Application of Fuzzy Logic to the Control of a Pedestrian Crossing Signal

JARKKO NIITTYMAKI AND SHINYA KIKUCHI

Fuzzy logic is known to be suited for dealing with a complex optimization problem with many objectives, many constraints, unclear input information, and vague decision criteria. Controlling the timing of a traffic signal falls in this category of problem. Fuzzy logic is introduced for controlling the timing of a pedestrian crossing signal. The controller is designed to emulate the decision process of an experienced crossing guard. The performance of this control is tested against two types of conventional demand-actuated control: one that uses the traditional green extension and the other that uses modified extension rules. The criteria for evaluation are delays to the pedestrians and the vehicles, and the percentage of vehicles that are stopped. The fuzzy logic controller finds a compromise between two conflicting objectives: minimization of pedestrian delay and minimization of vehicular delay and stops. The evaluation was performed using a microscopic simulation called HUTSIM developed at the Helsinki University of Technology. The fuzzy logic controller performs equally well as or better than conventional demandactuated control without requiring many parameter settings. Furthermore, the control rules are simple and a compilation of rational decision processes is expressed in natural language.

The efficiency and safety of traffic flow through a signalized intersection depends on the phases, sequence, and timing of the traffic signal installed. Control of these parameters is dictated by the technologies of the timing control and traffic flow detection, and also by the algorithm of control.

This paper introduces fuzzy logic as an algorithm for controlling the timing of a traffic signal. To test the idea, a fuzzy control scheme is developed for a pedestrian crossing signal. The performance of this control scheme is compared with that of conventional signal control. Our aim is to create a control scheme that performs like an experienced crossing guard at a pedestrian crossing. This study is motivated by successful application of fuzzy control to various control problems in recent years.

Several attempts to apply fuzzy logic to traffic signal control have been made in the past two decades. Among them, Pappis and Mamdani's (I) work in the mid-1970s is very significant, not only from the standpoint of the first application of fuzzy control to traffic signal but also from the standpoint of formulating the principle of the fuzzy control algorithm. Despite work by them and others, the application of fuzzy logic to traffic signals has only been known within a limited circle, primarily to electrical/electronic engineers. The main shortcoming of past work is that the problem environment assumed is not realistic from the traffic engineering point of view, and also that the performance measures used to compare a fuzzy controller with a conventional controller are not adequate.

This paper presents the nature of traffic signal control and reviews past work on the application of fuzzy logic to traffic signals. It then develops a series of control rules for a pedestrian crossing signal. The

J. Niittymaki, Laboratory of Transportation Engineering, Helsinki University of Technology, Espoo, Finland. S. Kikuchi, Transportation Engineering Program, Civil and Environmental Engineering Department, University of Delaware, Newark, DE 19716.

fuzzy controller's performance is tested using HUTSIM, a traffic flow simulation model developed at the Helsinki University of Technology. For various measures of performance, a comparison is made between the proposed fuzzy control and two types of conventional demand-actuated control.

PROBLEMS OF CONTROLLING TRAFFIC SIGNAL TIMING

Nature of Signal Control Problems

Controlling timing of a traffic signal means making the following evaluation constantly: whether to (a) terminate the current phase and change to the next most appropriate phase, or (b) continue the current phase. In other words, a controller (or a crossing guard) continuously (or at regular intervals) gathers information and evaluates the status of each approach and takes the most appropriate option.

Like most practical control problems, this control process involves the following elements: input, processor, output, the desired goal, evaluation criteria, and a feedback loop. In feedback control, input is the desired state of the system and the information about the current state. The processor is the knowledge base (or rule base) that provides the appropriate decisions given the input—that is, to continue or terminate the current phase. Output is the predicted consequences of the control prescribed by the processor. The desired goal is the target, and it establishes the tolerable conditions before the current phase needs to be changed. Evaluation criteria and feedback loop represent the process of comparing the output and the target; the output is then sent back to become part of the new input in the next time increment.

The difficulty of this control process lies in the fact that the process must be repeated with very short time intervals. Second, because traffic conditions in the immediate future cannot be predicted precisely, the control action is based on optimizing the current state only. As a result, the individual control actions do not necessarily yield the optimal condition in the long term. Third, the detectors cannot capture details of the prevailing conditions on the approaches (not as well as a human). When the intersection is complex in terms of its geometric design, channelization, and types of vehicles to be handled, the control process must consider many objectives—which are usually mutually conflicting, although safety is the most important requirement. Thus, signal control deals with a complex multiobjective/multiconstraint problem in which the optimization is performed based only on the most recent information.

Control of Pedestrian Crossing Signals

Perhaps the simplest and most fundamental traffic signal control problem is controlling a pedestrian crossing. The focus of this paper Niittymaki and Kikuchi Paper No. 98-0362

is to examine the effectiveness of fuzzy logic in controlling a pedestrian crossing signal. The main issue of control here is, given the prevailing conditions of vehicle arrivals on the roadway and of the pedestrians wishing to cross the street, when to terminate the current phase and give green to the other approach. The objectives are maximum safety (particularly to the pedestrian), minimum wait time to the pedestrian, and minimum delay to the vehicle movements.

Two types of control are commonly used: manual control by a pedestrian crossing guard and automated demand-actuated traffic signal. With the manual control, a crossing guard (or police officer) applies a set of common-sense rules. The rules are based on words rather than numerical values; for example, if a pedestrian has been waiting for a long time, then the current green to the roadway needs to be terminated and green should be given to the pedestrian. The entire control processes are executed based on the exchange of linguistic information including the evaluation of actual and target conditions.

The conventional demand-actuated control, on the other hand, relies on a number of rigid rules. When information on the prevailing conditions matches the antecedent of a rule, the rule is "fired." Because the conclusion of the rule is either to continue the current phase or to terminate it, the evaluation process is implicitly built into the rules. In this case, the doubt and hesitation experienced in the manual control do not exist. For this type of control, many rules are necessary to cover all possible situations. As a result, most signal controllers require the setting of a large number of parameters.

The fuzzy logic—based control that we attempt to develop here is to emulate the manual process as much as possible in a computerized environment. The aim is to "soften" the decision-making process by accepting human-like acquisition of information and executing "soft" decision rules. The premise of the study is that such a control will be robust and adaptive in terms of handling various objectives at the same time, yet keeping the parameter setting a simple task.

PRINCIPLES OF FUZZY CONTROL

Fuzzy control has been developed in the context of fuzzy inference. Fuzzy inference is the inference process based on multivalue logic: the truth values of input and the rules of the inference process are not singular (yes or no); rather, they are multivalued. The essence of this inference is the use of fuzzy sets for the representation of inputs and rules (relations). A number of reference materials are available on fuzzy sets, fuzzy inference, and control, including Klir and Yuan (2), Yager and Filev (3) and Zimmermann (4). Principles of traditional inference and fuzzy inference are compared for the following case:

Input: x is A and y is BRules: (1) if x is A1 and y is B1, then z is C1(2) if x is A2 and y is B2, then z is C2

Under traditional inference, the conclusion is drawn from a rule that has the exact match between the input and the premise (the "if ..." part of the rule), A = A1 and B = B1. This is the principle of *modus ponens* of classical logic. As a result, many rules are necessary to cover all possible inputs. Further, the output is singular (rigid decision process).

Under fuzzy inference, on the other hand, the conclusion is drawn based on the similarity between the input (A, B) and the premise (A1, B1; A2, B2). An exact match of the two is not necessary. The degree of similarity between them determines the degree of validity of the

conclusion. This process is called the *generalized modus ponens*. Under such a scheme, the input and the rules are represented by fuzzy sets and fuzzy relation, respectively. The degree of truth is measured by a set operation on three sets representing input, premise, and conclusion.

It is also important to note that a given input can apply to more than one rule because both the input and the antecedents of the rules refer to a range represented by a fuzzy set. As a result, conclusions from different rules are valid and they are aggregated. The final output (or conclusion) is a compromise between the conclusions of different rules.

In order to decide on a particular action (such as whether to terminate or continue a signal phase), we still need an operation to pinpoint the specific action because, after all, the final outcome still has to be binary. This process is called *defuzzification*. Various methods have been proposed to convert the fuzzy outcome to a binary decision. In our problem, the output of the rule that produces the maximum truth value is used for the conclusion.

PAST WORK

The first known attempt to use fuzzy control in traffic control was made by Pappis and Mamdani (1), who compared the fuzzy method to a traditional signal control with optimal cycle length at a simple isolated signalized intersection (two lanes in either direction, one-way streets, two phases). Nakatsuyama et al. (5) suggested that a fuzzy logic phase controller is valid for the control of two consecutive signalized intersections. Chiu (6) used fuzzy decision rules to adjust cycle time, phase split, and offset for traffic signal control at the network level and tested the model using a traffic flow simulation model. Kim (7) presented the problem of turning phases at an isolated intersection by taking into account possible blockage at the coordinated signals in heavy traffic conditions. Sayers et al. (8) tested fuzzy control in a real intersection in Germany. Pursula and Niittymaki (9) showed that fuzzy control (the Pappis-Mamdani algorithm) gives smaller delays than traditional vehicle-actuated signals, which use the "gap-seeking" method. Niittymaki (10) described the principles of controlling traffic signal using fuzzy logic under real-world conditions.

The problem found in past efforts is that comparison with traditional control is not conducted in a realistic environment. It is usually made with the classical fixed time signals, not with the advanced demand-actuated signal controls. Further, the simulation models used for comparison are rather crude considering the advanced traffic flow simulations available in the traffic engineering community today.

PROPOSED FUZZY LOGIC CONTROLLED PEDESTRIAN CROSSING SIGNAL: SYSTEM DESIGN

In this section, we establish the general features of the assumed pedestrian crossing signal: its basic parameters, objectives of control, and the rules and membership functions of the fuzzy sets used.

Assumed Pedestrian Crossing: Operating Environment and Requirements

Let us assume that a fuzzy logic-controlled pedestrian crossing is to be installed on a two-lane two-way roadway. The layout of the

assumed pedestrian crossing is shown in Figure 1. It has two vehicle detectors on each approach and a pedestrian push button on each side of the crossing. One vehicle detector is placed at the crossing; the other detector is 60 m from the crossing. These two vehicle detectors provide information on the number of approaching vehicles.

The signal operates as follows: when no pedestrian wishing to cross the street is present, the signal phase is green for the vehicular traffic—this is called the rest phase. When a pedestrian arrives and presses the button (or pad detector), the controller evaluates the state of vehicle arrivals and the waiting time of the pedestrian. If specified criteria are met, the current phase is terminated, and a green phase is provided for the pedestrian; otherwise, the current phase continues. The minimum green time for the roadway is set to 5 s; the green time for a pedestrian is 10 s.

Variables

The necessary variables are of two groups: input and output. The input variables are as follows:

- The cumulative waiting time of one or more pedestrians from the last signal change (WT).
- The approach volume measured by the number of vehicles present between the two detectors (the higher of the two approach volumes) (A).
- The discharge gap (time headway between two vehicles) in seconds (S). The smaller the value of S, the more the vehicles are packed (high density of flow). The smaller value between the two approaches is used.

The output variable is the decision variable to continue the same phase (E) or to terminate the current phase (T).

Objectives

The assumed fuzzy controller is a multiobjective controller. The objectives are as follows:

- Minimum Pedestrian Waiting Time—The pedestrian wishing to cross must be accommodated as soon as possible.
- Minimum Delay to the Vehicular Movement—The delay to the vehicles should be kept to a minimum. Vehicles should not be held up for an unreasonably long period.
- Maximum Safety to the Vehicles and Pedestrians-When a group of vehicles is approaching, the group should be preserved and passed. This is important from two aspects: (a) to avoid rear-end

collisions and (b) to create a gap in the downstream flow to provide better opportunities for the pedestrians at the downstream crossing to cross.

Rules of Control

The general format of the rules is the following:

If WT is (X1) and A is (X2) and S is (X3), then E (or T)

where X1, X2, and X3 are natural language expressions of the condition of respective variables.

WT is divided into three fuzzy sets: "short," "long," and "very long". A is divided into three fuzzy sets: "very few," "some," and "many." S is divided into two sets: "small" and "large." The number of rules depends on the number of combinations of fuzzy sets for X1, X2, and X3.

if WT is short and A is very few and S is small, then T or if WT is long and A is very few and S is small, then T or if WT is very long and A is very few and S is small, then T or if WT is long and A is some and S is small, then T or if WT is very long and A is some and S is small, then T or if WT is very long and A is many and S is small, then T or if WT is very long and A is very few and S is large, then T or if WT is very long and A is many and S is large, then T or if WT is very long and A is many and S is large, then T or if WT is short and A is some and S is small, then E or if WT is short and A is many and S is small, then E or if WT is long and A is many and S is small, then E or if WT is short and A is very few and S is large, then E or if WT is long and A is very few and S is large, then E or if WT is short and A is some and S is large, then E or if WT is long and A is some and S is large, then E or if WT is short and A is many and S is large, then E or if WT is long and A is many and S is large, then E

Table 1 summarizes these rules in matrix form.

In applying these rules, the basic signal system requirements described earlier must be considered. They are the fixed green time (10 s) for pedestrian and the minimum green time (5 s) for the roadway. Hence, once a command to terminate the current phase is issued, the phase will run for the minimum required time. Thereafter, the controller reviews the situation and executes the above rules every second.

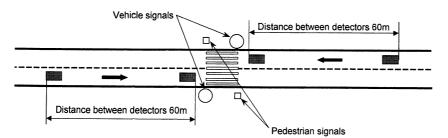


FIGURE 1 Layout of the signalized pedestrian crossing.

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TABLE 1 Summary of Fuzzy Control Rules

S (Discharge gap) = Small

		A (Vehicle Arrivals)		
		Very few	Some	Many
WT (Ped. Wait Time)	Short	Т	Е	Е
	Long	Т	Т	Е
	Very long	Т	Т	Т

S (Discharge gap) = Large

		A (Vehicle Arrivals)		
		Very few	Some	Many
WT (Ped. Wait Time)	Short	E	E	Е
	Long	Е	E	Е
	Very long	Т	Т	Т

E: Extend; T: Terminate

Membership Functions

The membership functions for expressions of WT (X1), A (X2), and S (X3) are shown in Figure 2. These are triangular membership functions, the most commonly used form. The membership functions for WT (short, long, very long) are prepared for three cases: (a) normal, (b) pedestrian friendly, and (c) vehicle friendly. As seen, "friend-liness" to either party can be controlled by sliding the membership function of the normal WT along the time axis. For the pedestrian-friendly case, the membership functions are moved to the left by 3 s; for the vehicle-friendly case, they are moved to the right by 3 s.

To determine the shape of the membership functions, many onsite interviews with pedestrians were made to capture the feeling of "short," "long," and "very long" wait times. Before interviewing individual subjects on site, the actual waiting time of the subject was also noted.

Control System Structure

The entire signal control system operates as follows: (a) the controller receives information from the vehicle detectors and pedestrian presence detectors (push buttons); (b) it applies the rules as listed above; (c) the output is a command either to continue the current phase or to terminate it; (d) if the output is to continue, then the phase extension criteria are checked, if they are met, the phase is extended; if not, then the phase is terminated (the phase extension criteria apply to the conventional demand actuated signals only as described later). The above activities repeat in short time increments.

SIMULATION ENVIRONMENT TO BE COMPARED WITH OTHER MODELS

The testbed used to evaluate the performances of the proposed fuzzy control is HUTSIM. It is a traffic flow simulation package developed at the Laboratory of Transportation Engineering of the Helsinki University of Technology, Finland. Since being developed in 1993, it has been used both for real-world applications and as a research tool. In HUTSIM, different control logics can be introduced as modules and, under individual control schemes, it simulates flow microscopically and produces various measures of performance. This feature is particularly useful for this study because it allows us to compare the performance of different control schemes for the same input.

PERFORMANCE TESTING ENVIRONMENT

The performance of the proposed fuzzy control signal is compared with that of two types of the conventional demand-responsive signal controller. This section discusses the measures of performance and the scheme of the comparison.

Control Schemes Compared

The proposed fuzzy control is tested against the following control schemes: (a) conventional demand-actuated control and (b) modified demand-actuated control. These controls are commercially available, with the former being more common than the latter. In principle, the former aims to provide convenience to vehicles (vehicle friendly); the latter aims to provide convenience to pedestrians (pedestrian friendly).

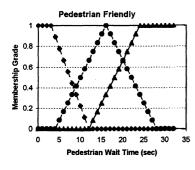
Both of these controls change the signal phase based on the gap between vehicles. When a pedestrian wishes to cross the street, the controller looks for a gap in the traffic flow, and as soon as a gap of prescribed size is known to exist, the traffic flow is stopped (red to the roadway) and a green phase to the pedestrians. The two controls differ in terms of how to weigh the approaching vehicles on the roadway against the waiting pedestrian.

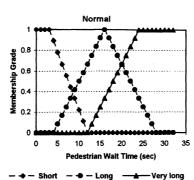
The conventional demand-actuated controller allows the green phase to be extended as long as the time headway of vehicle arrival is less than 4 s. If the gap (headway) becomes greater than 4 s, the green is terminated. Each extension increment is 4 s; the maximum time green can be extended is 30 s. In other words, when a pedestrian is waiting, the maximum he or she waits is 35 s (5-s minimum green plus 30-s extension).

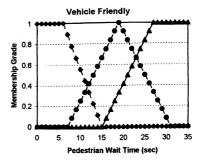
The modified demand-actuated control permits only one time extension of 4 s if the queue has been discharged (when the headway of the oncoming vehicle is greater than 4 s). If the queue is still discharging, the control mode is the same as the conventional mode. In other words, when a pedestrian arrives and pushes the button, the current green to the roadway can be extended by a maximum of 4 s. This is known as the "fast pedestrian signal".

It should be noted, however, that these rules of extension apply only when a pedestrian wishing to cross the street is present. If no pedestrian is present, the roadway receives a green (rest phase green). It should also be noted that the proposed fuzzy controller does not prescribe the extension explicitly; the extension of green to roadway or the termination of green extension is implicit in the rule.

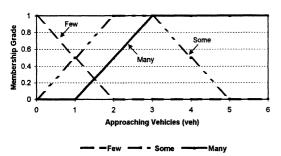








Membership Functions re: Approach Vehicle (A)



Membership Functions re: Discharge Gap (S)

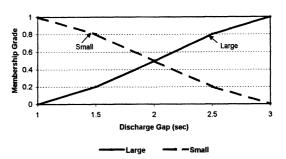


FIGURE 2 Membership functions for pedestrian wait time (WT), approach vehicles (A), and discharge gaps (S).

Measures of Performance

The measures of performance used to compare the fuzzy logic control with the conventional schemes are

- The average wait time of a pedestrian (measure of pedestrian delay).
- The average wait time of a vehicle (measure of vehicle delay).
- Percentage of stopped vehicles (measure of safety and driver inconvenience).

Of the above measures, pedestrian delay is the most important. A trade-off exists between the reduction of pedestrian delays and the reduction of vehicle delays. The percentage of stops is considered to be an indicator of traffic safety and of inconvenience to the driver.

Cases Analyzed

Various test cases are analyzed for the three control schemes (fuzzy, conventional demand-actuated, and modified demand-actuated controls). Each test case has different combinations of pedestrian volume, vehicle volume, and membership function of the rules. The specific cases and values assumed are presented below.

Pedestrian Volumes

There are three pedestrian volume levels:

- Low—15 pedestrians per hour (pph).
- Medium—50 pph.
- High—150 pph.

Vehicle Volumes

The vehicle volumes of EW (east—west) and WE (west—east) vary between 100 and 900 vehicles per hour (vph), with a total of 200 to 1,800 vph. For simulation purposes, the volumes for both directions are assumed to be the same.

Membership Functions

The membership functions for pedestrian waiting time (WT) for the fuzzy controlled signal are of three types: normal, pedestrian friendly, and vehicle friendly, as shown in Figure 2.

COMPARISON OF PERFORMANCES

The performances of the three controls are compared with respect to the three performance measures. The pedestrian volumes tested are 15 pph, 50 pph, and 150 pph; the membership function of WT is the normal case as seen in Figure 2:

• Low Pedestrian Volume (15 pph)—Figure 3 shows a comparison of the three controls. The average pedestrian delay is found to be significantly smaller for fuzzy control than for conventional

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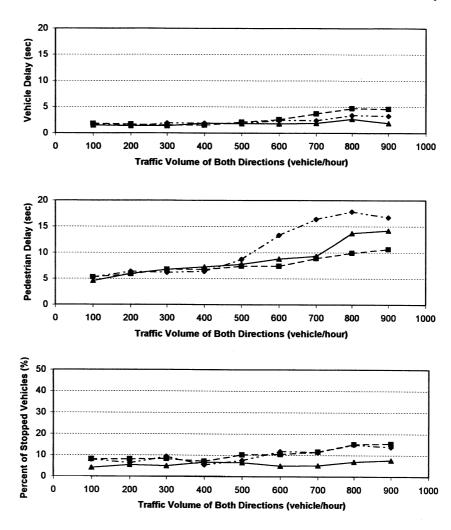


FIGURE 3 Comparison of performances for low pedestrian volume (15 pph).

Conventional

Modified

demand-actuated control. However, it is slightly higher than that of the modified demand-actuated control.

The vehicle delays are similar for all cases with the modified demand-actuated controller being slightly higher than the others; the fuzzy control case has the smallest delays. The average delay per vehicle is between 2 and 5 s. The percentage of vehicles stopped is smallest for the fuzzy control. The conventional demand-actuated and modified demand-actuated controls show very similar results.

• Medium Pedestrian Volume (50 pph)—Figure 4 shows a comparison of the three controls. When compared with Figure 3, vehicle delays and percentage of vehicles stopped are higher than for the case of the low pedestrian volume. Pedestrian delay, on the other hand, is nearly the same as with the case of low pedestrian volume for all three controls.

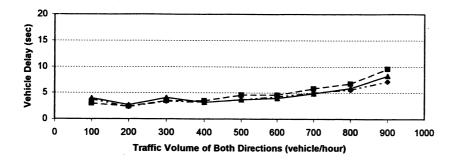
• High Pedestrian Volume (150 pph)—Figure 5 shows that vehicle delay and percentage of stopped vehicles are greater than for the previous two cases. These findings are expected because the higher the pedestrian volume, the greater the chance that traffic flow is interrupted. The pedestrian delay remains nearly unchanged from the above two cases (low and medium pedestrian volumes).

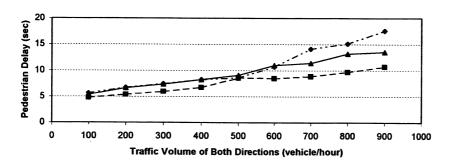
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Impacts of Different Membership Functions

Fuzzy

The three membership functions related to wait time (pedestrian friendly, normal, and vehicle friendly), shown in Figure 2, are tested for their effects on the performance measures. Figure 6 shows delays to pedestrians and vehicles for the three membership groups in the case of fuzzy control. When the pedestrian-friendly membership function is assumed, pedestrian delay is clearly lower. Vehicle delay varies





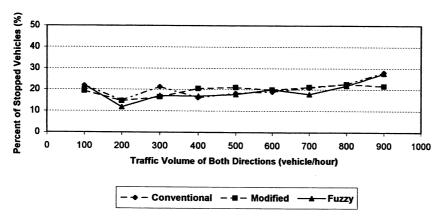


FIGURE 4 Comparison of performances for medium pedestrian volume (50 pph).

depending on pedestrian volume. Obviously, the higher the pedestrian volume, the greater the vehicle delay becomes; however, this is lower when the vehicle-friendly membership functions are used for WT. Correspondingly, the percentage of vehicles stopped increases with pedestrian volume and with the use of pedestrian-friendly membership functions.

Comparison of Pedestrian Wait Times

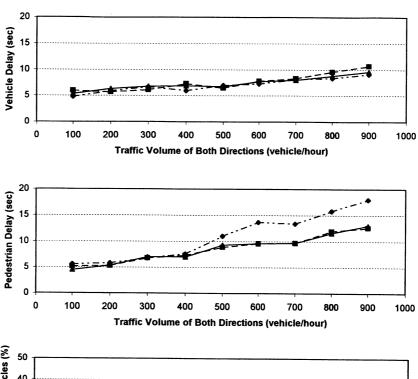
Figure 7 compares fuzzy and conventional controls with respect to pedestrian wait times. It shows that for the same arrival patterns of pedestrians and vehicles, 95 percent of the pedestrians wait no more than 20 s under fuzzy control; for the conventional control, only 56 percent of pedestrians wait no more than 20 s. The membership functions used for this analysis are the normal case.

DISCUSSION OF RESULTS

The results show that the fuzzy controller provides pedestrian-friendly control and keeps vehicle delay smaller than either of the conventional controls. In other words, it provides a compromise between the two opposing objectives of minimum pedestrian delay and minimum vehicle delay. This finding is consistent with the characteristics of most other applications of fuzzy control; fuzzy control looks for a compromise in the multiobjective problem environment. Also, such a compromise is executed without specific limits imposed in the rules of the fuzzy control.

Another aspect that is worth examining is robustness and adaptability. Conventional signal control requires the setting of a large number of parameters. This is because of the specificity of the rules; for each condition, one specific rule must be present. In the case of the fuzzy

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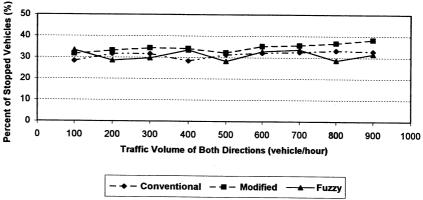


FIGURE 5 Comparison of performances for high pedestrian volume (150 pph).

logic controller, the number of parameters can be reduced significantly. This is possible due to the use of the membership function which covers a range. Further, fuzzy rules are modularly constructed, and hence, more rules can be added without changing the structure of the algorithm or altering the function of the preexisting rules. Furthermore, the fuzzy controller can be made adaptive to different situations as shown. The three sets of membership functions on WT provide contingency plans based on the priority between vehicles and pedestrians.

CONCLUSION

This study has developed a fuzzy logic algorithm for controlling the timing of a pedestrian crossing signal and tested its performance. The result shows that the algorithm not only effectively controls the

signal timing but also offers at least equal or better performance than conventional demand-actuated signal control. The logic is simple, and the number of parameters is small; yet the performance is found to be remarkably robust.

The traits of a fuzzy logic controller are well suited to dealing with a control problem at a complex intersection with many approaches and vehicle movement requirements. Therefore, the result in this paper should provide an impetus to develop fuzzy control logic for more complex intersections. Finally, the testing of a new control scheme such as the problem at hand requires not only the algorithm for control but also a microscopic simulation testbed. In this respect, a sophisticated simulation model such as HUTSIM is indispensable for the development and testing of an advanced signal control algorithm.

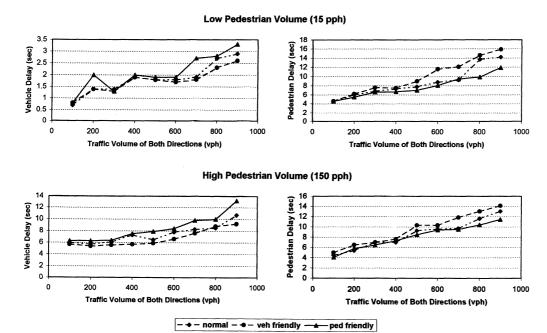


FIGURE 6 Effects of different membership functions on WT (normal, vehicle friendly, and pedestrian friendly).

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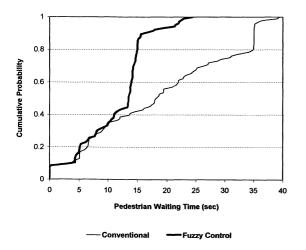


FIGURE 7 Comparison between fuzzy and conventional controls: cumulative distributions of pedestrian wait time. Approach vehicle volume (both directions = 800 vph; pedestrian volume = 50 pph).

go to Mitsuru Tanaka of the University of Delaware for his work on the graphs.

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