Intermittence and roughening of periodic elastic media

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We analyze intermittence and roughening of an elastic interface or domain wall pinned in a periodic potential, in the presence of random-bond disorder in $1+1$ and $2+1$ dimensions. Though the ensemble average behavior is smooth, the typical behavior of a large sample is intermittent, and does not self-average to a smooth behavior. Instead, large fluctuations occur in the mean location of the interface and the onset of interface roughening is via an extensive fluctuation which leads to a jump in the roughness of order λ, the period of the potential. Analytical arguments based on extreme statistics are given for the number of the minima of the periodicity visited by the interface and for the roughening crossover, which is confirmed by extensive exact ground state calculations.

I. INTRODUCTION

The properties of extended, elastic manifolds, like domain walls in magnets or contact lines of liquids on solid substrates become very varied if one introduces some disorder. Defects on a surface or impurities in a magnet often pin such interfaces. The recent interest in their physics follows from the observation that the energetics in the presence of randomness is obtained by optimizing the configuration of the manifold [1]. A competition between elasticity and the random potential arises. It results in a scale invariance described by a *roughness exponent* that measures the geometrical fluctuations, and an *energy fluctuation exponent* that measures the variation of the manifold energy around its mean. It is also related to the energy scales of excitations from the state of minimum energy. The experimental interest in these systems arises, in particular, due to the energetics: time-dependent phenomena like creep and coarsening (in magnets) follow slow, activated dynamics dictated by the energy barriers that can be described with such exponents [2].

Frequently manifolds also experience a periodic potential. In the case of superconductors, one periodicity is due to the rotational invariance of the phase. A second periodicity is induced when flux lines form a lattice. Similarly, in the case of charge density waves (CDW's) or domain walls in magnets, one periodicity is due to the underlying lattice structure, and a second is due to the self-organized periodicity of the CDW's or magnetic domains themselves. Generic models for these phenomena are called *periodic elastic media* (PEM), and are the focus of this work. As noted recently the asymptotic behavior of the PEM class depends on the type of periodicity, with the case of a periodic surface tension being in one universality class [3], while the case of an applied periodic potential is in another [4]. In this work we are interested in the case of an applied periodic potential, in particular the intermittent behavior of interfaces which experience a competition between pinning due to the periodic potential and pinning due to random bond disorder.

The paper is organized as follows. Section II introduces the Hamiltonian of periodic elastic media and describes two intermittent behaviors involved in PEM, when the amplitude of the applied periodicity is changed. This section also includes a discussion of the numerical method used. In Sec. III the first type of the intermittent behavior, jumps of manifolds, is analyzed using extremal statistics, and is demonstrated with numerical simulations. Section IV discusses the second type of the intermittent behavior, the roughening of the manifolds, with the aid of droplet arguments and further numerics. In Sec. V the roughening behavior is studied in {10}- and {100}-oriented lattices which have a lattice-induced periodicity; we compare these systems with other PEM. The paper ends with conclusions in Sec. VI.

II. PERIODIC ELASTIC MEDIA

The continuum Hamiltonian that describes the competition between elasticity, periodicity, and randomness is given by

$$\mathcal{H}_{pem} = \int \left[\frac{\gamma}{2} \{\nabla h(\vec{r})\}^2 + \eta\{h(\vec{r})\} + V_p\{h(\vec{r})\} \right] d\vec{r}. \quad (1)$$

Here $h(\vec{r})$ is a single-valued height variable, and $\vec{r}$ is a $(d-1)$-dimensional vector. V_p is a periodic potential (of amplitude V_0 and wavelength λ) in the height direction and $\eta\{h(\vec{r})\}$ is the disorder, which we take to be of the random bond type with delta-function correlations. The physics of manifolds described by Eq. (1) was discussed recently, since there may exist a *roughening transition* that separates an algebraically rough regime from a logarithmically rough one as the potential strength is varied [4]. However, in the dimensions considered here $[d=(1+1),(2+1)]$ these manifolds are always rough at large enough length scales [5] with the corresponding roughness exponents $\zeta=2/3$ and $\zeta=0.41 \pm 0.01$ for $d=(1+1)$ and $(2+1)$, respectively. The issues we raise here arise in all dimensions, and so we numerically illustrate them in $(1+1)$- and $(2+1)$-dimensional systems.

We calculate the exact location and morphology of interface ground states for a given configuration of bond disorder. For this configuration of bond disorder we vary the *ampli-*

tude of the periodic modulation. Interfaces which experience this combination of a periodic potential and random bond disorder show a variety of intermittent behaviors as the amplitude of the potential, V_0, is varied. Two types of intermittence which we study in detail are: intermittent jumps in the center of mass location of the interface, and intermittent jumps in the roughness of the interface. The first type is most easily discussed at strong pinning (large values of the key ratio $v = V_0 \lambda J / \delta J$), where the interface is always pinned near a minimum of the periodic potential, but it jumps between different minima as v is varied. It does this to maximize the energy gain due to small fluctuations about a flat interface. In the limit of large system sizes there can be an infinite number of such jumps with, of course, no overlap between the ground states of interfaces in different minima. We develop a scaling theory to demonstrate that the number of minima explored as v is varied over a finite range is of order $\ln(L_h)$, where L_h is the system size in the h direction in which the manifold fluctuates. Such intermittence is similar to the chaos seen in spin glasses (where it implies a vanishing overlap between spin configurations), and is related to replica symmetry breaking [6]. It is also a close cousin of the phenomenon that takes place if the disorder is changed randomly [7].

A second type of intermittence occurs when it becomes energetically favorable to form a large domain excitation. This means that a finite fraction of the interface is in one minimum of the potential, while another finite fraction is in an adjacent minimum. These large fluctuations are the classical ''Imry-Ma''-type droplets, and have a linear extension of the order of the system size. By slowly decreasing the potential, we are able to find the threshold at which the first domain excitation occurs, and to demonstrate its effect on the roughness $w(v)$. We observe that since the domain excitation is of the order of the sample size, the roughness produced by that domain fluctuation is proportional to λ. Thus there is a *first-order* jump in roughness. In contrast, a naive averaging of the roughness looks smooth and scales nicely. This is due to a scaling of the *probability* of a jump of the order λ occurring at v rather than being the self-averaging behavior of a typical sample. The exact numerical calculations are supported by scaling theories based on Imry-Ma and large fluctuation ideas, which account for the jumpy behavior of interfaces in a periodic potential.

The numerical calculations are carried out using Ising magnets with random bonds. For a given configuration of bond disorder, we find the ground state interface in square and cubic, nearest-neighbor, spin-half, ferromagnetic Ising models. An interface is imposed along the {11} or {10} directions of a square lattice, or along the {111} or {100} directions of a cubic lattice, by using antiperiodic boundary conditions. Periodic boundaries are used in directions parallel to the interface, unless otherwise mentioned. The average value of the exchange constant is $J = 1$, while the random-bond disorder is drawn from a uniform distribution of width δJ. The periodic potential $V_p = V_0 [0.5 \sin(2\pi h / \lambda) + 0.5]$ is added to the random bond disorder, where h is to be along a direction perpendicular to the average orientation of the interface. This is done for the {11} and {111} cases, while in

the other orientations ({10},{100}) we use the intrinsic lattice potential as discussed below. Note that if λ is small, the discrete representation of the potential will by necessity be rather coarse. The exact interface ground state in this random energy landscape is found using a mapping to the minimum-cut maximum-flow optimization problem [8]. We have developed a highly efficient (in both memory and speed) implementation of the push-and-relabel method for the maximum flow problem [9]. The exact ground state of a manifold in system with 1 000 000 sites can be found in about 1 min on a workstation.

III. JUMPS BETWEEN POTENTIAL VALLEYS

We first discuss the sensitivity of the ground state of model (1) to small variations in the amplitude V_0 of the potential V_p, with wavelength λ. A simple scaling theory captures many aspects of this sensitivity. The scaling theory begins with the central limit form for the energy of a flat interface located at a minimum of the periodic potential, $P_1(E)$. If the interface is exactly flat, the energy fluctuations are just due to the random bond disorder, so that

$$P_1(E) = \frac{1}{\sqrt{\pi}\sigma} \exp\left\{ -\frac{(E - JA)^2}{\sigma^2} \right\}, \tag{2}$$

where $A = L^{d-1}$ is the area of the manifold, and $\sigma^2 = 2A \delta J^2$ is the width of the Gaussian distribution.

Now consider a system in which there are N minima in the periodic potential. The probability $L_N(E)$ that the lowest minima has energy E is $L_N(E) = N P_1(E) \{1 - C_1(E)\}^{N-1}$, where $C_1(E) = \int_{-\infty}^{E} P_1(e) de$. The difference in energy, g, between the lowest energy state and the next lowest energy state of the manifold may also be simply calculated. We call this difference in energy the ''gap,'' and its distribution $G_N(g,E)$ is given by $G_N(g,E) = N(N-1) P_1(E) P_1(E + g) \{1 - C_1(E + g)\}^{N-2}$. Stated more precisely, $G_N(g,E)$ is the probability that if the lowest energy manifold has an energy E, then the gap to the next lowest energy level is g. The average lowest energy level is given by $\langle E_M \rangle = \int_{-\infty}^{\infty} E L_N(E) dE$. This is not analytically tractable. However, the typical value of this lowest energy is estimated from $\sigma N P_1(\langle E_M \rangle) \approx 1$, which yields

$$\langle E_M \rangle \sim JA - \sigma \{\ln(N)\}^{1/2}. \tag{3}$$

To estimate the typical value of the gap, we use $\sigma^2 N(N-1) P_1(\langle E_M \rangle) P_1(\langle E_M \rangle + \langle g \rangle) \approx 1$, which, with Eq. (3), and the fact that $|\langle g \rangle| \ll |\langle E_M \rangle|$, yields

$$\langle g \rangle \approx \frac{\sigma^2 \ln(\sigma)}{(JA - \langle E_M \rangle)} \approx \frac{\sigma \ln(\sigma)}{\{\ln(N)\}^{1/2}}, \tag{4}$$

where $\sigma = \sqrt{2A} \delta J$ and $A = L^{d-1}$. The gap between minima of the potential is thus of order $1/\{\ln(N)\}^{1/2}$, where $N \sim L_h / \lambda$ and L_h is the system size perpendicular to the interface. So the separation between minima grows increasingly small as L_h increases. Similar extreme statistics problems were discussed in Ref. [10].

Given the small gap between the metastable minima of the periodic potential, due to the presence of random bonds, we now need to find the typical change in V_0 which can cause a level crossing in which the global ground state changes from one minimum of the periodic potential to another [11]. The key effect that we must control is the fact that the interfaces are *not flat* even when confined to one minimum of the periodic potential. Instead they have a roughness which is determined by the interplay between the curvature of the periodic potential at its minima and the energy variations of a manifold due to confinement. We now develop a scaling theory for this phenomenon.

First we treat the confinement effect. Consider a manifold in the presence of random bond disorder, and which is confined in a slab of size $l \times L^{d-1}$. The energy of such a slab is given by

$$E(l,L) = \left(\frac{L}{L_x}\right)^{d-1} (c_1 L_x^{d-1} + c_2 L_x^{\theta}), \qquad (5)$$

where $L_x = l^{1/\zeta}$. This yields

$$\epsilon(l) = \frac{E(l,L)}{L^{d-1}} = c_1 + c_2 l^{-x}, \qquad (6)$$

where

$$x = (d - 1 - \theta)/\zeta. \qquad (7)$$

Note that x is *positive*, so that the confinement energy *decreases* as the confinement length l increases, as expected.

To include the effect of the confining potential, consider the behavior near a minimum of the periodic potential to be of the form

$$V(l) = \mathcal{V}_0 \left(\frac{l}{\lambda}\right)^y, \qquad (8)$$

where $\mathcal{V}_0 = V_0/\delta J$, and y is a positive exponent to ensure that the potential is confining. For example, a sinusoidal potential has $y = 2$. The behavior of a manifold in this confining potential, and in the presence of an additive random bond disorder, is estimated by considering its total energy as a function of l [i.e., combining Eqs. (5) and (8)]:

$$\epsilon_{total} = c_1 + c_2 l^{-x} + \mathcal{V}_0 \left(\frac{l}{\lambda}\right)^y. \qquad (9)$$

Finding the minimum of the total energy yields the manifold roughness,

$$l_c = \left(\frac{c_2 x \lambda^y}{y \mathcal{V}_0}\right)^{1/(y+x)}, \qquad (10)$$

with the energy of this optimal manifold being

$$\epsilon_{opt} = c_1 + c_3 \left(\frac{c_2^y \mathcal{V}_0^x}{\lambda^{xy}}\right)^{1/(y+x)}, \qquad (11)$$

where c_3 is a constant that depends on x and y [12].

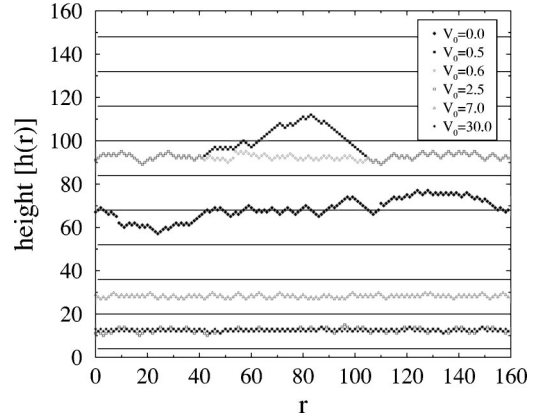Now the variation in the optimal energy, with a small variation in $\mathcal{V}_0$, is given by

FIG. 1. Interface configurations in $1+1$ dimensions for various V_0. In this calculation the disorder configuration and wavelength ($\lambda = 16$) are fixed at $\delta J = 1$. As V_0 is varied, the interface jumps between the minima of the periodic potential. The solid lines denote the position of the largest values of the sinusoidal periodic potential V_p. The lattice size is 160×160, and the interfaces are oriented along the $\{11\}$ direction. Note that the disorder is *exactly* the same for each value of V_0.

$$\epsilon_{opt}(\mathcal{V}_0 + \delta \mathcal{V}_0) - \epsilon_{opt}(\mathcal{V}_0) = \frac{\partial \epsilon_{opt}}{\partial \mathcal{V}_0} \delta \mathcal{V}_0. \qquad (12)$$

This *change in energy* also varies randomly from one minimum of the potential to another. If *the variation in the energy change* is of order the gap found in Eq. (4), then we expect the ground state location to change from one minimum of the potential to another. Thus we find the typical value of $\delta \mathcal{V}_0$ between jumps to be found from

$$\left(L^{d-1} \frac{\partial \epsilon_{opt}}{\partial \mathcal{V}_0} \delta \mathcal{V}_0\right)^{1/2} = \langle g \rangle. \qquad (13)$$

Thus, using Eq. (4),

$$\delta V_{0jump} = \frac{\langle g \rangle^2}{L^{d-1}} \left(\frac{\partial \epsilon}{\partial \mathcal{V}_0}\right)^{-1} = \frac{\langle g \rangle^2}{L^{d-1}} \frac{(x+y)}{c_3 x} \left(\frac{\lambda^x \mathcal{V}_0}{c_2}\right)^{y/(x+y)}$$

$$\sim \frac{\delta J^2 \{\ln(L^{d-1} \delta J^2)\}^2}{\ln(N)} \left(\frac{\lambda^x V_0}{\delta J}\right)^{y/(x+y)}, \qquad (14)$$

where $\mathcal{V}_0 = V_0/\delta J$. There are several interesting features of this equation. First, note that δV_{0jump} increases logarithmically with the area of the manifold, L^{d-1}. On the other hand, the number of minima $N \sim L_h/\lambda$, and δV_{0jump}, decrease logarithmically with L_h. The dependence of δV_{0jump} on λ and on V_0 is qualitative, as expected in that it increases monotonically with both of these factors.

The intermittence implied by result (14) is illustrated in Figs. 1 and 2. As a function of V_0, the manifold mostly stays almost unchanged in the current valley of minimum energy, and occasionally jumps to another, new minimum of the periodic potential. A useful way to illustrate this intermittence as a function of V_0 is to calculate the *configurational overlap* between the ground states as a function of V_0 (in analogy with the overlap used in spin glasses [13]). The overlap q is
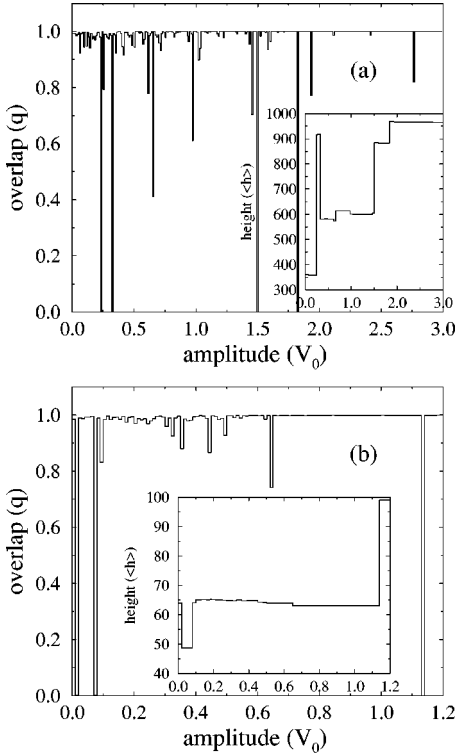
FIG. 2. The overlap $q = L^{-(d-1)}\Sigma_i \delta(h_i^1 - h_i^2)$ between ground states as the amplitude of the potential V_0 is varied ($\delta J = 1$). As V_0 is decreased, we calculate the overlap between the interface configuration at one value of V_0 (described by $\{h_i^1\}$) and the interface at the next value of V_0 (described by $\{h_i^2\}$). The corresponding mean heights $\langle h \rangle$ are shown in the insets. The calculations were carried out as for Fig. 1; however we used 300 different values of V_0 with $\Delta V_0 = 10^{-2}$ for the same realization of disorder, and the wavelength $\lambda = 4$. (a) Two-dimensional case, with the system size $L \times L_h = 1024 \times 1025$. (b) $(2+1)$-dimensional interfaces oriented along the $\{111\}$ direction for lattices of size $L^2 \times L_h = 100^2 \times 129$.

1 if the two configurations are the same and 0 if they have no bonds in common. Figure 2 presents the overlap as a function of the amplitude of the pinning potential, V_0, for interfaces in square and cubic lattices. The intermittent nature of periodic elastic media is clearly evident in these figures. Note that while the overlap and the interface roughness are intermittent, the *interface energy* (see Fig. 3) does not show any obvious signs of the jumps. Due to the logarithmic reduction in the gap size [Eq. (4)], the interface will only sample an infinitesimal fraction $[\ln(N)/\{\ln(L^{d-1})\}^2]$ of the available minima of the potential as we sweep v. Nevertheless a large number of different minima $[\sim \ln(L_h)]$ will be sampled by the system, in particular if L_h is increased while the transverse size L is kept fixed.

IV. ROUGHENING OF THE MANIFOLDS

The behavior of the roughness of interfaces seen in Fig. 3 is also strongly intermittent, especially in $1+1$ dimensions. The large jumps in roughness seen in this figure are easily understood from the Imry-Ma arguments [14] concerning the instability of interfaces to large fluctuations, as we now dem-
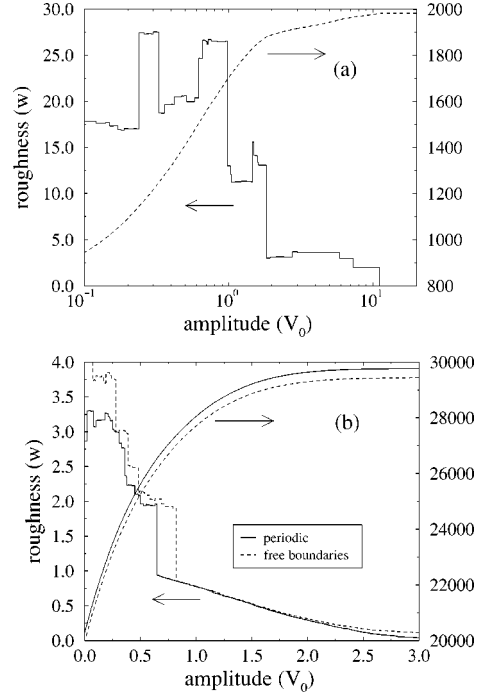

FIG. 3. The interface width $[w^2 = L^{-(d-1)}\Sigma_i(h_i - \langle h \rangle)^2]$, and the total energy as a function of V_0 for $\lambda = 4$ and $\delta J = 1$. The results are for a fixed disorder configuration and from the same calculations as Fig. 2. (a) $(1+1)$-dimensional system. (b) $(2+1)$-dimensional system. Systems with free and periodic boundaries have the same realization of randomness.

onstrate for the $(2+1)$-dimensional case.

The interface energy of a subregion a of the interface of the area A is, of course, also drawn from the Gaussian, $P_1(E) = (1/\sqrt{\pi}\sigma)\exp\{-(E - Ja)^2/\sigma^2\}$, but now with a standard deviation $\sigma^2 = 2a\delta J^2$. Some of these energy fluctuations are favorable while others are unfavorable. The largest favorable fluctuations are found by setting $A\sigma P_1(E) \approx 1$, similarly to the extreme statistics arguments as in deriving Eq. (3), and as the value of the energy gain this gives

$$\langle E_g \rangle \approx \sigma \{\ln(A)\}^{1/2}. \tag{15}$$

A flat interface would tend to "take advantage" of such large favorable energy fluctuations in adjacent minima of the periodic potential. However, this requires having segments of the interface crossing the barriers in the periodic potential. We define the barrier cost per bond to be ϵ_b^0, and this is given by the integral over the barrier, $\epsilon_b^0 = (1/\lambda)\{\int_0^\lambda V(x)dx\} = \epsilon V_0$. We shall use the last of these forms, as we shall often be interested in the dependence on V_0. We consider $(1+1)$- and $(2+1)$-dimensional systems of wavelength λ, length L, width B, and $A = BL$ so that $a_b \simeq \lambda B$ is the area of the part of the interface which crosses the energy barrier, and $a \simeq LB/2$ in order to maximize the energy gain. $B = 1$ is the two-dimensional case, and $B = L$ in the isotropic three-dimensional case. The barrier energy cost is given by
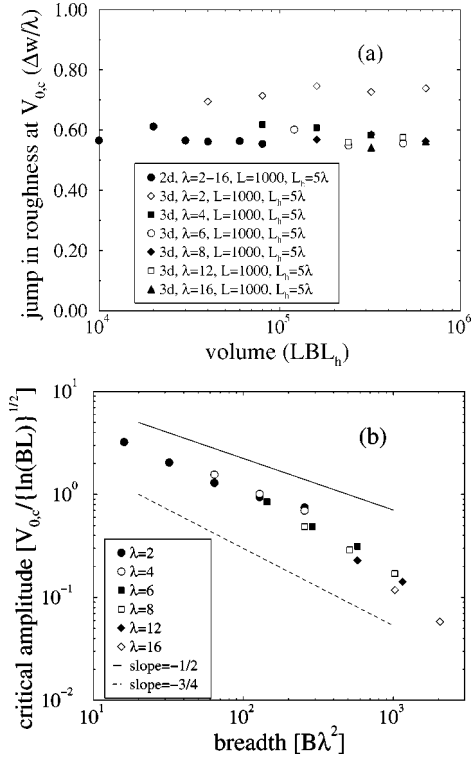
$$E_b = \epsilon V_0 \lambda B. \tag{16}$$

FIG. 4. (a) Average size of the first jump in roughness Δw, when $V_0 = V_{0,c}$, normalized using λ and calculated as the difference between roughness values just after a jump and before that, as a function of the volume of the systems. We have carried out simulations for a strips of dimension $L = 1000$, $B = 1$–64, and $L_h = 5\lambda$, for various values of λ. The number of realizations is 100. (b) Average value of the amplitude of the potential $V_0 = V_{0,c}$ at which the large-scale ''Imry-Ma'' fluctuation occurs ($\delta J = 1$). The data are from the same simulations as in (a) for the $(2+1)$-dimensional case, i.e., $B > 1$. The results are scaled using prediction (17).

Equating Eqs. (15) and (16) yields the estimate of the parameter values at which the first Imry-Ma jump in the manifold roughness occurs:

$$\left(\frac{\epsilon V_0 \lambda}{\delta J} \right)_1 \sim \left[\sqrt{\frac{L}{B}} \{\ln(BL)\}^{1/2} \right]_1 . \tag{17}$$

In the $(1+1)$-dimensional case the logarithmic correction drops out, by elementary considerations.

An Imry-Ma fluctuation of size a leads to a jump in the roughness, which is of order $\lambda \times a/A \simeq \lambda/2$. We emphasize that this is the expected outcome in any system with fixed disorder, when V_0 is varied. If $B \propto L$, there is an exponential dependence of the crossover length on the parameters; for example, for $B = L$,

$$L_1 \sim \exp\left[\left(\frac{\epsilon V_0 \lambda}{\delta J} \right)^2 \right], \tag{18}$$

an exponential dependence on v [14].

In Fig. 3, we present the numerically observed behavior of the interface roughness as a function of V_0. We observe that for very large V_0 the interfaces are flat, and are confined
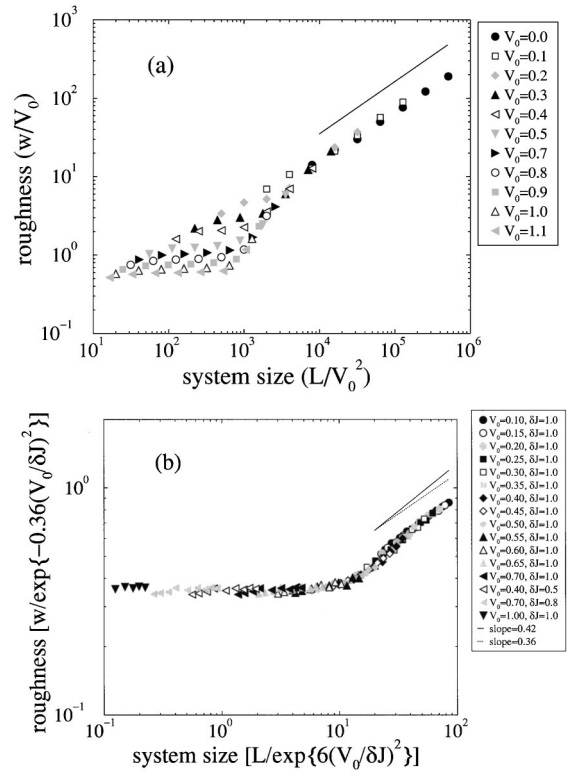

FIG. 5. Scaled roughness of interfaces oriented in $\{11\}$ and $\{111\}$ directions, for various values of V_0 and L. (a) $\{11\}$-oriented systems with $\lambda = 16$, $\delta J = 1$, and system sizes $L^2 = 20^2$–1280^2. The number of realizations is 200 for each system size and V_0. The solid line corresponds the slope $\zeta = 2/3$. (b) $\{111\}$-oriented systems with $\lambda = 4$ and system sizes $L^3 = 10^3$–90^3. The number of realizations is 200 for each system size, δJ and V_0. The solid line corresponds the slope $\zeta = 0.42$, while the dotted line is $\zeta = 0.36$.

to a minimum of the potential. For a large range of V_0 the roughness stays the same or increases slowly (in three dimensions), until finally at a critical value a discrete jump occurs due to the Imry-Ma nucleation process. This implies that the roughening process, as defined by the point at which the interfaces begin to fluctuate outside a single valley, has a *first-order character*. It is seen from Fig. 4(a) that the first jump is $\propto \lambda$, as expected for an extensive fluctuation. The critical value $V_{0,c}$, at which the first extensive fluctuation occurs [Fig. 4(b)], follows roughly the prediction of Eq. (17), though the slope is closer to 3/4 instead of 1/2.

The analysis of the last paragraph clearly demonstrates that the roughening of manifolds in periodic elastic media is via a first order jump in roughness, which is of the order of the wavelength of the periodic elastic medium. It is interesting to investigate whether this first order jump is observable in the ensemble-averaged behavior. Scaled, ensemble-averaged plots of the manifold roughness as a function of V_0 are presented in Fig. 5 for $\{11\}$-oriented interfaces [Fig. 5(a)] and for $\{111\}$-oriented interfaces [Fig. 5(b)]. These plots scale quite nicely with the characteristic length and roughness suggested by Eqs. (17) and (18). In the two-dimensional case, there is also a clear indication of the first order character of the transition. The three-dimensional data give little
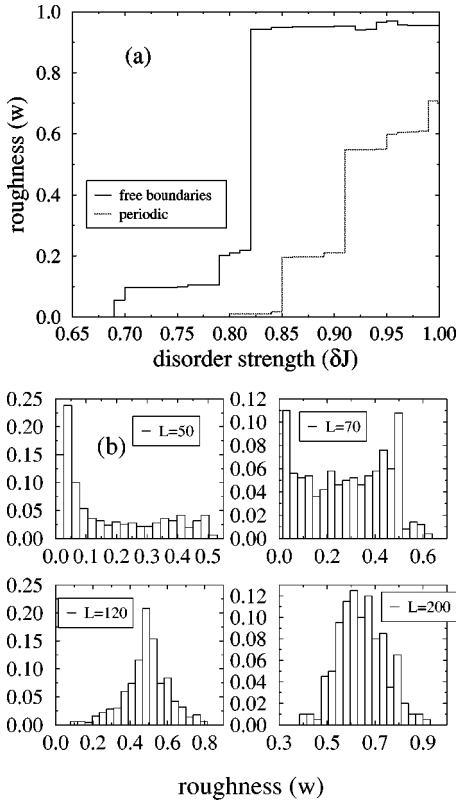
FIG. 6. Behavior of the roughness of interfaces oriented in the $\{100\}$ direction. (a) The intermittence of a single realization as a function of the amplitude of uniform disorder δJ. The disorder configuration is the same (with both free and periodic boundaries), but the ratio $\delta J/J$ is slowly increased in steps of 0.01. The system size is $L^3 = 100^3$. (b) The histograms of the roughness values w for system sizes $L^2 \times L_h = 50^3 \ldots 200^2 \times 100$. The peak of the distributions jumps from $w \simeq 0$ to $w \simeq 0.5$ when the system size increases. The number of realizations is 500 for smaller system sizes and 200 for $L^2 \times L_h = 200^2 \times 100$. $\delta J/J = 0.9$.

indication of the first order jump in roughness, and underscore the problems with a naive averaging of the data. However, we do not have any clear explanation, of why the roughness values in the plateau before the jump can be collapsed with the same prefactor as in the asymptotic roughness in the $\{111\}$ case, but not in the $\{11\}$ case.

V. PERIODICITY DUE TO THE LATTICE

It is of interest to see if the first order character of the roughening of PEM extends to manifolds in the $\{10\}$ and $\{100\}$ directions. In these directions, the lattice itself introduces a periodicity, which, for example, is the origin of the thermal roughening transition in lattice models in three dimensions. Thus we do not need to introduce an extra periodic potential, and instead we just study the roughness of these manifolds as a function of disorder. We have studied the roughness of $\{100\}$ manifolds as a function of disorder before, and in those studies we ensemble averaged the data [15]. In light of the understanding developed above, we have revisited this problem, and found that the typical behavior in both the $\{10\}$ and $\{100\}$ problems is very similar to that
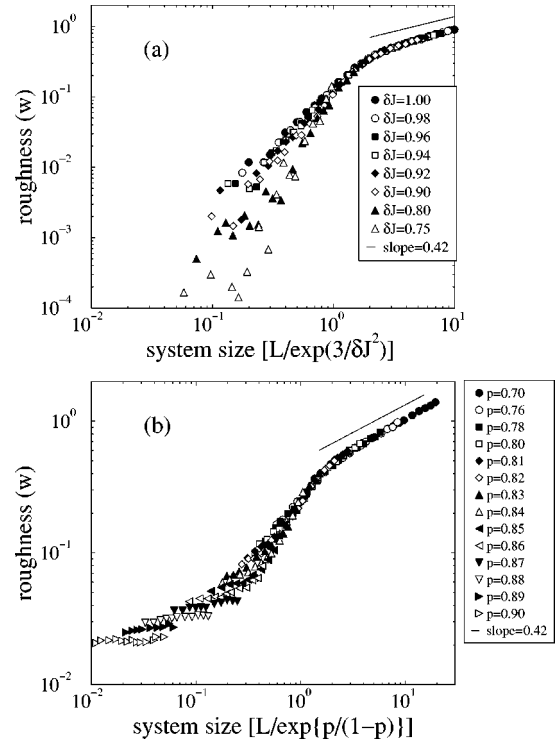

FIG. 7. (a) Scaled roughness with Eq. (18) for continuum disorder. The system sizes ranges from $L^2 \times L_h = 4^3$ to $200^2 \times 100$. The number of realizations ranges from 500 for system sizes $L^2 \times L_h < 140^2 \times 100$ to 200 for the larger ones. (b) Scaled roughness with Eq. (18) or dilution type of disorder. The system sizes ranges from $L^2 \times L_h = 4^3$ to $200^2 \times 100$ (and even up to 400^3 for $p = 0.90$). The number of realizations ranges from 500 for system sizes $L^2 \times L_h < 140^2 \times 100$ to 200 for the larger ones (with the exception of larger system sizes for $p = 0.90$).

suggested by the PEM model. That is, in a large sample the system roughens via a first order jump in the roughness due to an extensive fluctuation. The behavior of one sample as a function of disorder is presented in Fig. 6(a). The probability distributions of the roughness for several L's are presented in Fig. 6(b), in which we observe how one can pass through a *coexistence* region with both flat and rough samples as L is varied. The intermittent behavior typical of PEM is evident in Fig. 6(a), but is obscured by the averaging in Fig. 7(a). Though a jump transition from a flat phase to an algebraically rough phase occurs in both the periodic elastic model in the $\{111\}$ direction and for interfaces in the $\{100\}$ direction, there is an important difference in the behavior of these models [compare Figs. 5(b) and 7(a)]. In the PEM model in the $\{111\}$ direction, there is a pronounced plateau in the roughness due to the saturation of wandering within one well [Fig. 5(b)]. In contrast, in the $\{100\}$ direction, the interface remains flat until the transition to the algebraically rough phase [see Fig. 7(a)]. The extent of the plateau region can be tuned in the PEM model by varying the shape of the potential near the minimum and by varying the wavelength. We have also carried out calculations for the case of dilution disorder [Fig. 7(b)], and found a similar behavior, with the averaged behavior presented in Fig. 7(a). With dilution disorder the pronounced plateau is not due to any roughening

inside a valley, but because of rare ''bumps,'' whose occurrence is due to the Poissonian statistics of diluted bonds. The averaged data scale quite well with $(\delta J/J)^2 = p(1-p)J^2/(pJ)^2 = (1-p)/p$, where $J=1$, and the variance of the binomial distribution $\text{var} = std^2 = p(1-p)J^2$ with the corresponding mean pJ, and p is the occupation probability of a bond. Thus we find, in contrast to our earlier conclusions from similar data, that at large enough length scales interfaces in the $\{100\}$ orientation are algebraically rough, and are consistent with the PEM model.

A further important feature of the large fluctuation character of the roughening transition is that it is strongly dependent on the boundary conditions. This is illustrated in Figs. 2(b) and 5(a), in which the roughness is depicted as a function of the amplitude of the disorder for both periodic and free boundaries, and with the *same* disorder configuration. The threshold value of V_0 at which the first order jump in roughening occurs is typically smaller for the case of periodic boundaries. Large fluctuations can take advantage of the boundary to reduce the cost of crossing the energy barrier. This sensitivity to boundary conditions is a hallmark of the large fluctuation effects discussed here.

VI. CONCLUSIONS

To conclude, we have discussed the roughening of elastic manifolds in the presence of a competition between bulk randomness and a confining periodic potential. We have concentrated on the two- and three-dimensional cases, which are well known to have, asymptotically, an algebraic roughness scaling. However, a study of the system-by-system behavior reveals a much richer scenario in which each manifold makes intermittent jumps, finally culminating in a first-order change in its roughness. This process is also important, since it is related to the asymptotic scaling of the roughness. Recent experiments on the creep of $(1+1)$-dimensional systems [2] showed that scaling arguments of activation energy barriers can match real systems, using predictions based on rough manifolds. The time scales also depend crucially on the actual amplitude which is set in our picture by the roughening transition.

Also, the intermittence in the early stages would merit experimental consideration. Such jumps in the mean location of the interface could be studied in the asymptotic rough regime. In an independent study we have pointed out this mechanism for both fracture surfaces, arising from random fuse networks and from yield surfaces of perfectly plastic media which are equivalent to the minimum energy surfaces studied here [16].

The focus of renormalization group and variational calculations in this problem has been dimensions $d=(D+1)>4$, since there one encounters two asymptotic regimes separated by a transition. Of the two phenomena discussed here, at least the intermittent jumps in the center of mass location of the interface should persist in that case.

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