Effect of poloidal density variation on parallel viscosity for large Mach numbers

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Parallel viscosity $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ as a function of radial electric field E_r (or, equivalently, non equilibrium poloidal rotation) in a tokamak is studied using a 5D (3D in configuration space and 2D in velocity space) Monte Carlo particle following code. It is shown that including the poloidal density variation changes the qualitative behavior of the solution causing the parallel viscosity to change sign when the Mach number $M_p = |E_r/v_t B_\theta|$ is of the order of unity. Here, v_t is the thermal velocity and B_θ is the poloidal magnetic field.

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In the standard neoclassical theory of tokamaks, the neoclassical transport is automatically ambipolar and independent of the radial electric field E_r. As discussed in Ref.[1], this follows from momentum conservation and is valid only in the absence of momentum sources. In the presence of forces, such as an externally applied radial electric field [2] or torque by the orbit losses [3, 4], neoclassical transport depends on E_r, and various expressions for the neoclassical ion flux and parallel viscosity have been derived [4, 5, 6]. In the context of study of the L–H transition theory in Refs. [3, 7], it was important to expand the validity of expression for the parallel viscosity in the region where $M_p \geq 1$. Here, $M_p = |E_r/v_t B_\theta|$ is the Mach number, B_θ is the poloidal magnetic field, and $v_t = (2k_B T/m)^{1/2}$ is the thermal velocity, where m is the ion mass and T the zeroth order temperature (no poloidal variation). First, an expression for $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ was derived assuming an incompressible plasma flow and constant density in poloidal angle ($\mathbf{B}$ is the magnetic field, $\mathbf{\Pi}_i$ is the viscosity tensor, and $\langle\ \rangle$ indicates a flux surface average). It was found that the viscosity has a maximum at $M_p \approx 1$ and it decays to zero without changing sign when M_p increases. Similar result was obtained for $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i/n \rangle$ in Ref. [8], where the effect of poloidal variation of density n and compressibility were included. Here, the behavior of $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ was not investigated. Since $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ is the standard expression in the literature, and it appears in a majority of formulations of rotation dynamics and momentum balance in tokamak theory [2, 3, 9, 10], the study of $\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle$ including the poloidal density variation is of importance. In this Brief Communication, we show both by numerical and analytic methods that the standard expression for the parallel viscosity changes sign when the Mach number increases provided the variation of density in poloidal angle is taken into account consistently.

The standard expression for the parallel viscosity in terms of pressure anisotropy is

$$\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle = \left\langle (p_\perp - p_\parallel) \frac{\mathbf{B} \cdot \nabla B}{B} \right\rangle, \tag{1}$$

where $p_\parallel$ and $p_\perp$ are the parallel and perpendicular components of the pressure, respectively. Different expressions for the pressure components and pressure anisotropy exist in the literature. The analytic theory presented in Refs. [5, 11] includes poloidal variation of the electrostatic field, density and temperature, and is based on full velocity integrals. To simplify the problem, we here neglect the radial density and temperature gradients and, also, the poloidal electric field. Thus, we can write the density as $n = n_0 + n_1(\theta)$, where θ is the poloidal angle. With these simplifications and using the expressions for $p_\parallel$ and $p_\perp$ given in Eqs.(9) and (10) of Ref. [11], the pressure anisotropy can be written as

$$p_\parallel - p_\perp = T(2x^2 - 1) n_1(\theta) + \epsilon n_0 T \left[4x^2 + \frac{E_r(1-I)}{\mathrm{i} r \nu B + E_r} \right] \exp(\mathrm{i}\theta), \tag{2}$$

where the poloidal dependence of the density is

$$n_1(\theta) = -\epsilon n_0 [1 - (1 + 2x^2)\Lambda] \exp(\mathrm{i}\theta), \tag{3}$$

r is the radius, R_0 is the major radius, B_0 is the magnetic field at the axis, $\epsilon = r/R_0$, and n_0 is the zeroth-order density. The functions I and Λ are defined as

$$I(z) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \frac{w}{w - 2^{1/2} z} \exp(-\frac{w^2}{2}) dw, \qquad \Lambda = I[I + 2\mathrm{i} x y I + (1-I)x/z]^{-1},$$

where $z = x + \mathrm{i}y$, $x = M_p$, and $y = \nu_{*i} \epsilon^{3/2}$. Here, the normalized collisionality is $\nu_{*i} = \nu R_0 q / v_t \epsilon^{3/2}$ with the ion-ion collision frequency ν and the safety factor q.

An alternative expression for the pressure anisotropy is given in Refs. [6, 12]:

$$p_{\|} - p_{\perp} = -2\pi^{1/2} I_{ps} n m u_{\theta} v_t B / B_{\theta} \left[\frac{\partial}{\partial \theta}(\ln B) - \frac{2}{3}\frac{\partial}{\partial \theta}(\ln n) \right], \tag{4}$$

where $u_{\theta} = (u_{\phi} B_{\theta} - E_r)/B_{\phi}$ is the poloidal flow velocity. Neglecting the toroidal flow velocity u_{ϕ}, the integral I_{ps} is [12]

$$I_{ps} = \frac{1}{\pi} \int_0^{\infty} w^2 e^{-w} \int_{-1}^{1} (\frac{1}{2} - \frac{3}{2}\xi^2)^2 \frac{\chi}{(\xi - x/w^{1/2})^2 + \chi^2} d\xi dw, \tag{5}$$

where $\xi = v_{\|}/v$ is the velocity pitch, and $\chi = y\nu_T/\nu w^{1/2}$. Here, ν_T is the collision frequency for anisotropy relaxation. Using the expression for ν_T given in [10], the results in the Pfirsch-Schlüter regime obtained with a more complete collision operator have been reproduced to within 20 % [6]. In Refs.[3, 7], the second term in the brackets on the right hand side of Eq.(4) was neglected since it was not considered important. However, here we show that taking into account the poloidal density dependence clearly changes the qualitative behavior of the parallel viscocity.

In the numerical simulation, the 5D (3D in configuration space and 2D in velocity space) Monte Carlo particle following code ASCOT [13] is used. The guiding-centre orbits of the ions are followed in a tokamak geometry, and a binary collision model [14] is used to model ion-ion collisions. In this model, simulation region is divided into cells (in r and θ) small enough that the plasma parameters are approximately constant inside the cell. In each cell, particles are paired randomly and they are left to collide pairwise. These small angle collisions are performed in the whole test particle ensemble after each time step during which all particles are advanced along their guiding-centre orbits. During this time step the particles typically move only a small fraction ($\propto 1/100$) of their whole orbit. The guiding-centre equations in the ASCOT code are written in straight magnetic field line coordinates [15] using canonical Hamiltonian variables to avoid numerical drifts. The magnetic background is assumed stationary. In the Runge-Kutta integration of the the orbit, total energy, magnetic moment and toroidal momentum remain constant, but they are changed in collisions for each individual particle. However, for a group of particles in each cell, the chosen collision operator conserves the number of particles, the total momentum, and the total energy. Test particles are initially distributed uniformly in configuration space and pitch angle, and distribution in velocity is Maxwellian. Each test particle is weighted with a number that corresponds to the relative volume of its initial location in phase space. The parallel viscosity from Eq.(1) is calculated directly from the code in terms of the statistically measured pressure components $p_{\|} = \int m(v_{\|} - u_{\|})^2 f d^3v$ and $p_{\perp} = \int (m|\mathbf{v}_{\perp} - \mathbf{u}_{\perp}|^2/2) f d^3v$. Here, $v_{\|}$ and $v_{\perp}$ are the parallel and perpendicular velocity components, respectively, and $u_{\|}$ and $u_{\perp}$ are the corresponding flow velocities. All the flow velocity components, as well as $p_{\|}$ and $p_{\perp}$, are calculated from the code on a grid of θ and r as time and ensemble averages of particle velocities. Using the momentum conserving collision operator with a fixed radial electric field and excluding other forces generates a particle flow parallel to the magnetic field to compensate the poloidal rotation. This mean flow velocity, $U_{\|} = \int u_{\|}(\theta)(R/R_0) f d\theta$, is driven by viscous processes and its build up occurs on a collisional timescale. Thus, in order to compare the results to analytic estimates obtained for $U_{\|} \approx 0$, the measurement is here done before a significant mean parallel velocity has developed. The present assumption of a fixed E_r may correspond to situations in which E_r is externally applied by probe, or in which some other source of momentum such as Reynold's stress, ion orbit loss current or current due to magnetic ripple is present and sustains the rotation in equilibrium.

In the absence of momentum sources, E_r would evolve to a value in which the parallel viscosity vanishes on a time scale which is faster than the collisional time scale.

Parameters similar to those of ASDEX Upgrade [16], $a = 0.5$ m, $I_{pl} = 1$ MA and $B_0 = -2.5$ T, are used for the minor radius, plasma current, and magnetic field on the axis, respectively. Since the analytic results were derived in the large aspect ratio limit of a quasitoroidal configuration, a larger value ($R_0 = 3$ m) for the major radius and co-centric circular magnetic surfaces on a poloidal cross-section are chosen. In this geometry, magnetic field is of the form $\mathbf{B} = B_0(\mathbf{e}_\phi + \Theta(r)\mathbf{e}_\theta)/(1+\epsilon\cos\theta)$. In Fig. 1, the parallel viscosity calculated from Eq. (1) using the pressure anisotropy obtained from the simulations is compared to the viscosity obtained with the analytic pressure anisotropies of expressions (2) and (4). To show the influence of the poloidal density dependence, Eq. (4) is used both without a poloidal density gradient and with the density dependence of Eq. (3). Here, the densities and temperatures are chosen to correspond to the collisionalities of $\nu_{*i} = 46$, 12 and 2.5, respectively, the first one being in the Pfirsch-Schlüter regime and the other two in the plateau regime. Both in the Pfirsch-Schlüter regime and in the plateau regime, the parallel viscosity $\langle \mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_i\rangle$ changes sign when M_p approaches unity. This takes place both in the numerical simulation and for the analytic result with the poloidal density dependence. Only when the poloidal density variation is neglected, one obtains a positive definite result. For a small poloidal rotation, the density perturbation is insignificant and the standard neoclassical result is found with all methods. The effect of poloidal electric field has also been tested. With the analytic expressions of Refs. [5, 11] its contribution to the viscosity is of the same order as that of the other terms but it does not change results qualitatively. This was also checked by running the ASCOT code by solving the poloidal electric field from the assumption of quasi-neutrality and Boltzmann distribution of the electrons on a magnetic surface. Again, the results were not changed qualitatively.

It is interesting to take a closer look at the density variation. A further investigation of the real part of Eq. (3) shows that for small values of M_p, the $\sin\theta$ term dominates the poloidal density dependence. At $M_p \approx 1$, both $\sin\theta$ and $\cos\theta$ terms are important, and for large M_p, the $\sin\theta$ contribution vanishes leading to

$$n = n_0 + n_1(\theta) \overset{M_p\to\infty}{\longrightarrow} n_0(1 - 2\epsilon\cos\theta), \tag{6}$$

i.e., the density is peaked on the high field side. Indeed, this same dependence can be found also in Ref. [12], but there the expression $\langle \mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_i/n\rangle$ was used instead of $\langle \mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_i\rangle$ for the viscosity, and only the latter one changes sign because of the density variation when M_p increases. Now, if one assumes the magnetic field of the form

$$B \approx B_0(1 - \epsilon\cos\theta),$$

and uses the density dependence of Eq. (6) in Eq. (4), one sees that the $\sin\theta$ dependent part inside the parantheses clearly changes sign due to the inclusion of poloidal density variation. When introducing the pressure anisotropy of Eq. (4) into Eq. (1), only the $\sin\theta$ part becomes relevant for the sign and the value of parallel viscosity, since the $\cos\theta$ part vanishes in the integration. In Fig. 2, the poloidal density variation from the code and from Eq. (3) are compared. Good qualitative agreement between the analytic and numerical result is found both for small and large values of M_p.

In the present numerical and analytic study of parallel viscosity, it was found that the parallel viscosity changes sign when the Mach number is of the order of unity. Although one obtains qualitatively the same results from the analytic calculation

and the numerical simulation, quantitative differences exist due to simplified collision operators used in the analytic formulas. However, there is evidence from the present ASCOT simulations that the effect of convection term, defined as $\langle n\mathbf{B}\cdot\mathbf{u}\cdot\nabla\mathbf{u}_i\rangle$, is of importance for large Mach numbers and that the effective viscosity, which is, the sum of convection term and standard parallel viscosity, has a maximum at $M_p \approx 1$. For this sum, no change in sign is observed when M_p increases. Furthermore, one should remember that the qualitative behavior of viscosity term of Ref. [8], defined as $\langle \mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_i/n\rangle$, is different from the behavior of the standard parallel viscosity $\langle \mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_i\rangle$ for large Mach numbers.

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Figures

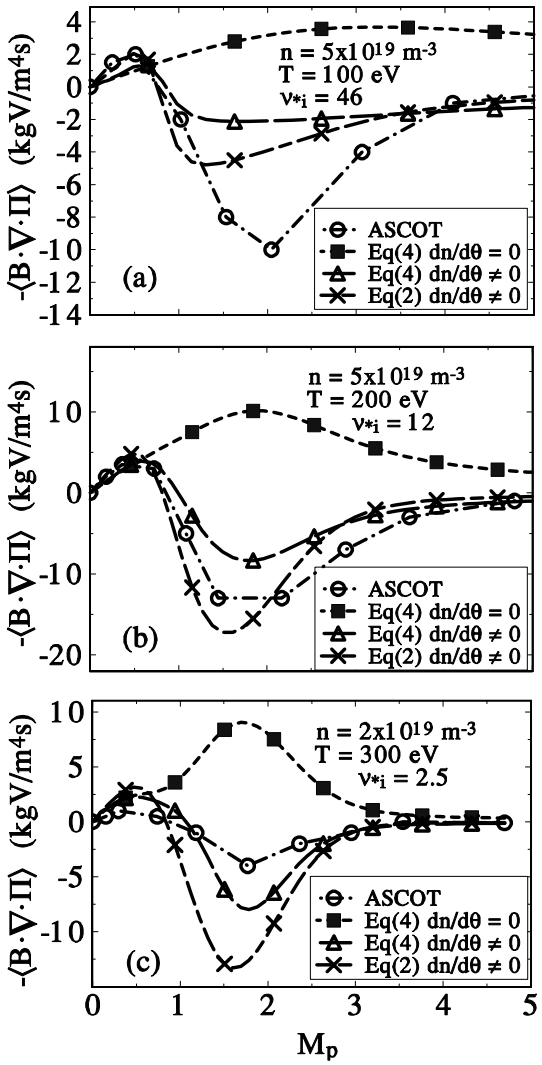

Figure 1: *Parallel viscosity as a function of the radial electric field for temperatures a)* 100 *eV and b)* 200 *eV, with the density* $n = 5 \times 10^{19}\ m^{-3}$*, and c) for low the collisionality case with* $T = 300\ eV$ *and* $n = 2 \times 10^{19}\ m^{-3}$ *calculated with ASCOT and from analytic formulas. Parallel viscosity changes sign at* $M_p \approx 0.5 - 1$ *in all cases where the poloidal density dependence is taken into account.*

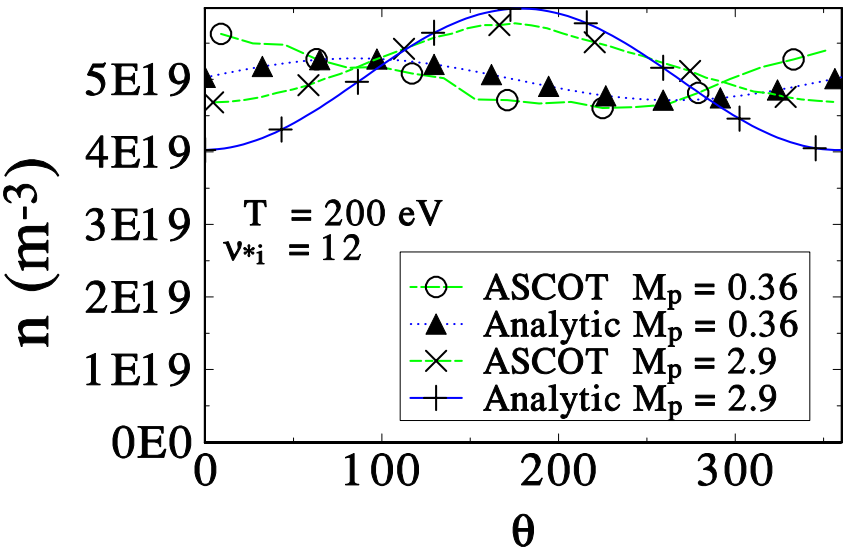

Figure 2: *Density variation for* $T = 200$ eV *and* $n_0 = 5 \times 10^{19}$ m^{-3} *from the numerical simulation and from Eq. (3) for* $E_r = 10$ kV/m $(M_p = 0.36)$ *and for* $E_r = 80$ kV/m $(M_p = 2.9)$*. For large* M_p*, the density is largest at the inboard equator.*