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Outage Probability in GSM-GPRS Cellular Systems With and Without Frequency Hopping

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Abstract. General packet radio service (GPRS) is designed for transmitting packet data and is supposed to take its radio resource from the pool of unused channels of GSM voice services. Obviously, the introduction of GPRS has an impact on the voice services. In this paper, we present a method to calculate the outage probability of the GSM-GPRS network for both non-frequency hopping and frequency hopping systems. This method takes into account Rayleigh fading, power control (with error), discontinuous transmission, and frequency hopping (if applied). The outage probability of voice services affected by the introduction of GPRS is discussed. The number of unused voice channels allocated to GPRS depends on the difference between the outage level of the existing GSM network and the maximum acceptable level. The frequency hopping system can accommodate more GPRS traffic than the non-frequency hopping system. The power control error has more impact on system performance when more channels are allocated to GPRS. Beyond our expectations, for the non-frequency hopping system, the channels provided for GPRS are not much different between high channel occupancy and low channel occupancy of voice services. In contrast, for the frequency hopping system, the system can provide more channels for GPRS at low channel occupancy. The cell service area decreases by about 10%∼20% for each additional channel allocated to GPRS.

Keywords: GPRS, outage probability, quality of service.

1. Introduction

Since cellular systems were introduced in the 1970s, the main application of most mobile communications systems has been voice-oriented service. Recently, the use of data applications in mobile terminals, such as facsimile transmission, Internet surfing, short message exchange, multimedia services, etc., is becoming more and more popular. As human mobility becomes even more important, the mobile communications systems will require all-embracing applications, such as wireless personal computers, mobile offices, mobile electronic funds transfer, road transport telematics, fleet management, and remote telematics, in addition to the existing services.

The current method of data transmission in the pan-European Global System for Mobile Communications (GSM) and the American Advanced Mobile Phone Standard (AMPS) cellular networks is circuit switching. This technique reserves the traffic channel for the entire communication time, which wastes the radio resource when data traffic occurs in bursts with long silent intervals. In the development of GSM phase 2+, the European Telecommunications Standard Institute (ETSI) has specified [1] a general packet radio service (GPRS) over the GSM to increase the utilization efficiency of the radio resource and provide a set of additional services. Similarly, in North America, Cellular Digital Packet Date is specified [2] to provide packet data services utilizing the AMPS infrastructure.

A cell supporting GPRS may allocate resources to one or several physical channels. Those physical channels, shared by the GPRS MSs, are taken from the common pool of physical channels available in the cell. The physical channels unused by circuit-switched services are allocated dynamically to the GPRS according to the need for actual packet transfers which is referred to as the "capacity on demand" principle. Up to eight packet data traffic channels (PDTCHs), with different timeslots but with the same frequency carrier, may be allocated to one MS at the same time.

Whenever the signaling is not carried by the existing GSM common control channel (CCCH), at least one packet data channel (PDCH, e.g., physical channel) is needed to act as a master. This master channel accommodates the packet common control channel (PCCCH), which carries all the necessary control signaling for initiating packet transfer, as well as user data and dedicated signaling. Other PDCHs act as slaves for user data transfer and for dedicated signaling. A new protocol, the medium access control (MAC) protocol, is introduced in order to multiplex up to eight different MSs onto one slave channel. Different packet data logical channels and the PDTCH can be multiplexed on the same physical channel.

When a circuit-switched network is operating at a blocking probability of 0.02, the average channel load is in the range of 60%–80%, depending on the total number of channels used in the cell. Thus, there are 20%–40% idle channels on average which may be used for data services. Both GPRS and CDPD are designed to utilize those unused channels to transmit short bursts of packet data. However, the introduction of the packet data services into existing circuit-switched networks without allocating a new spectrum may cause the degradation of either voice quality or voice capacity. More specifically, it may increase the blocking probability or degrade the service of voice quality and reduce the cell service area, defined as the area over which a specified outage probability limit is achieved.

The introduction of packet data services must have no effects or just very small effects on the existing voice services. In order to guarantee the quality of service (QoS), it is necessary to allocate dedicated channels to the packet data service, especially to the GPRS with multiple applications and multiple class services. However, such a scheme with dedicated channels will reduce the number of channels provided for voice services and increase the blocking probability to an undesirable level. The dynamic sharing of the channels between voice services and packet data services does not seem to have much impact on the capacity of voice services and creates an additional capacity for packet data services [3]. However, the system performance will be degraded, e.g., outage probability increase and cell service area decrease, due to the additional interference contributed by packet data transmission [4–6]. When the operator plans a system, the outage probability distributed in a cell is normally designed within some limit for a certain traffic load. There are two possible cases in existing circuit-switched cellular networks, i.e., (1) the outage level has reached the maximum acceptable value, and (2) the outage level is below the maximum acceptable value.

In the first case, since the voice services have already achieved the network capacity limit, overlaying the packet data services will degrade either voice quality or voice load due to the additional cochannel interference. Even though in the low channel occupancy, some channels unused by voice services might be used for GPRS, the obtained capacity is quite limited and unpredictable. In the second case, we may obtain additional capacity by overlaying the packet data services onto voice services, but the tolerable level of the outage limits the capacity extension so that it does not exceed the maximum acceptable value.

When a channel is allocated to GPRS, it is shared by a few data users simultaneously. The co-channel interference to the voice users might vary rapidly and dramatically in the time interval from 20 ms to a few seconds depending on the transmitted packet data size, since the locations of those packet data users could be very different. This effect could drive the system into an unpredictable and unstable situation, except for causing a degradation of the QoS of voice services. Therefore, the admission control of the packet data traffic is necessary to guarantee the quality of service for voice users. In this paper, we focus on discussions of the outage probability affected by GPRS traffic in order to provide some guidelines for system evaluation and planning, as well as the admission control. Numerous papers [7–10] have presented methods for outage probability calculation for different considerations separately, including Rayleigh fading and voice activity factor. However, very few papers can give a method integrating the considerations of Rayleigh fading, voice and data activity, and power control, with and without frequency hopping, especially for systems, where voice and packet data coexist. In Section 2, we present a detailed analytical approach for the outage probability calculation of GSM-GPRS networks. Power control, Rayleigh fading, and voice activity detection, with and without slow frequency hopping, are taken into consideration. In Section 3, numerical results are given and the QoS of voice services affected by GPRS is discussed using outage probability calculations. Finally, some conclusions are drawn in Section 4.

2. Outage Probability of the GSM-GPRS Network

2.1. ASSUMPTIONS

All signals are assumed to have experienced Rayleigh fading with respect to a local mean signal strength, while the local mean signal strength experiences log-normal slow fading based on a mean value, which is determined by the propagation loss law of an inverse n-th power of distance. The desired signal and interfering signals, and the different individual interfering signals are all statistically mutually independent. Slow fading is assumed to be uncorrelated and the variance is the same for all cells. Voice activity detection is assumed to be used in the system. Using the discontinuous transmission (DTX) method, the transmitted power of a user is reduced to a low level during speech pauses, and its interference to other users is assumed to be zero. For a voice user, the active time of speech is about 40% of his call duration on average, and the rest of the call duration is in listening state. Therefore, when a channel is used for transmitting voice, the activity factor is about 40%.

The selective automatic repeat request (ARQ) protocol is used for GPRS data transfer. The MAC protocol [13] was developed to multiplex up to eight different MSs of GPRS onto one physical channel. Different packet data logical channels and the PDTCH can also be multiplexed on the same physical channel. The MAC function provides collision avoidance, detection, and recovery procedures. On the downlink, multiplexing is controlled by scheduling. On the uplink, it is controlled by medium allocation to individual users. When a channel is reserved for an individual MS, it allows the transmission of a predetermined (limited or unlimited) amount of time without interruption. When the MS (or BS) has sent the last Radio Link Control (RLC) Data Block (perhaps not the final data packet), the channel is released and it waits for acknowledgment. A new Packet Channel Request must be issued in order to continue the transfer of the remaining data block and erroneous data block. When the service-initiating request of a GPRS user is accepted, it can be in two states: active and idle. In the active state, it may either be in a transmitting or a backlogged (retransmitting) state. In the idle state, it waits for acknowledgment or for transfer. However, a channel accommodating a few GPRS users can be assumed to transmit some kind of information if the number of users requesting service is large enough. The reason is that the time scheduling a user into the transmitting state is so small (one burst period) that the idle time of this channel can be neglected compared to its active time. The traffic load of GPRS may vary in a day; however, from the system planner's point of view, the network situation with a high GPRS load should be considered in order to guarantee the QoS of voice services. Therefore, the activity factor of a channel carrying the packet data traffic is considered 100% here.

2.2. PROBABILITY DENSITY FUNCTIONS OF THE SIGNAL AND INTERFERENCE

A signal from the transmitter to the receiver experiences path attenuation, slow fading, and fast fading. Thus, the received power S of a desired signal and the interference power I can be described as:

$$S = Ar_{00}^{-\eta}\xi_{00}Z_{00}P_{T_{00}} \tag{1}$$

$$I = \sum_{k=1}^{M}\sum_{i=1}^{N_k} I_{ik} = \sum_{k=1}^{M}\sum_{i=1}^{N_k} Ar_{ik}^{-\eta}\xi_{ik}Z_{ik}P_{T_{ik}}X_{ik}\,, \tag{2}$$

where P_T is the transmitted power, ξ is the slow fading variable, Z is the fast fading variable, r is the distance from the transmitter to the receiver, and η is a factor of transmitted power attenuated with distance. X is the channel activity factor and A is a proportionality constant. The indexes i and k refer to the i-th mobile in the k-th interfering cell and $i = k = 0$ corresponds to the desired mobile and the desired cell. M and N_k are the number of interfering cells and the number of mobiles in the k-th cell, respectively. One should note that, when a channel is allocated to GPRS, it is shared by a few data users. The co-channel interference to the voice users might fluctuate frequently in the time interval from 20 ms to a few seconds depending on the transmitted packet data size, since the locations of those packet data users could be very different.

r_{00} is the distance from the desired mobile to its host base station; r_{ik} is the distance from the desired mobile to the k-th interfering base station of the i-th mobile in downlink or the distance from the host base station of the desired mobile to the i-th mobile in the k-th interfering cell in uplink.

ξ_{00} and ξ_{ik} are log-normal random variables which refer to the slow fading of the desired signal and the interference signal from the i-th mobile in the k-th interfering cell, respectively.

Z_{00} and Z_{ik} are fast fading random variables of desired signal and interference signal from the i-th mobile in the k-th interfering cell, respectively. The distribution of those variables for Rayleigh fading is normally the exponential distribution, i.e., the probability of density function (PDF) is:

$$f_{Z_{ik}}(Z_{ik}) = \exp(-Z_{ik}), \quad Z_{ik} > 0, \quad i = 0, 1, \ldots, N_m \text{ and } k = 0, 1, \ldots, M\,. \tag{3}$$

X_{ik} is a binary random variable, which takes values in $\{0, 1\}$, and represents which channel used by the i-th mobile in the k-th cell is active ($X_{ik} = 1$) or inactive ($X_{ik} = 0$). The PDF of X_{ik} is

$$f_{X_{ik}}(X_{ik} = p_k^{X_{ik}}(1 - p_k)^{1-X_{ik}}, \quad X_{ik} = 0, 1\,, \tag{4}$$

where p_k is the probability that the i-th mobile in the k-th interfering cell is active at the same frequency and time slot as that of the desired mobile.

Power control compensates the long-term attenuation (i.e., attenuation due to path loss and shadowing) by different power level transmissions. The power control algorithm, which is based on the SIR measurement, has a risk of system instability [11]. Some radio links of the system may end up transmitting at, or alternating between, their maximum or minimum power. The power control algorithm for GSM and GPRS systems is recommended [12, 13] on the basis of the received strength power:

$$P(t) = \max\{P_{\min}, \min(\Gamma_0 - \alpha \cdot C, P_{\max})\} \quad \text{in dB}$$

or

$$P(t) = \max\left\{P_{\min}, \min\left(\frac{\Gamma_0}{C^{\alpha}}, P_{\max}\right)\right\}, \tag{5}$$

where $P_{\max}$ and $P_{\min}$ are the maximum and minimum transmitted power allowed by the system, respectively, Γ_0 is a predesigned constant, C is the received signal level, and α is a system parameter varying from 0 to 1. If the power control fully compensates the path loss and slow fading, α is equal to 1. However, if the path loss and slow fading are only partly compensated ($0 < \alpha < 1$), some gain in capacity is obtained [14]. More specifically, the transmitted power of the desired signal, $P_{T_{00}}$, is inversely proportional to $r_{00}^{-\alpha\eta}\xi_{00}^{\alpha}$. For user i in cell k, its transmitted power $P_{T_{ik}}$ is inversely proportional to $r_{0ik}^{-\alpha\eta}\xi_{ik}^{\alpha}$, where r_{0ik} and ξ_{0ik} refer to distance and slow fading on the path from mobile i in cell k to its own base station, respectively. However, the power control is imperfect in the practical system due to measurement error, and a power control error P^e may exist. The power control error P^e is normally considered as a log-normal random variable with a zero mean and a standard deviation σ_e [20, 22]. Therefore, we can rewrite Equations (1) and (2) as:

$$S = A' r_{00}^{-\eta(1-\alpha)} \xi_{00}^{1-\alpha} P_0^e Z_{00} = A' r_{00}^{-\eta(1-\alpha)} \beta_{00} Z_{00} \tag{6}$$

and

$$\begin{aligned} I &= \sum_{k=1}^{M}\sum_{i=1}^{N_k} I_{ik} = \sum_{k=1}^{M}\sum_{i=1}^{N_k} A' \left(\frac{r_{ik}}{r_{0ik}^{\alpha}}\right)^{-\eta} \xi_{ik}\xi_{0ik}^{-\alpha} P_{ik}^e Z_{ik} X_{ik} = \\ &= \sum_{k=1}^{M}\sum_{i=1}^{N_k} A' \left(\frac{r_{ik}}{r_{0ik}^{\alpha}}\right)^{-\eta} \beta_{ik} Z_{ik} X_{ik}, \end{aligned} \tag{7}$$

where $A' = A\Gamma_0$, $\beta_{00} = \xi_{00}^{1-\alpha} P_0^e$, and $\beta_{ik} = \xi_{ik}\xi_{0ik}^{\alpha} P_{ik}^e$. Because ξ_{00}, P_0^e, ξ_{ik}, ξ_{0ik} and P_{ik}^e are independent log-normal random variables, β_{00} and β_{ik} are independently log-normal random variables with a zero mean and the standard deviation $\sigma_{00} = \sqrt{(1-\alpha)^2\sigma_s^2 + \sigma_e^2}$ and $\sigma_{ik} = \sqrt{(1+\alpha^2)\sigma_s^2 + \sigma_e^2}$ (both in dB), respectively [15]. Here, we assume a slow fading factor and power control error with standard deviation σ_s and σ_e, respectively, for all mobile stations.

For Equation (6), we consider the log-normal random variable β_{00} to be superimposed on $A' r_{00}^{-\eta(1-\alpha)}$. This superimposition produces the local mean power $\overline{S}$ which is log-normally distributed with a standard deviation σ_{00} and a mean value $10\log(A' r_{00}^{-\eta(1-\alpha)})$, i.e.,

$$f_{\overline{S}}(\overline{S}|r_{00}) = \frac{1}{\sigma_{00}\sqrt{2\pi}(\overline{S}\ln 10/10)} \exp\left\{-\frac{[10\log(\overline{S}) - 10\log(A' r_{00}^{-\eta(1-\alpha)})]^2}{2\sigma_{00}^2}\right\}. \tag{8}$$

Applying the same principle in Equation (7), the local mean individual interference power $\overline{I}_{ik}$ is obtained by superimposing β_{ik} into the term of $A'[r_{ik}/(r_{0ik})^{\alpha}]^{-\eta}$. The PDF of the local mean individual interference power $\overline{I}_{ik}$ is

$$f_{\overline{I}_{ik}}(\overline{I}_{ik}|r_{ik}, r_{0ik}) = \frac{1}{\sigma_{ik}\sqrt{2\pi}(\overline{I}_{ik}\ln 10/10)} \exp\left\{-\frac{[10\log(\overline{I}_{ik}) - 10\log(A' r_{ik}^{-\eta}/r_{0ik}^{-\eta\alpha})]^2}{2\sigma_{ik}^2}\right\}. \quad (9)$$

Equations (6) and (7) can be rewritten as

$$S = \overline{S}Z_{00} \quad (10)$$

$$I = \sum_{k=1}^{M}\sum_{i=1}^{N_k} \overline{I}_{ik} Z_{ik} X_{ik}. \quad (11)$$

2.3. OUTAGE PROBABILITY

The outage probability of a system is the probability that the instantaneous signal power to interference power ratio (S/I) falls below a specified threshold γ and denoted as

$$P_o(outage) = \Pr\{S/I < \gamma\} \text{ i.e.,} \quad (12a)$$

$$P_o(outage) = \Pr\left\{\frac{\overline{S}Z_{00}}{\sum_{k=1}^{M}\sum_{i=1}^{N_k} \overline{I}_{ik} Z_{ik} X_{ik}} < \gamma\right\}. \quad (12b)$$

From Equation (12), we have

$$P_o(outage|\overline{S}, \overline{\boldsymbol{I}}, \boldsymbol{Z}, \boldsymbol{X}) = \Pr\left\{Z_{00} < \frac{\gamma}{\overline{S}}\sum_{k=1}^{M}\sum_{i=1}^{N_k} \overline{I}_{ik} Z_{ik} X_{ik}\right\}, \quad (13)$$

where $\overline{\boldsymbol{I}} = [\overline{I}_{11}, \overline{I}_{12}, \ldots \overline{I}_{N_k M}]$, $\boldsymbol{Z} = [Z_{11}, Z_{12}, \ldots Z_{N_k M}]$, $\boldsymbol{X} = [X_{11}, X_{12}, \ldots X_{N_k M}]$. Because Equation (12) refers to the probability of the signal-to-interference ratio (SIR) when applying Equations (8) and (9) to Equation (12), we can let $A' = 1$ without loss of generality. Using the PDF Z_{00} in Equation (3) in Equation (13), we have

$$\begin{aligned} P_o(outage|\overline{S}, \overline{\boldsymbol{I}}, \boldsymbol{Z}, \boldsymbol{X}) &= \int_0^{\frac{\gamma}{\overline{S}}\sum_{k=1}^{M}\sum_{i=1}^{N_k} \overline{I}_{ik} X_{ik} Z_{ik}} \exp(-Z_{00})dZ_{00} = \\ &= 1 - \prod_{k=1}^{M} \exp\left\{-\sum_{i=1}^{N_k} \frac{\gamma}{\overline{S}} \overline{I}_{ik} Z_{ik} X_{ik}\right\}. \end{aligned} \quad (14)$$

2.3.1. *Non-Frequency Hopping Systems*

For a non-frequency hopping system, there is only one co-channel user allowed in a cell. Thus, $N_k = 1$, and

$$P_o(outage|\overline{S}, \overline{\boldsymbol{I}}, \boldsymbol{Z}, \boldsymbol{X}) = 1 - \prod_{k=1}^{M} \exp\left\{-\frac{\gamma}{\overline{S}}\overline{I}_k Z_k X_k\right\}. \tag{15}$$

Averaging Equation (15) over Rayleigh fading and shadowing using Equations (3), (8), and (9) as shown in the Appendix, we have an average outage probability of

$$\begin{aligned}\overline{P}_o(outage) = 1 - \pi^{-\frac{1+N_d}{2}} \sum_{l=1}^{n} w_l \left\{ \prod_{k=1}^{N_v} \left[1 - p_k + \pi^{-\frac{1}{2}} p_k \sum_{j=1}^{n} w_j \frac{1}{1 + d_k \exp(cx_j - ax_l)} \right] \cdot \right.\\ \left. \cdot \prod_{q=1}^{N_d} \sum_{i=1}^{n} w_i \frac{1}{1 + d_q \exp(cx_i - ax_l)} \right\},\end{aligned} \tag{16}$$

where w_i is the weight of the n-point Gauss-Hermite quadrature formula and x_i is the abscissa of the i-th zero of Gauss-Hermite polynomial; N_v and N_d are the number of co-channels used by voice users and data users in M interfering cells, respectively. $a = \sqrt{2}\sigma_{00} \ln 10/10$, $c = \sqrt{2}\sigma_k \ln 10/10$, $d_k = \gamma(r_{00}^{1-\alpha} r_{0k}^{\alpha}/r_k)^{\eta}$, $d_q = \gamma(r_{00}^{1-\alpha} r_{0q}^{\alpha}/r_q)^{\eta}$, and r_{00} is the distance from the desired mobile to its host base station; r_k (or r_q) is the distance from the desired mobile to the k-th (or q-th) interfering base station of the voice (or data) user in downlink, or the distance from the host base station of the desired mobile to location of the co-channel voice (or data) user in the k-th (or q-th) interfering cell in uplink; r_{0k} (or r_{0q}) refers to the distance from the location of the co-channel voice (or data) user in the k-th (or q-th) interfering cell to its own base station. The outage probability is mainly dependent on the locations of mobiles, the frequency reused factor, and the channel load by GSM voice services and GPRS.

2.3.2. *Frequency Hopping Systems*

For frequency hopping systems, each channel in a cell is occupied by voice or data users with the same probability A_k

$$A_k = \frac{N_v(k) \cdot V_f + N_d(k)}{N_t \cdot N_{hop}(k)}, \tag{17}$$

where N_t is the number of time slots per TDMA frame and $N_{hop}(k)$ is the number of distinct frequency carriers in cell k. $N_v(k)$ and $N_d(k)$ are the number of channels used in cell k by voice users and data users, respectively V_f is the voice activity factor.

In a GSM frequency hopping system, each channel within a cell is orthogonal. Only the intercell interference is considered. Because of the non-uniform call traffic in practical networks, the number of frequency carriers allocated to different cells may not be the same. Therefore, the frequency set used in the desired cell may not be exactly the same as the frequency sets in the interfering cells. Let $N_{co}(k)$ be the number of frequency carriers used in both the desired cell and interfering cell k and N_{de} the total number of frequency carriers used in the desired cell. We define a fractional factor f_k as

$$f_k = \frac{N_{co}(k)}{N_{de}} \times \frac{N_{co}(k)}{N_{hop}(k)}. \tag{18}$$

Therefore, the interference probability caused by all users in cell k is $p_k = A_k \cdot f_k$. However, each $N_k = N_v(k) + N_d(k)$ co-channel user in cell k is equally likely to cause interference and they are mutually exclusive; the probability of interference from the i-th mobile in the k-th cell should use p_k/N_k. Combining Equations (4) and (14), we have

$$P_o(outage|\overline{S}, \overline{\boldsymbol{I}}, \boldsymbol{Z}) = 1 - \prod_{k=1}^{M} \left\{ (1 - p_k) + \frac{p_k}{N_k} \sum_{i=1}^{N_k} \exp\left(-\frac{\gamma}{\overline{S}} \overline{I}_{ik} Z_{ik} \right) \right\} . \tag{19}$$

Averaging Equation (19) over Rayleigh fading and shadowing using Equations (3), (8), and (9) as shown in the Appendix, we have an average outage probability of

$$\begin{aligned} \overline{P}_o(outage) &= \\ &= 1 - \pi^{-\frac{1}{2}} \sum_{l=1}^{n} w_l \left\{ \prod_{k=1}^{M} \left[1 - p_k + \pi^{-\frac{1}{2}} \frac{p_k}{N_k} \sum_{i=1}^{N_k} \sum_{j=1}^{n} w_j \frac{1}{1 + d_{ik} \exp(cx_j - ax_l)} \right] \right\} , \end{aligned} \tag{20}$$

where $d_{ik} = \gamma (r_{00}^{1-\alpha} r_{0ik}^{\alpha} / r_{ik})^{\eta}$, r_{00} is the distance from the desired mobile to its host base station, r_{ik} is the distance from the desired mobile to the k-th interfering base station of the i-th mobile in downlink or the distance from the host base station of the desired mobile to the i-th mobile in the k-th interfering cell in uplink, and r_{0ik} refers to the distance from mobile i in interference cell k to its own base station. Obviously, the outage probability is mainly dependent on the number of hopping frequencies, the locations of mobiles, the frequency reuse factor, and the channel load by GSM voice services and GPRS.

3. Numerical Results

Introducing GPRS into GSM networks may have an effect on the quality of service and capacity of the existing circuit-switched services. In this section, we will give some numerical results of the outage probability of the uplink affected by transmitting the packet data traffic in the GSM radio network resource.

The hexagonal cell cellular system with a 7-cell reused pattern is considered in this calculation. A central cell which is taken as the cell with desired mobiles has six interfering cells. Omnidirectional antennas are assumed to be used and the propagation loss exponent and slow fading factor are the same for all cells. The propagation loss exponent η and standard deviation σ_s of the slow fading are assumed to be 4 and 8 dB, respectively. The discontinuous transmission (DTX) method is assumed to be used in the system. The voice channel activity factor is assumed to be 0.4 and the packet data channel activity factor is used 100%, as discussed in Section 2. For power control, the algorithm partly compensating the path loss and shadowing is chosen $\alpha = 0.5$ in Equation (5).

A typical GSM system normally has 3 or 4 carriers per cell. Here, the number of carriers is assumed to be 4 in each cell. Thus, $4 \times 8 = 32$ physical channels are available in a cell. Suppose 3 channels are reserved for network signaling, then only 29 traffic channels are available for carrying the users' information in a cell. The mobile stations are uniformly distributed with an identical number in each cell.

Suppose the GSM network is operating at a blocking probability of 0.02 for voice services. For a cell with 29 traffic channels, an average traffic load of 21.04 Erlangs is supported. The average number of calls in the system is $E(n) = \rho(1 - P_b) = 21.04 \times 98\% = 20.62 \approx 21$.

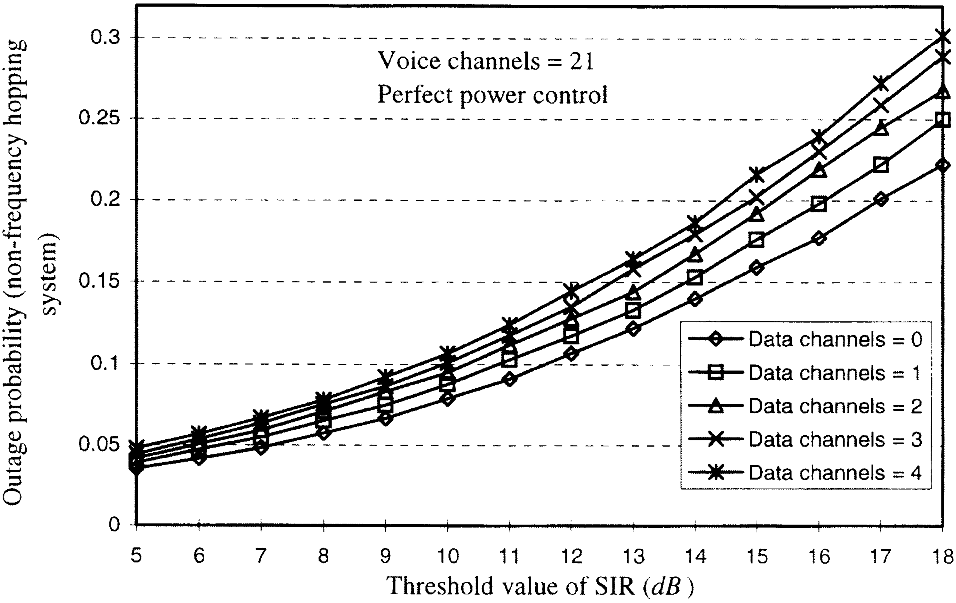

Figure 1a. Outage probability (90% worse case) of the non-frequency hopping system vs. SIR and different GPRS channel occupancy (21 channels occupied by voice services, perfect power control).

Our goal was to investigate how the performance of circuit-switched services are affected by the introduction of the GPRS traffic into existing GSM networks. Our interest is the interference statistics of voice services affected by GPRS. Therefore, we assume that the call-blocking probability of voice services is not affected by GPRS because of the dynamic resource allocation between voice services and GPRS.

The GSM specification [18] recommends a SIR threshold of 9 dB. A 2 dB implementation margin has been included in the simulated residual BER threshold. A threshold lower than 9 dB, such as 7 dB [19, 20], corresponding to a bit error rate of 10^{-3}, should be considered in the theoretical calculations. However, in practical network planning, operators normally use a SIR threshold higher than the recommended value of 9 dB, i.e., up to 12–14 dB. In addition, only one ring of the interference cells is considered in our calculation. Therefore, a threshold value of 10 dB is used in the following calculations.

The outage probability is calculated through a series of Monte Carlo simulations based on the generation of a large number of snapshots. In each snapshot, locations of the users are randomly generated, and a pure random channel allocation algorithm is used to assign channels to users in the central (desired) cell as well as in each interfering cell, according to the considered number of simultaneous users. Due to the random channel allocation, there are always some "good" channels with less co-channel users and "bad" channels with more co-channel users in the non-frequency hopping system, while for the frequency hopping system the quality of every channel is the same because of the interference diversity. From a system planner's point of view, the system should guarantee the quality of the "worst" channel with the largest number of co-channel users. Therefore, for the non-frequency hopping system, the outage probability of each snapshot is calculated from the "worst" channel. In a simulation, 10,000 snapshots are generated and the outage probability of each snapshot is calculated using Equations (16) or (20). The distribution of the outage probability in a given cell depends on the locations of the mobile stations in the cell and its interfering cells. Instead of giving

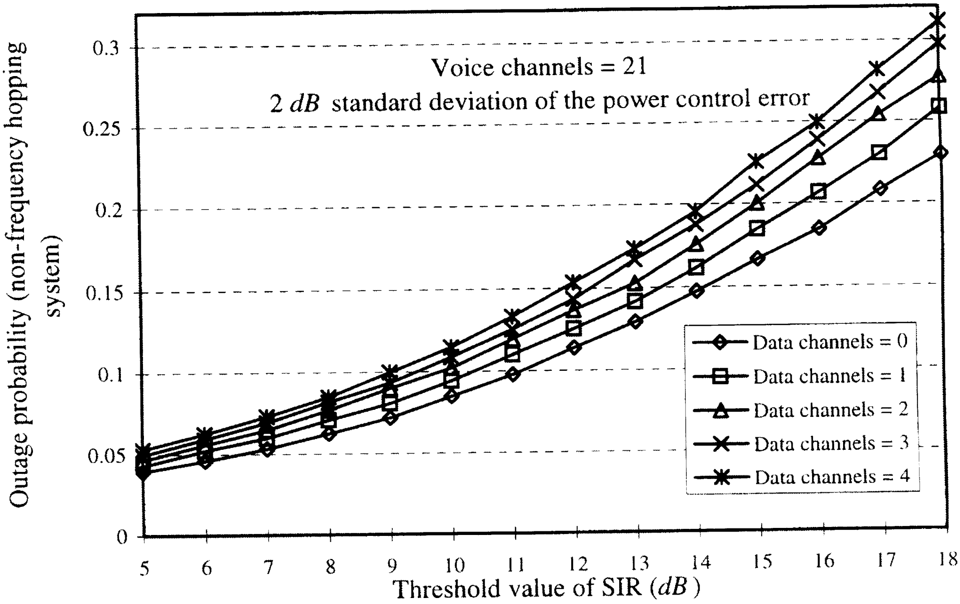

Figure 1b. Outage probability (90% worse case) of the non-frequency hopping system vs. SIR and different GPRS channel occupancy (21 channels occupied by voice services, standard deviation of the power control error = 2 dB).

a cumulative distribution function (CDF) of the outage probability, we only show the 0.9 percentile value [21] of the outage probability, called the 90% worst-case outage probability. The 90% worst-case value is obtained by sorting those 10,000 snapshots' values of the outage probability in increasing order and choosing the 9000th value.

For an average situation of 21 traffic channels used by voice services in each cell simultaneously, Figure 1 shows the 90% worst-case outage probability of the non-frequency hopping system with a SIR value for different GPRS channel occupancy. Figures 1(a, b) correspond to perfect power control and a 2 dB standard deviation of power control error in the system, respectively. As seen in Figure 2, with the 10 dB SIR threshold value, the outage probability increases by about 5%–10% whenever the number of channels used for GPRS increases by one. In addition, comparing Figures 1(a, b), about 0.5 dB gain for the SIR threshold value is found for the perfect-power control system.

Similarly, Figures 2(a, b) show the 90% worst-case outage probability of the frequency hopping system with the SIR value for different GPRS channel occupancy. Figures 2(a, b) correspond to perfect power control and 2 dB standard deviation of power control error in the system, respectively. The performance of the frequency hopping system in the outage probability is much better than that of the non-frequency hopping system. However, its characteristic of the outage probability affected by GPRS is similar to that of the non-frequency hopping system.

From Figure 1 and Figure 2, we find that the power control error has more impact on the system performance when more channels are allocated to GPRS.

Figure 3 shows the 90% worst-case outage probability of the non-frequency hopping system with a different number of channels occupied by voice services and GPRS. A SIR threshold value of 10 dB and perfect power control are assumed in the simulation. For the same number of channels used by GPRS, the outage probability does not vary very much from high channel occupancy to low channel occupancy of voice services. The main reason is that the

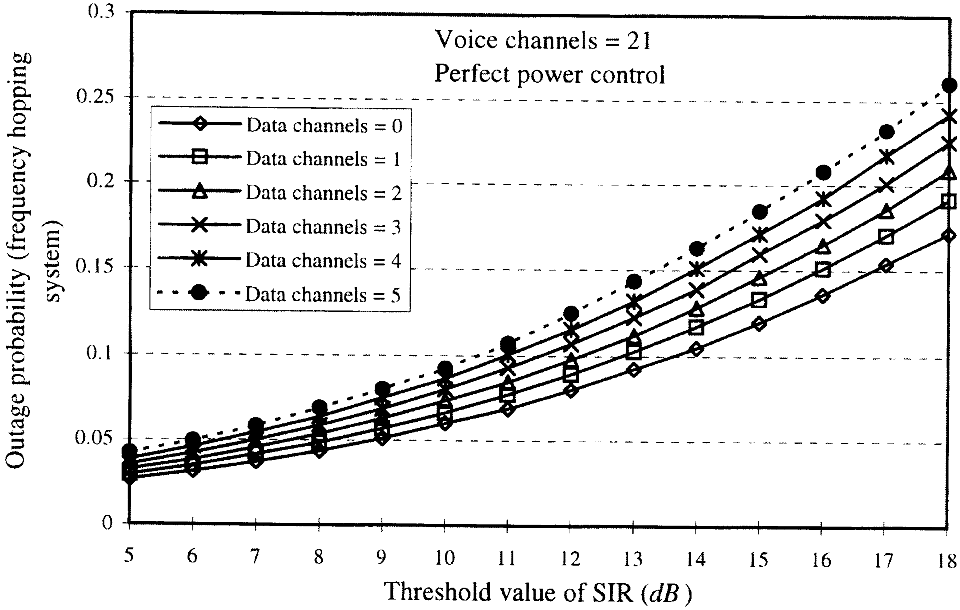

Figure 2a. Outage probability (90% worse case) of the frequency hopping system vs. SIR and different GPRS channel occupancy (21 channels occupied by voice services, perfect power control).

outage probability is calculated with the worst channel in each snapshot. This result implies that, for the non-frequency hopping system, the channels available to GPRS are not much different between high channel occupancy and low channel occupancy of voice services. The result is beyond of our expectations because we expected more resources available for GPRS at the low channel occupancy by voice traffic. Therefore, multislot GPRS services might cause a large degradation of the quality of existing voice services even at low channel occupancy of voice services. In order to guarantee the QoS of existing voice users, some observations, such as GPRS admission control, need to be taken before allocating unused channels to GPRS.

Figure 4 shows the 90% worst-case outage probability of the frequency hopping system with a different number of channels occupied by voice services and GPRS. For the same number of channels used by GPRS, the outage probability increases as the channel occupancy of voice services gets higher. From Figure 4, we find that the quality of voice services may not be affected by the introduction of GPRS into GSM with proper admission control of GPRS.

Cell coverage is normally determined by the received signal strength; however, the received value of the signal-to-interference ratio (SIR) is more relevant to describe the service area of a cell. Therefore, a parameter, the cell service area, is defined as the area over which a specified outage probability limit is achieved. In order to investigate the cell service area of existing voice services affected by GPRS, we simulate the outage probability distributed with the normalized radius (r/R, where R is the cell radius) in the desired cell. In each simulation, the location of the mobile station in the desired cell is restricted to a circle with a radius r, and locations of mobiles in interfering cells are randomly generated in those cells. Twenty-one simultaneous voice users are assumed in each cell and a SIR threshold value of 10 dB is used. Perfect power control is assumed in the system. Figures 5(a, b) show the 90% worst-case outage probability and average outage probability of the non-frequency hopping system distributed with normalized radius (r/R), respectively. Comparing the 90% worst-case outage

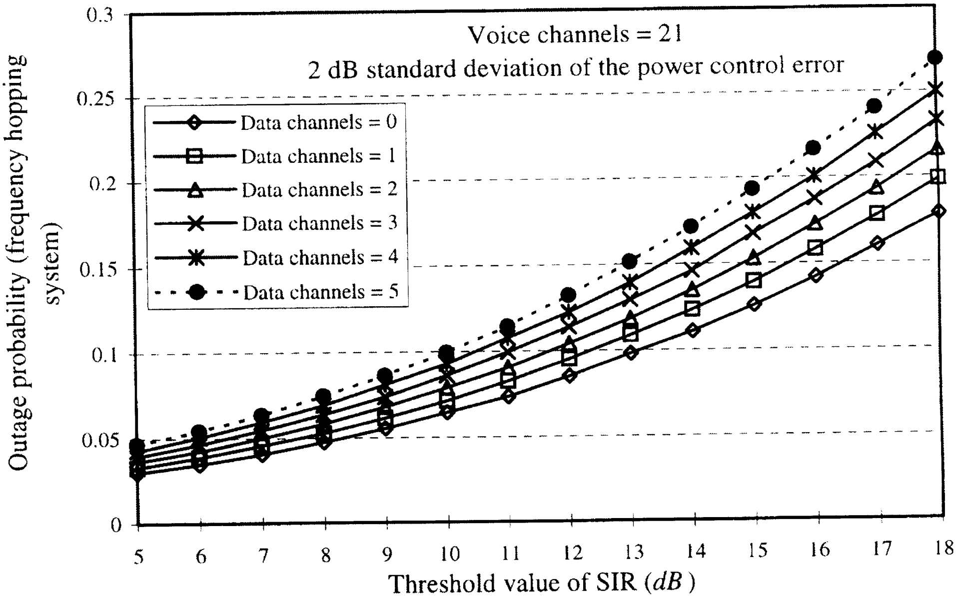

Figure 2b. Outage probability (90% worse case) of the frequency hopping system vs. SIR and different GPRS channel occupancy (21 channels occupied by voice services, standard deviation of the power control error = 2 dB).

probability with the average outage probability, we find that the 90% worst-case outage probability is much higher. With the same outage probability limit, both Figures 5(a, b) show that the cell service area decreases by about 10%≈20% whenever the number of channels used for GPRS increases by one. As more channels are allocated to GPRS, the cell service area decreases dramatically.

Figures 6(a, b) show the 90% worst-case outage probability and average outage probability of the frequency hopping system distributed with normalized radius r/R, respectively. Although the frequency hopping system has better performance in the cell service area than the non-frequency hopping system, the characteristics of the cell service area shrinkage because of the introduction of GPRS are similar to those of the non-frequency system. The cell service area decreases by about 15% whenever the number of channels used for GPRS increases by one.

4. Conclusions

GPRS is designed for transmitting packet data and is supposed to take its radio resources from the pool of unused channels by GSM voice services in order to increase the effective capacity of the digital cellular system. However, introducing GPRS into the existing GSM network risks degradation in either voice quality or voice capacity. In this paper, we present a method to calculate the outage probability of the GSM-GPRS network for both the non-frequency hopping and frequency hopping systems. This method takes into account fast (Rayleigh) fading, power control (with error), discontinuous transmission, and frequency hopping (if applied), and can be utilized to evaluate network performance and network planning for GSM-GPRS cellular systems. The analytic expressions of outage probability are obtained when considering a high load of GPRS (the data channel activity factor = 1); with a small change, they can be applied to the situation when the GPRS load is low (the data channel activity factor

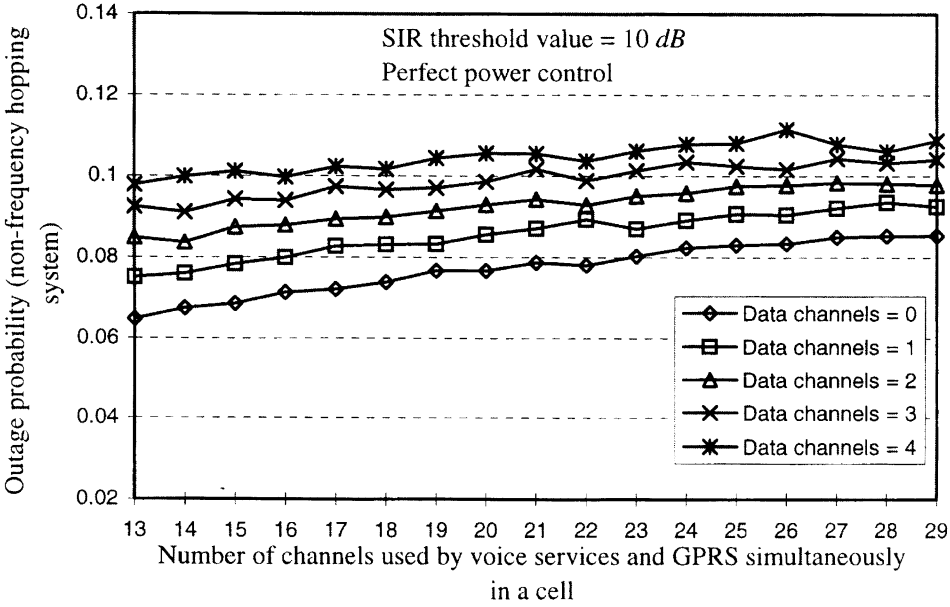

Figure 3. Outage probability (90% worst case) of the non-frequency hopping system vs. the different number of channels occupied by voice services and GPRS (SIR threshold value = 10 dB, perfect power control).

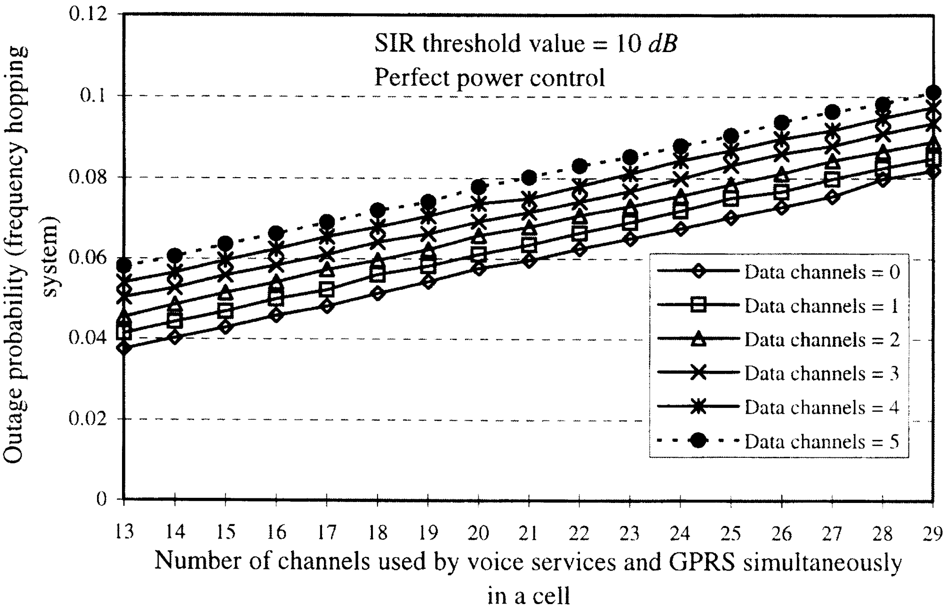

Figure 4. Outage probability (90% worst case) of the frequency hopping system vs. the different number of channels occupied by voice services and GPRS (SIR threshold value = 10 dB, perfect power control).

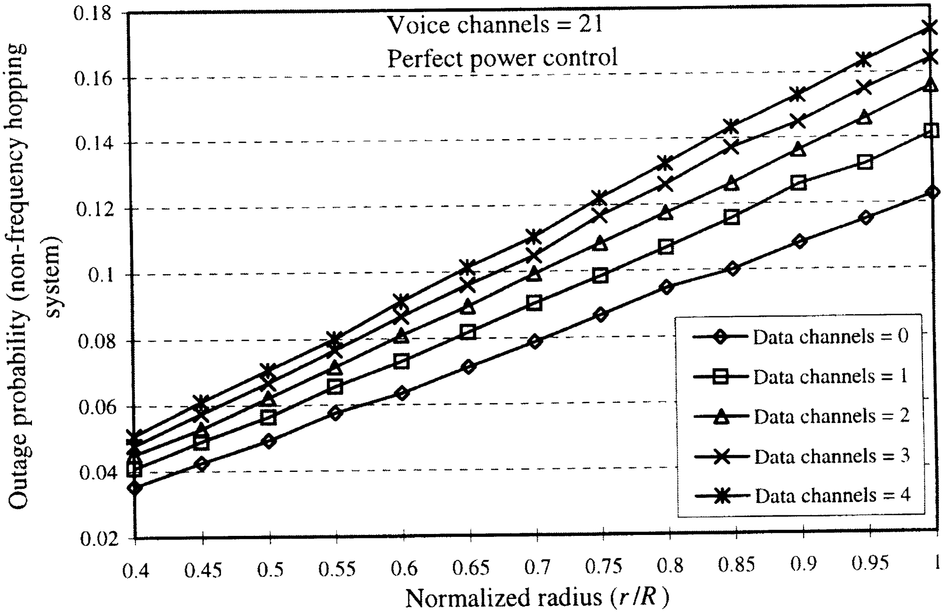

Figure 5a. Outage probability (90% worst case) of the non-frequency hopping system vs. normalized radius (r/R).

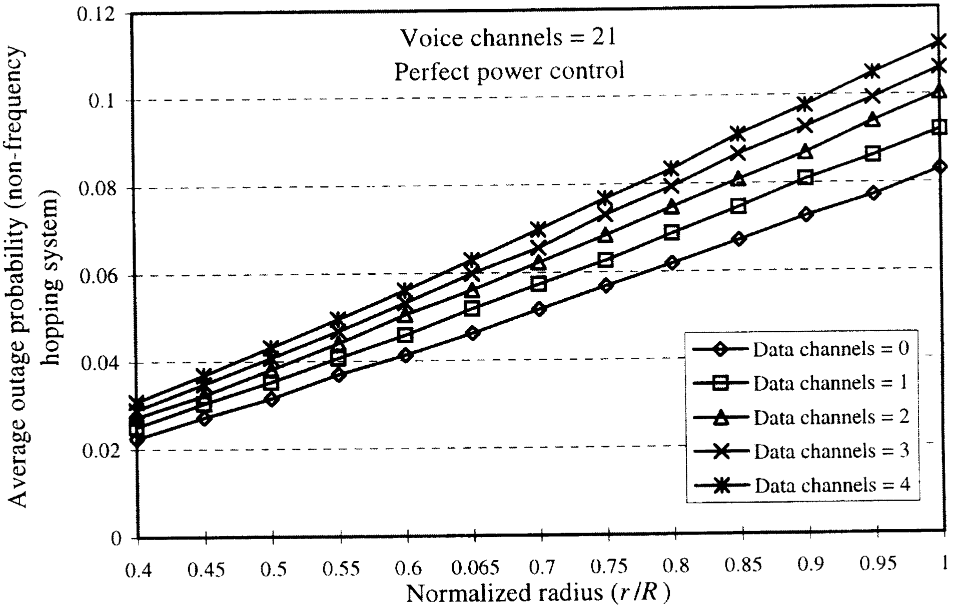

Figure 5b. Average outage probability of the non-frequency hopping system vs. normalized radius (r/R).

<1). Obviously, in the latter, the GPRS is less aggressive, but it still increases the interference statistics of voice services.

The effects on the quality of voice services due to the introduction of GPRS into the GSM network are evaluated by calculating the outage probability. Obviously, GPRS increases the outage probability of existing GSM voice services. Therefore, all those unused voice channels might not be used for carrying GPRS traffic. The number of unused voice channels allocated

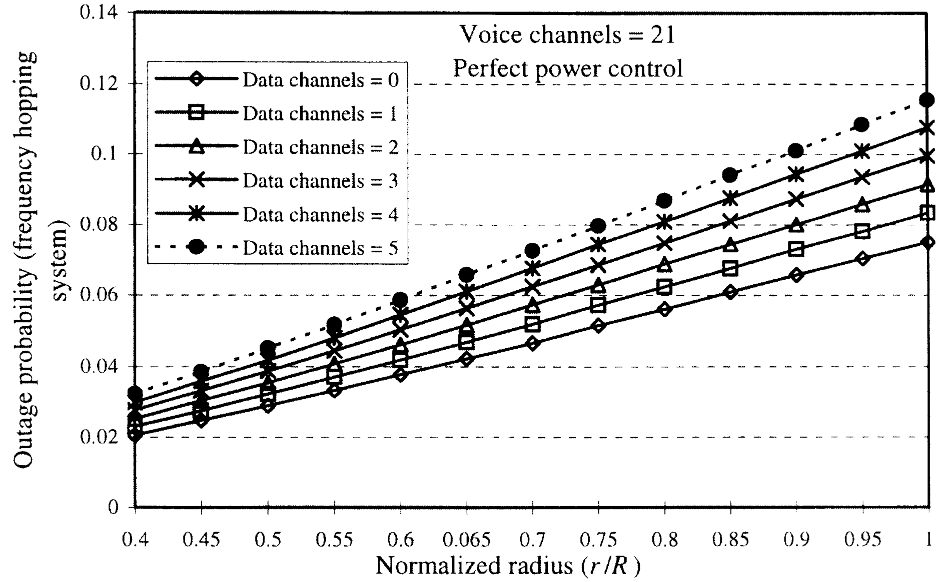

Figure 6a. Outage probability (90% worst case) of the frequency hopping system vs. normalized radius ($r/$).

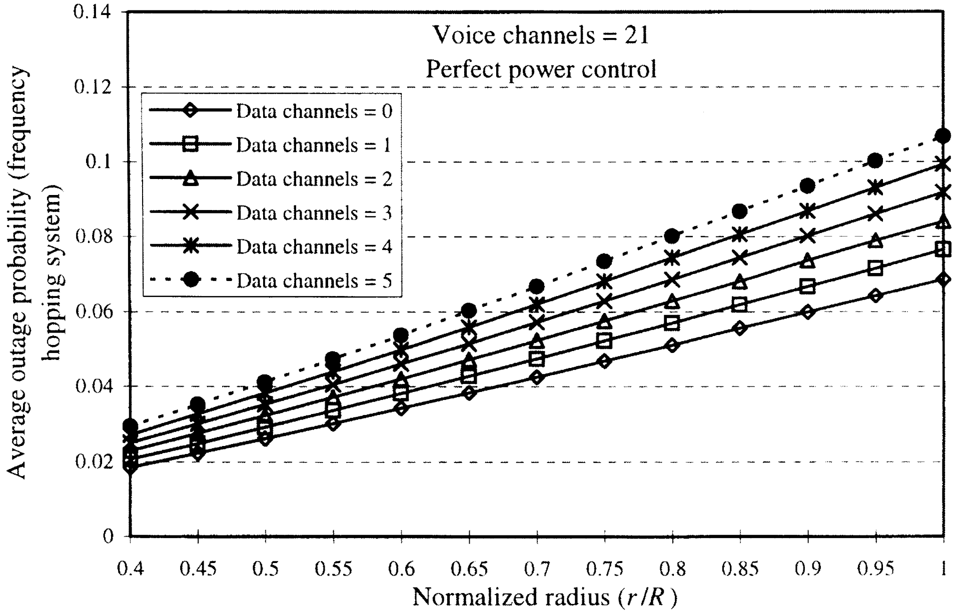

Figure 6b. Average outage probability of the frequency hopping system vs. normalized radius (r/R).

to GPRS depends on the difference between the outage level of the existing GSM network and the maximum acceptable level.

The frequency hopping system can accommodate more GPRS traffic than the non-frequency hopping system. The power control error has more impact on system performance when more channels are allocated to GPRS. Beyond our expectations, for the non-frequency hopping system, the channels provided for GPRS do not differ much between the high channel occupancy and the low channel occupancy of voice services. In contrast, for the frequency hopping system, the system may provide more channels for GPRS at the low channel occupancy. The cell service area is decreased by about 10%≈20% whenever the number of channels used for GPRS increases by one. As more channels are provided for GPRS, the cell service area decreases dramatically.

Appendix

A. Derivations of the Outage Probabilities

A.1. NON-FREQUENCY HOPPING SYSTEMS

Averaging Equation (15) over Rayleigh fading using the PDF of Z_k in Equation (3), we obtain

$$\begin{aligned} P_o(outage|\overline{S}, \boldsymbol{I}, \boldsymbol{X}) &= \\ &= 1 - \prod_{k=1}^{M} \int_0^{\infty} \exp\left\{ -\left(\frac{\gamma}{\overline{S}} \overline{I}_k X_k + 1 \right) Z_k \right\} dZ_k = 1 - \prod_{k=1}^{M} \frac{\overline{S}}{\gamma \overline{I}_k X_k + \overline{S}} \,. \end{aligned} \tag{A.1}$$

Let N_v and N_d be the total number of co-channel users used by voice and data in all interfering cells, respectively. For voice users, the activity factor V_f is about 40%, while for data users, the activity factor $V_d = 100\%$. Therefore, for data users, $X_k \equiv 1$, while for voice users, the probability p_k in Equation (4) is equal to the voice activity V_f. Combining Equations (A.1) and (4), we obtain

$$P_o(outage|\overline{S}, \overline{I}) = 1 - \prod_{k=1}^{N_v} \left\{ (1 - p_k) + p_k \frac{\overline{S}}{\gamma \overline{I}_k + \overline{S}} \right\} \prod_{q=1}^{N_d} \frac{\overline{S}}{\gamma \overline{I}_q + \overline{S}} \,. \tag{A.2}$$

Averaging Equation (A.2) over the shadowing by Equations (8) and (9), we have

$$\begin{aligned} \overline{P}_o(outage) = 1 - \int_{-\infty}^{\infty} &\frac{1}{\sigma_{00}\sqrt{2\pi}(\overline{S} \ln 10/10)} \exp\left\{ -\frac{[10\log(\overline{S}) + 10\eta(1-\alpha)\log(r_{00})]^2}{2\sigma_{00}^2} \right\} \cdot \\ \cdot \prod_{k=1}^{N_v} \int_{-\infty}^{\infty} &\frac{\left(1 - p_k + p_k \dfrac{\overline{S}}{\gamma \overline{I}_k + \overline{S}}\right)}{\sigma_k \sqrt{2\pi}(\overline{I}_k \ln 10/10)} \exp\left\{ -\frac{[10\log(\overline{I}_k) + 10\eta\log(r_k/r_{0k}^{\alpha})]^2}{2\sigma_k^2} \right\} d\overline{I}_k d\overline{S} \cdot \\ \cdot \prod_{q=1}^{N_d} \int_{-\infty}^{\infty} &\frac{\dfrac{\overline{S}}{\gamma \overline{I}_q + \overline{S}}}{\sigma_q \sqrt{2\pi}(\overline{I}_q \ln 10/10)} \exp\left\{ -\frac{[10\log(\overline{I}_q) + 10\eta\log(r_q/r_{0q}^{\alpha})]^2}{2\sigma_q^2} \right\} d\overline{I}_q d\overline{S} \,. \end{aligned} \tag{A.3}$$

Here, $\sigma_k = \sigma_q = \sqrt{(1+\alpha^2)\sigma_s^2 + \sigma_e^2}$. Let

$$\begin{aligned} &a = \sqrt{2}\sigma_{00} \ln 10/10,\, b = \eta(1-\alpha)\ln r_{00},\, x = (10\log \overline{S} + 10\eta(1-\alpha)\log r_{00})/\sqrt{2}\sigma_{00} \,, \\ &c = \sqrt{2}\sigma_k \ln 10/10,\, d_k = \gamma(r_{00}^{1-\alpha}/r_k)^{\eta},\, y = \{10\log \overline{I}_k + 10\eta\log(r_k/r_{0k}^{\alpha})\}/\sqrt{2}\sigma_k \,, \\ &d_q = \gamma(r_{00}^{1-\alpha} r_{0q}^{\alpha}/r_q)^{\eta},\, z = \{10\log \overline{I}_q + 10\eta\log(r_q/r_{0q}^{\alpha})\}/\sqrt{2}\sigma_q \,. \end{aligned}$$

We have

$$\overline{P}_o = 1 - \pi^{-\frac{1+N_d}{2}} \int_{-\infty}^{\infty} dx \cdot \exp(-x^2) \prod_{k=1}^{N_v} \left\{ 1 - p_k + \pi^{-\frac{1}{2}} p_k \int_{-\infty}^{\infty} \frac{\exp(-y^2)dy}{1 + d_k \exp(cy - ax)} \right\} \cdot \prod_{q=1}^{N_d} \int_{-\infty}^{\infty} \frac{\exp(-z^2)dz}{1 + d_q \exp(cz - ax)} . \tag{A.4}$$

Those integrals can be calculated approximately using the Gauss-Hermite polynomial method [16, 17], i.e.,

$$\int_{-\infty}^{\infty} e^{-x^2} f(x)dx = \sum_{i=1}^{n} w_i f(x_i) + R_n,\ R_n = \frac{n!\sqrt{\pi}}{2^n(2n)!} f^{(2n)}(\xi) \quad (-\infty < \xi < \infty), \tag{A.5}$$

where w_i is the weight of the n-point Gauss-Hermite quadrature formula and x_i is the abscissa of the i-th zero of Gauss-Hermite polynomial, which are tabulated in [16] or calculated according to [17]. It is not difficult to find that the remainder R_n is sufficiently small for $n > 10$ (in this paper, we use $n = 20$), if Equation (A.5) is applied in Equation (A.4). Applying Equation (A.5) in Equation (A.4), we obtain

$$\overline{P}_o(outage) = 1 - \pi^{-\frac{1+N_d}{2}} \sum_{l=1}^{n} w_l \left\{ \prod_{k=1}^{N_v} \left[1 - p_k + \pi^{-\frac{1}{2}} p_k \sum_{j=1}^{n} w_j \frac{1}{1 + d_k \exp(cx_j - ax_l)} \right] \cdot \prod_{q=1}^{N_d} \sum_{i=1}^{n} w_i \frac{1}{1 + d_q \exp(cx_i - ax_l)} \right\} . \tag{A.6}$$

A.2. FREQUENCY HOPPING SYSTEM

Averaging Equation (20) over Rayleigh fading by Equation (3), we obtain

$$P_o(outage|\overline{S}, \overline{\boldsymbol{I}}) = 1 - \prod_{k=1}^{M} \left\{ (1 - p_k) + \frac{p_k}{N_k} \sum_{i=1}^{N_k} \frac{1}{1 + \frac{\gamma}{\overline{S}} \overline{I}_{ik}} \right\} . \tag{A.7}$$

Comparing Equations (A.7) and (17), we can easily obtain the outage probability according to the last session result of Equation (A.4),

$$\overline{P}_o = 1 - \pi^{-\frac{1}{2}} \int_{-\infty}^{\infty} dv \cdot \exp(-v^2) \prod_{k=1}^{M} \left\{ p_k + \pi^{-\frac{1}{2}} \frac{p_k}{N_k} \sum_{i=1}^{N_k} \int_{-\infty}^{\infty} \frac{\exp(-u^2)du}{1 + d_{ik} \exp(cu - av)} \right\} , \tag{A.8}$$

where $d_{ik} = \gamma (r_{00}^{1-\alpha} r_{0ik}^{\alpha} / r_{ik})^{\eta}$. Using the integral result of Equation (A.5), we have

$$\overline{P}_o(outage) = \\ = 1 - \pi^{-\frac{1}{2}} \sum_{l=1}^{n} w_l \left\{ \prod_{k=1}^{M} \left[1 - p_k + \pi^{-\frac{1}{2}} \frac{p_k}{N_k} \sum_{i=1}^{N_k} \sum_{j=1}^{n} w_j \frac{1}{1 + d_{ik} \exp(cx_j - ax_l)} \right] \right\} . \tag{A.9}$$

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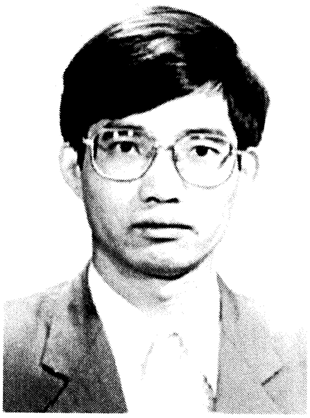

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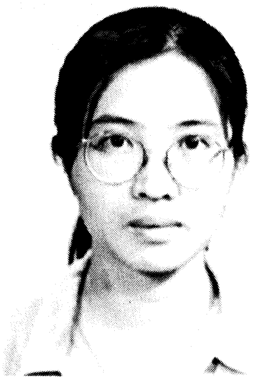

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