ADAPTIVE GENERAL PARAMETER EXTENSION FOR TUNING FIR PREDICTORS

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Abstract: A general parameter (GP) extension to expand the applicability of discrete time finite impulse response (FIR) filters designed for sinusoidal or polynomial signal prediction while attenuating white noise is proposed. These FIR filters are simple and powerful linear methods for filtering, predicting and modeling many real world phenomena. With only a single adaptive GP, it is computationally efficiently possible to extend prediction capabilities of polynomial and sinusoidal FIR predictors beyond their design input signal models, i.e., beyond polynomial input signals, or nominal design frequencies, respectively. This allows for more accurate prediction of signals with time varying characteristics.

Keywords: filter design, adaptive digital filters, digital filter structures, prediction methods, closed loop control, delay compensation

1. INTRODUCTION

Generally, signal processing methods assume input signal statistics for which they are optimized. In the real world, however, statistics of measured signals may either not be presentable by simple distributions, or at least determining the current distribution requires considerable computational resources. Adaptive filtering and soft computing methods, e.g., neural networks, address this problem of processing measured signals with unknown and/or varying signal characteristics. Many signal processing and control applications rely on identified signal statistics, or are forced to employ computationally complex adaptation schemes. In nature, many properly sampled measured signals are buried in noise and exhibit near polynomial or sinusoidal behavior, making polynomial or sinusoidal filtering an attractive choice. Prediction capabilities are most valuable in any control application in which the operation of a control loop is limited by unavoidable control loop delays, likewise in any application in which signal processing or other delays are to be avoided. Predictive FIR filtering is a simple and powerful linear approach but as such is fixed for the design input signal statistics; GP extension provides means for tuning the FIR basis predictors to allow operation beyond the design signal statistics.

The first application example for GP extended prediction is transmitter power control of a mobile communication system (Tanskanen *et al.*, 1995; Harju *et al.*, 1996), in which the employed closed loop control system operation is inherently delay limited and could benefit from an accurate prediction of the Rayleigh distributed attenuation, or fading, present in a radio channel under certain conditions (Parsons, 1995). Since a Rayleigh distributed signal can be approximated by an appropriate sum of sinusoids and in practice the exact distribution is unknown, an adaptive polynomial-predictive filtering (Heinonen and Neuvo, 1988; Väliviita *et al.*, 1999) scheme is a lucrative approach. Also, it would naturally be beneficial if the adaptation was computationally as inexpensive as possible. Sinusoidal signal prediction is applied to zero crossing detection of power line frequency (Vainio and Ovaska, 1995). With prediction, it is possible to overcome the signal processing delay in the zero crossing detection calculations, and thus provide more accurate zero crossing information for the power electronics switching control.

The basis FIRs employed are polynomial-predictive (Heinonen and Neuvo, 1988) and sinusoidal-predictive (Vainio and Ovaska, 1995; Väliviita *et al.*, 1999) FIRs. The extension that is attached to the FIRs consists of a single adaptive general parameter (GP) (Ashimov and Syzdykov, 1981), c.f. Fig. 1 for the block diagram of the GP extension and GP updating. The GP approach is fairly unknown technique even though it is very simple and capable of producing improved filtering results. It has been employed with radial basis function neural networks (Akhmetov and Dote, 1999). Here, the GP method is shown to extend the applicability of polynomial-predictive FIR filters beyond polynomial input signals, and that of sinusoidal-predictive FIR filters beyond their nominal frequencies. This allows for more relaxed requirements for the signal statistics estimation, the control loop delay estimation, or the sinusoidal frequency estimation need not be as accurate as they should without the GP extension.

The paper is organized as follows. In section 2, predictive FIR filtering is overviewed, and the general parameter extension is reviewed in section 3. In section 4, the stability condition of the GP approach is derived. Application examples are given in section 5, and section 6 concludes the paper.

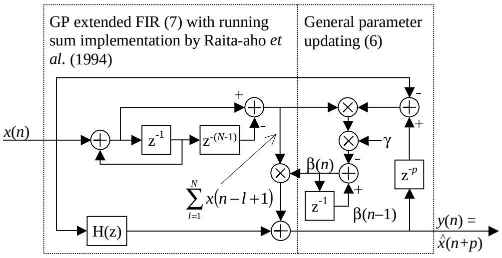

Fig. 1. Structure of the GP extended FIR. Notation: input signal $x(n)$, output signal $y(n)$, transfer function of the FIR H(z) with the coefficient vector $\mathbf{h}$, FIR length N, general parameter $\beta(n)$, p prediction step of the FIR, gain factor γ, unit delay z^{-1}, and hat denoting an estimate.

2. BASIS FILTERS FOR GENERAL PARAMETER EXTENSION

FIR predictors for polynomial and sinusoidal signals are employed as basis FIR filters, and both polynomial-predictive (Heinonen and Neuvo, 1988; Väliviita *et al.*, 1999) and sinusoidal-predictive (Vainio and Ovaska, 1995) FIR filtering theories are well established. Output of an ideally operating p-step-ahead FIR predictor is calculated by

$$\sum_{k=1}^{N} h(k)x(n-k+1) = x(n+p), \tag{1}$$

where $h(k)$ are filter coefficients, and $x(n)$ are input signal samples at discrete time instants n. With the coefficient vector $\mathbf{h} = [h(1) \quad h(2) \quad \cdots \quad h(N)]^{\mathrm{T}}$, and input signal sample vector $\mathbf{x}(n) = [x(n) \;\; x(n-1) \;\; \ldots \;\; x(n-N+1)]$, (1) can be written as

$$\mathbf{x}(n)\mathbf{h} = x(n+p). \tag{2}$$

In FIR design, after providing for the exact p-step-ahead prediction of a polynomial signal of a given degree, or of a sinusoidal signal with given frequency, the white noise gain given by

$$NG = \sum_{k=1}^{N} |h(n)|^2, \tag{3}$$

is minimized for achieving disturbance suppression outside the frequency band in which prediction is desired. A set of constraints on filter coefficients can be derived from the filter input-output relation (1), and from the constraint can closed form solutions for the FIR filter coefficients be calculated (Heinonen and Neuvo, 1988; Väliviita *et al.*, 1999; Vainio and Ovaska, 1995).

3. GENERAL PARAMETER EXTENSION TO FIR PREDICTORS

The GP method (Ashimov and Syzdykov, 1981) is employed by adding an adaptive GP $\beta(n)$ to each FIR coefficient in (2), resulting in the predictive filtering

$$\mathbf{x}(n)(\boldsymbol{\beta}(n)+\mathbf{h}) = x(n+p), \tag{4}$$

where $\boldsymbol{\beta}(n) = [\beta(n) \quad \beta(n) \quad \cdots \quad \beta(n)]^{\mathrm{T}}$ is the time dependent GP vector. Writing this as

$$\beta(n)\sum_{k=1}^{N} x(n-k+1) + \sum_{k=1}^{N} h(k)x(n-k+1) = x(n+p), \tag{5}$$

the GP is seen to act as an adaptive gain operating on average input signal value.

Let us denote the actual filter output sample as $y(n)$.

The GP is adapted to the input signal according to the following simple updating rule (Ashimov and Syzdykov, 1981) which, taking also the prediction step p into account, is given by

$$\beta(n+1) = \beta(n) - \gamma(y(n-p) - x(n))\sum_{l=1}^{N} x(n-l+1), \quad (6)$$

where $\gamma > 0$ is the adaptation gain factor. The filter output is now given by

$$\sum_{k=1}^{N} (\beta(n) + h(k))x(n-k+1) = y(n) = \hat{x}(n+p), \quad (7)$$

where hat denotes estimate, c.f. Fig 1. This adaptation resembles the well-known least mean square filter coefficient adaptation (Widrow and Stearns, 1985) but since there is only a single adaptive parameter $\beta(n)$ per filter and not adaptation of all the filter coefficients independently, input signal statistics are taken into account in a form of average over the filtering window.

From Fig. 1 it can be calculated that the GP extension requires 5 additions and 3 multiplications regardless of the FIR length. Also $N + p + 1$ memory locations for the delays are necessary. Thus, GP extension can thus be regarded computationally efficient.

4. STABILITY OF THE GENERAL PARAMETER FILTERS

Before the GP extension can be applied, stability of the resulting system must be considered. Stability of the GP extended filters can be analyzed similarly as the least mean square algorithm (LMS) (Kalouptsidis, 1997). Assume reference data sequences generated by the linear time varying model

$$y(n) = \mathbf{x}(n)\mathbf{\Theta}_0(n) + v(n), \quad (8)$$

where $v(n)$ represents noise, and $\Theta_{\mathbf{0}}(n)$ is a parameter vector obeying a model of the form

$$\mathbf{\Theta}_0(n+1) = \mathbf{\Theta}_0(n) + \xi(n) \quad (9)$$

with a corrective term $\xi(n) = [\xi_1(n) \ \cdots \ \xi_N(n)]^{\mathrm{T}}$. The GP extended filter has the coefficient vector

$$\mathbf{\Theta}(n) = \mathbf{h} + \boldsymbol{\beta}(n). \quad (10)$$

Considering the adaptation error

$$\tilde{\mathbf{\Theta}}(n) = \mathbf{\Theta}_0(n) - \mathbf{\Theta}(n), \quad (11)$$

$$\tilde{\mathbf{\Theta}}(n+1) = \mathbf{\Theta}_0(n) + \xi(n) - \mathbf{h} - \boldsymbol{\beta}(n) - \gamma[y(n) - \mathbf{x}(n)\mathbf{\Theta}(n)]\mathbf{S}, \quad (12)$$

where

$$\mathbf{S} = \left[\sum_{i=1}^{N} x(n-i+1) \ \cdots \ \sum_{i=1}^{N} x(n-i+1)\right]^{\mathrm{T}}. \quad (13)$$

Substituting (11) into (12), we obtain

$$\tilde{\mathbf{\Theta}}(n+1) = \tilde{\mathbf{\Theta}}(n) + \xi(n) - \gamma[y(n) - \mathbf{x}(n)\mathbf{\Theta}(n)]\mathbf{S}, \quad (14)$$

and inserting $y(n)$ from (8) yields

$$\tilde{\mathbf{\Theta}}(n+1) = \tilde{\mathbf{\Theta}}(n) + \xi(n) - \gamma[\mathbf{x}(n)\tilde{\mathbf{\Theta}}(n) + v(n)]\mathbf{S}. \quad (15)$$

This can be written in form

$$\tilde{\mathbf{\Theta}}(n+1) = [\mathbf{I} - \gamma\mathbf{S}\mathbf{x}(n)]\tilde{\mathbf{\Theta}}(n) + \xi(n) - \gamma v(n)\mathbf{S}, \quad (16)$$

where $\mathbf{I}$ is an N by N unit matrix. The stability of the system therefore depends on the eigenvalues λ_i of the matrix

$$\mathbf{I} - \gamma\mathbf{S}\,\mathbf{x}(n), \quad (17)$$

and it is required that $|\lambda_i| \le 1, i = 1, ..., N$. This guarantees the stability in the sense that the prediction error will always remain finite, if the gain factor γ is set accordingly. However, no guarantees are made of the prediction error approaching zero.

The eigenvalues of the matrix (17) are equal to one except the one given by

$$\lambda = 1 - \gamma\mathbf{x}(n)\mathbf{S}. \quad (18)$$

For stability it is therefore required that the gain factor is constrained as

$$0 \le \gamma \le \frac{2}{\mathrm{E}[\mathbf{x}(n)\mathbf{S}]}, \quad (19)$$

where $\mathrm{E}[\cdot]$ denotes expectation value. It is possible to formulate a normalized algorithm, as with LMS, by adjusting the gain factor as

$$\gamma = \frac{1}{\mathbf{x}(n)\mathbf{S}}, \quad (20)$$

which, by inserting the vectors $\mathbf{x}$ and $\mathbf{S}$ (13), is seen to be

$$\gamma = \frac{1}{[x(n) \ \cdots \ x(n-N-1)]\begin{bmatrix}\sum_{i=1}^{N} x(n-i+1) \\ \vdots \\ \sum_{i=1}^{N} x(n-i+1)\end{bmatrix}} = \frac{1}{\left(\sum_{i=1}^{N} x(n-i+1)\right)^2}. \quad (21)$$

Inserting (21) into (6) yields the normalized GP update equation

$$\beta(n+1)=\beta(n)-\frac{1}{\sum_{i=1}^{N}x(n-i+1)}\left(y(n-p)-x(n)\right). \tag{22}$$

As compared to (6), (22) involves division operation instead of multiplication, and therefore is in contrast with the original motivation of the GP method, i.e., adaptivity with very simple computations, if the application hardware does not support efficient division.

If the cost of the division is tolerable, the gain factor (21) could be updated continuously in order to operate with the maximum gain factor. While (19) only guarantees that the error remains finite, the normalized gain (20) is set to half of the stability bound, which may be sufficient to avoid diverging. Should it happen that $\mathbf{x}(n) = \mathbf{0}$, where $\mathbf{0}$ denotes a zero vector, (20) and thus (22) would be useless since the gain factor would become infinite. In practice, like also for LMS adaptive filters, a fixed gain factor can be estimated from the input signal to yield a sufficient probability of convergence by estimating the fixed gain factor (21) from the input signal. For example, a minimum gain factor according to (21) over a given period of time could be calculated periodically. Thus, stability of the GP system can be guaranteed, or a sound tradeoff between convergence probability and adaptation speed can be made with low computational cost.

5. SIMULATION RESULTS

Since the underlying signal model for the FIR predictors is assumed inaccurate as compared to the actual filter input signal, it is not expected that the GP extended filter should converge to a fixed general parameter value, but rather to continuously adapt and track the input signal even if the input signal is stationary. This can be seen in Fig. 2, in which the GP, appended to the first-degree one-step-ahead polynomial-predictive FIR of length $N = 3$, is seen to continuously vary with the one-step-delayed 50 Hz sinusoidal input signal sampled at 1.667 kHz. Note that here a sinusoid is filtered with a ramp predictor, which can be regarded as a large deviation in the signal statistics, especially as there are only 33 samples takes during a period of the sinusoidal signal. Also, in Fig. 2, the predictive FIR and the corresponding GP filter input and output signals are shown along with the desired output signal. In Fig. 2, the gain factor is set to $\gamma = 0.1138$ which is the minimum gain factor over one period of the input sinusoid calculated using (21). In the case seen in Fig. 2, the maximum prediction error with the fixed FIR predictor is 0.059 while with the GP extended predictor it is 0.0073. For reference, the maximum

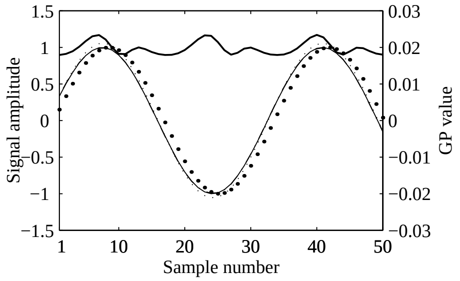

Fig. 2. One-step ahead sinusoid prediction with a first-degree polynomial predictor of length $N = 3$ (dotted), and with the corresponding GP extended polynomial predictor (dash-dot), along with the one-step delayed filter input (dark dotted) and desired output (solid) signals. Also shown is the GP value (dark solid).

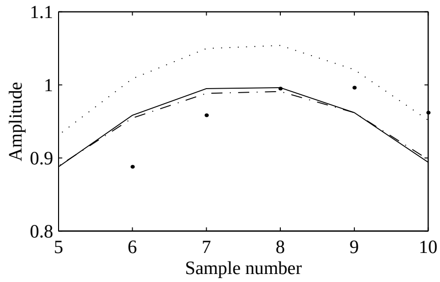

Fig. 3. Peak detail of the signals shown in Fig. 2.

error caused by the one-step delay is 0.188. In Fig. 3, a detail of the signals in Fig. 2 is shown to illustrate the effect of GP approach on the signal amplitude. In Figs. 4 and 5, the magnitude responses and group delays are shown for the same FIR predictor as employed in Figs. 2 and 3 with different GP values. In the sinusoid prediction case in Fig. 2, the GP value is seen to fluctuate between 0.017 and 0.024, and the frequency response and group delay for this GP extended FIR with the GP value 0.02 can be seen in Figs. 4 and 5, respectively. Here of interest is only the low frequency range since the examples are noiseless. From Fig. 4 it is seen that the DC gain of the predictor is deviated from unity by the GP, and also the group delay is adjusted, Fig. 5. By design the exact p-step-ahead prediction requires unity DC gain, but the GP extension is seen to have a complicated effect as both the DC gain deviation and group delay chances are concerned. Comparing Figs. 4 and 5, it is seen that in this case, for negative GP values, prediction step grows while the DC gain decreases, and vise versa for the positive GP values.

To demonstrate the effects of the general parameter extension, the sinusoidal and polynomial predictors are employed in sinusoidal or Rayleigh distributed signal prediction, respectively. The first example employs a two-step-ahead $p = 2$ sinusoidal FIR predictor of length $N = 12$ designed for predicting the 50 Hz line frequency with the sampling rate of 1.667 kHz, giving the nominal normalized prediction frequency of $\omega_0 = 0.06\pi$, as employed by Vainio and Ovaska (1995) for the power line frequency zero crossing detection problem. The results of applying this predictor to a set of two steps delayed sinusoidal signals with different frequencies are given in Figs. 6 and 7 with the GP gain factor set to $\gamma = 0.001$. In Figs 6 and 7, the maximum and mean square errors (MSE) are given as functions of input sinusoid frequency for the sinusoidal and GP extended sinusoidal predictors, along with the corresponding errors caused by the pure delay of the input signal. From the Figs 6 and 7, it is seen that both the fixed coefficient and the GP extended predictors are able to improve the signal quality, and that the GP extended sinusoidal predictor outperforms the corresponding fixed-coefficient predictor in both maximum and mean square errors, providing for extended applicable input signal frequency range at a given tolerable prediction error level. For example, if a system can tolerate the maximum prediction error of 0.1 in amplitude, the possible input frequency range for the sinusoidal predictor is 47~53 Hz while for the GP extended sinusoidal predictor the corresponding range is <40~57 Hz. Similar results are observed for MSE.

In the second example, presented in Figs. 8 and 9 as error reductions in percents gained by application of FIR and GP extended FIR predictors, a Rayleigh distributed signal with different delays is fed to a polynomial-predictive FIR filter, i.e., Heinonen-Neuvo (H-N) filter (Heinonen and Neuvo, 1988), of length $N = 2$, designed for one-step-ahead prediction of first degree polynomial signals, and into the corresponding GP extended filter with the gain factor set to $\gamma = 0.005$. From Fig 8, it is seen that GP is able to improve the maximum error performance of the pure H-N predictor only with input delay of 1 sample. In Fig. 9 for the MSE, the GP extended predictor again outperforms the fixed FIR predictor.

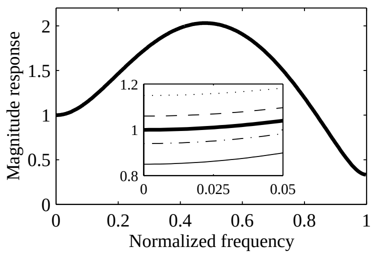

Fig. 4. Magnitude response of the one-step-ahead predictive H-N FIR of $N = 3$ (strong solid) and in detail the same the GP extension with the GP values –0.05 (solid), –0.02 (dash-dot), 0.02 (dashed), and 0.05 (dotted).

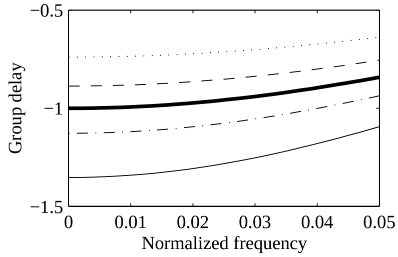

Fig. 5. Detail of the group delays of the one-step-ahead predictive H-N FIR of $N = 3$ (strong solid) and the same with GP extension with the GP values –0.05 (solid), –0.02 (dash-dot), 0.02 (dashed), and 0.05 (dotted).

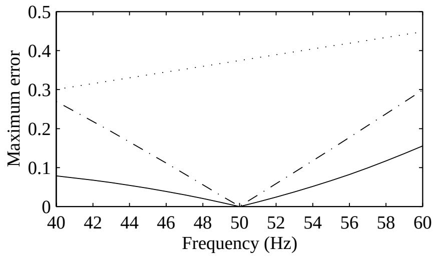

Fig. 6. Maximum errors of the sinusoid (dash-dot) and GP extended sinusoid (solid) predictors as functions of input frequency, along with the maximum errors at the filter input caused by the delay only (dotted).

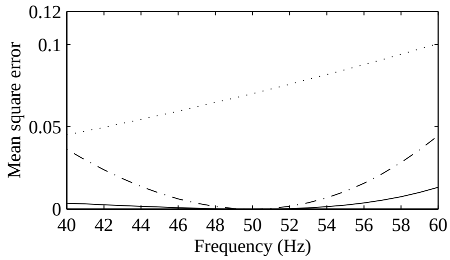

Fig. 7. MSEs of the sinusoid (dash-dot) and GP extended sinusoid (solid) predictors as functions of input frequency, along with the MSEs at the filter input caused by the delay only (dotted).

for all input signal delays. Thus, it can be concluded that the GP extension might be able to slightly expand the applicability range of the polynomial-predictive FIR filters in this application. Improved received power level prediction would yield relaxed requirements for the mobile radio system transmitter power control loop delay estimation.

The used Rayleigh distributed signal models the power level received by a mobile phone (Parsons, 1992) which operates at 1.8 GHz carrier frequency

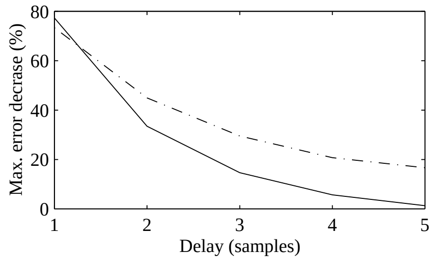

Fig. 8. Decrease of maximum error in percents; with the H-N (dash-dot) and GP extended H-N (solid) predictors when predicting a Rayleigh distributed signal with different delays.

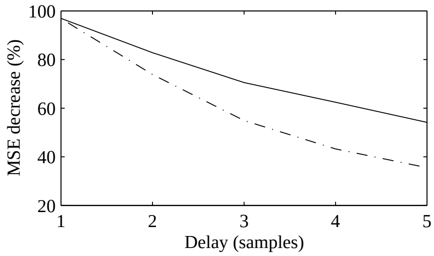

Fig. 9. Decrease of MSE in percents; with the H-N (dash-dot) and GP extended H-N (solid) predictors when predicting a Rayleigh distributed signal with different delays.

and is moving at 10 km/h, and whose received power level is measured at 1 kHz rate.

For the line frequency zero crossing application, it can be concluded that as the frequency deviates from the nominal line frequency of 50 Hz, and thus from the nominal design frequency of the predictor, GP extension is able to improve the prediction capabilities, and thus to contribute to the zero crossing detection accuracy.

6. CONCLUSIONS

It has been demonstrated that adaptive general parameter extension to sinusoidal-predictive and polynomial-predictive FIR filters is able to expand the applicability of these filters. GP extended filtering is of low computational complexity since it involves only a single adaptive parameter that is added to all the filter coefficients. The stability conditions of the GP systems are stated and corresponding normalized update equation for the general parameter is given with comments for practical application using a fixed adaptation speed factor estimated from the input signal, which provides for maximum adaptation speed with which the stability of the GP system can be guaranteed.

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