derived in Appendix A.1.2.

By using the results of Appendix B, we obtain the following closed-form expression for the Rayleigh fading channel:
\[ P_{\text{out, Rayleigh}}^{\text{CG}} = 1 - e^{-\gamma_{\text{th}} \left( \frac{1}{\sqrt{G \gamma_1}} + \sqrt{\frac{G}{\gamma_2}} \right)} \cdot \left( \frac{2 \gamma_{\text{th}}}{\sqrt{G \gamma_1 \gamma_2}} K_1 \left( \frac{2 \gamma_{\text{th}}}{\sqrt{G \gamma_1 \gamma_2}} \right) \right), \quad (5.8) \]

Where \( K(.) \) is the first-order modified Bessel function of the second kind defined in [21] Eq. (9.6.22). \( G \) is the transmit power normalization factor for CG relaying over a Rayleigh fading channel (4.14).

It is worth noticing that without the power normalization factor, i.e. for \( G \) equals to one, the outage probability expression for \textit{clipped gain} will reduce to the outage probability of \textit{unlimited gain} protocol (5.2).

### 5.1.6 Outage performance comparison over Rayleigh fading channel

In this section, we compute the outage probability over a Rayleigh fading channel of the single-user and single-relay protocols described above and compare their numerical results. The outage threshold, \( \gamma_{\text{th}} \), is taken to be 10 dB and the relay-destination (or second-hop) average SNR is taken to be five times the source-relay (or first-hop) average SNR: \( \gamma_2 = 5 \times \gamma_1 \).

In Figure 5.1 and Figure 5.2, we compare the outage probabilities of the FG, UG, VG, and CG protocols. The outage probability of the UG protocol is a tight lower bound for the VG protocol as mentioned in [40]. We verify also that FG has slightly better performance than VG at the low average SNR region because the VG gain takes into account the noise power at the relay and, therefore, is limited by a gain lower bound when the fading coefficient is too small [41].

One can notice that the \textit{clipped gain} relaying protocol outperforms the other protocols for low SNR and has similar performance than \textit{variable gain} and \textit{unlimited gain} for mid-range to high SNR. Indeed, the CG allows avoiding of non favorable event for transmission (i.e. when the realized first-hop SNR is below the outage threshold and leads the system to an outage event) which is a more frequent event for low average SNR. Therefore, for low SNR, the CG protocol yields a better performance than other practical protocols and the benchmark protocol UG.

We compare also the performance of the CG protocol to that of the DF protocol, where the relay fully decodes the message from the source before transmitting. In [41], the authors compare the outage performance of UG relay to that of DF relay and conclude that DF relay clearly improves the performance at low average SNR in exchange of an increased complexity.
Figure 5.2 compares the performance of CG relay to that of VG relay (or its tight upper bound given by the performance of UG relay) and DF relay. One can see that the CG relay is a tradeoff between the VG relay and DF relay. Indeed, the CG relay, with only little additional complexity to calculate the SNR of the first-hop, can get closer to the performance of the DF relay protocol at low average SNR; at high average SNR however, the performance of all the protocols are similar as stated also in [41, 40].

The CG protocol can, therefore, be taken to be a good option for practical relaying protocol.

\[ \text{outage probability comparison for single-user and relay protocols} \]

![Outage probability comparison for single-user and relay protocols](image)

Figure 5.1: Outage performance vs average SNR of FG, UG, VG, and CG over Rayleigh channel for \( \gamma_2 = 5 \ast \gamma_1 \) and outage threshold \( \gamma_{th} = 10dB \)

\[ \text{normalized average snr of the first hop in dB} \]

5.2 Performance over Nakagami fading channel

In this section, we first recall the outage probability closed-form expressions which have been derived in [55, 42] for the FG, VG, and UG protocols over a Nakagami fading channel. \( m_i \) \((i = 1, 2)\) denotes the factor describing the severity of fading of the first and second link.

Then, using similar arguments and mathematical derivations as in [40, 55], we derive the outage probability closed-form expressions for DF and CG protocols. We
further use the resulting expression for the CG relaying protocol to derive a new outage probability expression for the UG protocol. Indeed, the UG outage probability expression given in [40] is for a less general channel model which assumes that source-relay and relay-destination links are i.i.d distributed with the same average SNR (i.e. $\bar{\gamma}_1 = \bar{\gamma}_2$) and same factor describing the severity of fading (i.e. $m = m_1 = m_2$). As we want to compare the effect of different average SNR and factor m values for the first hop and the second hop, we will use the new UG outage probability formula.

Finally, we compute the numerical results using the closed-form expressions and compare the performance of those relaying protocols over Nakagami fading channel.

5.2.1 Outage probability of the fixed gain protocol

The outage probability for a general FG protocol with a relay gain depending on a constant C has been derived in [55]. We apply this result to the normalized fixed
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Gain defined by equation (4.10).

\[ P_{\text{out},\text{Nakagami}}^{\text{VG}}(\gamma_{th}) = 1 - \sum_{k=0}^{m_1-1} \sum_{l=0}^{m_2-1} \left\{ 2 \binom{k}{l} \frac{(m_1-1)!(\gamma_1+1)\gamma_1^{k+m_2-l}}{\Gamma(m_1)\Gamma(m_2)k!} \right. \]

\[ \left. \left( \frac{m_1}{\gamma_1} \right)^{k+m_2-l} \left( \frac{m_2}{\gamma_2} \right)^{k+m_2-l} e^{-\frac{m_1\gamma_{th}}{\gamma_1}} \frac{K_{k-m_2-l}}{\gamma_1\gamma_2} \right\} \] (5.9)

where \( K_{\mu}[\cdot] \) is the \( \mu \)th order modified Bessel function of the second kind defined in [21] eq. (8.432) and \( \binom{\cdot}{\cdot} \) is the binomial coefficient.

### 5.2.2 Outage probability of the variable gain protocol

In [55], the outage probability of the variable gain protocol over independent and non-identical Nakagami channel is given.

\[ P_{\text{out},\text{Nakagami}}^{\text{VG}}(\gamma_{th}) = 1 - \frac{2 \cdot m_2^{m_2-1} \cdot \frac{m_1^{m_1-1} \Gamma(m_1)\Gamma(m_2)}{\gamma_1^{m_1} \gamma_2^{m_2}}}{\Gamma(m_1)\Gamma(m_2)} \cdot \left\{ \binom{k}{l} \binom{m_2-1}{r} \right. \]

\[ \left. \frac{1}{k!} \frac{\Gamma(m_2-l)}{\Gamma(m_2)} \cdot \frac{\Gamma(1+r-l)}{\Gamma(1+r)} \cdot \frac{\Gamma(m_1-l)}{\Gamma(m_1)} \cdot \frac{\Gamma(l)}{\Gamma(l)} \cdot \frac{\Gamma(m_1)}{\Gamma(m_1)} \cdot \frac{\Gamma(m_2)}{\Gamma(m_2)} \right\} \] (5.10)

where \( K_{\mu}[\cdot] \) is the \( \mu \)th order modified Bessel function of the second kind defined in [21] eq. (8.432) and \( \binom{\cdot}{\cdot} \) is the binomial coefficient.

### 5.2.3 Outage probability of the unlimited gain protocol

In [42], the closed-form expression for outage probability of unlimited gain protocol is given. However, the channel model is less general and assumes that source-relay and relay-destination links are i.i.d distributed with same average SNR (i.e. \( \bar{\gamma} = \gamma_1 = \gamma_2 \)) and same factor describing the severity of fading (i.e. \( m = m_1 = m_2 \)). The outage probability in this case results in a compact formula is terms of the Meijer’s G function.

\[ P_{\text{out},\text{Nakagami}}^{\text{UG}}(\gamma_{th}) = \sqrt{\pi} \left( \frac{m\gamma_{th}}{\bar{\gamma}} \right)^{2l} G_{23}^{1} \left( \begin{array}{c} \frac{m\gamma_{th}}{\bar{\gamma}} \\ 0, \frac{1}{2} \\ m-1,2m-1,-1 \end{array} \right) \] (5.11)
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where $G_{mn}^{pq}(.)$ is the Meijer’s G function defined in [21] Eq. (9.301).

However, we wish to compare the performance of the studied protocols over an independent and non-identical Nakagami channel with different fading factor $m$ and average SNR for the source-relay and relay-destination hop. Therefore, we derive a new closed-form expression for the unlimited gain’s outage probability using the clipped gain outage probability calculations from the Appendix: B.2.

The closed-form expression for the outage probability is:

$$P_{\text{out}, \text{Nakagami}}^{\text{UG}} = 1 - \frac{(m_1 - 1)!}{(\frac{m_2}{m_2})^{m_2}} e^{-\gamma_1 (\frac{m_1}{m_2} + \frac{m_2}{m_2})} \cdot \sum_{k=0}^{m_1-1} \sum_{l=0}^{m_2-1} \frac{1}{k!} \left( \frac{m_1 \gamma_{th}}{\gamma_1} \right)^k \frac{k^l}{l^r} \left( \frac{m_2 - 1}{\gamma_{th}} \right)^{\gamma_{th}} \sum_{r=0}^{\gamma_{th} - (r+1)} 2 \cdot \frac{m_1 m_2 \gamma_{th}^2}{\gamma_1 m_2^2} \cdot K_{l-r-1} \left( 2 \sqrt{\frac{m_1 m_2 \gamma_{th}^2}{\gamma_1 \gamma_2}} \right).$$

where $\Gamma(\cdot, \cdot)$ and $\Gamma(\cdot)$ are respectively the gamma and incomplete gamma function defined in [21] Eq. 8.310.1 and in [1] Eq. (6.5.3).

This expression is simply obtained by replacing the power normalization factor $G_N$ in the clipped gain protocol outage probability by 1 (Section 5.2.5).

Figure 5.3 shows that the two expressions of the outage probability for the unlimited gain give the same numerical results given that $\gamma_1 = \gamma_2$ and that fading factor $m_1$ and $m_2$ have the same integer value.

5.2.4 Outage probability of decode-and-forward protocol

With similar arguments as in [40] to compute the outage probability of decode-and-forward protocol over Rayleigh fading channel, the outage probability of decode-and-forward protocol over Nakagami fading channel can be easily derived given that an outage occurs if either one of the two links is in outage.

$$P_{\text{out}, \text{Nakagami}}^{\text{DF}}(\gamma_{th}) = 1 - \int_{\gamma_{th}}^{\infty} \frac{m_1^{m_1}}{\Gamma(m_1)} \gamma_{th}^{m_1-1} e^{-\frac{m_1 \gamma_{th}}{\gamma_1}} d\gamma \cdot \int_{\gamma_{th}}^{\infty} \frac{m_2^{m_2}}{\Gamma(m_2)} \gamma_{th}^{m_2-1} e^{-\frac{m_2 \gamma_{th}}{\gamma_2}} d\gamma$$

$$= 1 - \left( \frac{\Gamma(m_1, \frac{m_1 \gamma_{th}}{\gamma_1})}{\Gamma(m_1)} \right) \left( \frac{\Gamma(m_2, \frac{m_2 \gamma_{th}}{\gamma_2})}{\Gamma(m_2)} \right).$$

where $\Gamma(\cdot, \cdot)$ is the incomplete gamma function defined in [1] Eq. (6.5.3).
5.2.5 Outage probability of the clipped gain protocol

The outage probability for the clipped gain protocol over a Nakagami fading channel can be derived similarly to the Rayleigh fading case. The involved derivation steps follow closely those in [55] for VG and FG protocol (detailed calculations are given in the Appendix B.2).

The resulting closed-form expression for clipped gain outage probability is:

\[
P_{\text{out}, \text{Nakagami}}^{\text{CG}} = 1 - \frac{(m_1 - 1)!e^{-\gamma_1/(\gamma_2^1 + \gamma_2^2 G_N)}}{\left(\frac{\gamma_2^2}{m_2} \Gamma(m_2) \Gamma(m_1)\right)} \sum_{k=0}^{m_1 - 1} \sum_{l=0}^{m_2 - 1} \frac{1}{k! \gamma_1^l} \left(\frac{m_1 \gamma_{th}}{\gamma_1}\right)^k \left(\frac{m_2 - 1}{r}ight) \left(\frac{\gamma_{th}}{G_N}\right) \frac{(-1-r+l)}{2} \cdot 2 \cdot \left(\frac{m_1 m_2 \gamma_{th}^2}{\gamma_1 \gamma_2^1 G_N}\right)^{(r-1)(r-1)} K_{1-r-1} \left(2 \sqrt{\frac{m_1 m_2 \gamma_{th}^2}{\gamma_1 \gamma_2^1 G_N}}\right) \tag{5.14}\]

Figure 5.3: Comparison of the numerical values given by the two closed-form expressions of UG outage probability over a Nakagami fading channel, \(\gamma_1 = \gamma_2 = 5\) dB, \(\gamma_{th} = 10\) dB, \(m_1 = m_2 = 2, 5, 10\)
where \( \Gamma(\cdot) \) and \( \Gamma_\nu(\cdot, \cdot) \) are respectively the gamma and incomplete gamma function defined in [21] Eq. 8.310.1 and in [1] Eq. (6.5.3). \( G_N \) is the transmit power normalization factor over a Nakagami fading channel (4.15).

Besides verifying the numerical results from the simulations, the above expression also verifies, with adequate parameter values, the previously published results and the CG outage probability expression over Rayleigh fading channel. Thus, for \( m_1 = m_2 = 1 \), the theoretical expression \( P_{\text{out}, \text{Nakagami}}^{\text{CG}} \) (5.14) reduces to \( P_{\text{out}, \text{Rayleigh}}^{\text{CG}} \) (5.8) using simple algebraic manipulations and properties of modified Bessel functions ( [1]Eq. 9.6.6). Additionally, for \( G = 1 \) and \( m_1 = m_2 = 1 \), \( P_{\text{out}, \text{Nakagami}}^{\text{CG}} \) reduces to \( P_{\text{out}, \text{Rayleigh}}^{\text{UG}} \) (5.2).

5.2.6 Outage performance comparison over Nakagami fading channel

In this section, we use the closed-form expressions above to compute numerical results of the outage probabilities over a Nakagami fading channel for different single-user and single-relay protocols, and compare their performances. The outage threshold, \( \gamma_{th} \) is set to 10 dB and the relay-destination (or second-hop) average SNR is taken to be five times the source-relay (or first-hop) average SNR: \( \gamma_2 = 5 \times \gamma_1 \). The factors describing the severity of fading for the first- and second-hop (resp. source-relay and relay-destination link), \( m_1 \) and \( m_2 \), vary depending on the simulation.

In the first numerical computation, \( m_1 \) is set to 1, i.e. the fading model of the first hop is Rayleigh fading, and \( m_2 \) varies from 1 to 30 meaning that the second hop fading is the same or less severe than that of the first hop. For \( m_1 = 1 \) and \( m_2 = 1 \), the Nakagami relay channel reduces to the Rayleigh fading channel and CG outage probability verifies the result of the Rayleigh fading outage probability expression. In Figure 5.4, we notice that when \( m_2 \) superior or equal to 5 there is no significant change in CG outage probability performance. The same observation can however be generalized to any case where the factor \( m_1 \) is inferior to the factor \( m_2 \) meaning that the second link between the relay and the destination is subject to reduced amount of fading compared to the first link between the source and the relay.

In the second numerical computation, the factors describing the severity of the fading of the first- and second link are respectively taken to be: \( m_1 = 1 \) and \( m_2 = 5 \). The observations can however be generalized to any case where the factor \( m_1 \) is inferior to the factor \( m_2 \) meaning that the second link between the relay and the destination is subject to reduced amount of fading compared to the first link between the source and the relay.

As for Rayleigh fading, we can notice from Figure 5.5 that the outage probability...
of the UG protocol is a tight lower bound to that of the VG protocol and FG has a slightly better performance at small average SNR values.

In Figure 5.5 that compares the outage probabilities of the FG, UG, VG, CG and DF relaying protocols, the difference in performance can be seen more clearly than the case of a Rayleigh fading channel. The CG relay protocol, by saving power and transmitting only when the SNR of the first hop crosses the transmission threshold, outperform the FG, UG, and VG protocol for low and middle-range SNR. The CG protocol's performance gets also closer to the DF protocol performance when compared to the other practical protocol FG and VG. Although the DF protocol is expected to improve the performance in exchange of increased complexity, the CG protocol can be a good trade-off in terms of additional complexity vs performance gain. Compared to the FG and VG protocols, the CG protocol needs only a little additional complexity at the relay (to calculate the first hop SNR) to improve outage probability performance.
Figure 5.5: Outage performance vs average SNR of FG, UG, VG, CG, and DF over Nakagami fading channel with $\gamma_2 = 5 \cdot \gamma_1$, $\gamma_{th} = 10dB$, $m_1 = 2$, and $m_2 = 5$. 
Chapter 6

Performance of multiuser and multirelay protocols

In this chapter, we assess the outage performance of the scheduling protocols introduced in Chapter 4. First, we derive the outage probability closed-form expression for the all multiuser and multirelay protocols described in Chapter 4: $\gamma_1$-selection scheduling and $\gamma_1, \gamma_2$-selection scheduling. Then we compute the numerical results and compare their outage performances. We carry the comparison separately for respectively the $\gamma_1$-selection schemes and $\gamma_1, \gamma_2$-selection schemes as the latter schemes are more demanding in terms of CSI information and hence are expected to give much better performance.

6.1 $\gamma_1$-selection scheduling

The $\gamma_1$-scheduling protocols have been introduced in Chapter 4. In those protocols, the scheduler gives transmission or receiving turn based on the SNR condition of the first-hop link. The SNR condition of every first-hop link is known at the scheduler under the form of one bit CSI information that indicates if the link SNR is below or above a predefined selection threshold.

The first-hop or source-relay link is respectively the BS-relay link in DL communication and the user-relay link in UL communication. In the following, we consider the DL case and use subsequently the corresponding notation, for instance $M$ is the number of relays in the system but also the number of first-hop links in DL. However, we stress that derivation steps are similar in DL and UL and basically by changing from DL to UL notation, we would obtain the closed-form expression for UL communication.
We recall that $\gamma_1$-selection scheduling protocols in DL communication result in the selection of one first-hop link SNR among $M$ links and the selected first-hop SNR is subsequently $\gamma_1$.

To derive the outage probability expressions of those $\gamma_1$-selection scheduling schemes, we define the following notation:

The predefined threshold can be expressed in terms of the outage threshold as:

$$\tau = d \cdot \gamma_{th}$$

(6.1)

where $d$ is a real positive number.

Thus, the values of $d$ for different $\gamma_1$-selection scheduling schemes are:

$$\begin{cases} 
  d < 1 & \text{for TIS} \\
  d = 1 & \text{for TES} \\
  d > 1 & \text{for TSS}
\end{cases}$$

(6.2)

In those scheduling schemes, the considered relay gain for deriving the outage probability is:

$$\beta_{\gamma_1} = \frac{G_1}{E_1|h_1|}$$

(6.3)

$G_1$ denotes the power normalization factor of the relay transmit power that ensures unit average transmit power for a fair comparison between protocols:

$$G_1 = \left( 1 + \frac{\frac{\tau}{\gamma_1}}{\gamma_1} \cdot E_1 \left( \frac{\gamma_{th}}{\gamma_1} \right) \right)^{-1},$$

(6.4)

where $E_1$ is the exponential integral defined in (Eq. (5.1.1) [1])

Note that, in practice, $G$ can be defined by this expression if the scheduler always choose a dual-hop link the first-hop SNR superior to $\tau$, i.e. for all the $\gamma_1$-selection scheduling schemes (TIS, TES, TSS) in the hypothetical case of infinite number of users and relays ($M$ infinite) and in the case where the system stops transmitting when there is no first-hop link SNR meeting the requirement of being superior to $\tau$.

The outage probability can be expressed as:

$$P_{out} = P \left[ \frac{G_1 \gamma_1 \gamma_2}{\gamma_1 + G_1 \gamma_2} < \gamma_{th} \right]$$

$$= \int_0^{\infty} P_{\gamma_1} \left[ \frac{G_1 \gamma_1 \lambda}{\gamma_1 + G_1 \lambda} < \gamma_{th} \right] P_{\gamma_2}(\lambda) d\lambda$$

(6.5)
where $\gamma_1$ and $\gamma_2$ are respectively the SNR of the first- and second-hop of the selected dual-hop.

The relay gain $\beta_{\gamma_1}$, though similar to UG gain, does not tend to infinity for bad channel condition. Indeed, the selection criterion on the first-hop discards the bad channel condition leading to small $|h_1|$. 

### 6.1.1 TES scheduling: $\tau = \gamma_{th}$

In this section, the threshold $\tau$ on the signal-to-noise ratio of the first link is set to be equal to the outage threshold $\gamma_{th}$. The factor $d$ is therein equal to one.

**Hypothetical case with $M = \infty$**

In the case of an infinite amount of users and relays, there is an infinite number of first-hop links to choose from for scheduling. As a result, the selected source-relay link SNR $\gamma_1$ has a shifted exponential distribution over a Rayleigh fading channel [23].

$$p_{\gamma_1}(\gamma) = \frac{1}{\gamma_1} e^{-\frac{\gamma}{\gamma_1}} \cdot e^{-\gamma_{th}}$$  \hspace{1cm} (6.6)

The PDF of second hop SNR $\gamma_2$, however, remains unchanged and follows an exponential distribution: $p_{\gamma_2}(\gamma) = \frac{1}{\gamma_2} e^{-\frac{\gamma}{\gamma_2}}$, where $\gamma_2$ is the average second hop SNR.

The outage probability (Eq. (6.5)) is evaluated in Appendix C.1.1 to yield the following expression of the outage probability for an infinite amount of users and relays when $\tau$ is set equal to $\gamma_{th}$:

$$P^{\infty}_{out, TES} = 1 - \frac{2\gamma_{th}}{\sqrt{G_1 \gamma_1 \gamma_2}} \cdot K_1 \left[ \frac{2\gamma_{th}}{\sqrt{G_1 \gamma_1 \gamma_2}} \right] \cdot e^{-\frac{\gamma_{th}}{\gamma_2}}.$$  \hspace{1cm} (6.7)

**Finite M case**

To derive the outage probability expression for the case of a finite number of first links, the selection threshold is considered on the BS-relay SNR in DL communication.

In our system model, the BS is transmitting to $N$ users through $M$ relays in DL. The scheduler chose a dual-hop link such that this later has a first-hop SNR is above a predefined threshold $\tau$ (the protocol is described in Chapter 4). The choice of the second link is of a secondary importance in this scheduling scheme, it is randomly chosen among the users in the partition of the selected relay in downlink
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(Algorithm 1). In the case where there is no dual-hop link that has BS-relay SNR above the threshold, we consider IT for the TES protocol, i.e. no scheduling and no transmission.

As we assume that the M BS-relay links’ SNRs are i.i.d. variables, the probability that the system transmits and the probability that the system stays idle (respectively $P_M^1(\tau)$ and $P_M^2(\tau)$) can be subsequently derived.

The probability that the system stays idle or that the set $\mathcal{R}_\tau$ is empty is:

$$P_M^2(\tau) = (P_{BS-R}[\gamma \leq \tau])^M,$$

$$= (P_{\gamma_1}[\gamma \leq \tau])^M,$$

(6.8)

The probability that system transmits or that the set $\mathcal{R}_\tau$ has at least one element is:

$$P_M^1(\tau) = 1 - P_2 = 1 - (P_{BS-R}[\gamma \leq \tau])^M,$$

$$= 1 - (P_{\gamma_1}[\gamma \leq \tau])^M,$$

(6.9)

Where $P_{BS-R}[\gamma \leq \tau]$ or $P_{\gamma_1}[\gamma \leq \tau]$ is the probability that the SNR of a given BS-relay link is inferior or equal to the threshold $\tau$. The BS-relay links SNR are i.i.d. variables and follow an exponential distribution $p_{BS-R}(\gamma) = p_1(\gamma) = \frac{1}{\tau_1} e^{-\frac{\gamma}{\tau_1}}$ where $\gamma_1$ is the average SNR; $\gamma_1 = \bar{\gamma}_{BS-R} = \bar{\gamma}_{BS-R}$. Therefore,

$$P_{BS-R}[\gamma \leq \tau] = P_1[\gamma \leq \tau] = 1 - e^{-\frac{\gamma}{\tau_1}},$$

(6.10)

To derive the outage probability for a system using TES protocols with M relays, we consider that the system is in outage where there is no dual-hop link with $\gamma_{BS-R}$ superior to $\tau$. Furthermore, for the case where the transmission takes place, i.e. $\mathcal{R}_\tau$ is not empty, the probability that the TES system is in outage is the same as $P_{out,TES}^{\infty}$ (the outage probability of TES system with infinite M (6.7)) as the selected dual-hop has a BS-relay link’s SNR ($\gamma_1$) always superior to the threshold $\tau$ in this case. Therefore,

$$P_{out,TES}^M = P_M^1(\gamma_{th}) \cdot P_{out,TES}^{\infty} + P_M^2(\gamma_{th}),$$

(6.11)
TES: simulation results

Figure 6.1 which shows both the case where M (the number of first-hop link) is infinite and the case where M is finite, demonstrates that simulation verifies the numerical results of the theoretical formulas derived above. The simulation parameters are: $\gamma_1 = 5dB$, $a = 5$, and $n = 5000$, where respectively, $\gamma_1$ is the first-hop average SNR, $a$ is the factor between the $\gamma_1$ and $\gamma_2$ ($\gamma_2 = a \cdot \gamma_1$), and $n$ is the number of iterations.

In Figure 6.2, we assess the impact of the factor M (number of first-hop link) on the system outage performance. The simulation parameters are: $\gamma_{th} = 10dB$ and $a = 5$, where $\gamma_{th}$ is the outage threshold. We notice that for a relatively small number of relays M (M=10), the TES system has already similar performance to the case of an infinite number of first-hop links M.

![Figure 6.1: TES protocol simulation with $\gamma_1 = 5dB$, $a = 5$, $n = 5000$](image)
6.1.2 TIS scheduling: $\tau < \gamma_{th}$

In this section, the threshold $\tau$ on the signal-to-noise ratio of the first link is set to be inferior to the outage threshold $\gamma_{th}$. The factor $d$ is therein inferior to one.

**Hypothetical case with $M = \infty$**

As the argued in the preceding section, the distribution of the first-hop SNR $\gamma_1$ is a shift exponential distribution (6.6) and the distribution of the second-hop SNR is an exponential distribution.

Therefore, the outage probability of the TIS system with a selection threshold $\tau$ inferior to $\gamma_{th}$ can be computed to give:

$$
P_{out,TIS}^\infty = 1 - e^{\frac{\tau}{\gamma_1} - \gamma_{th}(\frac{1}{\gamma_1} + \frac{1}{\gamma_2})} \cdot \left( \frac{2\gamma_{th}}{\sqrt{G_1 \gamma_1 \gamma_2}} \right) \left( \frac{2\gamma_{th}}{\sqrt{G_1 \gamma_1 \gamma_2}} \right) K_1 \left( \frac{2\gamma_{th}}{\sqrt{G_1 \gamma_1 \gamma_2}} \right),
$$

(6.12)

where $K_1$ is the modified Bessel function of the first order defined in 9.6 [1].
The derivation of the closed-form expression of $P_{\text{out}, \text{TIS}}^{\infty}$ is given in Appendix C.1.2.

**Finite M case**

For the TIS protocol, as the selection threshold on the first-hop SNR is inferior to the outage threshold, IT transmission is considered, i.e. the BS transmission is interrupted when all the $\gamma_{BS-R}$ fall below the threshold $\tau$ in DL (see a Algorithm 1). Actually, when the first-hop signal-to-noise ratio is below $\tau$, the general upper bound states (4.5) that outage happens whatever the condition of the second-hop is. Therefore, stopping the transmission in such cases allows power saving without impairing the performance of the overall system.

With similar arguments as for TES protocol in the precedent section, the closed-form expression for DL outage probability of TIS system with $M$ relays is given by:

$$P_{\text{out}, \text{TIS}}^{M} = P[\gamma_{eq} < \gamma_{th}] = P_{1}^{M} \cdot P_{\text{out}, \text{TIS}}^{M}[\gamma > \tau] + P_{2}^{M} \cdot P_{\text{out}, \text{TIS}}^{M}[\gamma \leq \tau]$$

$$= P_{1}^{M} \cdot P_{\infty, \text{TIS}}^{\infty} + P_{2}^{M}$$ (6.13)

Where $P_{1}$ and $P_{2}$ are respectively the probability that system transmits and the probability that the system stays idle defined in (6.9) and (6.8).

**TIS: simulation results**

Figure 6.3, for both the case where $M$ (the number of first-hop link) is infinite and the case where $M$ is finite, shows how the simulation verifies the numerical results of the theoretical formulas derived above. The simulation parameters are: $d = 0.7$, $\overline{\gamma}_{1} = 5\, \text{dB}$, $a = 5$, and $n = 5000$, where respectively, $d$ is defined by Equation (6.1), $\overline{\gamma}_{1}$ is the first-hop average SNR, $a$ is the factor between the $\overline{\gamma}_{1}$ and $\overline{\gamma}_{2}$ ($\overline{\gamma}_{2} = a \cdot \overline{\gamma}_{1}$), and $n$ is the number of iteration.

In Figure 6.4, we assess the impact of the factor $M$ (number of first-hop link) on the system outage performance. The simulation parameters are: $d = 0.7$, $\gamma_{th} = 10\, \text{dB}$ and $a = 5$, where $\gamma_{th}$ is the outage threshold. We notice that, similarly to the TES system results, for a relatively small number of relay $M$, the TIS system has already similar performance to the case of an infinite number of first-hop links $M$. The number $M$ of first-hop links or of relay in DL has less influence on the outage performance in TIS scheduling than in TES scheduling, i.e. for $M=5$, the M-finite TIS scheme gives already an outage performance close to that of the M-infinite TIS...
scheme. More generally, when the parameter \( d \) (Eq. 6.1) decreases, the outage performance of the TIS scheme for a finite number of relays or first hop links \( M \) gets closer to that of TIS scheme with an infinite number of relays \( M \).

![Figure 6.3: TIS protocol simulation with \( \gamma_1 = 5dB, a = 5, n = 5000 \)](image)

We also consider the impact of the parameter \( d \) on the TIS scheduling outage performance. In Figure 6.5, the outage performances have been computed for \( d = 0.2, 0.4, 0.6, 0.8 \) with \( M = 10, \gamma_{th} = 10dB \) and \( a = 5 \). As expected, when \( d \) is small, i.e. when the selection threshold is small compared to the outage threshold, the outage performance is poorer.

6.1.3 TSS scheduling: \( \tau > \gamma_{th} \)

In this section, the considered scheduling scheme (TSS) has a selection threshold \( \tau \) superior to the outage threshold \( \gamma_{th} \), thus, \( d > 1 \).

**Hypothetical case with \( M = \infty \)**

In this case, the selected first-hop SNR is again always superior to the predefined selection threshold; thus, its distribution is a shift exponential distribution (Section 6.1.1). However, the probability density of the signal-to-noise ratio of the second hop \( \gamma_2 \) remain unchanged: 

\[
p_{\gamma_2}(\gamma) = \frac{1}{2\sigma_2^2} e^{-\frac{\gamma}{\sigma_2^2}}
\]
The outage probability of TSS scheduling scheme with an infinite $M$ has been computed in Appendix C.1.3 to give:

$$P_{out,TSS}^{\infty}(\gamma_{th}) = 1 - e^{-\frac{\gamma_{th}}{\tau \gamma_{th}} \tilde{G}_1(\tau - \gamma_{th})} - J_2 \quad (6.14)$$

Where $J_2$ is the integral:

$$J_2 = \int_{\gamma_{th}}^{\tau \gamma_{th}} e^{-\frac{G_1 \gamma_{th} \lambda}{\tilde{G}_1(\tau \gamma_{th} - \gamma_{th})}} \cdot p_{\gamma_2}(\lambda) d\lambda \quad (6.15)$$

In order to compare the performance of TSS scheme to other scheduling schemes, a numerical integration or an approximation of the integral $J_2$ may be used.

We can get a simple upper bound of $J_2$ by using the fact that $e^{-\frac{G_1 \gamma_{th} \lambda}{\tilde{G}_1(\tau \gamma_{th} - \gamma_{th})}} \leq 1$ as $\left[\frac{G_1 \gamma_{th} \lambda}{\tilde{G}_1(\tau \gamma_{th} - \gamma_{th})} - \frac{\tau \gamma_{th}}{\tilde{G}_1} \right]$ is positive given the condition (C.18) (see Appendix C.1.3). Therefore, the following inequality is obtained:

$$J_2 \leq \int_{\gamma_{th}}^{\tau \gamma_{th}} p_{\gamma_2}(\lambda) d\lambda, \quad (6.16)$$
We refer to this upper bound as $J_b$ in the sequel for more convenient reading. $J_b$ can be computed to yield a lower bound expression $P_{\text{out}}^L$ of the TSS outage probability (6.14):

$$P_{\text{out, TSS}}^\infty, L(\gamma_{th}) = 1 - e^{-\frac{\gamma_{th}}{\gamma_2^2}},$$  \hspace{1cm} (6.17)

To derive an upper bound for TSS outage probability (6.14), an approximation of $J_2$ can be used. $J_2$ has a negligible value, especially for high average SNR of the second link $\overline{\gamma}_2$ or a high value of $d$ (6.1) due to the property of $\gamma_2$ exponential distribution over a Rayleigh fading channel and the fact that the upper integration bound $\frac{\gamma_{th}}{\gamma_1(\tau - \gamma_{th})}$ is tending to the lower integration bound $\frac{\gamma_{th}}{\gamma_1}$ for a high value of $d$. Therefore, by neglecting the contribution of $J_2$ the following upper bound for outage probability (6.14) can be derived:

$$P_{\text{out, TSS}}^\infty, U(\gamma_{th}) = 1 - e^{-\frac{\gamma_{th}}{\gamma_2^2}},$$  \hspace{1cm} (6.18)

To carry on the comparison between outage performance of different $\gamma_1$ scheduling schemes, the numerical integration of the expression or the tight upper bound

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**Figure 6.5**: TIS outage probability for different value of $d$ with $M = 10$, $\gamma_{th} = 10 dB$ and $a = 5$
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$P_{\text{out},TSS}^U$ may be used. Note that $P_{\text{out},TSS}^U$ gives a slightly worst outage performance than the numerical integration of (6.14), $P_{\text{out},TSS}^\infty$, depending on the value of the parameter $d$ (respectively, $P_{\text{out},TSS}^L$ gives a slightly better outage performance compared to the numerical integration method).

In Figure 6.6, we compare the upper bound and lower bound numerical results to the simulation of the TSS scheme with an infinite number of first-hop $M$. The upper bound gives a closer result to the simulation than the lower bound. Therefore, we take the upper bound in the following as an approximation to the outage performance of the TSS scheme with an infinite $M$.

![Figure 6.6: Simulation of TSS outage probability for infinite M, its upper bound, and its lower bound](image)

In Figure 6.7, the upper bound expression (6.18) is taken as an approximation to the TSS outage probability and its numerical result is compared to that of the TSS scheme simulation. The simulation parameters are: $\gamma_{\text{th}} = 10dB$, average SNR of the first hop $\overline{\gamma}_1 = 5dB$, and average SNR of the second hop $\overline{\gamma}_2 = 5 \ast \gamma_1$.

The Monte Carlo simulation of outage performance of the TSS system with an infinite number of first links $M$ gives a close result to the theoretical upper bound.
outage probability for \( d \) greater or equal to 5 (Figure 6.7).

![TSS outage probability simulation for infinite M](image)

Figure 6.7: Simulation of TSS outage probability for infinite \( M = 1.5, 3, 5, 10, 20 \)

**TSS scheme with IT transmission**

In this configuration, the transmission is considered only if \( R_\tau \) is not empty, otherwise there is no transmission. Therefore, when there is no \( \gamma_{BS-R} > \tau \) (\( R_\tau = \emptyset \)), the system is in outage. When there is a selected \( \gamma_1 \) superior to the threshold, the BS transmits to the corresponding relay and the outage probability in this case is the same as \( P_{out,TSS}^\infty \).

It follows that outage probability of the TSS scheme with IT transmission, i.e. transmitting only if at least there is one relay among \( M \) with \( \gamma_{BS-R} > \tau \) can be written as:

\[
P_{out,TSS,IT}^M = P_1^M \cdot P_{out,TSS}^\infty + P_2^M
\]

(6.19)
As defined earlier, \( P_M^1 \) represents the probability in DL of having at least one BS-relay link with a SNR \( \gamma_{BS-R} \) above the threshold among the \( M \) first hop links \((6.9)\) and \( P_M^2 \) represents the probability of having no \( \gamma_{BS-R} \) satisfying this condition among all the links \((6.8)\).

For comparing the performance of \( \gamma_1 \)-scheduling schemes, we use \( P_{U,\text{TSS}}^{\infty} \) as an approximation to \( P_{\text{out,TSS}}^{\infty} \). Hence, we obtain the following approximation for \((6.19)\):

\[
P_{\text{M,U,\text{TSS,IT}}} = P_M^1 \cdot P_{\text{out,TSS}}^{\infty} + P_M^2
\]

\((6.20)\)

Note that \((6.21)\) is an upper bound to \((6.21)\), i.e. \( P_{\text{M,U,\text{TSS,IT}}} \) gives a slightly worst performance result than \( P_{\text{M,\text{TSS,IT}}} \).

In Figure 6.8, both the approximations using the lower bound and the upper bound on the TSS outage probability with IT transmission are considered. The simulation parameters are: \( M=10, \gamma_1 = 5 \text{dB}, \) and \( a = 5 \).

As with the upper bound, \( P_{\text{M,L,\text{TSS,IT}}} \) can be derived:

\[
P_{\text{M,L,\text{TSS,IT}}} = P_M^1 \cdot P_{\text{out,TSS}}^{\infty} + P_M^2
\]

\((6.21)\)

We can notice that both approximations give close results to the simulation for \( d \geq 5 \). However, we take in the sequel the upper bound for comparing the outage performance as \( P_{\text{out,TSS}}^{\infty} \) is a better approximation of \( P_{\text{out,TSS}}^{\infty} \).

**TSS scheme with CT transmission**

TSS scheduling with CT transmission in DL consists of choosing one relay \( \in R_\tau \) if \( R_\tau \) is not empty (\( \gamma_1 > \tau \)) or one random relay \( \in R \) if \( R_\tau \) is empty (\( \gamma_1 < \tau \)) (see Algorithm 3).

When there is no link with a first-hop SNR above the threshold, the scheduler choice does not lead necessarily to the system outage: among the \( \gamma_{BS-R} \) inferior to \( \tau \), the \( \gamma_{BS-R} \) superior to \( \gamma_{th} \) are included (as \( \tau > \gamma_{th} \)) and the choice of one of those \( \gamma_{BS-R} \) such that \( \gamma_{th} < \gamma_{BS-R} < \tau \) may not lead to an outage.

The outage probability of the TSS-CT scheme is derived in Appendix C.1.3. Its closed-form expression can be expressed as follows:

\[
P_{\text{out,TSS,CT}} = \left(1 - \left( e^{-\frac{\tau}{\bar{\gamma}_1}} \right)^M \right) P_{\text{out,TSS}}^{\infty} + \left( e^{-\frac{\tau}{\bar{\gamma}_1}} \right)^{M-1} \left( P[\gamma_{eq} < \gamma_{th}] - P_{\text{out,TSS}}^{\infty} \cdot \left( e^{-\frac{\tau}{\bar{\gamma}_1}} \right) \right)
\]

\((6.22)\)
where $P_{\text{out,TSS}}^\infty$ is the outage probability in the case where the amount of users and relays are infinite such that the selected $\gamma_1$ is superior to the threshold $\tau$, and

$$
P[\gamma_{eq} < \gamma_{th}] = 1 - \frac{2\gamma_{th}}{\sqrt{\gamma_1 \gamma_2}} K_1 \left( \frac{2\gamma_{th}}{\sqrt{\gamma_1 \gamma_2}} \right) e^{-\gamma_{th} \left( \frac{1}{\gamma_1} + \frac{1}{\gamma_2} \right)}, \quad (6.23)
$$

As for TSS scheduling with IT transmission, the outage probability expression above (C.32) can be approximated by replacing $P_{\text{out,TSS}}^\infty$ by its upper bound $P_{\text{out,TSS}}^{\infty,U}$. Thus, we obtain the following approximation that will be used in the sequel:

$$
P_{\text{out,TSS,CT}}^{M,U} = \left( 1 - (P_{\gamma_1[\gamma < \tau])^M \cdot P_{\text{out,TSS}}^{\infty,U} + (P_{\gamma_1[\gamma < \tau])^{M-1} \cdot (P[\gamma_{eq} < \gamma_{th}] - P_{\text{out,TSS}}^{\infty,U} \cdot (P_{\gamma_1[\gamma > \tau])}, \quad (6.24)
$$

Note that we can also use the lower bound on the TSS outage probability with an infinite M $P_{\text{out,TSS}}^{\infty,L}$ for our approximation (6.25). The two bounds give besides

Figure 6.8: Simulation of TSS outage probability with IT transmission with $d = 1.5, 3, 5, 10, 20, M=10, \tau_1 = 5dB$, and $a = 5$
close numerical results for $d \geq 5$ (Figure 6.9). However, we use $P_{out,TSS,CT}^{MU}$ to approximate the TSS outage performance for CT transmission in order to avoid an over optimistic evaluation of the TSS scheme outage performance for low values of $d$.

$$P_{out,TSS,CT}^{ML} = P[\gamma_{eq} < \gamma_{th}] = \left(1 - (P_{\gamma_1}[\gamma < \tau])^M\right) \cdot P_{out,TSS}^{\infty,L} + (P_{\gamma_1}[\gamma < \tau])^{M-1} \cdot \left(P[\gamma_{eq} < \gamma_{th}] - P_{out,TSS}^{\infty,L} \cdot (P_{\gamma_1}[\gamma > \tau])\right),$$

Figure 6.9: Simulation of TSS outage probability with CT transmission $d = 3, 5, 10, 20$

**TSS: simulation results**

In Figure 6.11, the performance of TSS with CT transmission is compared to that of the TSS scheme with IT transmission for different values of $d$. The simulation parameters are $a = 5$, $\gamma_{th} = 10$, $M = 10$.

For TES and TIS schemes, a choice among a finite number of first hop $M$ does not
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affect much the system performance while compare to the infinite number of first hop scheme. However, for the TSS scheme, the finite $M$ case converges to the ideal infinite $M$ case more slowly than for the other scheduling scheme based on $\tau < \gamma_{th}$ (TIS) and $\tau = \gamma_{th}$ (TES).

The choice of $d$ may affect the convergence of the outage probability in the finite $M$ first links case to the infinite performance limit. Namely, a small value of $d$ ensures a faster convergence of the outage probability to the limit performance. This slower convergence for high $d$ values can be explained by the reduced probability of finding a SNR $\gamma_{BS-R}$ (in DL) above the threshold $\tau$ for small average SNR $\bar{\gamma}_1$. Obviously, the number of first-hop to schedule from also influences the convergence of the TSS scheme. In figure 6.10, for increasing values of $M$, the outage performance of the TSS scheme with a finite $M$ converges more rapidly to that of the TSS scheme with an infinite $M$.

![TSS outage probability comparison for M=100,50,10](image)

Figure 6.10: Simulation of TSS outage probability with $M = 10, 50, 100$, $a = 5$, $\gamma_{th} = 10$, $d = 5$

Additionally, from Figure 6.11, one can notice that for small values of $d$, there is less difference in terms of outage performance between the CT and IT transmission: even though the CT transmission still performs slightly better for small average
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SNR.

The value of $d$ also influences the M infinite performance: as the value of $d$ increases, the infinite M outage performance for TSS scheme improves. Note that this observation is only true for M infinite. Indeed, for M finite, as $d$ increases, the outage performance is getting worst when looking at a fixed normalized $\gamma_1$ (SNR) value. This can also be seen from Figure 6.7 and Figure 6.6 illustrating the M infinite case in contrast to Figure 6.8 and 6.9, respectively, the finite M case with IT and CT transmission.

![TSS outage probability comparison between M=∞ and M finite](image)

Figure 6.11: Comparison between TSS with CT transmission and IT transmission $d = 3, 5, 10, 20$

6.1.4 Outage performance comparison for $\gamma_1$-scheduling schemes

In this section, the performance of the $\gamma_1$-scheduling schemes are studied. In Figure 6.12, the outage probability of the TSS scheduling protocol is approximated by its upper bounds $P_{\text{out},TSS}^{\infty, U}$ and $P_{\text{out},TSS}^M$; those upper bounds are respectively used for TSS outage probability performance when scheduling is done among an infinite number of first-hop links and a finite number of first-hop links. In Figure 6.13, the accuracy of the upper bound approximation is assessed and compared to the lower bound approximation. The simulation parameters are: $M = 50$, $a = 5$, and
$\gamma_{th} = 10dB$.

![Comparison of $\gamma_1$-scheduling scheme](image)

Figure 6.12: Comparison of $\gamma_1$-scheduling scheme for different $d$ values, with $M = 50$, $a = 5$, and $\gamma_{th} = 10dB$

**Convergence to infinite M performance limit**

Depending on the value of $d$, the TSS performance in the case of $M$ first-hop links converges more or less rapidly to the infinite performance limit with increasing SNR values. Thus, for a small $d$, the outage performance of TSS scheduling with $M$ first-hop links is closer to the TSS infinite performance limit than for a high value of $d$. Meanwhile, it can be seen from Figure 6.12 that the value of $d$ impacts also the TSS infinite performance limit: as expected, the outage performance limit improves with increasing $d$ values. This result can be explained by the fact that a higher selection threshold allows the selection of a dual-hop communication link with a higher first-hop instantaneous SNR. Indeed, there is always such dual-hop that satisfies this selection criterion among the theoretical infinite possibilities to choose from. Finally, the selected dual-hop with higher first-hop SNR has less probability to lead to an outage.
Performance comparison at low to medium SNR values

While comparing the infinite performance limit (infinite M) of the three $\gamma_1$-scheduling schemes in Figure 6.12, one can notice that both the TES and TSS schemes outperform the TIS scheme for any value of the ratio selection threshold to outage threshold ($d$ for TSS and $1/d$ for TIS). Likewise, the TSS infinite performance limit should be better than that of the TES scheme. Obviously, for low SNR region, the TSS scheme outperforms the TES scheme for an infinite number of first-hop links as the upper bound approximation curves of TSS outage performance are below those of the TES scheme. However, for high SNR region, the upper bound approximation curves for the TSS infinite performance limit are above the TES outage performance curves. In this case, the upper bound approximation of TSS infinite limit might not be accurate.

Figure 6.13 compares both lower bound and upper bound approximation. For small values of $d$, the gap between the two approximations is larger than for high $d$ values; thence, the upper bound approximation is more accurate for high $d$ values as the exact TSS outage performance is located within the bounds.

Although the upper bound does not give a highly accurate approximation in some cases, it can still be used to draw precise conclusions on TSS outage performance at low SNR. Thus for low SNR values, the TSS infinite performance limit outperforms the TES scheme even for small $d$ values such as 3. Additionally, for small $d$, the outage performance in the case of M first-hop links converges much faster to the infinite outage performance limit with increasing SNR. For a medium value of $d$ such as 5 to 10, as the TSS outage performance with M first-hop links converges more slowly to the performance limit. Therefore, the TES scheme outperforms slightly the TSS scheme for the smallest SNR values. Meanwhile, the outage performance of the TSS scheme with M first-hop links rapidly reach and then exceed that of the TES scheme as the SNR increases. Thus, the TSS with $d = 5$ still outperforms the TES scheme for small to medium SNR values.

Performance comparison at high SNR values

Besides the good performance at small to medium SNR region, the M finite outage performance of the TSS scheme also converges to the infinite M performance limit at high SNR (Figure 6.12). The TSS infinite performance limit theoretically outperforms the TES scheme for any $d$ value: the scheduled dual-hop in the TSS scheme has less probability to lead to an outage than in the TES scheme as the TSS selection threshold is higher than the TES selection threshold.
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Figure 6.13: TSS outage probability approximations compared to TES and TIS schemes
The approximation of the TSS outage performance cannot render this comparison result for high SNR, as the upper bound approximation of TSS outage probability is not accurate enough and thus is above the TES outage probability curves. However, the simulation of TES and TSS infinite outage performance limits in Figure 6.14 shows that the TSS outage performance curves for an infinite $M$ are actually always below that of the TES scheme.

Figure 6.14: Comparison between $\gamma_1$-scheduling schemes for different $d$ values, with $M = 50$, $a = 5$, and $\gamma_{th} = 10dB$

**Impact of relay density $M$**

Besides the selection threshold, another parameter that has an impact on the comparison result of the $\gamma_1$-scheduling schemes is the relay and user density. Figure 6.15 assesses the influence of the parameter $M$ that relates this density. The simulation parameters are: $d = 5$, $a = 5$ and $\gamma_{th} = 10dB$.

When increasing the value of $M$, the finite $M$ outage performance converges faster to the infinite $M$ outage performance for increasing SNR. This observation is true for the three $\gamma_1$-schemes. Meanwhile, the convergence of the TSS scheme with increasing
SNR is once more slower than that of TES and TIS schemes for any M value. As a result, the value of the parameter M impacts more the outage performance of the TSS scheme. Thus, for a high M value, the TSS scheme outperforms the TES and TIS scheme for any d value. For a medium M value such as 10 and for d = 5 for instance, the TSS scheme already outperforms the TES scheme for medium to high SNR values. However, for the smallest SNR values, the outage performance of the TSS scheme may be even worse than the TIS scheme. For a higher M value, the TSS scheme outperforms the TES scheme also for lower SNR.

![Comparison of γ₁-scheduling schemes](image)

**Figure 6.15:** Comparison between γ₁-scheduling schemes for different M values, with d = 5, a = 5, and γ₁h = 10dB

**Conclusion**

The TSS scheme is the most beneficial γ₁-scheduling scheme for a high relay and user density area. For low to medium user density, a trade-off between the value of d and the density factor M should be considered to optimize the benefit of TSS scheme: for a low density area, a small d should be chosen. Thus, for a sufficiently low density, TES should be the favored scheme. Note that TIS performance is in...
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...general lower than that of the TES and TSS scheme.

6.2 \(\gamma_1, \gamma_2\)-selection scheduling

In this section, we assess the outage performance of the \(\gamma_1, \gamma_2\)-scheduling schemes introduced in Chapter 4. For that purpose, we derive the closed-form expressions of the DS, DSS, and CS outage probabilities.

The considered relay gain for the system using those schemes is the UG gain. However, on the contrary to the single-user and single-relay system with UG relay, those scheduling schemes are applicable in practice. Indeed, as the \(\gamma_1, \gamma_2\)-scheduling schemes allow the system to transmit only in the case where the first-hop SNR is above the outage threshold, the relay gain always remain finite.

The equivalent end-to-end SNR is:

\[ \gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \]

Therefore the outage probability can be calculated as:

\[
P_{out} = P \left[ \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} < \gamma_{th} \right] = \int_0^\infty p_{\gamma_1} \left( \frac{\gamma_1 \lambda}{\gamma_1 + \lambda} < \gamma_{th} \right) p_{\gamma_2}(\lambda)d\lambda \tag{6.26}
\]

Where \(\gamma_1\) and \(\gamma_2\) are respectively the SNR of the first- and second-hop of the selected dual-hop.

6.2.1 Distributed scheduling

Under Rayleigh fading, the SNRs of the link 1 and 2 that compose the selected dual-hop of the distributed scheduling scheme follow a shift exponential distribution:

\[
p_{\gamma_i}(\gamma) = \frac{1}{\gamma_i e^{-\gamma_i}} e^{-\frac{\gamma}{\gamma_i}}, \quad i = 1, 2 \tag{6.27}
\]

Infinite number of relays M and users N

The outage probability in the case of an infinite number of relays M and users N has been calculated in [23].
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\[ P_{\text{out,DS}}^\infty = 1 - e^{-\frac{\gamma_{th}}{\gamma_1}} \left( 1 - e^{-\frac{\gamma_{th}}{\gamma_2}} \right) \frac{2\gamma_{th}}{\sqrt{G_m \gamma_1 \gamma_2}} K_1 \left( \frac{2\gamma_{th}}{\sqrt{G_m \gamma_1 \gamma_2}} \right), \quad (6.28) \]

We now consider the DS scheme for a finite number of M relays and N users. The DS scheme algorithm is given in Chapter 4. This scheduling scheme selects separately the first-and second-hop of the dual-hop link. Scheduling is done in a decentralized manner in every transmitter’s partition: each relay selects one user in its partition that has \( \gamma_{R-U} > \gamma_{th} \); then, the BS selects one relay that has \( \gamma_{BS-R} > \gamma_{th} \) and a selected user in its partition with \( \gamma_{BS-R} > \gamma_{th} \).

\( \gamma_{BS-R} \) and \( \gamma_{R-U} \) follow exponential distributions while \( \gamma_1 \) (selected \( \gamma_{BS-R} \)) and \( \gamma_2 \) (selected \( \gamma_{R-U} \)) follow a shift exponential distribution. Therefore, the probability that one BS-relay SNR is below the threshold \( \tau_1 = \gamma_{th} \) is given by:

\[ P[\gamma_{BS-R} < \gamma_{th}] = 1 - e^{-\frac{\gamma_{th}}{\gamma_1}}, \quad (6.29) \]

Similarly, one can calculate the probability for one relay-user link SNR to be below the threshold \( \tau_1 = \gamma_{th} \):

\[ P[\gamma_{R-U} < \gamma_{th}] = 1 - e^{-\frac{\gamma_{th}}{\gamma_2}}, \quad (6.30) \]

When none of the M relays satisfy the two conditions of having \( \gamma_{BS-R} > \gamma_{th} \) and a user in its partition with \( \gamma_{R-U} > \gamma_{th} \), the system stops transmitting. Thus, the probability for the system to cease the transmission is:

\[ P_{\text{stop,DS}} = \left[ e^{-\frac{\gamma_{th}}{\gamma_1}} \cdot \left( 1 - e^{-\frac{\gamma_{th}}{\gamma_2}} \right)^\frac{N}{M} + \left( 1 - e^{-\frac{\gamma_{th}}{\gamma_1}} \right)^M \right]^N, \quad (6.31) \]

The outage probability of DS scheme in the case of M relays and N users can be expressed in terms of \( P_{\text{stop,DS}} \) (6.31) and \( P_{\text{DS}}^\infty \) (6.28) as presented below:

\[ P_{\text{out,DS}}^{M,N} = (1 - P_{\text{stop,DS}}) \cdot P_{\text{DS}}^\infty + P_{\text{stop,DS}} \quad (6.32) \]

DSS

In the simplified DSS scheme, the scheduler chooses randomly one relay with \( \gamma_{BS-R} > \gamma_{th} \) in the BS partition. The selected relay then chooses randomly one user with \( \gamma_{BS-R} > \gamma_{th} \). The number of relays and users are M and N, respectively.
CHAPTER 6. PERFORMANCE OF MULTIUSER AND MULTIRELAY PROTOCOLS

The probability of transmitting in the case of the DSS scheme is given by the probability of having at least one relay with BS-relay SNR superior to the threshold $\gamma_{th}$ and one user with relay-user SNR superior to $\gamma_{th}$ within the partition of the selected relay. This probability can be written in terms of $P[\gamma_{BS-R} < \gamma_{th}]$ (6.29) and $P[\gamma_{R-U} < \gamma_{th}]$ (6.30) given above as:

$$P_{\text{transmit,DSS}} = \left(1 - P[\gamma_{BS-R} < \gamma_{th}]^M\right) \left(1 - P[\gamma_{R-U} < \gamma_{th}]^N\right).$$

Therefore, the outage probability of DS scheme in the case of $M$ relays and $N$ users can be written as:

$$P_{\text{out,DS}}^{M,N} = P_{\text{transmit,DSS}} \cdot P_{\text{out,DS}}^{\infty} + (1 - P_{\text{transmit,DSS}})$$

Comparison between DS and DSS schemes

In this section, we compare the outage performance of the DS and DSS protocols. For a constant number of users in the system $N = 400$, we assess both the case of a small number of relays $M=10$ (Figure 6.17) and the case of a large number of relays $M=200$ (Figure 6.16). Another fixed parameter in our numerical computation is $a = 5$. Obviously, the DS performs better for high values of $M$. Actually, the DSS scheme does not consider all the relay fulfilling the criterion $\gamma_{BS-R} > \gamma_{th}$ but only one of them. However, the difference in terms of outage performance remains surprisingly small for high $M$ values and this difference is even negligible for small $M$ values. The $M$ value is most likely small in practice as the number of relay should be kept reasonable compared to the number of users in the system. Thus, the simplified scheme DSS can be taken as a good decentralized scheduling scheme for any $M$ values.

6.2.2 Centralized scheduling: CS

At the BS, one feedback bit indicating whether the equivalent end-to-end SNR is above or below the outage threshold is available for the $N$ dual-hop links. The BS chooses randomly one dual-hop link that has $\gamma_{eq} > \gamma_{th}$. If there is no dual-hop link that satisfies this criterion then no transmission takes place.
CHAPTER 6. PERFORMANCE OF MULTIUSER AND MULTIRELAY PROTOCOLS

Figure 6.16: Comparison of DS and DSS schemes for $M = 200$ and $N = 400$
Figure 6.17: Comparison of DS and DSS schemes for $M = 10$ and $N = 400$
Infinite number of relays \( M \) and users \( N \)

In the case of an infinite number of relays \( M \) and users \( N \), the outage probability of the system is obviously zero. The CS scheme ensures zero outage probability for the theoretical case of an infinite amount of relays and users, as it is always possible to find a dual-hop link that satisfies the selection condition leading to a non-outage case.

\[
P_{\text{out},\tau_{eq}=\gamma_{th}} = P_{\gamma_{eq} > \gamma_{th}}[\gamma_{eq} \leq \gamma_{th}] = 0, \quad (6.35)
\]

Finite number of relays \( M \) and users \( N \)

The system is in outage if there is no dual-hop link with \( \gamma_{eq} > \gamma_{th} \). In the case of \( N \) users, the outage occurs when no dual-hop link associated with those \( N \) users satisfy the transmission condition.

We assume that all the links in the same partition experience the same fading and have an independent identically distribution. The probability of having the end-to-end SNR below the outage threshold is then the same for every group of dual-hop links associated to different relay. Thus, the outage probability of CS scheme can be written as follow:

\[
P_{M,N}^{\text{out,CS}} = \left[ P_{\text{out,}R}^{N} \right]^{M} \quad (6.36)
\]

Where \( P_{\text{out,}R}^{N} \) is the outage probability associated with one relay \( R \).

\[
P_{\text{out},R}^{N} = 1 + \sum_{k=1}^{\frac{N}{M}} \left( \frac{N}{M} \right) (-1)^{k} e^{-\frac{k\gamma_{th}}{\gamma_{1}}} \frac{2}{\gamma_{1}\gamma_{2}} 2 \sqrt{\frac{\gamma_{th}^{2}}{\gamma_{1}\gamma_{2}}} K_{1} \left[ 2 \sqrt{\frac{\gamma_{th}^{2}}{\gamma_{1}\gamma_{2}}} \right] \quad (6.37)
\]

The calculations details of the expression of \( P_{\text{out,}R}^{N} \) can be found in Appendix C.

6.2.3 Comparison between DS, DSS, and CS scheduling schemes

In Figure 6.18, we compare the performance of the Decentralized scheduling schemes: DS and DSS, and the Centralized scheduling scheme CS. The number of users is set to be equal to 40 and \( a = 5 \). The number of relays \( M \) takes the following values: 1, 2, 4, 8, and 10.

The outage performances of the CS and DS schemes improve for an increasing number of relays \( M \). Note that the \( DS_{\infty} \) curve sets the limit of the Decentralized
scheduling schemes’ outage performance. On the other hand, the outage performance of the DSS scheme improves with increasing values of M when those M values remain small compared to the number of users. Indeed, numerical results show that the performance of the DSS scheme is slightly degraded when a large number of relays is used relative to the number of users.

Obviously, the CS scheme outperforms the DS and DSS schemes in terms of outage probability. However, the DS and DSS schemes are quite competitive with CS at small number of relays relative to the number of users. Besides, this case is a more common case in a practical network as the number of relays deployed would be kept small compared to the number of users.

As a conclusion, the CS outperforms the DS and DSS schemes as expected but since it requires more feedback at the BS, the DS and DSS schemes may still be considered as an interesting option at small number of relays.

Figure 6.18: Comparison of DS, DSS, and CS schemes for \( N = 40 \)
Chapter 7

Conclusion

This final chapter summarizes the contributions of the thesis and suggests some areas for further research.

7.1 Contribution

In this thesis, we have proposed new relaying protocols for single-user and single-relay networks and multiuser and multirelay networks. Exploiting two upper bounds on the equivalent end-to-end SNR of the dual-hop channel, we have introduced a new clipped gain protocol for single-user and single relay network and new scheduling schemes for multiuser and multirelay network. The clipped gain protocol takes advantage of the first-hop CSI available at the relay to save transmit power at low SNR region. As a result, the CG protocol can be a good trade-off in terms of additional complexity vs performance gain. Compared to the practical AF protocols, the CG protocol needs only little additional complexity at the relay (to calculate the first hop SNR) to yield outage probability performance close to that of a DF relay.

For a multiuser and multirelay network model, we have proposed new efficient power-saving protocols that set transmission thresholds on the first- and second hop SNR. Those protocols help to avoid the cases when the SNRs of the links are too poor. γ1-scheduling schemes set a transmission threshold on the first-hop SNR. Thus, those protocols select one dual-hop link for transmission among the links having a favorable first-hop SNR. Both the DL and UL transmissions have been considered in our algorithms. We have carried the performance comparison of those schemes in terms of outage probability. We have derived for that purpose the closed-form expressions for TES, TIS outage probabilities and a lower and upper bound
approximations for the TSS outage probability. The numerical results show that TSS scheme is the most beneficial $\gamma_1$-scheduling scheme for a high relay and user density area. For low to medium user density, the value of the threshold relative to the density should be adjusted to optimize the benefit of TSS scheme. Indeed, for sufficiently low density, TES should be the favored scheme. We have also shown that TIS performance is in general lower than that of the TES and TSS scheme.

Finally, in this thesis, we have also presented the $\gamma_1, \gamma_2$-scheduling schemes in which the quality of both links of the dual-hop transmission is taken into account. We have introduced new Distributed Scheduling schemes (DS and DSS) and Centralized Scheduling scheme (CS) based on one-bit feedback information on the first- and second link SNR and one-bit feedback information on the end-to-end SNR, respectively. We have proposed the DL transmission algorithms for applying those protocols in a cellular system. The performance comparison is based on the close-form expressions of DS, DSS, and CS outage probabilities derived in this thesis. The numerical results show that even though CS outperforms the DS and DSS schemes, the DS and DSS schemes may still be considered as an interesting option since those latter schemes require more feedback at the BS and yield quite competitive performance for a small number of relays.

The main contribution of this thesis is the new energy-efficient protocols and their corresponding close-form expressions of outage probabilities. On the other hand, a general overview of the existing power-saving protocols for both single-user and single-relay network and multiuser and multirelay network is also provided. Furthermore, for the first time, a rigorous comparison in terms of outage performance has been carried out between practical protocols for a single-user and single-relay network.

7.2 Future research

An immediate extension to our work could be to assess the performance of the schemes with other performance measures such as BER or FER. Additionally, the amount of saving in transmission power could be quantified more precisely for those schemes.

The feedback allowing the CSI knowledge at the transmitter has been shown to bring tremendous benefit to the relay system. It will be interesting to investigate more involve protocols with more feedback bits in order to extend the saving in transmission power. Namely, with more feedback bits the relay could adapt more accurately its transmit power to the channel quality.
Appendix A

Transmit power normalization

The gain $G$ allows to make a fair comparison between different amplify-and-forward protocols and scheduling schemes by setting the transmitted power after the relay to 1.

The transmitted signal after the relay is:

$$ t(t) = \beta(\sqrt{E_1}h_1x(t) + n1), $$

Setting the transmitted power to unity results in the following equation:

$$ \varepsilon[|t|^2] = \varepsilon[E_1|h_1\beta|^2] + \varepsilon[\sigma_1^2\beta^2] = 1 \quad (A.1) $$

A.1 Transmit power normalization for clipped gain

A.1.1 CG relay over Nakagami fading channel

The relay transmit power is set to unity. Thus,

$$ \varepsilon[|t|^2] = \varepsilon[E_1|h_1\beta|^2] + \varepsilon[\sigma_1^2\beta^2] = 1 \quad (A.2) $$

for $\gamma_1 > \gamma_{th}$, the relay gain is $\beta = \frac{G_N}{E_1|h_1|^2}$ (section 4.2.5): $E_1|h_1|^2 = G_N$ and $\sigma_1^2\beta^2 = \frac{G_N}{\gamma}$ then

$$ \varepsilon[\sigma_1^2\beta^2] = \varepsilon[f_{\gamma_1}(\gamma)] = \int_{\gamma_{th}}^{\infty} f_{\gamma_1}(\gamma)p_{\gamma_1}(\gamma)d\gamma = \int_{\gamma_{th}}^{\infty} f_{\gamma_1}(\gamma)p_{\gamma_1}(\gamma)d\gamma = \int_{\gamma_{th}}^{\infty} \frac{G_N}{\gamma}p_{\gamma_1}(\gamma)d\gamma \quad (A.3) $$
APPENDIX A. TRANSMIT POWER NORMALIZATION

with $p_{\gamma_1}(\gamma) = \frac{\gamma^{m_1-1}e^{-\gamma/m_1}}{(m_1)^{m_1}\Gamma(m_1)}$ as the SNR of the first hop is gamma distributed. By changing the integration variable, $t = \frac{1}{\gamma}$ the integral can be written in terms of $E_n(z) = \int_{1}^{\infty} \frac{e^{-zt}}{t} dt$ [1] 5.1.4. Then, with the formula given by [1]5.1.45, (A.4) becomes:

$$\varepsilon[\sigma_1^2\beta^2] = \frac{G_N}{\Gamma(m_1)} \frac{m_1^{m_1}}{\gamma_1^{m_1}}\Gamma(m_1 - 1, \frac{m_1\gamma_1}{\gamma_1})$$ (A.5)

where $\Gamma(., .)$ is the incomplete gamma function [1] 6.5.

By taking for integration variable $t = \frac{m_1}{\gamma_1}$, the second part of the expression (A.2) can be written in terms of incomplete gamma function:

$$\varepsilon[E_1 | h_1 \beta |^2] = \int_{\gamma_1}^{\infty} G_N p_{\gamma_1}(\gamma) d\gamma$$ (A.6)

$$= \frac{G_N}{\Gamma(m_1)} \frac{m_1^{m_1}}{\gamma_1^{m_1}} \Gamma(m_1, \frac{m_1\gamma_1}{\gamma_1})$$ (A.7)

Using A.2, A.4, and A.6 along with simple algebraic manipulations gives $G_N$:

$$G_N = \frac{\Gamma[m_1]}{(m_1)^{m_1} \Gamma(m_1 - 1, \frac{m_1\gamma_1}{\gamma_1}) + \Gamma[m_1, \frac{m_1\gamma_1}{\gamma_1}] + \Gamma[m_1, \frac{m_1\gamma_1}{\gamma_1}] + \Gamma[m_1, \frac{m_1\gamma_1}{\gamma_1}]}$$ (A.8)

### A.1.2 CG relay over Rayleigh fading channel

One can also derive the expression of power normalization factor $G$ for a Rayleigh fading channel by setting $m = 1$ in Equation A.8.

Thus,

$$G = \frac{1}{e^{-\gamma_1/\gamma} + \frac{1}{\gamma_1} E_1\left(\frac{\gamma_1}{\gamma_1}\right)}$$ (A.9)

### A.2 Transmit power normalization for multiuser and multirelay systems

#### A.2.1 The case where a threshold $\tau$ is consider on the first link SNR

The normalization factor derived here ensures a unit average transmit power in the case where the SNR of the selected first-hop link SNR $\gamma_1$ is always taken to be superior or equal to $\tau$. This normalization factor can be used for the hypothetic case where the infinite number of users and relays permits the scheduler to pick only the dual-hop link with $\gamma_1 > \tau$, but also in the case of a finite number of users and
APPENDIX A. TRANSMIT POWER NORMALIZATION

Relays in which the system cease transmitting whenever there is no first-hop link that meet the requirement.

The selection condition on $\gamma_1$ gives an shift exponential distribution under Rayleigh fading:

$$p_{\gamma_1}(\gamma) = \frac{1}{\gamma_1} e^{-\frac{\gamma}{\gamma_1}}$$  \hfill (A.10)

The relay gain is defined as:

$$\beta = \frac{G}{E_1 | h_1 |^2}, \quad \gamma_1 > \tau,$$

With the later expression of $\beta$, $E_1 | h_1 \beta |^2 = G$ and $\sigma_1^2 \beta^2 = \frac{G}{\gamma}$. The expectation from the transmitted power normalization equation (A.1) can be evaluated as follows:

$$\mathbb{E}[E_1 | h_1 \beta |^2] = \int_{\gamma_{th}}^{\infty} G p_{\gamma_1}(\gamma)d\gamma = G$$  \hfill (A.11)

$$\mathbb{E}[\sigma_1^2 \beta^2] = \int_{\gamma_{th}}^{\infty} \frac{G}{\gamma} p_{\gamma_1}(\gamma)d\gamma$$

$$= G \int_{\gamma_{th}}^{\infty} e^{\frac{\gamma}{\gamma_1}} \frac{1}{\gamma_1} e^{-\frac{\gamma}{\gamma_1}} d\gamma$$

$$= \frac{G}{\gamma_1} e^{\frac{\gamma_{th}}{\gamma_1}} E_1 \left( \frac{\tau}{\gamma_1} \right)$$  \hfill (A.12)

Thus, Equation (A.1) along with the results A.11 and A.12 gives:

$$G = \frac{1}{1 + \frac{1}{\gamma_1} e^{\frac{\gamma_{th}}{\gamma_1}} E_1 \left( \frac{\tau}{\gamma_1} \right)}$$  \hfill (A.13)
Appendix B

Outage probability for clipped gain relay

B.1 Rayleigh fading channel

In Chapter 5, we have shown that the outage probability for the relay channel can be written in terms of conditional probability given \( \gamma_1 \leq \gamma_{th} \) (Chapter 4).

\[
P_{CG_{out}}(\gamma_{th}) = P[\gamma_1 \leq \gamma_{th}] + P\left[\frac{G\gamma_1\gamma_2}{\gamma_1 + G\gamma_2} \leq \gamma_{th} \mid \gamma_1 > \gamma_{th}\right] \cdot P[\gamma_1 > \gamma_{th}],
\]

The probability that the SNR of the first hop is below the threshold \( \gamma_{th} \) is defined by

\[
P[\gamma_1 < \gamma_{th}] = \int_0^{\gamma_{th}} p_{\gamma_1}(\gamma)d\gamma \quad (B.1)
\]

As over a Rayleigh fading channel, the signal-to-noise ratio \( \gamma \) follows an exponential distribution: \( p_{\gamma_i}(\gamma) = \frac{1}{\gamma_i} e^{-\frac{\gamma}{\gamma_i}}, i = 1, 2 \), we get:

\[
P[\gamma_1 < \gamma_{th}] = 1 - e^{-\frac{\gamma_{th}}{\gamma_1}} \quad (B.2)
\]

\[
P[\gamma_1 \geq \gamma_{th}] = e^{-\frac{\gamma_{th}}{\gamma_1}} \quad (B.3)
\]

The conditional probability of outage knowing the good condition of the first link
can be broken into two parts regarding the condition on the SNR of the second link.

\[
\begin{align*}
P \left[ \frac{G\gamma_1 \lambda}{\gamma_1 + G\lambda} < \gamma_{th} \mid \gamma_1 \geq \gamma_{th} \right] & \cdot P[\gamma_1 > \gamma_{th}] \\
= & \int_0^\infty P\left[ \frac{G\gamma_1 \lambda}{\gamma_1 + G\lambda} < \gamma_{th} \mid \gamma_1 \geq \gamma_{th} \right] \cdot P[\gamma_1 > \gamma_{th}] \cdot p_{\gamma_2}(\lambda) d\lambda \\
= & \int_0^{\gamma_{th}} P\left[ \frac{G\gamma_1 \lambda}{\gamma_1 + G\lambda} < \gamma_{th} \mid \gamma_1 \geq \gamma_{th} \right] \cdot P[\gamma_1 > \gamma_{th}] \cdot p_{\gamma_2}(\lambda) d\lambda \\
+ & \int_{\gamma_{th}}^\infty P\left[ \frac{G\gamma_1 \lambda}{\gamma_1 + G\lambda} < \gamma_{th} \mid \gamma_1 \geq \gamma_{th} \right] \cdot P[\gamma_1 > \gamma_{th}] \cdot p_{\gamma_2}(\lambda) d\lambda \quad (B.4)
\end{align*}
\]

The first integral composing the conditional probability of outage (B.4) can be calculated as follows:

\[
\int_0^{\gamma_{th}} P\left[ \frac{G\gamma_1 \lambda}{\gamma_1 + G\lambda} < \gamma_{th} \mid \gamma_1 \geq \gamma_{th} \right] \cdot P[\gamma_1 > \gamma_{th}] \cdot p_{\gamma_2}(\lambda) d\lambda \\
= \int_0^{\gamma_{th}} P[\gamma_1 > \gamma_{th}] \cdot p_{\gamma_2}(\lambda) d\lambda \\
= e^{-\frac{\gamma_{th}}{\gamma_1}} \cdot (e^{-\frac{\gamma_{th}}{\gamma_1 + G\lambda}} + 1) \quad (B.5)
\]

The second part of the conditional outage can be devised by writing the joint probability using the Bayes’ rule:

\[
P[\frac{G\gamma_1 \lambda}{\gamma_1 + G\lambda} < \gamma_{th} \mid \gamma_1 \geq \gamma_{th}] \cdot P[\gamma_1 > \gamma_{th}] = P[\gamma_{th} \leq \gamma_1 < \frac{G\gamma_1 \lambda}{\gamma_1 + G\lambda}]
\]

\[
P[\gamma_{th} \leq \gamma_1 < \frac{G\gamma_1 \lambda}{\gamma_1 + G\lambda}] \]

\[
\int_{\gamma_{th}}^\infty P\left[ \frac{G\gamma_1 \lambda}{\gamma_1 + G\lambda} < \gamma_{th} \mid \gamma_1 \geq \gamma_{th} \right] \cdot P[\gamma_1 > \gamma_{th}] \cdot p_{\gamma_2}(\lambda) d\lambda \\
= \int_{\gamma_{th}}^\infty \left[ e^{-\frac{\gamma_{th}}{\gamma_1}} - e^{-\frac{\gamma_{th}(\gamma_1 + G\lambda)}{\gamma_1(\gamma_1 + G\lambda)}} \right] \cdot p_{\gamma_2}(\lambda) d\lambda \\
= J_1 + J_2 \quad (B.6)
\]

The first integral is an integral from \( \frac{\gamma_{th}}{G} \) to infinity over the distribution of the first hop SNR and the second integral can be derived using the change of variable
\[ x = \gamma_2 - \frac{\gamma_{th}}{G} \] and [21] to yield the result.

\begin{align*}
J_1 &= e^{-\left(\frac{\gamma_{th}}{\gamma_1} + \frac{\gamma_{th}}{\gamma_2}\right)} \\
J_2 &= -e^{-\gamma_{th}} \left(\frac{1}{\gamma_1 + \frac{1}{\gamma_2}}\right) \left(\frac{2\gamma_{th}}{\sqrt{G\gamma_1\gamma_2}} K_1 \left(\frac{2\gamma_{th}}{\sqrt{G\gamma_1\gamma_2}}\right)\right)
\end{align*}

The closed form expression for outage probability of the clipped gain protocol can be expressed using (B.2), (B.5), \(J_1\) (B.7), and \(J_2\) (B.8).

\[ P_{CG}^{out} = 1 - e^{-\gamma_{th}} \left(\frac{1}{\gamma_1 + \frac{1}{\gamma_2}}\right) \left(\frac{2\gamma_{th}}{\sqrt{G\gamma_1\gamma_2}} K_1 \left(\frac{2\gamma_{th}}{\sqrt{G\gamma_1\gamma_2}}\right)\right) \]

### B.2 Nakagami fading channel

The outage probability (2.8) for the clipped gain relay can be written as:

\[ P_{CG}^{out} = P[\gamma_{eq}^{CG} < \gamma_{th}] \] (B.9)

where \(\gamma_{eq}^{CG}\) is the end-to-end equivalent SNR (4.13).

One can calculate the outage probability of the clipped gain relay protocol over Nakagami fading similarly to the case over Rayleigh fading using the conditional probability (see Chapter 5). Meanwhile, given that the clipped gain protocol prevents transmission in the case where the first hop SNR drop below the outage threshold \(\gamma_{th}\) and that those cases with the first-hop SNR inferior to the outage threshold always lead to an outage event independently of the relay protocol (see general bound Eq. (4.5)), the outage probability (B.9) for Nakagami fading channel can be calculated simply as:

\[ P_{CG, Nakagami}^{out} = P[\frac{G_N\gamma_1\gamma_2}{\gamma_1 + G_N\gamma_2} < \gamma_{th}] \] (B.10)

\[ P_{CG, Nakagami}^{out} = \int_0^\infty P[\frac{G_N\gamma_1\gamma_2}{\gamma_1 + G_N\gamma_2} < \gamma_{th}|\gamma_2] p_{\gamma_2}(\gamma_2) d\gamma_2 \] (B.11)

where \(p_{\gamma_2}(\gamma_2)\) is the gamma probability density function of the second hop SNR.
B.11 can be written as the sum of two integrals $I_1$ and $I_2$:

$$P_{\text{out, Nakagami}}^{CG} = \int_{0}^{\gamma_{th}} P[\gamma_1 < \frac{G_N\gamma_2\gamma_{th}}{G_N\gamma_2 - \gamma_{th}} | \gamma_2] p_{\gamma_2} (\gamma_2) \, d\gamma_2 + \int_{\gamma_{th}}^{\infty} P[\gamma_1 < \frac{G_N\gamma_2\gamma_{th}}{G_N\gamma_2 - \gamma_{th}} | \gamma_2] p_{\gamma_2} (\gamma_2) \, d\gamma_2$$

$$= I_1 + I_2 \quad \text{(B.12)}$$

with:

$$I_1 = \int_{0}^{\gamma_{th}} \frac{1}{G_N} \cdot p_{\gamma_2} (\gamma_2) \, d\gamma_2$$

$$= 1 - \frac{\Gamma \left[m_2, \frac{m_2 \gamma_{th}}{G_N \gamma_2} \right]}{\Gamma[m_2]} \quad \text{(B.13)}$$

and

$$I_2 = \int_{\gamma_{th}}^{\infty} \left[ 1 - \frac{\Gamma \left[m_1, \frac{m_1 \gamma_{th} G_N \gamma_2}{\gamma_{th} G_N - \gamma_{th}} \right]}{\Gamma[m_1]} \right] \cdot p_{\gamma_2} (\gamma_2) \, d\gamma_2$$

$$= \frac{\Gamma \left[m_2, \frac{m_2 \gamma_{th}}{G_N \gamma_2} \right]}{\Gamma[m_2]} - J \quad \text{(B.14)}$$

where $\Gamma(.)$ and $\Gamma(., .)$ are respectively the gamma and incomplete gamma function defined in [21, Eq. 8.310.1] and in [1, Eq. 6.5.3], and $J = \int_{\gamma_{th}}^{\infty} \frac{\Gamma[m_1, \frac{m_1 \gamma_{th} G_N \gamma_2}{\gamma_{th} G_N - \gamma_{th}}]}{\Gamma[m_1]} \cdot p_{\gamma_2} (\gamma_2) \, d\gamma_2$.

With the following notation: $y = \frac{\gamma_{th}}{G_N}$, $b = \frac{m_2}{\gamma_{th}}$, $d = \frac{m_1 \gamma_{th}}{y}$, $a = m_2$, $c = m_1$, $e = 0$, $J$ can be written in a more general form as:

$$J = \int_{y}^{\infty} x^{a-1} e^{-bx} \Gamma \left[ c, d \left( \frac{x + e}{x - y} \right) \right] \, dx \quad \text{(B.15)}$$

As $d$ and $e$ are real numbers and restricting $c$ to be a positive integer, the incomplete gamma function in equation (B.15) can be written as follow using [21, Eq. 8.352.2, Eq. 1.111] :

$$\Gamma \left[ c, d \left( \frac{x + e}{x - y} \right) \right] = (c - 1)! e^{-d} e^{-\frac{d x + e}{x - y}} \sum_{k=0}^{c-1} \sum_{l=0}^{k} \frac{d^k}{k!} \left( \frac{y + e}{x - y} \right)^l \quad \text{(B.16)}$$
APPENDIX B. OUTAGE PROBABILITY FOR CLIPPED GAIN RELAY

Therefore, after transformation of the integration variable, $J$ becomes:

$$J = (c - 1)!e^{-(d+by)} \sum_{k=0}^{c-1} \sum_{l=0}^{a-1} \sum_{r=0}^{d} \frac{d^{k}}{k!} \binom{k}{l} (a - 1)^{l}(y + e)^{a-r-1} \int_{0}^{\infty} x^{r-l}e^{bx}e^{-d\frac{x}{y^{l}}} dx \quad \text{(B.17)}$$

The integral in the equation (B.17) can be solved using [21, Eq. 3.471.9]. Thus, $J$ can be expressed with the initial variables (replacing the notation $y = \frac{n_{1}}{\gamma_{1}}$, $b = \frac{n_{2}}{\gamma_{2}}$, $d = \frac{n_{1}\gamma_{2}}{\gamma_{1}}$, $a = m_{2}$, $c = m_{1}$, $e = 0$ in its expression) as:

$$J = \frac{(m_{1} - 1)!e^{\left(-m_{1}\gamma_{th}\gamma_{1}^{2}f_{th}\gamma_{2}G_{N}\right)}}{\left(\frac{n_{2}}{m_{2}}\right)^{m_{2}} \Gamma(m_{2})\Gamma(m_{1})} \sum_{k=0}^{m_{1}-1} \sum_{l=0}^{m_{2}-1} \sum_{r=0}^{d} \frac{1}{k!} \left(\frac{m_{1}\gamma_{th}}{\gamma_{1}}\right)^{k} \binom{k}{l} \left(\frac{m_{2} - 1}{r}\right) \left(\frac{\gamma_{th}}{G_{N}}\right)^{\frac{\gamma_{th}}{G_{N}} - \frac{1}{2} - \frac{r + 1}{2}} \cdot 2 \cdot \left(\frac{m_{1}\gamma_{th}^{2}}{\gamma_{1}m_{2}}\right)^{\frac{\gamma_{th}}{G_{N}} - \frac{1}{2}} K_{l-r-1} \left(2 \sqrt{\frac{m_{1}m_{2}\gamma_{th}^{2}}{\gamma_{1}\gamma_{2}G_{N}}}\right) \quad \text{(B.18)}$$

Therein, the outage probability expression over Nakagami fading channel is:

$$P_{CG_{out,Nakagami}} = 1 - \frac{(m_{1} - 1)!e^{\left(-m_{1}\gamma_{th}\gamma_{1}^{2}f_{th}\gamma_{2}G_{N}\right)}}{\left(\frac{n_{2}}{m_{2}}\right)^{m_{2}} \Gamma(m_{2})\Gamma(m_{1})} \sum_{k=0}^{m_{1}-1} \sum_{l=0}^{m_{2}-1} \sum_{r=0}^{d} \frac{1}{k!} \left(\frac{m_{1}\gamma_{th}}{\gamma_{1}}\right)^{k} \binom{k}{l} \left(\frac{m_{2} - 1}{r}\right) \left(\frac{\gamma_{th}}{G_{N}}\right)^{\frac{\gamma_{th}}{G_{N}} - \frac{1}{2} - \frac{r + 1}{2}} \cdot 2 \cdot \left(\frac{m_{1}\gamma_{th}^{2}}{\gamma_{1}m_{2}}\right)^{\frac{\gamma_{th}}{G_{N}} - \frac{1}{2}} K_{l-r-1} \left(2 \sqrt{\frac{m_{1}m_{2}\gamma_{th}^{2}}{\gamma_{1}\gamma_{2}G_{N}}}\right) \quad \text{(B.19)}$$
Appendix C

Multiuser and multirelay protocols

In this appendix, we evaluate the closed-form expression of the multiuser and multirelay scheduling schemes introduced in Chapter 4.

C.1 $\gamma_1$-selection scheduling

In this section, we derive the closed-form outage probability expression for the $\gamma_1$-selection scheduling protocols: TES, TIS, and TSS.

C.1.1 TES scheduling: $\tau = \gamma_{th}$

In this section, the threshold $\tau$ on the signal-to-noise ratio of the first link is set to be equal to the outage threshold $\gamma_{th}$. The factor $d$ is therein equal to one.

Hypothetical case with $M = \infty$

Outage probability is given by the following integral:

$$P_{out} = P \left[ \frac{G_1 \gamma_1 \gamma_2}{\gamma_1 + G_1 \gamma_2} < \gamma_{th} \right]$$

$$= \int_0^\infty P_{\gamma_1} \left[ \frac{G_1 \gamma_1 \lambda}{\gamma_1 + G_1 \lambda} < \gamma_{th} \right] p_{\gamma_2}(\lambda) d\lambda \quad (C.1)$$

Given that the scheduling threshold $\tau$ is equal to the outage threshold $\gamma_{th}$, the pdf of $\gamma_1$ (the selected first-hop SNR) $p_{\gamma_1}(\gamma)$ is a shift exponential distribution (6.6). The pdf function of $\gamma_2$ remains unchanged; $p_{\gamma_2}(\gamma)$ is an exponential distribution.
By rearranging the terms in the inequality: $\frac{G_1 \gamma_2}{\gamma_1 + G_1 \gamma_2} < \gamma_{th}$ and isolating the term $\gamma_1$, one obtains:

\[
\begin{cases}
\gamma_1 > \frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}, & \text{when } \lambda < \frac{\gamma_{th}}{G_1} \\
\gamma_1 < \frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}, & \text{when } \lambda > \frac{\gamma_{th}}{G_1}
\end{cases}
\] (C.2)

Therefore,

\[
P\left[\frac{G_1 \gamma_1 \lambda}{\gamma_1 + G_1 \lambda} < \gamma_{th}\right] = \begin{cases} 
1, & \lambda < \frac{\gamma_{th}}{G_1} \\
P[\gamma_1 < \frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}], & \lambda > \frac{\gamma_{th}}{G_1}
\end{cases}
\] (C.3)

where,

\[
P[\gamma_1 < \frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}] = \int_{\tau}^{\frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}} p_{\gamma_1}(\gamma) d\gamma
\]
\[
= e^\frac{\gamma_{th}}{G_1 \lambda - \gamma_{th}} \left[ e^{-\frac{\gamma_{th}}{G_1 \lambda - \gamma_{th}}} - e^{-\frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}} \right]
\]
\[
= 1 - e^\frac{\gamma_{th}}{G_1 \lambda - \gamma_{th}} \cdot e^{-\frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}}
\] (C.4)

The outage probability can now be written in terms of two integrals:

\[
P_{\text{out,TES}}^\infty = I_{1,TES} + I_{2,TES},
\] (C.5)

where,

\[
I_{1,TES} = \int_{0}^{\frac{\gamma_{th}}{G_1 \lambda - \gamma_{th}}} p_{\gamma_2}(\lambda) d\lambda = 1 - e^{-\frac{\gamma_{th}}{G_1 \lambda - \gamma_{th}}}
\] (C.6)

and,

\[
I_{2,TES} = \int_{\frac{\gamma_{th}}{G_1 \lambda - \gamma_{th}}}^{\infty} P[\gamma_1 < \frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}] \cdot p_{\gamma_2}(\lambda) d\lambda
\] (C.7)
\[
= \int_{\frac{\gamma_{th}}{G_1 \lambda - \gamma_{th}}}^{\infty} p_{\gamma_2}(\lambda) d\lambda + e^{\frac{\gamma_{th}}{G_1 \lambda - \gamma_{th}}} \int_{\frac{\gamma_{th}}{G_1 \lambda - \gamma_{th}}}^{\infty} e^{-\frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}} \cdot p_{\gamma_2}(\lambda) d\lambda
\] (C.8)
\[
= e^{-\frac{\gamma_{th}}{G_1 \lambda - \gamma_{th}}} + e^{\frac{\gamma_{th}}{G_1 \lambda - \gamma_{th}}} \cdot J_1,
\] (C.9)

The integral $J_1$ can be calculated using [21, Eq. 3.471.9]. With (C.5), we obtain the following expression for TES outage probability with an infinite amount of users and relays ($M = \infty$):
\[ P^{\infty}_{\text{out,TES}} = 1 - \frac{2\gamma_{th}}{\sqrt{G_1\gamma_1\gamma_2}} K_1 \left[ \frac{2\gamma_{th}}{\sqrt{G_1\gamma_1\gamma_2}} \right] \cdot e^{-\frac{2\gamma_{th}}{\gamma_1\gamma_2}}, \]  

(C.10)

C.1.2 TIS scheduling: \( \tau < \gamma_{th} \)

In this section, the threshold \( \tau \) on the signal-to-noise ratio of the first link is set to be inferior to the outage threshold \( \gamma_{th} \). The factor \( d \) is therein inferior to one.

**Hypothetical case with \( M = \infty \)**

The outage probability defined by Equation (C.1) can also be written in terms of two integrals in the case of TIS system:

\[
P^{\infty}_{\text{out,TIS}} = \int_{0}^{\frac{\gamma_{th}}{G_1}} P[\gamma_1 \geq \frac{G_1\lambda\gamma_{th}}{G_1 - \gamma_{th}}] \cdot p_{\gamma_2}(\lambda)d\lambda
+ \int_{\frac{\gamma_{th}}{G_1}}^{\infty} P[\gamma_1 < \frac{G_1\lambda\gamma_{th}}{G_1 - \gamma_{th}}] \cdot p_{\gamma_2}(\lambda)d\lambda
= I_{1,TIS} + I_{2,TIS},
\]  

(C.11)

Where \( P[\gamma_1 \geq \frac{G_1\lambda\gamma_{th}}{G_1 - \gamma_{th}}] \) is equal to 1 as \( \frac{G_1\lambda\gamma_{th}}{G_1 - \gamma_{th}} \) is negative and \( P[\gamma_1 < \frac{G_1\lambda\gamma_{th}}{G_1 - \gamma_{th}}] \) has been calculated earlier in (C.4).

As for the TES case:

\[ I_{1,TIS} = \int_{0}^{\frac{\gamma_{th}}{G_1}} p_{\gamma_2}(\lambda)d\lambda = 1 - e^{-\frac{\gamma_{th}}{\gamma_1\gamma_2}}, \]  

(C.12)

The integral \( I_{2,TIS} \) can be calculated with the help of the formulas [21, 3.471.9] to give:

\[
I_{2,TIS} = \int_{\frac{\gamma_{th}}{G_1}}^{\infty} e^{\frac{\gamma_{th}}{G_1\gamma_1\gamma_2}} \left[ e^{-\frac{\gamma_{th}}{\gamma_1\gamma_2}} - e^{-\frac{G_1\lambda\gamma_{th}}{G_1 - \gamma_{th}}} \right] p_{\gamma_2}(\lambda)d\lambda
= e^{-\frac{\gamma_{th}}{\gamma_1\gamma_2}} - e^{-\frac{\gamma_{th}}{\gamma_1\gamma_2}\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)} \cdot \left( \frac{2\gamma_{th}}{\sqrt{G_1\gamma_1\gamma_2}} K_1 \left[ \frac{2\gamma_{th}}{\sqrt{G_1\gamma_1\gamma_2}} \right] \right),
\]  

(C.13)

Where \( K_1 \) is the modified Bessel function of the first order defined in 9.6 [1].

The outage probability of the TIS system with the threshold \( \tau \) inferior to \( \gamma_{th} \) is then:

\[
P^{\infty}_{\text{out,TIS}} = 1 - e^{-\frac{\gamma_{th}}{G_1\gamma_1\gamma_2}\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)} \cdot \left( \frac{2\gamma_{th}}{\sqrt{G_1\gamma_1\gamma_2}} K_1 \left[ \frac{2\gamma_{th}}{\sqrt{G_1\gamma_1\gamma_2}} \right] \right),
\]  

(C.15)
C.1.3 TSS scheduling: \( \tau > \gamma_{th} \)

In this section, the selection threshold \( \tau \) on the first link signal-to-noise ratio is set to be superior to the outage threshold \( \gamma_{th} \). The factor \( d \) is therein superior to one.

**Hypothetical case with \( M = \infty \)**

In this case, the selected first-hop SNR is again always superior to the predefined selection threshold; thus, its distribution is a shifted exponential distribution. However, the probability density of the signal-to-noise ratio of the second hop \( \gamma_2 \) remains unchanged: \( p_{\gamma_2}(\gamma) = \frac{1}{\gamma_2} e^{-\frac{\gamma}{\gamma_2}} \)

By rearranging terms and isolating \( \gamma_1 \) in the inequality: \( \frac{G_1 \gamma_1 \lambda}{G_1 + G_1 \lambda} < \gamma_{th} \), the overall outage probability given by Equation (C.1) can be expressed for the two following cases:

\[
\begin{cases}
1, & \lambda < \frac{\gamma_{th}}{G_1} \\
\frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}, & \lambda > \frac{\gamma_{th}}{G_1}
\end{cases}
\]

(C.16)

Where,

\[
P\left[ \gamma_1 < \frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}} \right] = \int_{\tau}^{G_1 \gamma_{th} \lambda} p_{\gamma_1}(\gamma) d\gamma
\]

(C.17)

(C.17) is non zero for \( \tau < \frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}} \). Namely, as the variable \( \lambda \) varies with respect to the distribution \( p_{\gamma_2} \), this condition can be seen as an upper bound for the SNR of the second link \( \gamma_2 \)

\[
\gamma_2 < \frac{\tau \gamma_{th}}{G_1 (\tau - \gamma_{th})}
\]

(C.18)

If this condition (C.18) is fulfilled, the probability \( P[\gamma_1 < \frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}] \) can be calculated using the integral (C.17) to give the following result:

\[
P\left[ \gamma_1 < \frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}} \right] = e^{\frac{\gamma_{th}}{\lambda}} \cdot \left[ e^{-\frac{\gamma_{th}}{\lambda}} - e^{-\frac{G_1 \gamma_{th} \lambda}{G_1 \lambda - \gamma_{th}}} \right]
\]

(C.19)

Therein, the outage probability (C.1) can be written as a sum of two integrals \( I_{1,TSS} \) and \( I_{2,TSS} \). \( I_{1,TSS} \) and \( I_{2,TSS} \) result from slitting the outage probability integration into two cases (C.16). Therefore:
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\[
P_{\text{out,TSS}}^\infty(\gamma_{th}) = I_1 + I_2,
\]

\[
I_1 = \int_0^{\frac{\gamma_{th}}{\tau_1}} p_{\gamma_2}(\lambda) d\lambda = 1 - e^{-\frac{\gamma_{th}}{\tau_1} \cdot \frac{G_1}{\gamma_2} \cdot \lambda}.
\]

\[
I_2 = \int_{\frac{\gamma_{th}}{\tau_1}}^{\infty} e^{-\frac{\gamma_{th}}{\tau_1}} \cdot \left[ e^{-\frac{\gamma_{th}}{\tau_1}} - e^{-\frac{G_1 \gamma_{th}}{\gamma_2 \cdot (G_1 \lambda - \gamma_{th})}} \right] p_{\gamma_2}(\lambda) d\lambda
\]

\[
= - e^{-\frac{\gamma_{th}}{\tau_1} \cdot \frac{G_1}{\gamma_2} \cdot \lambda} + e^{-\frac{\gamma_{th}}{\tau_1} \cdot \frac{G_1}{\gamma_2} \cdot \lambda} - J_2
\]

where the first equality in (C.22) results from the upper limit on \(\gamma_2\) derived in (C.18) for the probability (C.17) to be non zero.

The outage probability (C.1) can be written in terms of \(J_2\) as follows:

\[
P_{\text{out,TSS}}^\infty(\gamma_{th}) = 1 - e^{-\frac{\gamma_{th}}{\tau_1} \cdot \frac{G_1}{\gamma_2} \cdot \lambda} - J_2
\]

where \(J_2\) is the integral:

\[
J_2 = \int_{\frac{\gamma_{th}}{\tau_1}}^{\infty} e^{-\frac{\gamma_{th}}{\tau_1} \cdot \frac{G_1}{\gamma_2} \cdot \lambda} \cdot p_{\gamma_2}(\lambda) d\lambda
\]

Finite M: TSS-CT

TSS scheduling with CT transmission in DL consists of choosing one relay \(\in R_{\tau}\) if \(R_{\tau}\) is not empty (\(\gamma_1 > \tau\)) or one random relay \(\in R\) if \(R_{\tau}\) is empty (\(\gamma_1 < \tau\)) (see Algorithm 3).

Note that here the first-hop SNR and the second hop SNR are exponentially distributed. The first-hop SNR of the selected dual-hop link is not necessarily above the selection threshold.

First, TSS scheme outage probability can be expressed in terms of two conditional probabilities: firstly, the outage probability given that the selected SNR \(\gamma_1\) is superior to \(\tau\) \(P_{\text{out,}\tau > \gamma_1} [\gamma_1 > \tau]\), and secondly, in terms of the outage probability given that \(\gamma_1\) is inferior to \(\tau\) \(P_{\text{out,}\tau > \gamma_1} [\gamma_1 < \tau]\):

\[
P_{\text{out,TSS,CT}}^M = P_1^M \cdot P_{\text{out,}\tau > \gamma_1} [\gamma_1 > \tau] + P_2^M \cdot P_{\text{out,}\tau > \gamma_1} [\gamma_1 < \tau]
\]
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Where $P_{1}^{M}$, $P_{2}^{M}$ are respectively defined in (6.9), (6.8).

The conditional outage probabilities $P_{\text{out,} r > \gamma_{th}}^{M} [\gamma_{1} > \tau]$ and $P_{\text{out,} r > \gamma_{th}}^{M} [\gamma_{1} < \tau]$ can be rewritten using Baye’s rules as:

$$P_{\text{out,} r > \gamma_{th}}^{M} [\gamma_{1} > \tau] = \frac{P[\gamma_{eq} < \gamma_{th} | \gamma_{1} > \tau]}{P_{\gamma_{1}}[\gamma_{1} > \tau]} = P_{\text{out,TSS}}^{\infty} \quad (C.26)$$

$$P_{\text{out,} r > \gamma_{th}}^{M} [\gamma_{1} < \tau] = \frac{P[\gamma_{eq} < \gamma_{th}, \gamma_{1} < \tau]}{P_{\gamma_{1}}[\gamma_{1} < \tau]} \quad (C.27)$$

Where $P_{\gamma_{1}}[\gamma < \tau]$ (resp. $P_{\gamma_{1}}[\gamma > \tau]$) is the probability that the SNR of a given BS-relay link is inferior (resp. superior) to the threshold $\tau$.

Note that the outage probability given that the selected BS-relay SNR is above $\tau$, $P_{\text{out,} r > \gamma_{th}}^{M} [\gamma_{1} > \tau]$ has the same analytical expression as the outage probability in the infinite amount of users case: $P_{\text{out,} r > \gamma_{th}}^{M} [\gamma > \tau] = P_{\text{out,TSS}}^{\infty}$

Moreover, $P[\gamma_{eq} < \gamma_{th}]$, the probability that the equivalent SNR is below the outage threshold with $\gamma_{1}$ and $\gamma_{2}$ exponentially distributed, can be expressed as:

$$P[\gamma_{eq} < \gamma_{th}] = P[\gamma_{eq} < \gamma_{th}, \gamma > \tau] + P[\gamma_{eq} < \gamma_{th}, \gamma < \tau], \quad (C.28)$$

And,

$$P[\gamma_{eq} < \gamma_{th}] = 1 - \frac{2\gamma_{th}}{\sqrt{\gamma_{1}\gamma_{2}}} K_{1} \left( \frac{2\gamma_{th}}{\sqrt{\gamma_{1}\gamma_{2}}} \right) e^{-\gamma_{th} \left( \frac{1}{\gamma_{1}} + \frac{1}{\gamma_{2}} \right)}, \quad (C.29)$$

Therefore, the outage probability given that the selected $\gamma_{1}$ is inferior to $\tau$ can be written as:

$$P_{\text{out,} r > \gamma_{th}}^{M} [\gamma < \tau] = \frac{P[\gamma_{eq} < \gamma_{th}] - P[\gamma_{eq} < \gamma_{th}, \gamma > \tau]}{1 - P_{\gamma_{1}}[\gamma > \tau]}, \quad (C.30)$$

By rearranging terms in the above expression, one can obtain an expression for the conditional outage probability given that $\gamma_{1}$ is inferior to the threshold depending on the outage probability in the infinite amount of users’ case:

$$P_{\text{out,} r > \gamma_{th}}^{M} [\gamma < \tau] = \left( \frac{P[\gamma_{eq} < \gamma_{th}] - P_{\text{out,TSS}}^{\infty}}{1 - P_{\gamma_{1}}[\gamma > \tau]} \right) \cdot \frac{P_{\gamma_{1}}[\gamma > \tau]}{1 - P_{\gamma_{1}}[\gamma > \tau]}, \quad (C.31)$$

By replacing the latter expression in (C.25) along with using the expression of $P_{1}$ and $P_{2}$ given in (6.9), (6.8), we obtain the outage probability of TSS system with CT transmission:
\[ P_{\text{out,TSS,CT}}^M = P[\gamma_{eq} < \gamma_{th}] = \left(1 - (P_{\gamma_1}[\gamma < \tau])^M\right) \cdot P_{\text{out,TSS}}^\infty + (P_{\gamma_1}[\gamma < \tau])^{M-1} \cdot \left(P[\gamma_{eq} < \gamma_{th}] - P_{\text{out,TSS}}^\infty \cdot (P_{\gamma_1}[\gamma > \tau])\right), \] (C.32)

Where: \( P_{\gamma_1}[\gamma > \tau] \) is the probability that the signal-to-noise ratio of a given first-hop link \( \gamma_{BS-R} \) is above the threshold \( \tau \). \( P_{\gamma_1}[\gamma > \tau] = 1 - P_{\gamma_1}[\gamma < \tau] = e^{-\tau} \).

\( P_{\text{out,TSS}}^\infty \) is the outage probability in the case where the amount of users and relays are infinite so the selected \( \gamma_1 \) is superior to the threshold \( \tau \).

And \( P[\gamma_{eq} < \gamma_{th}] \) is given by Equation (C.29).

### C.2 \( \gamma_1, \gamma_2 \)-scheduling

In this section, we derive the closed-form outage probability expression for the \( \gamma_1, \gamma_2 \)-selection scheduling protocol CS.

We assume that all the links in the same partition experience the same fading and have an independent identically distribution. The probability of having the end-to-end SNR below the outage threshold is then the same for every group of dual-hop links associated to different relay. Thus, the outage probability of the CS scheme can be written as follow:

\[ P_{\text{out,CS}} = \left[P_{\text{out,R}}^N\right]^M \] (C.33)

Where \( P_{\text{out,R}}^N \) is the outage probability associated with one relay R.

We enumerate the \( \frac{N}{M} \) users belonging to one relay R’s partition from 1 to \( \frac{N}{M} \).

The contribution of one relay R to the system outage is the probability of having all the dual-hop links corresponding to the \( \frac{N}{M} \) users in its partition in outage. Namely,

\[ P_{\text{out,R}}^\frac{N}{M} = P(\gamma_{eq,R,U_1} < \gamma_{th}, \gamma_{eq,R,U_2} < \gamma_{th}, \ldots, \gamma_{eq,R,U_{\frac{N}{M}}} < \gamma_{th}) \] (C.34)

The expression C.34 can be written in terms of the conditional probabilities as follow:
\[ P_{\text{out},R}^{\frac{N}{M}} = P(\gamma_{eq,R,U_1} < \gamma_{th}, \gamma_{eq,R,U_2} < \gamma_{th}, \ldots, \gamma_{eq,R,U_{\frac{N}{M}}} < \gamma_{th}|\gamma_{BS-R} \leq \gamma_{th}) P(\gamma_{BS-R} \leq \gamma_{th}) 
+ P(\gamma_{eq,R,U_1} < \gamma_{th}, \gamma_{eq,R,U_2} < \gamma_{th}, \ldots, \gamma_{eq,R,U_{\frac{N}{M}}} < \gamma_{th}|\gamma_{BS-R} > \gamma_{th}) P(\gamma_{BS-R} > \gamma_{th}) 
= 1 \cdot P(\gamma_{BS-R} \leq \gamma_{th}) 
+ P(\gamma_{eq,R,U_1} < \gamma_{th}, \gamma_{eq,R,U_2} < \gamma_{th}, \ldots, \gamma_{eq,R,U_{\frac{N}{M}}} < \gamma_{th}|\gamma_{BS-R} > \gamma_{th}) P(\gamma_{BS-R} > \gamma_{th}) \]

Taking the case of a relay R and 2 users per relay. (C.35) can be written as:

\[ P_{\text{out},R}^{2} = P(\gamma_{BS-R} \leq \gamma_{th}) + P(\gamma_{eq,R,U_1} < \gamma_{th}, \gamma_{eq,R,U_2} < \gamma_{th}|\gamma_{BS-R} > \gamma_{th}) P(\gamma_{BS-R} > \gamma_{th}) \]

\[ = P(\gamma_{BS-R} \leq \gamma_{th}) + \int_{\gamma_{th}}^{\infty} P\left(\frac{\lambda \gamma_{R-U_1}}{\lambda + \gamma_{R-U_1}} \leq \gamma_{th}, \frac{\lambda \gamma_{R-U_2}}{\lambda + \gamma_{R-U_2}} \leq \gamma_{th}\right) p_{\gamma_1}(\lambda)d\lambda \quad (C.36) \]

\[ = P\left(\frac{\lambda \gamma_{R-U_1}}{\lambda + \gamma_{R-U_1}} \leq \gamma_{th}, \frac{\lambda \gamma_{R-U_2}}{\lambda + \gamma_{R-U_2}} \leq \gamma_{th}\right) 
= P\left(\frac{\gamma_{R-U_1}}{\lambda - \gamma_{th}} \leq \frac{\gamma_{th}}{\lambda - \gamma_{th}}, \frac{\gamma_{R-U_2}}{\lambda - \gamma_{th}} \leq \frac{\gamma_{th}}{\lambda - \gamma_{th}}\right) \]

\[ = P\left(\frac{\gamma_{R-U_1}}{\lambda - \gamma_{th}} \leq \frac{\gamma_{th}}{\lambda - \gamma_{th}}\right) \cdot P\left(\frac{\gamma_{R-U_2}}{\lambda - \gamma_{th}} \leq \frac{\gamma_{th}}{\lambda - \gamma_{th}}\right) \quad (C.37) \]

Where the first equality results from algebraic manipulation and the second equality results the independency of the two event associated with two different relay-user link.

Besides, as all the relay-user links within a partition are i.i.d exponentially distributed:

\[ P\left(\gamma_{R-U_1} \leq \frac{\gamma_{th}}{\lambda - \gamma_{th}}\right) = P\left(\gamma_{R-U_2} \leq \frac{\gamma_{th}}{\lambda - \gamma_{th}}\right) = P\left(\gamma_{R-U_i} \leq \frac{\gamma_{th}}{\lambda - \gamma_{th}}\right) \quad (C.38) \]

And,

\[ P\left(\gamma_{R-U_i} \leq \frac{\gamma_{th}}{\lambda - \gamma_{th}}\right) = \int_{0}^{\frac{\gamma_{th}}{\lambda - \gamma_{th}}} p_{\gamma_2}(\gamma)d\gamma = 1 - e^{-\frac{\gamma_{th}}{\gamma_2(\lambda - \gamma_{th})}} \quad (C.39) \]
Therefore, (C.36) gives:

\[ P_{out,R}^2 = P(\gamma_{BS-R} \leq \gamma_{th}) + \int_{\gamma_{th}}^{\infty} \left( 1 - e^{-\frac{\gamma_{th}}{\pi_2(\lambda-\gamma_{th})}} \right)^2 p_{\gamma_1}(\lambda) d\lambda \]  

(C.40)

A generalization of the case of \( \frac{N}{M} = 2 \) gives the following outage probability expression for a general \( \frac{N}{M} \) users per relay’s partition:

\[ P_{out,R}^2 = P(\gamma_{BS-R} \leq \gamma_{th}) + I_{CS} \]  

(C.41)

where

\[ I_{CS} = \int_{\gamma_{th}}^{\infty} \left( 1 - e^{-\frac{\gamma_{th}}{\pi_2(\lambda-\gamma_{th})}} \right) \frac{1}{\gamma_{th}} p_{\gamma_1}(\lambda) d\lambda \]  

(C.42)

Using the Binomial theorem, \( I_{CS} \) can be expressed as:

\[ I_{CS} = \int_{\gamma_{th}}^{\infty} \sum_{k=0}^{\frac{N}{M}} \left( \frac{N}{M} \right) \left( -e^{-\frac{\gamma_{th}}{\pi_2(\lambda-\gamma_{th})}} \right)^k 1 e^{\frac{\lambda}{\gamma_{th}}} d\lambda \]  

(C.43)

\( I_{CS} \) can be computed using the change of variable \( u = \lambda - \gamma_{th} \) and [21, eq. 3.471.9] to give:

\[ I_{CS} = e^{-\frac{\gamma_{th}}{\pi_1}} + \sum_{k=1}^{\frac{N}{M}} \left( \frac{N}{M} \right) (-1)^k e^{-\frac{k\gamma_{th}}{\pi_1}} - \frac{\gamma_{th}}{\pi_1^2} 2 \sqrt{\frac{\gamma_{th}^2}{\pi_1 \pi_2}} K_1 \left[ 2 \sqrt{\frac{\gamma_{th}^2 k}{\pi_1 \pi_2}} \right] \]  

(C.44)

Thus, (C.35) and (C.45) give:

\[ P_{out,CS}^{M,N} = [P_{out,R}^{N,M}]^M \]  

(C.45)

Where \( P_{out,R}^{N,M} \) is the outage probability associated with one relay R.

\[ P_{out,R}^{N,M} = 1 + \sum_{k=1}^{\frac{N}{M}} \left( \frac{N}{M} \right) (-1)^k e^{-\frac{k\gamma_{th}}{\pi_1}} - \frac{\gamma_{th}}{\pi_1^2} 2 \sqrt{\frac{\gamma_{th}^2}{\pi_1 \pi_2}} K_1 \left[ 2 \sqrt{\frac{\gamma_{th}^2 k}{\pi_1 \pi_2}} \right] \]  

(C.46)
Bibliography


