HELSINKI UNIVERSITY OF TECHNOLOGY
Faculty of Electronics, Communications and Automation

Tapio Savunen

APPLICATION OF THE COOPERATIVE GAME THEORY TO GLOBAL STRATEGIC ALLIANCES

Master's Thesis submitted to the Faculty of Electronics, Communications and Automation at Helsinki University of Technology for fulfilment of the requirements for the degree of Master of Science.

Helsinki, 3 August, 2009

Supervisor Professor Jouko Lampinen

Instructor Heikki Salovuori, M.Sc.

ABSTRACT

Author: Tapio Savunen

Name of the thesis:
Application of the Cooperative Game Theory to Global Strategic Alliances

Date: 3 August, 2009 **Language:** English **Pages:** 129+10

Faculty:	Faculty of Electronics, Communications and Automation
Professorship:	Computational Engineering, code: S-114
Supervisor:	Professor Jouko Lampinen
Instructor:	Heikki Salovuori, M.Sc.

The scope of this thesis is the application of the cooperative game theory to global strategic alliances. The objective is to find out how the cooperative game theory can be applied to global strategic alliances. Global strategic alliances are studied to understand the major questions of the alliance formation and the long-term stability. Various concepts of the cooperative game theory are investigated to find a feasible way to apply them to the modelling of strategic alliances. Three examples from literature are presented in which the cooperative game theory is applied to strategic alliances. Also a case study is introduced in which the global mobile operators' alliances are modelled as cooperative games using the core solution concept.

The literature examples and also the case study demonstrate that the theory of cooperative games can be applied to model the global strategic alliances. The stability and likelihood of strategic alliances can be assessed by modelling them as cooperative games. With this modelling the stability of strategic alliances can be assessed in the alliance formation. Also the business dynamics and changes in the business environment can be taken into account in order to understand the alliance's long-term vitality. The case study demonstrates that the findings of the computational model of the strategic alliances reflect the respective findings made with qualitative methods.

In the case study a new computational method is introduced to estimate an alliance's long-term stability in the changing business environment. In this method the uncertain and changing business parameters are modelled with applicable probability distributions. By combining the Monte Carlo simulation and the core solution concept of the cooperative game theory, a measure called a stability indicator can be calculated. It reflects the stability of an alliance in the changing business environment.

Keywords: game theory, cooperative games, strategic alliances, mobile operators, computational modelling, Monte Carlo simulation, core solution concept

TIIVISTELMÄ

Tekijä: Tapio Savunen

Työn nimi:
Yhteistyön peliteorian soveltaminen globaaleihin strategisiin liittoutumiin

Päivämäärä: 3.8.2009 **Kieli:** Englanti **Sivumäärä:** 129+10

Tiedekunta: Elektroniikan, tietoliikenteen ja automaation tiedekunta

Professuuri: Laskennallinen tekniikka, koodi: S-114

Valvoja: Professori Jouko Lampinen

Ohjaaja: DI Heikki Salovuori

Tutkimuksen tavoitteena on selvittää yhteistyön peliteorian sovellusmahdollisuuksia globaaleihin strategisiin liittoutumiin. Globaalien strategisten liittoutumien kirjallisuustutkimuksella analysoidaan liittoutumien muodostumiseen ja pysyvyyteen liittyviä tekijöitä. Yhteistyön peliteorian peruskäsitteet ja ratkaisukonseptit kuvataan ja käsitellään niiden sovellusmahdollisuuksia strategisiin liittoutumiin. Kolme kirjallisuusesimerkkiä kuvaa erilaisia yhteistyön peliteorian sovellustapoja strategisiin liittoutumiin. Työhön sisältyy tapaustutkimus, jossa muodostetaan laskennallinen malli globaalien matkapuhelinoperaattoreiden strategisista liittoutumista soveltamalla yhteistyön peliteoriaa. Käytetty ratkaisukonsepti on yhteistyöpelin ydin.

Kirjallisuusesimerkit ja tapaustutkimus todentavat, että yhteistyön peliteoriaa voidaan soveltaa strategisten liittoutumien mallintamiseen. Liittoutumien muodostumisen todennäköisyyttä ja niiden stabiilisuutta voidaan arvioida yhteistyön peliteorian avulla. Myös liiketoiminnan dynamiikka ja liiketoimintaympäristön muutokset voidaan ottaa mallinnuksessa huomioon. Tämä mahdollistaa liittoutumien pitkän aikavälin elinkelpoisuuden tarkastelun. Tapaustutkimuksen avulla osoitetaan, että laskennallisen mallinnuksen avulla saatavat tulokset vastaavat strategisten liittoutumien kvalitatiivisen tutkimuksen tuloksia.

Tapaustutkimuksen yhteydessä esitellään uusi menetelmä liittoutumien pitkän aikavälin stabiilisuuden arvioimiseksi liiketoiminnan muutoksissa. Liiketoimintaa kuvaavat epävarmat ja muuttuvat parametrit mallinnetaan soveltuvalla todennäköisyysjakaumalla. Tutkimuksessa osoitetaan, että yhdistämällä Monte Carlo -simulointi ja yhteistyöpelin ydin -ratkaisukonsepti, voidaan muodostaa stabiilisuusindikaattori, joka kuvaa liittoutuman pitkän aikavälin stabiilisuutta muuttuvassa liiketoimintaympäristössä.

Avainsanat: peliteoria, yhteistyön pelit, strategiset liittoutumat, matkapuhelin-operaattori, laskennallinen mallintaminen, Monte Carlo -simulointi, yhteistyöpelin ydin

PREFACE

This thesis was completed at EADS Security & Communication Solutions in Helsinki, Finland. I would like to thank my superior Hannu Juurakko, M.Sc, for the assignments that have greatly contributed to my insight into practical cooperation between business partners.

I would like to express my gratitude to the supervisor of this thesis, Professor Jouko Lampinen at the Helsinki University of Technology, for supporting and examining this thesis. I would also like to thank my instructor, Heikki Salovuori, M.Sc., for his valuable comments and guidance while writing this thesis and also for inspiring discussions and a long friendship.

Also I would like to thank my dear wife Outi Susiluoto, M.Sc., for her patience, support and wise advice.

Helsinki, 3 August, 2009

Tapio Savunen

TABLE OF CONTENTS

SYMBOLS AND ABBREVIATIONS

Symbols

b^i_{max}	*i*'s maximum contribution to a coalition
b^i_{min}	*i*'s minimum contribution to a coalition
c	country
C	number of countries
C_t	expected future cash flow
$\mathcal{C}$	set of all coalitions
$\mathcal{C}(.)$	core
$\mathcal{C}_\varepsilon(.)$	strong ε-core
E_c	export of a country
G	game
hc	home country
ho	home operator
I_c	import of a country
$\mathcal{LC}(.)$	least core
$m(.)$	lower vector of τ-value
$M(.)$	upper vector of τ-value
M_c	number of inhabitants in a country
n_{Core}	number of samples in the core
$n_{samples}$	number of all samples
N	game theory: a set of players, a grand coalition; case study: number of proportionate subscribers
N_c	number of subscribers in a country
N_w	number of subscribers in a network
o	operator
O	number of operators
P	price
PV	present value
Q	quantity
R	(computational) revenue
ΔR	revenue impact (negative or positive)
$R_{R\text{-}Top10}$	total roaming revenue of top 10 global mobile operators
R_{RH}	home roaming revenue
R_{RV}	visited roaming revenue
r_t	discount rate%
$s\%_{\Delta R}$	percentual standard deviation of the additional revenue
$s(\Delta R)$	standard deviation of the additional revenue
s^*_i	player *i's* best response to the strategies of other players
s_i	element of the strategy set S_i
S	game theory: subset of N, $S \subseteq N$; case study: alliance

S_i	player *i's* strategy set
t	time in years
T	subset of N; $T \subseteq N$
$u_i(.)$	player *i's* pay-off function
$v(.)$	coalition (characteristic) function of a cooperative game
vc	visited country
vo	visited operator
w	network
W	number of networks
$\boldsymbol{x}$	payoff vector; $\boldsymbol{x} \in \mathbb{R}^S$
$x\ dom_S\ y$	imputation x dominates another imputation y via S
X^K	lexicographic center
X	set of (pre)imputations
X^*	set of feasible vectors
y	year
Γ	set of games
δ	roaming discount
ε_d	coefficient of elasticity of demand
ϕ	Shapley value
ϕ_H	operator's share of home roaming revenue
ϕ_V	operator's share of visited roaming revenue
η	home operator's retail margin
φ	stability indicator
λ	home operator's share of the roaming revenue
λ_S	balanced collection of weights
μ	mean
π^*	egalitarian solution
π^S	modified Shapley value
π^T	relative threats solution
π_c	mobile services penetration of a country
π_w	network level penetration of mobile services
σ	game theory: solution function; case study: standard deviation
$\tau(.)$	τ-value

Abbreviations

ACA	Alternate Cost Avoided
AM	America Movil
AMI	Asia Mobility Initiative
ARPU	Average Revenue Per User
BMA	Bridge Mobile Alliance
BT	British Telecommunications
CDMA	Code Division Multiple Access
CM	China Mobile Ltd
CU	China Unicom
DT	Deutsche Telekom
EDGE	Enhanced Data rates for GSM Evolution
ENSC	Egalitarian Non-Separable Cost
EU	European Union
EV-DO	Evolution Data Optimized
FCC	Federal Communications Commission
FMCA	Fixed Mobile Convergence Alliance
FT	France Telecom
GDP	Gross Domestic Product
GHz	Giga Hertz
GPRS	General Packet Radio Service
GSM	Global System for Mobile communications
HPAC	Heating, Plumbing and Air-Conditioning
ICT	Information and Communication Technologies
IP	Internet Protocol
MB	Mega Byte
MMS	Multimedia messaging System
MNO	Mobile Network Operator
MTS	Mobile TeleSystems
NTT	Nippon Telegraph and Telephone
OTA	Over-The-Air
PC	Personal Computer
R&D	Research and Development
SCRB	Costs Remaining Benefits
SIM	Subscriber Identity Module
SMS	Short Message Service
TI	Telecom Italia
TO2	Telefonica O2
UK	United Kingdom
UMTS	Universal Mobile Telcommunications System
US	United States of America
USD	United States Dollar
VF	Vodafone
W-CDMA	Wideband Division Multiple Access

WLAN	Wireless Local Area Network

Billion means one thousand million.

1 INTRODUCTION

The scope of this thesis is the application of the cooperative game theory to global strategic alliances. This work includes a case study of global mobile operators' alliances.

The importance of cooperative strategies and thus strategic alliances has increased as a consequence of global competition. Companies cannot create, produce or deliver everything by themselves anymore, but they need strategic partners in order to be competitive. In the past the strategic alliances were seen as a feasible alternative only for large international companies, but today they are a viable approach for companies of all sizes.

The research of strategic alliances has been wide-ranging and vital. A vast amount of studies, text books and best-seller business books have been published. Most of the work has its roots in industrial and business management and therefore also the research methodologies are inherited from those areas. Qualitative methods have been typical in the domain, for example when analysing the reasons for unsuccessful alliances. Also comparative and statistical methods have been used, e.g. to investigate the causalities between strategic alliances and business results.

Game theory is the study of mathematical modelling of multi-person decision problems. Decision makers are called players and they interact with each other in a context called the game. The roots of the game theory are in economics and mathematics. Academic research of the game theory has been active and also the progress has been remarkable since 1944, when the classical book of game theory was published by mathematician John von Neumann and economist Oskar Morgestern.

The pioneers of the game theory have been noticed in the highest academic level twice. In 1995 and 2005 the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel ("Nobel prize in economics") was awarded to game theoreticians.

Game theory has found its applications in the fields of economics, politics, sociology, biology and computer sciences. Even philosophers have found the game theoretical approach to be fruitful.

One sub-category of the games is the class of cooperative games. In cooperative games the players can form coalitions which can make binding agreements on the distribution of the payoff and also on the strategies to be chosen. The cooperative game theory is very much focused on the results of the cooperation and how the results are divided among the players. Questions such as what coalitions can be formed and how the results of the cooperation should be divided to ensure a sustainable agreement are among the basic topics of cooperative games.

The basic concepts of strategic alliances and cooperative games resemble each other. When companies establish a strategic alliance, they are expecting that it is a reasonable way to improve the economical success of all the companies that are involved in the alliance. They also bind an agreement which defines the terms of the cooperation and how the added value of the alliance is shared among the alliance partners. The very same concept can be found in cooperative games, the terminology is just different. The players of the game cooperate with each other and they need to agree on the common strategy and the way the payoff of the game is divided.

Even though there seems to be a relationship between strategic alliances and cooperative games, it is hard to find research which would investigate this relationship in the form of mathematical modelling, that is, using the methods which are the most evident from the game theory's point of view. In the literature of strategic alliances the game theory is frequently mentioned, its basic concepts are presented and even the results are described. Usually, however, the approach is qualitative as an opposite to the quantitative methods of the game theory.

The objective of the study is to find out how the cooperative game theory can be applied to global strategic alliances. What are the concepts of the cooperative games that can be used to model strategic alliances? What are the aspects of the strategic alliances that can be modelled with the game theory? Does game theoretic approach reveal some new angles about strategic alliances?

The hypotheses of the study are the following:

- The theory of cooperative games can be applied to model the global strategic alliances.

- Modelling strategic alliances as cooperative games contributes to the assessment of a strategic alliance's stability and likelihood.

- The business dynamics and changes in the business environment can be taken into account when modelling the global strategic alliances with the cooperative games.

- The findings of the computational model of strategic alliances reflect the respective findings made with qualitative methods.

The scope of the study is not limited to any particular industry but strategic alliances are contemplated in the general context. However, many of the examples are from the telecommunications domain to support the case study of mobile operators' alliances. The global aspect is taken into account in order to have all the angels of alliances in the richest form.

The cooperative game theory is elaborated in the scope of strategic alliances. The basic concepts and solutions of the games, which are typical and applicable in the context of alliance formation, are described. This has lead to the exclusion of the games with non-transferable utility and to focus on games with transferable utility. In the games with transferable utility a player can move a part of his or her own utility to another player without loss. Typically we can assume that the payoffs are money, which has the same value for all players. Hence the games with transferable utility are applicable to the games of money based transactions.

The work consists of three parts. The first part, Chapters 2 and 3, is the literature study in which the status of the existing research is described. Chapter 2 deals with strategic alliances and Chapter 3 deals with game theory in general, concentrating on cooperative games. The second part, Chapter 4, is the case study, and the third part, Chapters 5 and 6, is discussion and conclusions.

Chapter 2 starts with the definition of strategic alliances, following with the discussion about the motives to establish an alliance and further with the introduction of different forms of strategic alliances. The value creation of strategic alliances has its own sub-chapter. Finally the chapter ends with the description of the strategic alliances of telecommunications operators, both older and newer ones. This is to support the case study of mobile operators' alliances.

Chapter 3 of the game theory and cooperative games starts with the introduction to the game theory, which is followed by the classification of the games. Nash equilibrium has its own sub-chapter in order to discuss the basic elements of game theory. Cooperative games are introduced and their solution concepts are defined. Stable sets and the core have their own sub-chapters, and the Shapley value with some other one-point solution concepts are presented in one sub-chapter. Chapter 3 ends with the presentation of three different cases in which game theory and cooperative games have been applied to strategic alliances.

Chapter 4 is the case study in which cooperative game theory is applied to strategic alliances. Also the methods and knowledge from the fields of computational modelling, microeconomic modelling and industrial and business management have been applied in the case study. It is a computational model of global mobile operators' strategic alliances. The chapter starts with the introduction to the model, giving a summary about the contents of the model. The technical structure of the model is presented and the computational aids are described. The following sub-chapters present the model in detail: modelling the business of global mobile operators, modelling the strategic alliances and how to apply the concepts of cooperative games. The results of the model are described and the chapter ends with an analysis of the results.

Chapter 5 embodies the discussion about the results of the whole work which is presented in the light of the hypothesis of the study. In Chapter 6 the conclusions are provided.

2 STRATEGIC ALLIANCES

2.1 Definition of Strategic Alliances

Globalization and the global economy, in which goods, services, people, skills, ideas and capital move freely from one country to another, have significantly changed and complicated the competitive landscape of companies. Political changes, increasing rate of technological development, ubiquitous communication technology and growing knowledge intensity are constantly reforming most of the businesses (Hitt, Ireland & Hoskisson 2001). As a consequence the means to be competitive in global markets are not the same they used to be. Companies cannot any longer create, produce or deliver everything by themselves, but they need strategic partners in order to be competitive. Even the biggest companies, such as AT&T, IBM, Philips and Siemens cannot achieve leadership without forming alliances with domestic or multinational companies (Kotler, Keller 2006). Also competitors can cooperate to shape the market and to create a larger and more valuable market than they could do by acting independently. This kind of simultaneous competition and cooperation between rivals is called ‘co-opetition’ (Nalebuff, Brandenburger 1996) or ‘co-option’ (Doz, Hamel 1998).

The importance of cooperative strategies has increased since mid-1980's as a consequence of the global competition. Strategic alliances have become the dominating form of applying the cooperative strategy (Hitt, Ireland & Hoskisson 2001). In the past strategic alliances were seen as a feasible alternative only for large international companies, but also small firms can benefit from the alliance with large companies (Slowinski, Seelig & Hull 1996). A small firm can e.g. provide their technological expertise for a bigger company and the bigger party can provide their capital and organisational resources for the smaller firm, e.g. in the form of sales and marketing channels. Many big companies need strategic alliances also as a result of more focused business strategies. In order to increase the shareholder value, businesses have been focusing in their core competencies and core businesses. Companies are leaner, more productive and more selective about what they do. The vertically integrated companies largely do not exist anymore (Doz, Hamel 1998).

To get an insight into strategic alliances let us take some examples. The first one is from car industry. In 1982 Seat and Volkswagen established a strategic alliance. The scope of the alliance was production and marketing. Seat started manufacturing Volkswagen models in Spain in 1984. It sold the cars in Spain or exported them to other countries. Also the export of VW cars from Germany to Spain increased due to the alliance. Seat was able to renew its product and process technologies and thus to reduce the manufacturing costs. Seat also launched two new models during the alliance; Ibiza 1984 and Málaga 1985. Málaga was based on VW Jetta. The Seat brand’s image was also improved due to the high quality reputation of German cars. The alliance of Seat and Volkswagen ended in 1986 when VW acquired Seat. Seat was to become the third member in the Volkswagen consortium after VW and Audi (Gil, de la Fe, Pedro Gonzalez 1999).

The second example is from the telecommunications business. In the early 1990s Motorola initiated an alliance to create a global satellite communications network named Iridium. The system was based on 66 low-orbit satellites. Motorola planned to overcome the limitations of ground-based cellular phone systems and to outperform its competitors. The idea of the Iridium was to introduce a single global network for wireless communications. Motorola gathered a group of partners to provide all it needed: funds, traffic rights and technologies. Technologies were very demanding from ground and space-based communication networks to satellites and rocket launching systems. The alliance was comprised of seventeen equity-holding partners and a collection of industrial partners e.g. Raytheon, Lockheed Martin, Krunichev Enterprise, China Great Wall and Nippon Iridium (an alliance of eighteen Japanese partners) (Doz, Hamel 1998).

Commercial enterprises are not the only ones which are capable of forming alliances. The third example is from Swedish politics. Four Swedish conservative parties formed an alliance to win the election and to beat the Social Democrats in the Swedish general election in September 2006. The members of the alliance were the Moderate Party, the Christian Democrats, the Liberal Party and the Centre Party. The prime minister candidate of the alliance was Moderate Party’s leader Fredrik Reinfeldt. In the heart of alliance’s program was the individual worker, which is not so typical of the right-wing politics (Claude 2006). As we know, the alliance was successful and they won the election and Frederik Reinfeldt became the Prime Minister of Sweden.

These arbitrary examples demonstrate that the form and the qualities of strategic alliances can vary considerably. The number of partners can be two or more, the objectives of the partnership are very different from one alliance to another, and also the contribution of partners is varying in these examples. Partners of a strategic alliance can operate in the same field or business (car industry and politics) or in different businesses as in the Iridium example the technology partners do.

To get a more systematic approach to the nature of strategic alliances, some literature definitions are reviewed. The first definition considers alliances as partnerhips, where the partners combine their contributions in order to achieve mutual interests.

“Strategic alliances are partnerships between firms whereby their resources, capabilities, and core competencies are combined to pursue mutual interests in designing, manufacturing, or distributing goods or services.” (Hitt, Ireland & Hoskisson 2001)

The viewpoint of the second definition is the cooperative strategy of organisations, the motives for cooperation and how to establish and manage cooperative arrangements between partners. Strategic alliances are considered to be a means of implementing an organisation’s cooperative strategy. The interpretation is that typically strategic alliances are established as a consequence of major strategic challenges or opportunities the partners have encountered, that is, a kind of marriage of necessity.

"Cooperative strategy is the attempt by organizations to realize their objectives through cooperation with other organizations rather than in competition with them. … Alliances, which are partnerships between firms, are the normal agent for cooperative strategy. They are often 'strategic' in the sense that they have been formed as a direct response to major strategic challenge or opportunities which the partner firms face." (Child, Tallman & Faulkner 2005)

Although the partners of a strategic alliance have agreed on cooperation, they still remain legally independent and share the managerial control over the partnership. The benefits of the alliance are also shared and the contribution of all the partners is seen as a necessity.

"A strategic alliance involves at least two partner firms that:

- remain legally independent after the alliance is formed;
- share benefits and managerial control over the performance of assigned tasks;
- make continuing contributions in one or more strategic areas, such as technology or products" (Yoshino, Rangan 1995).

According to the partners' level of commitments, strategic alliances can be very dissimilar (Hill, Hwang & Kim 1990). That applies to both control and resources:

"Strategic alliances can be placed on a continuum where contractual agreements lie on one end of the continuum, representing low control and low resource commitment, whereas joint ventures lie on the other end of the continuum, representing high control and high resource commitment."

In addition to common goals partners can also have individual ones:

"Strategic alliances, a manifestation of inter-organisational cooperative strategies, entails the pooling of specific resources and skills by the cooperating organizations in order to achieve common goals, as well as goals specific to the individual partners" (Varadarajan, Cunningham 1995).

To gather all these different considerations, we can deduce the following statements to characterise strategic alliances:

- Strategic alliances are a form of implementing the cooperative strategy of organisations.
- In a strategic alliance there are at least two partners.
- Each of the partners contributes in one or more strategic areas of the alliance.
- In a strategic alliance partners have some shared objectives which they pursue.

- In addition to shared objectives, partners can have their own individual goals.
- Partners share the managerial control and benefits of an alliance.
- The binding of strategic alliances varies greatly in terms of partners' resource commitments and control over the partnership.
- In a strategic alliance partners remain legally independent.
- Often the motive to establish a strategic alliance is a major strategic challenge or an opportunity that the partners have met.

2.2 Motives for establishing a strategic alliance

Usually companies have several motives for establishing a strategic alliance. For example they may want to access technologies, to pursue a greater technical critical mass or to share the risk of technology development (Trott 2002). Generally there are at least two kinds of motives for an alliance. A company needs to react to changes in the external environment or the company's performance in certain areas of its internal operations needs to be improved (Child, Tallman & Faulkner 2005).

To continue with the previous example of Motorala and its Iridium alliance, the following motives can be identified (Doz, Hamel 1998).

- Cooperation with national telecommunication operators provides two benefits for the alliance. When operators are tied to the Motorola partnership, they are neutralised and not available for any rival's coalitions. Furthermore, they provide an access to national traffic rights.
- Each of the partners contributes their own unique and differentiated resources, such as skills, brands, relationships and tangible assets to the success of the alliance. Combining all these resources with a joint effort is much more valuable than keeping these separate. Also these separate resources are a necessity for the alliance. None of the partners would be able to create the new business alone due to lack of mandatory resources.
- The alliance supports also the learning and internalization of new skills, especially those which are tacit, collective and hard to obtain and internalize by other means. Gradually partners will able to create new core competencies in consequence of the alliance and thus will be able to leverage these into new businesses beyond those covered by the alliance.

Motives and reasons for establishing strategic alliances can be explained from different viewpoints and thus there are several classification schemes to interpret motives behind strategic alliances (Todeva, Knoke 2005). In the following some usual ones are introduced.

Markets can be divided into slow-cycle, standard-cycle and fast-cycle markets (Hitt, Ireland & Hoskisson 2001). The distinctive factor is the openness of the markets. Slow-cycle markets are shielded from the competition. Firms' resource positions are defended and "competition do not readily penetrate a firm's sources of strategic competitiveness". Examples of such sheltered or near monopolistic markets are railroads, utilities and, historically, telecommunications. A company having a unique product design may dominate its markets for decades. Also in the markets of rapid technological changes a firm can gain a strong long-lasting position. An example of such is Microsoft with its complex software products built around the Windows operating system.

Standard-cycle markets are often large and companies' strategy and organisation are targeted to serve high-volume mass markets. Economies of scale are an essential part of the firms' strategy in standard-cycle markets, e.g. in car industry and commercial aerospace.

Fast-cycle markets are characterised by entrepreneurial firms which offer new goods or services with a short life cycle. Usually it is very hard to gain a sustainable competitive advantage in fast-cycle markets since the fast counter-attack of rivals erodes the competitive advantage. Examples of such markets are mobile telecommunications industry and Internet businesses (Hitt, Ireland & Hoskisson 2001).

One motive for slow-cycle markets companies to cooperate is to develop standards, e.g. air or train traffic or telecommunications standards. Although the explicit reason for standards is to maintain market stability, they can also cooperate in order to reduce competition (Hitt, Ireland & Hoskisson 2001), for instance telecommunications standards are an efficient entry barrier for potential entrants with new technologies (Greenstein 1993). Companies in slow-cycle markets also form strategic alliances when seeking entry into markets that are restricted, or when trying to build franchises in new markets (Hitt, Ireland & Hoskisson 2001). E.g. utility companies have formed alliances with local players in the emerging markets in Eastern Europe, Russia, Latin American India and China. Telecommunications companies have also been active in establishing alliances in these countries. They have achieved an access to near-monopolistic markets. Local players have earned the expertise and technological know-how that firms from developed countries have been able to provide them. In addition to emerging markets, telecommunications firms form alliances in developed economies when targeting at a global position and to be able to better serve international business customers.

Companies of standard-cycle markets often cooperate to avoid industry overcapacity and to gain market power. Due to the orientation of economies of scale alliances are likely to occur between partners with complementary resources and core competencies. Increasing globalization has amplified opportunities and needs to combine resources at an international level. Local markets may be too small to gain the scale-effect of businesses. A good example of such a global trend is many joint ventures and similar alliances in the car industry. Firms also search pooling of resources via cooperation to meet their capital needs. In 1999 DuPont was seeking partners “to bring its small pharmaceuticals business to critical mass through strategic alliances." The target was to conclude one or more alliances before the end of the year (Warren, Tejada 1999). Other reasons for alliances in standard-cycle markets are to overcome trade barriers and to learn new business techniques (Hitt, Ireland & Hoskisson 2001).

In fast-cycle markets cooperative strategies are usually used to amplify the competitiveness by increasing the speed of product development or to accelerate the entry into the market (Hitt, Ireland & Hoskisson 2001). This is due to the nature of the fast-cycle markets. Product cycles are short and new products or services are quickly imitated. Another characteristic is concurrent cooperation and competition of partners. E.g. Yahoo has cooperated with its Internet rivals American Online and AltaVista. The reason for Yahoo’s co-opetition has been explained to be shaping and expanding the new market. "We're at a stage where there's a transformation of the economic infrastructure. It's too big a problem for any one company to solve. So you cooperate to create value" (Wysocki Jr 2000). Companies also form alliances to create new industry standards, share risky R&D expenses, or to overcome uncertainty.

The framework for strategic alliances by (Doz, Hamel 1998) explains reasons for cooperation based on two main motives; “race for the world” and “race for the future”. The former means building up a new position in an existing global or local market or leveraging existing external skills. The latter refers to the future opportunities, that is, to products or services that haven't yet been created.

Companies forming strategic alliances when attempting to build a new position in existing markets, usually have the following motives (Doz, Hamel 1998):

- Building critical mass globally or in a specific new market;
- Learning quickly about unfamiliar markets and become an insider;
- Accessing skills concentrated in another geographic area (e.g. fashion design skills of Italy).

Companies racing for the future through strategic alliances often have the following reasons (Doz, Hamel 1998):

- Building nodal positions in coalitions aimed at creating new markets;
- Creating new opportunities by combining skills and resources;
- Building new competencies faster than would be possible through internal efforts.

Te basic motivations for joint ventures between firms can be described from the perspective of three theoretical paradigms (Kogut 1988). These are i) the theory of transactions costs, ii) focusing on strategic position and market power and iii) the organizational theories and mechanisms for transferring organisational knowledge.

Transaction costs refers to costs of arranging, managing and monitoring transactions in the market, such as negotiation costs, preparing contracts and managing logistics. The transaction cost paradigm is a potential motive in relationships with small number bargaining, high asset specificity and high uncertainty about specifying and monitoring performance. A strategic alliance dilutes the potential costs of such situations because there is a mutual dependency on each other through joint commitment of financial or other assets which align the partners of the alliance (Child, Tallman & Faulkner 2005).

The strategic behaviour paradigm can be seen a complementary theory to the transaction cost paradigm. When the latter assumes that firms cooperate by minimizing the sum of production and transaction costs, the former assumes the reasoning to be based on maximizing the profits through improving a company's competitive position in the market (Kogut 1988). Thus the strategic behaviour paradigm emphasizes the partners' complementary assets and the synergies that arise as a result of introducing a strategic alliance. With an alliance the partners can reach a competitive advantage they could not have achieved alone without complementary assets (Child, Tallman & Faulkner 2005).

The third paradigm of (Kogut 1988) is organizational learning. It proposes that competence transfer and especially tacit knowledge are the driving motives for strategic alliances. Tacit knowledge cannot be transferred by contractual codified means, but by having teams working together (Child, Tallman & Faulkner 2005).

To summarise, the motives for establishing a strategic alliance vary to a great extent and a plethora of reasons can be found. It is likely that in many cases there is no single reason for the introduction of a strategic alliance, but a wide array of motives can be identified. Also the motives of the alliance partners are not necessary the same but may differ from one another.

2.3 Forms of Strategic Alliances

As the definition of strategic alliances or their motives do not easily fit into a single simplified model, the very same applies to their classification. There are a number of ways to categorize strategic alliances depending on the classification criteria. The following describes the most common ones.

Strategic alliances can take place either between partners from the same industry or from different industries. The former are called intra-industry alliances and the latter inter-industry alliances. For example an intra-industry strategic alliance is the one between US automobile manufacturers to develop technology for an electric car. The

UK pharmaceutical company Glaxo-Smithkline has established many inter-industry alliances with companies from a variety of industries, e.g. Matsusita, Canon, Fuji and Apple (Trott 2002).

One way to classify strategic alliances is based on the share of ownership between alliance partners (Hitt, Ireland & Hoskisson 2001). Joint ventures are alliances in which partners create an independent company by combining parts of their assets. Joint ventures are created especially for establishing long-term relationships and for transferring tacit knowledge. Usually partner firms own an equal part of the joint venture's equity. In equity strategic alliances partners own different percentages of equity in a venture. Motorola's Iridium alliance is a typical example in which partners own different shares of the new company. Also foreign direct investments are usually of this type, e.g. investments of US and Japanese companies in China. A nonequity strategic alliance is the third type of this category. These are agreements of the supply, production or distribution of goods without equity sharing between partners. Also cooperative arrangements concerning marketing and knowledge sharing can be in the scope. Because no separate venture is established or equity investments are needed, nonequity strategic alliances are less formal and demanding than the two other types of alliances. On the other hand, they are not suitable for complex projects requiring effective competence transfer between the partners.

Complementary strategic alliances are established to take advantage of market opportunities and to create new value by combining the partners' assets in a complementary way (Hitt, Ireland & Hoskisson 2001). *A vertical complementary strategic alliance* is an arrangement between companies that agree to use their assets in different stages of the value chain. Benetton is an example of a company which uses a number of vertical alliances in their operations. Benetton's core competencies are in the marketing and sales activities, not in the manufacturing of clothes. Instead, they have created alliances with companies that are highly skilled in manufacturing high-fashion, trendy clothes. *A horizontal complementary strategic alliance* is established between partners who agree to combine their assets in the same stage of the value chain. Usually firms use this type of alliance for long-term product and service technology development. Companies can also create horizontal alliances to jointly market their products and services. As an example airline companies have established alliances to reduce costs and to increase revenues. Also telecommunications operators have formed several horizontal alliances to be able to compete in worldwide telecommunications markets and to provide services for their global business customers.

When comparing horizontal and vertical complementary strategic alliances, the role of trust is very significant. In horizontal alliances partners are simultaneously competitors, which is not the usual case between vertical alliance partners. They use their competitive advantages in different parts of the value chain to create shared value. An example again is Benetton which has created trust in its relationships with manufacturing partners. This is due to the positive outcome from previous business transactions and to clearly separate roles and different positions in the value chain. An opposite example is the horizontal alliances of airline companies. The trust between the alliance partners may be quite low because they continue to compete on many routes. This can be one of the reasons for the low success of airline alliances; less than 30 percent have been successful (Hitt, Ireland & Hoskisson 2001).

In (Todeva, Knoke 2005) the strategic alliances are analysed from the point of view of formalization of the governance and partners' inter-organisational relationships. Governance here means the legal and social control mechanisms for co-ordinating the partners' resource contributions, responsibilities and the division of reward from the joint activities. The less demanding alliances are pure market relationships requiring no obligation for recurrent cooperation, co-ordination or collaboration among the partners. The most demanding alliances are hierarchy authority relations in which a company takes full control.

A citation from (Todeva, Knoke 2005), inter-organisational relations from the most binding to the loosest ones:

1. *"Hierarchical relations:* through acquisition or merger, one firm takes full control of another's assets and co-ordinates actions by the ownership rights mechanism.

2. *Joint ventures:* two or more firms create a jointly owned legal organization that serves a limited purpose for its parents, such as R&D or marketing.

3. *Equity investments:* a majority or minority equity holding by one firm through a direct purchase of shares in another firm.

4. *Cooperatives:* a coalition of small enterprises that combine, co-ordinate, and manage their collective resources.

5. *R&D consortia*: inter-firm agreements for research and development collaboration, typically formed in fast-changing technological fields.

6. *Strategic cooperative agreements*: contractual business networks based on joint multi-party strategic control, with the partners collaborating over key strategic decisions and sharing responsibilities for performance outcomes.

7. *Cartels:* large corporations collude to constrain competition by cooperatively controlling production and/or prices within a specific industry.

8. *Franchising*: a franchiser grants a franchisee the use of a brand-name identity within a geographical area, but retains control over pricing, marketing, and standardized service norms.

9. *Licensing*: one company grants another the right to use patented technologies or production processes on return for royalties and fees.

10. *Subcontractor networks:* inter-linked firms where a subcontractor negotiates its suppliers' long-term prices, production runs, and delivery schedules.

11. *Industry standard groups*: committees that seek the member organizations' agreements on the adoption of technical standards for manufacturing and trade.

12. *Action sets*: short-lived organizational coalitions whose members co-ordinate their lobbying efforts to influence public policy making.

13. *Market relations:* arm's-length transactions between organizations co-ordinated only through the price mechanism."

A significant category of strategic alliances are marketing alliances. These can be classified into four major groups (Kotler, Keller 2006):

- *Product or service alliances:* A company licenses its product to another, or companies jointly market their complementary products or a new product.

- *Promotional alliances:* A company carries a promotion for another company' product or service.

- *Logistics alliances:* A company provides logistical services for another company's product.

- *Pricing collaborations:* One or several companies join in a pricing collaboration.

Among the forms of strategic alliances one worth mentioning is innovation networks. The term itself is hard to define. To many it means a type of "virtual organization", others believe there is nothing new but a new name for a firm's range of supplier and market relationships. The driving force behind innovation networks and virtual organizations is their promise to reduce bureaucracy and expenses, and the capability to exploit more efficiently the fast technological change. A practical example is Nike, which owns and manages the brand and relies on networks to produce and distribute its products. Nike does not own all the manufacturing plants to produce its shoes nor all the retail outlets to sell the products. It takes care of research, design and development and has networks for manufacturing in Asia, India and South America. Furthermore, it has the network of distributors (Trott 2002).

2.4 Value creation of Strategic Alliances

The value creation of strategic alliances is not as widely discussed in the literature as other aspects, such as motives and forms of alliances. This is surprising since to be sustainable the alliance must provide additional value at least in the long term for all the partners. An extensive discussion and framework for value creation of strategic alliances can be found in (Doz, Hamel 1998). That has been used as a main source for this chapter and the discussion refers to it if not otherwise indicated.

When measuring the value of strategic alliances there are two perspectives. The first one, 'system performance', is the performance of the alliance itself as a business unit. The second one, 'goal performance', focuses on the alliance's value to its parent companies and whether their strategic goals for the alliance are met (Child, Tallman & Faulkner 2005). The 'system performance' is measured in general business terms and assuming that the ultimate goal is to increase the wealth of shareholders through increased profitability. Naturally this applies to all the alliance partners. The 'goal performance' is defined as the extent to which the partners' objectives for the alliance are realized. In a similar way it has been proposed that strategic alliances need to fulfil two criteria to be successful. The partners need to recover their financial capital costs and achieve their strategic objectives (Bleeke, Ernst 1993).

According to the framework for strategic alliances by (Doz, Hamel 1998), "race for the world" and "race for the future", explains that underlying value creation logic of alliances is based on three distinctive benefits:

1. Gaining competitive capabilities through co-option
2. Leveraging cospecialized resources.
3. Gaining competence through internalized learning

Co-option refers to cooperation between competitors. By cooperating with rivals, they are effectively neutralized by bringing them into the alliance and also their complementary assets are exploited.

These three benefits are also the main drivers for companies to establish alliances. Firms need to position themselves strategically and to exploit competitive capabilities through the co-option with competitors and complementary firms. They need to access new markets and to create new opportunities through cospecialization and with combined resources. Finally, firms need to learn to fill the gaps in needed skills and to create new competencies through alliances.

Value creation through co-option

In the race for the world, co-option is a means to reach the needed market share and critical mass for effective competition. Firms in such an alliance can be competitors or companies with complementary assets or both.

An example of building the critical mass is the alliance of Deutsche Telekom and France Telecom in 1997. They both were awakened by the liberalization of telecommunications services and the strength and entrepreneurship of their major competitors, especially British Telecom. BT was to challenge the two operators in their home markets. It also had a bunch of alliances with other European operators and was to have cooperation with MCI, a US long-distance provider. Deutsche Telekom and France Telecom established an alliance to merge some services, avoid rivalry between them and to counteract British Telecom. This alliance gave the partners the critical mass in Europe and secured their home markets. Also they gained access to the US markets via the ownership of Sprint.

When targeting new markets of new technologies, race for the world, co-option supports the building of nodal positions in emerging coalitions. This applies especially in standard based markets where network effects provide significant advantages for first-movers. The aim of the coalition is to make the own standard or de-facto solution the winner in the markets. The support of vendors of complementary services and products is very important when convincing markets of the own standard's superiority.

In alliances for new technologies and new markets two types of partners are usually needed: competitors and companies providing complementary technologies. A citation from (Doz, Hamel 1998):

1. "Exploit new opportunities and meld a wider set of differentiated resources than any company would possess on its own;

2. Co-opt "unaligned" competitors and complementers into their own camp, sometimes because they can contribute valuable cospecialized and differentiating resources, sometimes to prevent them from falling in with a rival coalition;

3. Build market leadership quickly. Being first matters most when the advantage that accrue to first movers are substantial (most often because technical compatibility needs – what economists call network externalities – leave little room for incompatible new solutions once the first one is established on the market);"

In Motorola's Iridium alliance the national telecommunications service operators were committed to the co-option in two ways. They provided Iridium the needed traffic rights (complementary co-option) and they supported Iridium's deployment and were not available to rival coalitions (competitor co-option).

Value creation through co-specialization

Co-specialization is a means for companies that race for the world to gain global market access and to get partners to complement their skills. Especially this applies to the companies in the oligopolistic industries, whose home markets are often mature and slow growing and who are searching access to new emerging markets. Alliances with local firms are a preferred mechanism to achieve the target.

Telecommunications markets are an arena of global and fierce competition. Global operators have established alliances with local players when targeting for new countries. For example in Latin America France Telecom has an alliance with Sprint to access the

Argentina and Mexico markets, the Spanish operator Telefónica has activities in Chile and Argentina, and US operators have had similar arrangements.

Big global operators have needed cooperation with local companies to have access to specialized capabilities required in unfamiliar new markets. When France Telecom took over part of Argentina's telecommunication market in 1990, it needed cooperation with many partners: Italian STET for complementary experience and competencies, J.P. Morgan for financial expertise and the Perez Company Group for having a credible local face. The alliance with the partners provided France Telecom the access to all these new and complementary skills and enabled fast operations in Argentina.

Alliances can be beneficial not only for global players but also for local companies searching for entry into global markets. In the 1960s Japanese companies used alliances with American and European partners to learn technologies and skills needed in foreign markets. Similarly Taiwan's Acer, nowadays a laptop manufacturer, had an alliance with Texas Instruments in the pursuit of global markets.

Co-specialization is also a favourite means for companies creating new products and services. Frequently skills needed for new opportunities are too broad for any single company and the cooperation between many contributors is needed. This is especially true these days when companies are leaner and more selective on their core competencies. Also new industries require an even greater set of different skills. As an example, today's laptop is an outcome of many specialized capabilities; high resolution flat screen technologies, high performance hard disk drives, high-energy batteries, CPUs and other specialized integrated circuits, operating systems etc. There is no single company in the world that would master all these technologies.

When new products are based on hybrid technologies combining results from different scientific fields, the knowledge needed is likely to be spread over several firms, universities and alike. Uncertainty and risks are high and thus the product creation is a second-order challenge. Take for instance biological computers. Results from the fields of biotechnology, optoelectronics and bionics are needed.

Even more challenging combination of capabilities and skills is needed in case of complex system solutions. These are integrated service and network-based systems with a blend of hardware, software, services and which require the integration of technical, political, regulatory and financial elements.

The alliances of high network externalities and first-mover advantages act as a convergence arena and a forum for validating technological choices by activating the knowledge and insights of partners. Also the partners can ensure that their interests and capabilities are taken into account in the design of the solution. Simultaneously these alliances drive the goals of strategic co-option by setting the terms of competition and resource and skills complementation.

Value creation through learning and internalization

Firms competing in global markets must have comparable skills with their rivals. Shortfalls in skills make companies vulnerable and they are not able to compete with their global competitors. If not rapidly filled, skill gaps can break weaker players, especially when their rivals continue to develop new skills and to use them in competition.

Companies often exploit alliances to learn and internalise new skills. This is usually faster than e.g. internal competence development. Also acquisitions have their own risks. Competencies are not necessary transferred from the acquired company. Key persons can leave the company after the acquisition or the different management cultures can hinder the transfer of competencies.

Siemens is an example of using strategic alliances systematically to acquire and develop new competencies. In the early 1980s Siemens was awakened to the growing importance of microelectronics and to the fact that they had fallen behind world leaders. Siemens introduced a series of strategic alliances to catch up with the competencies in microelectronics. The first alliance was with Toshiba on manufacturing technology. Others were to follow; an alliance with Philips on the development of memory chips, a broad alliance with European partners and alliances with IBM and Motorola. Siemens used these alliances as building blocks for creating new competencies and in 1995 Siemens was included among the world leaders in microelectronics.

2.5 Strategic Alliances of Telecommunications Operators

Drivers behind Telecommunications Alliances

There are two main drivers behind the strategic alliances of the telecommunications industry. The first one, convergence of ICT industry stretches the product portfolio of telecommunications operators. The second one, globalization with telecommunications flavours demands the geographical coverage of telecommunication services to be extended over national boundaries. These two drivers have significantly changed the telecommunications markets and competition. The observable revolution has been the rapid migration from national voice centric networks to a global integrated communications system with voice, data, video and data transmitted over diverse wire and wireless networks (Chan-Olmsted, Jamison 2001).

The convergence is melding the previously distinct industries into one ICT industry: telecommunications, media, computing and consumer electronics. The basic elements of the ICT mixture are customer devices (e.g. telephones, PCs, televisions), networks (e.g. wire/wireless, cellular, satellite, Internet), computing of network devices (i.e. servers) and the content (e.g. music, videos, TV programs) (Jamison 1999).

The role of traditional telecommunications incumbents has changed dramatically due to the convergence. Instead of providing voice services over a fixed network, competitiveness is now measured in terms of global coverage and services (Jamison 2000). Coverage and services have an effect on each other. To be able to leverage network effects, the operator must have attractive services and a critical mass of connected customers. A large customer base in turn enables the scale of economies and further development of new services and coverage (Chan-Olmsted, Jamison 2001).

The convergence between telecommunications and media markets takes place when originally separate offerings start to adapt elements from each other, e.g. operators adding content into their services and media firms extending their delivery networks to systems such as cable, satellite and Internet. Gradually the overlap between these two areas increases and firms of the two markets start to compete against each other (Colombo, Garrone 1998). Technologically the IP protocol used in Internet is the common denominator between diverse networks and user devices. IP networks provide a flexible platform for new kinds of services. (Vesa 2005).

A set of reasons has been behind the globalization of the telecommunications sector. Liberalization, privatization, international reciprocal agreements, technological development and customer demands have all been favourable for the globalization (Chan-Olmsted, Jamison 2001).

Liberalization and privatization are likely the two most significant reasons behind the telecommunications industry globalization. The opening of the markets has brought new business opportunities, and thus new entrants providing telecommunications services have emerged. Also the competition has increased and many incumbent telecom service providers have been forced to bargain over their profits. Many of the new opportunities due to the liberalization and privatization have emerged in developing countries, which has been a new challenge for traditional telecommunications companies. New markets have been unfamiliar to them and thus incumbents have adopted new entry strategies including local partners and strategic alliances rather than direct investments (Chan-Olmsted, Jamison 2001).

International reciprocity agreements have aided the impact of liberalization and privatization. World Trade Organization's Basic Agreement on Telecommunications and 1997 FCC Benchmarks Order have considerably reduced the fees that US operators pay international operators for terminating US originated calls (Chan-Olmsted, Jamison 2001).

Technological development also drives the globalization of telecommunications. It has had an impact on provided services, cost structure of the industry, customer demands and capital investments. Technological development has decreased prices and thus increases the demand for services and leveraged growth of global and domestic markets. Also the pricing principles have changed; in Internet distance means nothing and it is losing its meaning also in mobile communications (Chan-Olmsted, Jamison 2001).

The growth of Multinational Corporations and their global business has created demand for worldwide, integrated and seamless communications services. Also the hardening competition over the traditional industry boundaries creates multilateral competition and collaboration (Jamison 2000).

A telecom company has two different strategies when targeting growth in global markets. It can use its own resources by building services and channels in the target market or it can cooperate with other companies. The choice of using only the company's own resources is usually the slower and more expensive alternative. It also lacks the brand name recognition and local political and business expertise. In global telecommunications markets where time-to-market is an important factor and resource allocations to unfamiliar markets can have a high risk, this alternative may be undesirable (Joshi, Kashlak & Sherman 1998). This leaves the alternative of building the global growth strategy on the basis of strategic alliances with other companies.

High-profile failures – Concert, Global One and World Partners

In the late 1990s all the major telecommunications players were involved in strategic alliances of the industry. All the biggest alliances, namely Concert, Global One and World Partners, were failures and were later dissolved (Curwen, Whalley 2004). This raises the question of the reasons behind the failures. Why were they not flourishing although the partners in the alliances were the world leaders, having plenty of resources and the start of the cooperation was full of hope and good intentions?

The original *Concert* was an alliance of British Telecom and MCI. BT's intention was to establish a foothold in the US. The target customer segment of the Concert were international business customers (Roberts, Waters 2001). BT purchased a share of 20% in MCI. Other partners of the alliance were Spanish Telefónica and Portugal Telecom which gave Concert a strong hold on Latin America markets (Chan-Olmsted, Jamison 2001).

Later BT tried to purchase the rest of MCI, but failed due to MCI's losses in the US local exchange markets and the concerns they caused among BT's shareholders. The original Concert was dissolved when WorldCom outbid MCI and MCI WorldCom was established. BT started a second round and a new Concert was introduced with AT&T in 2000. The target market was multinational corporations (Chan-Olmsted, Jamison 2001). It did not take longer than a few months before problems emerged in the alliance and Concert stopped doing business with big corporations in its own name. Fairly soon in 2001 also the new Concert came to an end. (Roberts, Waters 2001).

The problems BT encountered in the original Concert were related to regulatory approvals and information asymmetries. US and EU regulators set limiting conditions due to BT's 20% stake in MCI, and UK regulator required BT to commit to domestic investments. The information asymmetries were between BT and MCI and between the BT management and shareholders. BT did not know about the MCI's losses in the US local exchange markets and BT's shareholders were not convinced that MCI was worth the purchase after the losses. This created distrust between the alliance partners and between BT's shareholders and management (Chan-Olmsted, Jamison 2001).

The problem of BT's and AT&T's new Concert was bad management. Sales and marketing were not managed by one organization but by separate sales forces of Concert, AT&T, AT&T Solutions and BT's Ignite. Furthermore, AT&T's executives were disappointed to find that BT was incapable of providing Concert a strong European presence. They claimed that the reason was BT's partial ownership in the telecommunications business in Europe. BT rejected the claims and stated: "The fact that these were not fully-owned shareholdings didn't really make any difference. It was always intended to be that way". To summarize, the feelings of one person involved in break-up discussion: "Concert is a classic tale of management failure. There were lots of other problems like the corporate structure but ultimately it was the management that were to blame" (Roberts, Waters 2001).

Global One was an equity alliance between Sprint, Deutsche Telekom and France Telecom. Deutsche Telekom and France Telecom purchased a 20% share in Sprint (Chan-Olmsted, Jamison 2001). Their motive was to avoid rivalry and to counter British Telecom. The alliance provided them the critical mass in Europe and secured their home markets. Furthermore, they also gained access to the US markets via Sprint (Doz, Hamel 1998).

Partners of the alliance expected Global One to be a profitable business unit. However, they were not able to agree on a single global strategy and thus the alliance did not come up to expectations. When 1999 MCI WorldCom and Sprint agreed on a merger, Sprint repurchased Deutsche Telekom's and France Telecom's shares in Sprint. Finally France Telecom purchased Deutsche Telekom's share in the alliance and thus became the only owner of Global One (Chan-Olmsted, Jamison 2001).

The partners of Global One were able to pursue their own business strategies. Also the alliance achieved some of the objectives: it won a large number of contracts of multinational companies. On the problem side we can find again distrust between partners. When Deutsche Telekom made a bid for Telecom Italia, it did not inform its partner France Telecom, a competitor of Telecom Italia, about it (Chan-Olmsted, Jamison 2001).

World Partners was an alliance that was based on equity shares and cooperation agreements. The partners were AT&T and Unisource, itself a European alliance of Telecom Italia, Telia from Sweden, Swiss Telecom and KPN from Netherlands. Other members were Japanese KDD, Australian Telstra and Canadian Unitel.

The formation of World Partners was a counteraction to Concert alliance of British Telecom and MCI. AT&T already had contracts with many telecommunications incumbents and basically World Partners was only a renaming of these existing agreements. Due to its history World Partners was mainly based on agreements and not so much on equity shares as Concert and Global One.

World Partners did not extend its service offering beyond the alliance’s starting point and thus was not able to deliver a strong global service portfolio. That was likely due to the loose structure of the alliance. World Partners is not an active alliance anymore (Chan-Olmsted, Jamison 2001).

According to the experience of Concert, Global One and World Partners, the intra-industry alliances with competitors are very challenging. As suggested by (Doz, Hamel 1998), an alliance usually has two dimensions. One is what partners can achieve together and the second is what partners gain for themselves. To find the balance between cooperation and competition is demanding for partners who quite often are rivals in the marketplace. To get the desired value from the alliance, a partner must contribute unique assets to the alliance if it aspires to maintain the influence in the alliance. As summarized by (Curwen, Whalley 2004), “companies involved in alliances often have as many good reasons to compete against one another as to collaborate, which creates a certain tension in the working relationship.”

New Strategic Alliances of Mobile Operators

The bitter experience from Concert, Global One and World Partners has not totally deterred telecommunications industry from introducing new alliances. Actually, in recent years mobile operators have formed a series of alliances (Curwen, Whalley 2004). To understand the motives for mobile operators' alliances, they are shortly introduced in the following discussion.

Mobile operator alliances according to (Rieck et al. 2005), (Curwen, Whalley 2004) and (NTT DoCoMo 2006):

- *Fixed Mobile Convergence Alliance* (FMCA): The founder members of the FMCA are British Telecom, Brazil Telecom, Korea Telecom, NTT, Rogers Wireless and Swisscom; established in July 2004. The customer base is 122 million fixed and 23 million mobile subscribers. The goals of the FMCA are to remove the distinction between fixed and mobile networks; to provide cost-efficient and seamless converged fixed-mobile services to consumers; influencing and promoting fixed-mobile convergence targeting to high quality, low-priced phones and to provide seamless services to customers.

- *Vodafone Group and Partner Network*: Vodafone Group was established in the 1990s when Vodafone extended its operations beyond the UK. The group’s strategy has been to avoid alliances and joint ventures and to grow by direct investments. Currently the group provides services in 26 countries in five continents. To further gain a foothold in Europe, Vodafone Group has formed a Partner Network comprising of mostly European operators. Vodafone’s targets have been to ensure the global presence, strong single brand and to have a worldwide mobile network with seamless global service offering. Partner Network also enables additional roaming fees from visiting subscribers and access to other international telecommunications markets. The resulting subscriber base is 112 million.

- *Asia Mobility Initiative* (AMI): AMI is a non-exclusive Asian alliance formed in April 2003 by CSL from Hong Kong, Malaysian Maxis, Mobile One from Singapore, Smart from Philippines and Australian Telstra. DTAC from Thailand and CTM from Macau increased the alliance's subscriber base to 31 million. AMI's targets are to provide simpler standardized access to customers, to offer improved user experience in mobile data services and new platforms and jointly produced devices. AMI has strengthened their competitiveness in regional niche markets by introducing entertainment services and delivering games.

- *Starmap Mobile Alliance:* Starmap alliance was established in October 2003 by nine European mobile operators: Spanish Amena, British O2, Austrian One, Hungarian Pannon GSM, Swiss Sunrise, Norwegian Mobil and Italian Wind. New members Eurotel from the Chech Republic and Danish Sonofon increased the alliance's subscriber base to 53 million. Their target is to provide "Feel at home whenever you go"; seamless access for business and consumer customers across Europe.

- *FreeMove Alliance*: FreeMove Alliance was established in 2003 by four leading mobile operators with a 230 million subscriber base: T-Mobile from Germany, Spanish Telefonica Moviles, Telecom Italia Mobile and French Orange. Their target is to deliver a seamless user experience at home and abroad. Members improve their competitiveness by cooperating to increase efficiency and economies of scale benefits in R&D and procurement. Simple and predictable roaming pricing plans are a significant improvement for customers. FreeMove has further expanded its alliance network.

- *Bridge Mobile Alliance* (BMA): BMA is an Asian-Pacific alliance which was formed in 2004 by seven operators of 56 million subscribers: Indian Bharti, Globe Telecom from Philippines, Malaysian Maxis, Australian Optus, SignTel from Singapore, Taiwan Cellular Corporation and Indonesian Telkomsel. The target of the alliance is to encourage their customers to use enhanced roaming services and thus to increase the revenues of the alliance members. Bridge Mobile also aims to develop new products and services and to create core competencies for the alliance members.

- *Asia-Pacific Mobile Alliance*: This is an Asian-Pacific alliance which was formed in April 2006 by seven mobile operators of 100 million subscribers: Far EasTone from Taiwan, Indian Hutchison Essar, Hutchison Telecom Hong Kong, KTF from South Korea, Japanese NTT DoCoMO, Indonesian Indosat and StarHub from Singapore. The target of the alliance is to enhance its members'

> competitiveness in international roaming and corporate mobile services in the Asian-Pacific region. The alliance offers their customers greater convenience, ease-of-use and value added mobile services when roaming in the alliance members’ networks. Their aim is to create a “Virtual Home Environment” for their customers.

When studying these mobile operators' alliances one outcome is obvious: the difference between Vodafone Group/Partner Networks and the rest of the alliances. Vodafone mainly relies on direct investments in expanding its global presence. Also Vodafone’s target is to leverage its own brand worldwide. Other alliances build the global coverage through alliance members' networks and they do not promote a single global brand but the members use their own brands. Clearly Vodafone’s alliance is orchestrated by Vodafone, while in other alliances the leader is not so evident.

Loose alliances (i.e. not Vodafone’s) appear to offer a number of benefits to their members, such as scale-related savings when purchasing handsets, better control over roaming revenues, cooperation benefits in product and service development and learning through alliances. It has been proposed, however, that Vodafone’s position and control via direct ownerships outperforms the loose alliances, which are introduced without equity stakes (Curwen, Whalley 2004).

Many of the alliances state that their target is to co-ordinate the roaming fees between the alliance members and to provide their customers a better transparency on roaming prices. The roaming revenues would be gained from increased roaming traffic. From that target’s point of view Vodafone outperforms other alliances. In (Rieck et al. 2005) mobile operators’ alliances are studied from the roaming charges’ viewpoint. One of the questions of the research was whether roaming prices are aligned between the alliance’s members. In other words, how successful an alliance has been in controlling the roaming fees between its members and to provide the best roaming prices for its customers. A citation from (Rieck et al. 2005):

“Our results also show that only Vodafone Group and its Partner Network showed typical ‘favoured partner’ behaviour within its alliance. Vodafone achieved this through its consistent application of its networking strategy and uniform distribution of the ‘alliance benefits’ without discrimination within its networks. This resulted in a high degree of reciprocation of best prices within the group, making the ‘Vodafone Group and its Partner Network’ networking strategy more effective and successful as compared to other existing alliance strategies. The other alliances, such as FreeMove, had a relatively low rate of reciprocation due to inconsistent pricing within their network. For instance, their lowest prices were not restricted to alliance members, and were given on a non-discriminatory basis.”

Starmap alliance is formed by a handful of European mobile operators of roughly the same size who do not compete in most of the markets. The only exception is Austria where One and Telenor are both operating. This provides Starmap a clear advantage. Its members are more likely to agree on common strategies, products and services. The situation with FreeMove alliance is totally different. Alliance partners have overlaps in their markets. Also T-Mobile and Orange have both aspirations at the pan-European, maybe also on global level. This is a great challenge to FreeMove and to its capability to make decisions (Curwen, Whalley 2004).

3 GAME THEORY AND COOPERATIVE GAMES

3.1 Theory of games and economic behaviour

Game theory is the study of mathematical modelling of multi-person decision problems. Decision makers are called players and they interact with each other in a context called the game. In general, one player's actions may influence the other players' decisions. It is assumed that players behave rationally when they pursue well-defined objectives and they reason strategically by taking into account their knowledge of other players' behaviour (Osborne 1994).

The roots of the game theory are in economics and mathematics. A solid foundation in the game theory was given in the book Theory of Games and Economic Behaviour, originally published in 1944, written by mathematician John von Neumann and economist Oskar Morgestern (Von Neumann, Morgenstern 1944). The book created the game theory paradigm of mathematically modelling the social behaviour of human beings who interact with each other in the game context. The contribution of two authors with a different background is visible in the book. It is said that the book was written twice; with mathematical symbols by von Neumann and with prose of economists by Morgestern (Kelly 2003). The value of von Neumann's and Morgestern's book can hardly be overestimated. It created the basis for a rich variety of different theories and applications in many areas.

Many pioneers of the game theory have been noticed in the highest academic level. Game theorists have twice been awarded the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel ("Nobel prize in economics"), in 1995 and 2005. In 1995 John Nash, John Harsanyi and Reinhard Selten shared the prize for their pioneering analysis of equilibria of the theory of non-cooperative games. In 2005 Robert Aumann and Thomas Schelling received their prize for the analysis of conflict and cooperation through game-theory (Kungl. Vetenskapsakademien 2005).

3.2 Classification of games

There are many different ways to categorize games. Categories and sub-categories can be formed based on the number of players, the nature of the cooperation, the amount of information known to players, the duration of the game and several other factors. Also the historical dimension has an impact; the names of categories and the way to form them can depend on the phase of the historical continuum of the game theory.

The classification of games, which is illustrated by Figure 1, is based on the categories defined in (Gibbons 1992) and (Peleg, Sudhölter 2003).

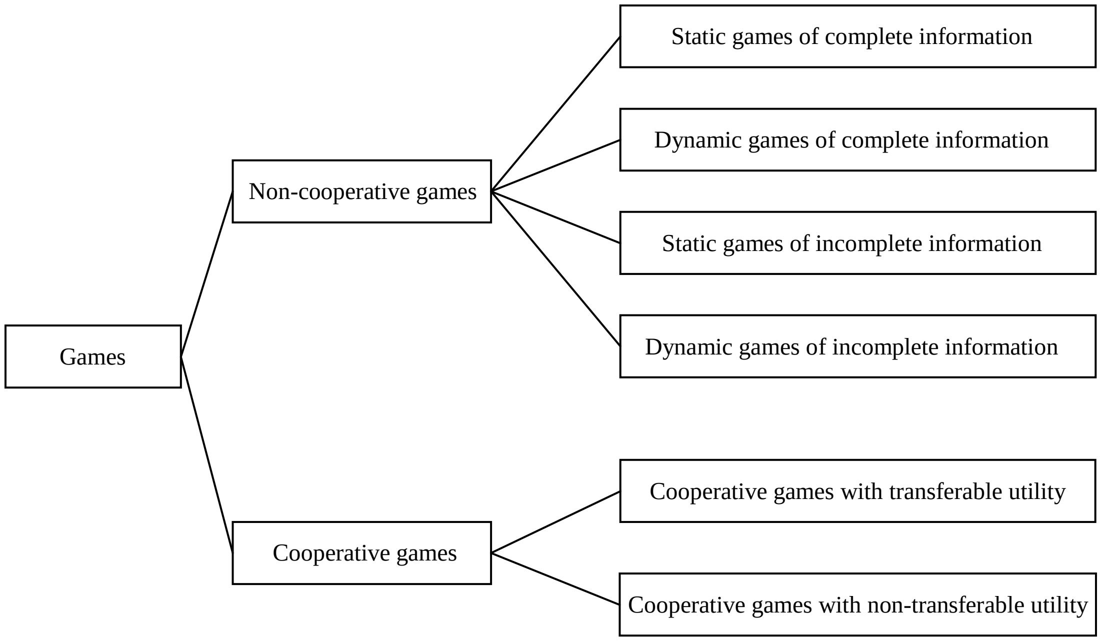


Figure 1.Classification of games

The main division is done between cooperative and non-cooperative games. Cooperative games are also called coalitional games due to the fact that in cooperative games a coalition makes the decisions about the strategies to be chosen instead of individual players as in non-cooperative games. In cooperative games the players can also form binding agreements about the division of pay-offs (Harsanyi, Selten 1988).

Cooperative games are divided into two sub-categories: games with transferable utility and games with non-transferable utility. In the games with transferable utility it is assumed that a player can move part of oneself's utility to another player without any loss. In cooperative games the payoffs of strategic moves are addressed to a coalition but not to individuals. If the utility is transferable it can be assumed that with all possible pay-off divisions between coalition members, the total utility received by the coalition is always the same. In a game with non-transferable utility, the utility is not transferable among the members of a coalition.

Non-cooperative games can be categorized by two criteria: the amount of information known to players and whether the game is static or dynamic. If all players know their own and also the other players' pay-off functions, it is a game of complete information. If a player is unsure about the other players' pay-off function, the question is about a game of incomplete information. A typical example is an auction where each bidder's willingness to pay for the object auctioned is not known to other bidders (Gibbons 1992).

When players participate in a static non-cooperative game, all participants choose their actions at the same time, without any knowledge about the other players' choices. After the decisions players receive their pay-offs and the game ends. In dynamic games players have some knowledge about earlier actions of the game. That knowledge can be complete or incomplete. Static games are also known as simultaneous-move games and, respectively, dynamic games as sequential games.

A typical and convenient way is to represent static games in normal-form (usually a matrix representation, Figure 2) and dynamic games in extensive-form (usually a tree representation, Figure 3) (Gibbons 1992). Because of this convention, also the names “games in normal form” (Von Neumann, Morgenstern 1944) and “extensive games” (Osborne 1994) are used.

		Player 2		
		Left	Middle	Right
Player 1	Up	1,0	1,2	0,1
	Down	0,3	0,1	2,0

Figure 2.An example of normal-form representation (Gibbons 1992)

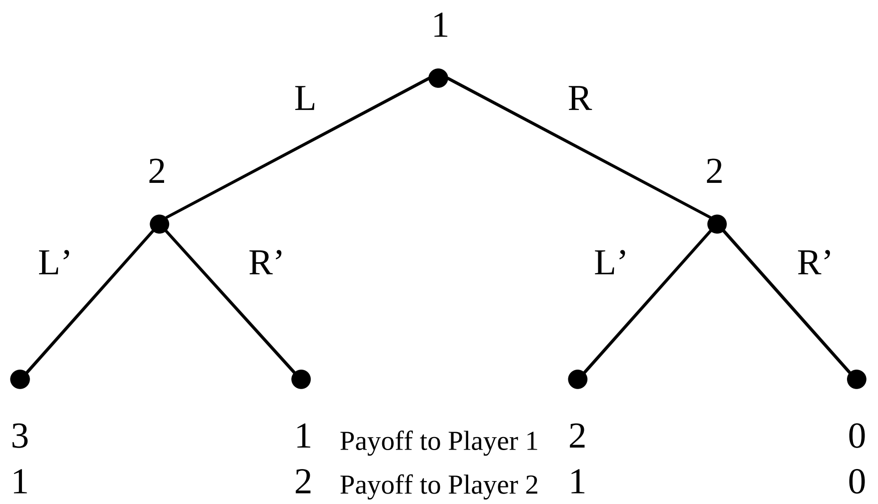


Figure 3. An example of extensive-form representation (Gibbons 1992)

In addition to the above presented taxonomy, there are also many other ways to classify games. Games can be two-person or n-person games, zero-sum games (total pay-off received by all players is always the same) (Davis 1997), repeated games (a sub-category of dynamic games of complete information) or bargaining games (Osborne 1994).

Applications of game theory are wide. In addition to economics, the game theory has been successfully applied in the fields of politics, sociology, biology and computer sciences. Even philosophers have found the game theoretical approach to be fruitful (Siegfried 2006). Due to its roots the economical applications of the game theory are wide-ranging. Phenomenon such as bargaining, labour markets, duopolies, oligopolies, international tariffs and auctions have got their game theoretical modelling (Gibbons 1992).

3.3 Nash equilibrium

A famous solution concept with economical applications is Nash equilibrium, named after John Nash. In his visionary paper Nash generalised the minimax solution concept of von Neumann and Morgestern (Kelly 2003). He was able to show that in any competitive finite game there is at least one equilibrium point (Nash 1950).

The basic assumption behind the game theory is that players behave rationally, i.e. that by using all information they have about their own alternatives and other players' choices, their objective is to maximize their own pay-off regarding their preferences (Osborne 1994). The same applies also to the Nash equilibrium. It predicts the strategies of all players when no one accepts any other solution but the one which maximizes his or her own pay-off when given the strategic decisions' of all other players (Gibbons 1992).

The basic idea behind the Nash equilibrium is general and widely applied in the game theory, especially with non-cooperative games. Although the Nash equilibrium is not usually applied in the class of cooperative games there is an analogue concept, the core, in cooperative games. For that reason the Nash equilibrium is presented here in more depth. The basic elements of game theory are also introduced at the same time. The introduction to Nash equilibrium presented below follows the text book (Gibbons 1992).

The *normal-form representation* of a game is comprised of 1) the players of a game, 2) the available strategies of each player and 3) the pay-off of each player for all possible combinations of strategies. In an n-player game there are n players which are numbered from 1 to n. The strategies available to player i are symbolized with set S_i with s_i as an element of this set, i.e. $s_i \in S_i$. The combination of strategies is denoted by $(s_1,\ldots,s_n)$ (one for each player) and the i player's pay-off function is $u_i(s_1,\ldots,s_n)$ for the strategy set of $(s_1,\ldots,s_n)$. This leads us to the following definition:

Definition: The normal-form n-player game is comprised of n players, their strategy spaces $S_1,\ldots,S_n$ and their pay-off functions $u_1,\ldots,u_n$. This game is denoted by $G = \{ S_1,\ldots,S_n; u_1,\ldots,u_n \}$.

Nash equilibrium can be defined in the following way:

Definition: In the normal-form n-player game $G = \{ S_1,\ldots,S_n; u_1,\ldots,u_n \}$, *the strategies* $(s^*_1,\ldots,s^*_n)$ *are a Nash equilibrium if, for each player i,* s^*_i *is player i's best response to the strategies of other n-1 players,* $(s^*_1,\ldots,s^*_{i-1}, s^*_{i+1},\ldots,s^*_n)$:

$$(s^*_1,\ldots,s^*_{i-1},s^*_i,s^*_{i+1},\ldots,s^*_n) \geq (s^*_1,\ldots,s^*_{i-1},s_i,s^*_{i+1},\ldots,s^*_n) \qquad (1)$$

for every feasible strategy $s_i \in S_i$.

The strategies predicted by the Nash equilibrium are stable because there is no reason for any player to deviate from the strategies of the equilibrium. If someone chose a strategy which is not according to the Nash equilibrium, either his or her payoff would be less than with the equilibrium strategy, or the solution would not be stable because the payoff of some other player would be less than with the solution of Nash equilibrium. This approach assumes that all players have complete information about the strategic alternatives of other players (Gibbons 1992).

3.4 Cooperative games

In non-cooperative games individual players make their own decisions about the strategy and the target of each player is to maximize his or her payoff regarding one's own preferences. This is not the case in cooperative games. In cooperative games the players can form coalitions which can make binding agreements about the distribution of payoffs and also about the strategies to be chosen.

The model of binding agreement has a close analogy in economics. Many business transactions are based on binding agreements and often a penalty will follow if the agreement is violated.

Even though the payoff of cooperative games is addressed to a coalition, i.e. a group of individual players, ultimately individual preferences count. In non-cooperative games the players usually focus on the stable strategies. For example the Nash equilibrium is a good example of a solution concept for non-cooperative games, in which the target is to find stable strategy sets. In cooperative games the solution concepts usually target at stable payoff distributions, which all members of the coalition can accept (Peleg, Sudhölter 2003).

The cooperative game theory is very much focused on the results of the cooperation and how the results are divided among the players rather than on the strategies, as is the case with non-cooperative games. Questions such as what coalitions can be formed and how the results of the cooperation should be divided to ensure a sustainable agreement are among the basic topics of cooperative games (Parrachino, Zara & Patrone 2006).

Cooperative games are divided into two sub-categories: games with transferable utility and games with non-transferable utility. In the games with transferable utility a player can move a part of his or her own utility to another player without any loss. Typically we can assume that the payoffs are money, which has the same value for all players. Of course also other utilities can be used if there is a common way to compare the players' utilities.

Cooperative games with transferable utility can be seen as a primitive and most common concept. In the following the basic concepts of cooperative games are introduced, the focus is on the games with transferable utility. Discussion follows (Peleg, Sudhölter 2003) and (Parrachino, Zara & Patrone 2006); originally basic concepts were introduced in (Von Neumann, Morgenstern 1944).

Definition: Let's assume that N is a set of players. An n-person cooperative game in the characteristic form is a pair (N, v) where v is a function that associates a real number v(S) with each subset $S \subseteq N$. Always $v(\emptyset) = 0$.

In this definition the *N* is a set of players, also called the grand coalition. *v* is the coalition function, also called characteristic function which defines the payoff for each coalition *S*, subsets of N. If the *S* is established, the members get *v*(S) amount of money (transferable utility). The *v(S)* can also be called the worth of *S*. It is required that the worth of an empty coalition is zero.

If a coalition is established, it can divide the worth of *v(S)* in all feasible ways among the members of the coalition, that is, all payoff vectors $\boldsymbol{x} \in \mathbb{R}^S$ are valid, which satisfy:

$$\sum_{i \in S} x^i \leq v(S) \tag{2}$$

Usually the players of cooperative games are persons or groups of persons, e.g. labour unions, political parties, municipal authorities, landowners, voters or business partners. To concretize the theory, let's take an example from (Brânzei, Dimitrov & Tijs 2005), called "Glove game":

Example 1. Let $N = \{1,\ldots,n\}$ be a set of gloves which is divided into two subsets $L \equiv$ *left hand gloves* and $R \equiv$ *right hand gloves*. Because one glove is either for left hand or right hand, $L \cup R = \emptyset$. The value of a pair of gloves is one euro, a single glow without its pair is worth of nothing. This can be described with a game *(N, v)*, where each coalition $S \subseteq N$ has the value $v(S) := min\{|L \cap S|, |R \cap S|\}$.

Next continue with some basic definitions which describe the quality of a game.

Definition: A game (N,v) is superadditive if, for all $S,T \subseteq N$, *with* $S \cap T = \emptyset$,

$$v(S \cup T) \geq v(S) + v(T) \tag{3}$$

If a game is superadditive, the value of all disjoint coalitions of *N* is not less than the worth of the coalitions' value separately. If a coalition $S \cup T$ is formed, at least the members of the coalition do not lose the total original value of *S* and *T*. If a game is not superadditive, there is no incentive for the players to establish coalitions because they can do better on their own. It can be claimed that if $S \cup T$ is formed, its members can continue as S and T would act separately, i.e. to follow the same strategies as earlier and to divide the results among the members of coalitions S and T as before the merger. This way they would gain $v(S) + v(T)$, which is according to (3). In real life situations this does not always hold. E.g. anti-trust laws can reduce the profits of a merger or large coalitions can be more inefficient than smaller ones.

Definition: A game (N,v) is convex if for all $S,T \subseteq N$,

$$v(S \cup T) + v(S \cap T) \geq v(S) + v(T) \tag{4}$$

A convex game is superadditive. The following expression is equivalent with the convex game definition. For a convex game, for all $i \in N$ and all $S \subseteq T \subseteq N$ not containing i, the following holds

$$v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T) \tag{5}$$

In a convex game the marginal contribution of a player to a coalition is monotone non-decreasing; the larger the coalition, the greater the marginal contribution of new members (or at least not less than with smaller coalitions).

Definition: A game (N,v) is constant-sum, if for every $S \subseteq N$,

$$v(S) + v(N \setminus S) = v(N) \tag{6}$$

Many political games are constant sum games. The competition is about the votes of electors, which is usually constant.

Definition: A game (N,v) is inessential if for every $S \subseteq N$,

$$v(S) = \sum_{i \in S} v(\{i\}) \tag{7}$$

An inessential game is trivial from the viewpoint of game theory. If every player i demands at least $v(\{i\})$, the inessential game's distribution of $v(N)$ is completely defined.

Let's assume that players of an n-person cooperative game in the characteristic form (N, v) have agreed to cooperate and to form the grand coalition N. The players need to agree on the distribution of the payoff $v(N)$ among the members of the grand coalition. First we shall assume that the way to divide the payoff must be *feasible*.

Definition: The set $X^*(N,v)$ is the set of feasible payoff vectors for the game (N,v), if following holds

$$X^*(N,v) = \{x \in \mathbb{R}^N \mid x(N) \leq v(N)\} \tag{8}$$

A payoff vector $x \in \mathbb{R}^N$, also called *an allocation*, defines the payoff for each player ***i***. Feasible payoff vectors $X^*(N,v)$ are simply all allocations, which do not exceed the total payoff of $v(N)$. If the vectors $X^*(N,v)$ satisfy the efficiency principle and the payoff vectors split exactly the total value of $v(N)$, they are called *efficient payoff vectors* or *(pre)imputations X(N,v)*:

$$X(N,v) = \{x \in \mathbb{R}^N \mid x(N) = v(N)\} \tag{9}$$

3.5 Solution concepts of cooperative games

To choose the appropriate allocations from the set of feasible payoff vectors, some agreements or rules are needed among the players. Such a set of rules is called *a solution* or *a solution concept* (Parrachino, Zara & Patrone 2006). Discussion follows (Peleg, Sudhölter 2003).

Definition: Let Γ be a set of games. A solution on Γ is a function σ which associates with each game $(N,v) \in \Gamma$ a subset $\sigma(N,v)$ of $X^*(N,v)$.

To be able to characterize different solution concepts, some properties of solutions are introduced.

Definition: A solution σ on Γ is individually rational if $(N,v) \in \Gamma$ and $x \in \sigma(N,v)$, then $x^i \geq v(\{i\})$ for all $i \in N$.

This says that every player *i* gets at least the worth he or she could get on his/her own. If a solution is not individually rational, the players do not have an incentive to join the grand coalition N.

Definition: A solution σ on Γ is efficient if $(N,v) \in \Gamma$ and $x \in \sigma(N,v)$, then $x(N) = v(N)$.

When the solution is efficient, the total value of $v(N)$ is exactly divided among the players.

Payoff vectors, which are individually rational and efficient, i.e. the total value of $v(N)$ is divided among the players and all the players get at least their individual value $v(\{i\})$, are called imputations.

In the following definitions, let's denote with $b^i_{max}(N,v)$ and $b^i_{min}(N,v)$ the maximum and minimum of *i*'s incremental contribution to a coalition in a game (N,v).

Definition: A solution σ on Γ is reasonable from above if for all $i \in N$,

$$((N,v) \in \Gamma \text{ and } x \in (N,v) \Longrightarrow x^i \leq b^i_{max}(N,v) \tag{10}$$

Definition: A solution σ on Γ is reasonable from below if for all $i \in N$,

$$((N,v)\in\Gamma \text{ and } x\in(N,v) \Longrightarrow x^i \geq b^i_{min}(N,v) \qquad (11)$$

Definition: A solution σ on Γ is reasonable from both sides if it is reasonable from above and reasonable from below.

It is not reasonable to pay any player more than his or her maximum contribution to any coalition. On the other hand a player *i* can demand at least $b^i_{min}(N,v)$ and that would not harm any coalition.

If a solution is individually rational it is also reasonable from below.

Solution concepts focus on the questions of stable coalitions and on the allocation of the payoff among the players. Many of the solutions do no give a one-point solution to the allocation question but a set of solutions. On the other hand, also one-point solution concepts exist. The most popular solutions in literature are the following:

Subset solutions:

- Stable sets (Von Neumann, Morgenstern 1944)
- Core (Gillies 1959)
- Bargaining sets (Aumann, Maschler 1964)
- Kernel (Davis, Maschler 1965)
- Least core (Maschler, Peleg & Shapley 1979)

One-point solutions:

- Shapley value (Shapley 1953)
- Nucleolus (Schmeidler 1969)
- τ-value (Tijs 1981)

3.6 Stable sets

The stable sets of a game were the first solution concept for n-player cooperative games (also known as von Neumann-Morgestern solution). The following introduction of the stable sets follows the book (Osborne 1994).

Let's assume to have a grand coalition N and a coalition $S\subseteq N$. The payoff vector $x(N)$ allocates the value of $v(N)$ among the players. The applicable payoff vectors shall be individually rational and efficient, i.e. the total value of $v(N)$ is divided among the players and all the players get at least their individual value $v(\{i\})$. Even though the members of S would get at least their own value $v(\{i\})$, they can still be unsatisfied. They can credibly object that by forming a coalition S, all the members of the coalition would get more by dividing the value of $v(S)$ on their own way. Thus they suggest allocate the value $v(N)$ in a new way. The suggested allocation is considered to be stable. If it were not, some other coalitions would come up with new objections and new suggestions, which in turn could create new objections. Ultimately there would be proposals and allocations which were not better for all the members of the coalition and the process would stabilize.

Summarizing the basic idea, we can say that a set of stable allocations satisfies two requirements. Firstly, some coalition always has a credible objection to an unstable allocation. Secondly, no coalition has a credible objection against any stable allocation.

For the formal definition the concept of dominance needs to be defined first:

Definition: Let (N, v) be a cooperative game with transferable utility. X is all imputations, i.e. all individually rational and efficient allocations of (N, v). An imputation $x \in X$ dominates another imputation $y \in X$ via S, if $x_i > y_i$ for all $i \in S$ and $x(S) \leq v(S)$, which is denoted with $x \; dom_S \; y$.

This definition simply states that the players of coalition S prefer the imputations x to y because all the members of S would get more by choosing x.

Definition: A subset Y of the set X of imputations of a cooperative game with transferable utility (N,v) is a stable set if it satisfies the following two conditions:

- *Internal stability: If $y \in Y$ then for no $z \in Y$ does exist a coalition S for which $z \; dom_S \; y$.*

- *External stability: If $z \in X \setminus Y$ then there exists $y \in Y$ such that $y \; dom_S \; z$ for some coalition S.*

This definition of a stable set requires that (a) no imputation in the stable set is dominated by another imputation in the stable set and (b) all imputations outside the stable set are dominated at least by one imputation in the stable set in some coalition. By having the requirements for internal and external stability, it is unlikely that either imputations outside the stable set would be established or that some imputation in the stable set would override other imputations of the set.

A game can have many stable sets or none at all and each set can contain several imputations.

Von Neumann's and Morgenstern's interpretation was to see stable sets as a "standard of behaviour", a kind of behaviour pattern acceptable in the society in general. Each stable set reflects some behaviour pattern, that is, a game having several stable sets as a solution would allow its players different behaviour options to choose from. If a stable set is comprised of many imputations, they reflect same behaviour pattern.

To clarify the theory of stable sets, let's take an example from (Davis 1997) and (Osborne 1994), which has many solutions and provides different behaviour patterns (the three-player majority game):

Example 2. A cooperative game *(N,v)*, $N = \{1,2,3\}$, $v(S) = 2$ *if* $|S| \geq 2$ *and* $v(S) = 0$ otherwise. We have a game of three players in which any coalition of two or three players get two units and a single player coalition gets nothing.

One solution and one stable set of this game is

$$Y = \{(1,1,0),(1,0,1),(0,1,1)\} \tag{12}$$

In this solution one of the players is ruled out and the two other players share the payoff of two units equally. This is one possible "standard of behaviour".

This solution fulfils the conditions of a stable set. Payoff vectors of the solution are imputations (individually rational and efficient) and the solution is internally and externally stable. The solution is internally stable because for all imputations x, $y \in Y$ only one player prefers x to y. When moving from any imputation of the solution to another, one player gains one unit, one player loses one unit and one player stays the same. In other words, there does not exist a coalition S in which any of the imputations of Y would be dominated by another imputation of Y.

To verify the external stability of Y let z be an imputation outside Y; $z \in X \setminus Y$. Because the sum of the payoff of two or three player coalitions is always two units, there are always two players i and j for whom $z_i < 1$ and $z_j < 1$ and thus there is an imputation in Y which dominates z via the coalition $\{i,j\}$.

Another solution and a "standard of behaviour" is a pattern in which two players merge, give the third player less that his or her fair share of 2/3 and divide the rest by themselves:

For any $c \in [0,1)$ and any $i \in \{1,2,3\}$ a stable set is

$$Y_{i,c} = \{x \in X : x_i = c\} \tag{13}$$

One example is $Y_{1,2/5} =$ {(2/5, 4/5, 4/5)}.

The internal stability of $Y_{i,c}$ is fulfilled because only one player prefers x to $y \in Y$. The payoff of the single player coalition would be zero and thus not the preferred solution. To show the external stability let $i = 3$ and $z \in X \setminus Y_{3,c}$. If $z_3 > c$ then $z_1 + z_2 < 2 - c$ and there exist $x \in Y_{3,c}$ such that $x_1 > z_1$ and $x_2 > z_2$, from which follows $x\ dom_{\{1,2\}}\ z$. If $z_3 > c$ and $z_1 \leq z_1$ then $(2-c,0,c)\ dom_{\{1,3\}}\ z$.

The solution concept of the stable sets is very general and can thus be applied to a wide variety of games. However, the concept has also its disadvantages. Stable sets can be difficult to find. Furthermore, there may be no solution at all (Lucas 1969).

3.7 The core

The reasoning behind the core solution concept of cooperative games is analogous to the Nash equilibrium of non-cooperative games. The stability is achieved if deviation is not profitable. In non-cooperative games the subject of profitability are individual players, that is, the outcome of a game shall be approved by every player. In cooperative games with the core this requirement needs to be extended to cover also all potential coalitions. An outcome is stable if no coalition wants to deviate due to the better payoff for all its members. A game is stable if no coalition can make a payoff that is more than the sum the current payoffs of its members (Osborne 1994).

The definition of the core introduces a new condition, “coalitional rationality”. In addition to efficiency and individual rationality, also coalitional rationality is demanded (Parrachino, Zara & Patrone 2006).

The introduction of the core is by Gillies (Gillies 1959). The following definition follows (Peleg, Sudhölter 2003).

Definition: Let (N,v) be a cooperative game with transferable utility and $X^*(N,v)$ *the set of feasible payoff vectors. The core of (N,v), denoted by* $\mathcal{C}(N,v)$ *is*

$$\mathcal{C}(N,v) = \{x \in X^*(N,v) \mid x(S) \geq v(S) \text{ for all } S \subseteq N\} \tag{14}$$

This includes that an imputation x is in the core if no coalition can provide its members a better payoff than x. This definition also implicates the condition of efficiency, because when $x \in X^*(N,v) \Longrightarrow x(N) \leq v(N)$ and $S=N \Longrightarrow x(N) \geq v(N)$, it follows that $x(N) = v(N)$.

The core allocations provide a clear incentive for cooperation in the grand coalition. No single player can get a better payoff alone and no coalition is able to provide all its players a better outcome than the grand coalition N. However, in the core solution there is usually an uncountable number of possible allocations. Also the core can be empty with no solution.

Let’s concretize the existence and non-existence of the core with two examples. The first one is a variant of the three-player majority game. It is from (Osborne 1994).

Example 3. A cooperative game (N,v), $N = \{1,2,3\}$, $v(S) = \alpha \in [0,1]$ *if* $|S|=2$, $v(S) = 1$ *if* $|S|=3$ *and* $v(S)=0$ otherwise. We have a game of three players in which any coalition of two players gets α and a three player coalition gets one unit. A single player gets nothing. The core solution of the game is the set of nonnegative allocations (x_1, x_2, x_3) for which its holds:

$$\begin{cases} x(N) = 1 \\ x(S) \geq \alpha, \forall S \subseteq N \wedge |S| = 2 \end{cases} \tag{15}$$

It is easy to see that the core does exist if and only if $\alpha \leq 2/3$.

Another example of the core, a simple majority game is from (Parrachino, Zara & Patrone 2006).

Example 4. A cooperative game *(N,v)*, *N = {1,2, ..,n}*, in which a coalition *S* gets always the same if the number of the coalition members exceeds the half of the grand coalition size:

$$v(S)=\begin{cases}1 \ if \ |S|>\frac{n}{2} \\ 0 \ if \ |S|\leq\frac{n}{2}\end{cases} \tag{16}$$

The core conditions do not provide any solution for this game, that is, the core is empty. When e.g. $n=3$ and $|S| = 2$, the two members of *S* could divide the payoff of one unit by themselves, e.g. (½,½). The grand coalition *N* would not gain anymore but the same payoff of one unit and thus its members would get less than when joining the *S*.

The Bondareva-Shapley theorem defines the necessary and sufficient conditions under which the core of a cooperative game with transferable utility is non-empty (Bondareva 1963). The following presentation of that theorem follows (Osborne 1994).

For a cooperative game *(N,v)* with transferable utility, $\mathcal{C}$ denotes the set of all coalitions. For any coalition *S* there is associated a characteristic vector $1_S \in \mathbb{R}^N$, such that

$$(1_S)_i=\begin{cases}1 \text{ if } \ i\in S \\ 0 \quad \text{otherwise}\end{cases} \tag{17}$$

A collection $(\lambda_S)_{S\in\mathcal{C}}$ of numbers *[0,1]* is a *balanced collection of weights* if for every player *i* the sum of λ_S over all the coalitions that contain *i* is 1

$$\sum\nolimits_{S\in \mathrm{C}} \lambda_S 1_S = 1_N \tag{18}$$

Definition: A game (N,v) is balanced, if for every balanced collection of weights:

$$\sum\nolimits_{S\in C} \lambda_S v(S) \leq v(N) \tag{19}$$

The Bondareva-Shapley theorem is the following:

Definition: A cooperative game with transferable utility has a non-empty core if and only if it is balanced.

For the balanced game the following interpretation can be given. Let us assume that each player has one unit of time and he or she shall divide that among all the coalitions he/she is a member of. The coalition S is active for the time λ_S and all the members of S must be active for that given time. Thus the coalition S provides the payoff $\lambda_S v(s)$ to its members. By following this interpretation we conclude that a game that balances the players' allocation of time must satisfy the condition that the total payoff to players shall not exceed $v(N)$ (Osborne 1994).

In his paper Shapley (Shapley 1967) also concludes a single condition, under which the core of a three-person superadditive cooperative game is not empty:

$$v(\{1,2\})+v(\{2,3\})+v(\{2,3\}) \leq 2v(\{1,2,3\}) \tag{20}$$

Because the core can be empty, Shapley and Shubik introduced a generalisation of the core which they called the strong ε-core (Shapley, Shubik 1966). In the following definition the notation is according to the one used in the definition of the core *(14)*.

Definition: Let (N,v) be a cooperative game with transferable utility, $X^*(N,v)$ the set of feasible payoff vectors and $\varepsilon \in \mathbb{R}$. The strong ε-core of (N,v), denoted by $\mathcal{C}_\varepsilon(N,v)$ is

$$\mathcal{C}_\varepsilon(N,v) = \{x \in X^*(N,v) \mid x(S) \geq v(S) - \varepsilon \text{ for all } S \subseteq N\} \tag{21}$$

If $\varepsilon = 0$, the strong ε-core and the core are the same, i.e. $\mathcal{C}_0(N,v) = \mathcal{C}(N,v)$. If ε is large enough the strong ε-core is non-empty and on the other hand if ε is small enough, the strong ε-core is empty (Peleg, Sudhölter 2003).

The interpretation of the strong ε-core can be the following. It is a set of efficient payoff vectors (or pre-imputations), which any coalition cannot improve by leaving the grand coalition, if the leaving coalition must pay the cost of ε for leaving. The value of ε can also be negative, which yields an extra commission if leaving the grand coalition (Peleg, Sudhölter 2003).

In their paper Shapley and Shubik interpret the parameter ε to represent a sociological factor which mirrors the organizational cost prerequisite to cooperative action. Also the existence of the real core or the strong ε-core with small values of ε is interpreted to implicate the stability of contracts: "… the profit to be gained from re-contracting out of an ε-core would be small and a near stability can be achieved." (Shapley, Shubik 1966). On the contrary, if ε-core does not exist with small values of ε, it predicts that the profit gained from re-contracting can be significant and thus the contractual stability can be weak.

Following the central ideas of the core and the ε-core, a new solution concept called the *least-core* was introduced by Maschler, Peleg and Shapley (Maschler, Peleg & Shapley 1979). As a solution concept, the least-core ensures existence and uniqueness. "If the core is not empty, then the least-core is a centrally-located point within the core. If the core is empty, then the least-core may be regarded as revealing the "latent" position of the core." The least-core can also be interpreted as the strong ε-core with the smallest value of ε that ensures the solution to be non-empty.

For the least core two equivalent definitions can be provided (Peleg, Sudhölter 2003):

Definition A: The least core of a cooperative game (N,v), denoted $\mathcal{LC}(N,v)$, is the intersection of all nonempty strong ε-cores of the game.

Definition B: Let (N,v) be a cooperative game and let $\varepsilon_0 = \varepsilon_0(N,v)$ be the smallest ε such that $\mathcal{C}_\varepsilon(N,v) \neq \emptyset$, that is,

$$\varepsilon_0 = \min_{x \in X(N,v)} \max_{0 \neq S \subseteq N} e(S,x,v) \qquad (22)$$

then $\mathcal{LC}(N,v) = \mathcal{C}_{\varepsilon 0}(N,v)$.

The applications of the core are mainly in the area of economics and it is likely that the core is the best known to economists among the cooperative game solutions. The major part of the applications is based on the core's equivalence principle, according to which every competitive allocation (a.k.a. price equilibrium) in an exchange economy is in the core. The basic requirements are perfectly competitive markets and that the number of traders is big enough to avoid the dominance of individual traders. Under these circumstances the core always exists, which expresses the idea that stability can be achieved within the price equilibrium. The other applications of the core are e.g. in public goods and in the share of the cost of public services (Aumann 1985).

3.8 Shapley value and other one-point solutions

Shapley value

The Shapley value (Shapley 1953) is a solution concept of cooperative games like the stable sets of von Neumann and Morgestern and the core. Deviating from these two, the Shapley value is a one-point solution and it assigns a unique outcome for each game. Also it exists always for finite superadditive transferable utility games, which is not the case with the stable set and the core. On the other hand the Shapley value does not contribute to the assessment of the stability, as do the two other solution concepts (Aumann 1985).

The Shapley value is introduced in an axiomatic way, that is, the solution fulfils a set of beforehand defined axioms. The following definition follows (Peleg, Sudhölter 2003) and (Parrachino, Zara & Patrone 2006).

In the following we are considering *G(N)* a class of superadditive cooperative games with *N* players. The Shapley value is a function, which maps

$$\phi : G(N) \longrightarrow \mathbb{R}^N \tag{23}$$

and satisfies the properties of efficiency (also called Pareto optimality), the dummy player property, anonymity and additivity.

The function ϕ satisfies the *efficiency property*, if for every $v \in G(N)$, it holds

$$\sum_{i \in N} \phi_i(v) = v(N) \tag{24}$$

This means that the total value of $v(N)$ is exactly divided among the players.

A player is a *dummy player*, if for every coalition S, such that $i \notin S$, it holds

$$v(S \cup \{i\}) = v(S) + v(i) \tag{25}$$

This means that a dummy player's contribution to S is only $v(i)$. The special case of a dummy player is a player which does not contribute anything and thus should receive nothing.

The function ϕ satisfies the *dummy player property*, if for every $v \in G(N)$ and every dummy player $i \in N$, it holds

$$\phi_i(v) = v(i) \tag{26}$$

This means that dummy players shall receive only their own value $v(i)$.

The function ϕ satisfies the *anonymity property*, if for every $v \in G(N)$ and for every player $i, j \in N$, which contributes equivalently in the game, it holds

$$\phi_i(v) = \phi_j(v) \tag{27}$$

This means that the players which contribute equally are also treated equally in a game.

The function ϕ satisfies the *additivity property*, if for every $v, w \in G(N)$, it holds

$$\phi(v+w) = \phi(v) + \phi(w) \tag{28}$$

The meaning of additivity property is debatable. Some argumentation is based on "the games that are two games played separately by the same players (e.g. at different times, or simultaneously using agents)" (Peleg, Sudhölter 2003). Also its said that the axiom "strikes us as a flaw in the concept of value" (Todeva, Knoke 2005).

The value ϕ, which satisfies all these four properties, is called the *Shapley value*. The Shapley value for every $v \in G(N)$ and for every $i \in N$, is given by

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n-|S|-1)!}{n!}(v(S \cup \{i\} - v(S)) \quad (29)$$

The interpretation which is given by Shapley: "The players in *N* agree to play the game *v* in a grand coalition, formed in the following way: 1. Starting with a single member, the coalition adds one player at a time until everybody has been admitted. 2. The order in which the players are to join is determined by chance, with all arrangements equally probable. 3. Each player, on his admission, demands and is promised the amount which his adherence contributes to the value of the coalition (as determined by the function *v*). The grand coalition then plays the game "efficiently" so as to obtain *v(N)* – exactly enough to meet all the promises."

The Shapley value has been very successful as a solution concept and it has been solved in many application areas. One of the areas of extensive investigation is voting games. It has been found out that the real voting power controlled by major parties can be significantly more than the membership ratios would necessitate. Shapley and Shubik analysed the United Nations Security Council. Until 1965 there were five permanent members and six non-permanent members. The analysis showed that the voting power of the permanent members was 98.7% because of their veto right. In 1965 the number of non-permanent members was increased to ten but still the permanent members controlled 98% of the voting power, that is, the change was only marginal (Shapley, Shubik 1954) (Kelly 2003).

Another important application of Shapley value is cost sharing. Often the target of the cost sharing applications is to find a fair allocation of costs between the members of a project. The project can be for example to produce public goods or to establish a common infrastructure. Cost allocation solutions can include water resource management, electric power, pollution treatment, allocation of taxes and public utility pricing (Aumann 1985).

An example of the cost sharing application is a method of calculating the airport landing charges by Littlechild and Owen (Littlechild, Owen 1973). The runway of an airport must be long enough to match the largest aeroplane that will land there. On the other hand it would not be fair to charge the same tariff from a Boeing 707 and a Fokker Friendship 27 (the biggest and the smallest plane presented in the paper). The presented method to calculate the landing fees is the following:

"Divide the cost of catering for the smallest type of aircraft equally among the number of landings of all aircraft. Divide the incremental cost of catering the second type of aircraft (above the cost of the smallest type) equally among the number of landings of all but the smallest type of aircraft. Continue thus until finally the incremental cost of the largest type of aircraft (above the cost of the second largest type) is divided equally among the number of landings made by the largest aircraft type."

Littlechild and Owen noticed that this algorithm is equivalent with the Shapley value of a cooperative game, in which the players are individual landings and the value of a coalition is the cost of building an airport that would match the landings.

Another interesting application of the Shapley value is the internal telephone billing rates by Billera, Heath and Raanan (Billera, Heath & Raanan 1978). In general the target is to divide the costs of bulk services among a big number of users. The service rates should be fair and they must cover all the costs of producing the services. Examples of such applications are the division of administration costs in large corporations, computer centre costs and communications costs over Wide Area Networks. In their paper the authors present a general solution in a form of a non-atomic game, which can be applied in a large number of similar applications. They also present a detailed case of internal telephone billing rates. Finally, the proposed scheme of billing rates was adopted for use by Cornell University. The unique element of the solution is that it was (according to the authors) the first time to apply the theory of the *non-atomic* game for such a purpose. The theory was introduced by Aumann and Shapley (Aumann, Shapley 1974). In this case the non-atomic game paradigm is justified by the nature of the application. For telephone calls a continuous model is the most appropriate.

A very significant aspect of the Shapley value is its mathematical flexibility. Its relationship to areas such as probability, measure theory and functional analysis enlarges its applicability to many complex application areas, which can be modelled and solved in a systematic way. Quite likely the theory built around the Shapley value is the richest in the game theory, although ultimately the value of the mathematical toolbox is in its power to solve different applications (Aumann 1985).

Nucleolus

The nucleolus of a game was introduced by Schmeidler (Schmeidler 1969). Like the Shapley value, also the nucleolus is a one-point solution and it always exists. Maschler, Peleg and Shapley (Maschler, Peleg & Shapley 1979) defined the Nucleolus by introducing a concept called “lexicographic center”, which they then showed to coincide with the nucleolus. The lexicographic center can be defined in the following way.

“We now introduce the lexicographic center of a game. Intuitively, it is an extension of the idea leading to the least-core. The procedure is as follows: First we find all the imputations that minimize the maximum excess; in general they will form a nonempty compact convex set. Then we put aside those coalitions whose excess never goes below this minimum in this set and “reminimize” the maximum excess over the remaining coalitions. This gives us in general a nonempty compact convex subset of the previous set, as well as some new coalitions whose excess cannot be further reduced. These coalitions in turn are put aside, and the process is repeated until there are no coalitions left.”

The term "excess" represents the gain to a coalition *S* if its members depart from an agreement that provides them *x* (usually the grand coalition) to form their own coalition:

$$e(S,x) = v(S) - \sum_{i \in S} x_i \tag{30}$$

The definition of the lexicographic center is the following (Maschler, Peleg & Shapley 1979):

Definition: Let G=(N,v) be a cooperative game with transferable utility and $X^0 \supset X^1 \supset \cdots \supset X^\kappa$ *a nested sequence of sets of payoff vectors and* $\sum^0 \supset \sum^1 \supset \cdots \supset \sum^\kappa$ *a nested sequence of sets of coalitions.* X^0 *is initiated to be imputations of (N,v) and* $\sum^0 = \{S \subset N : S \neq \varnothing, N\}$. *For k = 1,2,…,*$\kappa$ *define recursively:*

$$\varepsilon^k = \min_{x \in X^{k-1}} \max_{S \in \sum^{k-1}} e(S,x), \tag{31}$$

$$X^k = \left\{x \in X^{k-1} : \max_{S \in \sum^{k-1}} s(S,x) = \varepsilon^k \right\} \tag{32}$$

$$\sum_k = \left\{S \in \sum^{k-1} : e(S,x) = \varepsilon^k, \, all \; x \in X^k \right\}, \tag{33}$$

$$\sum^k = \sum^{k-1} \setminus \sum_k, \tag{34}$$

where κ *is the first value of k for which* $\sum^K = \varnothing$. *The set* X^K *is called the lexicographic center of the game G.*

The lexicographic center is shown to coincide with the nucleolus.

The Figure 4 illustrates the lexicographic center of a three-person game with a nonempty core.

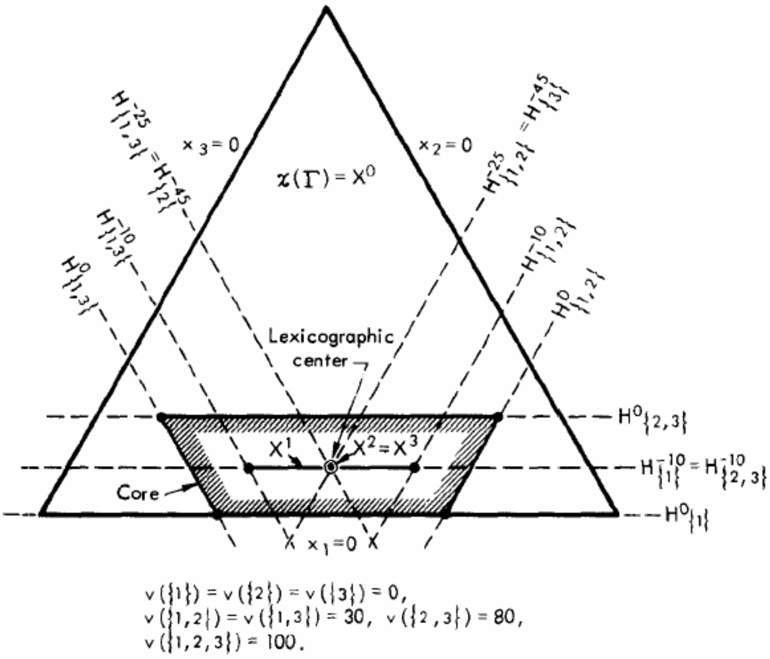


Figure 4. Reaching the lexicographic center in a game with a nonempty core (Maschler, Peleg & Shapley 1979)

Maschler, Peleg and Shapley also discussed the nucleolus as a fair division between the players. The nucleolus satisfies many desired properties, which also the Shapley value fulfils:

- A unique payoff vector for each game
- Individual and group rationality
- Symmetric players receive equal payments
- Desirable players receive at least as much as less desirable players
- A dummy player receives only his/her own value

(Actually the Shapley value does not fulfil the “group rationality”, even though listed by the authors. See the discussion below.)

The clear differences between the Shapley value and the nucleolus are the following:

- The nucleolus belongs to a nonempty core, which is not the case with the Shapley value.
- The Shapley value satisfies the monotonicity property and the nucleolus does not.

Because the Shapley value is not always in a nonempty core, the allocation proposed by it can be non-optimal for some coalitions and thus the solutions would not be stable. It is easy to find cost allocation examples, in which some coalitions should pay more when participating in the grand coalition than when making it by themselves (Carter 1993).

The benefit of the Shapley value is the monotonicity property. The monotonicity means that the outcome for the players in a coalition *S* follows the changes of the *v(S),* i.e. if the value of *v(S)* increases the members of *S* get more and vice versa. Because with the nucleolus the monotonicity property does not hold, there can be games whose nucleolus payoff decreases for some players when *v(N)* is increased without any other changes (Maschler, Peleg & Shapley 1979).

τ-value

τ-value was introduced by Tijs (Tijs 1981). It is a compromise between two vectors: the *upper vector M(N,v)* and the *lower vector m(v).* Also the names *utopia payoff* and *minimum right payoff* are used (Brânzei, Dimitrov & Tijs 2005). These two vectors can be defined for the player *i* in the following way (Parrachino, Zara & Patrone 2006):

$$M_i(N,v) = v(N) - v(N \setminus \{\mathrm{i}\}) \tag{35}$$

$$m_i(v) = \max_{S:i\in S}\left(v(S) - \sum_{j\in S\setminus\{\mathrm{i}\}} M_j(v) \right) \tag{36}$$

The upper vector *M(N,v)* can be interpreted to be the marginal contribution of a player to the grand coalition. For a player it is not realistic to hope for a bigger payoff than that. Otherwise it would be beneficial for other players to discard him/her.

Respectively, the lower vector *m(v)* can be seen as the minimum right for the player *i*, because the player can request at least that in the grand coalition. It is the remainder for *i* in the coalition *S*, if all other players receive their utopia payoff *M(N,v)*. Thus the player *i* can always rationalize that in the coalition *S* he/she would get at least *m(v)* (Brânzei, Dimitrov & Tijs 2005).

Definition: Let G=(N,v) be a cooperative game with transferable utility. The τ-value is defined by

$$\tau(v) = \alpha\, m(v) + (1-\alpha) M(N,v) \tag{37}$$

where $\alpha\in[0,1]$ is uniquely determined by $\sum_{i\in N} \tau_i(v) = v(N)$.

3.9 Cooperative games and Strategic alliances

When searching the studies in which strategic alliances are analysed by using a game theoretical framework, the results are quite thin. The number of papers of such an intersection is quite moderate. One gets the impression that the study of strategic alliances, which has its roots in industrial and business management, shuns the methodologies provided by the game theory. Study of strategic alliances operates mostly with qualitative methods and concepts familiar with social sciences, whereas the game theory has its roots deep in mathematics and is thus familiar with quantitative methods and the natural science approach. This bridge seems to be challenging to cross.

Three different examples are presented in the intersection of strategic alliances and cooperative game theory. The first two examples concentrate on a narrow and well-defined area of cooperation and they provide practical models and solution concepts. They also introduce numerical calculations to verify the proposed concepts. The first example is an application of cooperative game theory to the strategic alliances of liner shipping companies. The second example is a game theoretic approach about innovation incentives in enterprise networks. An example from the boat-building industry is introduced. The scope of the third example is broader than in the two first examples. It is a review of the game theoretic applications in water resource issues including many examples from a variety of diverse water infrastructure projects.

An application of cooperative game theory to liner shipping strategic alliances

The scope of the paper of Song and Panayides (Song, Panayides 2002) is strategic alliances of liner shipping companies. The alliances are assessed at a conceptual level by using a game-theoretic framework to better understand the inter-organisational relationships of liner companies and the strategic decision-making of the alliance members.

The liner shipping industry has been characterized both by the fierce competition and cooperation between the carriers. The cooperative relationships have led to the establishment of various strategic shipping alliances. Even though the number of companies participating in alliances is significant, there are also liners who have chosen to continue by themselves.

The reasons behind the strategic alliances of liner shipping companies in the 1990’s were the typical ones in all alliances: technological, operational, market and economic forces, the need to confront the challenges of uncertainty, the allocation of resources, and market penetration. The first alliance was established in 1994 by four major carriers: APL, OOCL, MOL and Royal Nedlloyd Lines. The name of the alliance was “Global Alliance” and the strategic intention was to establish an integrated Europe-Far

East service. After the Global Alliance several other alliances followed: the Hanjin/Tricon Alliance (Hanjin, DSR Senator and Cho Yang), Grand Alliance of Haplag-Lloyd, NYK, NOL and P&O and Maersk-Sealand. The alliances were not stable and some re-organizations took place due to the exits of key members, who merged even with the companies from opposing alliances.

The objectives of the modern liner shipping companies can be classified into five major areas: financial, economic, strategic, marketing and operational objectives. The motivation to form liner shipping alliances can be projected against these areas:

- *Financial objectives:* profit maximization, increased shareholder value, sharing investments and reduction of financial risk;

- *Economic objectives*: cost reduction, economies of scale;

- *Strategic objectives*: entry in new markets, wider geographical scope, increase in purchasing power;

- *Marketing objectives:* satisfy better customer requirements, e.g. higher frequency, reliability, variety of routes and destinations;

- *Operational objectives:* increase in frequency of services, vessel planning and coordination on a global scale.

Even though the advantages of strategic alliances of liner companies have been obvious, the instability and the changing strategic direction of some partners may have diluted the benefits of the alliances. The reasons for the instability of liner shipping alliances can be originated in the behaviour of the alliance members. The need to drive the individual objectives that have a negative impact on the cooperation and the existence of intra-alliance competition are factors that increase the instability of alliances. Other reasons that have a role in alliance stability are the number of partners in an alliance, the nature of their roles, members’ contribution to the alliance and the level of mutual trust.

In the paper of Song and Panayides game theory is applied to liner shipping alliances to better understand the prerequisites for stable alliances. The mapping between the cooperative game paradigm and the strategic alliances of liner shipping companies is presented in the Table 1.

Table 1. Cooperative games and liner shipping alliances (Song, Panayides 2002)

Cooperative game	**Liner shipping alliance**
Player	Liner shipping company
Coalition	Shipping alliance
Strategy	Daily operation plans to long-term development strategy: for example selecting ship types, operating routes and seeking a partner in the market
Outcome	Gaining economic benefits and know-how; penetrating new markets; improving and/or sustaining the reputation

An illustrative model (Figure 5), which is presented in the paper includes the liners and their pendulum routes "A-B" and "B-C". The goal of the model is to maximize the revenue generation capabilities of two shipping companies. It is supposed that the globalization leads shippers to require carriers to provide more efficient services between "AC". The liner companies have two options to react. They can either continue their pendulum service or they can form a strategic alliance. By coordinating their route schedules they can form a large-scale pendulum service, that is, "A-B-C-B-A". The new alliance would be able to keep the original volumes of freight and tariffs of the alliance partner and in addition to increase their business with the new pendulum service enabled by the alliance.

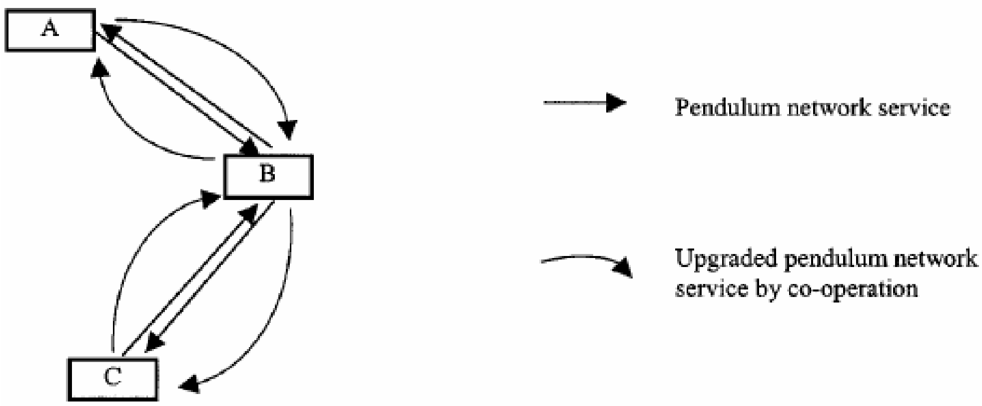


Figure 5. An example of a liner shipping service network (Song, Panayides 2002)

In the paper the authors present illustrative calculations which show that the presented alliance would be beneficial for both liner companies. They could increase their revenue and thus the solution would be in the core of the cooperative game. An assumption is that the payoffs (revenue) of the game would be shared according to the shipping capacities of the two alliance members. On the other hand, if the ratio of the shipping capacities would change due to the alliance, the solution would probably not be stable anymore because the other partner would get less than making the business on his own. From the game theoretical point of view this means that the solution would not be rational from the individual and coalitional point of view and thus the core would be empty.

An example of successful cooperation between two liner companies is the alliance of Maersk and Sea-Land. Maersk’s profits in 1996 rose 23% to USD 349 million and Sea-Land’s operating income increased by USD 80 million to USD 318 million despite the declining markets.

In the paper the authors enlarge their model to include more than two partners. They are able to demonstrate that with three partners the existence of the core is quite sensitive to diverse factors. In an alliance it is not possible to guarantee conditions that would distribute the freight equally among the alliance members and thus would keep the payoffs in the core of the cooperative game. In shipping industry there are external factors which can change the shipping demand sporadically. They can be seasonal, political or other external reasons. These changes are unavoidable in the industry, which implies that in practice it is challenging to form stable shipping alliances

The authors point out that the mechanics of strategic alliances are a multi-dimensional problem. When the cooperative game theory is mainly concerned with a single dimension, all the different aspects of shipping alliances are quite likely beyond the explanation capabilities of the cooperative game theory. Another limiting factor is that all elements that affect the strategic shipping alliances cannot be quantified. For example, the cultural differences of partners are often regarded as an important reason leading to the failure of shipping cooperation, but this is hard to estimate in quantitative models.

Even with these obvious limitations, the authors perceive the model based on the cooperative game theory as a valuable tool for understanding the relationships between partners of the strategic shipping alliances and also the impact of the shipping demand fluctuation to these alliances. One conclusion is that the external factors of the shipping markets can destroy the stability of strategic alliances in a case where liners address to great importance to their revenues.

The core is seen as a vital concept in cooperative game theory and it can explain certain aspects of the markets and strategic alliances which are similar to the liner shipping companies and markets described in the paper. On the other hand, if considering the dynamic aspects and strategic decisions regarding the market dynamics between different parties, the application of non-cooperative game theory could provide additional value and new interesting results.

Innovation Incentives in Enterprise Networks – A Game Theoretic Approach

In the paper of Jarimo, the innovation incentives in enterprise networks are studied with the game theoretic modelling approach (Jarimo 2004). Traditionally, enterprise networks and network economy have been studied by the qualitative means of industrial management. In the paper the new approach is to study enterprise networks from a mathematical perspective and especially with game theoretic tools. In the paper a game theoretic model is constructed for studying innovative incentives in an enterprise network.

The viewpoint and postulates of the paper are very classical to game theory and cooperative games. In the network of cooperative companies, e.g. a supplier network, the parties have both their own interests and the shared interests of the network which can be partly conflicting. Each of the companies has their own available strategies which they play to achieve their goals. If companies cooperate in a reasonable manner when acting as members of the network, it benefits the parties in the form of increasing global utility. The final question is the fair and motivating distribution of the increased utility among the companies of the network.

The aim of the game theoretic analysis is to predict the behaviour of players in situations of conflicting interests, and thus to provide tools for parties of networked business environment. The game theoretic models target to nurture optimal strategies, reveal and decrease possibilities for opportunistic behaviour and model the causal relationships between the dynamics of the players. Also the modelling is expected to bring up win-win situations among the cooperative parties and thus to improve the competitiveness of the companies working together in the enterprise networks.

In the study an example from the boat-building industry is presented from the viewpoint of innovation incentives. In the example there is a client and several suppliers. The client is a Finnish sailing-yacht manufacturer Nautor. Suppliers are well-known partners to Nautor. They provide components to the boats manufactured by Nautor. In the example there is a new type of boat, the Swan 45. There is one supplier who is the hull manufacturer of the boat. Another supplier is in charge of installing the heating, plumbing and air-conditioning equipment (HPAC).

With the first boats the HPAC installer has drilled the holes for the pipelines into the ready-made hull. Drilling the wholes in this phase has been time consuming and it has been proposed to drill the holes before the hull is finalized. This arrangement decreases the overall work effort, although at the same time moving the drilling from the HPAC supplier to the hull manufacturer. The evident question in this example is the total price paid by the client, which should decrease due to the reduced work amount. On the other hand, the interests of two suppliers seems to be conflicting with each other and with the overall target to reduce the total operating expenses of the client. Hence a predefined mechanism is needed for re-evaluating the fixed payments of suppliers and also for encouraging them to suggest such initiatives that decrease overall expenditure.

Three different solution concepts are introduced to model the efficiency-improving arrangement in enterprise networks. Also a numerical application of the introduced solution concepts is provided. The models presented in the paper are

- Egalitarian solution without threats;
- Relative threats solution;
- Modified Shapley value.

The main idea behind these models is that when the companies cooperating know beforehand that they will be compensated in the case of the efficiency-improving arrangement, they are willing to suggest and implement efficiency-improving initiatives. The models include an appropriate compensation payment in case of increasing work effort and in addition the additional payout obtained by the rationalisation is shared among the parties.

The three models deviate in the area of utility sharing. In the egalitarian solution the companies neither threaten nor ally against each other. The relative solution enables the use of threats, that is, actions that can harm a company if committed by another company. The modified Shapley value by Harsanyi (Harsanyi 1963) is the most general supporting threats and coalitions within the enterprise network.

In the numerical application the example from the boat-building industry is used to compare the three solution concepts. It is assumed that the drilling work is transferred from the HPAC supplier (Supplier 1) to the hull manufacturer (Supplier 2). When HPAC supplier performs the drilling, the cost of the work is 800 €. Respectively, the hull manufacturer can provide the drilling with the cost of 200 €. Thus by transferring the drilling work from supplier 1 to supplier 2, the enterprise network would gain 600 €. It is assumed that the Supplier 1 would not easily find a new customer whereas the client would be able to substitute the Supplier 1. The example is illustrated by Figure 6.

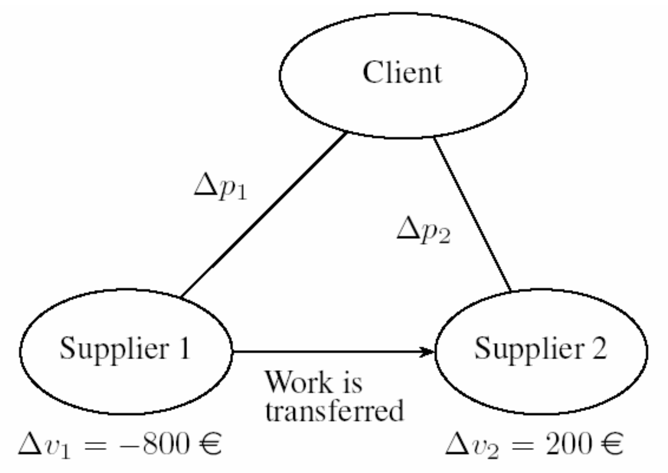


Figure 6. Enterprise Network example (Jarimo 2004)

All three solution concepts yield some benefit to all parties, that is, to both suppliers and to the client. The major difference between the models is how they penalize Supplier 1 due to his dependency of the client. Because Supplier 1 cannot easily find a new customer, the two other parties can take a bigger share of the utility without Supplier 1 being able to credibly object his share. The results of the numerical application are illustrated in Figure 7. In the figure π^* denotes the Egalitarian solution without threats, π^T the relative threats solution and π^S the modified Shapley value.

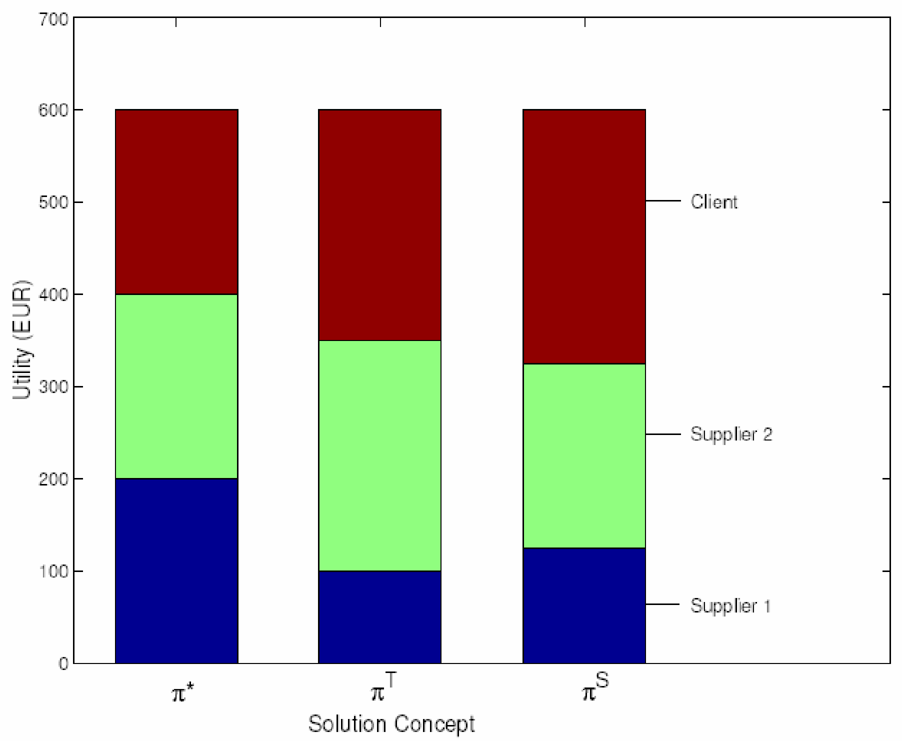


Figure 7. Comparison of different solution concepts (Jarimo 2004)

The commonality between all three solution concepts is that an efficiency-improving arrangement leads to a situation in which all the parties gain some benefit. When this is known to all parties of the network, the companies are willing to implement efficiency-improving ideas and thus the overall competitiveness of the network will increase.

In the conclusions of the paper the author summarizes that the game theoretic models have been presented to support decision making in the business-network environment. The models serve as a tool for leveraging innovation incentives in enterprise networks. Furthermore, these models support utility sharing among the companies of a network. Thus the assumption that “game theory is a suitable machinery for formal modelling of enterprise networks” was found to be applicable. Game theoretic approach can be used to assess the possibilities of networking and to find the presuppositions of successful cooperation. For example the cost-efficiency of the network can be improved and thus the overall competitiveness of the partnering companies will be sharpened.

The incentive models presented have also practical value. They can be implemented in contracts between companies and thus take into account optional rationalisation arrangements and their impact to the contractual partners. The persons implementing cooperation between companies are usually business managers who are in charge of the area in question. The application of the models does not require mathematical skills but the analytical capabilities are needed to understand the key concepts and the causalities of the model.

In the practical applications of the incentive models the most challenging element is probably the estimation of model parameters, especially regarding the immaterial areas such as quality, innovativeness or responsiveness. If the utility can be measured in monetary units, usually transferable utility can be assumed and the complexity of the model and calculus are relaxed. On the other hand, if the number of partners increases, that also usually calls for more complex models.

Cooperative game theory and its application to water resources

In the paper of World Bank’s Policy Research series by Parrachino, Dinar and Patrone the scope is cooperative game theory and its applications to water resource issues (Parrachino, Dinar & Patrone 2006). The paper provides a review on various water use cases, such as “multi-objective water projects, irrigation, groundwater, hydropower, urban water supply, wastewater and transboundary water disputes.” Cooperative solutions are described to many practical allocations resulting from cooperation in water projects.

According to the authors, game theoretic approach has not been widely applied in the field of water resources even though it has provided important input to policy making in many other sectors, e.g. communication, transportation, aviation and energy.

Cooperative approach is very natural to model the water resources due to the specific attributes of the field. It is more attractive to build big water projects instead of small ones because of economies of scale. Thus there is a clear incentive for cooperation among the diverse parties. Also the negative effects of the water utilization can be reduced by cooperation and by building larger projects.

The applications of the cooperative game theory are mostly about sharing benefits. The cooperation is the source of the benefits and new opportunities. Without cooperative behaviour these advantages would not be achievable. Benefits can be savings on investments and operational cost and increased welfare due to the utilization of shared resources. The solution concepts of cooperative games provide the allocation of the benefits.

In the paper a set of diverse cooperative water projects are reviewed. The list of projects includes categories “dams, water supply and wastewater treatment facilities, water exchange (water markets and water rights), groundwater, multipurpose water facilities, and transboundary water conflicts.” Respectively, the list of cooperative solution concepts includes the core, the Shapley value, the nucleolus, the generalized Shapley and Nash/Nash-Harsany solutions. Also some non-cooperative solutions are included: Egalitarian Non-Separable Cost (ENSC), Alternate Cost Avoided (ACA), Separable Costs Remaining Benefits (SCRB), Marginal Cost and Proportional Use.

One of the examples presented in the paper is a study about Indian hydroelectric plants. The objective is to allocate the electricity and its production cost among the Indian states which can share hydroelectric plants. In the example a new concept called “players’ propensity to disrupt a coalition” is introduced.

The main question of the example is to determine a mutually acceptable division of the savings provided by the cooperation of building up hydroelectric power in the Southern Electricity Region of India. The electricity region has been divided into three areas. Five possible coalition structures depending on the degree of cooperation are considered: complete cooperation among the states, independence of all states and partial cooperation among the states. The costs of building power plants are allocated to regions according to the different degrees of cooperation.

The payoff of a player (one of the areas) in a coalition is the difference of building the hydroelectric power independently and the cost allocated to the player in a given coalition. If this difference is negative, there is no incentive for the player to cooperate. The concepts introduced are the core and imputations; an imputation in the core supports the objective of “propensity to disrupt”. The “propensity to disrupt” is defined to be the ratio of how much the other two areas would lose if the area in question refused to cooperate to how much the area would lose if it refused to cooperate. This definition can be used to eliminate such core allocations, in which one area’s gain would be too small and thus the coalition would be unstable.

Six different solutions are presented to allocate the costs among the areas:

1. Equal shares;
2. Equal ratio of total final costs to costs of building alone;
3. Kernel;
4. Demand weighted shares;
5. Shapley value;
6. Equal propensity to disrupt.

The first two solutions are not in the core. The third one does not consider the different bargaining power of the areas. Three and four can be in the core, but some area's propensity to disrupt can be too high. Five is in the core and gives small propensities to disrupt. Number six leaves each areas equally satisfied from the viewpoint of the propensity to disrupt. Among these solutions the Shapley value and the Equal propensities to disrupt are mutually acceptable by all three areas.

In the conclusions of the paper the authors suggest that "the cooperation over scarce water resources is possible under a variety of physical conditions and institutional arrangements. In particular, the various approaches for cost sharing and for allocation of physical water infrastructure and flow can serve as a basis for stable and efficient agreement, such that long-term investments in water projects are profitable and sustainable. The latter point is especially important, given recent developments in water policy in various countries and regional institutions such as the European Union (Water Framework Directive), calling for full cost recovery of investments and operation and maintenance in water projects."

The cooperative game theory provides valuable tools for finding a solid basis for possible and stable cost sharing concepts regarding the water resource issues. The joint costs can be allocated covering the total expenditure and at the same time each component of large water projects is economically justified. This supports the transparency of investments and helps to demonstrate the viability of the entire project and its components. Cooperative game theory nurtures efficiency and equity, thus providing valuable cost sharing rules for policy makers and urban water developers. Finally, the applications to irrigation water cases have demonstrated the usefulness of also the non-transferable utility games, where the players can transfer resources or the benefits from using the resources as side payments.

4 CASE STUDY: STRATEGIC ALLIANCES OF GLOBAL MOBILE OPERATORS

4.1 Introduction to the model of mobile operators' alliances

This is an introduction to a model, which models the strategic alliances between global mobile operators. The model applies the cooperative game theory to assess the stability of operators' alliances and also to consider how the added value of an alliance could be distributed among the alliance partners. The model is in the intersection of the study of strategic alliances and the game theory. Also the knowledge of microeconomic modelling and industrial and business management has been applied.

The objective of the model is to consider the potential strategic alliances of mobile operators in mathematical means, that is, not only to use the qualitative methods but to be able to assess the potential alliances with quantitative measures. The model looks answers to questions in the areas of alliance formation, alliance stability and the division of the alliance's value among the alliance partners. Some strategic alliances and alliance partners can be more preferable and probable to others, e.g. on the basis of the alliance value creation capability. Also the number of the partners can have an effect on the alliance formation. The value creation capability impacts also the stability of alliances.

A strategic alliance must provide added value in the long run to all its partners in order to be stable. Many different factors in the business environment may change over the time and thus also the prerequisites for stability can change. An alliance which seems to be healthy and stable in the formation phase can turn out to be non-profitable in the long run to some of the partners and thus its stability is endangered. The division of the alliance's added value has an impact on the alliance formation and its long-term stability. If the value the alliance provides its partners significantly changes over time, there can be reasons to align the value distribution according to the changed outward circumstances. If there is no flexibility in the alliance to agree on such changes, the long-term existence of the alliance is questionable.

In the center of the model are global mobile operators and their business of delivering mobile communication services to their subscribers, who can be consumer or enterprise customers. The model is based on the real data of the ten largest mobile operators in the world, measured in the number of subscribers. The data collected from each of the operator describes their global business in 2005, which is the year zero from the model's point of view. That data includes the operators' various networks worldwide: the number of subscribers on each network and in each country and the cellular technology of each (e.g. GSM, CDMA and W-CDMA) network. The data of mobile operators is completed with the country specific statistics, such as the penetration of mobile services and the average revenue per user (ARPU). When we add the country specific mobile business forecasts over the next five years, the overall picture about the business of the ten biggest mobile operators is quite illustrative. The model estimates over six years the business of each operator and its distribution in each country in terms of the number of subscribers, penetration, ARPU, the share of cellular technologies and overall revenues. This defines the baseline of the model. If an operator continues its business alone, the model estimates the revenue stream it is able to capture.

International roaming is a fundamental element in the service portfolio of mobile operators. International roaming enables a subscriber of a particular network to use the mobile phone abroad on another operator's network. When a subscriber roams to a network abroad (visited network) and uses its services, both operators, the visited network's and the home network's operator gain from it. They share the roaming fee the subscriber pays for using mobile services abroad. The revenue stream that mobile operators get from roaming is already now considerable and it is estimated to increase due to the growth of travelling and global business.

The model estimates the revenue the mobile operators would get from roaming and how that would develop if the operators formed a strategic alliance of two or more operators. By forming an alliance the operators could direct their roaming customers to use the networks of the alliance partners abroad and thus the alliance would get a bigger share of the roaming traffic and fees. The alliance could also promote the roaming services to their customers to make them more favourable to use roaming services when travelling abroad. By decreasing the roaming fees the alliance could increase the overall volume of roaming services used by their customers and thus increase the overall roaming revenue gathered by the alliance even though the unit price were reduced. Hence the mobile operators could gain roaming revenues by forming an alliance in two ways. They could get a bigger share of the existing roaming traffic and increase the use of roaming services among their subscribers. When estimating these two components, the additional roaming revenue can be calculated. The model does not estimate any other potential benefits of alliances. The benefits include e.g. reduced transaction costs due to shared resources, improved bargaining position in negotiations with vendors and complementary resources of partners that can be used to develop new services. The potential costs of forming an alliance have not been estimated. The only element the model forecasts is the development of the roaming revenues among the partners of an alliance.

The model includes all possible alliances that can be formed among ten mobile operators. The roaming revenue impact is calculated for all alliances of two or more partners, up to the alliance of all ten operators. Altogether this set-up gives 1013 different alliances of two or more members.

To estimate the roaming revenues of an alliance, there are some parameters which can vary from case-to-case and also over time. A couple of examples are the division of the roaming fee between the visited and the home operator, and the price elasticity of roaming services. These parameters have been estimated with normal distribution and

Monte Carlo simulation is used to estimate the roaming revenue of each alliance partner. Since there are several parameters with a normal distribution in the model, also the result is distributed. The distribution of the roaming revenue can be explained partly due to uncertainties of the model and input data, partly due to changes in the business environment. This is very valuable information when assessing the stability of an alliance. To be vital in the long-term, an alliance must be profitable in a changing environment with changing parameters, not only with a high tuned start-up configuration.

The stability of all potential alliances is estimated by using the cooperative game theory and the core solution concept. In order to be a stable and a reasonable alternative for the partners of an alliance, all the criteria of the core should be fulfilled. The alliance needs to be rational from a single partner's viewpoint; if a partner can make it better alone, there is no reason to join an alliance. The alliance must also own the "coalitional rationality", that is, no subset of alliance partners could earn better by forming a smaller alliance. Finally the efficiency criterion is a very natural requirement, i.e. the outcome of the alliance as a whole is divided between the alliance partners.

The revenue creation capability of all potential alliances is verified against the core. When the core exists for an alliance, the alliance is capable of generating such a level of revenue that all coalition members have an incentive for the cooperation. What is not known is how the additional revenue enabled by the alliance should be divided among the members, that is, the allocation of imputations. The division of an alliance's additional roaming revenue is investigated by using two different schemes to distribute the outcome.

The first scheme is to allocate the revenue for each alliance partner in a direct way, that is, what a partner would earn when making business as a member of an alliance. This is the most natural way to behave in an alliance in which the partners' assets and revenues are separate. The second scheme is based on the overall revenues of the partners. The additional revenue due to an alliance is divided between the partners by sharing the benefit in proportion to each member's overall revenue. This would provide for big operators more than for smaller ones. On the other hand, quite likely also the contribution of big players is bigger than that of the smaller operators. By having two different schemes to divide the revenue, their existence in the core solution can be compared and thus their impact to the alliance formation can be estimated.

Because the results of Monte Carlo simulations are not a single figure but a distribution, also the stability of an alliance can vary from one simulation to another. This information is used to assess an alliance's long-term stability in the changing business circumstances. In the model a new concept called stability indicator is introduced. It is the proportion of samples in core to all simulations. The stability indicator varies between zero and one, one being always stable and zero always unstable. The values between them indicate the sensitiveness to various changes in the business environment and can thus reflect the long-term instability.

When all the potential alliances have been calculated and their existence in the core with different revenue divisions has been assessed, the probable alliance formation can be identified. Patterns for the alliance formation are investigated. Are there operators that do not seem to fit in the same alliances? How doe the stability change with different alliance configurations? What is the impact of different schemes to divide the additional revenue among the partners?

4.2 Structure of the model and computational aids

The structure of the model is illustrated in Figure 8. Each of the model's elements is described in the following chapters.

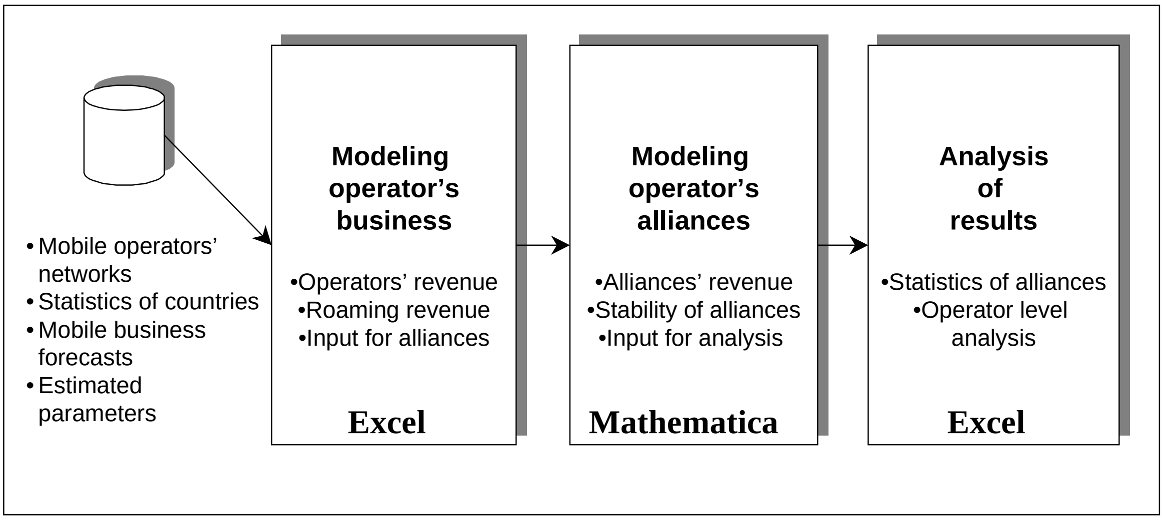


Figure 8. Structure of the model

All the computations are executed with a Hewlett-Packard lap-top computer ze5600. The computer is equipped with an Intel Celeron 2.6 GHz processor and with 512 MB of RAM memory.

The basic modelling of the mobile operators' business is done with Microsoft Excel spreadsheet program.

For the modelling of mobile operator alliances the Mathematica 5.2 of Wolfram Research is applied. Many of the Mathematica's additional packages are also applied, e.g. statistics, linear algebra and graphics packages. The Mathematica version 4.0 and its additional packages are documented in a book form in (Wolfram 1999) and (Wolfram Research 1999). The Mathematica code is listed in Appendix 2.

For the core analysis two additional Mathematica packages are applied. The first one, CooperativeGames, includes the basic tools to analyse cooperative games (Carter 1993). The second one, TuGames, extends the capabilities of the first package (Meinhardt 2005). It is available on the Wolfram Research Internet pages.

The analysis of the final results is done with Microsoft Excel.

4.3 Modelling the business of global mobile operators

Top global mobile operators and their business globally form the basis of the model. This chapter describes the contents, sources and the logic behind the economic model which incorporates the revenue creation of 10 biggest global mobile operators.

In the model real figures have been used. They are collected from diverse sources. The baseline year of the model is 2005. This is motivated by the availability of the statistical data and business forecast reports.

Services provided by mobile operators

Mobile operators provide wireless communication services to their consumer and enterprise customers. Wireless communication services can be categorized in many different ways. A topology proposed by Vesa is presented in Table 2 (Vesa 2005).

The basic service of mobile operators is still the plain person-to-person voice call, i.e. the mobile phone call service. In 2005 voice calls made up 88% of the mobile operators' global revenue, when SMS messages are included in data services (Reid, Sims & Gibney 2005). To the model the division of the services is not of great concern because the viewpoint is operators' overall revenue including the sales of all services.

Table 2. Topology of wireless communication services

WIRELESS COMMUNICATION SERVICES	
Conversation	
	Mobile voice
	- Voice call
	- Push-to-talk
	Person to person messaging
	- SMS
	- MMS
	- email
	- Instant messaging
	- Mobile chat
Content services	
	SMS based content services
	MMS based content services
	Browser based content services
	Downloadable applications
	Other content services
Data access	
	GSM data
	GPRS
	CDMA 1x
	EDGE
	UMTS
	CDMA EV-DO
	WLAN / WiFi
	Other methods

Top 10 global mobile operators in terms of proportionate subscribers

The basic figures of the mobile operators' global business are gathered from the report *World Cellular Investors* by Informa Telecoms & Media (Informa Telecoms & Media 2006). The following data is available about each operator and about all networks the operator controls:

- Operator's name (e.g. NTT DoCoMo)
- Region (e.g. Asia-Pacific)
- Country (e.g. Taiwan)
- Network's name (e.g. FarEasTone)

- Technology (e.g. GSM)
- Ownership (%)
- Number of subscribers in the network
- Proportionate subscribers in the network (= ownership * total subscribers)

When calculating over all networks, regions and countries, the global number of proportionate subscribers can be concluded for each operator. The proportionate subscriber is a measure which denotes the number of subscribers the operator can get revenue from. It is calculated by multiplying the operator's share of ownership in a network and the total number of subscribers in the network. Figure 9 illustrates the number of proportionate subscribers of the Top 10 global mobile operators. The size of the operator is valued in terms of proportionate subscribers. The distribution of the subscribers over geographical regions can also be found in the figure.

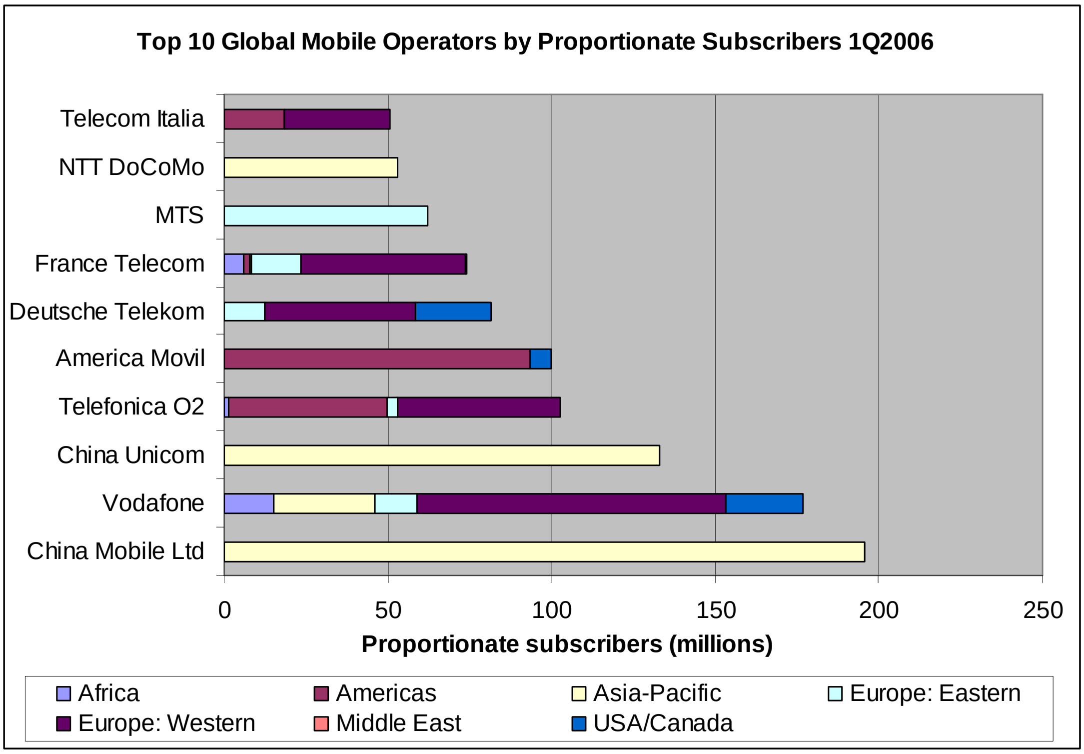


Figure 9. Top 10 Global Mobile Operators 1Q2006.

The distribution of the subscribers over geographical regions and countries varies to a great extent among the Top 10 operators. In the one end of the continuum are China Mobile and China Unicom which operate only in China. In the other end there is Vodafone with its operations in five regions and 35 countries.

Wireless technologies used by Top 10 global mobile operators

Wireless technologies used by mobile operators vary from operator to operator. The distribution of technologies is illustrated by Figure 10. There are still some analogue networks, about 0.1% of the total number of proportionate subscribers. GSM and its derivative W-CDMA hold the major share of the Top 10 operators' technology base, 84% of the proportionate subscribers. These are also the technologies which enable the global roaming between mobile operators. Thus from the roaming revenue's viewpoint the technology distribution of an operator is an essential element. For example since China Mobile has only GSM networks, 100% of its subscribers can contribute to roaming revenues when travelling abroad. On the other hand NTT DoCoMo's major technology is PDC and only 45.5% of its proportionate subscriber base uses GSM or W-CDMA. Thus NTT DoCoMo's potentiality to contribute to roaming revenues is much less than its size in terms of subscribers.

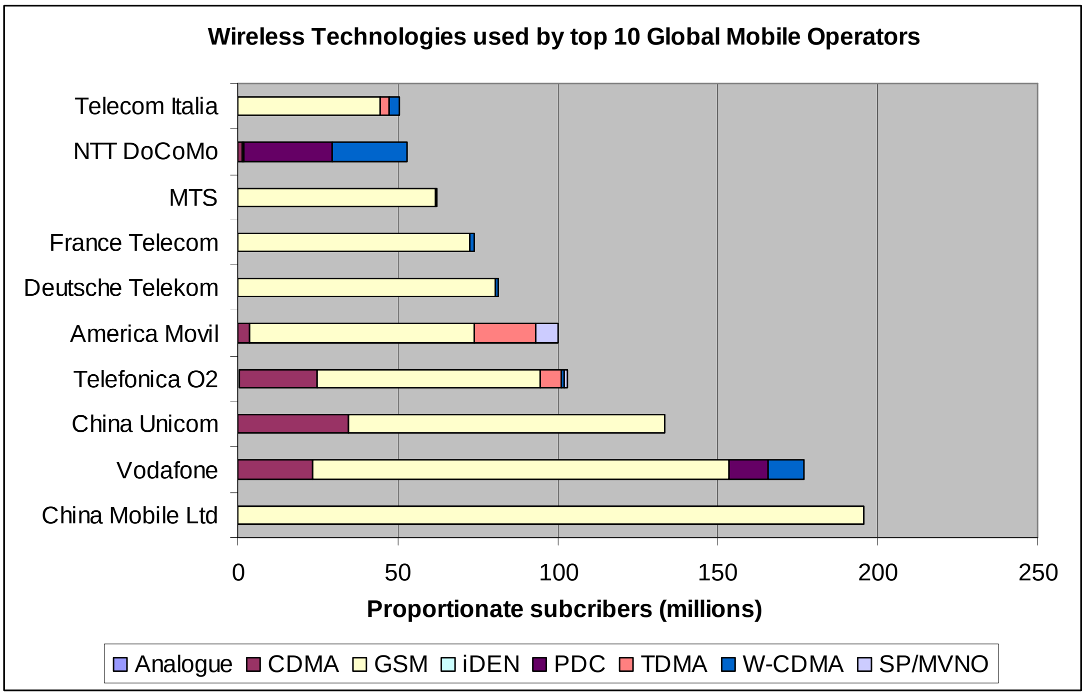


Figure 10. Wireless Technologies of Top 10 Global Mobile Operators 1Q2006.

Revenue of Top 10 global mobile operators in years 2005-2010

The typical way to calculate the revenue a mobile operator is able to gather from its subscriber base is an ARPU figure, average revenue per user. It denotes the amount of the money an operator is able to invoice for wireless services from all its subscribers on average. Usually it is calculated on a monthly basis. The ARPU varies as a function of country and time (Gruber 2005). The monthly ARPU over the whole world in 2005 was USD 25.63 (Reid, Sims & Gibney 2005).

A forecasting report, *Global Mobile Forecasts to 2010* (Reid, Sims & Gibney 2005) provides forecasts for global mobile markets in years 2005-2010. Forecasts are provided on country basis and when country specific data is not available, regional forecasts are included. To be able to calculate the basic revenue stream of the Top 10 mobile operators over the years 2005-2010, the following country specific data is collected to complete the model:

- ARPU monthly (USD); 2005-2010
- Subscriber growth (%); 2005-2010

For all countries the country specific data is not available. For such countries regional data is used, e.g. the general Africa forecast replaces missing Lesotho figures.

For an operator the yearly computational revenue can be calculated using the following expression:

$$R_{o,y} = \sum_{c=1}^{C}\sum_{w=1}^{W} N_{c,o,w,y} * ARPU_{c,y} * 12 \qquad (38)$$

In this expression $R_{o,y}$ denotes the computational revenue of an operator o in a year y; N the number of proportionate subscribers; $ARPU_{c,y}$ the average revenue per user in a country c in a year y. Indexes $c \in [1\ldots C]$, $o \in [1\ldots O]$, $w \in [1\ldots W]$ and $y \in [2005\ldots 2010]$ denote a country, an operator, a network and a year respectively. The constant *12* is due to the number of months in a year.

Figure 11 illustrates the computational revenue of the Top 10 global mobile operators. The computational revenue describes an operator’s revenue creation capability on the basis of its proportionate subscriber base and on forecasted ARPU of the markets the operator has networks in. It is a computational measure and thus does not necessarily conform to the operators’ real revenue figures. It is assumed that Top 10 operators’ subscriber growth follows the growth of the market.

Most of the operators have a declining revenue trend over the forecasting period. Chinese operators make a clear exception. The reason for that is evident. In most of the markets the ARPU is forecast to decline rapidly, e.g. in Brazil from USD 14.39 in 2005 to USD 9.3 in 2010. In Chinese markets the trend is also declining but no so rapidly: from USD 11.93 to 11.28. That combined with the reasonable subscriber growth produces a healthy growing revenue stream in China.

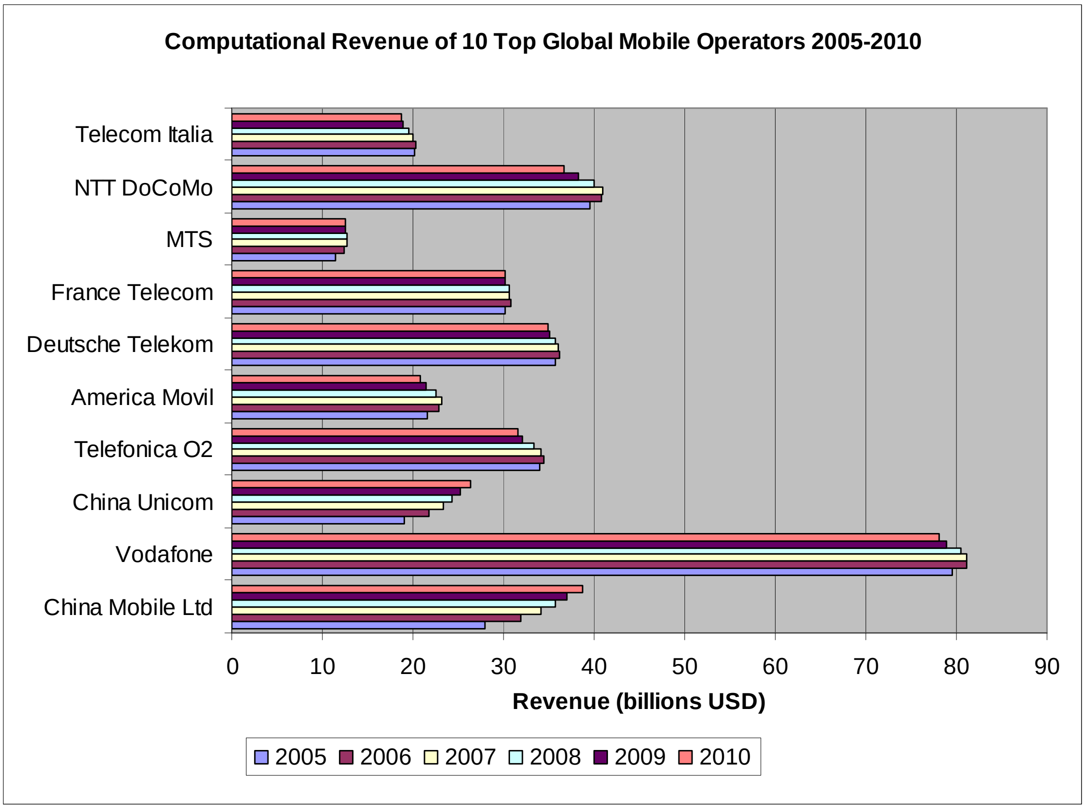


Figure 11. Computational revenue of Top 10 Global Mobile Operators 2005-2010

To have one reference figure which can be used to represent the overall business value of a mobile operator, the present value (PV) of computational revenue is calculated over the years 2005-2010. The present value is the value on a given date when future payments and a discount rate are given. Present value incorporates the time dimension of the money. *"A dollar today is worth more than a dollar tomorrow, because the dollar today can be invested to start earning interest immediately."* (Brealey, Myers 2003)

Present value is calculated with the following expression (Brealey, Myers 2003):

$$PV = \sum \frac{C_t}{(1+r_t)^t} \tag{39}$$

In the expression C_t denotes the expected future cash flow (revenues of coming years), r_t is the discount rate (cost of money) and t is time, usually in years. Figure 12 illustrates the present value of the computational revenue for Top 10 mobile operators in years 2005-2010. The discount rate $r_t=12\%$ is used and in year 2005 $t=0$, that is, no discount on that year revenue.

Clearly Vodafone outperforms all other operators with its present value of USD 368 billion due to the large amount of proportionate subscribers and a very healthy ARPU level. The next ones are NTT DoCoMo and Deutsche Telecom with USD 182 billion and USD 164 billion, respectively.

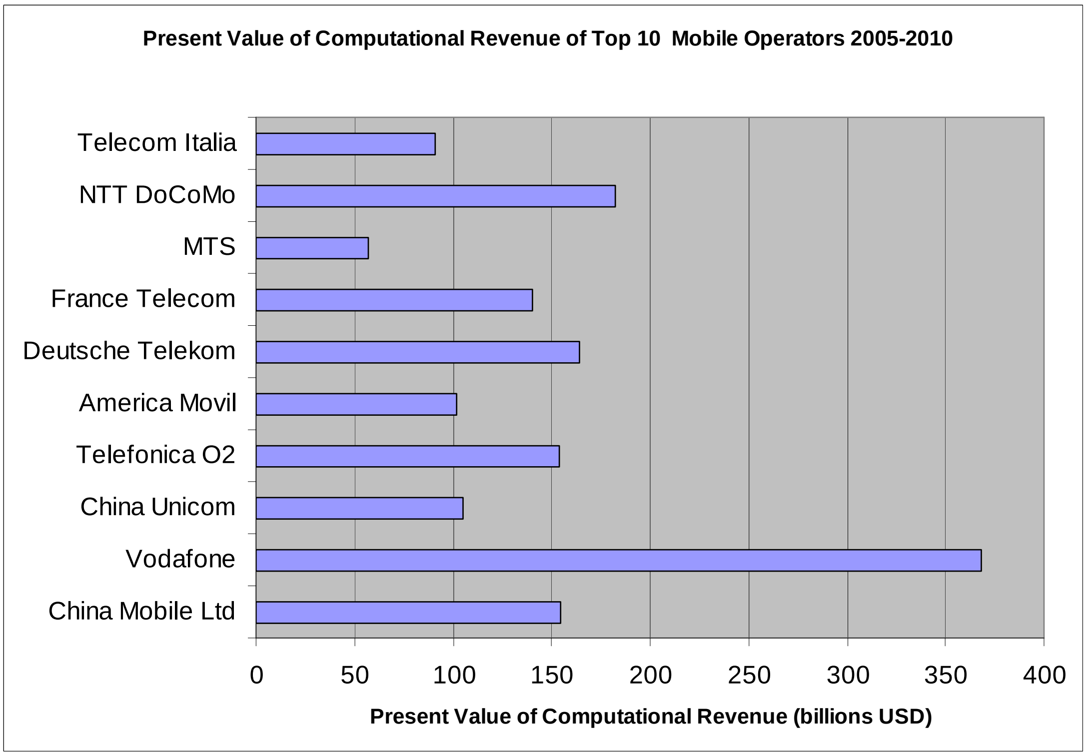


Figure 12. Present value of computational revenue of Top 10 Global Mobile Operators 2005-2010

Table 3 presents the distribution of the computational revenue's present value over the most important countries. The number of countries in which Top 10 Global Mobile operator have subscribers is 90, of which the ten most important countries make up 80.3% of their total revenue.

Chinese operators have subscribers only in China. NTT DoCoMo and the Russian operator MTS have operations only in a few countries (in four and five countries, respectively). When considering the operators' business in the most important countries clearly Vodafone's position is the best among its rivals. Vodafone's computational revenue is largest in these countries and also the distribution is the most even. It has subscribers in eight of the ten most important countries.

Table 3. Distribution of Top 10 Global Mobile Operators' computational revenue over the most important countries (billions USD)

	China	*Japan*	*USA*	*UK*	*Italy*	*Germany*	*Spain*	*France*	*Russia*	*Mexico*	*RoW*	***Total***
China Mobile Ltd	154,7										0,0	154,7
Vodafone	6,7	51,9	65,8	33,5	42,4	42,1	27,6	17,1			81,2	368,4
China Unicom	105,3										0,0	105,3
Telefonica O2				41,8		15,5	40,4			5,4	50,9	154,0
America Movil			18,9							33,2	49,8	102,0
Deutsche Telekom			63,5	27,6		47,4					25,6	164,1
France Telecom				34,8		0,1		49,1			56,1	140,1
MTS									42,0		14,8	56,8
NTT DoCoMo		178,8									3,4	182,2
Telecom Italia					73,1						17,7	90,7

International roaming of Top 10 global mobile operators in years 2005-2010

International roaming is an essential element in the service portfolio of mobile operators. International roaming enables a subscriber of a particular network to use the mobile phone abroad on another operator's network. When a subscriber roams to a network abroad (visited network) and uses its services, both operators, the visited network's and the home network's operator gain from that. They share the roaming fee the subscriber pays for using mobile services abroad.

The pricing of roaming services is a two-step procedure. The first step is wholesale pricing, where the visited network operator charges the user's home network operator for the services, usually by adding a mark-up of 15% on top of the normal tariff of the visited network. The second step, retail pricing, takes place when the home network charges the subscriber usually adding a margin of 10-35% on top of the wholesale price of the first step. Thus the roaming price charged from the subscriber is the result of double marginalisation (Gruber 2005).

When travelling abroad, there can be several mobile networks in the area where the roaming mobile phone is located in. If the home operator of a travelling subscriber has a roaming agreement with many operators with overlapping coverage, only one of the networks needs to be selected. There are two options to select a network: manual and automatic selection.

When using the manual selection, the mobile phone user can select a network from the list offered by the mobile phone. When the user has chosen one of the networks, the mobile phone uses that one until a new selection is done. According to a research report by Oftel 2002, most mobile subscribers do not choose networks manually but use the automatic selection: 63% in Great Britain, 75% in Northern Ireland and 66% in the Republic of Ireland (Ambjørnsen, Wasenden 2005).

The automatic network selection is based on the preferred roaming partners in a mobile terminal's SIM card. When a subscribers travels to another country, a visited network is selected according to the list of preferred partners if several networks providing the signal strength above a defined threshold are available. Earlier the memory capacity of SIM cards was quite limited and thus also the number of preferred roaming partners was limited. Today, when the capacity of SIM cards has increased, this is not a limiting factor anymore. Combining this with the Over-The-Air (OTA) technology which allows operators to update the information on SIM cards remotely, operators have good technical tools to control the selection of visited networks among their own subscribers (Ambjørnsen, Wasenden 2005).

The roaming revenue is included in the revenue and ARPU figures of mobile operators and usually operators do not publish the amount or the distribution of the roaming revenue. It is difficult to find credible information about roaming volumes and about its share in the mobile operators' business from any other sources. One reason for this is most likely the confidentiality of roaming agreements between mobile operators.

Agreements can be based on a standard framework provided by GSM Association (GSMA) or they can be bilaterally negotiated. Typically mobile operators have bilateral agreements with their major roaming partners, that is, counterparts in the countries where the roaming volumes are the largest (Ambjørnsen, Wasenden 2005). Another probable reason for operators' reticence is regulators increasing interest to investigate the end-user roaming fees in order to regulate them (Patterson 2005).

In a report published by International Telecommunication Union it is estimated that "in 2004, roaming generated USD 78.6 billion in revenues, representing over 15 per cent of global mobile revenues … this is predicted to rise by 2010 to USD 211.8 billion (28 percent of global revenues)" (ITU 2006). This is in line with typical estimations, e.g. "it is estimated that roaming services generate approximately 15 to 25 % of the total revenue for a mobile network operator (MNO) in Europe" (Dahlgren 2006).

Assuming that the Top 10 mobile operators get their share of the global roaming revenue according to the ratio of mobile subscribers, their total roaming revenue is USD 43.2 billion in 2005 and will rise to USD 95.2 billion by 2010. In 2005 this represents 13.5% of their total computational revenue and 28.9% in 2010. These figures are based on the following global figures of mobile subscribers: 2.14 billion subscribers in 2005, 3.01 billion in 2010 (Reid, Sims & Gibney 2005).

The next question to be solved is each Top 10 mobile operators' share of the roaming revenue. The sub-questions which follow are the roaming revenue's distribution over the countries in which operators have networks and over the networks in each country. Also the revenue division between home and visited operators needs to be answered.

Let us approach these questions via mobile subscribers. Are all users alike with each other in using roaming services? Quite likely this is not the case. The usual assumption is that business travellers consume the major amount of roaming services. They are forced to that and on the other hand they do not pay their mobile phone bills. Also "consumers are often too afraid to use roaming services, because they do not know what it costs and they find other ways to communicate" (Dahlgren 2006). According to (Patterson 2005) "business travellers, especially GSM contract users, dominate the roaming market".

The study of Rieck et al. (Rieck et al. 2005) seems to confirm the assumption about the business travellers' importance among the roaming service users. They analysed roaming alliances of mobile operators using social network analysis and statistical methods. The input data used in the study was roaming fees of mobile subscribers. It was assumed that if there is a partnership between two operators, they provide lower roaming fees to their customers in the country-to-country roaming markets. They found that some of the roaming alliances really behave in a coherent way in pricing the roaming services, while others did not demonstrate the existence of an alliance in their prices.

The study also analysed the significance of certain country specific variables to explain why some operators "are able to form more favoured partnerships and thereby leverage more benefits than others." The statistically significant variables were found out to be the operator's revenue, export and import of the country, whereas the variables of population size, GDP, GDP growth, and the number of international travellers did not explain a mobile operator's central role in the partnership network. The interpretation of the study is that "the country is viewed as being more commercially attractive due to its comparable healthy balance of trade." "A higher density of business travellers can be expected to travel out of these countries in order to close deals and expand their export networks. Hence, home carriers in major export countries become more attractive as the choice for favoured partnerships, due to the good prospects of demand for roaming."

Business travelling and trade figures seem to be an obvious pattern to model the mobile roaming between countries. When a business person travels from his/her home country to another country, e.g. to meet his/her company's customers, roaming revenue is generated every time he/she makes a phone call abroad. Both the home and visited operators get their share of that. Due to the findings of Rieck et al. it can be assumed that export and import figures reflect the distribution of roaming volumes and hence also roaming revenues of mobile operators. The bigger the export of a country, the bigger is the country's home operators' share in the roaming revenue. Respectively, the bigger the import of a country, the bigger the country's visited operators' share of the roaming revenue.

Considering a country's home operator's share of the global roaming revenue, the penetration of mobile services in the country has evidently a significant role. Even though the export figures of the country were notable but if the penetration is low, operators in the country could hardly expect a high level of home roaming revenue due to the low share of mobile phone owners among business travellers.

When considering the model from a country's visited operator's point of view, the country level penetration does not have a similar role to home operators' case. When a business traveller wants to take a mobile phone call abroad, he or she is not interested in the country's mobile penetration, once there is cellular coverage available. Usually mobile operators try to ensure the capacity and coverage of cellular networks at least in those geographical areas where business travellers usually travel. Hence it can be assumed that the country level penetration of mobile services does not limit a country's capability to capitalize the visited roaming revenue provided by business travellers from other countries.

To summarize the conclusions so far:

$$R_{RH,c} \propto \pi_c * E_c \tag{40}$$

$$R_{RV,c} \propto I_c \tag{41}$$

In these expressions R_{RH} and R_{RV} denote the home roaming revenue and visited roaming revenue, π_c the mobile penetration of a country, E export and I import of a country. Index c denotes a country. The term penetration refers to the ratio of number of subscribers per 100 of the population, usually shown as a percentage. It can be measured over a single network or over a whole country, that is, all cellular networks and their subscribers within the country.

The next level is the distribution of a country's home and visited roaming revenue among all cellular networks within the country. Here the solution is quite straightforward. It can be assumed that the home and visited roaming revenue of a country is divided among the different operators' networks according to the ratio of subscribers: the number of subscribers of a network to the total number of subscribers in the country. Here the wireless technologies which are taken into account are GSM and its derivative W-CDMA. They enable the global roaming between mobile operators (GSMA 2009). Cellular networks of other technologies are ignored in the roaming revenue calculations.

In the calculations the ratio of a network's number of subscribers to the total number of subscribers in a country can be replaced with the ratio of a network's penetration to the overall penetration of a country:

$$\frac{N_w}{N_c} = \frac{N_w / M_c}{N_c / M_c} = \frac{\pi_w}{\pi_c} \tag{42}$$

N denotes the number of (proportionate) subscribers, π_w the network level penetration, π_c the overall penetration of mobile services in a country and M the number of inhabitants in a country. Indexes *c and w* denote a country and a network in the country.

By combining the country and network level conclusions, the following can be written:

$$R_{RH,c,w} \propto \pi_c * E_c * \frac{\pi_{c,w}}{\pi_c} = E_c * \pi_{c,w} \tag{43}$$

$$R_{RV,c,w} \propto I_c * \frac{\pi_{c,w}}{\pi_c} \tag{44}$$

Symbols are as in (40) – (42).

In the model it is assumed that the revenue division between home and visited operators is constant. The home operator's margin added on top of the wholesale price is assumed to be 10-35% (Gruber 2005). This yields the home operator's share of 10%-25%. In the model 20% is used.

As a summary the following expressions can be written to calculate an operator's home and visited roaming revenue in a year:

$$R_{RH,o,y}=\frac{\sum_{c=1}^{C}\sum_{w=1}^{W}\left(\pi_{c,o,w,y}E_{c,y}\right)}{\sum_{c=1}^{O}\sum_{c=1}^{C}\sum_{w=1}^{W}\left(\pi_{c,o,w,y}E_{c,y}\right)}\lambda R_{R-Top10,y} \tag{45}$$

$$R_{RV,o,y}=\frac{\sum_{c=1}^{C}\sum_{w=1}^{W}\left(\frac{\pi_{c,o,w,y}}{\pi_{c,y}}I_{c,y}\right)}{\sum_{c=1}^{O}\sum_{c=1}^{C}\sum_{w=1}^{W}\left(\frac{\pi_{c,o,w,y}}{\pi_{c,y}}I_{c,y}\right)}(\lambda-1)*R_{R-Top10,y} \tag{46}$$

In these expression $R_{RH,o,y}$ ($R_{RV,o,y}$) denotes the home (visited) roaming revenue of an operator o in a year y; $\pi_{c,o,w,y}$ the network level penetration and $\pi_{c,y}$ the overall mobile penetration in a country; E export and I import of a country; $R_{R\text{-}Top10,y}$ the total roaming revenue of Top 10 global mobile operators; λ the home operator's share of the roaming revenue. Indexes $c\in[1\ldots C]$, $o\in[1\ldots O]$, $w\in[1\ldots W]$ and $y\in[2005\ldots2010]$ denote a country, an operator, a network and a year, respectively.

Expressions (45) and (46) can be simplified by introducing a new parameter ϕ. This parameter denotes an operator's share of home and visited roaming revenue in a given country in a given year among all Top 10 global mobile operators:

$$\phi_{H,c,o,y}=\frac{\sum_{w=1}^{W}\left(\pi_{c,o,w,y}E_{c,y}\right)}{\sum_{c=1}^{O}\sum_{c=1}^{C}\sum_{w=1}^{W}\left(\pi_{c,o,w,y}E_{c,y}\right)} \tag{47}$$

$$\phi_{V,c,o,y}=\frac{\sum_{w=1}^{W}\left(\frac{\pi_{c,o,w,y}}{\pi_{c,y}}I_{c,y}\right)}{\sum_{c=1}^{O}\sum_{c=1}^{C}\sum_{w=1}^{W}\left(\frac{\pi_{c,o,w,y}}{\pi_{c,y}}I_{c,y}\right)} \tag{48}$$

With these denotations expressions (45) and (46) can be written in the following more convenient way:

$$R_{RH,o,y}=\sum_{c=1}^{C}\phi_{H,c,o,y}\lambda R_{R-Top10,y} \tag{49}$$

$$R_{RV,o,y}=\sum_{c=1}^{C}\phi_{V,c,o,y}(\lambda-1)*R_{R-Top10,y} \tag{50}$$

To be able to calculate the roaming revenue of each operator, the model is completed with the additional country level data. The following data is from *International Telecommunication Union's World Telecommunication/ICT Indicators* (ITU 2006):

- Number of mobile subscriber in a country in 2005;
- Country level penetration in 2005 (%);

The following data is from *World Trade Organization's International Trade Statistics 2006* (WTO 2006):

- Export of a country in 2005 (USD);
- Import of a country in 2005 (USD);
- Export CAGR 2000-2005 (%);
- Import CAGR 2000-2005 (%);

Trade figures of some countries (e.g. Taiwan) are not available in WTO statistics. These figures are completed from *The World Factbook* of CIA (CIA 2006).

Export and import CAGR 2000-2005 (Compound Annual Growth Rate) are used to extrapolate export and import figures for years 2006-2010.

In addition to these figures the share of GSM/W-CDMA technologies in terms of subscribers in each country between years 2005-2010 is found in *Global Mobile Forecasts to 2010* (Reid, Sims & Gibney 2005).

With the described modelling and available data the roaming revenue can be estimated for each of the Top 10 global mobile operators. The results are illustrated by Figure 13 and Figure 14. In Figure 13 the present value of computational revenue is completed with home roaming revenue and visited roaming revenue. As it can be noticed, the absolute value of roaming revenue varies significantly among operators.
Figure 14 illustrates the roaming revenue's proportional contribution to the operators' revenue. This figure clearly shows that operators are in very different positions in their capability to capture roaming revenue from the market. For example Deutsche Telekom's roaming revenue is 37.2% of the overall computational revenue, whereas NTT DoCoMo's respective figure is 5.2%.

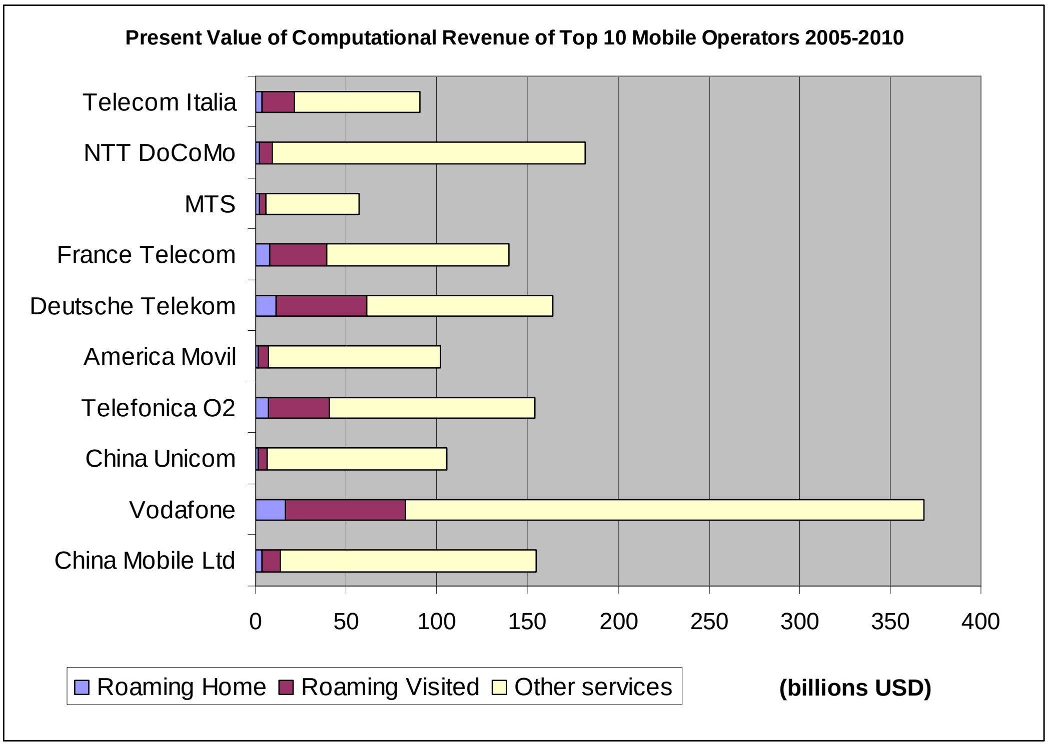


Figure 13. Present value of computational revenue of Top 10 Global Mobile Operators 2005-2010, roaming revenue displayed

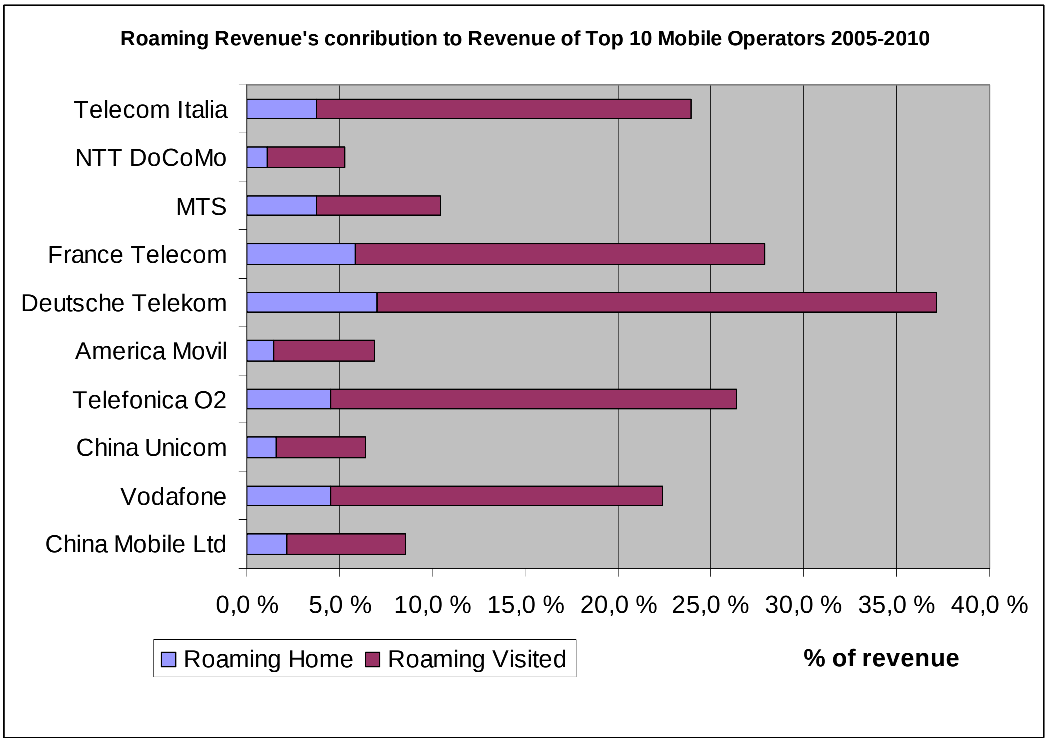


Figure 14. Roaming revenue’s contribution to the revenue of Top 10 mobile operators

4.4 Modelling mobile operator alliances

The model of mobile operator alliances estimates the revenue change the members of an alliance would gain by forming an alliance. By establishing an alliance the operators can direct their roaming subscribers to use the networks of the alliance partners abroad and thus the alliance can get a bigger share of the roaming traffic and fees of the market. Also an alliance can market its roaming services and thus increase the overall roaming services volume and also the roaming revenue. The objectives of the modelled alliances resemble the business objectives of real life mobile operators' roaming alliances (Rieck et al. 2005).

The model does not estimate any other potential benefits of an alliance, such as reduced transaction costs due to shared resources, improved bargaining position in negotiations with vendors, or complementary resources of partners that can be used to develop new services. Also the potential costs of the alliance forming are not estimated. The only element the model forecasts is the change of the alliance partners' roaming revenue.

It is assumed that an alliance can impact the behaviour of its subscribers in two ways. It can direct automatically subscribers to use the alliance partners' networks when roaming abroad by taking advantage of the automatic network selection as described in the previous chapter. Thus the alliance would get bigger share of the existing roaming volume. The second method is to promote the roaming services by decreasing the roaming fees and to make the subscribers more inclined to use roaming services when travelling abroad. When assuming that the demand of roaming services is elastic the volume of services increases and that compensates and can exceed the reduced unit prices. Hence the overall roaming revenue can increase.

By estimating these two components, a bigger share of the existing roaming traffic and the growth of roaming services among the subscribers, the additional roaming revenue of an alliance can be calculated.

In the model there are four parameters which can vary from case-to-case and also over time. Their distribution is modelled with the normal distribution. It is a common choice for parametric density estimations due to its many important properties (Bishop 1995):

- According to the *central limit theorem* under rather general circumstances the mean of M random variables tends to be distributed normally when M approaches the infinity.

- After any non-singular linear transformation of the coordinate system the distribution is normal, although with different mean and variance/covariance parameters.

- The marginal and conditional densities of a normal distribution are normal.

- Normal distribution is analytically simple to use.

The four varying parameters are the following:

- *Retail margin*: the retail margin a home operator adds on top of the roaming service's wholesale price; usually 10…35% (Gruber 2005)

- *Share of automatic selection*: share of subscribers who not choose the visited network manually but rely on automatic selection; typically over 60% (Ambjørnsen, Wasenden 2005)

- *Roaming discount:* the discount that alliance operators provide their customers to promote roaming services; assumed to be 0…45%

- *Elasticity of demand:* the price elasticity of roaming services demand; assumed to be between -0.3…-2.1

Elasticity of demand measures the responsiveness of quantity demanded to change in price. By letting ε_d to represent the coefficient of elasticity of demand, we can write

$$\varepsilon_d = \frac{\%\Delta Q}{\%\Delta P} \tag{51}$$

Here Q denotes quantity and P price, i.e. ε_d is equal to the percentage change in the quantity demanded divided by the percentage change in price. Usually the value of ε_d is negative because the changes of Q and P are opposed to each other. When the price declines, the demand increases and vice versa. If the absolute value of ε_d is less than *1*, demand is said to be inelastic, if it exceeds *1*, demand is elastic (Husted, Melvin 2004).

The elasticity of demand for roaming services is challenging to estimate. No figures can be found for roaming services and the data available for mobile services in general varies a lot. According to a paper published by Frontier Economics (Elliot 2004), the elasticity factor for mobile-originated calls varies between *-0.48…-0.8*. In a paper published by Coleago Consulting even such a high elasticity factors as *-3* can be read between UK Mobile retail minutes and revenues in years 1997-2003 (Duckworth 2005).

For the model the mean of demand of elasticity is chosen to be $\mu=-1.2$ and the standard deviation $\sigma=0.34$.

Varying parameters are illustrated in Figure 15.

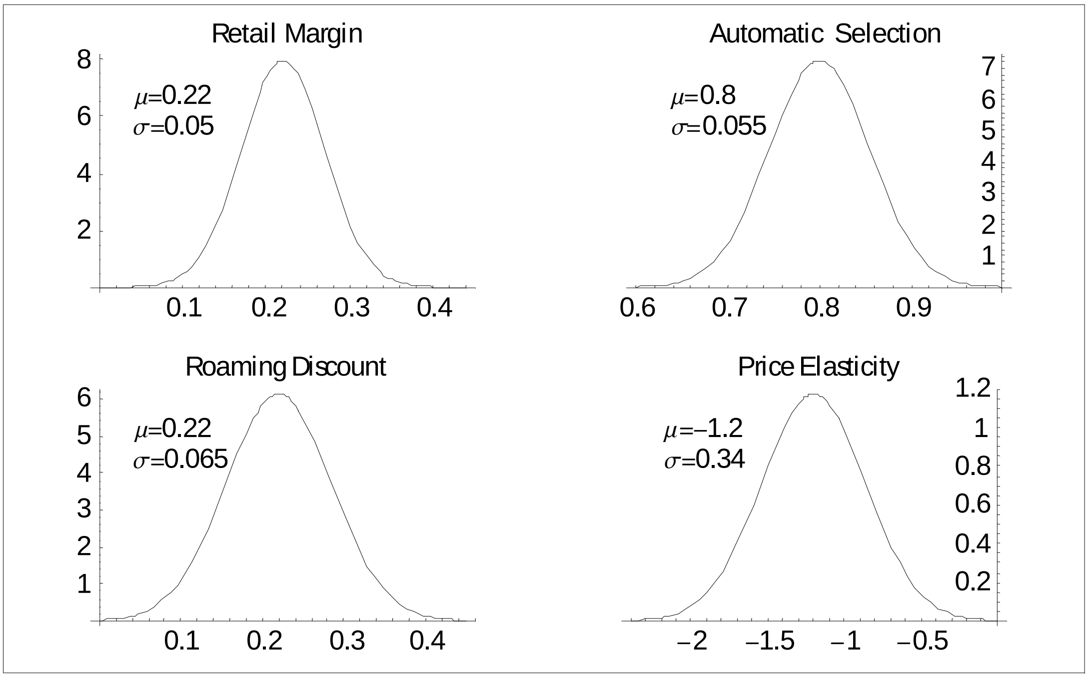


Figure 15. Parameters of the model with normal distribution

To estimate the revenue impact on each member of an alliance, the calculation can be divided into two parts: the impact on home roaming revenue and the impact on visited roaming revenue.

An alliance's impact on home roaming revenue has two components. A member of an alliance can lose home roaming revenue because of the discounted roaming fees. On the other hand if the roaming demand is elastic the volume of roaming services grows due to the discounted price and thus the roaming revenue can increase. The overall impact to the home operator's revenue, whether it is increasing or decreasing, depends on the elasticity factor and the discount percentage.

To consider closely the mechanism of an alliance's impact on home roaming revenue let us have a set-up of two alliance partners, one in the role of the home operator and the other as the visited operator:

- A home operator *ho* has customers in a home country *hc* and the operator's share of home roaming revenue in the home country among all Top 10 global mobile operators according to (47) is $\phi_{\mathrm{H,hc,ho}}$
- A visited operator *vo* has a network in a visited country *vc* and the operator's share of visited roaming revenue in the visited country among all Top 10 global mobile operators according to (48) is $\phi_{\mathrm{V,vc,vo}}$
- Home operator's retail margin is η and the roaming discount is δ
- Roaming services' elasticity of demand is ε_d

When home operator's customers travel to country *vc* they get a discount on roaming services because the home and visited operator are in the same alliance. The impact of the discount on the home operator's roaming revenue is negative:

$$\Delta R_{HD,hc,ho,vo,vc} = \frac{-\delta\eta\phi_{H,hc,ho}\phi_{V,vc,vo}R_{R-Top10}}{1+\eta} \tag{52}$$

In this expression $\Delta R_{HD,hc,ho,vo,vc}$ is the discount's revenue impact on the home operator *ho* in a home country *hc* when taking into account only one visited country *vc* and there only one visited operator *vo*. To get the whole impact, the calculation needs to be executed over all countries as home and visited country and over all members of the alliance. Also years need to be taken into account.

Using the same set-up we can construct the additional revenue produced by the additional demand of roaming services:

$$\Delta R_{HA,hc,ho,vo,vc} = \frac{(-\varepsilon_d\delta)(1-\delta)\eta\phi_{H,hc,ho}\phi_{V,vc,vo}R_{R-Top10}}{1+\eta} \tag{53}$$

In the expression the term ($-\varepsilon_d\delta$) denotes the additional traffic produced by the elasticity of demand and the price discount (the minus sign due to the negative elasticity factor). The term ($1-\delta$) is the price after the discount.

By combining (52) and (53) and executing the calculation over all countries as home and visited country and over all members of the alliance *S* can we construct the overall impact on home roaming revenue for an alliance member *ho* and for a given year *y*:

$$\Delta R_{H,ho,y} = \sum_{vo\in S}\sum_{hc=1}^{C}\sum_{vc=1,vc\neq hc}^{C}\frac{(-\delta-\varepsilon_d\delta(1-\delta))\eta\phi_{H,hc,ho,y}\phi_{V,vc,vo,y}R_{R-Top10,y}}{1+\eta} \tag{54}$$

The other part that needs to be estimated is the impact on visited roaming revenue. The same set-up as with the home roaming revenue can be applied. A home operator *ho* has customers in a home country *hc* and the operator's share of home roaming revenue in the home country among all Top 10 global mobile operators is $\phi_{H,hc,ho}$. A visited operator *vo* has a network in a visited country *vc* and the operator's share of visited roaming revenue in the visited country among all Top 10 global mobile operators is $\phi_{V,vc,vo}$. The standard visited roaming revenue produced by *ho*'s subscribers with the home country *hc* to *vo*'s network in the country *vc* is:

$$\Delta R_{VS,hc,ho,vo,vc} = \frac{(1-\eta)\phi_{H,hc,ho}\phi_{V,vc,vo}R_{R-Top10}}{1+\eta} \tag{55}$$

This is the amount that is included in the original revenue figure of the visited operator *vo* and thus it must be subtracted from the *vo*'s revenue to avoid the same revenue component to be included twice.

As a member of an alliance the amount of roaming traffic created by the home operator's subscribers is not the same as outside an alliance. Because of the discounted roaming services and elasticity of demand the volume is $(1+(-\varepsilon_d\delta))$ and the unit price is $(1-\delta)$. When considering first the subscribers that allow their mobile phone to automatically select the visited network, the following visited roaming revenue component can be constructed:

$$\Delta R_{VA,hc,ho,vo,vc} = \frac{\kappa(1-\eta)(1-\varepsilon_d\delta)(1-\delta)\phi_{H,hc,ho}\sum_{o=1}^{O}\phi_{V,vc,o}R_{R-Top10}}{1+\eta}\frac{\phi_{V,vc,vo}}{\sum_{o\in S}\phi_{V,vc,o}} \tag{56}$$

This is the visited roaming revenue component of the operator *vo*, which is produced by those subscribers of the home operator *ho* who rely on automatic selection of the visited network. The parameter κ is the share of the subscribers using the automatic selection. Because roaming subscribers are now automatically directed to the visited network owned by an alliance member instead of choosing the networks randomly, the alliance can create additional revenue to the visited operator. If the visited operator is the only alliance member in the visited country, it gets all the roaming traffic produced by the home operator's *ho* subscribers when they are travelling in *vc*. If there are several alliance members in the same country, they share the visited roaming revenue (the last term in the expression). Hence the optimal situation from the alliance's point of view is if in each country there is only one alliance member.

The last component of the visited roaming revenue is the subscribers who choose the visited network manually. Now it is assumed that they make the selection randomly and thus the visited roaming revenue is distributed evenly among all the operators in the country, i.e. not only among the members of an alliance:

$$\Delta R_{VM,hc,ho,vo,vc} = \frac{(1-\kappa)(1-\eta)(1-\varepsilon_d\delta)(1-\delta)\phi_{H,hc,ho}\phi_{V,vc,vo}R_{R-Top10}}{1+\eta} \tag{57}$$

By combining (55), (56) and (57) and executing the calculation over all countries as home and visited country and over all members of the alliance *S* can we construct the overall impact on the visited roaming revenue for an alliance member *vo* and for a given year *y*:

$$\Delta R_{V,vo,y} = \sum_{ho\in S} \sum_{hc=1}^{C} \sum_{vc=1,vc\neq hc}^{C} \left(-1 + \frac{\kappa(1-\varepsilon_d\delta)(1-\delta)\sum_{o=1}^{O}\phi_{V,vc,o,y}}{\sum_{o\in S}\phi_{V,vc,o,y}} + (1-\kappa)(1-\varepsilon_d\delta)(1-\delta) \right) *$$

$$\frac{(1-\eta)\phi_{H,hc,ho,y}\phi_{V,vc,vo,y}R_{R-Top10,y}}{1+\eta} \tag{58}$$

By performing (54) and (58) over all members of an alliance and over all years, the whole impact on the alliance's roaming revenue can be finally achieved. The present value over the years is calculated by using (39).

The model includes all potential alliances that can be formed among ten mobile operators. Altogether this gives 1023 different alliances. The number of members varies from one to ten. An empty alliance is not included. The alliances of only one member are trivial cases because actually they are not alliances but each of the operators doing business alone. Alliances are numbered from 1 to 1023 according to Table 4.

Table 4. Numbering of potential alliances

Number of members	Number of alliances	IDs
1	10	1-10
2	45	11-55
3	120	56-175
4	210	176-385
5	252	386-637
6	210	638-847
7	120	848-967
8	45	968-1012
9	10	1013-1022
10	1	1023

For each potential alliance the following figures are calculated:

- Revenue of each alliance member
- Additional revenue for each member due to the alliance
- Total revenue of the alliance
- Total additional revenue due to the alliance
- Percentual additional revenue

To estimate the revenue for each alliance partner, Monte Carlo simulation is used in computation. One simulation loop consists of 1000 independent samples. The result of one simulation loop is the mean of the 1000 independent computations. To get the distribution of the revenue figures of each alliance, the Monte Carlo simulation loops are performed 100 times. In practice this means that the revenue figures of all potential alliances are computed 1.000*100 = 100 000 times.

The results of the estimations are illustrated in Figure 16, Figure 17 and Figure 18. In each figure all the potential alliances are illustrated with a point. Coalition IDs can be found on the horizontal axis. Figures are means over the Monte Carlo simulations.

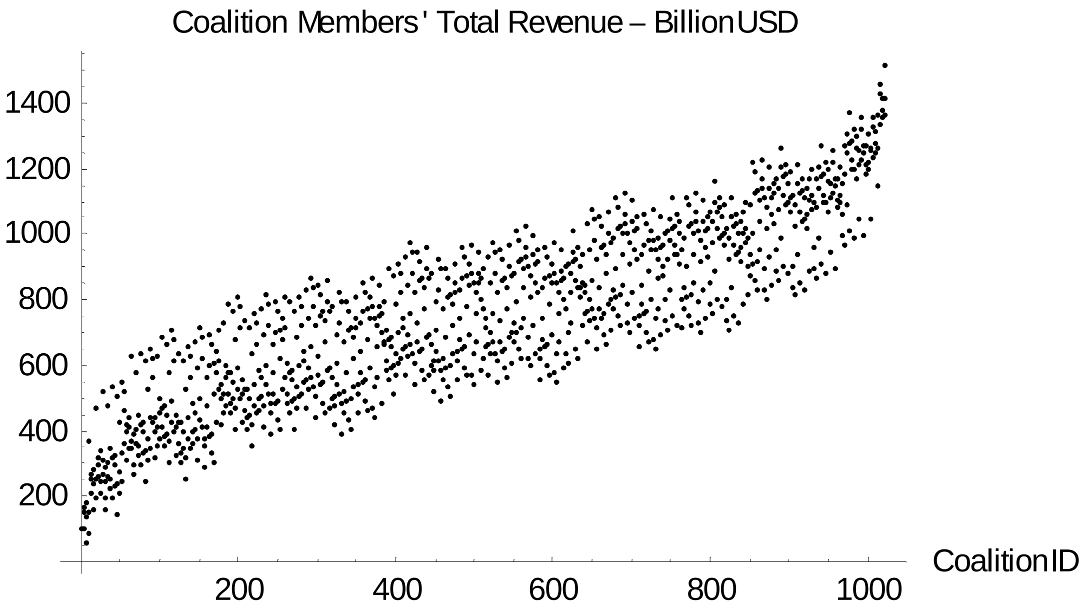


Figure 16. Present value of potential coalitions' total revenue

Figure 16 illustrates the present value of the potential coalitions' total revenue. This is calculated by summing up the revenues of all members of the potential alliance. The figure has a clear increasing trend. The more members in the alliance, the bigger the total revenue is. The mean is USD 760 billion, the minimum USD 56.8 billion and the maximum is USD 1518 billion.

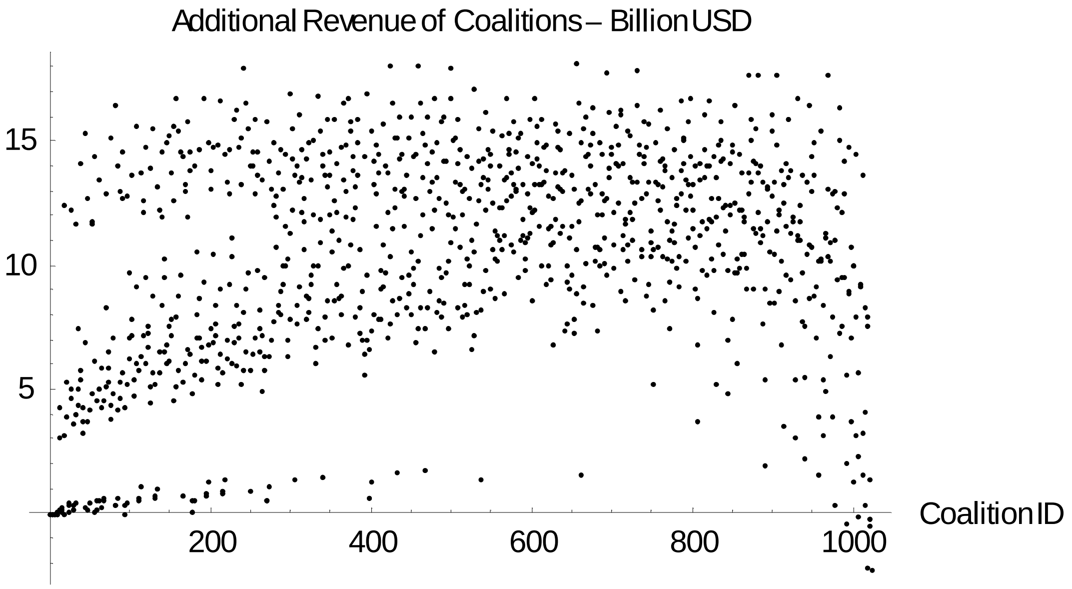


Figure 17. Present value of additional revenue of potential coalitions

Figure 17 illustrates the present value of the additional revenue of potential coalitions. This is calculated by summing up the additional revenue of all members of the potential alliance. Points spread over the figure and high and low additional revenues can be found both in small and large alliances. Some of the points are below the zero level, that is, some potential alliances create negative value for their members. The mean is USD 10.3 billion, the minimum USD -2.3 billion and the maximum is USD 18.1 billion.

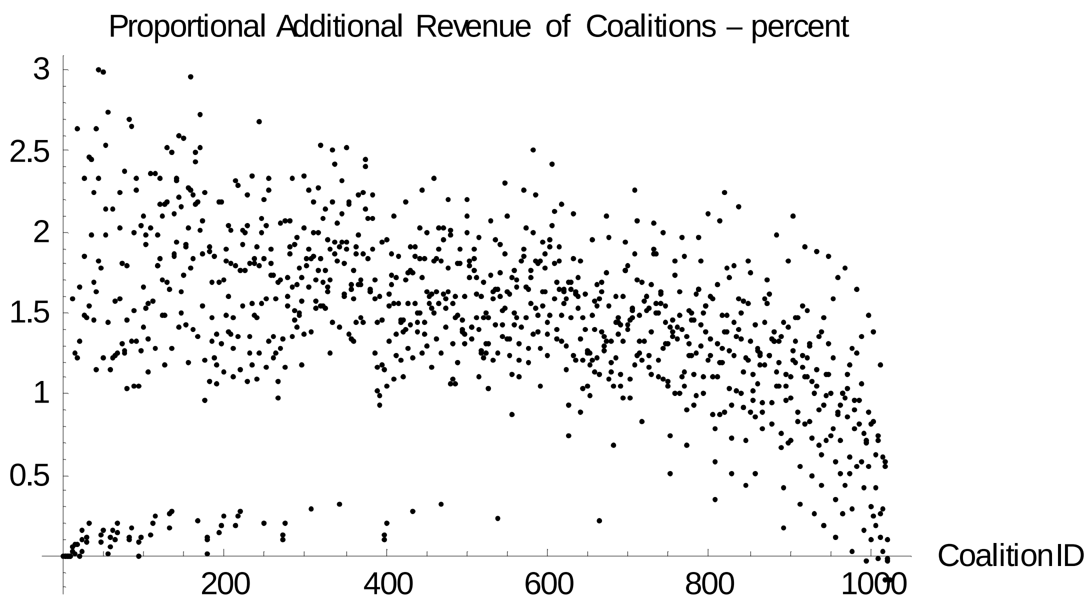


Figure 18. Percentual additional revenue of potential coalitions

Figure 18 illustrates the percentual additional revenue of potential coalitions. This is calculated by taking the percentual share between the total revenue of an alliance and the additional revenue of the alliance. There is a declining trend, that is, smaller alliances provide better percentual additional revenue than bigger ones. The mean is 1.4%, the minimum -0.15% and the maximum is 3.0%.

Because results are based on the Monte Carlo simulations and several independent simulations are performed, the revenue figures vary from one simulation to another. The basis for each alliance's revenue is the additional revenue estimated for each alliance member. When taking into account the present value of additional revenue of each member of all potential alliances, the number of individual figures is 5120, that is on average 5.0 members on each 1023 potential alliances. The variance of 100 simulations is measured by computing the standard deviation of these 5120 revenue figures.

The percentual standard deviation of each revenue figure is obtained by taking the percentual share of the additional revenue's mean and its standard deviation:

$$s\%_{\Delta R} = \frac{s(\mathbf{\Delta R})}{\overline{\mathbf{\Delta R}}} 100\% \tag{59}$$

In this expression $s\%_{\Delta R}$ denotes the percentual standard deviation of the additional revenue, $s(\mathbf{\Delta R})$ the standard deviation of the additional revenue of independent simulations and $\overline{\mathbf{\Delta R}}$ the mean of the additional revenue of independent simulations.

The percentual standard deviations of revenue figures are not equal but they vary greatly. The basic statistics of the 5120 different percentual standard deviation figures is the following:

- Mean of $s\%_{\Delta R}$: 14.0%
- Median of $s\%_{\Delta R}$: 1.50%
- Minimum of $s\%_{\Delta R}$: 0%
- Maximum of $s\%_{\Delta R}$: 3926%

The mean of the percentual standard deviation is not exceptionally high when considering the variance of four uncertain parameters. On the other hand, among the revenue figures the variance itself varies obviously quite much. The median and the mean are quite far from each other and also the maximum is really big. The distribution of percentual standard deviations confirms this assumption, see Figure 19.

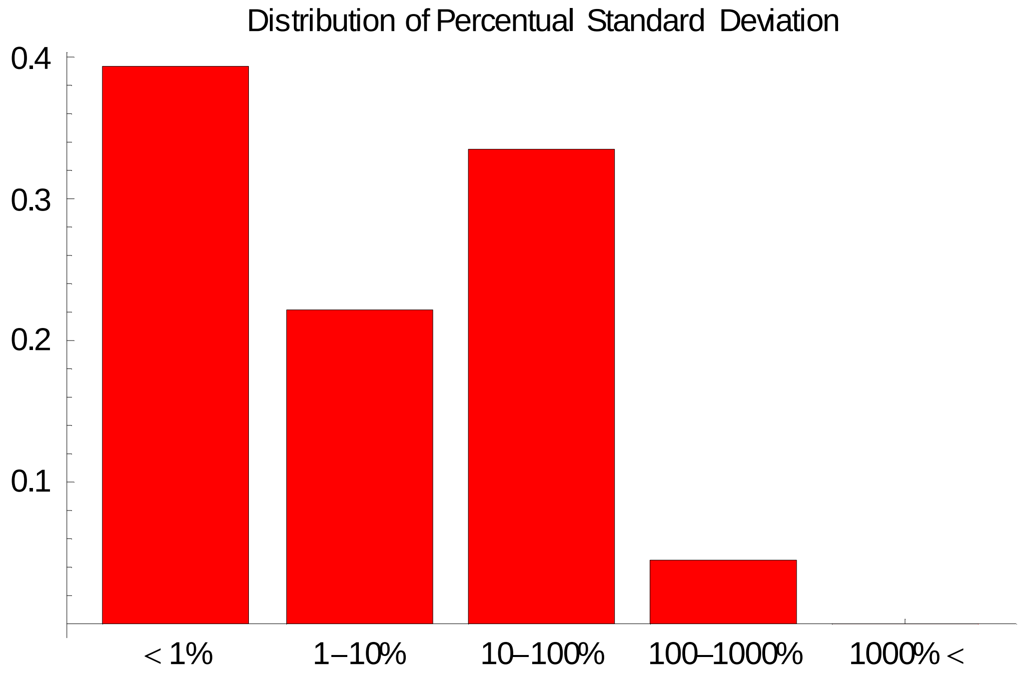


Figure 19. Distribution of percentual standard deviation of additional revenue figures of potential alliance members

The major part of the figures is within 10% of the maximum percentual standard deviation (61.7%, 3159 of 5120). 33.6% of the figures are between 10-100% (1720 of 5120) and 4.6% are between 100-1000%. Four figures are over 1000%.

The variance of the estimated revenue figures can be interpreted partly due to uncertainties of the model and input data, partly due to changes in the business environment. This is very valuable information when assessing the stability of an alliance. To be vital in the long-term, an alliance must be profitable in a changing business environment with changing parameters, not only within a highly tuned start set-up.

4.5 An alliance as a cooperative game

The stability of alliances is estimated by using the cooperative game theory and the core solution concept. The conditions for the core are clearly the ones that an alliance needs to fulfil to be stable in the long-term. According to Chapter 3.7, the criteria for the core are individual rationality, efficiency and coalitional rationality. The alliance needs to be rational from a single partner's viewpoint; if a partner can make it better alone, there is no reason to join an alliance. The alliance also must own the coalitional rationality, that is, no subset of alliance partners could earn better by forming an alliance of their own. This requirement also prevents the formation of cliques within the coalition once it has been established. Finally, the efficiency criterion is an evident requirement, i.e. the whole outcome of the alliance is divided between the alliance partners.

Each of the potential alliances of mobile operators is considered as an own n-person cooperative game with transferable utility and with *N* players. The game is in the characteristic form with a characteristic function *v*, which associates a real number *v(S)* with each subset $S \subseteq N$. Here the value of *v(S)* is the additional revenue of potential alliances. By combining (54) and (58) the expression for *v(S)* is:

$$v(S)_y = \sum_{ho \in S} \Delta R_{H,ho,y} + \sum_{vo \in S} \Delta R_{V,vo,y} \tag{60}$$

The present value over the years is calculated by (39).

As it is stated in Chapter 4.4, there is variance among the simulated revenue figures of potential alliances and thus for each *v(S)* there are 100 slightly different figures (number of simulations). Hence it is obvious that also the existence of the core can vary from one Monte Carlo simulation to another.

The roots of the variance are partly in the uncertainties of the model and input data, partly in the changes of the business environment. Because the existence of the core represents the stability of an alliance, the interpretation here is that the proportion of the simulation results indicating the existence of the core to all simulations represents the overall stability of an alliance. If all simulation results indicate the existence of the core, it can be assumed that the alliance in question is stable in varying business conditions. On the contrary not a single simulation being in the core, it is quite evident that the alliance is not capable of living.

To be able to measure the stability of an alliance, a stability indicator φ is introduced, which is the proportion of samples in the core to all simulations:

$$\varphi = \frac{n_{Core}}{n_{samples}} \tag{61}$$

If $\varphi = 1$ for an alliance, it is absolute stable. $\varphi = 0$ indicates respectively absolute instability. Values between *1* and *0* indicate sensitiveness to various changes in the business environment and thus can reflect the long-term instability.

When the core exists for an alliance, it is known that the alliance is capable of generating such a level of revenue that all coalition members have an incentive for cooperation. In other words, the alliance's revenue generation capability exceeds the revenue generation of individual members and all sub-alliances. What is not known is how the additional revenue enabled by the alliance should be divided among the members, that is, the allocation of imputations. Usually the core solution includes an uncountable number of possible allocations (ref. to Chapter 4.4).

The core solution sets limitations for applicable allocations. It is not evident that all feasible or even efficient and individually rational allocations would be within the boundaries of the core. Thus the outcome can be that even though the core exists for an alliance, alliance members are not able to agree on the revenue division which would be in the core and thus the stability of the alliance is threatened.

Applicable revenue allocations are investigated by using two different schemes to divide the outcome among the alliance members. The first scheme (later “revenue share scheme 1”) is to allocate the revenue for each alliance member in a direct way, that is, what a member would earn when making business as a partner of an alliance. This means that there is no re-allocation of the additional revenue between alliance partners but each keeps the benefits they are able to gain from an alliance. This is also the most natural way to behave in an alliance in which the partners’ assets and revenues are separate. The second scheme (later “revenue share scheme 2”) is based on the overall revenues of the partners. The additional revenue of an alliance is divided between the partners by sharing the benefit in proportion to each member’s overall revenue. This would provide for big operators more than for smaller ones. On the other hand, quite likely also the contribution of big players is bigger than the one of smaller operators. By having the different schemes to divide the revenue, their existence in the core solution can be compared and thus their impact to the alliance formation.

To summarize the above-told, for each of the mobile operator alliances the following figures describe the stability of the alliance:

- φ_{Core} Stability indicator between 1-0, which describes the existence of the core
- $\varphi_{RevShare1}$ Stability indicator between 1-0, which describes if the revenue share scheme 1 is in the core; no re-allocation of revenues, all members keep the additional revenue they are able to gain from the alliance
- $\varphi_{RevShare2}$ Stability indicator between 1-0, which describes if the revenue share scheme 2 is in the core; re-allocation of the additional revenue by sharing the benefit in proportion to each members overall revenue

4.6 Results of the model

Appendix 1 describes the results of the analysis of the mobile operators’ strategic alliances. All potential alliances are not listed due to the constraint on space. Some examples are listed to help the reader to follow the discussion.

For each potential alliance the following data is calculated:

- Alliance ID (1-1023);
- Alliance members;

- Total revenue of the alliance (millions USD);
- Additional revenue generated by the alliance (millions USD);
- The proportion of the additional revenue to the total revenue (%);
- Core stability indicator, describes the existence of the core (%);
- Revenue share 1 stability indicator which describes if the revenue share scheme 1 is in the core (%);
- Revenue share 2 stability indicator which describes if the revenue share scheme 2 is in the core (%);
- Additional revenue of each member according to revenue share scheme 1 (million USD);
- Additional revenue of each member according to revenue share scheme 2 (million USD);
- The proportion of the additional revenue of each member to the member's total revenue; revenue share scheme 1 (%)
- The proportion of the additional revenue of each member to the member's total revenue; revenue share scheme 2 (%)

Table 5 describes the basic statistic of potential alliances. On the table the mean figures are given for all potential alliances, for the alliances having the core and for alliances in which the revenue share scheme 1 and 2 are in the core. A potential alliance is considered to have the core if its φ_{Core} is not zero. The same logic applies for other two categories, revenue share scheme 1 and 2.

Table 5. Statistics of potential alliances

		Mean values				
Category	***Number of alliances***	***Members***	***Total revenue***	***Add Revenue***	***Add Rev%***	***Stability***
All alliances	1023	5,0	759 916	10 327	1,40 %	
Core exists	424	4,0	535 572	8 620	1,52 %	99,1 %
Revenue share 1 in Core	145	3,1	440 040	6 311	1,31 %	87,7 %
Revenue share 2 in Core	78	2,2	305 160	3 268	0,91 %	99,1 %

For approximately 41% (424) of all alliances the core exists. When considering the revenue share 1 and 2 schemes, only 14% (145) respectively 7.6% (78) of alliances are within the core. Also the average number of members in alliances declines clearly according to the same pattern. One can assume that the boundaries set by the core for stable cooperation are more demanding in large alliances than in smaller ones. An interesting finding is also that the "natural" share of revenues, in which all members keep the additional value they are able to gain from the alliance, is more favourable to be in the core than the one in which the alliance's benefits are re-allocated in proportion to each member's overall revenue.

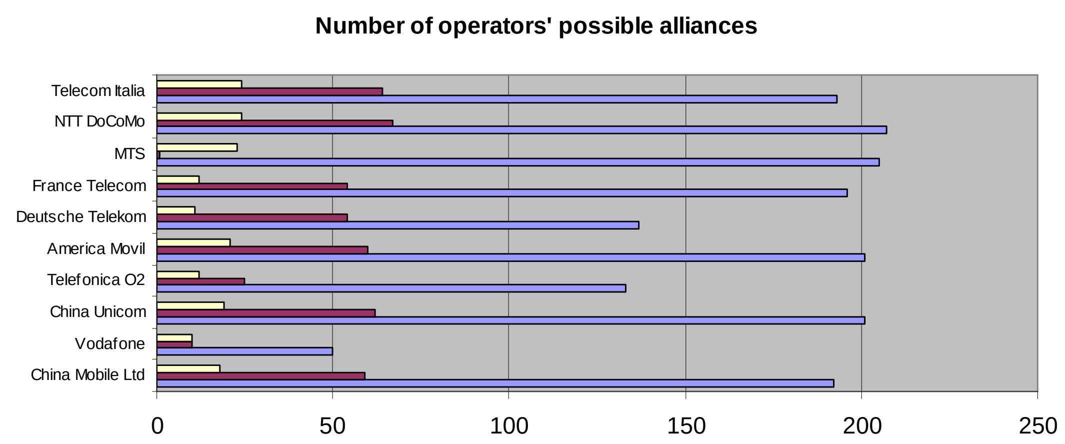
The trends in revenue figures are very much aligned with other findings. When the average number of alliance members decreases, it is natural that also the average total revenue declines. The alliances with the core are able to catch most of the additional revenue in proportion to the total revenue; on an average 1.52% which is in any case quite a humble figure. In alliances with the revenue share 1 and 2 in the core, the proportional additional revenue goes towards even smaller figures; on an average 1.31% respectively 0.91%. When the largest core alliances are not taken into account because of the revenue share limitations, also part of the revenue generation capability is lost. If alliances' members were able to agree on the revenue share within the core boundaries, also on average they could earn more value to be shared between them.

Table 6. Stability of alliances per number of members

	Mean stability per number of members ================================								
Category	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***All***
Core exists	100,0 %	100,0 %	99,8 %	99,5 %	99,2 %	97,8 %	91,3 %	100,0 %	99,1 %
Revenue share 1 in Core	100,0 %	100,0 %	91,1 %	79,8 %	68,3 %	48,0 %			87,2 %
Revenue share 2 in Core	100,0 %	100,0 %	96,2 %	100,0 %					99,1 %

According to Table 6 the categories Core exists and Revenue share 2 are very stable, both having the average figure 99.1%. The average stability 87.7% of the revenue share 1 category is also quite good although clearly on a lower level than within the other two. When looking in this category the distribution of the stability over the size of alliances, there is a clear trend. The decreasing number of alliance members improves the stability of alliances. Alliances with six members are quite unstable (48.0%) and yet the alliances with five members are not able to demonstrate a good stability (68.3%) on average. This is an important observation because the revenue share 1 scheme is the way the alliances would share the benefits of the cooperation if there was not any other agreement in place. All in all it is evident that the way the additional revenue is shared among the alliance members and the number of members has an impact on the stability.

It is good to notice that these are average figures over different categories and within each category there are also alliances with a 100% stability.



	China Mobile Ltd	Vodafone	China Unicom	Telefonica O2	America Movil	Deutsche Telekom	France Telecom	MTS	NTT DoCoMo	Telecom Italia
RevShare2	18	10	19	12	21	11	12	23	24	24
RevShare1	59	10	62	25	60	54	54	1	67	64
Core	192	50	201	133	201	137	196	205	207	193

Figure 20. Number of operators' possible alliances in categories Core exists, Revenue share 1 in Core and Revenue share 2 in Core

Figure 20 presents the number of the alliances in which each of the operators could participate as a member. Some deviations can be immediately noticed. First, MTS has only one possible alliance in the revenue share 1 category. In fact, it is an alliance of its own, that is, an alliance of only one member. MTS is incapable of being a member of any alliance within the revenue share 1 scheme.

The reason is the distribution of MTS' subscribers in different countries. Over 73% of MTS' revenue is from Russia and the rest mostly from Ukraine. Russian's export and import figures are not favourable for the model of calculating the roaming revenue distribution among the coalition members. Russian's import is not at the same level as for other big countries and on the other hand its export has been quite significant due to its energy industry. This leads to the situation in which MTS as an alliance member needs to provide discount for its own subscribers when they travel abroad but is not able to gain enough new roaming revenue from the other alliance members. MST would make losses in all alliances within the revenue share scheme 1 and therefore its revenue is not in the core. In the revenue share scheme 2 the situation is different because the re-allocation of the additional revenue by sharing it in proportion to each member's overall revenue would give MTS a positive benefit.

Another significant deviation is the number of Vodafone's alliances. In all categories it is clearly smaller than the mean of other operators. When looking closer, also the number of members in Vodafone's alliances is less than in other operator's alliances. It seems that when there are more members in an alliance where also Vodafone is involved, Vodafone is able to capture more additional revenue but at the same time one or several members are making losses. The explanation is Vodafone's significantly higher level of overall revenue among all other operators and also its very healthy distribution of the business over the most important countries (ref. to Table 3). If there is any additional roaming revenue available in an important country, it is probable that Vodafone is also there to capture its share and thus dilutes the revenue making opportunities of other alliance members. Vodafone often dominates other alliance members with its high market share. On the other hand, because it already has business in many significant countries over the world, Vodafone does not obviously need alliances to complete its distribution channel.

An interesting point is Vodafone's behaviour found in the model and Vodafone's behaviour in its real life alliance, Vodafone Group and Partner Network. In both domains Vodafone behaves differently than its competitors. Vodafone has demonstrated extraordinary favoured partner behaviour within its alliance when pricing roaming services. Vodafone also mainly relies on direct investments and its own brand when expanding the global presence as an opposite to the rest of the operators (ref. to New Strategic Alliances of Mobile Operators in Chapter 2.5).

Revenue share scheme 1 is the most probable alternative to share the benefits of an alliance, because it does not require any additional agreement between alliance members but each member keeps the additional revenue they are able to gain. Thus it is an evident choice when investigating alliances closer from each operator's point of view and the inter-operator impacts.

Table 7. Inter-operator impacts of revenue share scheme 1 alliances

	CM	*VF*	*CU*	*TO2*	*AM*	*DT*	*FT*	*MTS*	*NTT*	*TI*
China Mobile Ltd										
- Number of alliances	59	2	23	10	22	20	19	0	29	27
- Average stability	70 %	100 %	24 %	66 %	76 %	67 %	71 %		68 %	65 %
- Average add. revenue%	0,15 %	0,33 %	0,00 %	0,12 %	0,16 %	0,23 %	0,21 %	0,00 %	0,16 %	0,17 %
Vodafone										
- Number of alliances	2	10	1	1	3	1	2	0	0	1
- Average stability	100 %	100 %	100 %	100 %	100 %	100 %	100 %			100 %
- Average add. revenue%	3,32 %	2,89 %	2,90 %	3,01 %	3,40 %	3,20 %	3,32 %	0,00 %	0,00 %	3,15 %
China Unicom										
- Number of alliances	23	1	62	10	25	24	23	0	31	29
- Average stability	24 %	100 %	72 %	66 %	78 %	74 %	77 %		70 %	69 %
- Average add. revenue%	0,00 %	0,93 %	0,51 %	0,37 %	0,62 %	0,80 %	0,77 %	0,00 %	0,53 %	0,57 %
Telefonica O2										
- Number of alliances	10	1	10	25	6	1	1	0	11	8
- Average stability	66 %	100 %	66 %	86 %	100 %	100 %	100 %		85 %	78 %
- Average add. revenue%	3,88 %	2,11 %	3,55 %	3,11 %	2,86 %	1,83 %	3,14 %	0,00 %	3,61 %	4,00 %
America Movil										
- Number of alliances	22	3	25	6	60	24	24	0	29	25
- Average stability	76 %	100 %	78 %	100 %	91 %	87 %	91 %		89 %	91 %
- Average add. revenue%	0,11 %	0,19 %	0,11 %	0,01 %	0,10 %	0,15 %	0,14 %	0,00 %	0,11 %	0,10 %
Deutsche Telekom										
- Number of alliances	20	1	24	1	24	54	20	0	26	26
- Average stability	67 %	100 %	74 %	100 %	87 %	88 %	99 %		88 %	87 %
- Average add. revenue%	3,91 %	1,37 %	3,61 %	1,55 %	3,58 %	3,30 %	3,02 %	0,00 %	3,62 %	3,77 %
France Telecom										
- Number of alliances	19	2	23	1	24	20	54	0	26	25
- Average stability	71 %	100 %	77 %	100 %	91 %	99 %	90 %		88 %	90 %
- Average add. revenue%	3,58 %	2,51 %	3,36 %	1,44 %	3,30 %	3,24 %	3,13 %	0,00 %	3,45 %	3,56 %
MTS										
- Number of alliances	0	0	0	0	0	0	0	1	0	0
- Average stability								100 %		
- Average add. revenue%	0,00 %	0,00 %	0,00 %	0,00 %	0,00 %	0,00 %	0,00 %	0,00 %	0,00 %	0,00 %
NTT DoCoMo										
- Number of alliances	29	0	31	11	29	26	26	0	67	32
- Average stability	68 %		70 %	85 %	89 %	88 %	88 %		86 %	85 %
- Average add. revenue%	0,46 %	0,00 %	0,46 %	0,37 %	0,45 %	0,60 %	0,54 %	0,00 %	0,43 %	0,49 %
Telecom Italia										
- Number of alliances	27	1	29	8	25	26	25	0	32	64
- Average stability	65 %	100 %	69 %	78 %	91 %	87 %	90 %		85 %	85 %
- Average add. revenue%	1,02 %	0,05 %	1,00 %	0,79 %	0,97 %	1,35 %	1,21 %	0,00 %	1,02 %	0,93 %

Table 7 describes the inter-operator impacts within the revenue share scheme 1 alliances. In the table three different aspects are illustrated: the number of alliances between all operator pairs, the average stability of those alliances and the average of the percentual additional revenue of such alliances. Let us have an example and look at the figures under China Unicom. We can notice that China Unicom has 24 different possible alliances with Deutsche Telekom (column “DT”) and the average stability of those is 88%. They contribute additional revenue to China Unicom on an average 0.80% of its total revenue. It is worth of mentioning that the number of possible alliances and their stability are the same for both alliance members and thus the same figures can be found under the line Deutsche Telekom in column “CU” (China Unicom). This is not the case with the figure of percentual additional revenue. When China Unicom on average gains 0.80% from the alliances with Deutsche Telekom, the respective figure from Deutsche Telekom’s viewpoint is 3.61%. Hence Deutsche Telekom gets more benefit from the alliances with China Unicom than vice versa, when measuring the additional revenue proportional to an operator’s total revenue.

Additional information which can be read in Table 7 is each operator's own statistics. They can be found on the crossing line and column with an operator's name in a small box, e.g. for China Unicom at column "CU". The total number of China Unicom's revenue share scheme 1 alliances is 62, the average stability of those is 72% and on average they contribute to China Unicom 0.51% of its total revenue. Thus we can conclude that China Unicom's alliances with Deutsche Telekom are on average better from China Unicom's viewpoint than its other revenue share scheme 1 alliances.

In the following some remarks are collected from each operator's viewpoint.

China Mobile Ltd

- Preferred partners are Vodafone, Deutsche Telekom and France Telecom.
- Non-preferred partners are China Unicom and MTS.
- There are two possible alliances with Vodafone: 1) Vodafone and China Mobile and 2) Vodafone, China Mobile and America Movil.
- Alliances in which China Mobile and China Unicom are both involved do not provide any benefit for them and they are also quite unstable.

Vodafone

- Preferred partners are America Movil, China Mobile, France Telecom, Deutsche Telekom, Telecom Italia, Telefonica O2 and China Unicom.
- Non-preferred partners are NTT DoCoMo and MTS.
- Alliances are quite profitable from Vodafone's viewpoint (on average 2.89%).
- Most of Vodafone's alliances are alliances with two members. The only three-member alliances are 1) Vodafone, China Mobile and America Movil and 2) Vodafone, France Telecom and America Movil. Thus America Movil is the most flexible for Vodafone. This is because of their complementary assets in the most important countries (ref. Table 3).
- All Vodafone's alliances are 100% stable.

China Unicom

- Preferred partners are Vodafone, Deutsche Telekom and France Telecom and America Movil.

- Non-preferred partners are China Mobile Ltd and MTS.
- The only possible alliance with Vodafone is the two-member alliance of China Unicom and Vodafone (ref. China Mobile and its two possible Vodafone alliances).

Telefonica O2

- Preferred partners are Telecom Italia, China Mobile Ltd, NTT DoCoMo and China Unicom.
- Non-preferred partners are MTS and Deutsche Telekom.
- Alliances are quite profitable from Telefonica O2's viewpoint (on average 3.11%).

America Movil

- Preferred partners are Vodafone, Deutsche Telekom and France Telecom.
- Non-preferred partners are MTS and Telefonica O2.
- America Movil is the only operator that can establish three different alliances with Vodafone.

Deutsche Telekom

- Preferred partners are China Mobile Ltd, Telecom Italia, NTT DoCoMo, China Unicom and America Movil.
- Non-preferred partners are MTS and Vodafone.
- Deutsche Telekom enjoys the best additional revenue on average (3.30%).

France Telecom

- Preferred partners are China Mobile Ltd, Telecom Italia, NTT DoCoMo, China Unicom, America Movil and Deutsche Telekom.
- Non-preferred partners are MTS and Telefonica O2.

- Alliances are quite profitable from France Telecom’s viewpoint (on average 3.13%).

MTS

- MTS is incapable of being a member in any alliance within the revenue share 1 scheme. The reason is the distribution of MTS’ subscribers in different countries (ref. the discussion earlier).

NTT DoCoMo

- Preferred partners are Deutsche Telekom, France Telecom, Telecom Italia, China Mobile, China Unicom and America Movil.

- Non-preferred partners are Vodafone and MTS.

Telecom Italia

- Preferred partners are Deutsche Telekom, France Telecom, NTT DoCoMo, China Mobile Ltd, China Unicom and America Movil.

- Non-preferred partners are MTS and Vodafone.

The total number of revenue share scheme 1 alliances is 145. Among those there are 22 quite unstable alliances (stability 30% or less). In all of those China Mobile Ltd and China Unicom are both involved. Hence we have 123 different very stable alliances of the revenue share scheme 1 in the core. The largest of them is the alliance number 712 with six members: China Unicom, America Movil, Deutsche Telekom, France Telecom, NTT DoCoMo and Telecom Italia. Hence the operators outside of this alliance are China Mobile Ltd, Vodafone, MTS and Telefonica O2.

The stability of this alliance is 100% and it provides its members USD 16.3 billion additional revenue which is 2.1% of the alliance’s total revenue. The benefits of the alliance are distributed quite unevenly among the members, see Table 8.

Table 8. Alliance number 712, revenue share scheme 1

Alliance number 712, revenue share scheme 1		
Operator	***Add. rev. billions USD***	***Add. rev. %***
China Unicom	1,78	1,69 %
America Movil	0,18	0,18 %
Deutsche Telekom	5,95	3,62 %
France Telecom	5,31	3,79 %
NTT DoCoMo	1,47	0,80 %
Telecom Italia	1,57	1,73 %

According to the definition of the core the largest stable alliance provides its members the best solution when considering the potential alliances which can be formed between the members. Hence all sub-coalitions of the alliance number 712 would get less than if they were involved in the grand coalition. This does not mean that some of the members could not get better outcome in other operators' alliances which are not involved with the alliance number 712. For example, the alliance number 63 of China Mobile Ltd, America Movil and Vodafone would provide America Movil USD 186 million which is slightly more that it gets in the alliance 712.

The best alliance (i.e. the biggest additional revenue) which can be formed between the outsiders of the alliance 712, is the alliance number 55. It is an alliance of Vodafone and Telefonica O2. It provides USD 14.4 billion additional revenue, which is 2.75% of the alliance's total revenue. Vodafone would get the lion's share, USD 11.1 billion (3.0%) but also Telefonica O2 would make quite well with its USD 3.3 billion (2.1%).

4.7 Stability indicator and behaviour patterns

In the case study a new concept called stability indicator is introduced. According to the definition of the stability indicator φ, it is the proportion of samples in the core to all simulations. It reflects the overall stability of an alliance. An alliance with $\varphi = 1$ is absolute stable and $\varphi = 0$ indicates respectively absolute instability. Values between *1* and *0* indicate sensitiveness to various changes in the business environment and thus can reflect the long-term instability.

When considering the inter-operator impacts within the revenue share scheme 1 alliances and focusing on the stability indicator figures (Table 7), two exceptional patterns can be identified. These are the behaviour of two Chinese operators and Vodafone's stability figures.

Alliances in which both Chinese operators are involved are unstable, on average $\varphi = 24\%$. When taking into account the alliances in which only one of the Chinese operators is involved, the view is totally different. Alliances in which China Mobile Ltd is a member but not with China Unicom, have $\varphi = 99.2\%$ on average. Similarly China Unicom's alliances without China Mobile's involvement have all $\varphi = 100.0\%$. This demonstrates very clearly that the instability is not due to China Mobile or China Unicom but because of the joined alliance of these two. When both of these operators join an alliance, the alliance becomes unstable regardless of whoever the other members are.

This behaviour pattern of Chinese operators can be explained with their identical market distribution. Both of them have subscribers in China but not in any other country. Hence they are not able to complement their assets in an alliance; instead they compete with each other. This behaviour pattern can be observed in horizontal alliances when the alliance members combine their assets in the same stage of the value chain. Hence this finding can be confirmed by the qualitative research of strategic alliances (ref. 2.3).

All Vodafone’s revenue share scheme 1 alliances are 100% stable. This can be explained to a large extent with Vodafone’s dominance in the market and also with its very healthy distribution of business over the most important countries. The number of members in Vodafone’s alliances is less than in other operators’ alliances. When there are more members in an alliance where also Vodafone is involved, Vodafone is able to capture more additional revenue but at the same time one or several members are making losses. This limits the set-up of Vodafone’s alliances typically to two members; Vodafone and another operator. On the other hand, all the tiny Vodafone alliances are 100% stable. Vodafone’s wide market distribution evidently protects these alliances against changing business circumstances and market turbulence.

Vodafone’s behaviour attested by the model and the stability indicator is typical to the market leaders, which have a dominating position in the market. Usually they form alliances with smaller partners to get access to some specific resources they lack. It is far less typical for dominating companies to form an alliance with partners close to their own size.

These findings demonstrate the value of the stability indicator. If the stability indicator was not included in the model, the exceptional behaviour of these three operators and their alliances would not have been found out. Also the behaviour patterns revealed by the model can be explained with the general findings of the strategic alliance research and hence the evidence about the stability indicator’s applicability gains more strength.

4.8 Analysis of the results

Based on the results of the model the following conclusions can be summarized:

- The core solution limits the number of applicable alliances; all the potential alliances do not fulfil the core requirements and thus are not stable and probable.

- A decreasing number of alliance members increases the probability for the core solution to exist and hence an alliance to be stable.

- Different schemes to allocate an alliance’s revenue among its members limit the number of applicable alliances and have an impact on the alliances’ stability. They also have a decreasing impact on the average size of alliances.

- The “natural” share of revenues, in which all members keep the additional value they are able to gain from the alliance, is more favourable for the stability than the one in which the alliance’s benefits are re-allocated in proportion to each member’s overall revenue.

- Changes in the business environment have an impact on the alliances’ stability. Some alliances are more sensitive to changes in the business environment than others. As an example the alliances in which both Chinese operators are involved are quite unstable whereas all Vodafone’s alliances are 100% stable.

- The size of an alliances members’ market share and the distribution over different countries have an impact on a potential alliance’s stability. Hence some alliance configurations are more probable than others

- Alliances which are based on the complementary assets of the members are typically more stable than the alliances of members with competing and overlapping assets.

- The additional revenue an operator is able to gain is dependent on the operator’s market share and its distribution over countries.

- The stability indicator provides information about the behaviour of operators and their alliances which would not have been otherwise achieved. The behavioural patterns revealed by the stability factor can be explained with the general findings of the strategic alliance research.

The conclusion is that the model is able to demonstrate and to describe the conditions for alliance formation between global mobile operators. The behaviour of each operator can be recognized when they act as a member in diverse alliances. Depending on the operators’ overlapping and complementary assets, an alliance can be totally stable, not stable at all, or the stability can vary according to the changing business factors. Thus the model describes not only the static situation of an alliance formation but also shows the dynamics of the business environment and the long-term stability of the alliance.

5 DISCUSSION

A strategic alliance is an agreement between two or several organizations, typically companies, to implement a shared cooperative strategy. In a strategic alliance partners have some shared objectives which they pursue. At the same time partners usually have their own individual goals. Often the motive to establish a strategic alliance is a major strategic challenge or an opportunity that the partners have met. The reasons to go into an alliance do not need to be the same for all partners. Each of the partners contributes in one or more strategic areas of the alliance. Usually the managerial control and benefits of an alliance are shared by the partners. In a strategic alliance partners remain legally independent.

One of the major dilemmas in strategic alliances is the balance between partner organizations' own individual goals and the shared goals of the strategic alliance. When partners allocate resources for the shared operations of the alliance, they need to simultaneously take care of their own businesses, which are only in their own interest. If resource or some other conflicts occur, it is quite natural that a partner's own business is prioritized over the shared objectives of the alliance. This is even more evident if the question is about some strategically important aspects for the business, e.g. if a partner's and alliance's markets overlap in some areas.

The quality of a strategic alliance has a great importance for the stability of an alliance. Horizontal alliances are likely to be the most demanding ones because the partners of the alliance are also competitors. This is not usually the case in vertical alliances, in which companies agree to use their assets in different stages of the value chain. Partners of a horizontal strategic alliance combine their assets in the same stage of the value chain, e.g. to jointly market their products and services. This combination of cooperation and competition at the same time is challenging and it can cause unsolvable tensions between the partners, as attested by many failures among the horizontal alliances in airline and telecommunications industry.

Even though the starting point for an alliance may be promising, it does not automatically guarantee a long and successful relationship between the partners. Changes in the business environment or the original assumptions that do not hold may change the premises of an alliance and thus the long-term viability of the alliance can be jeopardized. In such a situation trust between alliance partners and other stakeholders, as well as the capabilities of the partners’ upper management are challenged. As the experience from many failed alliances demonstrates, even big alliances which are created with bells and whistles do not last in the changing business circumstances (see High-profile failures in Chapter 2.5).

Game theory provides concepts and tools for mathematical modelling of multi-person decision problems. The game theory in general and especially the theory of cooperative games can be applied to model strategic alliances. From the strategic alliances' viewpoint the theory of cooperative games can contribute from two different directions.

First we can ask if a given alliance is stable or not, that is, do partners have an incentive for sustainable cooperation? The second question is about sharing the benefits of an alliance. What would be a fair and acceptable way to share the additional value of the alliance? The same question can be applied if the alliance is to share some costs, e.g. to build a shared infrastructure. What would be the acceptable way to allocate the cost among the alliance partners?

The stability of an alliance can be assessed with solution concepts that set some requirements for the solution's existence. Often these are concepts which do no give a one-point solution to the allocation question but a set of solutions. Likely the best known and also very useful solution among the subset solution concepts is the core. The requirements of an alliance that can be verified with the core are: individual rationality, efficiency and coalitional rationality. This a minimum set of requirements that an alliance needs to fulfil to be stable. The example of liner shipping alliances in Chapter 3.9 demonstrates how the core can be applied to verify the stability of alliances.

One-point solution concepts can be used to find answers to the benefit or cost allocation questions. The best known concept is probably the Shapley value. It assigns a unique outcome for each game. The Shapley value exists always for finite superadditive transferable utility games and thus it does not contribute to the assessment of the stability. One important application of the Shapley value is cost sharing when the target is to find a fair allocation of costs between the members of an alliance. Two examples in Chapter 3.9, innovation incentives in enterprise networks and cooperative water projects, describe the application of the Shapley value. In the cooperative water projects' example the two approaches, cost allocation and an alliance's stability are combined. Several solution concepts are first used to provide different models to allocate the cost among the partners, Shapley value is among those. The core is then used to assess the stability of these different allocation schemes and hence to rate their value.

In the case study of global mobile operators' horizontal alliances the core is applied in different ways. In the first place, the core is used to categorize all potential alliances into two classes, depending on whether the core exists or not. This grades an alliance's general capability to survive. If the core does not exist, even the basic conditions are not met. The second way to test the potential alliances with the core is to verify the stability with two different schemes for allocating the additional revenue of an alliance. The third way is to test the stability of alliances in the changing business circumstances by combining the Monte Carlo simulation and the core solution concept.

All these three testing methods uncover different aspects about the conditions to establish sustainable cooperation between the partners.

In approximately 59% of alliances the core does not exist at all. All potential alliances do not fulfil the basic requirements of stability and are thus not probable. It is also obvious that the lower number of alliance members, the higher the probability of the alliance to be stable.

Different schemes for allocating an alliance’s revenue among its members further limit the number of stable alliances. The share of the alliances that are not stable when applying revenue allocation schemes 1 and 2 is 86% and 92%, respectively. Also the average size of alliances is reduced. An interesting point is that the “natural” way to allocate revenues, in which all alliance members keep the additional revenue they are able to catch from the alliance (revenue scheme 1), is more favourable for the stability than the other way, in which the alliance’s benefits are re-allocated in proportion to each member's overall revenue (revenue scheme 2).

To measure the long-term stability of an alliance a new concept called stability indicator is introduced in the case study. It is the proportion of samples in the core to all simulations and it reflects the overall stability of an alliance. An alliance can be absolutely stable, absolutely unstable or its stability can change according to the changes in the business environment. The stability indicator reveals two exceptional patterns when considering revenue scheme 1 alliances. These are the behaviour of two Chinese operators and Vodafone’s stability figures. These patterns can also be explained with the general findings of the strategic alliance research.

Revenue scheme 1 alliances in which Chinese operators are both involved are quite unstable. This demonstrates the behaviour of horizontal alliances in which partners operate in same markets and do not have complementing assets. This applies also in general. Alliances, which are based on the complementary assets of the members, are typically more stable than the alliances with competing and overlapping assets between the members.

Vodafone is a very interesting partner. All the revenue scheme 1 alliances of Vodafone are 100% stable. This is explained partly by the small number of Vodafone’s alliances and partly by the exceptionally strong assets of Vodafone. Vodafone’s networks are well spread over the most important countries in terms of markets and it has the biggest revenue. Vodafone clearly often dominates other alliance members with its strong assets. Vodafone’s uniqueness is an interesting parallelism between the real life and the results of the model. In both domains Vodafone behaves differently to its competitors and expresses individual behaviour (see the discussion in Chapter 4.6).

Not only with Vodafone, but also in general the size of alliance members’ market share and its distribution over different countries have an impact on potential alliances’ stability. Hence some alliance configurations are more probable than others.

The findings of the study verify the original hypotheses:

- The theory of cooperative games can be applied to model the global strategic alliances. The game theoretical approach provides a valuable method to model the formation of strategic alliances and the behaviour of the members of an alliance. The solution concepts of the cooperative games theory can be used to assess the different aspects of strategic alliances.

- Modelling strategic alliances as cooperative games contributes to the assessment of a strategic alliance's stability and likelihood. Some of the solution concepts are more applicable to assess the stability of an alliance, while others are suitable to model the allocation of the benefits or costs of the alliance among the alliance's members. The core solution concept can be used to assess the stability of the alliances. Also the benefit or cost allocation schemes' impact on the stability can be estimated with the core. If the minimum stability requirements of the core are not fulfilled, the alliance members do not have an incentive to join the alliance and hence it is most unlikely that the alliance is formed.

- The business dynamics and changes in the business environment can be taken into account when modelling the global strategic alliances with the cooperative games. In the case study a new concept called stability indicator is introduced. The uncertain and changing business parameters have been modelled with a normal distribution. By combining the results of the Monte Carlo simulation and the core solution concept the stability indicator can be produced. It describes an alliance's stability in the changing business circumstances.

- The findings of the computational model of strategic alliances reflect the respective findings made with qualitative methods. In the case study many behavioural patterns have been found which match up with the research of strategic alliances. The challenge to establish horizontal alliances is well demonstrated. This is documented also in the literature. The impact of the partners' overlapping markets to the alliance's stability is clearly negative in the model. On the other hand, the complementing assets contribute to the alliance formation. The experience from the real horizontal strategic alliances coincides with these findings. Also the behaviour patterns revealed by the stability indicator can be explained with the strategic alliance research. Finally, Vodafone's uniqueness is an interesting parallelism between real life and the results of the model.

6 CONCLUSIONS

The scope of this thesis is the application of the cooperative game theory to global strategic alliances. The objective has been to find out how the cooperative game theory can be applied to global strategic alliances. Global strategic alliances have been studied to understand the major questions of the alliance formation. Various concepts of the cooperative game theory have been investigated to find the feasible way to apply them to the modelling of strategic alliances. Three examples from literature were presented in which the cooperative game theory is applied with strategic alliances. Finally a case study is included in which the global mobile operators' alliances are modelled as cooperative games using the core solution concept.

In the study all the hypotheses are verified. The literature examples and also the case study demonstrated that the theory of cooperative games can be applied to model the global strategic alliances. The stability and likelihood of strategic alliances can be assessed by modelling them as cooperative games. With this modelling the stability of strategic alliances can be assessed in the alliance formation. Also the business dynamics and changes in the business environment can be taken into account to understand the alliance's long-term vitality. The case study has also demonstrated that the findings of the computational model of the strategic alliances reflect the respective findings made with qualitative methods.

In the case study a new computational method is introduced to estimate an alliance's long-term stability in the changing business circumstances. In this method the uncertain and changing business parameters are modelled with applicable probability distributions. By combining the Monte Carlo simulation and the core solution concept of the cooperative game theory, a measure called stability indicator can be calculated. It reflects the stability of an alliance in the changing business environment.

The question which follows is whether the modelling based on the cooperative games or game theory in general can be applied in the practical business decision making. Would it be possible to build up a model that could help business managers in the strategic decisions concerning strategic alliances? Could mathematics help make critical business decisions?

It is evident that the elementary concepts of cooperative games provide a framework for strategic alliances and make the causalities in the alliance formation visible and hence more understandable. Also the partners' behaviour predicted by the cooperative games could help to foresee the potential problems in an analytical way. Also the better understanding on the conditions of the long-term stability would be very useful for decision making.

Creating computational models for practical business questions is a much more challenging task. At least it can be argued that the real-life decision making situations are too complex and thus mathematical models do not reach them, or that model building and estimating the parameters is too challenging and requires too much time and effort. A counterargument is that the basic conditions for an alliance’s stability must be understood before going into the alliance formation. If it is not possible to estimate the essential parameters to create a model, is it possible to start an alliance with big uncertainties? These questions could help to assess the risk level of decision making.

From the further research’s point of view the stability indicator is an area which could provide many interesting questions to be studied. What are the business parameters that can be modelled with the stability indicator? What results can be achieved by investigating real strategic alliances from different business domains? Can the findings of the case study be verified with additional research? Could the stability indicator be used as a practical tool for business management?

REFERENCES

Ambjørnsen, T. & Wasenden, O.C. 2005, *Competition in and regulation of the wholesale market for international roaming,* Telenor R&I, Norway.

Aumann, R.J. 1985, "What Is Game Theory Trying to Accomplish?" in *Frontiers of Economics,* eds. K.J. Arrow & S. Honkapohja, Blackwell, Oxford, pp. 28-76.

Aumann, R.J. & Maschler, M. 1964, "The bargaining set for cooperative games" in *Advances in Game Theory,* eds. M. Dresher, R.J. Aumann, L.S. Shapley & A.W. Tucker, Princeton University Press, Princeton.

Aumann, R.J. & Shapley, L.S. 1974, *Values of non-atomic games,* Princeton University Press, Princeton, N.J.

Billera, L.J., Heath, D.C. & Raanan, J. 1978, "Internal Telephone Billing Rates - A Novel Application of Non-Atomic Game Theory", *Operations Research,* vol. 26, no. 6, November-December, pp. 296-265.

Bishop, C.M. 1995, *Neural networks for pattern recognition,* Clarendon, Oxford.

Bleeke, J. & Ernst, D. 1993, *Collaborating to Compete: Using Strategic Alliances and Acquisitions in the Global Marketplace,* Wiley, New York.

Bondareva, O.N. 1963, "Some applications of linear programming methods to the theory of cooperative games", *Problemi Kibernitiki,* vol. 10, pp. 119-139.

Brânzei, R., Dimitrov, D. & Tijs, S. 2005, *Models in cooperative game theory - Crisp, fuzzy, and multi-choice games,* Springer, Berlin.

Brealey, R.A. & Myers, S.C. 2003, *Principles of corporate finance,* McGraw-Hill Irwin, Boston.

Carter, M. 1993, "Cooperative Games" in *Economic and financial modeling with Mathematica*, ed. H.R. Varian, Springer-Verlag, New York.

Chan-Olmsted, S. & Jamison, M. 2001, "Rivalry through Alliances: Competitive Strategy in the Global Telecommunications Market", *European Management Journal,* vol. 19, no. 3, pp. 317.

Child, J., Tallman, S.B. & Faulkner, D. 2005, *Cooperative strategy - managing alliances, networks and joint ventures,* Second edn, Oxford University Press, Oxford.

CIA 2006, January-last update, *The World Factbook* [Homepage of Central Intelligence Agency], [Online]. Available: https://www.cia.gov/library/publications/the-world-factbook/ [2006, January 13] .

Claude, N. 2006, 26 May 2006-last update, *Swedish election race hots up* [Homepage of Sweden.se], [Online]. Available: http://www.sweden.se/templates/cs/Article____14663.aspx [2006, September 3] .

Colombo, M.G. & Garrone, P. 1998, "Common carriers' entry into multimedia services", *Information Economics and Policy; Amsterdam,* vol. 10, no. 1, pp. 77.

Curwen, P. & Whalley, J. 2004, *Alliances and Joint Ventures in the Telecommunications Industry: The Case of the Mobile and Vendor Sector*s, ITS 2004 - 15th Biennial Conference of International Telecommunications Society, Berlin, Germany.

Dahlgren, K. 2006, "Regulation of Roaming Services", *Innovation in Telecommunicatio*n, ed. S. Luukkainen, Helsinki University of Technology; Telecommunication Software and Multimedia Laboratory, Espoo, pp. 49.

Davis, M.D. 1997, *Game theory - A nontechnical introduction,* Dover Publications, Mineola, N.Y.

Davis, M.D. & Maschler, M. 1965, "The Kernel of a Cooperative Game", *Naval research logistics quarterly,* vol. 12.

Doz, Y.L. & Hamel, G. 1998, *Alliance advantage the art of creating value through partnering,* Harvard Business School Press, Boston (MA).

Duckworth, M. 2005, *Maximising ARPU Price Elasticit*y, Coleago Consulting Ltd., Blackpool.

Elliot, D. 2004, *The importance of price elasticities in the regulation of mobile call terminatio*n, Frontier Economics Ltd., London.

Gibbons, R. 1992, *A primer in game theory,* Harvester Wheatsheaf, New York.

Gil, M.J.A. & de la Fe, Pedro Gonzalez 1999, "Strategic Alliances, Organisational Learning and New Product Development: The Cases of Rover and Seat", *R & D Management; Oxford,* vol. 29, no. 4, pp. 391.

Gillies, D.B. 1959, "Solutions to General Non-Zero-Sum Games" in *Contributions to the theory of game, Vol.* 4, eds. R.J. Aumann, A.W. Tucker & R.D. Luce, Princeton University Press, Princeton.

Greenstein, S. 1993, "Markets, standards, and the information infrastructure", *IEEE Micro,* vol. 1993; 13, no. 6, pp. 36-51.

Gruber, H. 2005, *The economics of mobile telecommunications,* Cambridge University Press, Cambridge.

GSMA 2009, January-last update, *GSM Roaming* [Homepage of GSM Association], [Online]. Available: http://gsmworld.com/technology/roaming/ [2009, 11 January] .

Harsanyi, J.C. 1963, "A simplified bargaining model for the n-person cooperative game", *International economic review,* vol. 4, no. 2, pp. 194-220.

Harsanyi, J.C. & Selten, R. 1988, *A general theory of equilibrium selection in games,* MIT Press, Cambridge, Mass.

Hill, C.W.L., Hwang, P. & Kim, W.C. 1990, "An Eclectic Theory of the Choice of International Entry Mode", *Strategic Management Journal,* vol. 11, no. 2, pp. 117-128.

Hitt, M.A., Ireland, R.D. & Hoskisson, R.E. 2001, *Strategic Management: Competitiveness and Globalization Cases*, 4th edn, South-Western, Mason, OH.

Husted, S. & Melvin, M. 2004, *International Economics,* 6th edn, Addison Wesley, Boston.

Informa Telecoms & Media 2006, *World Cellular Investors - Quarterly Datasheet June 2006*, Informa UK Limited, London.

ITU 2006, *World Telecommunication/ICT Indicators*, International Telecommunication Union, Geneva.

Jamison, M.A. 1999, *Business Imperatives. The new Global Telecommunications Industry & Consumers*, Penn State University, Institute for Information Policy, College Park, PA.

Jamison, M.A. 2000, *Industry structure and pricing : The New Rivalry in Infrastructure*, Kluwer Academic Publishers, Boston.

Jarimo, T. 2004, *Innovation incentives in enterprise networks - A game theoretic approach*, VTT, Espoo.

Joshi, M.P., Kashlak, R.J. & Sherman, H.D. 1998, "How alliances are reshaping telecommunications", *Long Range Planning; London,* vol. 31, no. 4, pp. 542.

Kelly, A. 2003, *Decision making using game theory - An introduction for managers,* Cambridge University Press, Cambridge, UK.

Kogut, B. 1988, "Joint Ventures: Theoretical and Empirical Perspectives", *Strategic Management Journal,* vol. 9, no. 4, pp. 319-332.

Kotler, P. & Keller, K.L. 2006, *Marketing Management*, 12th edn, Pearson Prentice Hall, Upper Saddle River, NJ.

Kungl. Vetenskapsakademien, S. 2005, "Robert Aumann' and Thomas Schelling's Contribution to Game Theory: Analyses of Conflict Cooperation", *Nobel Foundation, Prize in Economics documents,* [Online], vol. 2005, , pp. 2005. Available from: http://nobelprize.org/nobel_prizes/economics/laureates/2005/ecoadv05.pdf. [August 1, 2008].

Littlechild, S.C. & Owen, G. 1973, "A simple expression for the Shapley value in a special case", *Management science,* vol. 20, no. 3, pp. 370-372.

Lucas, W.F. 1969, "The Proof That a Game May Not Have a Solution", *Transactions of the American Mathematical Society,* vol. 137, pp. 219-229.

Maschler, M., Peleg, B. & Shapley, L.S. 1979, "Geometric Properties of the Kernel, Nucleolus, and Related Solution Concepts", *Mathematics of Operations Research,* vol. 4, no. 4, pp. 303-338.

Meinhardt, H.I. 2005, *Graphical Extensions of the Mathematica Package TuGame*s, University of Karlsruhe, Institute of Statistics and Economic Theory, Karlsruhe.

Nalebuff, B.J. & Brandenburger, A.M. 1996, *Co-opetition - A Revolutionary Mindset that Combines Competition and Cooperation,* HarperCollins, London.

Nash, J. 1950, "Equilibrium points in n-Person Games", *Proceedings of the National Academy of Sciences of the United States of America,* vol. 36, no. 1, pp. 48-49.

NTT DoCoMo 2006, April 24, 2006-last updat*e, Seven Leading Asian Mobile Operators Form Alliance for Global Roaming and Corporate Mobile Service*s [Homepage of NTT DoCoMo Inc.], [Online]. Available: http://www.nttdocomo.com/pr/2006/001261.html [2007, August, 2] .

Osborne, M.J. 1994, *A course in game theory,* MIT Press, Cambridge (MA).

Parrachino, I., Dinar, A. & Patrone, F. 2006, "Cooperative Game Theory and its Application to Natural, Environmental, and Water Resource Issues: 3. Application to Water Resources", *World Bank Policy Research Working Paper Series,* vol. November, 2006, no. WPS4074.

Parrachino, I., Zara, S. & Patrone, F. 2006, "Cooperative Game Theory and its Application to Natural, Environmental, and Water Resource Issues: 1. Basic Theory", *World Bank Policy Research Working Paper Series,* vol. November, 2006, no. WPS4072.

Patterson, G. 2005, *Mobile Industry Outlook 200*6, T&F Informa UK Limited, UK.

Peleg, B. & Sudhölter, P. 2003, *Introduction to the theory of cooperative games,* Kluwer Academic, Boston, Mass.

Reid, R., Sims, G. & Gibney, O. 2005, *Global Mobile Forecasts to 2010 - 5th Editio*n, T&F Informa UK Limited, London.

Rieck, O., Yinzhi, C., Habib, H. & Junzheng, X. 2005, *Alliances of Networks and Networks of Alliances: International Cooperation in Mobile Telecommunication*s, ITS 2005 - 16th European Regional Conference of International Telecommunications Society, Porto, Portugal.

Roberts, D. & Waters, R. 2001, *Concert sounded off key almost from opening bars: The alliance of AT&T and BT hit trouble early on, say Richard Waters and Dan Robert*s, 16 October edn, Financial Times, London.

Schmeidler, 1969, "The nucleolus of a characteristic function game", *SIAM journal on applied mathematics,* [Online], vol. 17, no. 6, pp. 1163.

Shapley, L.S. 1967, "On balanced sets an cores", *Naval Research logistic quarterly,* vol. 14, no. 4, pp. 453-460.

Shapley, L.S. & Shubik, M. 1954, "A method for evaluating the distribution of power in a committee", *The American political science review,* vol. 48, no. 3, pp. 787-792.

Shapley, L.S. 1953, "A Value for n-Person Games" in *Contributions to the theory of games, Vol.* 2, eds. R.J. Aumann, A.W. Tucker & R.D. Luce, Princeton University Press, Princeton.

Shapley, L.S. & Shubik, M. 1966, "Qusi-cores in a monetary economy with nonconvex preferences", *Econometrica,* vol. 34, no. 4, pp. 805-827.

Siegfried, T. 2006, *A beautiful math - John Nash, game theory, and the modern quest for a code of nature,* Joseph Henry Press, Washington, D.C.

Slowinski, G., Seelig, G. & Hull, F. 1996, "Managing technology-based strategic alliances between large and small firms", *S.A.M.Advanced Management Journal; Cincinnati,* vol. 61, no. 2, pp. 42.

Song, D. & Panayides, P.M. 2002, "A conceptual application of cooperative game theory to liner shipping strategic alliances", *Maritime policy and management,* vol. 29, no. 3, pp. 285-301.

Tijs, S. 1981, "Bounds for the core and the t-value ", *Game theory and mathematical economics*, eds. O. Moeschlin & D. Pallaschke, North-Holland, Amsterdam, 1981, pp. 123.

Todeva, E. & Knoke, D. 2005, "Strategic Alliances and Models of Collaboration", *Management decision,* vol. 43, no. 1, pp. 123.

Trott, P. 2002, *Innovation management and new product development,* Pearson Education, Harlow.

Varadarajan, P.R. & Cunningham, M.H. 1995, "Strategic Alliances: A Synthesis of Conceptual Foundations", *Journal of the Academy of Marketing Science,* vol. 23, no. 4, pp. 282.

Vesa, J. 2005, *Mobile Services in the Networked Economy,* IRM Press, Hershey PA.

Von Neumann, J. & Morgenstern, O. 1944, *Theory of games and economic behaviour,* John Wiley and Sons, New York.

Warren, S. & Tejada, C. 1999, "DuPont Plans `Tracking' Stock for Life Sciences", *Wall Street Journal; Eastern edition; New York, N.Y.,* , no. Mar 11, pp. A3.

Wolfram Research 1999, *Mathematica 4.0 Standard Add-on Packages,* Wolfram Media, Champaign, IL.

Wolfram, S. 1999, *The mathematica book,* Wolfram Media, Champaign, IL.

WTO 2006, *International trade statistics 2006*, World Trade Organization, Geneva.

Wysocki Jr, B. 2000, "Cyberspace Inc.: More Companies Cut Risk by Collaborating With Their `Enemies' --- Many Internet Operations Co-Invest in Bid to Ensure At Least a Slice of Success --- The Real Giants Just Devour", *Wall Street Journal; Eastern edition; New York, N.Y.,* , no. Jan 31, pp. A1.

Yoshino, M.Y. & Rangan, U.S. 1995, *Strategic Alliances: an Entrepreneurial Approach to Globalization,* Harvard Business School Press, Boston, Mass.

APPENDIX 1. RESULTS

Data included

ID	Alliance ID (1-1023);
xx	Alliance members;
	AM America Movil
	CM China Mobil Ltd
	CU China Unicom
	DT Deutsche Unicom
	FT France Telecom
	MTS Mobile TeleSystems
	NTT Nippon Telegraph and Telephone
	TI Telecom Italia
	TO2 Telefonica O2
	VF Vodafone
	Alliance members;
R	Total revenue of the alliance (millions USD);
ΔR	Additional revenue generated by the alliance (millions USD);
$\Delta R\%$	The proportion of the additional revenue to the total revenue (%);
φ_{Core}	Core stability indicator, describes the existence of the core (%);
φ_{RS1}	Revenue share 1 stability indicator which describes if the revenue share scheme 1 is in the core (%);
φ_{RS2}	Revenue share 2 stability indicator which describes if the revenue share scheme 2 is in the core (%);
ΔR_{RS1}	Additional revenue of each member according to revenue share scheme 1 (million USD);
ΔR_{RS2}	Additional revenue of each member according to revenue share scheme 2 (million USD);
$\Delta R\%_{RS1}$	The proportion of the additional revenue of each member to the member's total revenue; revenue share scheme 1 (%)
$\Delta R\%_{RS2}$	The proportion of the additional revenue of each member to the member's total revenue; revenue share scheme 2 (%)

Results of the model (all alliances not presented due to the limited space)

ID:0001 AM

R:101 982m$ ΔR: 0m$ ΔR%:0.00% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	0									
ΔR_{RS2}	0									
$\Delta R\%_{RS1}$	0.00%									
$\Delta R\%_{RS2}$	0.00%									

ID:0002 CM

R:154 702m$ ΔR: 0m$ ΔR%:0.00% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}		0								
ΔR_{RS2}		0								
$\Delta R\%_{RS1}$		0.00%								
$\Delta R\%_{RS2}$		0.00%								

ID:0003 CU

R:105 328m$ ΔR: 0m$ ΔR%:0.00% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}			0							
ΔR_{RS2}			0							
$\Delta R\%_{RS1}$			0.00%							
$\Delta R\%_{RS2}$			0.00%							

ID:0004 DT

R:164 131m$ ΔR: 0m$ ΔR%:0.00% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}				0						
ΔR_{RS2}				0						
$\Delta R\%_{RS1}$				0.00%						
$\Delta R\%_{RS2}$				0.00%						

ID:0005 FT

R:140 057m$ ΔR: 0m$ ΔR%:0.00% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}					0					
ΔR_{RS2}					0					
$\Delta R\%_{RS1}$					0.00%					
$\Delta R\%_{RS2}$					0.00%					

ID:0006 MTS

R: 56 833m$ ΔR: 0m$ ΔR%:0.00% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}						0				
ΔR_{RS2}						0				
$\Delta R\%_{RS1}$						0.00%				
$\Delta R\%_{RS2}$						0.00%				

ID:0007 NTT

R:182 162m$ ΔR: 0m$ ΔR%:0.00% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}							0			
ΔR_{RS2}							0			
$\Delta R\%_{RS1}$							0.00%			
$\Delta R\%_{RS2}$							0.00%			

ID:0008 TI

R: 90 735m\$ ΔR: 0m\$ ΔR%:0.00% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}								0		
ΔR_{RS2}								0		
$\Delta R\%_{RS1}$								0.00%		
$\Delta R\%_{RS2}$								0.00%		

ID:0009 TO2

R:154 004m\$ ΔR: 0m\$ ΔR%:0.00% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}									0	
ΔR_{RS2}									0	
$\Delta R\%_{RS1}$									0%	
$\Delta R\%_{RS2}$									0%	

ID:0010 VF

R:368 414m\$ ΔR: 0m\$ ΔR%:0.00% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}										0
ΔR_{RS2}										0
$\Delta R\%_{RS1}$										0.00%
$\Delta R\%_{RS2}$										0.00%

ID:0019 AM, VF

R:470 396m\$ ΔR:12 384m\$ ΔR%:2.63% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	159									12 225
ΔR_{RS2}	2 685									9 699
$\Delta R\%_{RS1}$	0.16%									3.32%
$\Delta R\%_{RS2}$	2.63%									2.63%

ID:0023 CM, MTS

R:318 834m\$ ΔR: 5 298m\$ ΔR%:1.66% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}		69				-4				
ΔR_{RS2}		48				18				
$\Delta R\%_{RS1}$		0.04%				-0.01				
$\Delta R\%_{RS2}$		0.03%				0.03%				

ID:0055 TO2, VF

R:522 418m\$ ΔR:14 352m\$ ΔR%:2.75% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}:100%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}									3 253	11 099
ΔR_{RS2}									4 231	10 121
$\Delta R\%_{RS1}$									2.11%	3.01%
$\Delta R\%_{RS2}$									2.75%	2.75%

ID:0063 AM, CM, VF

R:625 098m\$ ΔR:13 429m\$ ΔR%:2.15% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	186	532								12 711
ΔR_{RS2}	2 191	3 323								7 915
$\Delta R\%_{RS1}$	0.18%	0.34%								3.45%
$\Delta R\%_{RS2}$	2.15%	2.15%								2.15%

ID:0093 CM, CU, FT

R:400 087m\$ ΔR: 4 217m\$ ΔR%:1.05% φ_{Core}:100% φ_{RS1}: 26% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}		-1	-1		4 219					
ΔR_{RS2}		1 632	1 110		1 476					
$\Delta R\%_{RS1}$		0.00%	0.00%		3.01%					
$\Delta R\%_{RS2}$		1.05%	1.05%		1.05%					

ID:0182 AM, CM, CU, VF

R:730 426m\$ ΔR:13 957m\$ ΔR%:1.91% φ_{Core}:100% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	199	-37	-19							13 813
ΔR_{RS2}	1 949	2 956	2 013							7 039
$\Delta R\%_{RS1}$	0.20%	-0.02%	-0.02%							3.75%
$\Delta R\%_{RS2}$	1.91%	1.91%	1.91%							1.91%

ID:0229 AM, DT, FT, VF

R:774 584m\$ ΔR:15 833m\$ ΔR%:2.04% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	335			1 291	3 221					10 986
ΔR_{RS2}	2 085			3 355	2 863					7 531
$\Delta R\%_{RS1}$	0.33%			0.79%	2.30%					2.98%
$\Delta R\%_{RS2}$	2.04%			2.04%	2.04%					2.04%

ID:0586 CU, DT, FT, NTT, TI

R:682 413m\$ ΔR:15 257m\$ ΔR%:2.24% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}			1 677	5 574	5 018		1 388	1 601		
ΔR_{RS2}			2 355	3 669	3 131		4 073	2 029		
$\Delta R\%_{RS1}$			1.59%	3.40%	3.58%		0.76%	1.76%		
$\Delta R\%_{RS2}$			2.24%	2.24%	2.24%		2.24%	2.24%		

ID:0658 AM, CM, CU, FT, NTT, TO2

R:838 235m\$ ΔR:11 761m\$ ΔR%:1.40% φ_{Core}:100% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	26	-3	-2		3 289		1 234		7 217	
ΔR_{RS2}	1 431	2 171	1 478		1 965		2 556		2 161	
$\Delta R\%_{RS1}$	0.03%	0.00%	0.00%		2.35%		0.68%		4.69%	
$\Delta R\%_{RS2}$	1.40%	1.40%	1.40%		1.40%		1.40%		1.40%	

ID:0712 AM, CU, DT, FT, NTT, TI

R:784 395m\$ ΔR:16 256m\$ ΔR%:2.07% φ_{Core}:100% φ_{RS1}:100% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	180		1 775	5 948	5 313		1 466	1 574		
ΔR_{RS2}	2 114		2 183	3 402	2 903		3 775	1 880		
$\Delta R\%_{RS1}$	0.18%		1.69%	3.62%	3.79%		0.80%	1.73%		
$\Delta R\%_{RS2}$	2.07%		2.07%	2.07%	2.07%		2.07%	2.07%		

ID:0938 CM, CU, DT, FT, NTT, TI, TO2

R:991 119m\$ ΔR:13 649m\$ ΔR%:1.38% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}		-7	-3	3 617	2 529		1 930	2 222	3 360	
ΔR_{RS2}		2 130	1 450	2 260	1 929		2 509	1 249	2 121	
$\Delta R\%_{RS1}$		0.00%	0.00%	2.20%	1.81%		1.06%	2.45%	2.18%	
$\Delta R\%_{RS2}$		1.38%	1.38%	1.38%	1.38%		1.38%	1.38%	1.38%	

ID:0982 AM, CM, CU, DT, NTT, TI, TO2, VF

R:1 321 458m\$ ΔR: 7 286m\$ ΔR%:0.55% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	-50	-87	-44	955			-67	-142	2 118	4 604
ΔR_{RS2}	562	853	581	905			1 004	500	849	2 031
$\Delta R\%_{RS1}$	-0.05	-0.06	-0.04	0.58%			-0.04	-0.16	1.38%	1.25%
$\Delta R\%_{RS2}$	0.55%	0.55%	0.55%	0.55%			0.55%	0.55%	0.55%	0.55%

ID:1013 AM, CM, CU, DT, FT, MTS, NTT, TI, TO2

R:1 149 934m\$ ΔR:13 630m\$ ΔR%:1.19% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	-43	-7	-3	4 054	2 792	-35	2 123	2 362	2 388	
ΔR_{RS2}	1 209	1 834	1 248	1 945	1 660	674	2 159	1 076	1 825	
$\Delta R\%_{RS1}$	-0.04%	0.00%	0.00%	2.47%	1.99%	-0.06%	1.17%	2.60%	1.55%	
$\Delta R\%_{RS2}$	1.19%	1.19%	1.19%	1.19%	1.19%	1.19%	1.19%	1.19%	1.19%	

ID:1014 AM, CM, CU, DT, FT, MTS, NTT, TI, VF

R:1 364 344m\$ ΔR: 8 286m\$ ΔR%:0.61% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	324	-93	-47	1 830	4 357	-44	-71	-146		2 175
ΔR_{RS2}	619	939	640	997	851	345	1 106	551		2 237
$\Delta R\%_{RS1}$	0.32%	-0.06%	-0.04%	1.12%	3.11%	-0.08%	-0.04%	-0.16%		0.59%
$\Delta R\%_{RS2}$	0.61%	0.61%	0.61%	0.61%	0.61%	0.61%	0.61%	0.61%		0.61%

ID:1015 AM, CM, CU, DT, FT, MTS, NTT, TO2, VF

R:1 472 613m\$ ΔR: 4 067m\$ ΔR%:0.28% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	60	-100	-50	-439	-302	-47	-77		-298	5 320
ΔR_{RS2}	291	441	300	468	399	162	519		439	1 050
$\Delta R\%_{RS1}$	0.06%	-0.06%	-0.05%	-0.27%	-0.22%	-0.08%	-0.04%		-0.19%	1.44%
$\Delta R\%_{RS2}$	0.28%	0.28%	0.28%	0.28%	0.28%	0.28%	0.28%		0.28%	0.28%

ID:1016 AM, CM, CU, DT, FT, MTS, TI, TO2, VF

R:1 336 185m\$ ΔR: 323m\$ ΔR%:0.02% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	-59	-103	-52	-454	-312	-49		-169	-312	1 833
ΔR_{RS2}	25	37	25	40	34	14		22	37	89
$\Delta R\%_{RS1}$	-0.06%	-0.07%	-0.05%	-0.28%	-0.22%	-0.09%		-0.19%	-0.20%	0.50%
$\Delta R\%_{RS2}$	0.02%	0.02%	0.02%	0.02%	0.02%	0.02%		0.02%	0.02%	0.02%

ID:1017 AM, CM, CU, DT, FT, NTT, TI, TO2, VF

R:1 461 515m\$ ΔR:-2 169m\$ ΔR%:-0.15% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	-59	-104	-52	-455	-313		-79	-169	-312	-626
ΔR_{RS2}	-151	-230	-156	-244	-208		-270	-135	-229	-547
$\Delta R\%_{RS1}$	-0.06%	-0.07%	-0.05%	-0.28%	-0.22%		-0.04%	-0.19%	-0.20%	-0.17%
$\Delta R\%_{RS2}$	-0.15%	-0.15%	-0.15%	-0.15%	-0.15%		-0.15%	-0.15%	-0.15%	-0.15%

ID:1018 AM, CM, CU, DT, MTS, NTT, TI, TO2, VF

R:1 378 291m\$ ΔR: 7 588m\$ ΔR%:0.55% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	-52	-91	-46	1 002		-43	-70	-149	2 222	4 815
ΔR_{RS2}	561	852	580	904		313	1 003	500	848	2 028
$\Delta R\%_{RS1}$	-0.05%	-0.06%	-0.04%	0.61%		-0.08%	-0.04%	-0.16%	1.44%	1.31%
$\Delta R\%_{RS2}$	0.55%	0.55%	0.55%	0.55%		0.55%	0.55%	0.55%	0.55%	0.55%

ID:1019 AM, CM, CU, FT, MTS, NTT, TI, TO2, VF

R:1 354 216m$ ΔR: 7 933m$ ΔR%:0.59% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	-49	-84	-42		1 380	-40	-65	-137	2 154	4 814
ΔR_{RS2}	597	906	617		820	333	1 067	531	902	2 158
$\Delta R\%_{RS1}$	-0.05%	-0.05%	-0.04%		0.99%	-0.07%	-0.04%	-0.15%	1.40%	1.31%
$\Delta R\%_{RS2}$	0.59%	0.59%	0.59%		0.59%	0.59%	0.59%	0.59%	0.59%	0.59%

ID:1020 AM, CM, DT, FT, MTS, NTT, TI, TO2, VF

R:1 413 020m$ ΔR: -469m$ ΔR%:-0.03% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	-60	1 534		-458	-314	-49	-80	-170	-314	-588
ΔR_{RS2}	-34	-51		-54	-46	-19	-60	-30	-51	-122
$\Delta R\%_{RS1}$	-0.06%	0.99%		-0.28%	-0.22%	-0.09%	-0.04%	-0.19%	-0.20%	-0.15%
$\Delta R\%_{RS2}$	-0.03%	-0.03%		-0.03%	-0.03%	-0.03%	-0.03%	-0.03%	-0.03%	-0.03%

ID:1021 AM, CU, DT, FT, MTS, NTT, TI, TO2, VF

R:1 363 645m$ ΔR: 1 329m$ ΔR%:0.10% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	-58		3 069	-433	-305	-47	-77	-164	-304	-340
ΔR_{RS2}	99		103	160	137	55	178	88	150	359
$\Delta R\%_{RS1}$	-0.06%		2.91%	-0.27%	-0.22%	-0.08%	-0.04%	-0.18%	-0.20%	-0.09%
$\Delta R\%_{RS2}$	0.10%		0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%

ID:1022 CM, CU, DT, FT, MTS, NTT, TI, TO2, VF

R:1 416 366m$ ΔR: -196m$ ΔR%:-0.01% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}		-105	-53	-459	-314	-49	-80	-30	1 526	-632
ΔR_{RS2}		-21	-15	-23	-19	-8	-25	-13	-21	-51
$\Delta R\%_{RS1}$		-0.07%	-0.05%	-0.28%	-0.22%	-0.09%	-0.04%	-0.03%	0.99%	-0.17%
$\Delta R\%_{RS2}$		-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%	-0.01%

ID:1023 AM, CM, CU, DT, FT, MTS, NTT, TI, TO2, VF

R:1 518 348m$ ΔR:-2 302m$ ΔR%:-0.15% φ_{Core}: 0% φ_{RS1}: 0% φ_{RS2}: 0%

	AM	CM	CU	DT	FT	MTS	NTT	TI	TO2	VF
ΔR_{RS1}	-61	-107	-54	-473	-324	-50	-82	-176	-324	-650
ΔR_{RS2}	-155	-235	-160	-249	-212	-86	-276	-138	-234	-559
$\Delta R\%_{RS1}$	-0.06%	-0.07%	-0.05%	-0.29%	-0.23%	-0.09%	-0.04%	-0.19%	-0.21%	-0.18%
$\Delta R\%_{RS2}$	-0.15%	-0.15%	-0.15%	-0.15%	-0.15%	-0.15%	-0.15%	-0.15%	-0.15%	-0.15%

APPENDIX 2. MATHEMATICA CODE

```
(* Beginning of the code *************************************)

(************************************************************)
(************************************************************)
(* Strategic Alliances of Global Mobile Operators          *)
(* Computational model                                     *)
(*                                                         *)
(* Tapio Savunen                                           *)
(* 2009                                                    *)
(************************************************************)
(************************************************************)


(************************************************************)
(* NOTEBOOK INITIALIZATIONS                                *)
(************************************************************)


(* Loading Mathematica's own packages *)

Remove["Global`*"];
<<Statistics`DataManipulation`
<<Graphics`Graphics`
<<Graphics`MultipleListPlot`
<<LinearAlgebra`MatrixManipulation`
<<Graphics`Graphics3D`
<<Statistics`DescriptiveStatistics`
<<Graphics`MultipleListPlot`
<<Statistics`ConfidenceIntervals`
<<Statistics`ContinuousDistributions`

(*Loading packages TuGames and CooperativeGames *)

<<coop`CooperativeGames`;
Off[ConstrainedMin::deprec]
<<TuGames`TuGames`
====================================================
Loading Package 'TuGames' for Windows
====================================================
TuGames V1.1 by Holger I. Meinhardt
Release Date: 27.07.2005
Program runs under Mathematica Version 3.0 or later
====================================================
====================================================
Package 'TuGames' loaded
====================================================


(************************************************************)
(* INITIALIAZATION OF DATA STRUCTURES                      *)
(************************************************************)


(****** Reading input data ******)

d = Import["oper_datat.txt","Table" ];
```

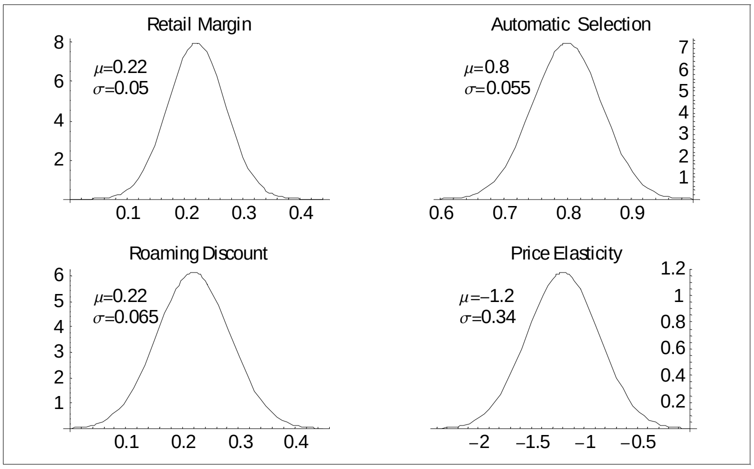
```
(* Structure of the operator data

- continuos lines from higher to lower hierarchical level:
   1) Mobile operators in alphabetical order
      2) Number of 1000s subscribers
      2) Revenue of 1000 USD
      2) Share of home roaming revenue (< 1)
      2) Share of visit roaming revenue (< 1)
      2) Share of GSM/WCDMA subscribers
         3) Year; 2005-2010
- each figure in lines represents one country

an example:
- lines 1-30 are for America Movil
- lines 1-6: Number of 1000s subscribers, years 2005-2010
- lines 7-12:  Revenue of 1000 USD, years 2005-2010
- lines 13-18: Share of home roaming revenue, years 2005-2010
- lines 19-24: Share of visit roaming revenue, years 2005-2010
- lines 25-30: Share of GSM/WCDMA subscribers, years 2005-2010
- each line has 91 figures, each fifure represents one country

length should be 10*5*6 = 300; 10:operators, 5:data types, 6:years *)

Length[d]

Out[]= 300

(****** Fixed parameters ******)

(* Internal discount rate *)
internrate = 0.12; (* 12% *)

(* Number of countries, years, operators *)
countries = Dimensions[d][[2]]

Out[]= 90

years = 6; operators = 10; cycle = 30;


(****** Operator data structures for computation ******)

(* Mobile Operator Data lists; structure of each list
1) operator
2) year
3) country  *)

(* Number of 1000s subscribers *)
subs = Table[d[[(i-1)*cycle+j]],{i,operators},{j,years}];

(* Revenue of 1000s USD *)
revs = Table[d[[(i-1)*cycle+j+6]],{i,operators},{j,years}];

(* Share of home roaming revenue *)
homeshare = Table[d[[(i-1)*cycle+j+12]],{i,operators},{j,years}];

(* Share of visit roaming revenue *)
visitshare = Table[d[[(i-1)*cycle+j+18]],{i,operators},{j,years}];

(* Share of GSM/WCDMA subscribers *)
gsmshare = Table[d[[(i-1)*cycle+j+24]],{i,operators},{j,years}];
```

```
Dimensions[subs]     (* all lists have same dimensions *)

Out[]= {10,6,90}


(* Countries' yearly visit share *)
(* structure: 1) year, 2)country *)
countryvisitshr = Apply[Plus,visitshare];

(* Operators' yearly revenues    *)
(* structure: 1) operator, 2)year*)
operrevens = ZeroMatrix[operators,years];
For[i=1,i <= operators, i++, operrevens[[i]] =
   Apply[Plus,Transpose[revs[[i]]]]];

(* Operators' revenue over all years – present value *)
totalvalue = Table[0,{operators}];
For[i=1, i <= operators,i++, totalvalue[[i]] =
   Sum[operrevens[[i,k]]*1/(internrate+1)^(k-1),{k,years}]];

totalvalue

Out[]= ({1.01982*^8, 1.54702*^8, 1.05328*^8,
    1.64131*^8, 1.40057*^8, 5.68332*^7,
    1.82162*^8, 9.07346*^7, 1.54004*^8,
    3.68414*^8}


(* Operators' roaming revenue *)

(* 10 big operators' share of global roaming revenue; yearly 2005-2010 *)
bigopershare = {0.481730011, 0.470297219,0.460946743, 0.454455256,
0.450641956, 0.449333931};

(* global roaming revenue yearly 2005-2010 *)
roamrev = {43181257,49997545,58118140,67957403,79921193,95173420};

(* 10 big operators' roaming revenue *)
bigoperroamrev = bigopershare * roamrev;


(****** Coalition data structures for computation ******)

(* Creation of possible coalitions; an empty coalition not included *)
coals = Subsets[Range[operators],{1,operators}];

(* Coalition impact to coalition members' revenue *)

(* Beginning of the pseudo code *)
(*

Loop over all years = y
   Loop over all coalitions = c
      Loop over all coalition members as home operator = ho
         Loop over coalition members as visited operator = vo
            Loop over all countries as home country =hc
               If home operator’s share in the country is zero, continue
               Loop over all countries as visited country = vc
                  If visited country is same as home country, continue
                  If visited operator’s share in the visited country is zero,
                     continue
```

```
                Calculate coalition’s_total_visited_share_in_country(y,vc)

                Calculate revenue impact to home operator(y,c,ho)
                   Lost revenue due to the discount:
                   roam_discount * big_oper_roam_revenue(y) *home_oper_share *
                   home_share(y,ho,hc) * visited_share(y,vo,vc)

                Additional revenue due to additional traffic
                   (because of the price elasticity and discount):
                   additional_traffic * (1-roamdiscount) *
                   big_oper_roam_revenue(y) * home_oper_share *
                   home_share(y,ho,hc) * visited_share(y,vo,vc)

                Summary of the home operator impact:
                   [(-roam_discount + additional_traffic * (1-roamdiscount)) *
                   home_oper_share] * [big_oper_roam_revenue(y) *
                   home_share(y,ho,hc) * visited_share(y,vo,vc)] (coalimpact A)

                Calculate revenue impact to visited operator(y,c,vo)
                   - Standard roaming revenue to visited operator:
                   - big_oper_roam_revenue(y) * visited_oper_share *
                   home_share(y,ho,hc) * visited_share(y,vo,vc)

                   + Revenue from active subscribers and additional traffic:
                   big_oper_roam_revenue(y) * visited_oper_share *
                   (1 + additional_traffic) * (1 - roamdiscount)*
                   automatic_selection * home_share(y,ho,hc) *
                   country_visited_share(y,c) * visited_share(y,vo,vc) /
                   coalition’s_total_visited_share_in_country(y,c)

                   + Revenues from passive subscribers and additional traffic:
                   big_oper_roam_revenue(y) * visited_oper_share *
                   (1 + additional_traffic) * (1 - roamdiscount)*
                   (1-automatic_selection) * home_share(y,ho,hc) *
                   visited_share(y,vo,vc)

                Summary of the visited operator impact:
                   - visited_oper_share * [big_oper_roam_revenue(y) *
                   home_share(y,ho,hc) * visited_share(y,vo,vc)] (coalimpact B)

                   +[visited_oper_share * (1 + additional_traffic) *
                   (1 - roamdiscount)* automatic_selection] *
                   [big_oper_roam_revenue(y) * home_share(y,ho,hc) *
                   visited_share(y,vo,vc) * country_visited_share(y,c) /
                   coalition’s_total_visited_share_in_country(y,c)]
                   (coalimpact C)

                   +[visited_oper_share * (1 + additional_traffic) *
                   (1 - roamdiscount)* (1-automatic_selection)] *
                   [big_oper_roam_revenue(y) * home_share(y,ho,hc) *
                   visited_share(y,vo,vc)] (coalimpact B)
*)

(* End of the pseudo code *)


(* Calculation of the structures coaliampact A, B and C *)
(* These are used in in the coalition computation *)

(* structure: 1) coalition 2) member 3)year *)

coalimpactA = Map[ZeroMatrix[Length[#],years]&,coals];
```

```
coalimpactB = Map[ZeroMatrix[Length[#],years]&,coals];
coalimpactC = Map[ZeroMatrix[Length[#],years]&,coals];

(* Loop over all years *)
For[y=1,y <= years, y++,

   (* Loop over all coalitions *)
   For[c=1,c <= Length[coals], c ++, coalition = coals[[c]];

     (* Coalitions of only one member do not create any additional value:
        leaving the zeros untouched *)
      If[Length[coalition] <2, Continue[]];

      (* Loop over all coalition members as home operator *)
      For[ho=1,ho <= Length[coalition], ho++,

         (* Loop over all coalition members as visited operator *)
         For[vo=1,vo <= Length[coalition], vo++,

            (* Loop over all countries as home country *)
            For[hc=1,hc <= countries, hc++,

               (* If home operator's share in home country is zero, continue *)
               If[homeshare[[coalition[[ho]],y,hc]] == 0.0, Continue[]];

               (* Loop over all countries as visited country*)
               For[vc=1,vc <= countries, vc++,

                  (* If visited country is same as home country, continue *)
                  If[vc == hc, Continue[]];

                  (* If visited operator's share in visited country is zero,
                     continue *)
                  If[visitshare[[coalition[[vo]],y,vc]] == 0.0, Continue[]];

                  (* Calculate coalition's total visited share in each
                     country; structure: 1) year 2) country *)

                  coalvisitshare = Apply[Plus,Map[visitshare[[#]]&,coalition]];

                  (* Pseudo code: [big_oper_roam_revenue(y)*home_share(y,ho,hc)*
                     visited_share(y,vo,vc)] *)
                  tmp= bigoperroamrev[[y]] * homeshare[[coalition[[ho]],y,hc]] *
                     visitshare[[coalition[[vo]],y,vc]];

                  (* Home operator impact *)
                  coalimpactA [[c,ho,y]] += tmp;

                  (* Visited operator impact *)
                  coalimpactB [[c,vo,y]] += tmp;
                  coalimpactC [[c,vo,y]] += tmp * countryvisitshr[[y,vc]] /
                     coalvisitshare[[y,vc]] ;

               ]; (* Visited country loop *)
            ]; (* Home country loop *)
         ]; (* Visited operator loop *)
      ];(* Home operator loop *)
   ];(* Coalition loop *)
]; (* Year loop *)
```

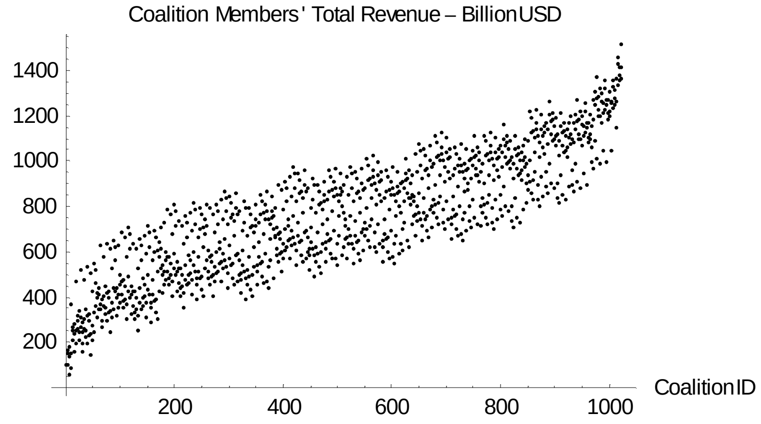
```
(*************************************************************)
(* COMPUTING COALITIONS                                      *)
(*************************************************************)


(****** Parameters of the model ******)

(* retailmargin: retail margin between home operator and subscribers;
   usually 10-35% *)
(* automselection: share of subscribers who not choose visited network
   manually but rely on automatic selection; typically over 60% *)
(* roamdiscount: roaming discount that alliance operators use to promote
   roaming services; assumed 0-45% *)
(* elasticity: price elasticity of roaming services; assumed
   between -0.3 - -2.1 *)

(* Define the distribution of each parameter *)
(* Parameters are assumed to be distributed normally *)
(* Parameters of NormalDistribution: 1) Mean 2) Standard Deviation *)

retmargdist=NormalDistribution[0.22, 0.05];
atmseldist = NormalDistribution[0.8, 0.055];
roamdiscdist = NormalDistribution[0.22, 0.065];
elastdist= NormalDistribution[-1.2, 0.34];

(* Drawing the Probability Density Functions of parameters *)

$TextStyle = {FontFamily -> "Arial", FontSize -> 10};
g1 = Plot[PDF[retmargdist,x],{x,0.0,0.44},DisplayFunction -> Identity,
      PlotLabel -> "Retail Margin",
      Epilog -> {Text["μ=0.22",Scaled[{0.1,0.83}],{-1,0}],
          Text["σ=0.05",Scaled[{0.1,0.7}],{-1,0}]}];
g2 = Plot[PDF[atmseldist,x],{x,0.6,1.0},DisplayFunction -> Identity,
      PlotLabel -> "Automatic Selection",
      Epilog -> {Text["μ=0.8",Scaled[{0.1,0.83}],{-1,0}],
          Text["σ=0.055",Scaled[{0.1,0.7}],{-1,0}]}];
g3 = Plot[PDF[roamdiscdist,x],{x,0.0,0.45},DisplayFunction -> Identity,
      PlotLabel -> "Roaming Discount",
      Epilog -> {Text["μ=0.22",Scaled[{0.1,0.83}],{-1,0}],
          Text["σ=0.065",Scaled[{0.1,0.7}],{-1,0}]}];
g4 = Plot[PDF[elastdist,x],{x,0.0,-2.4},DisplayFunction -> Identity,
      PlotLabel -> "Price Elasticity",
      Epilog -> {Text["μ=-1.2",Scaled[{0.1,0.83}],{-1,0}],
          Text["σ=0.34",Scaled[{0.1,0.7}],{-1,0}]}];

Show[GraphicsArray[{ {g1,g2},{g3,g4} }, ImageSize -> 460],Frame -> True]
```




```
(****** Monte Carlo simulation ******)

(* Number of independent simulations *)
m = 100;

(* Number of samples in a simulation *)
n = 1000;

(* Loop of independent simulations; m simulations *)

(* Output is in a list: *)
simulations = Table[{},{i,m}];

For[i=1, i <= m, i++,

   (* Loop of a simulation; n samples *)
   (* Output is coalrevs *)
   (* structure: 1) coalition 2) member 3)year *)

   coalrevs = Map[ZeroMatrix[Length[#],years]&,coals];

   For[j=1, j <= n, j++,

      (* Let's generate ramdom parameters of the sample *)
      retailmargin = Random[retmargdist];
      automselection  = Random[atmseldist];
      roamdiscount = Random[roamdiscdist];
      elasticity     = Random[elastdist];

      (* Calculated parameters - don't change *)
      homeoprshare = retailmargin/(1+retailmargin);
      visitoprshare = 1-homeoprshare;

      (* Additional roaming traffic due to the roaming discount and
         price elasticity *)
      addtraffic = -elasticity*roamdiscount;

      (* Calculating coalition's revenue impact to home operator*)
      tmp = ((-roamdiscount + addtraffic * (1-roamdiscount))*homeoprshare)*
         coalimpactA;
```

```
      (* Calculating coalition's revenue impact to visited operator*)
      tmp += (-visitoprshare + (visitoprshare * (1+addtraffic) *
         (1- roamdiscount) * (1-automselection)))*coalimpactB;
      tmp += (visitoprshare * (1+addtraffic) * (1-roamdiscount) *
         automselection) *coalimpactC;

      coalrevs += tmp;
   ]; (* Loop of a simulation *)

  (* Calculating the mean of samples; scaling with the number of samples *)

   coalrevs = coalrevs/n;
   simulations[[i]] = coalrevs;

]; (* Loop of several independent simulations *)

(* Write results to files *)
simulations >> simulations.txt;

(* Read results from files *)
(*
  simulations= <<"simulations.txt";
*)


(************************************************************)
(* ANALYSIS OF THE COALITIONS                               *)
(************************************************************)


(* Create Coalition Descriptors for simulation results *)
(*
Structure:
1) coalition ID
2) coalition members
3) coalition's PV of each member; revenue distribution based on
   members' revenue
4) coalitions total PV;
5) coalitions members' total PV
6) ratio: coalitions total PV / coalition member's total PV
7) coalition's PV of each member;  distribution based on the ratio:
   member's total PV / all members' total PV
*)

(* Create one coalition descriptor for each simulation *)

descriptors = Table[{},{i,Length[simulations]}];

For[i=1, i <= Length[simulations], i++,

   coalrevs = simulations[[i]];

   coaldesc = Table[{c,coals[[c]],coals[[c]]*0 ,0,0.0,0.0,coals[[c]]*0.0},
      {c,1,Length[coals]}];

   For[c=1,c <= Length[coaldesc], c++,
      For[j=1, j <= Length[coalrevs[[c]]],j++,
         coaldesc[[c,3,j]] =
            Sum[coalrevs[[c,j,k]]*1/(internrate+1)^(k-1),{k,years}];

         (* Rounding to nearest integer *)
         coaldesc[[c,3,j]] = Round[coaldesc[[c,3,j]]];
```

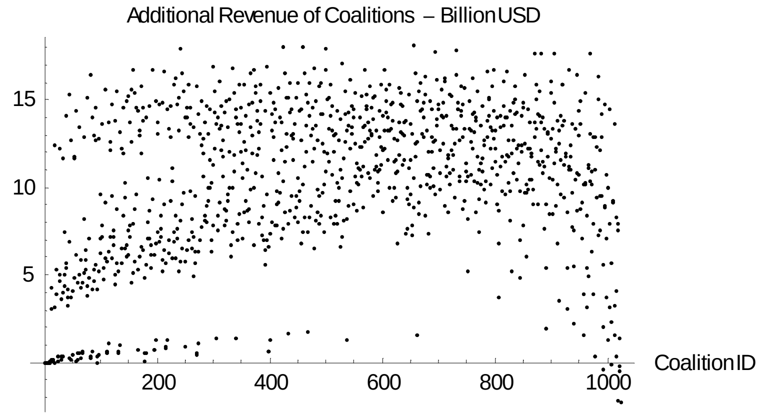
```
      ];

      coaldesc[[c,4]] = Apply[Plus,coaldesc[[c,3]]];

      tmp =0.0;
      For[j=1,j <= Length[coaldesc[[c,2]]], j += 1,
         tmp += totalvalue[[coaldesc[[c,2,j]]]];];
      coaldesc[[c,5]] = tmp;

      coaldesc[[c,6]] = coaldesc[[c,4]] / coaldesc[[c,5]];

      (* If only one member, the share of coalition's PV is identical
         with different revenue distributions *)

      If[Length[coaldesc[[c,2]]] == 1,
         coaldesc[[c,7,1]] = coaldesc[[c,3,1]]; Continue[]];

      tmp =0;
      For[j=1,j <= Length[coaldesc[[c,2]]]-1, j++, coaldesc[[c,7,j]] =
         Round[totalvalue[[coaldesc[[c,2,j]]]]/ coaldesc[[c,5]]*
            coaldesc[[c,4]]];
         tmp += coaldesc[[c,7,j]];
      ];

      coaldesc[[c,7,Length[coaldesc[[c,2]]]]] = coaldesc[[c,4]] - tmp;

   ]; (* coalitions *)

   descriptors[[i]] = coaldesc;

]; (* simulations *)

(* Release memory *)
simulations =.;

Dimensions[descriptors]

Out[]= {100,1023,7}


(* Mean values *)

coalmean = descriptors[[1]];
coalmean[[All,4]] = Mean[descriptors[[All,All,4]]]//N;
coalmean[[All,6]] = Mean[descriptors[[All,All,6]]]//N;
coalmean[[All,3]] = Table[Mean[descriptors[[All,i,3]]]//N,{i,Length[coals]}];
coalmean[[All,7]] = Table[Mean[descriptors[[All,i,7]]]//N,{i,Length[coals]}];

(* Summary figures *)
(* Coalition Member's total Revenue - PV 2005-2010 *)

$TextStyle = {FontFamily -> "Arial", FontSize -> 12};
ListPlot[coalmean[[All,5]]/1000000,ImageSize -> 460,
    PlotLabel -> "Coalition  Members’ Total Revenue - Billion USD",
    AxesLabel -> {"Coalition ID", None}];
```




```
Print["Mean:   ",Mean[Mean[descriptors[[All,All,5]]]/1000000]];
Print["Min:    ",Min[Mean[descriptors[[All,All,5]]]/1000000]];
Print["Max:    ",Max[Mean[descriptors[[All,All,5]]]/1000000]];
```

Out[]= Mean: 759.916
Out[]= Min: 56.8332
Out[]= Max: 1518.35

```
(* Coalitions' Additional Revenue - total PV 2005-2010*)

$TextStyle = {FontFamily -> "Arial", FontSize -> 12};
ListPlot[coalmean[[All,4]]/1000000,ImageSize -> 460,
    PlotLabel -> "Additional Revenue of Coalitions  - Billion USD",
    AxesLabel -> {"Coalition ID", None}];
```

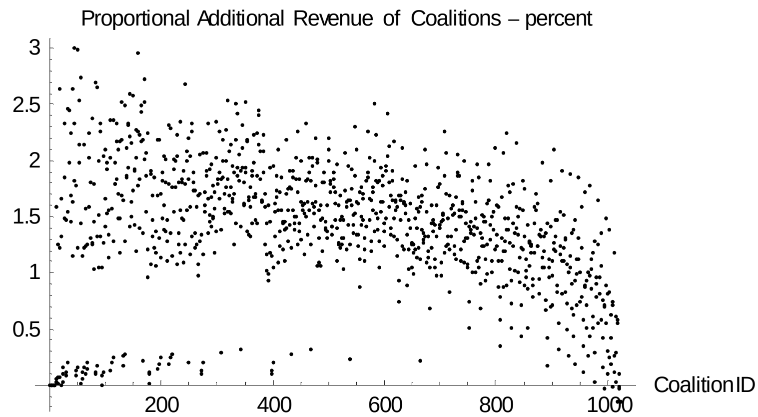



```
Print["Mean:   ",Mean[Mean[descriptors[[All,All,4]]]/1000000]//N];
Print["Min:    ",Min[Mean[descriptors[[All,All,4]]]/1000000]//N];
Print["Max:    ",Max[Mean[descriptors[[All,All,4]]]/1000000]//N];
```

Out[]= Mean: 10.3271
Out[]= Min: -2.30235
Out[]= Max: 18.1003

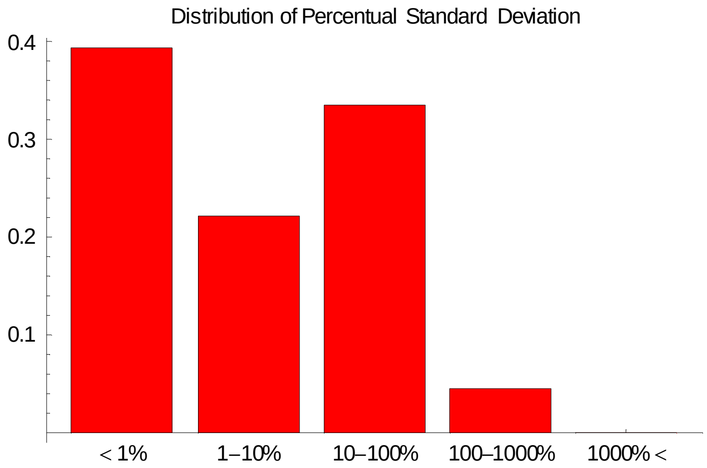
```
(* Coalitions ' Additional Proportional Revenue (%)*)
```

```
$TextStyle = {FontFamily -> "Arial", FontSize -> 12};
ListPlot[coalmean[[All,6]]*100,ImageSize -> 460,
    PlotLabel ->
      "Proportional  Additional  Revenue  of  Coalitions  - percent",
    AxesLabel -> {"Coalition ID", None}];
```




```
Print["Mean:   ",Mean[Mean[descriptors[[All,All,6]]]*100]];
Print["Min:    ",Min[Mean[descriptors[[All,All,6]]]*100]];
Print["Max:    ",Max[Mean[descriptors[[All,All,6]]]*100]];

Out[]= Mean:   1.40217
Out[]= Min:    -0.151636
Out[]= Max:    3.00508


(****** Standard Deviation of results ******)

(* Flat the structure of alliance member's PV *)
simulflat= Map[Flatten,descriptors[[All,All,3]]];
Dimensions[simulflat]

Out[]= {100,5120}

(* Calculating for all simulation elements:
      abolute value of standard deviation proportional to mean *)
simuldev = StandardDeviation[simulflat];
simulmean = Mean[simulflat];

(* Let's replace 0 with 1 to avoid division by zero *)
simulmean = simulmean /.(x_/;x == 0)-> 1;
simuldevprop = Abs[(simuldev/simulmean)//N];
Dimensions[simuldevprop]

Out[]= {5120}

(* Calculate the distribution of simulated fifures's proportional
   standard deviation *)
tmp = RangeCounts[simuldevprop,{0.01,0.1,1,10}]

Out[]= {2019,1140,1720,237,4}

TableForm[Table[{{"[0-1%)","[1-10%)","[10%-100%)","[100%-1000%)",
   "[1000%-∞)"}[[i]],tmp[[i]]}, {i,Length[tmp]}],
```

```
   TableHeadings-> {None,{"Category","Number of samples"}}]

Category             Number of samples
[0-1%)               2019
[1-10%)              1140
[10-100%)            1720
[100-1000%)          237
[1000-∞)             4


$TextStyle = {FontFamily -> "Arial", FontSize -> 12};
BarChart[tmp / Total[tmp], BarLabels ->
   {"<1%", "1-10%", "10-100%", "100-1000%", "1000% <"}, PlotLabel ->
   "Distribution  of Percentual  Standard  Deviation", ImageSize -> 460 ]
```




```
(* Basic statistics *)

TableForm[{{"Mean",Mean[simuldevprop]},{"Median",Median[simuldevprop]},
   {"Min",Min[simuldevprop]},{"Max",Max[simuldevprop]}}]

Mean       0.139721
Media      0.0150442
Min        0.
Max        39.2566

(* Release memory *)
simulflat =.;
simulmean = .;
simuldevprop = .;
simuldev =.;


(****************************************************************)
(* ANALYSIS OF THE CORE                                         *)
(****************************************************************)


coreanalysis = Table[{{},{},{}},{i,Length[descriptors]}];

For[j=1, j <= Length[descriptors], j++,

   coaldesc = descriptors[[j]];
```

```
   (* One item per coalition; 0 = core empty, 1 = core not empty *)
   corenotempty = Table[1,{Length[coaldesc]}];

   (* One item per coalition; 0/1 = imputation isn't/is in core *)
   imputincore1 = Table[1,{Length[coaldesc]}];
   imputincore2 = Table[1,{Length[coaldesc]}];

   For[c=1,c <= Length[coaldesc], c++,
      coalition = coaldesc[[c,2]];

      (* Coalitions of only one member fullfil Core requirements:
         leaving the "1" untouched *)

      If[Length[coalition] <2, Continue[]];

      (* subsets of the coalition *)
      coalsubsets = Subsets[coalition,{1,Length[coalition]}];
      coalval = Table[0,{2^Length[coalition]}];

      For[i=1,i <= Length[coalsubsets], i++,
         coalss = coalsubsets[[i]];
         pos = Position[coals,coalss][[1,1]];
         (* position one is empty coalition *)
         coalval[[i+1]] = coaldesc[[pos,4]];)
      ];

      Game1:=(DefineGame[coalition,coalval]);
      corenotempty[[c]] = If[CoreQ[Game1],1,0];

      (* Imputations in Core? *)
      imputincore1[[c]] = If[corenotempty[[c]] == 1 &&
         InCoreQ[coaldesc[[c,3]],Game1],1,0];
      imputincore2[[c]] = If[corenotempty[[c]] == 1 &&
         InCoreQ[coaldesc[[c,7]],Game1],1,0];

   ]; (* coalitions *)

   coreanalysis[[j,1]] = corenotempty;
   coreanalysis[[j,2]] = imputincore1;
   coreanalysis[[j,3]] = imputincore2;

]; (* simulations *)

(* Write results to files *)
coreanalysis >> coreanalysis.txt;

(* Read in from files *)
(*
  coreanalysis= <<"coreanalysis.txt";
*)


(***********************************************************)
(* EXPORT RESULTS OF THE MODEL TO AN EXCEL FILE               *)
(***********************************************************)

(* Summarizing the results of Core Analysis *)

coreresults = {{},{},{}};

coreresults[[1]] = Table[Total[coreanalysis[[All,1,i]]] /
   Length[coreanalysis]//N, {i,Length[coaldesc]}];
```

```
coreresults[[2]] = Table[Total[coreanalysis[[All,2,i]]] /
   Length[coreanalysis]//N, {i,Length[coaldesc]}];
coreresults[[3]] = Table[Total[coreanalysis[[All,3,i]]] /
   Length[coreanalysis]//N,{i,Length[coaldesc]}];

(*
Record structure:
1:      Coalition ID
2-11:   Coalition members; one position per each member
12:     Total revenue; M$
13:     Additional revenue; M$
14:     Additional revenue%
15:     Stability - Core exists
16:     Stability - Revenue share 1 in Core
17:     Stability - Revenue share 2 in Core
18-27:  Additional revenue per member - Revenue share 1; M$
28-37:  Additional revenue per member - Revenue share 2; M$
38-47:  Additional revenue% per member - Revenue share 1
48-57:  Additional revenue% per member - Revenue share 2
*)

modelresults = Table[{
   0,
   0,0,0,0,0,0,0,0,0,0,
   0.0,
   0.0,
   0.0,
   0.0,
   0.0,
   0.0,
   0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
   0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
   0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,
   0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0
   },{i,Length[coalmean]}];

For[i=1, i <= Length[coalmean],i++,

   (* Coalition ID *)
   modelresults[[i,1]] = coalmean[[i,1]];

   (* Total Revenue *)
   modelresults[[i,12]] = coalmean[[i,5]]/1000;

   (* Additional revenue *)
   modelresults[[i,13]] = coalmean[[i,4]]/1000;

   (* Additional revenue% *)
   modelresults[[i,14]] = coalmean[[i,4]] / coalmean[[i,5]];

   (* Stability - Core exists *)
   modelresults[[i,15]] = coreresults[[1,i]];

   (* Stability - Revenue share 1 in Core *)
   modelresults[[i,16]] = coreresults[[2,i]];

   (* Stability - Revenue share 2 in Core *)
   modelresults[[i,17]] = coreresults[[3,i]];

   For[j=1, j <= Length[coalmean[[i,2]]],j++,
      ndx = coalmean[[i,2,j]];
```

```
      (* Coalition members;one position per each member *)
      modelresults[[i,1+ndx]] = ndx;

      (* Additional revenue per member - Revenue share 1 *)
      modelresults[[i,17+ndx]] = coalmean[[i,3,j]]/1000;

      (* Additional revenue per member - Revenue share 2 *)
      modelresults[[i,27+ndx]] = coalmean[[i,7,j]]/1000;

      (* Additional revenue% per member - Revenue share 1 *)
      modelresults[[i,37+ndx]] = coalmean[[i,3,j]] / totalvalue[[ndx]];

      (* Additional revenue% per member - Revenue share 2 *)
      modelresults[[i,47+ndx]] = coalmean[[i,7,j]] / totalvalue[[ndx]];
   ];
];

(* Writing a file - an excel sheet *)

Export["modelresults.xls", modelresults,"XLS"];


(* End of the code *******************************************)
```