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Macro Element Method for Modelling Eddy Currents in the Multi-conductor Windings of Electrical Machines

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Abstract

The computation of the magnetic field in electromagnetic devices - especially rotating electrical machines - usually excludes the effect of eddy currents in the multi-conductor windings. Eddy-current losses are frequently calculated from a magnetic field solution in which the effect of eddy currents is not considered. The aim of this research has been the development of efficient methods to consider the effect of eddy currents in multi-conductor windings accurately within electromagnetic field computation. There are methods, which take the effect of eddy currents on the magnetic field into account in multi-conductor windings but all those approaches have serious drawbacks. Analytical formulations lead to very complicated models while numerical analysis leads to enormously large problem size for many conductors or in small penetration depth cases. The thesis suggests a novel combination of the elimination of inner nodes method with the conventional finite element technique. Based on the magnetic linearity of the winding regions the creation of numerical macro elements is suggested. The Gauss elimination of the nodes inside the finite element model of the multi conductor winding resulted in significantly decreased problem size and in accelerated solution of the system of equations in the nonlinear iteration process. The new macro element method, has been developed by the creation of constant macro elements for the duration of the whole time-stepping analysis. The macro element method has been verified with test computations and it is found to be faster in nonlinear problems and it requires significantly less memory than the traditional approach. The advantages and limitations of the method are presented in this thesis.

Preface

This thesis is the fruit of a research project conducted in the Laboratory of Electromechanics, Helsinki University of Technology. The purpose of the work has been to develop methods, which are capable to model eddy-current effects in multi-conductor windings efficiently. Such windings can be found in rotating electrical machines and in other electromagnetic devices like transformers.

After thanking God, I have to mention the help I have received from many people who have helped me to complete this thesis.

I would like to express my gratitude to my supervising professor Tapani Jokinen, head of the Laboratory of Electromechanics whose help and guidance made the start, development and completion of this work possible. Without him the fruits of this work would have never been born.

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Áron Szűcs

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List of principal symbols:

Scalar variables are denoted by plane letters. Boldface symbols are used for vectors.

$\boldsymbol{A}$	vector potential
A	z-component of **A,** approximate solution for the vector potential field
$\boldsymbol{B}$	magnetic flux density
c	specific heat
$\boldsymbol{D}$	electric flux density
$\boldsymbol{E}$	electric field strength
$\boldsymbol{H}$	magnetic field strength
I	electric current
$\boldsymbol{J}$	current density
P	power
S	integration surface
U	voltage
ε	permitivity
η	logical operator with values: 0 and 1
μ	permeability
μ_0	permeability of free space
ν	reluctivity
δ	skin depth
σ	conductivity

1. INTRODUCTION

Increasing demand for accurate computation methods

Electrical machine designers and researchers in the industry face increasing demand to consider many physical effects more accurately, when they give birth to the next generation of electrical energy converters, such as electrical machines.

Industrial competition and the desire to increase efficiency has always inspired the engineers and researchers to design electromagnetic devices with higher power densities. In order to achieve this task, classical techniques in machine design have been applied for decades and the simple analytical and empirical formulations have been perfected to their limits. Later with the arrival of the "digital age", cheap, fast and reliable computers have come to life and they brought with themselves the possibility to perform computationally very heavy tasks in a short period of time. Machine designers started to rely on their computers and used them to perform designs with their analytical and empirical methods fast and elegant.

Today the need for more sophisticated computational tools has become transparent. Although analytical and empirical tools still provide the basis for the designs, in many cases they alone are not sufficient any more, and in increasing number of new designs, some of them are hardly applicable. Another example; new technologies and demands – such as inverter load and special working conditions, like in case of high speed, high power machines – represent an unsolvable challenge with the old tools.

Numerical methods such as the finite element method (FEM) applied in the electromagnetic design of electrical machines have been present in the industry for some time now. However the real daily use of FEM techniques by design engineers in big machine companies, is becoming a common practice only recently. The finite element (FE) modeling of the electromagnetic properties of electrical machines has become a basic tool. It is expected to boom in the next few years as faster and more user friendly commercial computation tools become available for machine designers. Many engineers condemn these new tools saying they are too slow and too complicated compared to traditional methods. However a couple of engineering "disasters", like unexpected harmonics, which could have been eliminated by a simple numerical analysis, become very convincing arguments. Accurate numerical analysis can also reduce manufacturing and material costs by eliminating “too safely, over designed” parts with the increased accuracy of the models.

Efficiency optimization of rotating machines and transformers is a very important task. A successful optimization of the efficiency requires high accuracy, when predicting the losses. The better we can calculate the losses, the higher the quality and reliability of the optimization. In the followings, the emphasis is placed on rotating electrical machines, but most of the observations can be applied to transformers and other devices as well.

The increased demands require an improved knowledge of the electromagnetic phenomena, including the effect of eddy currents. Eddy currents can contribute to the normal operation of electrical machines, but they can also be harmful by causing additional losses. The proper analysis of eddy-current effects and the losses dissipated in the windings of electromagnetic devices is the major objective of this thesis.

In electrical machinery, we can recognize two major kind of harmful eddy currents with respect to their place of appearance. Eddy currents induced in the iron core - usually limited by the lamination - and those induced in the winding of the machine. This work is focusing on the eddy currents flowing in and between the conductors of the windings. In laminated windings, eddy loss originates from currents circulating within and between strands. Usually the later can be totally eliminated by following certain design rules like the most well known Roebel bars method. The eddy currents flowing in the strands are classically limited by choosing small enough dimensions for the geometry of the conductors with respect to the skin depth which depends on the operating frequency and the material parameters. The use of such filamentary conductors increases the price of the winding and thus the machine significantly. There are also cases when this approach is not appropriate for example filamentary conductors are not applicable in high voltage machines. In such cases when the selection of filamentary conductors is not allowed, or it is not clear what will be the advantage and effect of their use it can be necessary to model the winding in details, to study carefully skin and proximity effects. Such study can also highlight consequences of the eddy currents like the heat up of the conductors exposed to high eddy current density for example those closest to the air gap in rotating machines, or local heat spots in transformers.

Consideration of eddy currents in multi-conductor windings is a classical but still a recent and exciting research topic. There are plenty of multi-conductor applications in electrical machinery and devices where the proper calculation of eddy currents is sorely needed for a prosperous design. The most classical problems are connected with the design and optimization of the windings in transformers and in rotating electrical machines. There are also new technologies such as high-speed high-power machines, which highlight the importance of the eddy-current effect in the windings.

The effect of eddy currents is traditionally neglected in the windings when solving the magnetic field, and eddy-current losses are calculated from a field solution which was gained by neglecting the effect of them. This kind of approach has its limitations and should be used carefully. One should see it clearly if the effect of eddy currents on the magnetic field can or cannot be ignored. Sometimes it is difficult to tell it in advance especially in the presence of nonlinear magnetic materials and in special transient problems. However if the amount of high frequency components of the field are known to be small in advance the computation the Faraday law might be neglected. When it is ignored, skin and proximity effects are not considered in the field computation.

Classical electrical machine design defines the geometry of the conductors in the windings in such a manner that skin effect should be limited. This purpose is superb but the applied tools to reach it are usually poor. Classically the geometrical parameters are determined by empirical formulae, which can be applied only for certain well-known designs. These formulae do not help when studying essentially new constructions.

1.1 Mathematical modelling of eddy currents

In order to avoid the empirical approach and to generalize a solution, case independent mathematical models should be built. The mathematical models can be applied in analytical, in numerical or in nowadays more popular combined analysis. In this subchapter, these analyses are introduced shortly. A deeper literature study is presented in Chapter 2.

1.1.1 Analytical computation

Analytical computation of eddy currents is usually applied for simple and special cases. Due to the simplifications or simplifying assumptions about the actual physical picture we usually arrive to similar limitations as when using empirical formulae. An analytical solution is usually valid only for certain special constructions, and/or conditions but it is still more flexible than the empirical formulae. There are plenty of works considering the analytical solution of eddy currents in simple geometries and several methods were applied for multi-conductors. Unfortunately it is practically impossible for a design engineer to obtain the general analytical solution for an arbitrary multi-conductor problem with a high number of conductors. However with the simplifying assumptions analytical computation can become a powerful tool.

1.1.2 Numerical analysis

A multi-conductor stator winding can be modeled by finite element method (FEM) with a detailed mesh and eddy-currents can be modeled properly when solving the magnetic field. The consideration of eddy-currents in the rotor winding (rotor bars) of cage induction machines is common using the FEM while the study of eddy-currents in stator windings is still an avoided task because of certain practical problems. The finite element modeling of the stator winding could result in thousands of elements per stator slot, which can make practical machine design very difficult or impossible within the finite element method.

Boundary element method (BEM) is of growing importance in recent years. Many scientific papers are dealing with the applications of this method. However there is little sign that BEM can be applied more efficiently for modeling an arbitrary multi-conductor winding for transient problems than the FEM recently, but the intensive research about boundary element method might offer this possibility in the near future. According to an in-depth literature study, multi-conductor eddy-current problems by the boundary element method are solved in time harmonic analysis, and nonlinear materials in the surroundings can hardly be considered. Boundary element method does not offer a superior way over the FEM to consider large skin depth transient problems. Time harmonic analysis can be performed with serious advantages compared to FEM for large and small skin depth problems but transient BEM analysis is advantageous only for small skin depths with surface currents.

In the integral equation (IE) formulation, when only the conducting and the magnetic regions are meshed, we arrive to a full matrix connecting the nodes on the boundary of the stator slot and on the boundary of the conductors. It is very similar to the "case of islands" during a badly selected elimination sequence as it is presented in Publication 4. The number of non-zeros in the system matrix increases dramatically for such formulation. It is not suggested for those kinds of problems where the number of nodes which, are mutually coupled is very high. This is the case in the IE model of the stator slot with many conductors.

1.1.3 Combined numerical - analytical analysis

Combined numerical-analytical methods are lively and promising subjects of research. Analytical methods live their renesans due to their speed and simplicity in certain applications. The coupling of numerical and analytical methods can overcome the weaknesses of both analysis and can utilize the advantages from both. Despite the great promises of the coupling, there are only a few references for coupling numerical and analytical methods for the modeling of the stator slots, or multi-conductor windings and those are usually heavily simplified. This is probably due to the fact that the general analytical modeling of multi-conductor systems is a very difficult task requiring heavy simplifications.

1.2 Summary

The consideration and calculation of eddy-current effects in multi-conductor windings is a difficult but necessary task in many fields of electrical engineering especially on the field of electrical machines and devices. Presently the practice is to ignore eddy-current effects during the magnetic field computation or to approximate them by simple empirical formulae because there is not any convenient solution for design engineers in the every day design work. The research on this topic is very lively and the present state of the art has to be studied very carefully before future trends could be observed.

1.3 Aim of the work

The aim of the work has been to develop methods, which are efficient in the modeling of eddy-current effects in multi-conductor windings in the electromagnetic field analysis of rotating electrical machines. The developed method should be easy to implement and use in electrical machine design.

The development and testing of proper measuring techniques to evaluate the new methods has also been a goal of the work.

1.4 Structure of the work

The research work accomplished for this thesis can be divided into the following major steps:

1. A detailed literature review in Chapter 2 is given to present the state of the art of the eddy current computation methods for multi-conductor windings.
2. Overview of the 2D finite-element eddy-current model in Chapter 3.
3. Verification of the finite element model for multi-conductor eddy-current problems by comparison to measurements in Chapter 4.
4. Study of the effects caused by the “elimination” of the winding region, with respect to the size and complexity of the system of equations in Chapter 5.

5. Determining the right place and order for the “elimination of inner nodes” method inside the nonlinear iteration process in Chapter 5.

6. Verification of the advantages and limitations of the pure elimination of inner nodes method in Chapter 6.

7. The development of the novel macro element method – which requires only one elimination of the system matrix and uses constant macro elements in the creation of the eliminated system of equations during the whole time stepping analysis – is presented in Chapter 7.

8. The development of advanced techniques for the application of macro elements in electrical machines presented in Chapter 8. Memory requirements have been decreased and the periodicity of the machine geometry has been utilized.

9. Test computations with several machine models to evaluate the effectiveness of the macro element method presented in Chapter 9.

In the following chapters these steps will be presented and explained in details as separate entities. The steps from 3 to 9 are based on publications. These publications are reprinted in the Publications chapter at the end of the thesis.

The content of the publications and the involvement of the author of this thesis in those articles is the following:

Publication 1

The significance of Publication 1 from the point of view of this thesis is that the verification of the finite element multi-conductor, eddy-current model - and the corresponding code - has been presented. The code, which has been verified in this paper, has provided the basis for further research. The skin resistance of a multi-turn winding has been modeled by 2D FEA. The calculated results were in good agreement with measurements.

The paper has been written by Áron Szücs. The finite element model of the coil used for the verifications has been built by Áron Szücs, and the test measurements and test computations have been performed by him also. Antero Arkkio contributed with his insight for the finite element model, which has largely been developed by him.

Sakari Palko has given invaluable help in the broken rotor bar analysis. Ivan Yatchev developed the multi-conductor, eddy-current code - as it existed that time - and which has been tested in the paper. Invaluable comments from Julius Saitz have helped the work presented in the paper.

Publication 2

In Publication 2 one of the fundamental ideas of this thesis - the "elimination of inner nodes" method - has been evaluated. When the effect of eddy currents in the multi-conductor stator windings is included in the finite element model the number of nodes in the linear stator slot regions is very high.

By the Gauss elimination of the variables corresponding to nodes inside the stator slots the size of the system matrix has been reduced. The reduction created several small full matrices in the system matrix. The major question in the paper was: Will the solution time decrease if we eliminate the inner nodes from the stator slot? The conclusion was that the efficiency of the elimination depends on the ratio of the number of nodes inside and on the boundary of the eliminated region. In some test cases the solution of the reduced system was 20 times faster than that of the original.

The paper has been written by Áron Szücs. Antero Arkkio has been a co-author of the paper. He contributed with valuable comments during the evaluation of the elimination method and helped with comments about the paper.

Publication 3

In Publication 3 the place of the elimination process inside a nonlinear iteration process has been studied. The paper concludes that it is enough to perform the elimination only once during a nonlinear iteration. The system matrix has been divided into linear and nonlinear parts where only the linear part needs to be eliminated. The eliminated linear part is created in the first step of the iteration and remains constant during the whole iteration. The nonlinear part is changed as needed by the iteration process and it is added to the constant eliminated linear part in every iteration step. This way the elimination has to be done only in every time step and not in very iteration step. It is an important step forward the utilization of the elimination but the ultimate goal - to create a constant macro element, which models the winding region in the whole time stepping analysis - has not been reached in this paper.

The work for the paper has been accomplished by Áron Szücs and the paper has been written also by him. Antero Arkkio who has been a co-author of the paper, contributed with his valuable comments and insight.

Publication 4

Publication 4 presents how to create a constant macro element once and to use that during a whole time stepping analysis. The idea behind the macro element method is that the linear part of the system matrix, which belongs to the multi-conductor stator windings, remains structurally unchanged during the whole analysis. This means that the same eliminated matrix can be included in all the time steps. Combining this conclusion with those presented in Publication 3, one can conclude that only one elimination of the inner nodes is required. The paper also investigates the effect of the elimination sequence on the speed of the elimination and suggests several algorithms for the proper selection of elimination sequence.

The paper has been written by Áron Szücs without co-authors. The paper acknowledges Prof. Tapani Jokinen and Dr. Antero Arkkio for their valuable help during the work.

Publication 5

Publication 5 presents the advanced use of the elimination method. The major conclusion of this work is that the macro element approach can not only increase computation speed but it can have several other advantages. The macro element method cuts a large problem to smaller, separately solvable parts, which still remain strongly coupled despite their separation. The division of the problem in such ways leads to much lower demand for memory size especially when the symmetry of the repetition of the stator slot geometry can be utilized. The paper concludes that the reduction in memory requirements by the macro element approach can be more important than any speed improvement. The macro element method can allow the solution of large problems, which would be unsolvable with limited computing resources. A new explanation of the matrix structure - when using macro elements - is also given in the paper.

Áron Szücs is the principal author of the paper. The co-authors Tapani Jokinen and Antero Arkkio contributed with precious comments for the paper.

Publication 6

Publication 6 presents a real life test of the macro element method. Two test machines have been analyzed with and without the use of the macro elements. All together 14 time stepping analyses have been performed and the results have been compared to evaluate the effect of the macro element method. The computation results have been the same but there was a significant difference in computation time and in memory requirements. The paper concluded that with the increase of the non-linearity of the problem the macro element method become more efficient. The speed improvement in the best case was 20 percent. The significantly decreased memory requirements for these test cases have also been noticed.

Áron Szücs is the principal author of the paper. The co-authors Tapani Jokinen and Antero Arkkio contributed with help and comments for the paper.

2 OVERVIEW OF EDDY-CURRENT COMPUTATION METHODS FOR MULTI-CONDUCTOR WINDINGS

State of the art of multi-conductor eddy-current modeling

A short summary about the basic theory behind eddy currents and the most significant analytical, numerical and combined methods in concern with the topic are presented in this chapter. The selection of the related papers is a hard task, because after the few directly connected publications we can find thousands of partly related works which can play important role in future research but they do not fall on the straight line of this specific research. The purpose of the following literature review is to highlight the major lines of research and achievements concerning the topic.

Eddy currents are induced in conducting materials due to the time variation of the magnetic field. This fact is well known as Faraday's law represented by (2.1).

Before going deeper into the mathematical descriptions and equations of eddy currents we can state some basic assumptions. The most common assumptions when investigating electrical machinery are the followings:

- the displacement current is neglected,
- isotropic conducting media are considered,
- no atomic effect is taken into account (macroscopic model),
- homogenous media structure (no domain theory),
- thermal nonlinearity is neglected, (lately there is an intensive research about this topic)
- in magnetically linear materials permeability μ is constant.

The validity of the first assumption is really questionable when the frequency of the source is very high or in case of fast transients. This topic seems to be encouraging for further studies, but we can suppose that in an ordinary electrical machine the effect of capacitive currents do not significantly influence the appearance and distribution of eddy currents in the windings. This statement is usually true even for machinery connected to inverters with small switching rise time. The amount of displacement currents compared to the phase currents is normally two or three orders of magnitude less. Taking capacitive effects into account requires the computationally very heavy three-dimensional analysis.

Under the listed assumptions the following fundamental laws of electromagnetism (most of Maxwell equations) represent the equations describing eddy currents.

$\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t}$	Faraday’s law	(2.1)
$\nabla \times \boldsymbol{H} = \boldsymbol{J}$	Ampere’s law	(2.2)
$\nabla \cdot \boldsymbol{B} = 0$	Gauss’s law	(2.3)
$\boldsymbol{B} = \mu \boldsymbol{H}$	constitutive relation	(2.4)
$\boldsymbol{J} = \sigma \boldsymbol{E}$	constitutive relation, Ohm’s law	(2.5)

The variables $\boldsymbol{H}$, $\boldsymbol{E}$, $\boldsymbol{B}$ stand for the magnetic field strength, the electric field strength and the magnetic flux density, respectively. The term $\boldsymbol{J}$ is the current density, μ and σ are the magnetic permeability and electric conductivity, respectively.

In the finite element formulation of electromagnetic field problems, an important role can be played by the magnetic vector potential. It is commonly used in the solution of two-dimensional magnetic fields because in that case it reduces to a single-component variable. To satisfy the non-divergence of the magnetic field, the magnetic vectorpotential is defined so that it’s curl is equal to magnetic field density

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \ . \tag{2.6}$$

Accordingly $\nabla \cdot (\nabla \times \boldsymbol{A}) = 0$ is satisfied for any $\boldsymbol{A}$. In order to define the vector field $\boldsymbol{A}$ uniquely, (2.6) alone is not enough. In magnetostatic field problems the Coulomb gauge is usually used to specify the magnetic vector potential (2.7).

$$\nabla \cdot \boldsymbol{A} = 0 \tag{2.7}$$

Faraday’s law described in (2.1) can be formulated by the use of magnetic vector potential (2.8).

$$\nabla \times \boldsymbol{E} = -\frac{\partial(\nabla \times \boldsymbol{A})}{\partial t} = -\nabla \times \frac{\partial \boldsymbol{A}}{\partial t} \tag{2.8}$$

This expression can be written into (2.9) where the appearance of $-\nabla\phi$ is no surprise since $\nabla \times (\nabla\phi) \equiv 0$. The symbol ϕ stands for the reduced electric scalar potential.

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\phi \tag{2.9}$$

Substituting (2.9) into (2.5), we get the equation concerning the current density (2.10).

$$\boldsymbol{J} = -\sigma\frac{\partial \boldsymbol{A}}{\partial t} - \sigma\nabla\phi \tag{2.10}$$

Combining (2.10) with (2.2) we arrive to (2.11). This is the field equation which considers the reverse effect of eddy currents on the magnetic field which causes them (Lenz’s law).

$$\nabla \times (\nu\nabla \times \boldsymbol{A}) + \sigma\frac{\partial \boldsymbol{A}}{\partial t} + \sigma\nabla\phi = 0 \tag{2.11}$$

From the point of view of this thesis, the skin effect and proximity effect in current carrying multi-conductor windings are the major topics. Skin effect in multi-conductor systems is referred to the phenomenon when the current flowing in a conductor is crowded out to the surface by its own time varying magnetic field. Proximity effect occurs when the current density in a conductor is influenced by the varying magnetic field created by the current flowing in neighboring conductors. Skin effect and proximity effect are both due to eddy currents, caused by the time varying magnetic field and their separation is purely hypothetical. This logic of separation can be found more frequently in the corresponding earlier literature.

2.1 Analytical methods to consider eddy currents

Eddy currents were tackled purely by analytical methods before the advent of the digital computer. The eddy-current effect was recognized very often as skin effect and in the literature they referred to it by this name. The problems investigated analytically were limited to simple geometrical configurations such as planes, cylinders, or spheres and the problems had to be linear. Analytical computation of eddy currents has a long history and it is often referred as an old approach, but it should not be considered useless in the shadow of numerical methods. Analytical methods are still widely used and probably they will always be considered as the basis for electromagnetic field computation. It is difficult to tell who is the founder of eddy-current calculation but the first valuable results, which are still in use nowadays were presented by A. B. Field in 1905 [17]. An excellent overview of analytical methods can be found in [51] by Stoll R. L., 1974 and in [27] by Lammeraner J. and Stafl M. from 1966.

Despite their limitations, when applicable, analytical methods are usually fast and accurate and maintain close contact with the physical reality.

The most widely used analytical design method is based on the single conductor approach. The basic idea is to define the geometrical parameters of the conductor so, that eddy currents are negligible at the operational frequency ϖ. The geometry is designed in accordance with the skin depth δ, which should be much larger than the width and/or height of the conductor. The skin depth, in case of sinusoidaly varying field, is defined in (2.12) where σ is the electric conductivity, μ_0 is the magnetic permeability of free space and μ_r is the relative magnetic permeability of the material of the conductor.

$$\delta = \sqrt{\frac{2}{\varpi\sigma\mu_0\mu_\mathrm{r}}} \tag{2.12}$$

As an example, at 50 Hz supply frequency the skin depth is about 9 mm for copper and 2 mm for saturated mild steel [51]. In synchronous generators, the 9 mm skin depth is a well known important parameter when designing the winding of the machine. The size - usually the height - of the conductors is selected typically less than 5 mm but this depends on the allowed amount of additional Joule losses. The amount of these losses is usually between 10-30% of the resistive losses in the practice.

The most popular analytical approach for multi-conductor eddy-current analysis is the network approach. In 1988 Iseli et al. presented an approximation of a solid conductor by a fictious strand structure [22]. This way the problem is reduced to the determination of the self and mutual inductances of the conductors involved. An improved model was presented by Wong 1994 in [67]. The use of such analytical approach becomes very complicated in case of high number of conductors.

An analytical circuit model of a DC machine winding was presented by Milykh V. I. 1993 in [31] in order to consider eddy currents due to the saturation of the armature-core teeth and to the relatively large cross-section of the wires. A very detailed explanation about the usage of the method is given in this paper but the references are difficult to reach and only published in Russian. The paper deals with six large conductors and uses network approach similar to that of Iseli et al.. The use of this approach would be similarly painful as it was mentioned before for a higher number of conductors for every day design.

An interesting investigation is conducted by Salon at. al. 1993 in [42] where a closed analytical formula was compared to finite element analysis. The analytical formula - which ignores the effect of eddy currents on the field - studied in this paper is widely used in machine design to find the eddy factor. The formula (2.13) gives the eddy-current loss in a rectangular conductor (height is 2b, length is *l*) as a function of the geometry and the alternating flux density defined as $B_0\sin(\omega t)$. Variable ρ stands for the resistivity of the conductor.

$$W = \frac{lb^3 B_0{}^2 \omega^2}{3\rho} \tag{2.13}$$

This is the eddy loss only and the loss due to the load current must be added to it to get the total loss in the conductor. In the paper, the strand height had been chosen to be much smaller than the skin depth at rated frequency, this way, the loss is resistance limited and the effect of eddy currents to the field is negligible. The resistance limited case predicts higher losses than actually occur because it ignores the decrease of the actual flux due to eddy currents.

A coupled magneto-thermal analysis of additional Joule losses in the copper conductors of induction motors was published in 1996 by Autier V. et al. in [5]. They investigate the effect of the temperature on the eddy currents in an electrical motor. Analytical formulae are applied and a simple model is build for modeling the thermal effect. The thermal effects in Roebel bars and Punga bars were presented.

A method for the analysis of eddy-current loss caused by the main and slot leakage flux in the bars with hollow conductors of electrical machine winding is presented by Demenko A. et al. in 1996 in [15]. Analytical approach is compared to FEM and the results are commented from the point of view of loss calculation. Special attention has been paid to Roebel bars composed of hollow and full copper conductors. They concluded that in the analysis of the power loss, the circulating currents can be neglected in Roebel bars using either hollow or full conductors. The analytical formula presented for the calculation of stray load loss was found to be more accurate than those in previous studies. They performed time harmonic analysis and the saturation effect was neglected.

An analytical method was presented in [32] 1990 by Namjoshi K.V. and Biringer P. P. to calculate the magnetic field when two long, perfectly conducting non-magnetic cylinders are placed in a uniform transverse magnetic field. The expressions derived in the paper may be used to estimate the proximity effect and the skin effect.

There are papers dealing with analytical methods applied in the analysis of eddy-currents in the rotor bars of induction machines. However, the application of these methods presented for rotor bars are not applicable directly for multi-conductor windings. Among these one can find [23] which will be explained more thoroughly in the coupled analytical - numerical section.

2.2 Numerical methods to consider eddy currents

Although there are several applications where analytical computation is sufficient, the majority of electromagnetic machinery and devices nowadays cannot be investigated purely analytically due to their complicated structure and due to the materials with nonlinear properties. This is the reason for the extensive use of numerical methods in the research and development and every day design work nowadays. Numerical methods can handle well nonlinearities and practically any configuration with complicated boundaries. The weakness of numerical analysis can be characterized by the long iterative computations and by the "blurred" picture of the physical reality. The information about the physical picture can be maintained partly by backing the numerical analysis with dimensional analysis. This can save time when calculating a large set of operational points.

A great source of information about numerical analysis of eddy currents is [26] by Krawczyk A. and Tegopoulos J. A. 1993. In this book, an advanced and in-depth picture about the topic is given, and numerical methods are explained in details. Special cases typical in engineering practice like the open boundary problem are also discussed. The book was published in 1993 and gives a good summary. However to get a recent picture about the newest developments especially about the handling of external circuit connections one should study literature written after 1993.

2.2.1 Finite Element Analysis

The most convenient way to model eddy currents in multi-conductor windings is to perform finite element analysis (FEA). There are two very popular one-step finite element formulations to consider eddy currents. They are called one-step methods because they realize a strong coupling of the eddy-current effect with the field solution. In contrary with the numerous earlier studies which used weak coupling and so iterations, or modal network solutions and superposition, (such approaches can be found in [27], [51], [60], [30], [45-50], [2], [11], [9-10], [13], [65]) in the one-step formulation the result is obtained in one single solution of the system of equations.

A. Konrad was first to present an integrodifferential approach 1981 in [25]. The other approach was published a year later in 1982 by Weiss J. and Csendes Z. J. in [64]. Their formulation is based on adding an extra equation for each conductor involved. The integrodifferential approach results in more dense system matrix while the addition of extra equations increases the size of the system. In both of these cases, a detailed finite element mesh is required to model the conductors especially the penetration region.

An in-depth assay about the perspectives of the eddy-current calculation in electrical machines has been given in [38] by Reichert et al. in 1988. They emphasized that the finite element meshing - in small skin depth problems - can increase the problem size so much, that it can limit the use of conventional finite element techniques. This observation is a major driving force from the point of view of this thesis.

The circuit equations connected with the finite element problem were included into the solution by Bourmanne P. et al. in 1991, [8]. A multi-conductor winding embedded into a linear magnetic slot was investigated in the paper. The calculation of the skin and the proximity effects in multi-conductor systems was the main concern of the work. The boundary element method was used to model the magnetic field in linear magnetic nonconducting media and finite element equations were used in the nonlinear magnetic and/or conducting media. The magnetic field in the multi-conductor winding was basically modeled by finite element method. The paper explains methods to deal with both serial and parallel connected conductors. This work is also a good example of joining boundary elements with finite elements.

In 1992 Sturgess J. P. presented an interesting work [52], where he investigated the three dimensional effects in two dimensional field modeling. He explained that the proper circuit connection modeling can be crucial for accurate analysis. The most interesting example is the rotor of a salient pole generator. The field plots of two solutions are compared. The first was calculated by the damping bars connected and the second the damping bars disconnected. The effect is very significant, although in both cases eddy currents were considered in the bars. Only time harmonic analysis was presented.

A method to model circuit connections in time dependent eddy-current problems was presented by Tsukerman I. A. et al. in [61] in 1992. The paper deals with 2D finite element analysis of eddy currents based on Konrad’s integrodifferential approach but in this work external circuit connections of the conductors are also taken into account. Konrad A. is a co-author of this paper. Conductors with eddy currents were treated like circuit elements with terminal voltages implicitly defined by the field equation. The finite element method was combined with the conventional loop current method of circuit theory to formulate the circuit equations. They show, how this method allows the consideration of outer regions such as end effects. The problems investigated by Reichert et al. in 1988 were not studied.

In 1993, Tsukerman I. A. et. al. (Konrad A. is again a co-author in this paper) present an overview of coupled field-circuit problems, with a “subtitle” : “Trends and Accomplishments” [62]. The paper reviews the formulations and numerical methods for two-dimensional eddy-current problems in which conductors with eddy currents are connected by external circuits. In the historical notes, they explain how eddy-current problems were handled first without and later with circuit connections. They explain how the so called filamentary problems were first to be solved with circuit connections and later eddy-current problems followed. The differentiation between filamentary and eddy-current problems may require some explanation. The first means that the conductors are composed of thin filaments and so the current distribution is predetermined. In such cases, the current density is not a function of the vector potential so the problem can be seen as a magnetostatic problem. The eddy-current problem in their definition is the case where the current distribution is unknown as it is in solid conductors and so it is related to the magnetic vector potential: $\boldsymbol{A}$. They suggest finally a similar approach as it was presented in [61] before by two of the principal authors. The real merit of this work is that it collects and summarizes different approaches for coupled field-circuit eddy-current problems, but again the problems mentioned by Reichert et al. in 1988 were not resolved.

Just ten pages further in the same volume (1993) there is an other in-depth study about transient eddy-current problems [63] written by Tsukerman I. A. with the co-authorship of Konrad A. again and two other authors. This time, they present a survey of numerical methods to consider transient eddy-current problems. The work starts from the finite element formulations, goes through the time stepping methods and system solvers for implicit schemes, and finally includes some numerical examples. The finite element formulations considered are

- the integrodifferential formulation in 2D
- the $\boldsymbol{H}-\boldsymbol{\Omega}$ method
- the $\boldsymbol{T}$-$\boldsymbol{H}$ method
- the $\boldsymbol{A}^{*}-\boldsymbol{\Omega}$ formulation.

A very recent work published by Weeber K. in 1998 in [66] deals with a similar topic as Sturgess J. P. 1992. However, Weeber applied time stepping analysis with Crank-Nikolson time discretization in the FEM to study the amortisseur windings of single-phase synchronous generators. He modeled the rotation of the rotor and applied a strong coupling between field and circuit equations. He finally identifies the main cause of the mechanical failures of the rotor as the temperature difference between adjacent amortisseur windings, and the large mechanical stresses caused by the differential axial expansions.

Another recent publication by Johan Driesen et. al. [16] in 1998 deals with the thermal effects in transformer windings. They consider the changes in the current density distribution arising from the thermal nonlinearity of the conductors. They used time harmonic analysis of the current harmonics. The magnetic nonlinearity is taken into account by approximating the time varying magnetic reluctivity by a fictious constant function. This way, only time harmonic analysis is required for the field computation. The computational problems connected to this kind of approach are the instability of the nonlinear iterations and the heavy computational requirements. Another important drawback is that motion and so rotating electrical machines can not be modeled this way. Although this approach has some computational problems it could be a very promising method. As it will be explained in the boundary element section, in case of time harmonic problems boundary element method (BEM) is superior over the FEM from the computational efforts point of view. This fact could mean that such an approach for considering nonlinearities combined with the BEM can be more efficient than the time stepping FEM for certain problems. This is an important observation for future research.

Ivan Yatchev, Antero Arkkio, Asko Niemenma has published a laboratory report in 1995 in the Helsinki University of Technology, Laboratory of Electromechanics. The title of the report was "Eddy-current losses in the stator winding of cage induction motors" [69]. In this study, they have investigated the effect of eddy currents in the stator winding, for the finite element analysis of induction machines. In sinusoidal case at the nominal frequency they did not notice significant change in the computed losses in their test machine. They concluded that only in case of inverter supply the effect of eddy currents has been significant in steady state. The most interesting conclusion was that the losses in the iron have decreased - due to the reduced total current by the eddy currents - and the losses have "redistributed" in the machine when the eddy current model was included. This loss redistribution is one of the most interesting topics to be investigated with a proper model of the eddy currents in the stator winding. The drawback of the model used in this report was that the same as the problem with many previous studies. The size of the finite element problem has increased very much compared to the size of the problem without the eddy current model. To keep the model size small, only a very simple model of the conductors has been used. This work was a major inspiration to launch a research project on this subject. The research work summarized in this doctoral thesis has been initiated by the recognition of the need for a reduced but accurate model for stator winding eddy current analysis.

2.2.2 Conclusions of the Finite Element subchapter

The finite element method is the most convenient way to consider eddy currents in multi-conductor windings. The finite element analysis is usually easy to formulate but in practical applications a detailed finite element mesh for a high number of conductors and / or small skin depth may require thousands of elements and nodes. Due to that, the size of the problem can be much higher compared to the simplified conducting region model [3] where current density is supposed to be homogeneous and the whole multi-conductor winding -like the stator slot of an electric machine - is modeled with one large conductor. This is the widely spread practice in today’s R&D of electrical machines. The increased problem size due to the detailed mesh for the conductors can make practical machine design very painful or impossible, as it was indicated by Reichert et al. [38] in 1988.

2.2.3 Boundary Element Method

In recent years the boundary element method (BEM) has become very popular. Several applications in connection with multi-conductor windings can be found.

A paper entitled the “Calculation of two-dimensional eddy-current problems with the boundary element method” was presented in 1983 by Rucker W. M. et al. in [37]. Two dimensional eddy-current problems are formulated by a set of boundary integral equations and were solved by the boundary element method. Time harmonic analysis was performed for the study of an infinitely long circular conductor. The results proved that the boundary element method provides high accuracy of the solution using quadratic elements.

In 1986 Zhou P. B. and Lavers J. D. published their work [71], in which they consider skin and proximity effects for single and multiple conductors. The equations are formulated by an integrodifferential expression and the problem was solved by the boundary element method. They studied the accuracy of the computation for linear and constant elements. The losses in single and multiply connected, solid and hollow conductor coils were studied. The goal of the paper was to calculate the time harmonic 2D magnetic vector potential $\boldsymbol{A_z}$ resulting from one or more long conductors carrying prescribed total currents. They reached this goal and showed that the constant boundary elements proved to be surprisingly good for problems where the ratio of characteristic dimension to penetration depth was less than 20. The constant elements proved to be superior to linear elements for such problems. The authors also developed a hollow conductor correction factor using their results and a single turn coil. The paper was limited to time harmonic analysis only.

In 1990, Ahmed T. M. presented a new formulation [1] for skin and proximity effect eddy-current problems. The approach is called H-formulation boundary-element method. The advantage of this method is that its highest singularity is a weak one and it is more accurate than previous formulations. Using the suggested formulation, results in a diagonally dominant matrix without creating a strong singularity term in the boundary integral equation.

Rucker W. M. and Richter K. R. present a software package based on a BEM code for 3D eddy-current calculations [36] in 1990. They applied time harmonic analysis to solve coupled boundary integral equations for the magnetic vector potential and the electric scalar potential within the conductors and the magnetic vector potential in non eddy-current regions. The results from boundary element analysis were compared with those of the finite element method and perfect agreement was found.

A sophisticated study was presented by Zhou P. et al. in 1991 in [73] to solve nonlinear eddy-current problems with the BEM. They suggest an iterative scheme to solve multi-conductor eddy-current problems where conductors have nonlinear permeabilities. Their results agreed well with the corresponding finite element analysis. This study was also limited to time harmonic analysis, and the magnetic flux density was assumed to be sinusoidal, despite the presence of nonlinearity. This assumption is a serious limiting factor for applications in electrical machinery.

All of the listed papers above restrict their studies to time harmonic analysis. This is due to the fact, that in time stepping analysis of transient problems, the superiority of the boundary element method over the finite element method disappears. A well placed explanation can be found in the abstract of [59] by Tanaka M. et al. from 1993:

"In the sinusoidal steady-state analysis, it is an advantage of boundary element method that eddy current analysis can be performed by only the surface integral without the volume integral. However, the volume integral and the surface integral are necessary for boundary element analysis in the transient problem. Therefore the advantage of the boundary element method vanishes."

They propose a Fourier approach for the transient problem to arrive to a situation in which the use of boundary element method is sufficient. After gaining the results by BEM those are transformed back to time domain by the inverse Fourier transformation. They study linear problems only so the problem of magnetic nonlinearities is not dealt.

Another area where boundary element method can be highly utilized is the field of high frequency small skin depth problems. Yuferev S. V. and Yuferev V. S. published two interesting papers [70], [71], in 1992 and 1994, respectively. In the first paper, they study a problem in which a short current pulse passes through a conductor. In such a case, the electric current flows only in the surface layer of the conductor, and the magnetic field penetrates only into a very narrow layer from the surface also. They investigate the current redistribution due to inductive coupling of so called current elements. They however clearly state that the current redistribution due to the direct diffusion of the magnetic field in all directions is not considered because it would require the use of the complete diffusion equation for the field inside the conductor. In the second paper, they formulate a three dimensional problem in a two dimensional boundary integral formulation extended with an additional term reflecting the axial dependence of the potential distribution resulting from the finite length of the conductors. Also in this paper, the short current pulse case with only surface currents was studied.

2.2.4 Conclusions of the Boundary Element subchapter

One can conclude that the intensive research on the BEM in the future might result in such a method, which can handle transient and nonlinear problems better and faster than the FEM, but recently the FEM is more convenient for such problems. From the point of view of this thesis, this is an important observation.

2.3 Combined numerical and analytical analysis of eddy currents

The majority of numerical analysis of electrical machines and devices is combined with analytical computations. In case of eddy-current loss computation in the stator winding of electrical machines, the use of analytical, and empirical formulas is a common practice. The eddy-current losses can be calculated from a magnetic field, which is actually not influenced by the effect of eddy currents. This is proper only in the filamentary case, however, because of the simplicity of this approach it is widely used in non-filamentary cases as well with some loss of accuracy. In such cases, the analytical methods are not used to consider the effect of eddy currents on the magnetic field.

The combination of numerical and analytical methods to consider the effect of eddy-currents on the magnetic field is more important from the point of view of this thesis. In certain cases, the flexibility of numerical analysis can be combined with the advantages of analytical approach.

The combined analysis is widely used when the problem can be divided into regions among which some can be modeled sufficiently analytically and some others which are more suitable to model numerically.

A good example for the application of combined methods is [23] by Kladas A. and Razek A. from 1988. This paper describes a combined analytical macro element and finite element method for the consideration of eddy currents in rotor bars. An asynchronous machine is modeled by finite element method except the rotor bars which are represented by analytical macro elements. Macro elements were connected to the finite element mesh by their boundary nodes (in this case the nodes on the boundary of the rotor bar). The advantage gained by combining the numerical and analytical methods in this paper is that a wide range of supply frequencies can be considered by the analytical macro element without any mesh refinement. In case of a finite element rotor bar model, the consideration of increased frequency would require the refinement of the rotor bar mesh. The application of this approach for stator windings looks very difficult, because it requires the accurate modeling of the winding by analytical means. This is the same obstacle as in the case of purely analytical methods, although here the rest of the machine like the stator iron could be modeled by the FEM.

In 1990 Ngnegueu Tr. et al. proposed a separate treatment of thin and thick conductors in electromagnetic devices [33]. They suggest that the Joule losses caused by the eddy-currents in the thin conductors should be taken into account by analytic formulae because the effect of the eddy currents to the flux distribution can be neglected in their region. They propose to treat thick conductors by FEM. The accuracy obtained was found to be good comparing FEM with the analytical solution for thin conductors. Time harmonic analysis has been performed for all components of the field. The limitation of this method is that the selection of the regions where the inverse effect of eddy currents on the field is neglected should be determined beforehand. This can be difficult in case of non-sinusoidal supply with high harmonic components. Transient analysis with this approach looks even more difficult in the presence of non-linearities. The proper description of eddy-current effect on the magnetic field would be done by the FEM as this paper suggests also.

Kladas A. G. and Tegopoulos J. A. presented a new analytic element method in their work [24]. The advantage of this method is that more complex geometries can be modeled by analytic elements and the need to use finite elements disappears. The whole solution domain can be modeled by analytic elements. Unfortunately, the paper deals only with a very simple geometry, sinusoidal supply and linearized magnetic properties, and there is no indication, that transient nonlinear problems could be modeled by this approach.

A semi-analytical method was proposed by Mayergoyz et. al. 1986 [28] for the calculation of the magnetic field in slotted structures of A.C. machines. The method is based on applying a slot impedance boundary condition. It leads to the analytical representation of magnetic field in terms of spatial harmonics. This approach is limited to a certain kind of rectangular slots whose depth are much larger than their widths. The magnetic field is also supposed to be varying in a predominant direction namely from the top to the bottom of the slot. This is essential for treating the slots in the impedance boundary way. The analysis of saturated slots and transients is inadequate with this approach.

In 1994, Gyimesy M. et. al. presented a paper [21] about the application of external circuit connections for multiply connected regions using the impedance boundary condition. The paper gives a good overview of the recent state of the art about the use of impedance boundary conditions. They state that impedance boundary conditions have been used together with FEM and BEM as well. The basic idea when using impedance boundary conditions is that the eddy-current domains are left out of the solution region, and only their effect is considered by a special boundary condition. The use of impedance boundary conditions requires time harmonic analysis and the consideration of nonlinearities is neglected in the paper similarly as in the work by Mayergoyz et. al. 1986 [28]. The paper does not deal with cases where the excluded region is also an active source of the field. Transient problems were also ignored.

2.3.1 Conclusion of the combined analysis chapter

According to the result of the literature survey, no combined numerical-analytical method has been found which would treat a transient, 2-D, multi-conductor eddy-current problem in the presence of nonlinear materials in such a way that the effect of eddy currents on the magnetic field would properly be taken into account. The problem seems to be the same as in case of purely analytical solutions, namely the analytical model of the multi-conductor winding is very complex.

2.4 Conclusions

We can conclude that the precise consideration of skin and proximity effects caused by eddy currents is an important task but yet unsolved for the every day design of multi-conductor windings like the stator winding of electrical machines. Either analytical or numerical approaches alone lead to serious problems such as the difficult formulation for many conductors in the analytical approach or the large problem size and the long computation times in the numerical approach. Unfortunately it seems that combined methods are also still hard to apply for transient multi-conductor problems. However the single conductor problem (rotor bar) has been solved and there are a large number of papers in recent years dealing with the problem.

One would wish to have the speed and simplicity of analytical methods and the flexibility and relatively straightforward applicability of the finite element method in one package. The most desired solution of the problem could be a so-called FEM combined with macro element method approach. The stator slot of an electrical machine - or the multi-conductor winding of some other device - should be modeled by a macro element that is an analytical or numerical function between the nodes of the region containing the multi-conductor winding.

It has been noticed in the literature review, that the analytical modeling of eddy currents in the multi-conductor windings requires serious simplifications. These kind of simplifications can undermine the very reason why one would take the eddy current effects into account in the magnetic field analysis.

In this thesis, the creation of the macro element is suggested by numerical means. This way the accuracy of the multi-conductor-winding model can be maintained. The macro elements can be utilized to replace the finite element model of the multi-conductor winding during the time consuming nonlinear iterations. From these iterations, the linear winding regions can be excluded if their effect is properly taken into account by the macro elements through the boundary nodes of the eliminated regions. The formulation of this novel method is derived, discussed and verified in this thesis. In the following chapter, the fundamental 2D finite-element eddy-current model is introduced and discussed.

3 FINITE ELEMENT MODEL FOR THE ANALYSIS OF EDDY CURRENTS IN MULTI-CONDUCTOR WINDINGS

The merits and limitations of FEM

In this chapter, a state of the art 2D finite-element eddy-current model for multi-conductor systems is presented which has been considered as a starting point for this work. The corresponding equations of the method are presented first. They are followed by the discussion of the basic mathematics and characteristics of the method with respect to the structure of the system matrix. Finally, the conclusions are drawn about the abilities and limitations of this well-known and well-published approach.

A finite element solver was written in the Laboratory of Electromechanics by Ivan Yatchev in 1995 which takes eddy currents into account in coupled field and circuit analysis [69]. His program is based on the theory described in [62], [63], [64] and on a code resulted by a long lasting research project conducted in the Laboratory of Electromechanics at the Helsinki University of Technology. The achievements of the previous studies are summarized in a 2D finite element program package, which is suitable for the analysis of rotating electrical machines, especially, cage induction motors. Non-linearity of iron is modeled by a single valued magnetization curve. Finite element equations are solved together with the circuit equations. The eddy currents in the windings are taken into account by adding an extra voltage equation for each serial connected conductor and for parallel connected conductors an extra current equation is needed.

3.1 Theoretical background

The model to consider eddy currents in the stator winding requires three kind of basic equations. The finite element equation (3.1) governing the electromagnetic field, the circuit equations for rotor (3.2) and stator (3.3), and extra equations (3.4) to account for eddy currents in the stator conductors. This formulation is based on theory described in [62], [63], [64] and was applied for cage induction motors in [69] where the details of the time and space discretization can also be found. For a deeper understanding of the problem, one can study the finite element formulation of the cage induction motor, which is explained and discussed thoroughly in [3] and [4] by Antero Arkkio.

$$\nabla \times (\nu \nabla \times \boldsymbol{A}) + \sigma \frac{\partial \boldsymbol{A}}{\partial t} - \left(\frac{1}{l_{\mathrm{e}}} \sum_{j=1}^{Q_{\mathrm{r}}} \sigma \eta_j^{\mathrm{r}} u_j^{\mathrm{r}} \right) \boldsymbol{e}_z - \left(\frac{1}{l_{\mathrm{e}}} \sum_{j=1}^{Q_{\mathrm{c}}} \sigma \eta_j^{\mathrm{c}} u_j^{\mathrm{c}} \right) \boldsymbol{e}_z = 0 \tag{3.1}$$

$$u_p^{\mathrm{c}} = R_{\mathrm{r}} i_p^{\mathrm{r}} + R_{\mathrm{r}} \int_{S_p} \sigma \frac{\partial \boldsymbol{A}}{\partial t} \cdot \boldsymbol{dS}_p ;\; p=1,..,Q_{\mathrm{p}} \tag{3.2}$$

$$u_m^{\mathrm{s}} = Q_{\mathrm{s}} \sum_{j=1}^{Q_{\mathrm{c}}} \eta_{jm}^{\mathrm{s}} u_j^{\mathrm{c}} + L_{\mathrm{b}} \frac{di_m^{\mathrm{s}}}{dt} + R_{\mathrm{b}} i_m^{\mathrm{s}} ;\; m=1,..,Q_{\mathrm{m}} \tag{3.3}$$

$$u_n^{\mathrm{c}} = R_{\mathrm{c}} \sum_{j=1}^{Q_{\mathrm{c}}} \eta_{nj}^{\mathrm{s}} i_j^{\mathrm{s}} + R_{\mathrm{c}} \int_{S_n} \sigma \frac{\partial \boldsymbol{A}}{\partial t} \cdot \boldsymbol{dS}_n ; \quad n=1,..,Q_{\mathrm{c}} \tag{3.4}$$

The variables $\boldsymbol{A}, u_m^s, i_m^s, u_n^c$ stand for the magnetic vector potential, the voltage and current of phase *m*, and the voltage on the conductor number *n* respectively. $L_{\mathrm{b}}, R_{\mathrm{b}}$ stand for the inductance and DC resistance of the end-winding of one phase. R_{c} is the DC resistance of one conductor in the slot. Q_{m}, Q_{c}, Q_{p} and Q_{s} are the number of phases, stator conductors and rotor bars in the solution sector and the symmetry sectors respectively. Reluctivity is represented by ν, and σ is conductivity. The η is a logical operator with the values: 0 and 1 describing the relationship of conductor voltages with a certain phase voltage or phase current, or the connection of the nodes in the finite element mesh to the rotor bars and to the stator conductors. The value 1 is applied if there is a connection and 0 otherwise. The effective air gap length of the machine is l_{e}. S_n is the cross sectional area of the n-th conductor.

Generalized first order finite difference procedure is used for the time stepping method. According to the study made by Ivan Yatchev, the Backward Euler method has proved to be more stable than the Crank-Nicholson (trapezoidal) method. Similar suggestion can be found in other works. This is proved by the analysis of the conductor voltages during the time stepping process. Non-linearity of iron is modeled by a single valued magnetization curve. The nonlinear system is solved by Newton - Raphson iteration. The discretization of these equations in time and space is not presented here because it is thoroughly explained in the work of Ivan Yatchev in [69] and more available by Tsukerman at al. in [61-63]. From the point of view of this thesis, it is more interesting how the structure of the discretized system of equations looks like.

3.1.1 Matrix structure

The structure of the equation system in matrix form - formulated for time stepping analysis in the k+1 -th time step and in the n-th nonlinear iteration step (Newton - Raphson) - can be seen in (3.5).

$$\begin{bmatrix} \boldsymbol{P}\left(\boldsymbol{a}_{\boldsymbol{k}+1}^{(\boldsymbol{n})}\right) & \boldsymbol{D}^{\mathrm{r}^{\mathrm{T}}} & 0 & \boldsymbol{D}^{\mathrm{c}^{\mathrm{T}}} \\ \boldsymbol{D}^{\mathrm{r}} & \boldsymbol{C}^{\mathrm{r}} & 0 & 0 \\ 0 & 0 & \boldsymbol{C}^{\mathrm{s}} & \boldsymbol{K}\boldsymbol{G}^{\mathrm{c}^{\mathrm{T}}} \\ \boldsymbol{D}^{\mathrm{c}} & 0 & \boldsymbol{G}^{\mathrm{c}}\boldsymbol{K}^{\mathrm{T}} & \boldsymbol{C}^{\mathrm{c}} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{a}_{\boldsymbol{k}+1}^{(\boldsymbol{n})} \\ \Delta\boldsymbol{u}_{\boldsymbol{k}+1}^{\mathrm{r}(\boldsymbol{n})} \\ \Delta\boldsymbol{i}_{\boldsymbol{k}+1}^{\mathrm{s}(\boldsymbol{n})} \\ \Delta\boldsymbol{u}_{\boldsymbol{k}+1}^{\mathrm{c}(\boldsymbol{n})} \end{bmatrix} = - \begin{bmatrix} \boldsymbol{f}^{\mathrm{a}}\left(\boldsymbol{a}_{\boldsymbol{k}+1}^{(\boldsymbol{n})}, \boldsymbol{u}_{\boldsymbol{k}+1}^{\mathrm{r}(\boldsymbol{n})}, \boldsymbol{u}_{\boldsymbol{k}+1}^{\mathrm{c}(\boldsymbol{n})}\right) \\ \boldsymbol{f}^{\mathrm{r}}\left(\boldsymbol{a}_{\boldsymbol{k}+1}^{(\boldsymbol{n})}, \boldsymbol{u}_{\boldsymbol{k}+1}^{\mathrm{r}(\boldsymbol{n})}\right) \\ \boldsymbol{f}^{\mathrm{s}}\left(\boldsymbol{i}_{\boldsymbol{k}+1}^{\mathrm{s}(\boldsymbol{n})}, \boldsymbol{u}_{\boldsymbol{k}+1}^{\mathrm{c}(\boldsymbol{n})}\right) \\ \boldsymbol{f}^{\mathrm{c}}\left(\boldsymbol{a}_{\boldsymbol{k}+1}^{(\boldsymbol{n})}, \boldsymbol{i}_{\boldsymbol{k}+1}^{\mathrm{s}(\boldsymbol{n})}, \boldsymbol{u}_{\boldsymbol{k}+1}^{\mathrm{c}(\boldsymbol{n})}\right) \end{bmatrix} \tag{3.5}$$

The first, second, third and fourth rows of this matrix equation system represent equations (3.1), (3.2), (3.3), (3.4), respectively. The $\boldsymbol{P}$ matrix stands for the sum of the Jacobian of the traditional finite element stiffness matrix plus the time dependent part, the so called mass matrix (3.6) representing time dependent terms.

$$\boldsymbol{P} = \boldsymbol{S} + \boldsymbol{T} \tag{3.6}$$

The stiffness matrix depends on the nonlinearity of the iron, implemented in the iron element matrices. The mass matrix includes the influence of the time derivative of the magnetic vector potential in conductor elements or in other words the effect of the eddy currents. $\boldsymbol{D}^{\mathrm{r}}$ and $\boldsymbol{D}^{\mathrm{r}^{\mathrm{T}}}$ represent the coupling between the rotor circuit (rotor voltages) and the corresponding rotor nodes (in the rotor bar) while $\boldsymbol{C}^{\mathrm{r}}$ accounts for rotor connections. Similarly $\boldsymbol{D}^{\mathrm{c}}$ and $\boldsymbol{D}^{\mathrm{c}^{\mathrm{T}}}$ stand for the coupling between the stator circuit (conductor voltages) and the conductor nodes, and $\boldsymbol{C}^{\mathrm{c}}$ account for the connections of the conductors. Among the remaining unexplained matrices $\boldsymbol{C}^{\mathrm{s}}$ contains the parameters accounting for the end windings. The matrices $\boldsymbol{G}^{\mathrm{c}}\boldsymbol{K}^{\mathrm{T}}$, $\boldsymbol{K}\boldsymbol{G}^{\mathrm{c}^{\mathrm{T}}}$

represent the connections of the conductors to the phase voltages and the connections of the phases to each other (star, delta or other).

On the right hand side, $\boldsymbol{f}^{a}$, $\boldsymbol{f}^{r}$, $\boldsymbol{f}^{s}$ and $\boldsymbol{f}^{c}$ are the corresponding residual vectors in connection with the finite element, the rotor, the stator and the conductor equations, respectively. The vector $\boldsymbol{f}^{s}$ includes the voltage sources (phase voltages).

3.1.2 Observations about the system matrix

The major part of the system matrix is represented by the Jacobian *P*. The size of this matrix is N x N where N - is the number of nodes in the finite element mesh. The total size of the system matrix is M x M, where M is the sum of the number of the nodes, rotor bars, phases and conductors in the stator slots. In a practical case studied in [69], N=11301 while M=11301+42+3+324=11670. The ratio M/N =1.033.

Another important notice is that the number of nodes connected with the stator winding mesh compared to the total number of nodes is usually very high even in case of a simple mesh. Using the same example the total number of nodes was: N = 11301, while nodes in the stator slots: Ns= 5022. Almost the half of the nodes are located in the stator slots even in the case of a sparse winding mesh. With increased number of conductors and more refined mesh the number of nodes can become enormously high.

3.2 Conclusions

We have arrived to a similar conclusion what was reached in Chapter 2. Despite its straightforward applicability, the 2D finite element modeling of eddy currents in multi-conductor windings carries with itself practical limitations. The high number of nodes inside the multi-conductor winding leads to these limitations. The reasons for the high node numbers are the high number of conductors or the small skin depth and these can turn the finite element approach practically useless.

In the next chapter, the finite element formulation is tested by comparing computed results to measured ones. Following that in Chapter 5, the Macro Element Method is introduced, which has been developed to overcome the limitations of the finite element approach.

4. VERIFICATION OF THE FINITE ELEMENT MODEL FOR MULTI-CONDUCTOR EDDY-CURRENT PROBLEMS BY COMPARISON TO MEASUREMENTS

Theory in the test of Mother Nature

The development of new accurate methods to model eddy currents requires the ability to verify them. Two major ways of verification are given. The most obvious and usually the easiest way is to compare a newly developed method to the existing ones, preferably to analytical methods which carry a close connection with the physical reality. Unfortunately, this kind of testing cannot always be performed, or sometimes it does not give convincing results proving the capabilities of the new method. A problem can arise that although the existing methods are accurate in certain cases, we might want to study new presently uncovered problems. If the researchers cannot utilize their results from previous methods for testing, it is suggested to go back to mother nature and to perform the experiments. Generally speaking, a new method can be trusted when properly measured results agree with the calculated ones.

The fundamental goal of this thesis has been to develop a method, which can be connected to, and utilized in the finite element analysis of electrical machines. The verification of the finite element model - and the finite element code - developed in the Laboratory of Electromechanics for multi conductor windings has been necessary to insure that it is a proper foundation to build on. Although many papers can be found where FEM is used for multi-conductor winding problems and one could believe those publications with a clear conscience [2, 11, 15, 26, 38, 62], the finite element code required first hand testing in order to be selected for further development. The accuracy of modeling eddy-current effects in multi-conductor windings of electrical machines, transformers, and other devices can be evaluated by comparing the calculated and measured eddy-current losses in the conductors. A verification of the model is presented in Publication 1 under the "First application: Determination of the Skin Resistance in a Stator Winding " section in the paper.

In addition to the same method explained in Publication 1, this chapter gives a short introduction to the separation of the eddy-current loss in the conductors from the resistive iron losses if the conductors are embedded into iron. A measuring method for this case is also introduced and evaluated.

Measurement of eddy-current losses in the windings of electrical machines is a very challenging task. The separation of eddy-current losses from the total losses in an electrical machine is very difficult, but it can be crucial for accurate testing of mathematical methods. The major problem is that looking at an electrical machine or transformer from its terminals does not allow us to tell the amount of the resistive losses dissipated in the windings. The core losses in the iron parts of the machine appear also as resistive losses from the terminals point of view. The separation of the core and the copper losses by measurements is a tough task, if not impossible in several cases.

In the absence of the iron losses or in other words in a device without iron, the eddy-current losses in the multi-conductor windings can be measured easily from the terminals by electrical means.

4.1 Measurement in linear media

If a multi-conductor winding is surrounded by air, the eddy-current losses can be measured directly from the terminals of the coil. The corresponding equation is (4.1) in which the product of the instantaneous voltage u and the instantaneous current i is integrated over one period of the supply voltage. The active power in this case is the same as the resistive losses.

$$\frac{1}{T}\int_0^T u i dt = P \tag{4.1}$$

In case of sinusoidal supply (4.1) becomes (4.2) where U_{RMS} and I_{RMS} are the RMS values of voltage and current measured at the terminals and $\cos(\varphi)$ is the power factor.

$$U_{\mathrm{RMS}} I_{\mathrm{RMS}} \cos(\varphi) = P \tag{4.2}$$

Using the measured active power and the RMS value of the current, the AC resistance can be calculated by (4.3). This characterizes the equivalent resistance of the test object at a certain frequency.

$$R_{\mathrm{AC}} = \frac{P}{I^2{}_{\mathrm{RMS}}} \tag{4.3}$$

4.1.1 The eddy factor

The eddy factor is used to identify the eddy-current effect, especially skin effect. The increase of the resistance due to eddy currents can be seen very well from the ratio of AC and DC resistance, which is defined as the eddy factor (4.4).

$$e = \frac{R_{\mathrm{AC}}}{R_{\mathrm{DC}}} \tag{4.4}$$

4.2 Measurement in the presence of iron

When an iron core is present in the surroundings of the studied winding, the measured active power is not the same as the resistive losses in the windings, but the sum of those and of the iron losses. The separation of these losses can be solved by thermal measurement. The resistive losses dissipated in the winding can be computed from the heat up process during a load. The temperature rise ($\Delta \boldsymbol{T}$), the mass of the coil (*m*) and the specific heat of the conducting material (*c*) defines the dissipated heat according to (4.5)

$$Q = c \cdot m \cdot \Delta T \tag{4.5}$$

The active power can be calculated from (4.6), where the dissipated heat is expressed as the time integral of the active power during the heat up process.

$$Q = \int P \cdot dt \tag{4.6}$$

If we suppose that P is constant during the measurement, then from (4.5) and (4.6) we get (4.7) where t is the duration of the heat up process, and ΔT is the temperature rise.

$$P = \frac{c \cdot m \cdot \Delta T}{t} \tag{4.7}$$

From (4.7), (4.3) and (4.4) the AC resistance and the eddy factor can be calculated.

4.3 The test object

The test object was designed to perform the eddy-current loss measurements on a multi-conductor winding, which can be surrounded by either magnetically linear or nonlinear media. The plan was created by Antero Arkkio. The structure of the test rig is shown on Fig. 4.1. The test coil consists of seven turns. Between the turns thermocouples have been placed to measure the temperature rise.

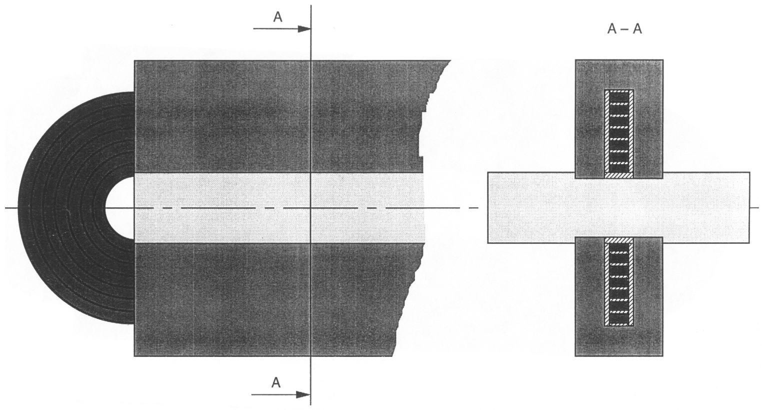


Fig 4.1 The geometrical construction of the test coil

The location of the thermocouples is presented in Fig. 4.2 There are 7 thermocouples (one on each turn) at all the 5 places along the length of the coil. The distances measured from one end of the coil are 10, 55, 95, 145 and 190 mm. Basically the losses at all of these places should be about the same due to the small third dimensional effect, but this arrangement gives a possibility to check it by measurements. All together 35 thermocouples were inserted into the winding.

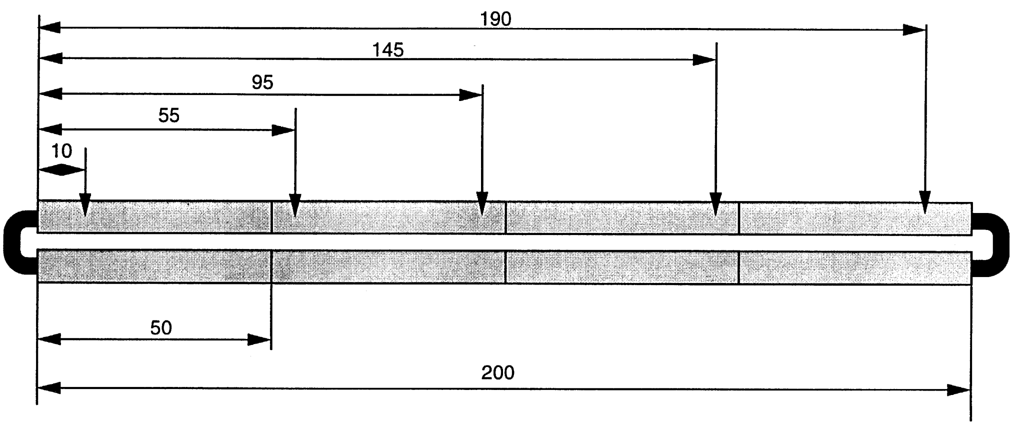


Fig. 4.2 The location of the thermocouples shown by the arrows over the test coil

The length of the coil is much larger than its height so the end winding effects are well damped and can be ignored. The linear measurements were performed with the plastic cover on the coil. The plastic cover was necessary to give a mechanical stiffness to the winding to hold it. Four plastic covers were used as can be seen from Fig. 4.2 with the length of 50 mm each. The nonlinear measurement tests can be performed using a laminated iron cover around the coil.

4.4 Results of the measurements

The electric and thermal measurements were performed based on the theoretical background explained in chapter 4.1 and 4.2, respectively. The results were evaluated with respect to the eddy factor, which was introduced in chapter 4.1. The supply voltage in these measurements was sinusoidal.

The voltage was supplied by a synchronous generator driven by a DC motor. The frequency (the speed of the generator) was controlled by the DC machine. The frequency, voltage, current, effective power loss, total loss and power factor were recorded.

4.4.1 Eddy-current effect measured from electric properties

In order to insure an accurate measurement, the AC and DC resistances of the coil were measured parallel to avoid the effect of the changes in the resistance - on the skin factor - caused by the heating-up of the coil during the measurement. The change in the DC resistance was at the order of milliohms until the completion of the measurement but it had to be considered for the determination of the eddy factor.

The maximum errors of the measured current, voltage, and resistive power values depending on the power factor and frequency were between 0.5 - 2.5 %. The lower the power factor or higher the frequency, the worse is the accuracy. In our case when increasing the frequency the power factor decreases. This causes that the accuracy of the measuring system is significantly worse at higher frequencies.

The measured and calculated eddy factor against the frequency curves were in a good match. The difference between the measured and calculated eddy factors was well inside the measurement accuracy. Similarly good agreement has been observed and published by the author of this thesis in Publication 1 and [57] in 1996. The measured and calculated values for the test rig can be compared in Fig. 4.4. The extended results of Publication 1 can be seen on Fig. 4.5. In Fig 4.5, the measured values are compared to the results obtained by time stepping and time-harmonic finite-element analysis. The later two gives identical results as they should and the discrepancy between the measured and calculated values is small.

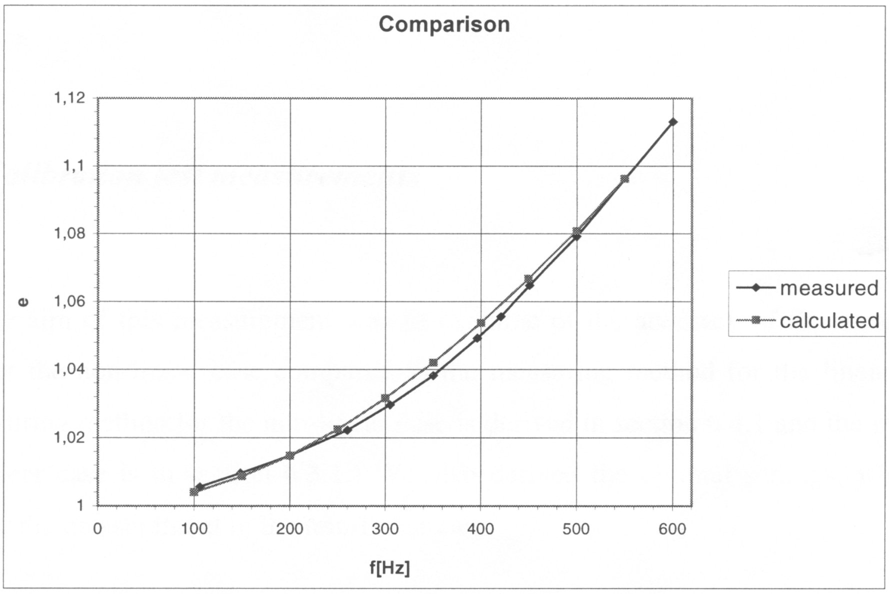


Fig 4.4 Comparison of measured and calculated skin factor versus the frequency for the test rig. The calculation was performed by the time stepping method.

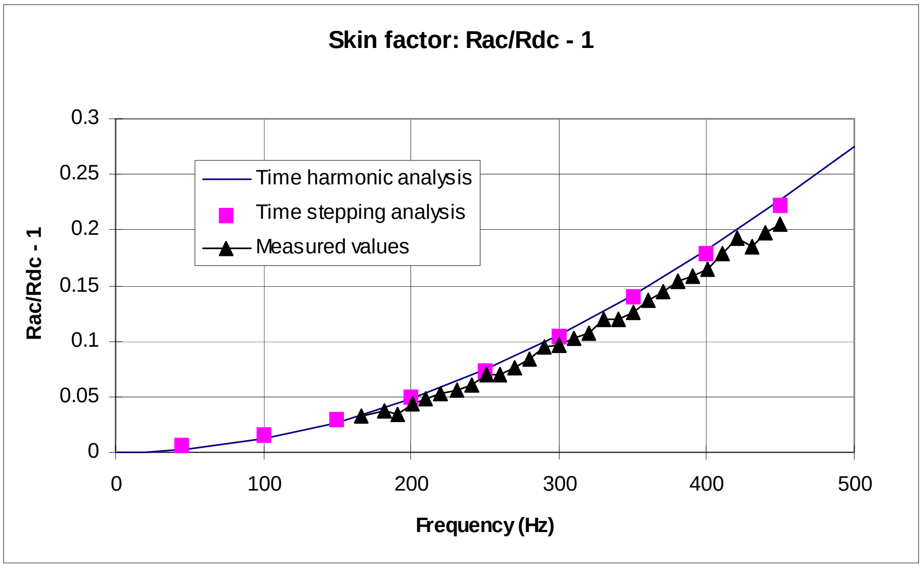


Fig 4.5 Measured and calculated results from a previous study involving another multi turn coil. The calculations were performed by time stepping and time harmonic analysis as well. The perfect match of the two methods is obvious for linear sinusoidal problems.

4.4.2 Eddy-current effect measured by thermal properties

A test measurement was performed to evaluate the usability of the thermal measurement for the estimation of eddy-current losses. In the test measurement the linear cover was used so the results could be compared to the values measured from the electric properties.

The heat up test measurements could clearly indicate the increased effect of eddy currents with the increase of the frequency but the accuracy of the heat measurement limited this study significantly. The measurement took too much time compared to the heat transfer procedures inside the test rig, and the different turns of the coil cooled down at different speeds decreasing the accuracy of the measurement. Taking each turn into account - including the fast cooled turns - when calculating the loss, the error of the measurement was under 9 % compared to the electric measurement. However this error was still too high to calculate the eddy factor precisely.

4.5 Summary

The measured results agreed well with the calculated ones for the linear case when the eddy-current losses could be measured from the electrical properties. The difference between the measured and calculated results was in the range of the measurement error. The measurement proved that the finite element model and the code used in this analysis is a proper tool for the study of multi-conductor eddy-current problems. One can conclude that the finite element model used in this work is a proper tool for multi-conductor eddy-current problems and the electrical measurements confirmed this statement.

An analytical computation for the studied problem would not be impossible due to the complexity, but it would require laborious modeling work.

The thermal measurement was a success from the theoretical point of view, but the measuring devices and the long measuring time due to them limited the accuracy of the measurement significantly. With better devices the measurement promises the possibility to separate eddy-current losses from the iron losses in a simple manner.

The improvement of the measuring setup is required to increase the accuracy of the measurement.

Numerical methods always carry the danger, that the result obtained is only a couple of meaningless numbers, because the direct connection to the physical world is blurred by the numerical techniques applied. These measurements proved that the tested finite element model and code is adequate for the analysis of multi-conductor eddy-current problems and the finite element analysis was formulated and performed properly.

The finite element model and code tested in this work has been proved to be a strong basis to develop it further and to build new methods on it.

5. ELIMINATION OF INNER NODES IN THE MODELING OF EDDY CURRENTS IN MULTI-CONDUCTOR WINDINGS

Problem size reduction with the Gauss elimination of inner nodes

This chapter introduces the elimination of inner-nodes method in the modeling of eddy currents in multi-conductor windings surrounded by or close to magnetically nonlinear media. It can be used to reduce the highly increased number of unknowns when modeling multi-conductor windings with the FEM. Using elimination method, the size of the system can be decreased and the solution of the system of equations can be accelerated very significantly.

Traditional finite element formulation is combined with the elimination of inner nodes to achieve the problem size reduction. The basic idea of the method is that the finite element system is divided into linear and nonlinear parts. The linear parts - where the multi-conductor winding also belongs - can be eliminated for the duration of the nonlinear iteration process, resulting in accelerated calculation. It promises significant improvement in the calculation speed for applications in electrical machines and devices. The elimination method is explained in the following manner according to the time sequence of the development process:

1. Study of the effects caused by the “elimination” of the winding region, with respect to the size and complexity of the system of equations. Evaluation of the expectations and the reality when using elimination. This study has been published in Publication 2.
2. Determination of the right place and order for the “elimination of inner nodes” method inside the nonlinear iteration process. This problem area has been addressed in Publication 3.

5.1 Elimination of inner nodes

As the slot of an electrical machine consists of linear materials only, the variables associated with the nodes inside the slot mesh can be eliminated from the system of equations for the duration of the nonlinear iteration process. In this formulation, the variables describing the voltages of the conductors could similarly be eliminated. In the given example, the sub matrix S_{11} depends on vector u representing non-linearity in the system. Performing Gauss elimination the number of variables and the size of the system matrix decreases (5.1), (5.2). It is enough to iterate with (5.2) and after reaching convergence linear variables in vector v can be calculated in one step.

$$\begin{bmatrix} \boldsymbol{S}_{11}(u) & \boldsymbol{S}_{12} \\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{v} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_u \\ \boldsymbol{R}_v \end{bmatrix} \tag{5.1}$$

$$\left[\boldsymbol{S}_{11} - \boldsymbol{S}_{12}\boldsymbol{S}_{22}{}^{-1}\boldsymbol{S}_{21}\right][\boldsymbol{u}] = \left[\boldsymbol{R}_u - \boldsymbol{S}_{12}\boldsymbol{S}_{22}{}^{-1}\boldsymbol{R}_v\right] \tag{5.2}$$

The eliminated system is expected to be faster to solve but it is not obvious, because the sparse system matrix becomes relatively denser. The effectiveness of the elimination has to be studied. This study is performed and presented in Chapter 6.

5.2 Expectations about the effect of elimination

Performing the elimination of the inner nodes from a selected region decreases the number of variables in the system. While the size of the system matrix decreases, a B x B full matrix is inserted into it due to the elimination, where B - is the number of nodes on the boundary of the eliminated region. The appearance of this full matrix is due to the fact, that by the elimination, we define a function between the nodes on the boundary. This function - represented by the full matrix - can be recognized as a macro element in the stator slot. Keeping these facts in mind, the expectations about the efficiency of the elimination before performing the tests are the following:

1. Higher number of eliminated nodes with the same number of nodes on the boundary results in higher efficiency because of the decrease in the size of the system is higher.

2. The larger the number of nodes on the boundary, the lower efficiency of the elimination we can expect because the size of the full matrix - inserted into the eliminated system - increases, and so the complexity of the whole system matrix.

5.3 Method of elimination in the solution process

The role and place of the elimination in the solution process is explained in this chapter and a novel technique is suggested to create the eliminated matrices.

The solution process consists of the following sub-processes:

- time steps,
- iteration steps,
- elimination,
- solution steps (of the linear system of equations),
- recovery of eliminated variables.

In the most simple case, the method of elimination is located in the solution process as it is depicted in Fig. 5.1. In this explanation, iteration steps include the solution steps.

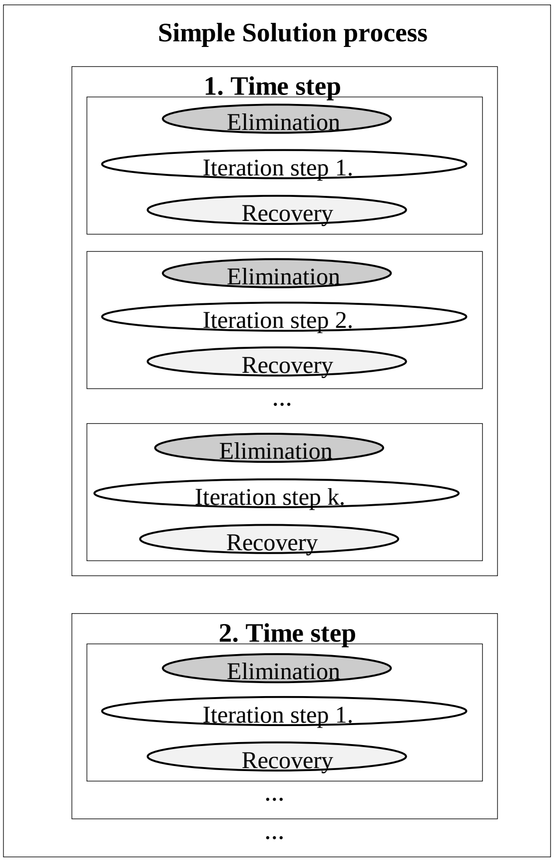


Fig. 5.1 Simple solution process. Elimination and recovery of the linear variables are performed in each iteration step. This kind of solution can already be faster than performing the solution without elimination. This has been indicated by other studies.

This kind of solution can already be faster than without elimination. Similar approach was found to be very efficient in [14] by Degeneff et al. who performed the elimination by pre-calculated matrices. A more advanced way of the solution is depicted in Fig. 5.2. Here the recovery of the variables is performed only at the end of each iteration process. In this formulation, the Jacobian before elimination is created so that the linear variables are kept constant during the iteration so their values are not recovered in each iteration step. It can be proved, that keeping the “linear variables” constant during Newton-Raphson iteration, does not influence the convergence. Only variables connected with nonlinear dependency affect the rate and way of the convergence. Utilizing this fact, the values of linear variables which are not connected with the non-linear dependency in the Jacobian can be set to “arbitrary” values. If the system matrix and the residual vector are set up properly using these “arbitrary” values, the iteration will converge exactly the same way as it would if linear variables are also “iterated”. This statement will be proved later in the mathematical validation subchapter.

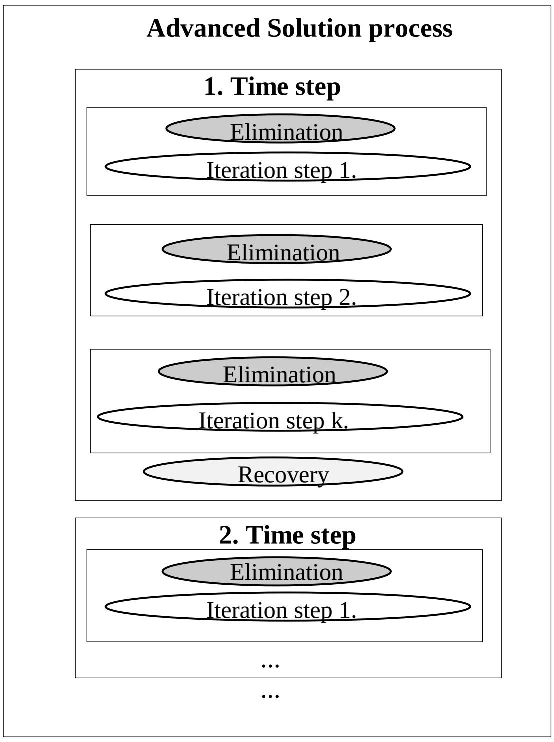


Fig. 5.2 Advanced solution process. The Jacobian is created in each iteration step so that the linear variables are kept constant. These variables are not iterated and they are only recovered after the iteration has converged.

5.3.1 The suggested construction of the eliminated system of equations

The reason why the previously discussed “advanced solution process” would require the elimination in each iteration step is that the eliminated system matrix and the eliminated residual changes in each iteration step. The need to eliminate in each iteration step was due to the fact, that after the elimination we get a complex system which contains nonlinear terms, time dependent terms and terms created by the elimination. In the “advanced solution process” the eliminated system is created by ordinarily constructing the system matrix and the residual vector and then elimination is performed.

In the suggested solution process, however, we create the eliminated system without the need to perform the elimination. The flowchart of the proposed technique is presented in Fig. 5.3.

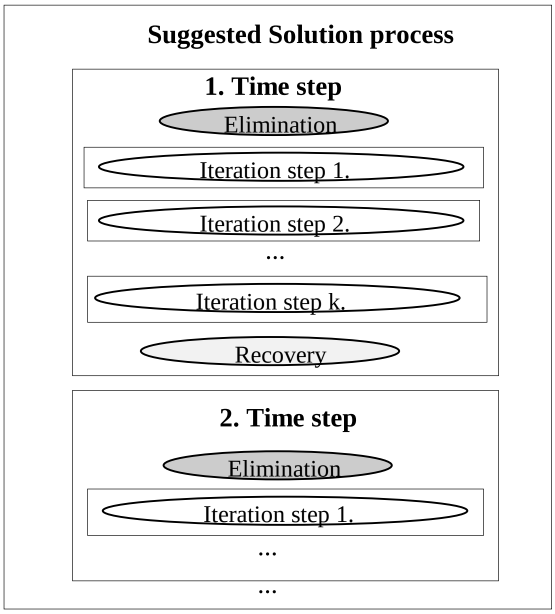


Fig. 5.3 The suggested flowchart for the solution. Elimination is performed only once in a time step.

The idea behind the technique how to handle the matrices in the Newton-Raphson iteration is depicted in a flow diagram in Fig. 5.4. The principle of the method is to create the constant linear part of the Jacobian, eliminate it, store it, and add the nonlinear parts to it in each iteration step. The residual can be set by the use of the stored eliminated matrix, and by adding the nonlinear parts to it in each iteration step. This way, the elimination is performed only once at the beginning of each iteration.

1. Create the linear part of the Jacobian matrix and set the sources on the right hand side!
2. Eliminate this system and **STORE** the result!
Iteration started here!
3. Create correct eliminated Jacobian by adding the non-linear parts in each iteration step to the stored linear system!
4. Create the correct eliminated residual by adding to the stored sources on the right hand side, the linear part of the residual, using the stored eliminated linear Jacobian. Add the non-linear parts by recalculating them in each iteration step!
5. Solve the system!
6. Check convergence! If it is not converged go to step 3! If converged, recover and calculate linear variables (vector-*v*)!

Fig. 5.4 Flow diagram of the technique for a time step, including the construction of the eliminated Jacobian matrix in each iteration step. Elimination is performed only once before the first iteration step.

5.3.2 Mathematical validation of the method

The validity of such an algorithm is proven in the followings. Steps 3 and 4 of Fig. 5.4. are investigated closely and are clarified.

Equation (5.3) represents a nonlinear system of equations and (5.4) is formulated for Newton Raphson iteration. A_{11} represents the nonlinear part depending on vector-*u*. In our case, the non-linearity of the surrounding iron is modeled by nonlinear elements in the mesh. Eliminating (5.4), we get (5.5). In (5.5), the Jacobian consists of a nonlinear and an eliminated linear part. Note that performing an elimination on the proposed purely linear structure (5.6) results in (5.7) and has the same Jacobian as in (5.5) except the nonlinear part. This proves the validity to handle the Jacobian in the way described steps 1,2 and 3 in Fig. 5.4.

However, the method is not complete without creating the residual. From (5.5), we can see that the first two terms on the right hand side considering the sources remain constant during the iteration. These are the same terms as in the right hand side of (5.7). The third term of the right hand side in (5.5) can be created by using the eliminated Jacobian of (5.7) and by adding the nonlinear parts. This proves the validity of step 4 in Fig. 5.4.

$$\begin{bmatrix} \boldsymbol{A}_{11}(\boldsymbol{u}) & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{v} \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_u \\ \boldsymbol{S}_v \end{bmatrix} \tag{5.3}$$

$$\begin{bmatrix} \boldsymbol{A}_{11} + \dfrac{\partial \boldsymbol{A}_{11}(\boldsymbol{u})}{\partial \boldsymbol{u}}\boldsymbol{u} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{u} \\ \Delta\boldsymbol{v} \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_u - \boldsymbol{A}_{11}\boldsymbol{u} - \boldsymbol{A}_{12}\boldsymbol{v} \\ \boldsymbol{S}_v - \boldsymbol{A}_{21}\boldsymbol{u} - \boldsymbol{A}_{22}\boldsymbol{v} \end{bmatrix}; \tag{5.4}$$

$$\left[\boldsymbol{A}_{11} + \frac{\partial \boldsymbol{A}_{11}(\boldsymbol{u})}{\partial \boldsymbol{u}}\boldsymbol{u} - \boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{A}_{21}\right][\Delta\boldsymbol{u}] = \left[\boldsymbol{S}_u - \boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{S}_v - (\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{A}_{21})\boldsymbol{u}\right] \tag{5.5}$$

$$\begin{bmatrix} 0 & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{u} \\ \Delta\boldsymbol{v} \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_u \\ \boldsymbol{S}_v \end{bmatrix} \tag{5.6}$$

$$\left[0 - \boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{A}_{21}\right][\boldsymbol{u}] = \left[\boldsymbol{S}_u - \boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{S}_v\right] \tag{5.7}$$

As it was mentioned in this chapter before, the convergence of the iteration depends only on the nonlinear variables, which influence the nonlinear parts of the Jacobian matrix. The validity of the iteration process without the linear variables can be proved from (5.4) and (5.5). The change of vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ in the current iteration step is given by (5.4). Eliminating vector $\Delta\boldsymbol{v}$ we get (5.5) where only the change of $\boldsymbol{u}$ is expressed. Unlike (5.4), this equation does not depend on vector $\boldsymbol{v}$! This means that the variables in vector $\boldsymbol{v}$ do not influence the iteration.

5.4 Realisation of the elimination method

In this chapter the realization of the suggested method is discussed. The necessary mathematical and preprocessing algorithms to handle and utilize the elimination method inside the finite element analysis are presented here and a short overview of the programming is given.

5.4.1 Methods for pre-processing

The selection of the variables which should be eliminated from the system can be a painful job. A simple and convenient method is explained in this section. The basic idea of this method is to set a special boundary condition for the free nodes in the eliminated region. This special condition can be used to find and differ them in the finite element code. The proposed preprocessing method can be used with any graphical preprocesor. If the eliminated region contains other than free nodes (fixed or periodic boundary) this method should be modified. Researchers, developing their own finite element preprocessors, can find this method interesting. With a sophisticated commercial preprocessor, the nodes to be eliminated might be selected easier on a different way. Creation of a separate file with the list of eliminated nodes could be a convenient solution if the preprocessor supports it.

The assumption for this method is that we can create a proper mesh for an electrical machine with a multi-conductor winding and in the winding we define only free nodes. The steps of the preprocessing are described in a flow chart in Fig. 5.5.

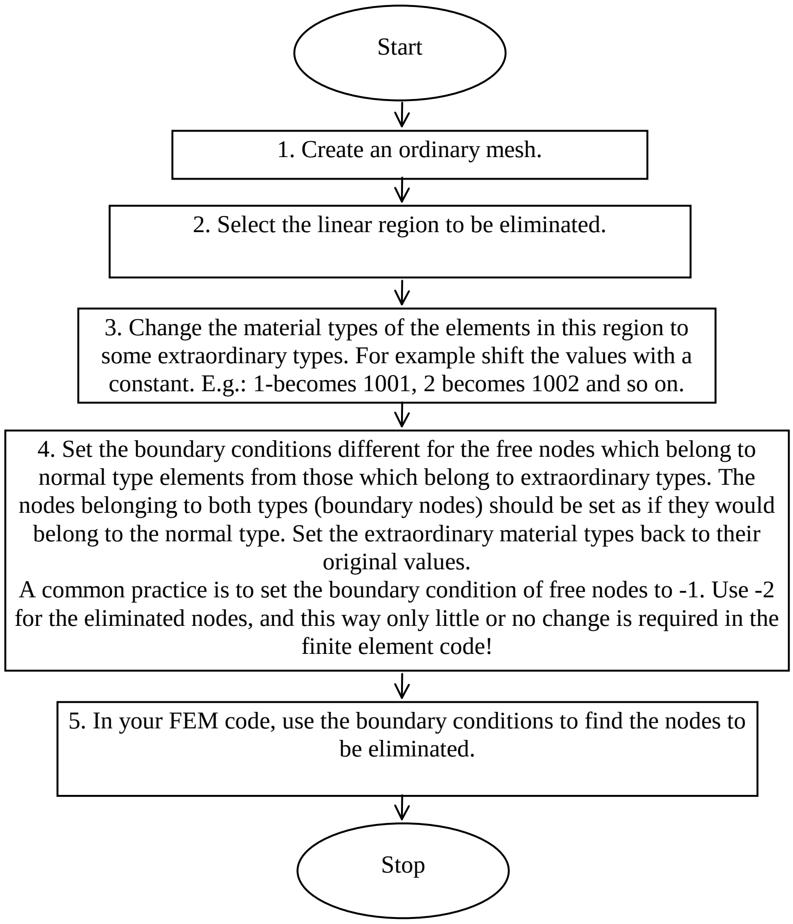


Fig. 5.5 The flow chart of the technique to create an ‘eliminated’ mesh.

The most important advantage of this process is that after step 3 we get such a mesh which we can modify and refine as we like with ordinary preprocessors, because the information about the ‘eliminated’ region is stored in the material types in standard format. This way it is enough to select the eliminated region once, and then we can easily refine it. After refinement, we can proceed to step 4 and our selection of the nodes which should be eliminated is complete for the refined mesh.

Another important advantage is that this method can be easily automated because of its nature. When setting the boundary conditions in step 4, a simple way to proceed is to change the boundary conditions only for free nodes, which belong to normal material type elements. This way the nodes on the boundaries of the eliminated region are set correctly. In case of higher order elements, this method works also properly and no complicated programming is required.

The third advantage is that the eliminated region in step 2 can be easily selected by most of the preprocessors. The material types can be modified conveniently - for example - on a graphical user interface.

5.4.2 Construction and handling of eliminated matrices

When the nonlinear components of the Jacobian are inserted into the eliminated system (Fig. 4. step 3) and when the eliminated residual is being created (Fig. 4. step 4), the location of certain components can be complicated. In order to avoid complicated programming, an indexing technique is suggested in this section.

The basic idea is to create a mapping vector $\boldsymbol{m}$. It is a simple one-dimensional array containing information about the variables moved by the elimination. The length of the vector is equal to the original number of variables in the system. The n-th value in vector $\boldsymbol{m}$, referred as $\boldsymbol{m}(n)$ is equal to the new serial number of the n-th variable after elimination. As an example if only the variable number 10 is eliminated from the system, $\boldsymbol{m}(n) = n$ if $n < 10$ and $\boldsymbol{m}(n) = n-1$ if $n > 10$. For $\boldsymbol{m}(10)$ we can define, for example, $\boldsymbol{m}(10) = 0$ which means that variable number 10 is eliminated.

The advantage of this kind of mapping is that every working FEM code can be easily modified by replacing the serial number of the n-th variable by the $\boldsymbol{m}(n)$ mapping vector. Before elimination by default $\boldsymbol{m}(n) = n$. During elimination the mapping vector can be easily updated. In this way, every component will be located and added properly to the system independently from when it is added (before, after or even during elimination) because the location of components in the system matrix is defined by the serial number of the variables.

5.4.3 Programming structure

The program utilizing the suggested technique consist of three major parts. First of those written earlier in FORTRAN to analyze cage induction machines, secondly the modules added by Ivan Yatchev also in FORTRAN and finally the new modules utilizing the elimination of inner nodes in FORTRAN and in C-language. There is a sophisticated data transfer in between the FORTRAN and C program side. The need for the C language routines was that it is convenient to construct sparse matrices using pointer structures which were not supported well enough in FORTRAN. The handling of the sparse matrices is done by the commercial SPARSPAK matrix solver on the FORTRAN side and by a separate code handling the elimination, written by the author of this thesis. The first version of the Gauss elimination software written in C with the pointer structure did not prove to be efficient, and this part of the code required revision. When the new data structure avoiding pointers was written in FORTRAN, the speed of the elimination increased enormously.

5.5 Summary

The suggested method of elimination creates a numerical macro element replacing the finite element nodes in the eliminated linear multi-conductor region. The size of the problem can be dramatically reduced with this method and the solution time for the linear system of equations can be decreased. The novelty of this proposal is that the method of “elimination of inner nodes” has been applied for the study of multi-conductor windings, and this has not been done before.

A new time stepping scheme is suggested and derived to reduce the required number of elimination processes per time step to one. The technique utilizes the fact that in a Newton-Raphson iteration process the “linear” variables do not have to be iterated.

The elimination process takes a lot of time by itself. This fact gives the motivation for the further development of a macro element approach where the macro elements are created only once in a whole time stepping analysis and consequently the elimination of the system matrix has to be performed only once.

The performance and effectiveness of the suggested elimination method is investigated in Chapter 6, and the macro element method is presented in Chapter 7.

6. RESULTS OF THE ELIMINATION OF INNER NODES
The advantages and limitations of the pure elimination approach

In this chapter, the effectiveness and the “behavior” of the elimination method is studied and some results are presented. First, the method to evaluate the effectiveness of the suggested technique is explained. This is followed by the description of the test cases and finally the effect of the elimination is shown by numerical results.

6.1 Method to evaluate the effectiveness

The effectiveness of the elimination was measured by comparing the system of equations before elimination to the system after elimination. Studying the structural changes of the system matrix due to elimination gives some hints about the complexity of the eliminated system. In this study, we introduce a term called the efficiency of the elimination. It is defined as a ratio of the solution times required before and after elimination. Solution time is the time needed to solve a system of equations once. In our case, it means one iteration step. From the definition of efficiency, it is clear that the higher the efficiency, the more time we can save by calculating with the eliminated sysem. The equation systems were solved by MATLAB (matrix laboratory) and computation times were measured. The matrices were handled in sparse form and the calculation was performed with a direct method with LU decomposition.

6.2 Test samples

The test cases for the study are two imaginary electrical machines. The solution sector in both cases contains one rotor and one stator slot. In the first case, the stator slot contains 24 conductors (Fig. 6.1) and in the second case 32 conductors (Fig. 6.2). The stator slot in the second case is more realistic in electrical machinery.

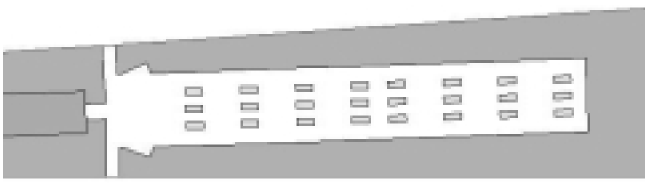


Fig. 6.1. The geometry of case A.

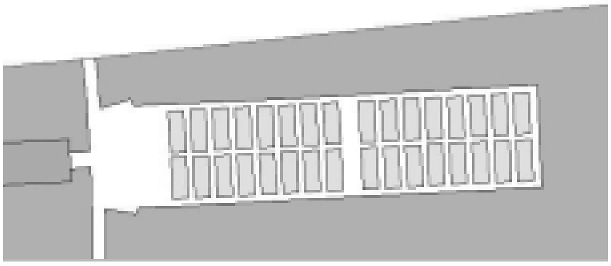
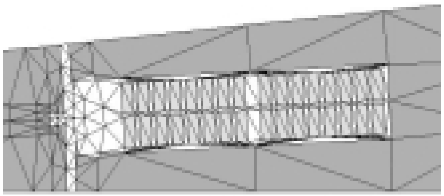


Fig. 6.2. The geometry in case B.

In both cases, two kind of second order meshes were studied. Sparse meshes (7 in both cases) were investigated with increasing number of boundary nodes while the number of eliminated nodes was kept about constant. Dense meshes (1 and 2 in case *A* and *B*) were studied with a small number of boundary nodes and with increased number of inner nodes.

The test meshes for case *B* are presented in Fig. 6.3. The structures of the meshes are similar for case *A*. For the sake of space and simplicity, only the meshes of case *B* are presented in this chapter, but some results from case *A* are also given. The eliminated region is presented in Fig. 6.4.

1,



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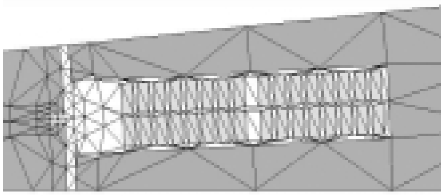

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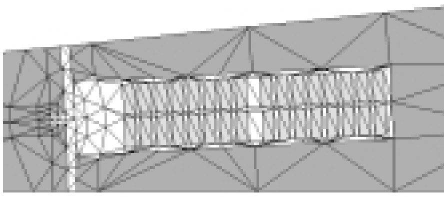

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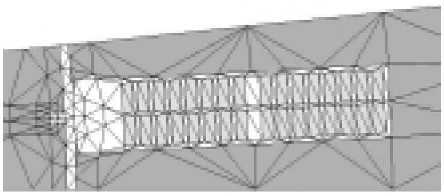

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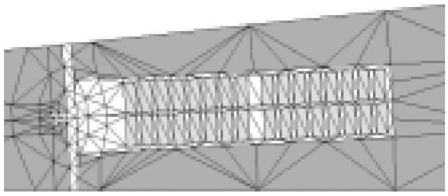

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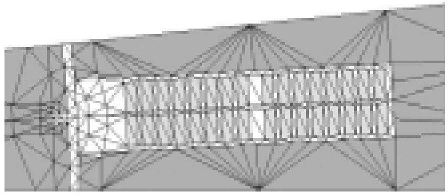

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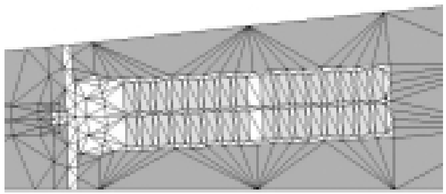

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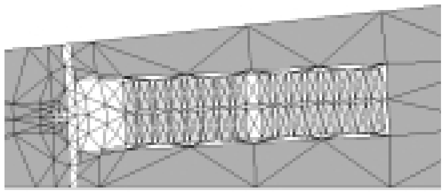

9,

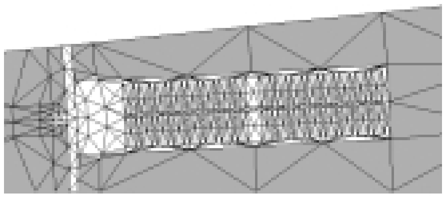

Fig. 6.3 The test meshes for case B. *The meshes 1-7 were used to test the effect of the number of boundary nodes. Meshes number 8 and 9 differ from number 2 in the denser mesh in their stator slots and they were used to test the effect of the increased number of inner nodes.*

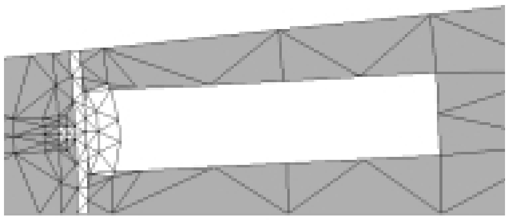

Fig. 6.4 The representation of the eliminated region by removing the nodes from the mesh which belong to variables removed after elimination.

6.3 Results - The efficiency of the elimination

Table I shows the “efficiency” of the elimination for case *B*. The efficiency is defined as the ratio of the calculation times needed to solve the system in one iteration step before and after the elimination. It can be seen, that the calculation times decreased dramatically after the elimination. The relatively long time spent for elimination is not included here.

TABLE 6.1: EFFICIENCY OF THE ELIMINATION IN CASE B.

Mesh parameters: N: number of nodes, Bn: number of boundary nodes, Ne: number of eliminated nodes	Solved in (s) O: original E: eliminated	Efficiency: Original / Eliminated
Sparse 1: N: 857, Bn: 22, Ne: 553 Fig. 6.3 / 1	O: 1.26 E: 0.17	12.45
Sparse 2: N: 873, Bn: 30, Ne: 557 Fig. 6.3 / 2	O: 2.64 E: 0.17	8.56
Sparse 3: N: 889, Bn: 38, Ne: 561 Fig. 6.3 / 3	O: 1.42 E: 0.21	6.76
Sparse 4: N: 905, Bn: 46, Ne: 565 Fig. 6.3 / 4	O: 1.42 E: 0.28	5.07
Sparse 5: N: 941, Bn: 64, Ne: 574 Fig. 6.3 / 5	O: 1.53 E: 0.60	2.55
Sparse 6: N: 989, Bn: 88, Ne: 586 Fig. 6.3 / 6	O: 1.76 E: 1.26	1.40
Sparse 7: N: 1029, Bn: 108, Ne: 596 Fig. 6.3 / 7	O: 2.30 E: 2.15	1.07
Dense 1: N: 1073, Bn: 30, Ne: 757 Fig. 6.3 / 8	O: 1.81 E: 0.16	11.31
Dense 2: N: 1589, Bn: 30, Ne: 1273 Fig. 6.3 / 9	O: 3.46 E: 0.16	21.63

Similar results were obtained for case *A.* These are embedded into Fig. 6.5 where the efficiency vs. the node ratio is plotted. The node ratio is defined as a ratio of the total number of nodes (in the whole mesh) over the number of nodes on the boundary of the eliminated region. The numerical values for case *B* are taken from Table I, sparse meshes 1-7 which mainly differ from each other in the number of boundary nodes. The figure shows that increasing the node ratio improves the efficiency of the method. In other words: the more nodes placed (required) on the boundary of the slot, the worse the efficiency of the method becomes.

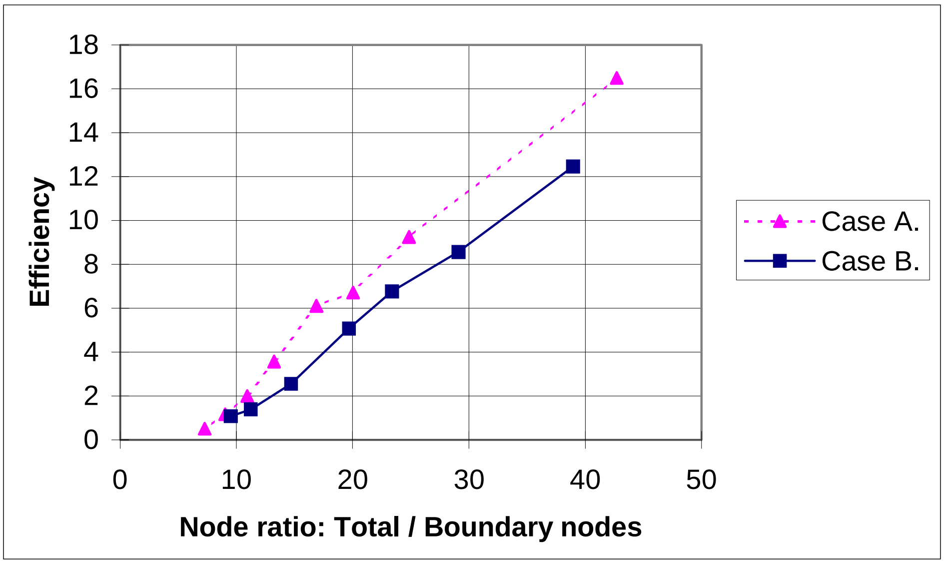

Fig. 6.5 The efficiency of the elimination vs. the node ratio. The node ratio is defined as the ratio of total number of nodes and the number of nodes on the boundary of the eliminated region.

6.4 Evaluation of the results

The results agree well with our expectations (described in chapter 3.3) and we can state the followings.

Our first expectation seems to be proven in Table I comparing the sparse mesh number 2 with the dense meshes. In case of more eliminated nodes in the same geometry and with the same amount of boundary nodes, the method of elimination offers greater improvement. In the sparse case, the efficiency was 8.56 while in the dense meshes, 11.31 and 21.63 increasing with the number of nodes in the slot. For case *A*, a similar effect was experienced.

Our second expectation is proved by Fig. 6.5 which clearly shows that the higher the number of nodes on the boundary, the worse the efficiency becomes. In both of the studied cases (*A* and *B*), the same characteristics can be seen in the presented figure.

The representation of the results in Fig. 6.5 shows a close to linear dependency of the efficiency vs. the node ratio. Despite the nice appearance of these results, general conclusion should be drawn carefully.

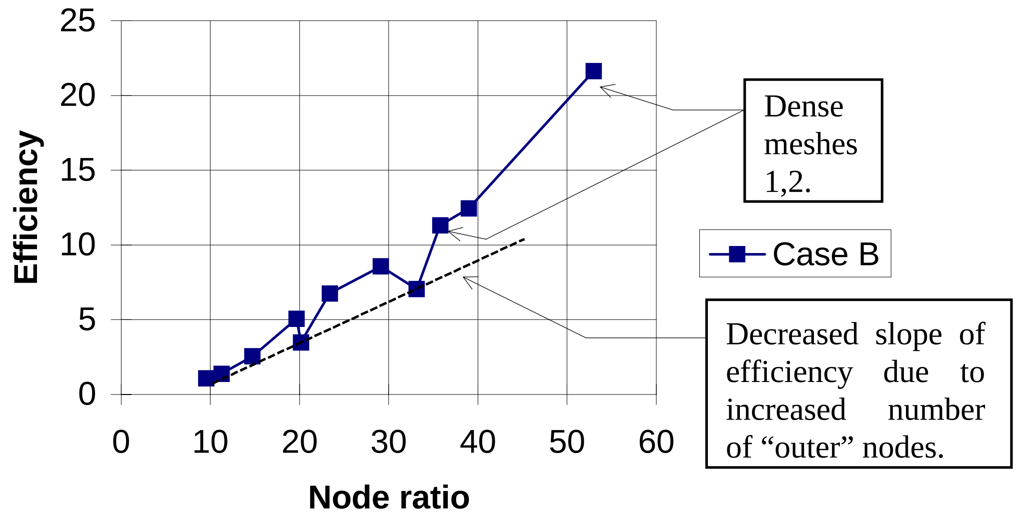

Fig. 6.6 The efficiency of the elimination vs. the node ratio is derived by using the meshes in Fig 3/ 1-9 and two extra meshes with significantly more nodes outside the eliminated region.

Fig. 6.6 shows the same function for case *B* as Fig. 6.5 for both cases but in Fig 6.6 four more points (meshes) are added. Two of these clearly fall out of the trend line of the previous results and two fits there perfectly. The meshes which fit properly are the dense meshes depicted in Fig 6.2. with number 8 and 9, and the results are given in Table I as “Dense 1” and “Dense 2” in the last two rows. The other two meshes which do not fit, have significantly higher number of nodes outside the eliminated region which causes the overall efficiency to fall. These nodes result in a larger system of equations before elimination and after it as well. It influences the relative change of the complexity of the equation system due to elimination and thus the efficiency of the elimination. Based on this consideration we can expect the slope of the efficiency curve to be lower if the number of nodes outside the eliminated region is higher. This decrease of the slope can be seen by connecting the non-fitting points in Fig. 6.6.

6.5 Conclusions

The elimination of variables corresponding to the nodes inside the linear slot region is proved to be a promising and powerful method for the study of multi-conductor windings surrounded by nonlinear media. It is clear from the results that the higher the density of nodes in the eliminated linear region, the more the elimination improves the solution time of the equation system.

Another important observation is that the lower the number of nodes on the boundary of the eliminated region, the higher the efficiency of the elimination becomes. The dependency of the efficiency vs. the node ratio for some certain geometries was experienced to be linear for meshes having close to the same number of nodes outside the eliminated region. The number of nodes outside the eliminated region influences the slope of the characteristic (Fig. 6.6.)

The elimination process takes a lot of time, so it is wise to build the system matrix during an iteration in such a way that elimination is performed only once. This is accomplished by the technique described in chapter 5.4. and the speed of the solution is increased significantly.

It has been explained in chapter 5.6 that the Gauss elimination - to reduce the system size (not to solve the system) - is proved to be a very time consuming process.

Although the test calculations proved that the elimination of inner nodes promises great improvements in the solution time of the reduced system of equations, the number of eliminations in a time stepping analysis is the same as the number of time steps. The time spent for the elimination process is too high. It could be that there are more efficient ways to create the eliminated system than to use the Gauss elimination, but the ultimate goal has been to create a constant macroelement for a whole time stepping analysis. It has been the major motivation for further research.

We can conclude that the development of a method, which requires less eliminations during the time stepping analysis is inevitable for the efficient use of the elimination of inner nodes method. In the next chapter the macro element method is presented, which requires only one time-consuming elimination of the system matrix in the full duration of a time stepping analysis.

7. MACRO ELEMENTS IN THE FINITE ELEMENT ANALYSIS OF MULTI-CONDUCTOR EDDY-CURRENT PROBLEMS

Only one elimination of the system matrix is required

The use of macro elements, which replace the multi-conductor winding in the time-stepping finite element analysis of electrical machines is presented in this chapter (Publication 4). The elimination method is incorporated into time stepping analysis in such a way, that the elimination – and so the creation of the numerical macro element – is performed only once in a whole time stepping analysis.

The novelty in this approach is that the creation of the time dependent right hand side vector can be created by a stored elimination vector and it does not require the elimination of the whole system matrix.

7.1 The finite element formulation for the macro element approach

The finite element formulation is the same as it has been described in Chapter 3 and the same as it has been used for the pure elimination of inner nodes approach. The fundamental idea of the elimination is also the same as described in Chapter 5 and the same equations used in Chapter 5 can be utilized to explain the new macro element technique.

7.2 The novelty of the macro element method

In this section, a time stepping technique is suggested by which it is enough to perform the matrix elimination only once in a whole time stepping analysis. It has been shown in Chapter 5 that the creation of the full eliminated system matrix on the left hand side of (5.5) $\left[\boldsymbol{A}_{11}+\frac{\partial \boldsymbol{A}_{11}(\boldsymbol{u})}{\partial \boldsymbol{u}}\boldsymbol{u}-\boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{A}_{21}\right]$ is possible by the use of the eliminated linear part of the system matrix - as shown on the left hand side of (5.7) $\left[0-\boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{A}_{21}\right]$- and by adding the nonlinear components $\boldsymbol{A}_{11}+\frac{\partial \boldsymbol{A}_{11}(\boldsymbol{u})}{\partial \boldsymbol{u}}\boldsymbol{u}$ in each iteration step. It can be seen from (5.5) and (5.7) that inside a nonlinear iteration, the residual $\left[\boldsymbol{S}_u-\boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{S}_v-(\boldsymbol{A}_{11}-\boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{A}_{21})\boldsymbol{u}\right]$ can similarly be created using the stored eliminated linear part of the system matrix $\left[0 - \boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{A}_{21}\right]$ and by adding the nonlinear contributions $(\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{A}_{21})\boldsymbol{u}$ in each iteration step. The sources including the time dependent terms due to the eddy currents have to be updated on the right hand side of (5.5) in each time step once.

Equations (5.5) and (5.7) clearly show that the linear part of the eliminated system matrix does not contain time dependent terms and so it is not necessary to perform the elimination of the system matrix in each time step. This is due to the eddy current formulation where the eddy effect adds time dependent terms only on the right hand side – represented by a source vector – and it adds a constant matrix to the system matrix. The real reason for the elimination in every time step is to create the eliminated residual properly. The eliminated residual should contain the time-dependent eddy-current source terms inside the eliminated region when modeling eddy currents. These time dependent terms represented by $\boldsymbol{S}_{\mathrm{v}}$ have to be taken into account in order to consider the eddy current effect properly in the windings.

7.2.1 The creation of a macro element

The elimination of the linear part of the system of equations in each time step can be avoided if we store the eliminating matrix $\boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}$ for the source vector $\boldsymbol{S}_{\mathrm{v}}$. The eliminated source term in the right hand side of (5.5) represented by: $\boldsymbol{S}_u - \boldsymbol{A}_{12}\boldsymbol{A}_{22}{}^{-1}\boldsymbol{S}_v$ cannot be created using the stored linear part of the system matrix. If the eliminated $\boldsymbol{S}_{\mathrm{v}}$ source term - representing the eddy current effect - is created by the stored elimination matrix, the rest of the residual seen in (5.5) can easily be constructed. Using the stored linear part of the system matrix (5.7), the nonlinear non-eliminated components and the source terms connected with the non-eliminated variables $\boldsymbol{S}_u$ the residual vector can be created without the need for elimination.

The macro element replacing the FEM model of the multi-conductor winding in Fig. 7.1 can be fully represented by the eliminated linear part of the system matrix together with the eliminating matrix of the source terms.

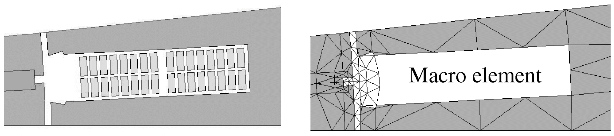

Fig. 7.1 A slot of an electrical machine and its macro element representation

7.2.2 Advantage of the suggested macro element representation

The number of operations to eliminate the whole system is significantly higher than the number of operations needed for eliminating only the residual. In a practical application without renumbering the mesh for optimal elimination, 14 341 730 operations were needed for the elimination of the whole system while only 76 435 operations were required for the elimination of the residual. The number of operations required for different meshes with increasing mesh density in the eliminated slot region are given in Table I and Fig. 7.2.

TABLE I
NUMBER OF OPERATIONS REQUIRED FOR ELIMINATION

Number of nodes	Whole system	Residual	Ratio
873	14 341 730	76 435	187.63
1073	22 453 817	101 834	220.49
1257	38 284 012	143 078	267.57
1353	46 573 378	161 866	287.73
1589	83 125 897	247 528	335.82

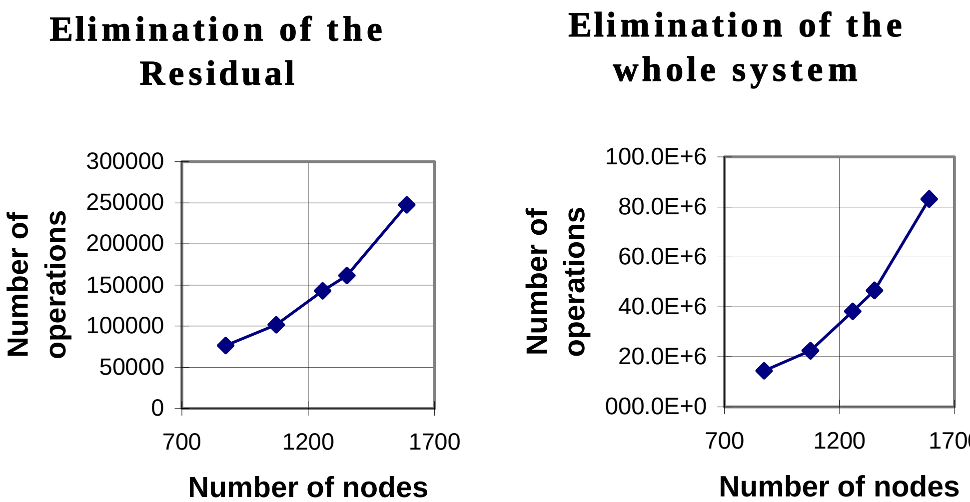

Fig. 7.2 Number of operations required for the elimination of the whole system and the residual vs. the node number.

The exponential behaviour of the number of operations with increasing number of nodes is a warning sign. If this exponential increase would continue with the same rate, it could limit the applicability of the macro element method to low node numbers. Fortunately, with the optimization of the mesh for elimination, this exponential behavior disappears.

7.3 Optimisation of the elimination sequence

In light of the macro element formulation, the aim of the elimination process is to create a macro element which is a function between the nodes on the boundary of the eliminated region. This function is represented by a full N x N matrix where N is the number of nodes on the boundary of the eliminated region. During the elimination, the number of non-zeros vary in the originally sparse system matrix. Depending on how the elimination proceeds and which nodes are eliminated first, we can obtain different amount of non-zero elements in our system matrix during the elimination. It could be low or high depending only on the sequence how the nodes are being eliminated from the macro element region. In an unlucky situation where the nodes closer to the boundary of the macro element are eliminated first, we can arrive to a system of “islands”. This problem has been tackled in Publication 4 and several optimization algorithms are suggested.

The most crucial result of the optimization of the elimination sequence is that the number of operations vs. the number of nodes curve become linear after optimized elimination sequence Fig 7.3.

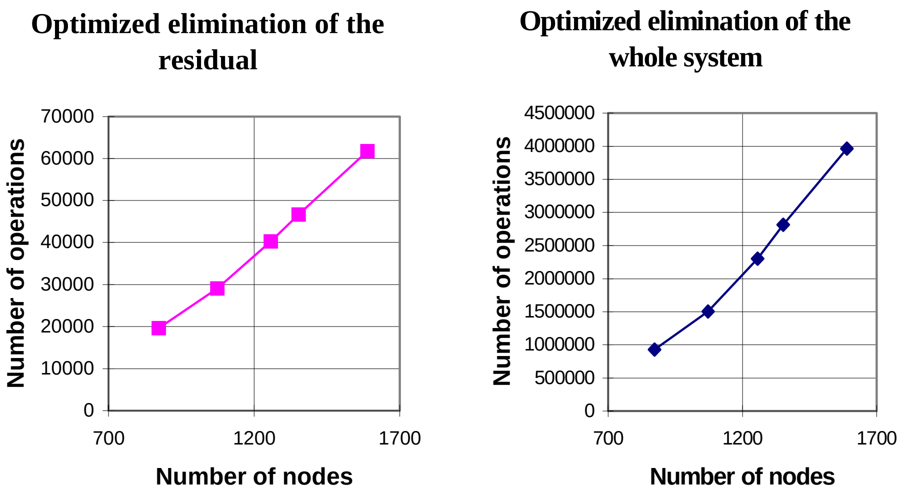

Fig. 7.3 Number of operations needed for the optimized eliminations versus the number of nodes

7.4 Discussion about the macro element method

The goal of the macro element approach is to accelerate and simplify the solution of the eddy current problem in the multi-conductor windings of electrical machines. It has been shown in previous chapters that the eliminated winding region results in a reduced system of equation which requires much smaller computation effort to solve than the original system, if certain conditions are met. These conditions were:

1. High number of eliminated nodes (variables)
2. Low number of nodes on the boundary of the eliminated region (macro element)

The computation time does not consist only of the solution of the system of equations but also a significant amount of time is used for the construction of them when using finite element analysis. The total time used for the macro element approach can be divided to the following parts:

1. Gauss elimination of the macro element regions, for the creation of the macro elements. It should be performed only once in a time stepping analysis, or more exactly, once after the winding region is constructed. The macro element can be saved and loaded unchanged when the same winding mesh is inserted into a different machine model. The elimination can take a long time, but it only has to be performed once, and its significance is decreased by creating the macro element to be time independent.

2. Elimination of the right hand side vector - due to time dependent sources - once in each time step. It takes much less time than the elimination of the whole system, but it is a time factor, which does not appear without macro elements.

3. Construction of the system outside the macro element, or in other words the construction of the nonlinear regions. It takes the same amount of time with or without macro elements.

4. Solution of the system of equations in each iteration step. It can take much less time when using macro elements. For a huge multi-conductor winding mesh, the difference can be very large. Depending on the nonlinear behavior of the problem, this step can become an important time factor. The “more nonlinear” the problem, the more iteration steps are needed.

5. Recovery of the eliminated variables when the iteration converged. This step is also unique for the macro element approach and does not appear in the classical finite element analysis. This step should be performed once in each time step in order to model the time derivative and so the eddy currents properly.

Studying these steps one fact can be seen clearly. In case of high number of time steps in nonlinear problems - like electromagnetic devices with iron cores and with a huge finite element model for the multi-conductor winding in them, to consider eddy currents - the method promises huge improvement over the classical approach. It has been experienced that even with a few steps of nonlinear iteration in each time step the method is superior over the classical approach if the winding mesh is dense enough.

One can conclude that the macro element method is greatly advantageous if the problem is nonlinear, non-sinusoidal, and complicated from the eddy current point of view (multi conductor). In that case, there is little sense to apply boundary element method combined with Fourier analysis. The problem can be conveniently formulated by the finite element method and - as it was discussed before - macro elements should be useful to reduce computational needs.

7.5 Conclusion

The suggested macro element representation allows us to construct the eliminated system of equations very conveniently and fast, and to utilize numerical macro elements in the finite element computation of eddy currents in multi-conductor windings of electrical machines. One needs to perform the elimination of the system matrix only once during a whole time stepping analysis. The macro element is fully represented by the eliminated linear part of the system matrix together with the eliminating matrix of the source terms. The study has shown that the number of operations required for the suggested technique - which is the same as the number of operations required to eliminate the source vector - are much less then for the elimination of the whole system.

It has also been shown that the optimized elimination process allows much more efficient creation of the macro element than the elimination without optimization. The number of operations needed for the elimination in the optimized case is significantly lower. In one case it appeared to be 16 times lower.

It is important to observe that the exponential behavior of the number of operations vs. the node number has changed to linear dependency when the elimination has been optimized. This means that larger problems only require proportionally larger amount of computation effort.

8. ADVANCED UTILIZATION OF MACRO ELEMENTS

Separately solved but strongly coupled problem parts

Macro elements – created by the elimination of inner nodes and certain circuit equations – can not only increase computation speed but with advanced use they can have many other advantages such as the significant decrease of the required computer memory in the finite element analysis of electrical machines. Macro elements allow us to divide the problem into smaller parts and to solve them separately while still maintaining a strong coupling between them. With the advanced use of macro elements, this fact can be utilized in several ways. A detailed explanation of several special advantages can be found in Publication 5.

8.1 The structure of the system of equations with macro elements

The linearity of the stator slots allows the elimination of linear nodal and circuit variables from the system and so the creation and use of the macro elements during the nonlinear iterations. The field inside the windings has to be computed only once in each time step. A technique to create constant macro elements for the duration of the whole time stepping analysis has been presented in Chapter 7. By this method, the elimination of the unnecessary “linear” variables from the system matrix – mostly those associated with the inner nodes inside the stator slots and circuit equations corresponding with the stator conductors – has to be performed only once during a whole time stepping analysis. Equations (8.1-8.4) explain the macro element idea.

In the following description, the term “boundary variables” is understood as the variables corresponding to nodal values of the nodes and circuit variables “surrounding” the macro element regions in the mathematical sense described bellow. Similarly “inner variables” represent the variables which from the mathematical point of view are inside the macro elements. These terms are easy to understand when looking at the nodal variables. Voltages associated with the conductors inside the slots have been eliminated too for the duration of the nonlinear iterations and in this respect they can be considered “inner variables”. However, due to the fact that the stator currents have remained in the analysis during the nonlinear iterations, and these currents are in connection with the eliminated voltages, the stator currents behave like "boundary variables". $\boldsymbol{S}$ stands for the parts of the system matrix, which are not affected by the macro element manipulations. Vector $\boldsymbol{a}$ represents the variables which are unaffected by the macro elements. Vectors $\boldsymbol{b}_1$, $\boldsymbol{b}_2$… $\boldsymbol{b}_n$ and $\boldsymbol{i}_1$, $\boldsymbol{i}_2$… i_n stand for the "boundary variables" and "inner variables" respectively in macro elements number 1,2… n.

$$\begin{bmatrix} \boldsymbol{S} & \boldsymbol{B}_1 & 0 & \boldsymbol{B}_2 & 0 & \boldsymbol{B}_n & 0 \\ \boldsymbol{B}_1 & \boldsymbol{O}_1^{11} & \boldsymbol{O}_1^{12} & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{O}_1^{21} & \boldsymbol{O}_1^{22} & 0 & 0 & 0 & 0 \\ \boldsymbol{B}_2 & 0 & 0 & \boldsymbol{O}_2^{11} & \boldsymbol{O}_2^{12} & 0 & 0 \\ 0 & 0 & 0 & \boldsymbol{O}_2^{21} & \boldsymbol{O}_2^{22} & 0 & 0 \\ \boldsymbol{B}_n & 0 & 0 & 0 & 0 & \boldsymbol{O}_n^{11} & \boldsymbol{O}_n^{12} \\ 0 & 0 & 0 & 0 & 0 & \boldsymbol{O}_n^{21} & \boldsymbol{O}_n^{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b}_1 \\ \boldsymbol{i}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{i}_2 \\ \boldsymbol{b}_n \\ \boldsymbol{i}_n \end{bmatrix} = \begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{r}_{b1} \\ \boldsymbol{r}_{i1} \\ \boldsymbol{r}_{b2} \\ \boldsymbol{r}_{i2} \\ \boldsymbol{r}_{bn} \\ \boldsymbol{r}_{in} \end{bmatrix} \tag{8.1}$$

$\boldsymbol{B}_1$, $\boldsymbol{B}_2$, $\boldsymbol{B}_n$ stand for the matrices representing the connection between the "boundary variables" and the unaffected variables of the system. The $\boldsymbol{O}$ matrices represent the original sub-matrices in the system matrix, which are influenced by the application of the macro elements. The $\boldsymbol{M}$ matrices represent the macro element matrices after the elimination of the "inner variables" in (8.2) and can be derived as shown in (8.3).

$$\begin{bmatrix} \boldsymbol{S} & \boldsymbol{B}_1 & \boldsymbol{B}_2 & \boldsymbol{B}_n \\ \boldsymbol{B}_1 & \boldsymbol{M}_1 & 0 & 0 \\ \boldsymbol{B}_2 & 0 & \boldsymbol{M}_2 & 0 \\ \boldsymbol{B}_n & 0 & 0 & \boldsymbol{M}_n \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{b}_n \end{bmatrix} = \begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{r}_{\mathrm{m}1} \\ \boldsymbol{r}_{\mathrm{m}2} \\ \boldsymbol{r}_{\mathrm{m}n} \end{bmatrix} \tag{8.2}$$

$$\begin{aligned} \boldsymbol{M}_1 &= \boldsymbol{O}_1^{11} - \boldsymbol{O}_1^{21}\boldsymbol{O}_1^{22^{-1}}\boldsymbol{O}_1^{12} \\ \boldsymbol{M}_2 &= \boldsymbol{O}_2^{11} - \boldsymbol{O}_2^{21}\boldsymbol{O}_2^{22^{-1}}\boldsymbol{O}_2^{12} \\ \boldsymbol{M}_n &= \boldsymbol{O}_n^{11} - \boldsymbol{O}_n^{21}\boldsymbol{O}_n^{22^{-1}}\boldsymbol{O}_n^{12} \end{aligned} \tag{8.3}$$

Vectors denoted by $\boldsymbol{r}$ represent the corresponding right hand side vectors. The variables $\boldsymbol{r}_{\mathrm{m}1}$, $\boldsymbol{r}_{\mathrm{m}2}$… $\boldsymbol{r}_{\mathrm{m}n}$ are created by the appearance of the macro elements and can be derived as shown in (8.4).

$$\boldsymbol{r}_{\mathrm{m}1} = \boldsymbol{r}_{b1} - \boldsymbol{r}_{i1}\boldsymbol{O}_1^{22^{-1}}\boldsymbol{O}_1^{12}$$

$$\boldsymbol{r}_{\mathrm{m}2} = \boldsymbol{r}_{b2} - \boldsymbol{r}_{i2}\boldsymbol{O}_2^{22^{-1}}\boldsymbol{O}_2^{12} \qquad (8.4)$$

$$\boldsymbol{r}_{\mathrm{m}n} = \boldsymbol{r}_{bn} - \boldsymbol{r}_{in}\boldsymbol{O}_n^{22^{-1}}\boldsymbol{O}_n^{12}$$

8.1.1 Recovery of variables inside the macro elements

When the nonlinear problem has been solved by iterative means the macro elements can be extracted and the “inner variables” can be recovered inside the linear macro elements in one step. The corresponding equations are in (8.5).

$$\begin{bmatrix} 1 & 0 \\ \boldsymbol{O}_1^{21} & \boldsymbol{O}_1^{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{i}_1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{r}_{i1} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ \boldsymbol{O}_2^{21} & \boldsymbol{O}_2^{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_2 \\ \boldsymbol{i}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_2 \\ \boldsymbol{r}_{i2} \end{bmatrix} \qquad (8.5)$$

$$\begin{bmatrix} 1 & 0 \\ \boldsymbol{O}_n^{21} & \boldsymbol{O}_n^{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_n \\ \boldsymbol{i}_n \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_n \\ \boldsymbol{r}_{in} \end{bmatrix}$$

8.1.2 One elimination of the system matrix during a whole time stepping analysis

As it was mentioned earlier, it is possible to eliminate the “inner variables” only once in a time stepping analysis. By storing $\boldsymbol{M}_1$, $\boldsymbol{M}_2$… $\boldsymbol{M}_n$, and $\boldsymbol{O}_1^{22^{-1}}\boldsymbol{O}_1^{12}$, $\boldsymbol{O}_2^{22^{-1}}\boldsymbol{O}_2^{12}$ … $\boldsymbol{O}_n^{22^{-1}}\boldsymbol{O}_n^{12}$, all components of (8.2) can be created without repeating the elimination of the system matrix. However, the right hand side vectors $\boldsymbol{r}_{\mathrm{m}1}$, $\boldsymbol{r}_{\mathrm{m}2}$… $\boldsymbol{r}_{\mathrm{m}n}$ containing time dependent source vectors due to the eddy-current effect have to be eliminated once in each time step.

8.1.3 Remarks concerning the system structure and matrix inversion

If the variables and the system of equations cannot be separated into such parts as presented above then some of the advantages (or all of them) described in this chapter – resulting from the advanced use of macro elements – may vanish.

In the description of the macro elements, one might be alerted by the use of large inverted matrices. This, is however, only a representation of the method and the creation of the inverted matrices in the traditional sense is never required, only the Gauss elimination of the “inner variables”.

8.1.4 Macro elements and sub-structuring

If the system of equations for an absolutely different problem – e.g. structural analysis – can be separated into similarly behaving sub-structures, the macro element approach is applicable. Sub-structuring is a well known method to replicate linear parts of the system and it is often used in structural analysis. However, the consideration of time dependent sources like eddy currents in this example – or speed dependent damping in structural analysis – is not so straightforward in the traditional sense of sub-structuring, and in most cases, such variables are not eliminated from the substructure but remain in the system as “boundary variables”. The reason for this is the need to eliminate the system in every time step in order to create the time dependent right hand side vector. Using macro elements suggested in this thesis, the eliminating terms are stored and the elimination of the whole system is unnecessary. In Chapter 7, a huge difference between the computational efforts required for the elimination of the whole system and the elimination of the source vectors only, has been observed. The macro element method is superior over traditional sub-structuring because the macro elements can account for time dependent quantities during the time stepping while still avoiding the need to eliminate the whole system of equations.

8.2 Test Case

The application to demonstrate the advanced use of macro elements is a large model of a three phase high voltage electrical machine with 18 stator slots in the computed region and with 18 conductors in each slot. The computed region of the machine is presented in Fig. 1a. The appearance of the macro elements in the structure of the system matrix is shown in Fig. 1b.

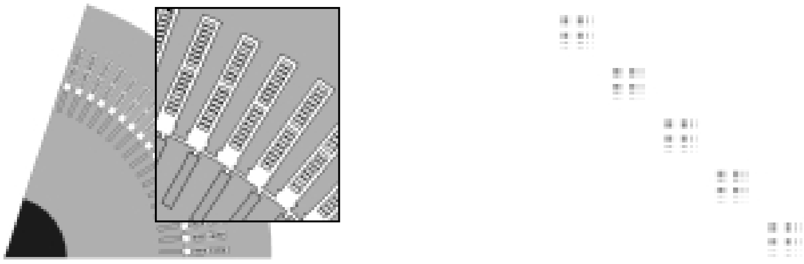

Fig. 1a The geometry of the test machine. The winding region is zoomed for better view.
Fig. 1b The structure of the system matrix after the inclusion of the macro elements. It is zoomed to show how the same structure repeats itself for each slot.

8.3 Reduced Memory Requirements by the Use of Advanced Macro Elements

The identical construction of the stator slots in the test machine allows us to use the same macro element in the place of all the slots. The same matrices are repeated for each slot, like if in (8.1) the ***O*** matrices with the same superscript would all be equal independently from their subscript being 1,2… *n*. This demonstrates clearly the reason why it is enough to use one slot to model all. Only one macro element and its connections to the boundary nodes of all the slot walls has to be stored. More significantly, the data required to store for the computation of the field inside the stator slots is also reduced to one slot data.

In one test case, the finite element mesh included 10 595 nodes and 5266 second order elements. The size of the system matrix before the application of the macro elements was 10 922x10 922 with 127 134 nonzero elements. After the application of the macro elements the necessary maximum size of the system matrix during the computation was 5270x5270 with 65 952 non-zeroes. This is about half of the original. The solver statistics show similar ratio for the maximum required storage, expressed in the numbers: 596 512 and 377 152 respectively.

The effect of the advanced use of macro elements has been discussed in details in Publication 5.

9. TEST COMPUTATIONS WITH THE MACRO ELEMENT METHOD

Effectiveness of the macro element method in time stepping analysis

In order to test the behavior of the macro element method in real life situations, two test machine structures have been studied. For both cases, several time-stepping analyses were performed with different supply voltage levels, but in all cases twice. Once without and once with the use of macro elements. The expectation was that for more non-linear cases the macro elements would be more efficient. The results discussed in this chapter have been published in Publication 6.

The test cases to evaluate the effect of the macro elements were two high voltage asynchronous machines with similar structures. The major difference between the two machines was in the number of conductors in one slot. It was 18 in the first case and 28 in the second. The rated voltage was 6000 V and the rated power 355kW. In each case, the solution sector for the machines contained 18 slots.

A startup transient was computed with different voltages. The transients produced extreme currents and losses in the first few periods of the line voltage. There were 200 time steps in one period and 800 time steps in each computation.

Computation times were measured in order to evaluate the effect of the macro elements on the computation procedure. The structure of the test machines are presented in Fig. 9.1 and 9.2. The major properties of the meshes are listed in Table 9.1.

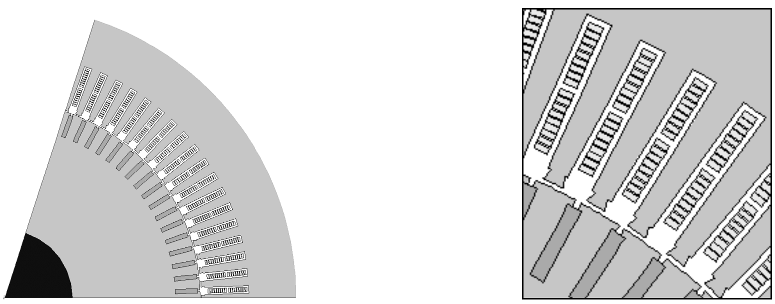

Fig. 9.1 Structure of the first test machine with 18 conductors in each slot, 18 slots in the solution sector. Version A.

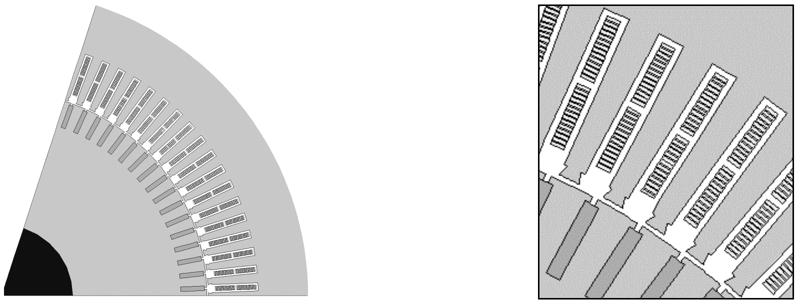

Fig. 9.2 Structure of the second test machine with 28 conductors in each slot, 18 slots in the solution sector. Version B.

TABLE 9.1 NUMBER OF NODES IN DIFFERENT PARTS OF THE TEST MACHINES

Number of nodes	Machine with 18 conductors in stator slots	Machine with 28 conductors in stator slots
In Stator winding	2916	4536
In Rotor winding	1037	1037
In Iron core	3446	3446

9.1 The results of the test computations

The numerical results when using the macro element method are identical with the results obtained by the traditional finite element formulation. Table 9.2 shows the computation times with and without the use of macro elements at different supply voltage levels and different machine versions A & B. All results are connected with start up transient computation with 200 time steps in one period and 800 total number of time steps. This means that the first four periods of the line voltage have been computed. In case A, the elimination at the beginning of each computation with macro elements took 3 minutes while in case B it took 6 minutes. The elimination is required only once for each mesh, however, to give an apples to apples comparison it is included in the values presented in Table 9.2.

TABLE 9.2 COMPUTATION TIMES AT DIFFERENT VOLTAGE LEVELS WITH AND WITHOUT THE USE OF MACRO ELEMENTS. M.E. STANDS FOR MACRO ELEMENTS. THE DIFFERENCE IS MEASURED IN MINUTES.

Version & Voltage	Computation time with M.E.-s	Computation time without M.E.-s	Difference
A 1000	1 hour 19 min	1 hour 7 min	-12
A 2000	1 hour 27 min	1 hour 25 min	-2
A 3000	1 hour 27 min	1 hour 38 min	11

A 5000	1 hour 28 min	1 hour 46 min	18
A 6000	1 hour 28 min	1 hour 51 min	23
B 6000	1 hour 42 min	1 hour 58 min	16
B 10000	1 hour 46 min	2 hours 10 min	24

The field plot at the end of the computed 800 time-step transient is shown in Fig. 9.3. It can be observed that high amount of flux penetrates the winding in the machine and crosses the conductors. This picture proves also the importance of the eddy current model in the windings.

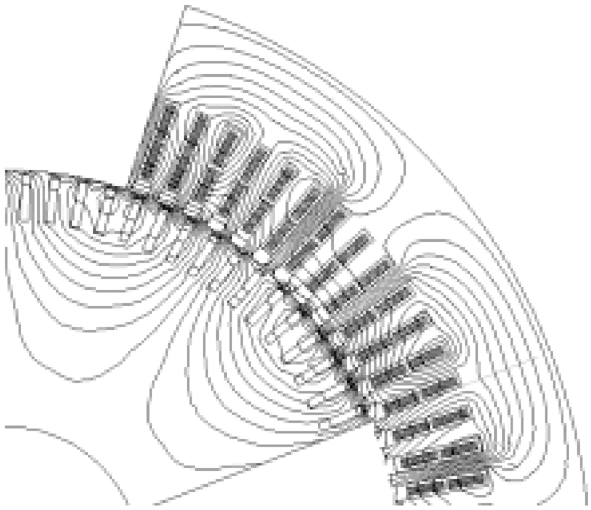

Fig. 9.3 Field plot of test machine B at the end of the 800th step of the transient at 6000 V. The high amount of flux penetrating the slots and crossing the stator and rotor conductors can be noticed.

9.2 Discussion about the results

It has been observed that the macro element method proves to be more efficient if the number of iteration steps per time steps is higher. With the increase of the line voltage, the system became more non-linear and the number of iteration steps has been increased and so the time gain, when using macro elements.

It is interesting to notice that the efficiency of the macro element method decreased at the same voltage level for version B compared to version A. One could conclude that the increased number of conductors resulted in lower efficiency. However this kind of conclusion can not be made simply because the two machines behaved significantly differently. Due to the difference in the winding of the machines, in case of version B the iron core was less saturated than in case of version A. This means that the number of iteration steps per time steps have been lower so it is not possible to make an apples to apples comparison between such test machines. In order to compare the effect of the increased number of nodes in the stator winding, one should ensure the same average number of iteration steps per time step. The same machine A or B at the different voltage levels can be compared to evaluate the effect of the non-linearity. In other words, the effect of the number of iteration steps (per time steps) on the effectiveness of the macro element method can be evaluated.

9.3 Conclusions

The results clearly indicate that with the increase of the nonlinear behavior and so with the increase of the average number of iterations required in each time step, the use of macro elements becomes more and more beneficial. It has been conformed for both test cases. With the use of macro elements up to 20 % gain in computation speeds could be reached in some case.

If the number of iteration steps would increase to very high values as in the case of hystresis modeling [39] – around 50-100 iteration steps per time step – then the time gain with macro elements would be more significant than in these computations with 4-10 iteration steps per time step. Macro elements promise larger improvements in computation speeds for those cases.

One would expect that with the increase of the complexity of the stator winding – like the increased number of conductors - macro elements “automatically” become also more and more efficient. This was not true for these tests, because the nonlinear behavior of the machines differed significantly due to the difference in their winding structure. The basic question in any case is; what will grow faster with the complexity, the increase of computation speed due to the reduced problem size or the computation time required for constructing the macro elements.

The macro element method has proved to be faster than the original FEM with the increase of the non-linearity. An advantage of the macro elements in these cases has surfaced again, namely the decreased memory requirements, as it has been presented in Chapter 8. These test cases have also proved that the general efficiency of the macro element method depends on several influencing factors and should be evaluated carefully.

10. CONCLUSIONS

Macro element method

In the magnetic field analysis of electrical machines, the modeling of eddy currents in multi conductor windings is a challenging and complicated task.

This work concludes that efficient and convenient time stepping analysis of electrical machines – including the modeling of eddy currents in multi conductor windings – can be made possible by the finite element method combined with a so called macro element method. This method uses the elimination of inner nodes, and the elimination of other “inner variables”, in a novel way for the linear region of the multi conductor windings, to form the macro elements and to reduce problem size. The macro elements are fully represented by the eliminated linear parts of the system matrix together with the eliminating matrices of the source terms. With the special formulation of the macro element method, the time consuming elimination of the system matrix has to be performed only once in a whole time stepping analysis.

The results have shown that the suggested method gives improvement in computation speed, significantly decreases memory requirements and has several other advantages for most practical cases of multi-conductor eddy-current problems. These advantages include easy adaptation of different nonlinear models and utilization of otherwise unused computation time to create the macro elements.

This research offers techniques to pre-process, create and handle eliminated system matrices, and a method to create the eliminated right hand side vector by allowing to perform the elimination of this vector only once during a non-linear iteration process.

The study of the elimination sequence of the linear variables has concluded that by optimizing the elimination sequence, the number of operations required for the elimination can be reduced significantly. In one test case it has been reduced by a factor of 16. It is important to observe that the exponential behavior of the number of operations vs. the node number has changed to linear dependency when the elimination has been optimized. This means that larger problems only require proportionally larger amount of computation effort. Several elimination strategies have been proposed.

The advantage of the macro element method is obvious for highly nonlinear problems, where many iteration steps are required in every time step, like in the case of hysteresis modeling. Using hysteresis models the number of iteration steps per time step may be 50-100.

The most exciting feature of the macro element method is its capability to separate a large coupled problem into one nonlinear and many linear problems which can be solved separately, while still maintaining strong coupling between these separately solved regions. This feature promises huge computation time gains when combined with parallel computing techniques.

A technique called sub-structuring – used frequently in mechanical analysis – has a very similar idea to the macro element method. The major difference between the macro element technique and sub-structuring is that the regions with time varying sources and effects like eddy currents – which have been modeled by the macro element method suggested in this thesis – are generally considered “outer” regions for sub-structuring. These regions are traditionally excluded from the super element regions with sub-structuring because of similar problems experienced with the pure elimination method.

A thermal measuring method has been tested and evaluated for the separation of eddy-current losses in windings surrounded by iron, but its accuracy has been found insufficient with the available tools. A much faster measuring system would be required.

The eddy-current loss measurements performed by electrical means in the absence of iron have confirmed that the finite element model was adequate for the study of multi conductor windings.

The macro element method has been developed and tailored to suit the finite element analysis of multi-conductor eddy-current problems in rotating electrical machines, but it can be adapted for similar problems in other fields of science.

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