Aspects in analysis and model-based sound synthesis of plucked string instruments[1]

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Abstract

This thesis consists of an introduction and nine articles. The general framework of the articles is the analysis and synthesis of plucked string instruments. The articles document acoustical analysis and model-based sound synthesis of the tanbur, the kantele, the lute, and the ud that are all traditional or historical plucked string instruments. In addition, the author's contributions to the development of an integrated model-based plucked string synthesis system is presented. As a key study, the integrated system concentrates on model-based sound synthesis of the acoustic guitar. However, the sound synthesis models of other plucked string instruments are also implemented within the system. Finally, two alternative techniques that can be used in model-based sound synthesis are covered. The first technique focuses on the loop filter design based on a polynomial damping model. The second technique presents a synthesis model based on finite differences that can be used in real-time sound synthesis.

Keywords acoustic signal processing, computer music, musical acoustics, physical modeling of musical instruments

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Preface

This work has been carried out during 1998-2002 at Helsinki University of Technology, Laboratory of Acoustics and Audio Signal Processing. From the beginning on, I experienced incredible help and support from each and every member of the lab. This preface aims to indicate the motivation and the mood of the research, the team sprit, and close collaborations spanning a long period of life.

My supervisor, Prof. Vesa Välimäki deserves my sincere gratitude for his support, advice, and guidance of this work. I also feel privileged to carry out this work in the Acoustics Labs, where Prof. Matti Karjalainen is a ”super” adviser in any audio-related matter, including my research. Matti and Vesa are the co-authors of many articles bundled in this thesis, and I thank them for their enthusiasm, expertise, positive feedback, and patience. Furthermore, I'd like to thank the pre-examiners of my thesis, Prof. Perry R. Cook (Princeton University) and Prof. Tapio Saramäki (Tampere University of Technology) for their comments, suggestions, and again, for their patience.

Dear reader, this thesis would not be in thy hands, if Dr. Tero Tolonen's help, support, motivation, and friendship would cease. From the beginning to the end, his scientific being shaped my research and in turn, this thesis, while his persona devised myself. I have seen Tero in five countries and on seven seas, and experienced a friendship beyond any prescribed limits. While this may suggest that I should express my appreciation to Tero within the family section of this preface, his scientific impact on this thesis compels me to do so somewhere here.

Integration into the group that has the unofficial name *sound source modeling team* took zero time due to the efforts, suggestions,, and general know-how of Tero, Matti, and Vesa. I also thank Mr. Sami Brandt, who helped the development of the analysis tools just before my arrival to Finland. The codes of Tero and Sami provided an initial acceleration for my research.

I enjoyed the collaboration with Dr. Mikael Laurson in full respect, and I am delighted about the goals achieved, the path taken, and the experience gained. I look forward to see that Mikael's PWSynth simulates a broader range of instruments; conventional and virtual ones. I also expect a big chart success from the upcoming CDs made by Mikael and his package. Special thanks also go to Mika Kuuskankare; the author of the ENP, and (according to some rumors) a trumpet player.

Ms. Patty Huang appeared in the lab during the sunny days, and spent the winters in California, which I consider a wise decision. In her presence, she carried out seemingly endless sets of experiments on the kantele, and in her absence, she went over and over on the results, methods, and the

presentation of the material. I thank to Patty for all her efforts, and above all, for her patience. I look forward to browse trough her upcoming thesis, and also to keep my promise about a celebration dinner.

The financial support of CIMO, Academy of Finland, Jenny and Antti Wihuri Foundation, and EU IST-2000-33059 project ALMA are highly acknowledged. I feel lucky to have Ms. Lea Söderman around to handle the logistics in the lab. During all these years, she scored top ranks in crisis management including last minute flights, registrations, official document requests, etc. I am grateful to Lea for her help, support, and again, patience. Moreover, I wish to thank to Martti Rahkila and Jussi Hynninen for their help and advice on many issues related to computers, software, and network.

Besides my overall feelings I tried to express about the whole lab crew, some *akulaiset* deserve special thanks. Among them are Poju Antsalo (wonderful music support and afternoon talks), Dr. Aki Härmä (LaTeX and thesis procedure know-how), Paulo Esquef (roommate of boto), Hanna Järveläinen (humor and directness), Henri Penttinen (support and fun, in and out), and Dr. Ville Pulkki (Pelle M. and other support). Natural akulaiset Dr. Jyri Huopaniemi, Matti Airas, and Miikka Huttunen get also special thanks for their help and friendship. Finally, I would like to direct a huge gratitude towards one member of the *naistenhuone*; to Ms. Riitta Väänänen, for being such a great sister to me. I enjoyed Riitta's company during the shiniest and the darkest days, and her energy and warmness kept me going.

Many exchange students and researchers visited the lab during the writing of this thesis. I thank Balász Bank, Yannick Delmaire, and João Martins for sharing their time and ideas with me. Dr. Federico Fontana still keeps me informed about interesting concerts here in Helsinki! Dr. Federico Avanzini, Dr. Tony Verma, and Mr. Miltiadis Daniil deserve special thanks for being awesome flatmates.

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Cumhur Erkut, Otaniemi, Espoo, Finland.

List of symbols

Scalar variables and parameters

a	Loop filter coefficient
c_l	Longitudinal wave propagation speed
c_t	Transversal wave propagation speed
f	Frequency
f_0	Fundamental frequency
f_s	Sampling frequency
g	Loop gain
l_{nom}	Nominal string length
m	Set index
m_{i}	Input mixing gain of a string model
m_{o}	Output mixing gain of a string model
n	Set index
t	Time
x, y, z	Axes of Euclidean space $\mathbb{R}^3$
ρ	Linear mass density
σ	Frequency-independent decay rate
τ	Frequency-independent decay time
ω	Angular frequency
A	Tension modulation depth
E	Young's elastic modulus
L	String length
N	Dimension of a vector or space
S	Source surface, cross-sectional area
T	Tension

Vectors

$\mathbf{e}_{\mathrm{z}}$	Unit vector in the vertical direction
$\mathbf{f}$	Force vector
$\mathbf{r}$	Position vector
$\mathbf{r}_s$	Source vector
$\mathbf{v}$	Velocity vector
$P_N[n]$	N-dimensional parameter vector of a discrete system

Matrices

$\mathbf{C} \in \mathbb{R}$	Coupling matrix of the model $h[n]$
$\mathbf{Y} \in \mathbf{L^2}(\mathbb{R})$	Admittance matrix

Continuous functions

$f(t)$	Force distribution
$h(t)$	An acoustical system
$x(t)$	Input signal of an acoustical system
$v(t)$	Velocity
$y(t)$	Output signal of an acoustical signal
$\kappa(\mathbf{r}_s, \mathbf{r})$	Spatial shaping function
$\sigma(\omega)$	Frequency-dependent decay rate
$\tau(\omega)$	Frequency-dependent decay time
$H(\omega)$	Frequency response of $h[n]$
$H_{\mathrm{B}}(\omega)$	Frequency response of a plucked string instrument body
$H_{\mathrm{BR}}(\omega)$	Frequency response of a plucked string instrument bridge
$H_{\mathrm{LFR}}(\omega)$	Low-frequency guitar function
$H_{\mathrm{HFR}}(\omega)$	High-frequency guitar function
$Y(\omega)$	Admittance function

Discrete functions

$d[n]$	Fractional delay control signal
$e_y[n]$	Approximation error
$h[n]$	A sound synthesis model
$x[n]$	Input signal of a sound synthesis model
$y[n]$	Output signal of a sound synthesis model

Spaces

$\mathbf{l}^2(\mathbb{Z})$	Space of square-summable discrete signals
$\mathbb{C}$	Space of complex numbers
$\mathbf{L}^2(\mathbb{R})$	Space of square-integrable continuous signals
$\mathbb{R}$	Space of real numbers
$\mathbb{Z}$	Space of integer numbers

List of abbreviations

BC	Before Christ
CWS	Commuted Waveguide Synthesis
DC	Direct Current
DSP	Digital Signal Processing
DWG	Digital Waveguide
ENP	Expressive Notation Package
FDTD	Finite Differences Time Domain
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
FPE	Final Prediction Error
GCV	Generalized Cross–validation
IIR	Inifinite Impulse Response
LTI	Linear, Time-Invariant
MSE	Mean Square Error
NN	Neural Network
PC	Personal Computer
RMS	Root Mean Square
SDL	Single Delay Loop
SMS	Shibata's Model Selector, Spectral Modeling Synthesis
SNR	Signal–to–Noise Ratio
PS-STFT	Pitch Synchronus STFT
PW	PatchWork
STFT	Short-Term Fourier Transform
STK	Synthesis ToolKit
TMDF	Tension Modulation Driving Force
URL	Universal Resource Locator

List of publications

This thesis summarizes the following articles and publications, referred to as [P1]-[P9]:

[P1] C. Erkut, T. Tolonen, M. Karjalainen, and V. Välimäki, ”Acoustical analysis of tanbur, a Turkish long-necked lute,” in Proceedings of the 6th International Congress on Sound and Vibration (ICSV6), Lyngby, Denmark, July 5-8, 1999, vol. 1, pp. 345-352.

[P2] C. Erkut and V. Välimäki, ”Model-based sound synthesis of tanbur, a Turkish long-necked lute,” in Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP’00), Istanbul, Turkey, June 5-9, 2000, vol. 2, pp. 769-772.

[P3] C. Erkut, M. Karjalainen, P. Huang, and V. Välimäki, ”Acoustical analysis and model-based sound synthesis of the kantele”, Journal of the Acoustical Society of America, vol. 112, no. 4, October, 2002, pp. 1681-1691.

[P4] M. Laurson, J. Hiipakka, C. Erkut, M. Karjalainen, V. Välimäki, and M. Kuuskankare, ”From expressive notation to model-based sound synthesis: a case study of the acoustic guitar,” in Proceedings of the International Computer Music Conference (ICMC’99), Beijing, China, October 22-28, 1999, pp. 1-4.

[P5] C. Erkut, V. Välimäki, M. Karjalainen, and M. Laurson, ”Extraction of physical and expressive parameters for model-based sound synthesis of the classical guitar,” presented at the AES 108th International Convention, Paris, France, February 19-22, 2000, preprint no. 5114, 17 p.

[P6] M. Laurson, C. Erkut, V Välimäki, and M Kuuskankare, ”Methods for modeling realistic playing in acoustic guitar synthesis,” Computer Music Journal, vol. 25, no. 3, pp. 38-49, 2001.

[P7] C. Erkut, M. Laurson, M Kuuskankare and V. Välimäki, ”Model-based synthesis of the ud and the renaissance lute,” in Proceedings of the International Computer Music Conference (ICMC 2001), Havana, Cuba, September 17-23, 2001, pp. 119-122.

[P8] C. Erkut, ”Model order selection techniques for the loop filter design of virtual string instruments,” in Proceedings of the 5th World Multi-

Conference on Systemics, Cybernetics and Informatics (SCI 2001), Orlando, FL, USA, July 22-25, 2001, vol. 10, pp. 529-534.

[P9] C. Erkut and M. Karjalainen, ”Virtual strings based on a 1-D FDTD waveguide model”, Proceedings of the Audio Engineering Society 22nd International Conference, Espoo, Finland, June 15-17, 2002, pp. 317-323.

Table of Contents

1. Introduction

This thesis focuses on specific sound sources known as plucked string instruments and synthesis of their sounds using simple, computationally efficient algorithms known as sound source models. The sound synthesis method that mimics the sound production mechanism of a sound source is called *physical modeling synthesis* or *model-based sound synthesis* [1, 2, 3]. In this thesis, the latter designation will be used.

The history of the plucked string instruments arose 6 millenia ago in Mesopotamia, where the instruments were built by limited technical skills, craftsmanship, and material knowledge of the ancient times. In the course of the history, plucked string instruments were first constructed by joining natural environmental objects such as wooden sticks, gourds, and turtle shells, and later, gradually by manufacturing selected, processed, and specifically designed parts. Motivated by different musical needs and shaped by the technology available at the time, various species of the plucked string instrument family emerged. Some members became the subject of modern studies in *musical acoustics* only after converging to their final form and function. The interaction of the science, technology, and craftsmanship eventually provided new methods to construct plucked string instruments [4].

It is instructive to consider the development of the violin family momentarily. The modern violin was developed in Italy in the sixteenth century, and in the eighteenth century Antonio Stradivari set the standards of the violin craftsmanship [5]. From nineteenth century onwards, distinguished scientists[1] contributed to the science of violin acoustics, and physical characteristics of the violin have been gradually revealed [6, 7]. The connection between the construction and the tonal quality has been addressed [8] and a new family of the fiddles has been constructed by scaling the characteristics of a fine violin [9]. An equivalent analog electrical circuit model of the violin has been formulated [10] and the instrument has been disembodied [11], i.e.,

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[1] Felix Savart, Hermann von Helmholtz, Lord Rayleigh, C. V. Raman, and many others.

its body has been replaced by an analog electronic circuit. The advent of fast modern digital processors, combined with efficient model-based sound synthesis algorithms [12, 1], allowed the instrument to be virtually reconstructed [13], eliminating both the resonating body and the strings.

A similar path may be traced in other families. The technical evolution of the plucked string instruments is outlined in Section 2. Prior to the research reported in this thesis, the acoustical properties of the representative plucked string instrument, that is, those of the acoustical guitar, have been widely studied, well-understood, and well-documented (see, for example, [14]). Sound source models of various instruments have been constructed and the status of each family has been reviewed [2]. The problems related to the control of the synthesis models have been addressed in [15, 16]. Open-source code implementation of sound source models that could be run and controlled in real-time has been available [17] and model-based sound synthesis methods have been standardized [18, 19].

The status and framework of, and future perspectives on the model-based sound synthesis of plucked string instruments have been presented in [20] just before the initiation of this thesis. From these statements, one could conclude that the potential and expressiveness of model-based sound synthesis could be fully explored by developing an integrated system that consists of an accurate, repeatable control scheme, and a synthesis engine based on a generic plucked string model, e.g., the model presented in [20]. The synthesis engine should be general enough to model different plucked string instrument (e.g. lutes') tones, yet be accurate enough to allow the model-based and expressive synthesis of the representative instrument of the family (i.e., the acoustical guitar). Moreover, the system should be capable of implementing alternative sound synthesis structures. These conclusions are the main motivations of this thesis and correspond to three aspects in analysis and model-based sound synthesis of the plucked string instruments:

1. Analysis and model-based sound synthesis of traditional and historical plucked string instruments.
2. High-quality and expressive sound synthesis of the acoustical guitar.
3. Investigation of alternative model-based sound synthesis techniques.

In order to cover all these aspects in a nutshell, a formal mathematical description is presented in the next subsection. Subsection 1.2 summarizes the scope, Subsection 1.3 outlines the contents, and Subection 1.4 presents the structure of the thesis.

1.1 Mathematical description of this thesis

This thesis deals with continuous and discrete finite energy signals $y(t) \in \mathbf{L}^2(\mathbb{R})$ and $y[n] \in \mathbf{l}^2(\mathbb{Z})$, respectively. In general, $y(t)$ has a spatial dependence, i.e., it is a function of the position vector $\mathbf{r}$ as well as the time t. In many cases, however, the spatial coordinates are prescribed so that the spatial dependence may be suppressed. $y(t)$ and $y[n]$ are not arbitrary signals but responses of systems $h(t)$ and $h[n]$ to the excitations $x(t)$ and $x[n]$, respectively. The systems are not arbitrary either. $h(t)$ corresponds to a physical system, i.e., to an acoustical plucked-string instrument, whereas $h[n]$ is its real-time sound synthesis model. Typically an N-dimensional parameter vector $P_N[n]$ and an excitation signal $x[n]$ are associated with $h[n]$.

$h(t) \in \mathbf{L}^2(\mathbb{R})$ is the only assured property of a plucked-string system. $h(t)$ may be intricate, nonlinear, and time-varying. A method to analyze complicated, nonlinear, time-varying systems is to decompose them into interacting functional blocks and simplify the function of each block, their interaction, or both. For some instruments the properties and the interaction of the blocks are well understood. For some others, they are unknown. The first aspect in this thesis is the analysis of $h(t)$ of two traditional instruments from the latter group. In this respect, if available, $h[n]$ is used to justify the decomposition and simplifications.

Let $y_h(t)$ be a recored sound signal produced by the acoustical sound source system $h(t)$, and $y_d[n]$ be its discrete representation[2]. A goal in sound synthesis is to minimize the approximation error $e_y[n] = \|y_d[n] - y[n]\|$ between the recorded signal $y_d[n]$ and the synthetic signal $y[n]$. The norm can be the L^2 norm or an induced perceptual one. Some synthesis techniques minimize $e_y[n]$ directly. Model-based techniques rather focus on the systems and try to match an $h[n]$ to a given $h(t)$. If $h(t)$ would be a simple LTI system, then one could solve this approximation problem by minimizing $e_h[n] = \|h_d[n] - h[n]\|$. Since it is not, then the match is evaluated by considering $e_y[n]$ instead of $e_h[n]$.

Additional desirable features of $h[n]$ are related to its computational cost, and to the dimension of its parameter vector $P_N[n]$. A desirable sound synthesis model $h[n]$ minimizes them both. Some models $h[n]$ are proved to fit a well-understood $h(t)$, to be efficient, and to have a $P_N[n]$ of a small dimension N. The only remaining problem is the calculation of $P_N[n]$, where N is assigned *a priori*. This is the second aspect in this thesis. Solution of this problem is discussed, and a parameter extraction method is presented. The focus is on the acoustical guitar, for which $h(t)$ is available and well

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[2] Usually obtained by an analog-to-digital conversion.

understood. Similar methods have been applied to two other lutes, assuming that they are similar acoustical systems to the guitar and to each other.

More degrees of freedom are introduced when the a priori constraint on N is relaxed for $h[n]$, or for some subblocks of it. Still more is introduced when the choice of $h[n]$ is relaxed. These additional degrees of freedom may increase one or more of the desired features of the model. However, there is no known method to optimize the synthesis quality, the efficiency, and the dimension of the parameter space of an $h[n]$ altogether. Therefore, there is ongoing research on alternative models or submodels. This is the third aspect of this thesis. A method to relax the constraint on N for a subblock of $h[n]$ and mathematical properties of an alternative $h[n]$ have been presented.

These are the three aspects of this thesis. The following parts elaborate these aspects.

1.2 Scope of this thesis

The acoustics of five plucked instruments are studied in this thesis. These instruments are the tanbur, the kantele, the acoustical guitar, the Renaissance lute, and the ud. The primary focus on the acoustical analyses is the string vibrations. The analysis results are described in detail only for the tanbur and the kantele, in [P1] and [P3], respectively. The acoustical guitar measurements are used to verify the measurement procedure, and the measurement data of all five instruments are used in model-based sound synthesis.

The term *simulation* addresses directly the sound synthesis procedure in this thesis. In fact, this thesis is devoted to the simulation of string instruments' tones, since the spatial representatation of the instrument is not taken into account. The spatial representation is suppressed by describing some parts of the acoustical system as black boxes with fixed transfer functions between prescribed measurement points.

In every measurement reported in this thesis, the sound recordings were made 1 m above the top plate of the corresponding instrument ($\mathbf{r} = \mathbf{e}_\mathrm{z}$) in an anechoic chamber. No attempt has been made to measure or model the radiation characteristics of the instruments. For the theory of the radiation characteristics of plucked string instruments, the reader is referred to [14] and [21], and for the source directivity synthesis models to [22] and [23].

The approach of this thesis in model-based sound synthesis is physically oriented. The informal listening tests mentioned in this summary and in the articles are the subjective statements of the authors of the articles. Although an important subject, the perceptual criteria to evaluate the quality of synthetic tones and simplify the synthesis models are otherwise not discussed in this thesis. More information on this topic can be found in [24, 25, 26].

Publications [P4], [P5], [P6], and [P7] report the development of an integrated system. Elements of system are the real-time synthesis engine PW-Synth and the Expressive Notation Package (ENP) that controls the PW-Synth. It should be emphasized that the implementation and coding of the system are outside of the scope of this thesis. Except simple code testing on circular buffers and fractional delay filters [27], the author's contribution to the development of the system is limited to extraction of the physical and expressive model parameters of the system.

1.3 Contents of this thesis

This doctoral thesis consists of this summary and nine articles. The articles investigate three aspects of the general framework, i.e., analysis and model-based sound synthesis of plucked string instruments. The articles in the first group ([P1]–[P3]) present investigations on the acoustics of two traditional instruments, namely, the tanbur and the kantele, and sound synthesis models associated with them. The articles in the second group ([P4]–[P7]) document the development of an integrated model-based plucked string synthesis system, with the acoustical guitar being the representative of the family. The developed methods have been also tested on two acoustically similar lutes; a traditional and a historical one. The third group of articles introduce alternative techniques that can be used in model-based sound synthesis of plucked string instruments. In [P8], a polynomial damping model has been used to increase the degree of freedom of a specific subblock of the model $h[n]$, and in [P9] a different model $h[n]$ that is based on the numerical integration has been utilized for real-time sound synthesis.

1.4 Structure of this thesis

Section 2 encapsulates the history of the plucked string instruments and summarizes the previous research on the acoustics and model-based sound synthesis of plucked-string instruments. It thus provides a background to evaluate the contributions of this thesis presented in Section 3. In Section 4, the main results of articles are summarized and the contribution of the author to each article is indicated. The rest of the thesis consists of the articles.

2. Background

2.1 A capsule history of plucked string instruments

This section provides a historical perspective of the plucked string technology. An excellent and comprehensive reference about the history of the plucked string instruments is [28]. Here, only an overview is given.

Generically, *lute* refers to any *chordophone*, whose strings are parallel to its soundboard, and run along a distinct neck. Lutes can be structurally classified according to their neck-to-body ratio as short-necked or long-necked. In the family of *zithers*, there is no neck and the string length cannot be varied. Among thesis publications, [P1] and [P2] are related to long-necked lutes, [P4], [P5], [P6], and [P7] are related to short-necked lutes, and [P3] is related to zithers.

Sumerians in the 4th Millennium BC were the first civilization technically capable of designing musical instruments. The knowledge about earliest lutes are mostly derived from historical drawings. The oldest drawing of a lute was found on a clay tablet in Uruk[1], which dates back to 2400 BC. The instrument had an oval-shaped body 20 cm long and 16-18 cm wide, and its neck was approximately 70 cm long. The proportions indicate that it was a long-necked lute.

2.1.1 Long-necked lutes

The Sumerian long-necked lute had a single string and there were no tuning pegs. The method how the string was fastened is not exactly known. The strings were excited with a plectrum, probably a 10-20 cm long piece of wood or bone. Egyptians and Persians were familiar with the instrument around

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[1] Ancient Mesopotamian city located in southeastern Iraq.

1700 BC, indicating two main branches of the spread. In both branches, the long-necked lute slowly evolved into the short-necked lute.

In the eastern path of spread through Persia, the know-how about the ancient Indian instrument *ravanastron* transformed the Sumerian lute. The tuning pegs were introduced in Turkestan around 500 BC. The eastern neighbors, Mongolians and Chinese, adopted Turkestan lute forms around 200 BC. A short-necked lute, *yueh k´in*, known in Japan as *gekkin*, eventually appeared in China.

Hyksos soldiers brought the long-necked lute towards west to Egypt around 1700 BC. These original lutes were called *nefer* in Egypt, and they were soon being built in several sizes. Professional musicians preferred a large lute, up to 120 cm long, with three strings and elongated body of 35-40 cm length and 12-18 cm width. Singers and dancers used two types of a smaller lute. One of the lutes had an elongated body (35 cm long) and a 50 cm neck.

The first well-preserved lute has been found in the tomb of Egyptian musician Har-mosê (1500 BC). The body has been made from a single piece of wood, and it is 38 cm long and 17 cm wide. The length of the neck is 65 cm. The soundboard was made of animal skin fastened to the rim with nails. The original gut strings were preserved, enabling the method of termination at both ends to be determined. There is no tuning peg in the lute. At one end, the strings are bound to the neck and raised by a nut. At the bottom end, the strings were attached to a holder. By moving a tuning block, the player could alter the tension of the strings, and hence tune the lute.

2.1.2 Short-necked lutes

Although short-necked lutes were invented in Ancient Egypt around (1200 - 1000 BC), they underwent further evolution in Sassanian times (226-646) in Persia. Many of the short-necked lutes that are still in use today grew out from two Sassanian lutes: the slender *mizhar* and the wide *barbat*. The mizhar had three strings, whereas the barbat had four.

Both of the Sassanian instruments spread towards the Arab peninsula in the 7th century. In the post-Sassanian period the Arabs created the prototype of the present day form of the lute from the mizhar and the barbat. This prototype had a wooden soundboard and a characteristic staved, wood-vaulted back design, and its name was based on the Arabic word for "the wood" - *al'ud*.

By the Moorish period in Spain, different lute types based on the al'ud have been used. These lutes had three strings and different tuning ranges according to their sizes. The earliest detailed lute drawings was found in Alfonso el Sabio's "Cantigas de Santa Maria", published in 1257-1275. These drawings indicate that the Spanish *laud* was an almost unaltered form of al'ud. The name al'ud is also thought to be the origin of the name of the new European instrument, the lute.

In Europe the lute underwent continuous adaptation and evolution according to the requirements of European music. European lutes are now divided into three general categories according to their form and history:

1. The medieval lute (up to the middle or late fifteenth century). It is the direct descendant of the ud, has 4 or 5 courses and it is played with a plectrum.
2. The Renaissance lute (late fifteenth to early seventeenth century). It has 6-8 or 9 courses and it is played with the fingers. Different sizes of the Renaissance lute form a consort (soprano, alto, tenor, and bass). The most common instrument for solo playing is the tenor lute.
3. The Baroque lute (early seventeenth to late eighteenth century). It is also played with the fingers and has 11-13 courses.

The popularity of the Renaissance lute has been somewhat reduced by the spread of the Baroque lute, and both of the lutes were gradually superseded by the widespread use of guitars.

2.1.3 Kantele

The *kantele* refers to a group of plucked string instruments that have been common in traditional folk music in Finland, its neighboring region in Northwest Russia, and the Baltic countries [29]. The instrument and its variations are called the *kannel* in Estonia, the *kanklės* in Lithuania, the *kokle* in Latvia, and the *gusli* in Russia [29]. The 5-string Finnish kantele has a significant role in Finnish folklore as the instrument of rune-singers, and in Finnish mythology, especially in the Kalevala, the collection of ancient Finnish runes. It is estimated that the origins of the kantele are more than 1000 years old.

The end of the 18th century was a start for new kantele forms. A larger design and increased number of strings (9 to 15) allowed the player to elaborate the playing possibilities. Even more possibilities are achieved by a *concert kantele* developed since 1920's. It contains up to 45 strings and has a playing range of approximately five octaves.

2.2 Acoustics of plucked string instruments

Acoustically, a plucked string instrument is a coupled, spatially distributed, and continuous system, as illustrated in Fig. 2.1. In this system, every subblock of the interacts with every other subblock, thus makes the functional description of system very difficult. One way to proceed with a coupled, spatially distributed, and continuous system is to decouple[2] the subblocks of the system by crude approximations and then separately discuss the inherent coupling mechanisms and other assumptions made.

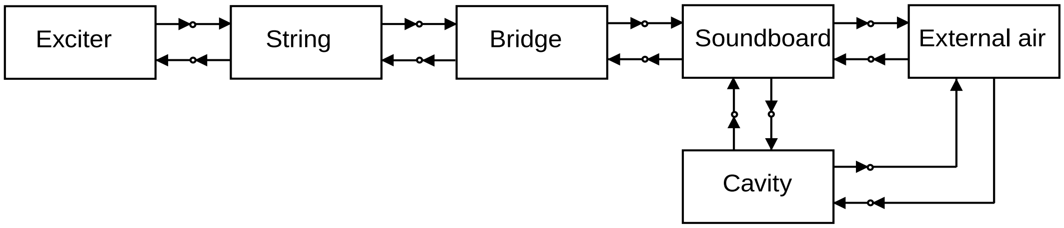

Figure 2.1: Illustration of a plucked string instrument as an acoustical system.

This thesis follows this approach; a simplistic description is provided in the following, and a more realistic description is presented in Subsection 2.2.5. It is important to note that the primary focus in the acoustical studies and model-based sound synthesis models presented in this thesis is the string vibrations. A very detailed discussion of coupling mechanisms and spatial representation of a plucked string instrument is beyond the scope of this thesis, as justified in Subsection 1.2. The reader may refer to the following references for more information. An introduction to and an excellent coverage of the research field can be found in [5] and in [14], respectively. In [30], a concise technical review is given. Important aspects regarding the physics and numerical simulations of stringed instruments are discussed in [31].

The simplified block diagram of a generic acoustical plucked string system (corresponding to that of a guitar-like monochord) is illustrated in Fig. 2.2. This simplified diagram is obtained by neglecting many coupling mechanisms between the blocks. In fact, for a more complete discussion of the system, the unidirectional signal flow between the blocks in Fig. 2.2 should be replaced by bidirectional interactions and interaction between non-consecutive blocks should also be considered.

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[2] In electrical engineering, decoupling is established by a circuit element that is a lossless path for direct current and an infinite impedance for alternating current. Decoupling prevents the undesired signal exchange between the subblocks.

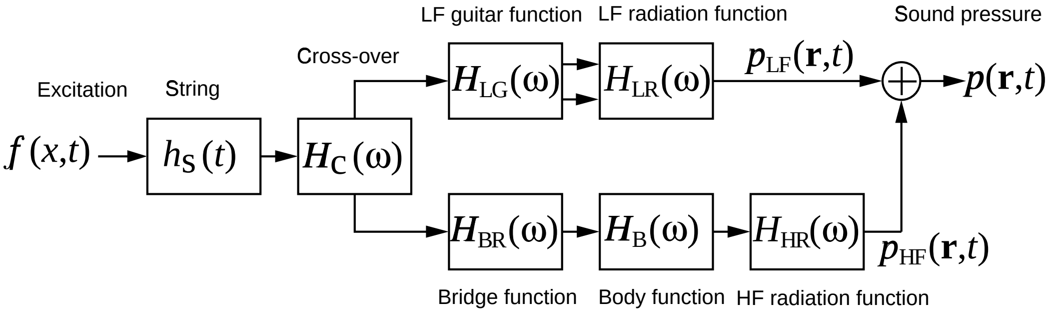

Figure 2.2: Illustration of a plucked string instrument as a simplified and decoupled acoustical system. After [14].

The operation of the simplified system is as follows. The player excites the system by imposing a force distribution $f(x,t)$ on the string $h_s(t)$, that in turn makes the string vibrate.[3] The vibration energy of the string is partly dissipated and partly transmitted to the body $H_B(\omega)$ via the bridge $H_{BR}(\omega)$, which responds to the forces exerted by the string. The subsystems bridge and body determine a conceptual cross-over function $H_C(\omega)$ so that the simplified system operates in two different ways depending on the frequency ω. This conceptual cross-over is included in the simplified model in order to refer to a reflex enclosure model that is a very crude approximation and is valid only up to 300 Hz (see Subsection 2.2.3).

In either case, the vibrations are spatially filtered according to the radiation functions $H_{LR}(\omega)$ or $H_{HR}(\omega)$ that essentially perform the following operation

$$p(\mathbf{r},\omega) = \int_S v(\mathbf{r}_s,\omega)\kappa(\mathbf{r}_s,\mathbf{r})H_R(\omega)dS \tag{2.1}$$

where $v : \mathbb{R}^3 \times \mathbb{R}^+ \to \mathbb{C}$ is the volume velocity over the source surface S, $\kappa : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{C}$ is a spatial shaping term that depends on the source and receiver position, and $H_R(\omega)$ is a general radiation function. In time, the filtering of Eq. 2.1 is equivalent to a convolution. In the following, the subsystems of a plucked string instrument are explained in detail.

2.2.1 String vibrations

Like many other acoustical systems, the governing equation of string vibrations is a second-order hyperbolic partial differential equation, i.e., the *wave*

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[3] ξ is the coordinate axis along the string. In this case, the string is along the axis $\xi = x$.

equation. Essentially the wave equation is nonlinear, but by certain assumptions it can be linearized [32]. The linearized wave equation in a lossless medium yields

$$\ddot{\mathbf{r}} - c_t^2 \mathbf{r}'' = f(\xi, t) \tag{2.2}$$

where $\mathbf{r}$ is the transversal[4] position vector, and $\dot{\mathbf{r}}$ and $\mathbf{r}'$ are its temporal and spatial derivatives, respectively. A similar equation may be derived for longitudinal vibrations by replacing the transversal propagation speed c_t with the longitudinal propagation speed c_l that are given by

$$c_t = \sqrt{\frac{T}{\rho}}\ ,\ c_l = \sqrt{\frac{EA}{\rho}} \tag{2.3}$$

where T is the tension, ρ is the linear mass density, E is Young's elastic modulus, and A is the cross-sectional area of the string. It can be verified that the transversal waves

$$\mathbf{r}(\xi, t) = \mathbf{r}_0(\xi \pm c_t t) \tag{2.4}$$

are the traveling wave solutions of the homogeneous equation, where ξ is a longitudinal coordinate so that $\mathbf{r}(\xi, t)$ lies in the transversal plane for all t, and $\mathbf{r}_0$ is an initial position vector. Longitudinal waves that propagate by c_l may be formulated similarly. The traveling wave solution of the wave equation was first proved by d'Alembert in [33].

A closed-form solution can be obtained by orthogonal decomposition of Eq. (2.2) in the space $(0, L) \in \mathbf{L}^2(\mathbb{R})$, where for the fixed boundary conditions the eigenvalues and eigenfunctions yield

$$\omega_n = \sqrt{\frac{T}{\rho}} \frac{\pi}{L} n \tag{2.5}$$

and

$$a_n(\xi) = \sqrt{\frac{2}{\rho L}} \sin \frac{n\pi\xi}{L} \tag{2.6}$$

respectively. The decomposition was first proved by Fourier in 1807.

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[4] In this thesis, the following convention is used to refer to three orthogonal vibration directions. The x-axis is along the string so that the string is stretched between $x = 0$ and $x = L$, where L is the string length. The y-axis is parallel to the top plate of a plucked string instrument, and z-axis is in the direction of the normal of the top plate. The *longitudinal*, *horizontal*, and *vertical* directions are aligned with the unit vectors of the x, y, and z axes, respectively. When referring to a vibration in the plane spanned by horizontal and vertical unit vectors, the term *transversal* has been used. The same convention is used in [P3].

Both solutions need to be modified to account for dispersion, dissipation, and energy transfer from the string to the rest of the system. The mathematical methods for such modifications have been described in detail in [34, 35]. Dispersion is not discussed in this thesis. Dissipation, energy transfer at the bridge, and the loading by the soundboard in effect modify the traveling waves in Eq. (2.4) and the eigenfunctions in Eq. (2.6) by a multiplicative frequency-dependent exponential decay term $e^{-t/\tau(\omega)}$. In addition, the eigenvalues in Eq. (2.5) are slightly modified. These modifications can be viewed equivalently as *apparent* modifications of the string length. An overview of damping mechanisms is the topic of the next subsection.

Damping mechanisms in string vibrations

Like in any other physical system, the total energy of the string dissipates by reversible or irreversible processes, i.e., mechanical energy transfer to the rest of the system, and thermodynamic dissipation, respectively [36, 14]. Three main damping mechanisms in string vibrations are *internal damping*, *air damping*, and *energy loss through supports*. In effect, as shown in [14], they introduce corrections to the decay time $\tau(\omega)$ so that

$$\frac{1}{\tau_{\text{Tot}}(\omega)} = \frac{1}{\tau_{\text{int}}(\omega)} + \frac{1}{\tau_{\text{air}}(\omega)} + \frac{1}{\tau_{\text{sup}}(\omega)} \; . \tag{2.7}$$

Internal damping includes the viscoelastic and thermoelastic losses. Viscoelastic losses are caused by a small delay between strain and stress. Their effect can be modeled by a complex Young modulus E_{c} and a loss angle δ_{ve} as

$$E_{\text{c}} = E(1 + \jmath\,\delta_{\text{ve}}) \; . \tag{2.8}$$

Strictly speaking, this relation is valid only in the frequency domain unless the loss angle has specific properties, in which case the equation yields causal functions in the time-domain. Thermoelastic losses are caused by the thermal conduction between compressed (heated) and stretched (cooled) regions of the string. Their effect can be as well modeled by a complex Young modulus and another loss angle δ_{tm} as

$$E_{\text{c}} = E(1 + \jmath\,\delta_{\text{tm}}) \; . \tag{2.9}$$

Same note of caution about the validity domain of the equation applies here. Usually, the viscoelastic and thermoelastic damping model predictions are in qualitative agreement with the measurements [36], see also [37] and [38]. This is in contrast with the third internal damping mechanism, i.e., the dry friction that is caused by the slip-stick friction between two contiguous turns in a wound string. There is currently no model to account for the dry friction.

In effect, viscoelastic damping is active at high frequencies, and thermoelastic damping is active around a resonance frequency at mid-frequencies, with respect to the hearing range. Measurements indicate that the dry friction is effective within the whole audible frequency range.

Air damping includes the direct radiation of the strings, and viscous damping. The strings are poor radiators, thus the effect of direct radiation is negligible. Viscous flow of air around the moving string, on the contrary, may be a major cause of damping under some conditions [14]. Viscous damping is modeled by a mechanical resistance that has a constant and a frequency-dependent term. The viscous decay time τ_{air} in Eq. (2.7) is a nonlinear function of the string radius and frequency.

The energy loss through supports depends on the mechanical characteristics of the support, which is determined by the bridge and the body functions as illustrated in Fig. 2.2. The mechanical characteristics of the support is usually given a mechanical admittance function $Y : \mathbb{R}^+ \to \mathbb{C}$ defined as

$$Y(\omega) = \frac{F(\omega)}{V(\omega)} \tag{2.10}$$

where $F(\omega)$ is the Fourier transform of the forces exerted on the support, and $V(\omega)$ is the velocity response of the support. The real part of the admittance function adds to the damping, whereas the imaginary part changes the effective length of the string. Several measurement techniques to obtain the admittance function have been discussed in [39] and [40] for the case of the violin.

Nonlinear string vibrations

Unlike an ideal flexible string given in Eq. (2.2), a real string is linear to the first order approximation only. The major cause of nonlinearities is that any small transverse displacement of the string makes a second-order change in its length and therefore in its tension.

Carrier [41] and Harrison [42] reported the first systematic analytical and experimental investigations of the nonlinear string vibrations, respectively. Since then, many other studies have been reported (see [43, 24] for a review). By assuming fixed boundary conditions and using an excitation force of a frequency close to that of the first mode of the string, the tension modulation is shown both analytically and experimentally to cause a fundamental frequency descent [41, 36], a whirling motion [44, 45, 46, 47], coupling between different modes and directions [44, 48, 49, 36], and amplitude jumps [50, 51].

Starting from a 3D elasticity model, a coupling torsion term has been obtained in [52]. Some of these studies deserve special attention. Therefore a brief discussion is provided below.

Narashima established a unifying framework of all the earlier research in [53] and emphasized the importance of the longitudinal displacement. Anand took the longitudinal displacement into account in [45], and showed that the transversal and longitudinal vibrations can be separated if the propagation speeds given by Eq. (2.3) satisfy $c_l \gg c_t$. This property is satisfied in all practical cases concerning the musical strings so that *Anand's argument* is the backbone of the follow-up studies including [P3]. Using Anand's argument, it can be shown [36] that the nonlinear wave equation is expressible as

$$\ddot{\mathbf{r}} - c_t^2 \left(1 + \frac{ES}{T_0} \ell_{\text{dev}}(t) \right) \mathbf{r}'' = f(\xi, t) \tag{2.11}$$

where $\ell_{\text{dev}}(t)$ is the instantaneous elongation of the string defined as

$$\ell_{\text{dev}}(t) = \frac{1}{L} \int_0^L \frac{\left(\mathbf{r}(\xi, t)'\right)^2}{2} d\xi \tag{2.12}$$

Note that this nonlinear equation would have been obtained from the linear case given by Eq. (2.2) if c_t is replaced by

$$c_t(t) = c_t^2 \left(1 + \frac{ES}{T_0} \ell_{\text{dev}}(t) \right) \tag{2.13}$$

Analogous to the linear case, the traveling waves then would satisfy

$$\mathbf{r}(\xi, t) = \mathbf{r}_0(\xi \pm c_t(t) t) \tag{2.14}$$

The existence and uniqueness of this kind of solutions has been proved (see for instance [54]), although representing them in closed-form is usually a challenging task. Therefore, numerical approaches have been favored since the earliest analytical studies of the nonlinear string vibrations [41].

2.2.2 String-bridge interaction

As mentioned in Section 2.2, energy transfer between the string and the body is generally determined by the response of the bridge to the forces exerted by the string (see [55] for a particular alternative path). Both linear and nonlinear string vibrations exert a transversal force $F_\xi(t)$ on the bridge that can be further decomposed into the horizontal and vertical components (see the footnote 2 in Subsection 2.2.1 for directional conventions). In addition to the transversal force component, the nonlinear vibrations of a string result in a longitudinal force component [36, 14] (see also [P3]), given by

$$F_x(T) \approx -T|_{x=L} = -\left(T_0 + ES\frac{\ell_{\mathrm{dev}}}{L}\right) \tag{2.15}$$

which has been termed in [56] as the *tension modulation driving force* (TMDF).

Usually, the bridge response differs according to the direction of the excitation force, and it is customary to generalize the concept of an admittance function given by Eq. (2.10) by an admittance matrix [57]:

$$\mathbf{v}(\omega) = \mathbf{Y}\mathbf{f}(\omega) \tag{2.16}$$

where $\mathbf{v}(\omega)$ and $\mathbf{f}(\omega)$ are the velocity and force vectors of three orthogonal components and $\mathbf{Y}$ is a 3×3 functional matrix. An element $Y_{i,j}(\omega)$ accounts for the velocity response of the bridge in the ith direction to the force in the jth direction, where $\{i, j\}$ are one of the three orthogonal components. As a convention, $n = \{1, 2, 3\}$ corresponds to longitudinal, horizontal, and vertical directions, respectively. Note that this formulation has a close resemblance with the 2D admittance matrix at the bridge of a guitar, reported earlier in

If $Y_{2,2}(\omega) \neq Y_{3,3}(\omega)$, a two-stage decay is caused by the superposition of the transversal velocity components with different decay rates [58]. In plucked string instruments $Y_{1,1}(\omega)$ is negligible usually at low frequencies, but becomes, however, comparable in magnitude to $Y_{2,2}(\omega)$ and $Y_{3,3}(\omega)$ for the middle frequencies. The proper cross-over frequency is around 1 kHz for the guitar [30].

If $Y_{i,j}(\omega)|_{i\neq j} \neq 0$, an interdirectional energy transfer occurs. This coupling is of paramount importance for nonlinear string vibrations. Feng has proved in [49] that intermodal coupling does not take place in the nonlinear string vibrations under the fixed boundary conditions. Legge and Fletcher have confirmed in [48] this result by proving that the nonlinear transfer of energy among modes of different frequencies on a vibrating string is only possible if $Y_{i,j}(\omega)|_{i\neq j} \neq 0$, in which case the TMDF is coupled back to the string.

A recent study performed in [59] provides experimental results in the presence of a longitudinal force component around the frequency ω at which $|Y_{1,1}(\omega)| \approx |Y_{2,2}(\omega)|$ or $|Y_{1,1}(\omega)| \approx |Y_{3,3}(\omega)|$. These cases correspond to the transmission of the TMDF to the body of an instrument. Instead of an analytic formulation of the TMDF, a nonlinear mixing model has been presented in [59]. The effects of the nonlinear mixing have been demonstrated for the acoustic guitar, orchestral harp, and piano tones. The partials thus generated are termed as *phantom partials* and it has been concluded that any plucked-string or struck string instrument that is susceptible to longitudinal string forces could produce phantom partials.

2.2.3 The body response

With a reference to Fig. 2.2, from the bridge onwards, the body of a plucked string instrument can be considered a linear system[5].

In the low-frequency range, where the size of the bridge is negligible compared to the wavelength, the bridge may be conceptualized as acting as a part of the top plate, and the top plate is coupled to the back plate through the enclosed air in the body. This coupling is modeled by a linear, coupled mechanical oscillator that is equivalent to a reflex enclosure model [61, 62]. The transfer function of the model $H_{\mathrm{LG}}(\omega)$ is called the *low-frequency guitar function* [62].

At higher frequencies, the bridge filter $H_{\mathrm{BR}}(\omega)$ shapes the string forces while transmitting them to the body of the instrument. An additional filtering is performed by the body function $H_{\mathrm{B}}(\omega)$ that accounts for the top or back plate resonances. The plate vibrations tend to break up into small patches, separated by nodal lines. These vibration modes have been studied using various techniques such as holographic interferometry [14, 63] and modal analysis [64, 65, 66] within the valid frequency range of the techniques.[6]

The radiation functions and the consequent spatial filtering given by Eq. (2.1) are usually tedious to measure or calculate because of the distributed multimode body vibrations $v(\mathbf{r}_s, \omega)$ and complex source geometry S. In this thesis, the pressure field is sampled at a single location in space[7] so that $\mathbf{r}$ is prescribed and the spatial dependence of Eq. (2.1) is suppressed. In this case, a radiation function may be consolidated with the spatial filtering term into a single transfer function. A general solution method has been discussed in [21] that decomposes the radiation integral of Eq. (2.1) into elementary radiators such as monopoles, dipoles, and quadrupoles. Usually, monopole and dipole elements are enough to approximate the radiation integral with a good accuracy. This decomposition yields a canonic description for the body characteristics of plucked string instruments [67].

2.2.4 Systems with multiple strings

So far the discussion was limited to a monochord shown in Fig. 2.2. In a multiple-string instrument, the lumped admittance matrix in Eq. (2.16) is

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[5] Despite strong objections, an opposite opinion has been presented in [60] based on an experimental evidence. Due to the lack of the physical explanation, this opinion is not further discussed in this thesis.

[6] For instance, the modal analysis technique can only be performed on guitars up to 1 to 2 kHz. For higher frequencies, the modes are coupled by the damping terms.

[7] 1 m above the top plate in the vertical direction.

to be replaced by a distributed matrix function. In effect, the distribution transfers energy from a vibrating string to the others. The energy transfer can set the resting neighbor strings into motion. This phenomenon, in which a string vibrates without any direct excitation but is driven only by vibrations of another string, is called a *sympathetic vibration*. There are relatively few analytical treatments of sympathetic vibration mechanisms [58, 68], and these treatments usually focus on two strings coupled to the body through a common bridge. In measurements reported in this thesis, only one string is set into motion while all the others are kept damped, thereby eliminating the sympathetic vibration effects. The complete analysis of sympathetic vibrations is a challenging future task.

2.2.5 Discussion of assumptions

This section is devoted to the discussion of the assumptions made in order to obtain the simplified system shown in Fig. 2.2 from the more realistic and complete system shown in Fig. 2.1. In the following, the physical nature and consequences of these assumptions are discussed. Their consequences in the framework of sound synthesis will be further discussed in Subsection 2.3.2.

The decoupling of the exciter block is relatively easy to conceptualize: one has to simply consider the system at time t_0 when the exciter is completely dissattached from the string. At this time, the complete history of interaction is contaminated within the initial state of the string, and the string vibrates freely until the next excitation event. Because of this assumption, perceptually important initial transients of a plucked string tone are lost. Fortunately, there is a simple sound synthesis method and associated analysis method to recover these transients up to some extent, as will be described in Subsection 2.3.1.

The bridge is assumed the primary source of energy transfer from the strings to the rest of the body. No effort has been made for direct verification of this assumption in this thesis. The resonances of an isolated bridge are within the kHz range. Therefore, the string-bridge coupling effects primarily the dynamic behavior of the higher harmonics. Since higher harmonics have relatively fast decay rates, it can be concluded that the error introduced by this assumption would be most pronounced in the initial segment of a tone, where the higher harmonics have their maximum energy. Again, the synthesis and associated analysis method recover these dynamic behavior up to some extent, as will be described in Subsection 2.3.1.

The reflex enclosure model outlined in Subsection 2.2.3 is valid only up to 300 Hz. This model is a crude simplification of the coupled system of two variables, represented by the motion of the air in the soundhole and the first mode of the soundboard. In reality, the cavity modes are complex and coupled to all plate modes. Moreover, the external sound pressure interacts with both the soundboard and the cavity. These coupling effects are totally neglected in the simplified system illustrated in Fig. 2.2.

2.3 Model-based sound synthesis of plucked-string instruments

Model-based sound synthesis methods mimic the sound production mechanisms of traditional or virtual musical instruments with the aid of digital processors, preferably in real time. Real-time constraints are challenging ones, requiring some simplifications in sound synthesis models. The criteria, which give an optimal balance between model complexity and sound quality, are usually determined by taking the perception into account.

As discussed in Subsection 2.2, the dynamically coupled systems of the plucked string instruments are either governed by the linear or by the nonlinear wave equation, as given by Eqs. (2.2) and (2.11), respectively. The dynamical properties, together with the computational constraints mentioned above, favor the time-domain modeling techniques over their frequency-domain counterparts. There are the following two main approaches in time-domain model-based sound synthesis:

1. Numerical integration of governing Eq. (2.2) or Eq. (2.11).
2. Digital waveguide synthesis [1] that is based on the traveling wave solution of the linear wave equation, given by Eq. (2.4).

Numerical integration methods approximate the wave equation by replacing the partial derivatives either by finite differences [69], or mechanical elements like masses, springs, and dampers [70]. The digital waveguide (DWG) synthesis method, on the other hand, relies on the system description illustrated in Fig. 2.2, where the interactions between the subblocks take place at isolated spots so that the rest of the system can be efficiently modeled by bidirectional delay lines, i.e., digital waveguides [71].

Historically, the time-domain finite difference (FDTD) approximation provided the first physical modeling synthesis scheme [72, 73], whereas the first real-time model-based sound synthesis system has been the CORDIS system [74]. Due to its computational burden, the FDTD has been generally used for analysis and non-real time purposes[8] [76, 75]. The real-time applications have been reported so far just for simplistic cases [77].

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[8] A typical computing time for a xylophone sound lasting one second was 10^3 s on a

In terms of real-time processing, methods that are based on the traveling wave solution have been more successful. The first notes on the real-time implementation appeared in [78], and a very efficient implementation is presented by Smith in his doctoral thesis [12]. In the same year, by physical reformulation and extension of an abstract sound synthesis algorithm, plucked string instrument tones were synthesized in real-time [79]. Smith generalized these ideas to formulate the digital waveguide theory [71]. Ever since, the DWG synthesis became the most effective and widespread model-based sound synthesis method [1, 2, 3, 20, 80, 81].

2.3.1 DWG plucked string models

Smith developed efficient synthesis techniques for sound synthesis by mimicking the sound production systems $h(t)$ of various instrument families, including the plucked string system shown in Fig. 2.2 [1, 12, 80]. In his formulation a simplified plucked string system $h(t)$ is approximated by a digital system $h[n]$ consisting of the following subblocks. The traveling waves of each polarization in Eq. (2.4) are modeled by a pair of lossless delay lines, the damping in Eq. (2.7) is modeled by low-order digital filters, the admittance matrix of Eq. (2.16) and multiple string terminations (see Subsection 2.2.4) by *one-multiply scattering junctions* [82], and the body response (see Subsection 2.2.3) by a high-order digital filter.

In a typical real-time model-based sound synthesis task, a considerable amount of processing power is devoted to implementating high-order body filters. The *commuted waveguide synthesis* (CWS) technique that relies on the commutativity due to linearity and time invariance of $h(t)$ and $h[n]$ reduces the computational cost [83, 84]. In the CWS the ordering of the subsystems in Fig. 2.2 is changed conceptually, all the subsystems except the string $h_s(t)$ are pre-convolved with the force distribution $f(\xi, t)$, and the resulting aggregate excitation signal is fed into the string $h_s(t)$. In the model $h[n]$, the aggregate excitation signal $x[n]$ can be extracted from the measurements or recorded tones, and stored in a look-up table. Note that $x[n]$ contains partly the dynamical information lost due the crude assumptions outlined in Section 2.2.5. Moreover, it contains *frozen snapshots* of the neglected coupling effects.

In this canonic representation, the basic LTI string model $h_s[n]$ is composed of a delay line, a *fractional delay filter* $F(z)$ that fine-tunes the overall delay [27], and a one-pole *loop filter* $H_l(z)$ that accounts for the losses. This

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Sun-Sparc10 workstation in 1996 [75]. It takes today nearly 10^2 s on a desktop computer. The ever-increasing processing power will surely allow to run the detailed finite difference simulations in real-time in the near-term future.

basic string model is illustrated in Fig. 4 in [P2] and presented in Fig. 2.3 for convenience. The associated parameter vector P_N with the basic string model in Fig. 2.3 has only seven elements consisting of four coefficients of $F(z)$, two coefficients of $H_\mathrm{l}(z)$, and the integer delay line length L.

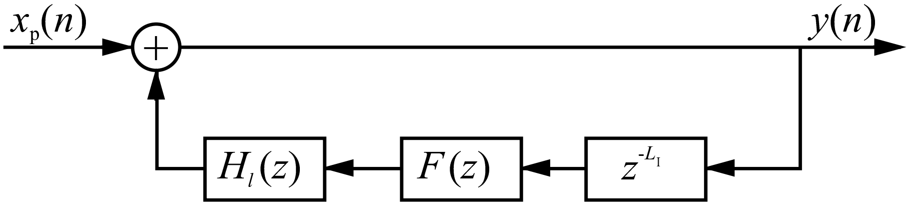

Figure 2.3: The basic string model after [79, 85].

In [20], a generic plucked string instrument model that is based on the basic string model $h_s[n]$ has been described. The generic model is illustrated in Fig. 5 in [P2], and presented in Fig. 2.4 for convenience. This generic string model has been used in [P2], [P6], and [P7] for real-time sound synthesis of various plucked string instruments. In the model, an aggregate excitation signal $x[n]$ has been stored in a wavetable. The *pluck-shaping* filter $E(z)$ and *pluck-position* filter $P(z)$ shape the initial eigenfunctions (see Eq. (2.6)). One basic string model $h_s[n]$ of Fig. 2.3 is used for each transversal polarization, and their transfer functions are indicated in Fig. 2.4 as $S_\mathrm{h}(z)$ and $S_\mathrm{v}(z)$, corresponding to horizontal and vertical polarizations, respectively. The coupling coefficient g_c is the gain of an inherently stable energy transfer between the transversal polarizations, and can be related to the admittance matrix discussed in Subsection 2.2.2 around a predefined frequency ω_1 as

$$g_c \approx \sqrt{|Y_{2,3}(\omega_1)Y_{3,2}(\omega_1)|} \tag{2.17}$$

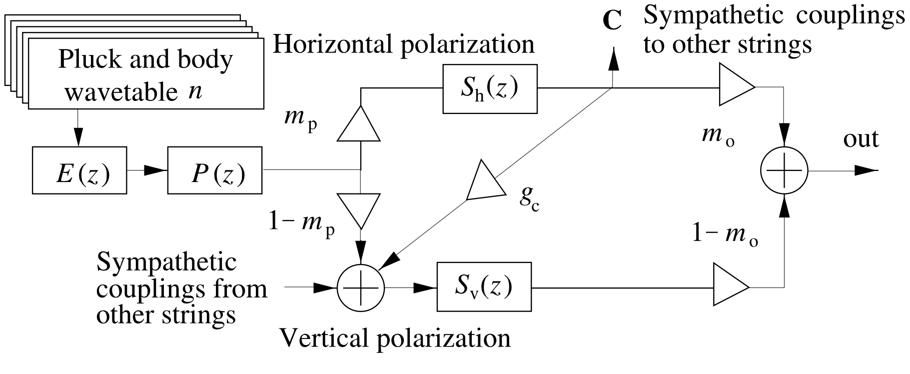

Figure 2.4: The generic plucked string instrument model after [20].

The matrix C simulates the sympathetic vibrations, and its elements are real-valued scalars. The generic instrument model gives realistic replicas of a broad range of plucked string instruments only if the model is calibrated with a good accuracy. The calibration of the DWG models with the main emphasis of the basic string model and the generic model discussed here is the topic of the next subsection.

2.3.2 Calibration of DWG plucked string models

A choice for extracting the parameters $P_N[n]$ of a synthesis model $h[n]$ is based on the use of the physical measurements on the system $h(t)$ [12]. Then, a synthesized tone $y[n]$ is compared with a recorded tone $y_d[n]$, and the approximation accuracy of the model $h[n]$ is evaluated[9]. Since a recorded tone $y_d[n]$ has to be stored for comparison in any case, it is desirable to reduce the need for the physical measurement data and to extract the parameters $P_N[n]$ directly from $y_d[n]$.

This aspect has been systematically explored within the last decade. Laroche and his colleagues utilized deconvolution-based techniques for obtaining the model parameters $P_N[n]$ and excitation signals $x[n]$ of plucked string tones [86] and a broader class of exponentially decaying tones [87]. A similar inverse-filtering approach has been used in [85] to extract the aggregate excitation signal $x[n]$ for resynthesis using the CWS method. Using inverse filtering, any detailed mechanism or interaction that a simple model cannot account for is represented within the excitation signal $x[n]$ as frozen snapshots of the actual dynamic behaviour manifested in recorded tones.

The basic string model parameters (see Fig. 2.3) have been obtained by extracting the fundamental frequency and the decay characteristics of the partials by the pitch-synchronous Short-time Fourier Transform (STFT), whereas in [88] a heterodyne filtering has been used for the partial tracking. Tolonen decomposed $y_d[n]$ into its deterministic and residual components, as described in [89], and obtained $x[n]$ by adding the residual with the inverse-filtered and equalized deterministic signal [90, 88]. This system is illustrated in Fig. 2 of [P5], and explained in detail in [P5]. [P5] and [P6] report extension to this calibration system, and the extended system is used for extracting the model parameters in [P2], [P4], [P6], and [P7].

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[9] Usually by informal listening tests.

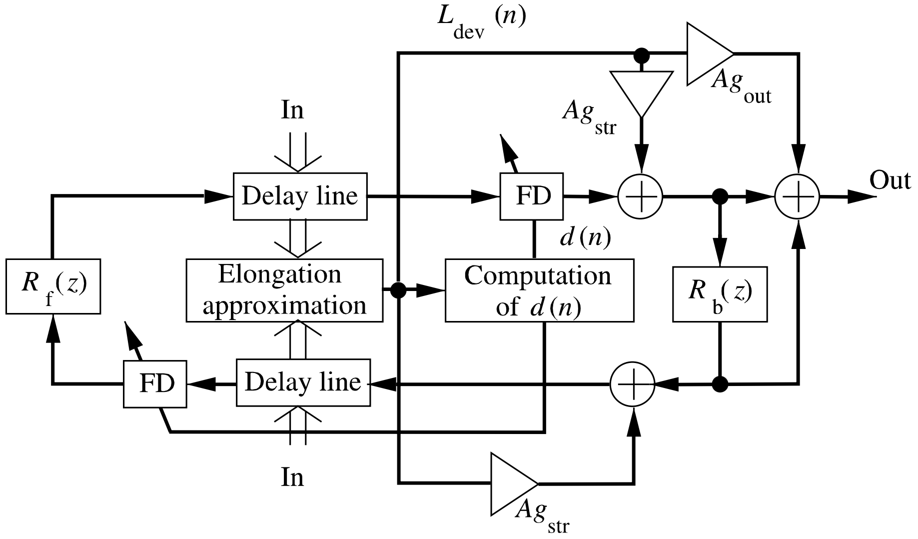

Figure 2.5: Dual-delay-line waveguide model of a string including tension modulation driving force output and coupling. After [56].

2.3.3 Nonlinear plucked string synthesis models

The first nonlinear plucked string synthesis model was an FDTD model presented by Chaigne [91]. The model discretized the nonlinear wave equation given by Eq. (2.11). It also accounted for the longitudinal wave propagation.

An early example of a nonlinear DWG model is the kantele string model [92]. This model first calculates the elongation of the string according to Eq. (2.12) and, then, the TMDF according to Eq. (2.15). The TMDF is filtered by an approximation of the measured $Y_{1,1}(\omega)$ and summed with transversal forces in order to obtain the model output signal $y[n]$. However, the nonlinear transversal wave propagation given by Eq. (2.14) is not implemented in the model. Other types of nonlinearities caused by the nonlinear boundary conditions have been presented in [93] and in [94].

Recently, linear DWG plucked string models have been extended to simulate the nonlinear traveling waves as given by Eq. (2.14) [43]. The extension is based first on the calculation or approximation of the elongation of Eq. (2.12) and, then, on the simulation of the instantaneous transversal velocity $c_t(t)$ by time-varying fractional delay filters. The simulation of the TMDF of Eq. (2.15) is eventually included in the model described in [56]. The complete model that accounts for both the TMDF and the transversal nonlinear wave propagation is shown in Fig. 6 in [P2], and presented here in Fig. for convenience.[10] This generic nonlinear string model has been shown to simulate many essential properties of nonlinear string vibrations as described in [P2] and [P3].

2.3.4 FDTD models

The FDTD is a simple yet powerful technique to numerically integrate all types of differential equations in the time domain [95]. The mathematical properties of the difference operators, stability and accuracy analysis of the numerical schemes, and the effect of the boundary operators have been studied in detail in [96, 95]. FDTD-based simulations have been reported to tackle challenging problems in electromagnetics [97] and in acoustics [98].

Chaigne systematically investigated FDTD simulations for analysis and sound synthesis purposes in musical acoustics. The framework and the synthesis aspects of his research were reported in [91, 99, 100]. In [69], the FDTD simulations of plucked string instruments have been studied extensively. Starting from the ideal homogeneous linear wave equation, the effects of the losses, dispersion, string-bridge interaction, the low-frequency guitar function, and body function have been investigated both numerically and physically. The study covers all the important acoustical properties of a plucked instrument outlined in Subsection 2.2, and extends the earlier work reported in [72, 73]. The FDTD techniques were then extended to analyze struck strings of piano [101, 102], vibrating bars and resonators of a xylophone [76, 75], and damped impacted plates [103, 104].

The FDTD simulations in many cases demand a considerable computational power. Usually, two or three dimensional FDTD simulations cannot be run in real-time. This fact is one of the motivations behind the research on *digital waveguide meshes*, where the DWG theory has been extended to higher dimensions [105, 106, 107]. Like in the one-dimensional strings, FDTD and digital waveguide formulations are equivalent for the lossless ideal case.

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[10] The physical wave variable conversions (slope-to-force, and vice versa) have not been explicitly shown in the model. For a more details, see Fig. 14 in [P3].

3. Contributions of this thesis

3.1 Contributions to musical acoustics

The acoustics of the acoustical guitar is a widely studied topic [14, 30]. However, some historical and traditional string instruments exhibit important and unique acoustical characteristics so that they deserve a special attention.

3.1.1 Acoustics of traditional plucked string instruments

So far, only a few studies on the acoustical properties of traditional and historical plucked string instruments have been reported. With the exception of Indian plucked string instruments, the reported studies have been experimental in nature. In Indian plucked string instruments (e.g. sitar, tanpura, and vina) nonlinearities created by unilateral boundary conditions have been subject to mathematical analysis and time-domain numerical modeling [108, 109, 36]. In these studies, the numerical methods have been used to validate the physical and mathematical models.

Firth reported some acoustical measurements on the lute in [110]. A Chinese plucked string instrument, the P'i P'a, has also been subject to a similar experimental study [111]. More recently, the holographic interferometry measurements were conducted on the mandolin [112], the Russian balalaika [113] and a class of Baltic psalteries [63]. With the exception of [63], zithers in general, and psalteries in particular do not appear to have been a subject of acoustical studies [14].

Table 3.1 summarizes the lowest body resonances of the plucked string instruments analyzed in this thesis. In this table, Kantele 1 refers to the five string kantele without the sound hole, and Kantele 2 is the traditional kantele with a sound hole. The measurements were conducted in an anechoic chamber.

The guitar and the lute results can be compared to other studies reported in the literature to verify the measurement accuracy. It is known that the lowest body resonances of the guitar are monopoles around 102 and 204 Hz [14], which are confirmed by the measurements presented in this thesis. In his study of the lute, Firth identified the resonance around 132 Hz as the *Helmholtz air mode* and the resonance around 304 Hz as the resonance of the top plate. His results have also been discussed in [14]. The measured lute in [P7] was smaller in size compared to Firth's lute, and its resonances are found around 144 Hz and 356 Hz. The other results concerning the kantele, the ud and the tanbur were obtained by the same measurement set-up and analysis tools as the guitar and the lute.

Instrument	f_{B1} (Hz)	Q_1	f_{B2} (Hz)	Q_2
Guitar	100	15.06	204	12.90
Lute	144	10.63	356	8.97
Ud	113	9.91	182	10.06
Kantele 1	457	7.24	817	5.01
Kantele 2	522	8.62	732	7.43
Tanbur	191	10.27	344	11.21

Table 3.1: The lowest body resonances of the plucked string instruments analyzed in this thesis.

3.1.2 Nonlinear string vibrations and TMDF

The TMDF effects even the lowest harmonics, thus the timbre of the kantele and the tanbur, as demonstrated in [P3] and [P1], respectively. The synchronous multi-channel measurement method presented in [P1] allowed a detailed analysis of the TMDF for the tanbur, and a similar measurement method has been used in the analysis of the kantele in [P3]. The physical mechanism of the TMDF has been fully studied in [P3] and it has been shown to explain the nonlinear effects observed in a broader class of plucked string instruments [59]. An analysis method that is based on standard DSP operations such as filtering and dilation has been developed. This method can also be used in extracting the parameters of the nonlinear string model from the recorded tones. Extraction of model parameters are further discussed in Subsection 3.2.1.

The simulation of TMDF has been carried out using a generic nonlinear string model, and the simulation results have been compared to actual measurements. In this respect, a sound source model has been used as a numerical method besides its primary function of sound synthesis. A similar approach has been reported recently in [114].

3.2 Contributions to model-based sound synthesis

The techniques described in this thesis have been driven by the practical needs of the integrated system described in [P6]. The system is controlled by a high-level musical representation, i.e., by the expressive notation. The low-level DSP control of the system is achieved by predefined rules about parameters and their trajectories. Therefore, a robust and reliable parameter estimation technique, and analysis of performance characteristics have been of paramount importance in this research. In this respect, the algorithms developed in this thesis can be considered as object-based analysis tools that fulfill specific needs in model-based sound synthesis of plucked string instruments.

3.2.1 Object-based analysis tools

In this thesis, the calibration process for a classical guitar synthesizer has been reported, and the extraction of physical and expressive parameters are discussed separately. They roughly correspond to the future extensions and directions outlined in [20]. Especially, the iterative overall-decay matching block presented in [P5] improves the quality of the synthesized sounds, as informal listening tests reveal.

A useful object-based analysis tool is the vibrato analysis developed in [P4] and then applied to the beating analysis in [P3]. Based on the nonlinear least-squares method, the technique has been tested on different plucked-string instruments and proved to be robust and reliable.

Parameter extraction of the nonlinear string models has usually been carried out by analyzing the linear and nonlinear parts of the model separately [115]. In practice, however, the nonlinearities effect the linear parameters of the model and introduce an additional damping or coupling. There is no technique available to extract the coupling gains in the generic nonlinear string model, although the TMDF analysis method reported in [P3] could potentially be facilitated for this purpose.

3.3 Future work

Beside the traditional and historical plucked string instruments presented in this thesis, there are many others yet to be studied. An acoustical study may be combined with the model-based sound synthesis in order to verify the acoustical characteristics of the instrument in focus.

Although the expressive parameter extraction methods presented in this thesis try to span a considerable area of the performance possibilities during measurements and analysis, there is still a vast amount of performance characteristics left to be explored. Ideally, these characteristics should be extracted from recordings rather than isolated experiments.

Similarly, physical parameter extraction techniques need to be extended. Promising results about plucking point estimation and parameter extraction of coupled strings have been reported in [116] and [117], respectively. Specifically, parametric techniques, as well as wavelet-based methods can be utilized for model calibration. Incorporation and further elaboration of these techniques are left as future work.

The results of perceptual research can potentially have a huge impact on the model-based sound synthesis [25, 26]. The use of these results is also a challenging future task. Another field that is considered outside of the scope of this thesis is to simulate the directivity of the virtual instruments. Motivated by the immersive and virtual reality applications, the directivity research and modeling is expected to be probably one of the major challenging problems in model-based sound synthesis in the next decade.

4. Conclusions

This chapter summarizes the main results of the publications and clarifies the contribution of the author to each of them. It should be noted that many initial ideas and developments presented in this work were originated from a fruitful team work with Prof. Matti Karjalainen, Prof. Vesa Välimäki, Dr. Tero Tolonen, and Dr. Mikael Laurson.

4.1 Main results of this thesis

- Acoustical analysis of the tanbur and the kantele are carried out in [P1] and [P3], respectively. The nonlinearities and effects of the TMDF are also demonstrated.[P3] is one of the few studies about the acoustical properties of the kantele, and [P1] is the first study of the tanbur, to the author's knowledge.
- The TMDF formulation is extended and related to broader family of plucked string instruments in [P3].
- Publications [P4], [P5], and [P6] describe a chronologic progress report about the development, calibration, and implementation of an advanced model-based sound synthesis system. The parameter extraction methods presented in [P5] are used to calibrate the synthesis engine (PWSynth) of the system.
- Acoustical properties of the lute and the ud are analyzed in [P7], treating them as acoustical systems similar to the acoustical guitar. Comparison with the lute results presented in [110] justifies the accuracy of the measurements. This paper presents the first systematic analysis of the ud, to the author's knowledge.
- The computationally demanding FDTD is re-evaluated with a DSP-oriented approach for real-time sound synthesis purposes in [P9]. A

sufficient condition for the numerical stability of the model is obtained.

- The traveling slope waves are obtained in [P9]. This indicates the possibility of interaction between different types of model-based sound synthesis structures based on a dual-wave formulation.

4.2 Contribution of the author

[P1]

With the exception of Section 4, the author wrote the article, carried out the relevant measurements and analyzed the measurement data. He contributed to Section 4 by collaborating with Dr. Tero Tolonen in measurements. The companion webpage is prepared by the author.

[P2]

The author implemented two generic models presented in the paper for model-based sound synthesis of tanbur and extracted the model parameters from the recorded tones and other measurements. The article was written and the companion webpage was prepared by the author. Dr. Tero Tolonen's help in computation and description of the nonlinear model is highly acknowledged.

[P3]

The author wrote Section 3 and Section 5, and combined the other sections written by co-authors. The measurements reported in the article were carried out in collaboration with Ms. Patty Huang. A novel analysis method and the physical TMDF formulation are the contributions of the author. In addition to analysis, the author calculated the synthesis examples reported in Section 4 and prepared the companion webpage.

[P4]

The author wrote Section 4, and performed the measurements and analysis discussed in the section. In particular, he produced the vibrato analysis and resynthesis example. This paper initiated the follow-up research that ended up by the integrated system presented in [P6].

[P5]

The author wrote the article, performed the measurements, extended the methods for extraction of the physical parameters, and developed techniques for expressive parameter extraction. The companion webpage was prepared by the author, where most of the sound examples were kindly provided by Dr. Mikael Laurson using the techniques described in the paper.

[P6]

The contribution of the author in this article is in Section 2, Section 5, and Section 6. Section 2 describes the calibration of the synthesizer, and it was written by the author. He summarized the extraction of the model parameters from recorded tones using the system described in [P5] in detail, and presented an improvement in pluck-shaping filter design over the deconvolution-based method presented in [P5]. Section 5 is a collaboration between the author and Dr. Mikael Laurson. The author described the physical mechanisms of different playing styles and drew general implementation guidelines, and Dr. Laurson described how the plucking styles are actually implemented in the present system. Section 6 is a conclusion section and the author indicated some future directions of the research.

[P7]

The author wrote the article except Section 4. He conducted the measurements, extracted the model parameters and body modes from recorded tones, and experimented with higher-order loop filter design and simulation of glissandi.

[P8]

The author was responsible for this research.

[P9]

The author wrote the article. The article presents an analysis of mathematical and physical aspects of a synthesis method suggested by Prof. Matti Karjalainen [118], and the author performed the analysis.

Bibliography

[1] J. O. Smith, "Physical modeling using digital waveguides," *Computer Music J.*, vol. 16, no. 4, pp. 74–91, 1992.

[2] J. O. Smith, "Physical modeling synthesis update," *Computer Music J.*, vol. 20, no. 2, pp. 44–56, 1996.

[3] M. Karjalainen, T. Tolonen, V. Välimäki, C. Erkut, M. Laurson, and J. Hiipakka, "An overview of new techniques and effects in model-based sound synthesis," *Journal of New Music Research*, vol. 30, no. 3, pp. 203–212, 2001.

[4] M. Kasha, *Scientific Development of a New Classical Guitar.* Florida, USA: Tallahassee, 1971.

[5] T. D. Rossing, R. F. Moore, and P. Wheeler, *Science of Sound.* Addison Wesley Longman, Inc., 3rd ed., 2001.

[6] C. M. Hutchins, "The physics of violins," *Sci. Am.*, vol. 207, no. 5, p. 78, 1962.

[7] L. Cremer, *The Physics of the Violin.* Cambridge, Massachusetts, USA: Massachusetts Institute of Technology, 1984. Traslated by J. S. Allen from *Physics der Geige.* 1981. S. Hirzel Verlag, Stuttgart, Germany.

[8] H. Meinel, "Regarding the sound quality of violins and a scientific basis for violin construction," *J. Acoust. Soc. Am.*, vol. 29, p. 817, 1957.

[9] C. M. Hutchins, "Founding a family of fiddles," *Physics Today*, vol. 20, no. 2, p. 23, 1967.

[10] J. C. Schelleng, "The violin as a circuit," *J. Acoust. Soc. Am.*, vol. 35, p. 326, 1963.

[11] M. V. Mathews and J. Kohut, "Electronic simulation of violin resonances," *J. Acoust. Soc. Am.*, vol. 53, Feb. 1973.

[12] J. O. Smith, *Techniques for digital filter design and system identification with application to the violin.* PhD thesis, Stanford University, California, USA, June 1983.

[13] D. Trueman and P. R. Cook, "BoSSA: The deconstructed violin reconstructed," *Journal of New Music Research*, vol. 29, no. 2, pp. 121–130, 2000. See also `http://silvertone.princeton.edu/~dan/BoSSA/`.

[14] N. H. Fletcher and T. D. Rossing, *The Physics of Musical Instruments.* New York, USA: Springer-Verlag, 2nd ed., 1998.

[15] C. Chafe, "Simulating performance on a bowed instrument," in *Current Directions in Computer Music Research* (M. V. Mathews and J. R. Pierce, eds.), vol. 2 of *System Development Foundation Benchmark Series*, ch. 15, pp. 185–198, Cambridge, MA: MIT Press, 1989.

[16] P. R. Cook, "A hierarchical system for controlling synthesis by physical modeling," in *Proc. Int. Computer Music Conf.*, (Banff, Canada), pp. 108–109, International Computer Music Association, Sept. 1995.

[17] P. R. Cook, "Synthesis toolkit in c++," in *SIGGRAPH 1996*, 1996. For an updated version of STK see `http://ccrma-www.stanford.edu/software/stk/`.

[18] B. L. Vercoe, W. G. Gardner, and E. D. Scheirer, "Structured audio: Creation, transmission, and rendering of parametric sound representations," *Proc. IEEE*, vol. 86, pp. 922–940, May 1998.

[19] "ISO/IEC International Standard 14496-3 Information technology–Coding of audiovisual objects," 1999.

[20] M. Karjalainen, V. Välimäki, and T. Tolonen, "Plucked-string models: From the Karplus-Strong algorithm to digital waveguides and beyond," *Computer Music J.*, vol. 22, no. 3, pp. 17–32, 1998.

[21] G. Weinreich, "Vibration and radiation of structures with application to string and percussion instruments," in *Mechanics of Musical Instruments* (A. Hirschberg, J. Kergomard, and G. Weinreich, eds.), pp. 79–114, Wien - New York: Springer, 1995.

[22] J. Huopaniemi, *Virtual Acoustics and 3-D Sound in Multimedia Signal Processing.* PhD thesis, Helsinki University of Technology, Espoo, Finland, 1999. Available online at `http://www.acoustics.hut.fi/publications/theses.html`.

[23] P. R. Cook and D. Trueman, "NBody: Interactive multidirectional musical instrument body radiation simulators, and a database of measured impulse responses," in *Proc. Int. Computer Music Conf.*, (Ann Arbor, Michigan, USA), pp. 353–356, International Computer Music Association, 1998.

[24] T. Tolonen, *Object-based sound source modeling.* PhD thesis, Helsinki University of Technology, Espoo, Finland, Oct. 2000. Available online at `http://lib.hut.fi/Diss/2000/isbn9512251965/`.

[25] H. Järveläinen and T. Tolonen, "Perceptual tolerances for decay parameters in plucked string synthesis," *J. Audio Eng. Soc.*, vol. 49, pp. 1049–1059, Nov. 2001.

[26] H. Järveläinen, V. Välimäki, and M. Karjalainen, "Audibility of the timbral effects of inharmonicity in stringed instrument tones," *Acoustics Research Letters Online*, vol. 2, pp. 79–84, July 2001. See also `http://www.acoustics.hut.fi/publications/`.

[27] T. I. Laakso, V. Välimäki, M. Karjalainen, and U. K. Laine, "Splitting the unit delay–Tools for fractional delay filter design," *IEEE Signal Processing Magazine*, vol. 13, pp. 30–60, Jan. 1996.

[28] F. Jahnel, *Manual of Guitar Technology.* Frankfurt am Main, Germany: Verlag Das Musikinstrument, 1981.

[29] S. Sadie, ed., *The New Grove Dictionary of Musical Instruments.* London, UK: MacMillan Press, 1984.

[30] B. E. Richardson, "String instruments: Plucked," in *Encyclopedia of Acoustics* (M. J. Crocker, ed.), vol. 4, pp. 1627–1634, John Wiley & Sons, Inc., 1997.

[31] A. Chaigne, "Numerical simulations of stringed instruments – today's situation and trends for the future," *Catgut Acoustical Society Journal*, vol. 4, pp. 12–20, May 2002.

[32] W. C. Elmore and M. A. Heald, *Physics of Waves.* New York, USA: Dover, 1969.

[33] J. l. d'Alembert, "Investigation of the curve formed by a vibrating string (1747)," in *Acoustics: Historical and Philosophical Development* (R. B. Lindsay, ed.), pp. 119–123, Dowden, Hutchinson & Ross, 1973.

[34] P. M. Morse, *Vibration and Sound.* American Institute of Physics for the Acoustical Society of America, 1976. (1^{st} ed. 1936, 2^{nd} ed. 1948).

[35] P. M. Morse and U. K. Ingard, *Theoretical Acoustics.* Princeton, New Jersey, USA: Princeton University Press, 1968.

[36] C. Valette, "The mechanics of vibrating strings," in *Mechanics of Musical Instruments* (A. Hirschberg, J. Kergomard, and G. Weinreich, eds.), pp. 116–183, Wien - New York: Springer, 1995.

[37] A. J. Chaigne, "Spectral distribution and damping factors measurements of musical strings using FFT techniques," in *Proc. ICASSP*, 1987.

[38] A. Chaigne, "Viscoelastic properties of nylon guitar strings," *Catgut Acoustical Society Journal*, vol. 1, pp. 21–17, May 1991.

[39] A. Moral and E. Jansson, "Eigenmodes, input admittance and the function of the violin," *Acustica*, vol. 50, pp. 329 – 337, May 1982.

[40] E. Jansson, I. Bork, and J. Meyer, "Investigations into the acoustical properties of the violin," *Acustica*, vol. 62, pp. 1–15, Nov. 1986.

[41] G. F. Carrier, "On the non-linear vibration problem of the elastic string," *Quarterly of Applied Mathematics*, vol. 3, pp. 157–165, 1945.

[42] H. Harrison, "Plane and circular motion of a string," *J. Acoust. Soc. Am.*, vol. 20, pp. 874–875, Nov. 1948.

[43] T. Tolonen, V. Välimäki, and M. Karjalainen, "Modeling of tension modulation nonlinearity in plucked strings," *IEEE Trans. Speech and Audio Processing*, vol. 8, pp. 300–310, May 2000.

[44] J. W. Miles, "Stability of forced oscillations of a vibrating string," *J. Acoust. Soc. Am.*, vol. 38, pp. 855–865, 1965.

[45] G. V. Anand, "Large-amplitude damped free vibration of a stretched string," *J. Acoust. Soc. Am.*, vol. 45, no. 5, pp. 1089–1096, 1969.

[46] J. W. Miles, "Resonant, nonplanar motion of a streched string," *J. Acoust. Soc. Am.*, vol. 75, pp. 1505–1510, 1984.

[47] C. E. Gough, "The nonlinear free vibration of a damped elastic string," *J. Acoust. Soc. Am.*, vol. 75, pp. 1770–1776, 1984.

[48] K. A. Legge and N. H. Fletcher, "Nonlinear generation of missing modes on a vibrating string," *J. Acoust. Soc. Am.*, vol. 76, pp. 5–12, July 1984.

[49] Z. C. Feng, "Does nonlinear intermodal coupling occur in a vibrating stretched string?," *J. Sound and Vibration*, vol. 182, no. 5, pp. 809–812, 1995.

[50] N. Tufillaro, "Nonlinear and chaotic string vibrations," *American Journal of Physics*, vol. 30, no. 8, pp. 408 – 414, 1989.

[51] R. J. Hanson, J. M. Anderson, and H. K. Macomber, "Measurements of nonlinear effects in a driven vibrating wire," *J. Acoust. Soc. Am.*, vol. 96, pp. 1549–1556, Sept. 1994. See also `http://www.physics.uni.edu/hanson/Hanson.html`.

[52] A. Watzky, "Non-linear three dimensional large-amplitude damped free vibration of a stiff elastic streched string," *Journal of Sound and Vibration*, vol. 153, pp. 125–142, feb 1992.

[53] R. Narasimha, "Non-linear vibration of an elastic string," *Journal of Sound and Vibration*, vol. 8, no. 1, pp. 134–146, 1968.

[54] G. P. Menzala, "On classical solutions of a quasilinear hyperbolic equation," *Nonlinear Analysis, Theory, Methods & Applications*, vol. 3, no. 5, pp. 613–627, 1979.

[55] R. R. Boullosa, "Admittance measurements in the neck of a classical guitar," in *Proc. Int. Symposium on Musical Acoustics (ISMA)*, vol. 2, (Perugia, Italy), pp. 425–429, Sept. 2001.

[56] T. Tolonen, C. Erkut, V. Välimäki, and M. Karjalainen, "Simulation of plucked strings exhibiting tension modulation driving force," in *Proc. Int. Computer Music Conf.*, (Beijing, China), pp. 5–8, Oct. 1999.

[57] X. Boutillon and G. Weinreich, "Three-dimensional mechanical admittance: Theory and new measurement method applied to the violin bridge," *J. Acoust. Soc. Am.*, vol. 105, pp. 3524–3533, July 1999.

[58] G. Weinreich, "Coupled piano strings," *J. Acoust. Soc. Am.*, vol. 62, pp. 1474–1484, Dec. 1977.

[59] H. A. Conklin, "Generation of partials due to nonlinear mixing in a stringed instrument," *J. Acoust. Soc. Am.*, vol. 105, pp. 536–545, Jan. 1999.

[60] C. Besnainou, V. Gibiat, J. Frelat, and J. Curtin, "The sound qualities of string instruments: A new approach by body's nonlinearities," in *Proc. Int. Symposium on Musical Acoustics (ISMA)*, vol. 1, (Perugia, Italy), pp. 161–164, Sept. 2001.

[61] G. Caldersmith, "Guitar as a reflex enclosure," *J. Acoust. Soc. Am.*, vol. 63, pp. 1566–1575, May 1978.

[62] O. Christensen and B. B. Vistisen, "Simple model for low-frequency guitar function," *J. Acoust. Soc. Am.*, vol. 68, pp. 758–766, Sept. 1980.

[63] A. Peekna and T. D. Rossing, "The acoustics of Baltic psaltery," in *Proc. Int. Symposium on Musical Acoustics (ISMA)*, vol. 2, (Perugia, Italy), pp. 437–442, Sept. 2001.

[64] K. D. Marshall, "Modal analysis of a violin," *J. Acoust. Soc. Am.*, vol. 77, pp. 695–709, Feb. 1985.

[65] D. J. Ewins, *Modal Testing: Theory and practice.* 58B Station Road, Letchword, Herts. SG6 3BE, England: Research Studies Press Ltd., 3rd ed., 1986.

[66] M. J. Elejabarrieta, A. Ezcurra, and C. Santamaria, "Evolution of the vibrational behavior of a guitar soundboard along successive construction phases by means of the modal analysis technique," *J. Acoust. Soc. Am.*, vol. 108, pp. 369–378, July 2000.

[67] T. J. W. Hill, B. E. Richardson, and S. J. Richardson, "Measurements of acoustical parameters for the classical guitar," in *Proc. Int. Symposium on Musical Acoustics (ISMA)*, vol. 2, (Perugia, Italy), pp. 417–420, Sept. 2001.

[68] C. E. Gough, "The theory of string resonances on musical instruments," *Acustica*, vol. 49, pp. 124–141, Oct. 1981.

[69] A. Chaigne, "On the use of finite differences for musical synthesis. Application to plucked stringed instruments," *Journal d'Acoustique*, vol. 5, no. 2, pp. 181–211, 1992.

[70] C. Cadoz, A. Luciani, and J. Florens, "Responsive input devices and sound synthesis by simulation of instrumental mechanisms: The CORDIS system," *Computer Music J.*, vol. 8, no. 3, pp. 60–73, 1983.

[71] J. O. Smith, “Music applications of digital waveguides,” Tech. Rep. STAN-M-39, CCRMA, Dept. of Music, Stanford University, California, USA, May 1987. 181 p.

[72] L. Hiller and P. Ruiz, “Synthesizing musical sounds by solving the wave equation for vibrating objects: Part 1,” *J. Audio Eng. Soc.*, vol. 19, pp. 462–470, June 1971.

[73] L. Hiller and P. Ruiz, “Synthesizing musical sounds by solving the wave equation for vibrating objects: part 2,” *J. Audio Eng. Soc.*, vol. 19, no. 7, pp. 542–550, 1971.

[74] J.-L. Florens and C. Cadoz, “The physical model: Modeling and simulating the instrumental universe,” in *Representations of Musical Signals* (G. D. Poli, A. Piccialli, and C. Roads, eds.), pp. 227–268, Cambridge, Massachusetts, USA: The MIT Press, 1991.

[75] V. Doutaut, D. Matignon, and A. Chaigne, “Numerical simulations of xylophones. II. Time-domain modeling of the resonator and radiated sound pressure,” *J. Acoust. Soc. Am.*, vol. 104, pp. 1633–1647, Sept. 1998.

[76] A. Chaigne and V. Doutaut, “Numerical simulations of xylophones. I. Time-domain modeling of the vibrating bars,” *J. Acoust. Soc. Am.*, vol. 101, pp. 539–557, Jan. 1997.

[77] M. Kurz, “Klangsynthese mittels physicalisher Modellierung einer schwingenden Saite durch numerische Integration der Differentialgleichung,” Master’s thesis, Technical University Berlin, 1995.

[78] M. E. McIntyre, R. T. Schumacher, and J. Woodhouse, “On the oscillations of musical instruments,” *J. Acoust. Soc. Am.*, vol. 74, pp. 1325–1345, Nov. 1983.

[79] D. A. Jaffe and J. O. Smith, “Extensions of the Karplus-Strong plucked-string algorithm,” *Computer Music J.*, vol. 7, no. 2, pp. 56–69, 1983. Also published in Roads C. (ed). 1989. *The Music Machine*, pp. 481–494. The MIT Press. Cambridge, Massachusetts, USA.

[80] J. O. Smith, “Principles of digital waveguide models of musical instruments,” in *Applications of Digital Signal Processing to Audio and Acoustics* (M. Kahrs and K. Brandenburg, eds.), pp. 417–466, Boston, Massachusetts, USA: Kluwer Academic Publishers, 1998.

[81] T. Tolonen, V. Välimäki, and M. Karjalainen, “Evaluation of modern sound synthesis methods,” Tech. Rep. 48, Helsinki University of Technology, Laboratory of Acoustics and Audio Signal Processing, Espoo, Finland, Mar. 1998.

[82] J. D. Markel and A. H. Gray, *Linear Prediction of Speech.* Springer Verlag, New York, 1976.

[83] J. O. Smith, “Efficient synthesis of stringed musical instruments,” in *Proc. Int. Computer Music Conf.*, (Tokyo, Japan), pp. 64–71, Sept. 1993.

[84] M. Karjalainen, V. Välimäki, and Z. Jánosy, “Towards high-quality sound synthesis of the guitar and string instruments,” in *Proc. Int. Computer Music Conf.*, (Tokyo, Japan), pp. 56–63, Sept. 1993.

[85] V. Välimäki, J. Huopaniemi, M. Karjalainen, and Z. Jánosy, “Physical modeling of plucked string instruments with application to real-time sound synthesis,” *J. Audio Eng. Soc.*, vol. 44, pp. 331–353, May 1996.

[86] J. Laroche and J.-M. Jot, “Analysis/synthesis of quasi-harmonic sound by use of the Karplus-Strong algorithm,” in *Proceedings of the 2*$^{\text{nd}}$ *French Congress on Acoustics*, (Archachon, France), 1992.

[87] J. Laroche and J.-L. Meillier, “Multichannel excitation/filter modeling of percussive sounds with application to the piano,” *IEEE Trans. Speech and Audio Processing*, vol. 2, pp. 329–344, Apr. 1994.

[88] V. Välimäki and T. Tolonen, “Development and calibration of a guitar synthesizer,” *J. Audio Eng. Soc.*, vol. 46, pp. 766–778, Sept. 1998.

[89] X. Serra and J. O. Smith, “Spectral modeling synthesis: a sound analysis/synthesis system based on a deterministic plus stochastic decomposition,” *Computer Music J.*, vol. 14, no. 4, pp. 12–24, 1990.

[90] T. Tolonen, “Model-based analysis and resynthesis of acoustic guitar tones,” Master’s thesis, Helsinki University of Technology, Espoo, Finland, Jan. 1998. See also `http://www.acoustics.hut.fi/publications/`.

[91] A. Chaigne, A. Ramdane, and A. Watzky, “Computer synthesis of string-like sounds by means of nonlinear physical models,” in *Proc. 13th International Cong. Acoustics*, pp. 117–120, 1989.

[92] M. Karjalainen, J. Backman, and J. Pölkki, "Analysis, modeling, and real-time sound synthesis of the kantele, a traditional Finnish string instrument," in *Proc. ICASSP*, vol. 1, (Minneapolis, Minnesota, USA), pp. 125–128, Apr. 1993.

[93] J. R. Pierce and S. A. Van Duyne, "A passive nonlinear digital filter design which facilitates physics-based sound synthesis of highly nonlinear musical instruments," *J. Acoust. Soc. Am.*, vol. 101, pp. 1120–1126, Feb. 1997.

[94] E. Rank and G. Kubin, "A waveguide model for slapbass synthesis," in *Proc. ICASSP*, vol. 1, (Munich, Germany), pp. 443–446, The Institute of Electrical and Electronics Engineers, IEEE Computer Society Press, 1997.

[95] J. C. Strikwerda, *Finite difference schemes and partial differential equations*. California, US: Wadsworth, Brooks & Cole, 1989.

[96] F. B. Hildebrand, *Finite-difference equations and simulations*. Prentice-Hall, Englewood Cliffs, NJ, USA, 1968.

[97] A. Taflove, *Computational electrodynamics: The finite-difference time-domain method*. Artech House, Boston, MA, USA, 1995.

[98] D. Botteldooren, "FDTD-simulation of low frequency room acoustic problems," *J. Acoust. Soc. Am.*, vol. 98, pp. 3302–3308, June 1995.

[99] A. Chaigne, A. Askenfelt, and E. V. Jansson, "Temporal synthesis of string instrument tones," in *Quarterly Progress and Status Report*, no. 4, pp. 81–100, Stockholm, Sweden: Speech Transmission Laboratory, Royal Institute of Technology (KTH), 1990.

[100] A. Chaigne, A. Askenfeld, and E. Jansson, "Time domain simulations of string instruments. a link between physical modeling and musical perception," *Journal de Physique*, vol. 2, pp. C51–C54, Apr. 1992.

[101] A. Chaigne and A. Askenfelt, "Numerical simulations of piano strings. I. A physical model for a struck string using finite difference methods," *J. Acoust. Soc. Am.*, vol. 95, pp. 1112–1118, Feb. 1994.

[102] A. Chaigne and A. Askenfelt, "Numerical simulations of piano strings. II. comparisons with measurements and systematic exploration of some hammer-string parameters," *J. Acoust. Soc. Am.*, vol. 95, pp. 1631–1640, Mar. 1994.

[103] A. Chaigne and C. Lambourg, "Time-domain simulation of damped impacted plates. I. Theory and experiments," *J. Acoust. Soc. Am.*, vol. 109, pp. 1422–1432, Apr. 2001.

[104] C. Lambourg, A. Chaigne, and D. Matignon, "Time-domain simulation of damped impacted plates. II. Numerical models and results," *J. Acoust. Soc. Am.*, vol. 109, pp. 1433–1447, Apr. 2001.

[105] S. V. Duyne and J. O. Smith, "Physical modeling with the 2-D digital waveguide mesh," in *Proc. Int. Computer Music Conf.*, (Tokyo, Japan), pp. 40–47, 1993.

[106] F. Fontana and D. Rocchesso, "A new formulation of the 2D-waveguide mesh for percussion instruments," in *Proceedings of the XI Colloquium on Musical Informatics*, (Bologna, Italy), pp. 27–30, Nov. 1995.

[107] L. Savioja, *Modeling Techniques for Virtual Acoustics.* PhD thesis, Helsinki University of Technology, Espoo, Finland, 2000. Available online at `http://www.tml.hut.fi/%7Elas/publications/thesis/`.

[108] R. Burridge and J. B. Keller, "Peeling, slipping and cracking-some one-dimensional free-boundary problems in mechancis," *SIAM Review*, vol. 20, pp. 31–61, Jan. 1978.

[109] R. Burridge, J. Kapraff, and C. Morshedi, "The sitar string, a vibrating string with a one-sided inelastic constraint," *SIAM Journal of Applied Mathematics*, vol. 42, pp. 1231–1251, Dec. 1982.

[110] I. Firth, "Some measurements on the lute," *Catgut Acoust. Soc. Newsletter*, vol. 27, p. 12, 1977.

[111] S. Y. Feng, "Some acoustical measurements of the chinese musical instrument P'i P'a," *J. Acoust. Soc. Am.*, vol. 75, pp. 599–602, 1984.

[112] D. Cohen and T. D. Rossing, "The acoustics of mandolins," in *Proc. Int. Symposium on Musical Acoustics (ISMA)*, vol. 2, (Perugia, Italy), pp. 431–436, Sept. 2001.

[113] A. Morrison and T. D. Rossing, "Modal analysis of the Russian balalaika," in *Proc. Int. Symposium on Musical Acoustics (ISMA)*, vol. 2, (Perugia, Italy), pp. 421–424, Sept. 2001.

[114] G. Essl and P. R. Cook, "Measurements and efficient simulations of bowed bars," *J. Acoust. Soc. Am.*, vol. 108, pp. 379–388, July 2000.

[115] V. Välimäki, M. Karjalainen, T. Tolonen, and C. Erkut, "Nonlinear modeling and synthesis of the kantele–a traditional Finnish string instrument," in *Proc. Int. Computer Music Conf.*, (Beijing, China), pp. 220–223, Oct. 1999.

[116] C. Traube and J. O. Smith, "Estimating the plucking point on a guitar string," in *Proc. of DAFx-00*, (Verona, Italy), pp. 153–158, 2000.

[117] M. Aramaki, J. Bensa, L. Daudet, P. Guillemain, and R. Kronland-Martinet, "Resynthesis of coupled piano string vibrations based on physical modeling," *Journal of New Music Research*, vol. 30, no. 3, pp. 212–218, 2001.

[118] M. Karjalainen, "1-D digital waveguide modeling for improved sound synthesis," in *Proc. IEEE Int. Conf. Acoustics, Speech and Signal Proc.*, vol. 2, (Orlando, Florida, USA), pp. 1869–1872, May 2002.

5. Publications

PUBLICATION 1

C. Erkut, T. Tolonen, M. Karjalainen, and V. Välimäki, ”Acoustical analysis of tanbur, a Turkish long-necked lute,” in Proceedings of the 6th International Congress on Sound and Vibration (ICSV6), Lyngby, Denmark, July 5-8, 1999, vol. 1, pp. 345-352.

PUBLICATION 2

C. Erkut and V. Välimäki, "Model-based sound synthesis of tanbur, a Turkish long-necked lute," in Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP'00), Istanbul, Turkey, June 5-9, 2000, vol. 2, pp. 769-772.

PUBLICATION 3

C. Erkut, M. Karjalainen, P. Huang, and V. Välimäki, ”Acoustical analysis and model-based sound synthesis of the kantele”, Journal of the Acoustical Society of America, vol. 112, no. 4, October, 2002, pp. 1681-1691.

PUBLICATION 4

M. Laurson, J. Hiipakka, C. Erkut, M. Karjalainen, V. Välimäki, and M. Kuuskankare, ”From expressive notation to model-based sound synthesis: a case study of the acoustic guitar,” in Proceedings of the International Computer Music Conference (ICMC’99), Beijing, China, October 22-28, 1999, pp. 1-4.

PUBLICATION 5

C. Erkut, V. Välimäki, M. Karjalainen, and M. Laurson, ”Extraction of physical and expressive parameters for model-based sound synthesis of the classical guitar,” presented at the AES 108th International Convention, Paris, France, February 19-22, 2000, preprint no. 5114, 17 p.

PUBLICATION 6

M. Laurson, C. Erkut, V Välimäki, and M Kuuskankare, ”Methods for modeling realistic playing in acoustic guitar synthesis,” Computer Music Journal, vol. 25, no. 3, pp. 38-49, 2001.

PUBLICATION 7

C. Erkut, M. Laurson, M Kuuskankare and V. Välimäki, ”Model-based synthesis of the ud and the renaissance lute,” in Proceedings of the International Computer Music Conference (ICMC 2001), Havana, Cuba, September 17-23, 2001, pp. 119-122.

PUBLICATION 8

C. Erkut, "Model order selection techniques for the loop filter design of virtual string instruments," in Proceedings of the 5th World Multi-Conference on Systemics, Cybernetics and Informatics (SCI 2001), Orlando, FL, USA, July 22-25, 2001, vol. 10, pp. 529-534.

PUBLICATION 9

C. Erkut and M. Karjalainen, "Virtual strings based on a 1-D FDTD waveguide model", Proceedings of the Audio Engineering Society 22nd International Conference, Espoo, Finland, June 15-17, 2002, pp. 317-323.