Outage Probability for GPRS over GSM Voice Services

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Abstract—**General packet radio service (GPRS) is designed for transmitting packet data and supposed to take its radio resource from the pool of channels unused by GSM voice services. Obviously, the introduction of GPRS affects on the voice services. In this paper the outage probability of non-frequency hopping and frequency hopping systems with different frequency reuse factor has been calculated in order to investigate those effects. The number of unused voice channels allocated to GPRS depends on the difference between the outage level of the existing GSM network and the maximum acceptable level. For both non-frequency hopping and frequency hopping systems, GPRS affects the QoS of voice services of the network with small reuse factor are higher than that of the network with large reuse factor. GPRS will reduce the cell service area, but the reduction percentage of the cell service area for the system with small reuse factor is higher than that for the system with large reuse factor.**

I. Introduction

The current method of data transmission in the pan-European Global System for Mobile Communications (GSM) and the American Advanced Mobile Phone Standard (AMPS) cellular networks is circuit switching. This technique reserves the traffic channel for the entire communication time, and wastes the radio resource when data traffic occurs in bursts with long silent intervals. In the development of GSM phase 2+, the European Telecommunications Standard Institute (ETSI) has specified [1] a general packet radio service (GPRS) over the GSM to increase the utilization efficiency of the radio resource and provide a set of additional services.

The GPRS is designed to utilize those channels unused by the circuit switched services to transmit short bursts of packet data. However, the introduction of GPRS into GSM networks without allocating new spectrum will increase the interference probability of circuit switched services. In addition, the physical channel allocated to GPRS is shared by a few data users simultaneously. The cochannel interference to the voice users might vary rapidly and dramatically in the time interval from 20 ms to a few seconds depending on the transmitted packet data size, because the locations of those packet data users could be largely different. This effect could drive the system into an unpredictable and unstable situation except for causing a degradation of the QoS of voice services. Therefore, a preliminary resource planning for GPRS is required to guarantee the quality of service for voice users. A method to calculate the outage probability of the GSM-GPRS network for both the non-frequency hopping and the frequency hopping systems has been presented in [4], but the paper only discussed the outage probability for the system with a frequency reuse factor of 7. In this paper, we focus on discussions of the outage probability for systems with different frequency reuse factors in order to provide some guidelines for system evaluation and planning, as well as the admission control. This paper is organized as: in section II, the calculation principles of the outage probability for the GSM-GPRS network are explained; in section III, the numerical results of the outage probability for systems with different frequency reuse factors are presented and the quality of service (QoS) of voice services affected by GPRS is discussed; finally the conclusions are given in section IV.

II. Outage Probability

A. System model

All signals are assumed to experience Rayleigh fading with respect to a local mean signal strength, while the local mean signal strength experiences log-normal slow fading (with a standard deviation σ_s) based on a mean value, which determined by the propagation loss law of an inverse η*-th* power of distance. The desired signal and interfering signals, and among the different individual interfering signal are statistically mutually independent. The slow fading is assumed to be uncorrelated and the variance is the same for all cells. The power control algorithm is assumed to compensate the pathloss and slow fading partly ($0<\alpha<1$) [2, 3]. The power control error is considered as a lognormal random variable with a zero mean and a standard deviation σ_e.

Voice activity detection is assumed to be used in the system. Using the discontinuous transmission (DTX) method, the transmitted power of a user is reduced to a low level during speech pauses, and its interference to other users is assumed to be zero. For a voice user, the active time of speech is about 40% of his call duration in average, and the rest of the call duration is in listening state. Therefore, when a channel is used for transmitting voice, the activity factor is about 40%. The selective automatic repeat request (ARQ) protocol is used for GPRS data transfer. The MAC protocol [2] is developed to multiplex up to eight different MSs of GPRS onto one physical channel. Different packet data logical channels and the PDTCH can also be multiplexed on the same physical channel. Due to the number of channels available for GPRS and the multiplexing mechanism, a channel accommodating a few GPRS users, can be assumed to be in the transmitting state all the time, if the users requesting service are large enough. Therefore, the activity factor of a channel carrying the packet data traffic is used as 100% here as considering a high load of GPRS traffic.

B. Outage Probability

The outage probability of a system is the probability that the instantaneous signal power to interference power ratio (*S*/*I*) falls below a specified threshold γ and denoted as

$$P_o(\text{outage}) = \Pr\{S / I < \gamma\} \tag{1}$$

A method to calculate the outage probability of the GSM-GPRS network for both the non-frequency hopping and the frequency hopping systems has been presented [4]. This method takes into account the Rayleigh fading, power control error, discontinuous transmission, and frequency hopping. According to [4], the average outage probability for without frequency hopping system is:

$$\begin{aligned}\overline{P}_o(\text{outage}) = 1 - p^{-\frac{1+N_d}{2}} \sum_{l=1}^{n} w_l \{ \prod_{k=1}^{N_v} \Big[\ 1 - p_k \\ + \pi^{-\frac{1}{2}} p_k \sum_{j=1}^{n} w_j \frac{1}{1 + d_k \exp(cx_j - ax_l)} \ \Big] \\ \cdot \prod_{q=1}^{N_d} \sum_{i=1}^{n} \ w_i \frac{1}{1 + d_q \exp(cx_i - ax_l)} \} \end{aligned} \tag{2}$$

where w_i is the weight of the *n*-point Gauss-Hermite quadrature formula and x_i is the abscissa of the *i*-th zero of Gauss-Hermite polynomial [5]; N_v and N_d are the number of cochannels used by voice users and data users simultaneously in *M* interfering cells respectively,

$$\sigma_k = \sigma_q = \sqrt{(1+\alpha^2)\sigma_s^2 + \sigma_e^2}\ ,\ \sigma_{00} = \sqrt{(1-\alpha)^2\sigma_s^2 + \sigma_e^2}\ ,$$
$$a = \sqrt{2}\sigma_{00} \ln 10 / 10\ ,\ c = \sqrt{2}\sigma_k \ln 10 / 10\ ,$$
$$d_k = \gamma(r_{00}^{1-\alpha} r_{0k}^{\alpha} / r_k)^{\eta}\ ,\ d_q = \gamma(r_{00}^{1-\alpha} r_{0q}^{\alpha} / r_q)^{\eta}\ ,$$

r_{00} is the distance from the desired mobile to its host base station; r_k (or r_q) is the distance from the desired mobile to the *k*-th interfering base station of voice (or data) user in downlink, or the distance from the host station to location of the cochannel voice (or data) user in the *k*-th interfering cell in uplink; r_{0k} (or r_{0q}) refers to distance from location of the cochannel voice (or data) user in the *k*-th interfering cell its own base station. The outage probability is mainly dependent on the locations of mobiles, the frequency reuse factor and the channel load by GSM voice services and GPRS.

For frequency hopping systems, each channel in a cell is occupied by voice or data users with the same probability p_k,

$$p_k = \frac{N_v(k) \cdot V_f + N_d(k)}{N_t \cdot N_{hop}(k)} \tag{3}$$

where N_t is the number of time slots per TDMA frame and $N_{hop}(k)$ is the number of distinct frequency carriers in cell *k*; $N_v(k)$ and $N_d(k)$ are the number of channels using in cell *k* by voice users and data users respectively. Here we assume the number of frequency carriers allocated to different cells are the same.

Because each of $N_k = N_v(k) + N_d(k)$ co-channels in cell *k* is equally likely to cause interference and they are mutually exclusive, the probability of interference from the *i*-th mobile in the *k*-th cell should use p_k/N_k.

The average outage probability with frequency hopping [4] is

$$\begin{aligned}\overline{P}_o(\text{outage}) = 1 - \pi^{-\frac{1}{2}} \sum_{l=1}^{n} w_l \{ \sum_{k=1}^{M} \Big[1 - p_k \\ + \pi^{-\frac{1}{2}} \frac{p_k}{N_k} \sum_{i=1}^{N_k} \sum_{j=1}^{n} w_j \frac{1}{1 + d_{ik} \exp(cx_j - ax_l)} \ \Big] \} \end{aligned} \tag{4}$$

where $d_{ik} = \gamma(r_{00}^{1-\alpha} r_{0ik}^{\alpha} / r_{ik})^{\eta}$; r_{00} is the distance from the desired mobile to its host base station; r_{ik} is the distance from the desired mobile to the *k*-th interfering base station of the *i*-th mobile in downlink, or the distance from the host base station of the desired mobile to the *i*-th mobile in the *k*-th interfering cell in uplink; r_{0ik} refers to distance from mobile *i* in interference cell *k* to its own base station.

III. Numerical Results

In this section, we will give some numerical results of the outage probability of the uplink affected by transmitting the GPRS traffic in the GSM radio network resource. The effects of GPRS on the outage probability for systems with different frequency reuse factors are our main focus.

A. System model

A hexagonal cell's cellular system with omnidirectional antennas is assumed to be used. A central cell which is taken as the cell with desired mobiles has six interfering cells. The propagation loss exponent η and standard deviation σ_s of the slow fading are same for all cells and assumed to be 4 and 8 *dB* respectively. The voice channel activity factor is assumed to be 0.4, and the packet data channel activity factor is 100%. For power control, the algorithm partly compensating the path loss and shadowing is chosen with α = 0.5 [2, 3]. The threshold value of SIR is 10 *dB*.

The number of physical channels are 32 (4×8) available in a cell. It is supposed that 3 channels are reserved for the network signaling, thus, only 29 channels are available for traffic in a cell. Suppose the GSM network operated at a blocking probability of 0.02 for voice services. For a cell with 29 traffic channels, an average traffic load of 21.04 Erlangs is then supported. The average number of voice calls in the system is $E(n) = \rho(1-P_b) = 21.04 \times 98\% = 20.62 \approx 21$. This average number of voice calls is considered in our calculations only. Based on the average situation with 21 channels used for voice services, a few channels are assumed to be allocated to GPRS and the outage probability is calculated. The system parameters are listed in Table 1.

TABLE 1. System parameters.

frequency reuse factor	from 3 to 19
propagation loss exponent	$\eta = 4$
std. deviation of the slow fading	$\sigma_s = 8\ dB$
voice channel activity factor	0.4
GPRS channel activity factor	100%
power control (partly compensation)	α = 0.5 [2, 3]
std. deviation of power control error	σ_e=0 and 2 *dB*
SIR threshold value	$\gamma = 10\ dB$
total number of traffic channels	29
voice traffic load (P_b=2%)	21.04 Erlangs
channels simultaneously used by voice	21

The mobile stations are uniformly distributed with an identical number in each cell. The outage probability is calculated through a series of Monte Carlo simulations based on generating a large number of snapshots. In each snapshot locations of users are randomly generated, and a pure random channel allocation algorithm is used to assign channels to users in the central (desired) cell as well as in each interfering cell, according to the considered number of simultaneous users. Due to the random channel allocation, for the non-frequency hopping system there always exist some "good" channels with less cochannel users and "bad" channels with more cochannel users, but for the frequency hopping system the quality of every channel is the same because of the interference diversity. From the system planner's point of view, the system should guarantee the quality of the "worst" channel with the largest number of cochannel users. Therefore, for the non-frequency hopping system the outage probability of each snapshot is calculated from the "worst" channel. In a simulation, 10000 snapshots are generated and the outage probability of each snapshot is calculated by (2) or (4). The distribution of the outage probability in a given cell depends on locations of the mobile stations in this cell and its interfering cells. Instead of giving a cumulative distribution function (CDF) of the outage probability, we only show the 0.9 percentile value, called 90% worst case outage probability. The 90% worst case outage probability is obtained by sorting those 10000 snapshots' values in increasing order and choosing the 9000th value.

B. Results

The following results are calculated for a SIR threshold value of 10 *dB* and the average situation of 21 traffic channels used by voice services in each cell simultaneously.

Fig. 1 and Fig. 2 show the 90% worst case outage probability of the non-frequency hopping system as function of the frequency reuse factor and with the number of channels used for GPRS as a parameter. Fig. 1 and Fig. 2 correspond to the perfect power control (σ_e=0 *dB*) and the power control with an error of 2 *dB* standard deviation (σ_e=2 *dB*) in the system respectively. As seen from the figures, the outage probability increases by about 5%-10% whenever the number of channels used for GPRS increases by one. The outage probability increase of the network with a small frequency reuse factor caused by GPRS is higher than that of the network with a large frequency reuse factor. Therefore, the GPRS affects the QoS of voice services of the network with small size reuse factor more.

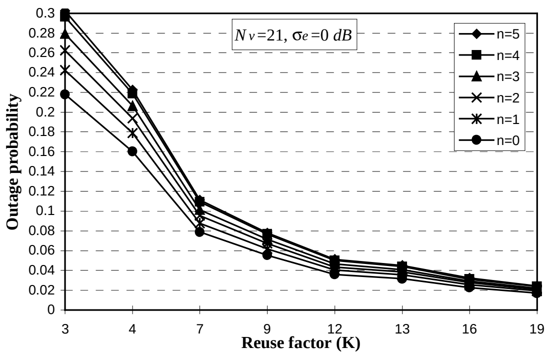

Fig. 1 The 90% worst case outage probability of the non-frequency hopping system (perfect power control, σ_e=0 *dB*). N_v and n denote the number of channels simultaneously used by voice services and GPRS respectively.

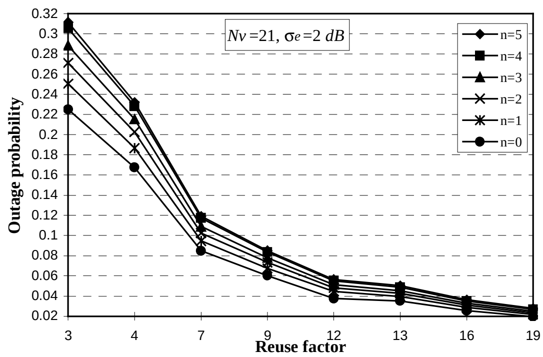

Fig. 2 The 90% worst case outage probability of the non-frequency hopping system (power control with an error of 2 *dB* standard deviation, σ_e=2 *dB*). N_v and n denote the number of channels simultaneously used by voice services and GPRS respectively.

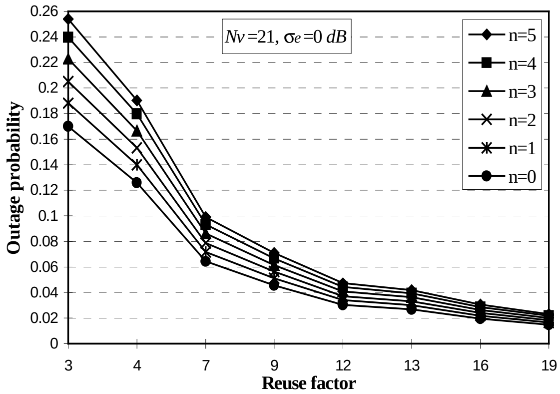

Fig. 3 The 90% worst case outage probability of the frequency hopping system (perfect power control σ_e=0 *dB*). N_v and *n* denote the number of channels simultaneously used by voice services and GPRS respectively.

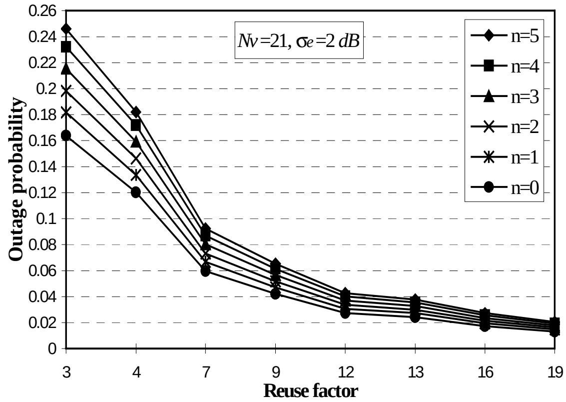

Fig. 4 The 90% worst case outage probability of the frequency hopping system (power control with an error of 2 *dB* standard deviation, σ_e=2 *dB*). N_v and *n* denote the number of channels simultaneously used by voice services and GPRS respectively.

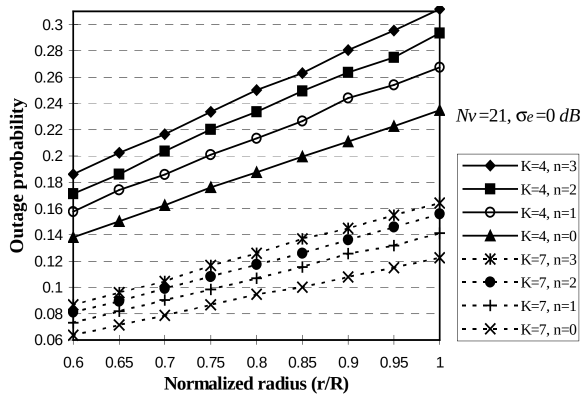

Fig. 5 The 90% worst case outage probability distributed with normalized radius (*r*/*R*) for the non-frequency hopping systems with the reuse factor (*K*) of 4 and 7 respectively.

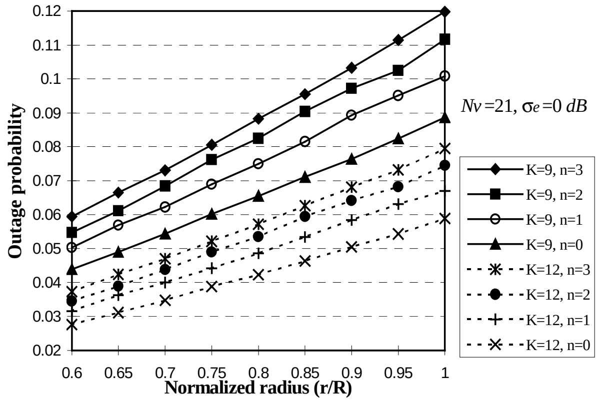

Fig. 6 The 90% worst case outage probability distributed with normalized radius (*r*/*R*) for the non-frequency hopping systems with the reuse factor (*K*)of 9 and 12 respectively.

By comparing the outage probability of the same reuse factor and same number of channels used by GPRS in Fig. 1 and 2, we find that the power control error causes the increase of the outage probability in the system with large number of GPRS channels higher than that in the system with small number of GPRS channels. Therefore, the power control error has more impact on the system performance when more channels are allocated to GPRS. This feature is also mentioned in the paper [4].

Fig. 3 and 4 show the 90% worst case outage probability of the frequency hopping system as function of the frequency reuse factor and with the number of channels used for GPRS as a parameter. Fig. 3 and 4 correspond to the perfect power control and the power control with an error of 2 *dB* standard deviation in the system respectively. The performance of the frequency hopping system in the outage probability is much better than that of the non-frequency hopping system. However, the general behavior of the outage probability affected by GPRS is similar to that of the non-frequency hopping system.

In this paper, a parameter, the cell service area, is defined as the area over which a specified outage probability limit is achieved. In order to investigate the cell service area of existing voice services affected by GPRS, we simulate the outage probability distributed with the normalized radius (*r*/*R*, *R* is the cell radius) in the desired cell. In each simulation, the location of the mobile station in the desired cell is restricted to a circle with a radius *r*, and locations of mobiles in interfering cells are randomly generated in those cells. Fig. 5 and Fig. 6 show the 90% worst case outage probability distributed with normalized radius (*r*/*R*) for the non-frequency hopping systems with the reuse factor of 4, 7, 9 and 12 respectively. The figures show that GPRS will reduce the cell service area, but the reduction percentage of the cell service area for the system with small reuse factor is higher than that for the system with large reuse factor.

Fig. 7 and Fig. 8 show the 90% worst case outage probability distributed with normalized radius (*r*/*R*) for the frequency hopping systems with the reuse factor of 4, 7, 9 and 12 respectively. The behavior of the cell service area shrinking as the introduction of GPRS is similar to that of the non-frequency hopping system.

In summary, GPRS which dynamically shares radio resource with GSM voice services may not reduced the capacity of voice services, however, the system performance will be degraded, e.g., outage probability will increase and cell service area decrease, due to the additional interference contributed by packet data transmission. The physical channel allocated to GPRS is shared by a few GPRS users simultaneously. The cochannel interference to the voice users might vary rapidly and dramatically in the time interval from 20 ms to a few seconds depending on the transmitted packet data size, because the locations of those packet data users could be largely different. This effect could drive the system into an unpredictable and unstable situation except for causing a degradation of the QoS of voice services. Therefore, a preliminary resource planning for GPRS is required to guarantee the quality of service for voice users. The number of unused voice channels allocated to GPRS depends on the difference between the outage level of the existing GSM network and the maximum acceptable level.

IV. CONCLUSIONS

In this paper, the effects on the quality of voice services due to the introduction of GPRS into GSM network are investigated by calculations of the outage probability. The outage probability of non-frequency hopping system and frequency hopping system with different frequency reuse factors has been calculated. Since GPRS increases the outage probability of existing GSM voice services, all those unused voice channels might not be used for carrying GPRS traffic. The number of unused voice channels allocated to GPRS depends on the difference between the outage level of the existing GSM network and the maximum acceptable level.

For both non-frequency hopping system and frequency hopping system, GPRS affects on the QoS of voice services of the network with small reuse factor are higher than that of the network with large reuse factor. The power control error has more impact on the system performance when more channels are allocated to GPRS. GPRS will reduce the cell service area, but the reduction percentage of the cell service area for the system with small reuse factor is higher than that for the system with large reuse factor.

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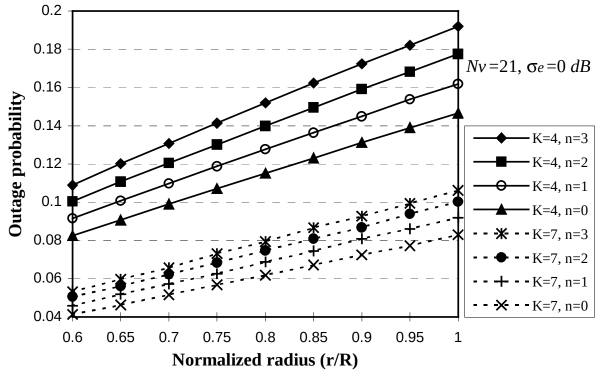

Fig. 7 The 90% worst case outage probability distributed with normalized radius (r/R) for the frequency hopping systems with the reuse factor (K) of 4 and 7 respectively.

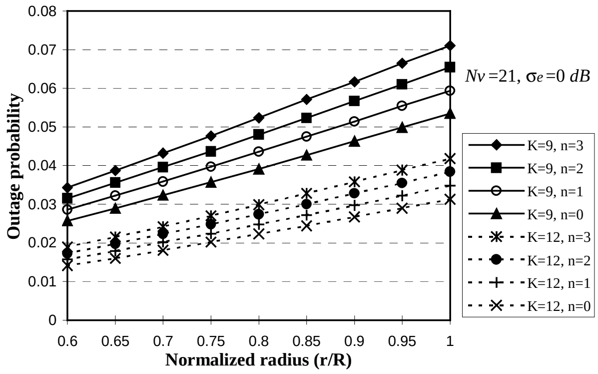

Fig. 8 The 90% worst case outage probability distributed with normalized radius (r/R) for the frequency hopping systems with the reuse factor (K) of 9 and 12 respectively.