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# OPTIMUM DESIGN OF COLD-FORMED STEEL PURLINS USING GENETIC ALGORITHMS

**Wei Lu**

TEKNILLINEN KORKEAKOULU
TEKNISKA HÖGSKOLAN
HELSINKI UNIVERSITY OF TECHNOLOGY
TECHNISCHE UNIVERSITÄT HELSINKI
UNIVERSITE DE TECHNOLOGIE D'HELSINKI

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**Wei Lu**

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Helsinki University of Technology
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Distribution:
Helsinki University of Technology
Laboratory of Steel Structures
P.O. Box 2100
FIN-02015 HUT
Tel. +358-9-451 3701
Fax. +358-9-451 3826
E-mail: srt-sihteerit@hut.fi



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# ABSTRACT

An important advantage of cold-formed steel is the great flexibility of cross-sectional shapes and sizes available to the structural steel designer. However, the lack of standard optimized shapes makes the selection of the most economical shape very difficult. This task is further complicated by the complex and highly nonlinear nature of the rules that govern their designs. In this thesis, genetic algorithms are used to carry out the optimization of cold-formed steel purlins, which are assumed to be continuous over two spans subjected to a gravity load.

A genetic algorithm based optimum design method for cold-formed steel purlins is developed firstly. This method obtains the optimum dimensions for purlins with the highest load efficiency subjected to the geometrical and strength constraints provided in Eurocode 3, Part 1.3, and fabrication constraints. The design of cold-formed steel purlins is based on Eurocode 3, Part 1.3. The integrated design method is general for any shape of cold-formed steel purlins. However, in this thesis, the investigations are concentrated on Z-shape and Σ-shape cold-formed steel purlins. With this design method, an optimization tool and a set of optimum sections that can be easily accessed are provided for structural steel designers and steel manufacturers.

An integration of a modified Eurocode 3 method into genetic algorithm optimization is carried out. Currently, the design of cold-formed steel purlins in Eurocode 3 relies on the effective width approach. The method works by considering the strength reduction due to local plate buckling as an effective width for each element of the cross-section and to a distortional buckling as a reduced thickness for the stiffener. In the modified Eurocode 3 method, the numerical elastic buckling stresses are introduced into the calculation of the effective section properties. Via this integration, it is shown that such numerical method as the finite strip method may also be integrated into the genetic algorithm optimization process. In addition, the modified Eurocode 3 method depends on the buckling types which differs from the variations of the dimension. Thus, the typical buckling types of the Z-shape and Σ-shape cold-formed steel purlins are investigated.

**Keywords:** cold-formed steel, optimization, genetic algorithms, Z-shape purlin, Σ-shape purlin.

# PREFACE

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Espoo, Otaniemi, June 29, 2002

Wei Lu

# LIST OF SYMBOLS

| | |
|---|---|
| $A_{eff}$ | Effective area |
| $A_g$ | Gross area of cross-section |
| $A_s$ | Effective area of the stiffener |
| $b$ | Width of the flange for Σ-shape purlin |
| $b_1$ | Width of top flange for Z-shape purlin |
| $b_2$ | Width of bottom flange for Z-shape purlin |
| $b_p$ | Width of the flange |
| $b_{p,c}$ | Width of the lip |
| $b_{p,i}$ | Notational width of the plane element $i$ |
| $c$ | Depth of the lip |
| $CC$ | Sum of constraint violation function |
| $C_D$ | Rotational stiffness |
| $c_i$ | Constraint violation function |
| $e$ | Shear center of the cross-section |
| $F$ | Fitness function |
| $\bar{f}$ | Average fitness for the whole population |
| $f_i$ | Fitness for each individual |
| $f_y$ | Yield stress |
| $f_{yb}$ | Basic yield stress |
| $g_j(X) \leq 0$ | Inequality constraints |
| $h$ | Height of the cross-section |
| $H$ | A schema |
| $h_d$ | Developed height of a purlin web |
| $h_j(X) \geq 0$ | Equality constraints |
| $I_s$ | Effective moment of inertia of the stiffener |
| $K$ | Spring stiffness per unit length |
| $K_A$ | Lateral stiffness corresponding to the rotational stiffness of the connection between the sheeting and purlin |
| $K_B$ | Lateral stiffness due to distortion of the cross-section of purlin |
| $KK$ | Coefficient in fitness function |
| $k_\sigma$ | Plate buckling coefficient |
| $L$ | Span of the purlin |
| $L_{strip}$ | Length of the strip |

| | |
|---|---|
| $m(H,t)$ | Expected numbers of the particular schema H at generation t |
| $M_{fz}$ | Bending moment about z-z axis in the free flange due to lateral load |
| $M_y$ | Bending moment capacity |
| $M_{y,Sd}$ | In-plane bending moment |
| $N$ | Number of the individuals in the whole populaton |
| $o(H)$ | Order of a schema H |
| $p_c$ | Probability to execute crossover |
| $p_i$ | Selection probability |
| $p_m$ | Probability to execute mutation |
| $q$ | Applied load |
| $q_{h,Fd}$ | Lateral load applied on the free flange |
| $t$ | Thickness of the cross-section |
| $W_{eff,y}$ | Effective section modulus for bending about y-y axis |
| $W_{fz}$ | Gross section modulus of the free flange plus $1/6$ web height for bending about z-z axis |
| $Z = F(\ )$ | General objective function |

## Greek symbols

| | |
|---|---|
| $\alpha_i$ | Normalized constraints |
| $\chi$ | Reduction factor for flexural buckling of the free flange |
| $\delta(H)$ | Length of a schema H |
| $\gamma_M, \gamma_{M1}$ | Partial safety factor |
| $\bar{\lambda}$ | Relative slenderness |
| $\bar{\lambda}_p$ | Plate slenderness |
| $\nu$ | Poisson's ratio |
| $\rho$ | Reduction factor to determine the effective width |
| $\sigma_1$ | Compression stress |
| $\sigma_2$ | Tension stress |
| $\sigma_{cr}$ | Elastic local buckling stress |
| $\sigma_{cr,s}$ | Elastic distortional buckling stress |
| $\psi$ | Ratio of compression stress to tension stress |

# CONTENTS

# 1 INTRODUCTION

## 1.1 Background

Cold-formed steel members provide substantial savings due to their high strength-to-weight ratio. As a result, they have become very popular in the construction of industrial, commercial, and agricultural buildings. An important advantage of cold-formed steel is the greater flexibility of cross-sectional shapes and sizes available to the structural steel designer. Through cold-forming operations, steel sheets, strips or plates can be shaped easily and sized to meet a large variety of design options. Such a large number of design possibilities create a very important challenge of choosing the most economical cold-formed shape in design of steel structures.

A search through structural engineering and computational journals turned out only a few papers on the optimization of cold-formed steel structures. Seaburg et al. (1971) used the direct and gradient search techniques for the minimum weight design of hat-shaped cold-formed light gauge steel members (Adeli et al., 1997). Park et al. (1995) developed a new neural dynamics model for optimization problems with highly nonlinear and complicated constraints. Subsequently, the model is applied to minimum weight design of simply supported cold-formed steel beams based on the AISI Specifications (Adeli et al., 1996; Karim et al., 1999; Adeli et al., 1999; and Karim et al., 2000). Nagy (2000) used the genetic algorithm to obtain the optimum shape of trapezoidal sheeting profiles.

The optimization problem in structural engineering can be defined mathematically as (Kirsch, 1981):

$$g_j(\{X\}) \le 0 \qquad j = 1, \ldots, m, \tag{1.1.1}$$

$$h_j(\{X\}) = 0 \qquad j = 1, \ldots, k \text{, and} \tag{1.1.2}$$

$$Z = F(\{X\}) \to \min . \tag{1.1.3}$$

In the above formula, $\{X\}$ is the vector of the design variables such as material, topological, configurational or geometric layout, and cross-sectional design variables; $g_j(\{X\}) \le 0$ and $h_j(\{X\}) = 0$ are the inequality and equality constraints, respectively; $m$ and $k$ are the number of inequality and equality constraints, respectively. $Z = F(\{X\})$ is the objective function and may represent the weight, the cost of the structure, or any other criterion. The essential of optimization problem is to find the design variables in the design space under the constraints, which minimize the objective function. In practice, the design variables of the optimization problem in structural engineering are discrete variables. Many mathematical nonlinear and linear programming methods have been developed for solving these optimization problems. However, no single method has been found to be entirely efficient and robust for all different kinds of engineering optimization problems. With these methods, if there is more than one local optimum in the problem, the result will depend on the choice of the starting point, and the global optimum cannot be guaranteed. Furthermore, when the objective function and constraints have multiple or sharp peaks, gradient search becomes difficult and unstable (Adeli et al., 1993).

In this thesis, a genetic algorithm is applied to solve the optimization problem. The Genetic Algorithms (GAs) were invented by John Holland in the 1960s and developed by Holland and his students and colleagues at the University of Michigan in the 1960s and 1970s (Mitchell, 1996). A GA performs a multi-directional search by maintaining a population of potential solutions and encourages information formation and exchange between these directions. As compared to other search and optimization algorithms, GA has the following features (Pohlheim, 1994-1997):

1. GAs search a set of points in parallel, not only at a single point;
2. GAs do not require derivative information or other auxiliary knowledge. Only the objective function and corresponding fitness affect the direction of search;
3. GAs use probability rules; and
4. GAs provide a number of potential solutions to a given problem. The final choice is left to user.

In structural engineering, GAs have been mainly applied to the optimization of steel trusses and steel frames (Rajeev, et al., 1992; Adeli et al. 1994 (a) (b); Koumousis et al. 1994; Adeli et al., 1995; Jenkins, 1995; Rajeev et al., 1997; Liu et al., 1999; Erbatur et al., 2000; Pezeshk, et al., 2000; Sarma et al., 2000; Kameshki et al., 2001; and Schilling et al., 2001), to sandwich plates (Sciuva, et al., 2001), and to reinforced concrete member and frames (Koumousis, et al., 1998; and Rajeev et al. 1998).

In this research, the GAs are used to optimize the dimensions of cold-formed steel purlins. When used as purlins in roof assemblies, cold-formed steel sections are usually attached along one flange to the roofing material. Screw fastener are used to attach the sheeting to the purlin, with the purlin in turn bolted to cleats welded to the main load-bearing members of the frame. Cold-formed steel purlins are normally manufactured by roll forming a thin strip of steel to the required section shape. The steel is usually coated with zinc or zinc/aluminum and may have a yield stress in the range of 250-550 MPa (Papangelis et al. 1998).

In contrast to isolated beams and columns, thin-walled roof purlins are often loaded through the member that they support and these, in turn, provide both rotational and translational restraint to the purlins (Jiang et al., 1997). Because of the low torsional stiffness, purlins may undergo flexural distortional buckling or non-linear twisting during bending. Bracing at discrete points along purlins is often used to increase the flexural-torsional buckling resistance. In addition, distortional buckling may occur in the unsheathed compression flanges of braced purlins under wind uplift or at the internal support under gravity load (Papangelis, et al., 1998).

The design of the purlin in this study is mainly based on Eurocode 3, Part 1.3 (1996). Currently the design of cold-formed steel purlins in Eurocode 3 relies on the effective width approach. The method works by considering the strength reduction due to local plate buckling as an effective width for each element of the cross-section and to a distortional buckling as a reduced thickness of the stiffener. The details of the design process are described in Chapter 3. However, research shows that the elastic distortional buckling stresses calculated using Eurocode 3 are higher than those using the numerical method due to not considering the reduction in the flexural restraints provided by the buckling web in Eurocode 3 for a C-shape column (Kesti et al., 1999 and Kesti, 2000). Thus, a modified Eurocode 3 method is chosen by integrating the numerical elastic buckling stress into the calculation of the effective section properties. With this modified Eurocode 3 method, the interactions amongst the flange, the web and the lip are considered; all buckling modes are taken into account; and the lengthy effective width calculation is reduced especially for the cross-section with multiple intermediate flange stiffener and web stiffeners.

## 1.2 Objective of Present Study

The main objective of this thesis is to show the capability of the optimization algorithm, GA, in finding out the optimum dimensions of cold-formed steel purlins with various cross-sectional shapes, such as Z-shape and Σ-shape. The main task is to develop an optimization tool that not only can be applied to the optimization of purlins mentioned above but also can be applied further to more complicated cross-sections with a large amount of design variables and more complex constraints. With this tool, the behavior of two types of the cold-formed steel purlins are investigated and a set of the optimum dimensions are provided for both structural steel designers and structural steel manufactures. In addition, another purpose of this research is to indicate such that numerical method as the Finite Strip Method (FSM) may also be integrated into the GA optimization process. By integrating this numerical method to the design process, the interactions amongst the different elements of the cross-section are taken into account in the design process, and the design process is simplified especially for cross-sections with multiple web stiffeners or flange stiffeners.

## 1.3 Outline of the Thesis

The design process of Eurocode 3, Part 1.3 and the basic idea of the modified Eurocode 3 method are described in Chapter 2. With this modified Eurocode 3 method, the interactions among different elements of the cross-section are taken into account.

Since the GAs are a relatively new area in the field of cold-formed steel structures, the main schemes of GA including the principles, operators and algorithms are introduced in Chapter 3.

In Chapter 4, the optimization problem for the design of cold-formed steel purlins is defined and an optimization tool called ODSP-GA is developed to integrate design of purlins into the GA optimization process. The capability of this optimization tool has been evaluated.

The effects of the dimensions on the behavior of the cold-formed Z-shape purlin are investigated in Chapter 5. These investigations provide the understandings of the optimum dimensions. Moreover, in this chapter, the purlins are designed based on the modified Eurocode 3 method as well and the results are compared with those based on the general Eurocode 3 method.

Chapter 6 provides the optimum dimensions of Z-shape purlins. The optimizations are carried out based on integrating both Eurocode 3 method and modified Eurocode 3 method into the GA optimization process. In order to simplify the optimization process, two relationships of flanges to lips are defined. From these calculations, the nominal optimum dimensions are provided for application purposes.

In Chapter 7, the effects of the position and size of the web stiffener on the behavior of Σ-shape purlins are investigated. With such numerical method as FSM, the elastic buckling modes are defined and the elastic buckling stresses are calculated so as to integrate into the modified Eurocode 3 method. The results are compared with the Eurocode 3 method.

Chapter 8 provides the optimum dimensions for the Σ-shape purlin. These optimizations are carried out for the cross-sections both with double edge stiffeners and with single edge stiffeners. With the similar simplified formula given in Chapter 6, the nominal optimized dimensions for the Σ-shape purlins are provided.

The summaries and conclusions are given in Chapter 9. One advantage of the GA is that it provides the various alternative solutions during the searching process and the final decision is left to the users. The solutions for each case calculated in this study are listed in the Appendices so as to provide enough information for the industrial designers and manufacturers to find the proper nominal dimensions, with the consideration of other practical constraints such as manufacturing tools and processes.

# 2 DESIGN OF COLD-FORMED STEEL PURLINS

## 2.1 Purlin Design Based on Eurocode 3

In Eurocode 3, Part 1.3, the free flange of cold-formed purlin is considered as a beam on an elastic foundation and the resistance of the cross-section should be verified as indicated in Figure 2.1.1.

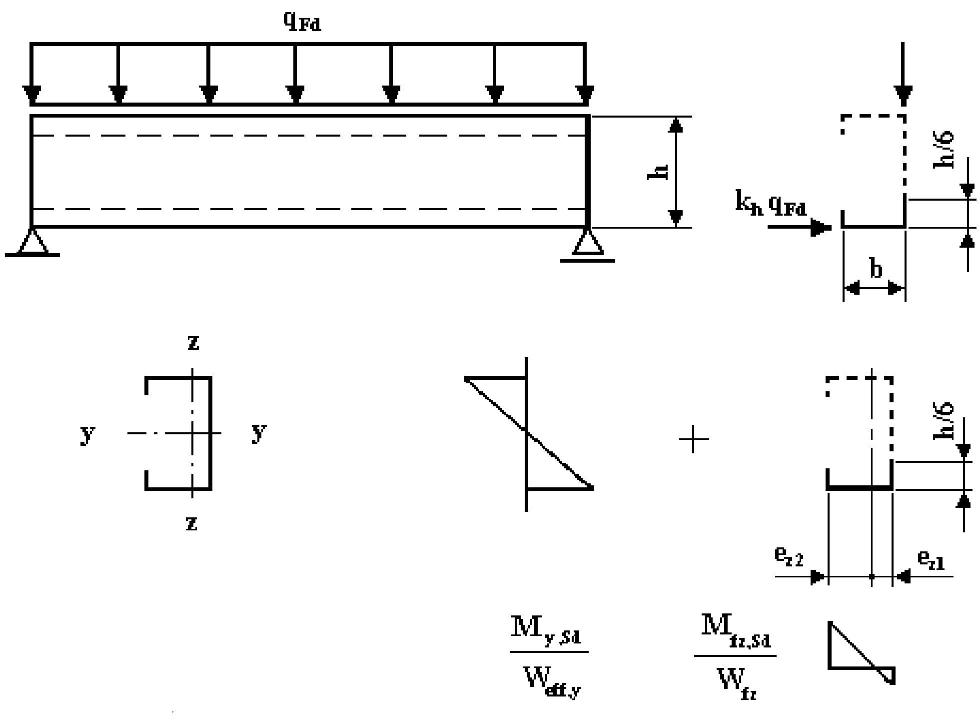



**Figure 2.1.1** Beam on an elastic foundation, model of Eurocode 3

When a purlin is continuous over two spans subjected to the gravity load, it should satisfy the following criteria for cross-section resistance:

$$\frac{M_{y,Sd}}{W_{eff,y}} \leq \frac{f_y}{\gamma_M}, \tag{2.1.1}$$

for the restrained flange, and

$$\frac{M_{y,Sd}}{W_{eff,y}} + \frac{M_{fz,Sd}}{W_{fz}} \leq \frac{f_y}{\gamma_M}, \tag{2.1.2}$$

for the free flange. The stability of the free flange at the internal support should be checked using the following equation:

$$\frac{1}{\chi} \cdot \frac{M_{y,Sd}}{W_{eff,y}} + \frac{M_{fz,Sd}}{W_{fz}} \leq \frac{f_y}{\gamma_{M1}}, \tag{2.1.3}$$

where $M_{y,Sd}$ is the in-plane bending moment; $W_{eff,y}$ is the effective section modulus of the cross-section for bending about y-y axis; $M_{fz}$ is the bending moment in the free flange due to the lateral load; $W_{fz}$ is the gross elastic section modulus of the free flange plus 1/6 of the web height, for bending about the z-z axis; $\chi$ is the reduction factor for flexural buckling of the free flange and $\gamma_M$ , $\gamma_{M1}$ are the partial safety factors.

### 2.1.1 Elastic buckling stress

The determination of the elastic local buckling stresses in Eurocode 3 is based on the element method. The formula for calculating the elastic local buckling stress is provided by

$$\sigma_{cr} = \frac{\pi^2 \cdot E \cdot k_\sigma}{12 \cdot (1-\nu^2)} \cdot \left(\frac{t}{b_{p,i}}\right)^2, \tag{2.1.4}$$

where $k_\sigma$ is the relevant buckling factor; $\nu$ is the Poisson's ratio; and $b_{p,i}$ is the notational width of the plane element $i$ for a cross-section with sharp corners.

The buckling factor, $k_\sigma$, of the flange is 4, which is chosen from Table 4.1 in Eurocode 3, Part 1.3. If the flange is stiffened with a single edge stiffener, the buckling factor, $k_\sigma$, of the lip is provided by

$$k_\sigma = \begin{cases} 0.5 & \text{if } b_{p,c}/b_p \le 0.35, \\ 0.5 + 0.83 \times \sqrt[3]{(b_{p,c}/b_p - 0.35)^2} & \text{if } 0.35 < b_{p,c}/b_p \le 0.6, \end{cases} \tag{2.1.5}$$

where $b_{p,c}$ and $b_p$ are widths of the lip and flange, respectively. If the flange is stiffened with a double edge stiffener, the buckling factor, $k_\sigma$, is obtained from Eurocode 3, Part 1.3 treating the first lip as a doubly supported element in Table 4.1 and the second lip as an outstand element in Table 4.2. The buckling factor of the web is calculated from:

$$k_\sigma = \begin{cases} 7.81 - 6.29 \cdot \psi + 9.78 \cdot \psi^2 & -1 < \psi < 0, \\ 23.9 & \psi = 0, \text{ and} \\ 5.98 \cdot (1-\psi)^2 & -3 < \psi < -1, \end{cases} \tag{2.1.6}$$

where

$$\psi = \frac{\sigma_2}{\sigma_1}, \tag{2.1.7}$$

in which $\sigma_1$ and $\sigma_2$ are the compression and tension stresses, respectively.

The calculation of the elastic distortional buckling stress in Eurocode 3 is based on flexural buckling of the stiffener. The stiffener behaves as a compression member with continuous partial restraint. This restraint has a spring stiffness that depends on the boundary conditions and the flexural stiffness of the adjacent plane elements of the cross-section. The distortional buckling stress, $\sigma_{cr,s}$, is calculated as:

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s}, \qquad (2.1.8)$$

where $I_s$ is the effective second moment of inertia of the stiffener and can be calculated by taking the effective area, $A_s$, about the centroidal axis of the effective area; $K$ is the spring stiffness per unit length and is given by:

$$K = \frac{3 \cdot E \cdot t^3}{12 \cdot (1-\nu^2) \cdot b_p{}^2 \cdot (b_p + h)}, \qquad (2.1.9)$$

where $h$ is the height of the web.

### 2.1.2 Eurocode 3 for effective cross-section properties

The effective cross-section properties such as effective section modulus, $W_{eff,y}$ and the effective area, $A_{eff}$ are determined based on effective cross-section obtained by effective width for the element and effective thickness for the stiffener under compression. The effective width of the plane element $i$ for the cross-section, $b_{eff,i}$ is obtained using

$$b_{eff,i} = \rho \cdot b_{p,i}, \qquad (2.1.10)$$

where $\rho$ is the reduction factor by which the post-buckling behavior is considered and can be obtained from the following formula

$$\begin{array}{ll} \bar{\lambda}_p \le 0.673 & \rho = 1.0, \\ \bar{\lambda}_p > 0.673 & \rho = (1.0 - 0.22/\bar{\lambda}_p)/\bar{\lambda}_p, \end{array} \qquad (2.1.11)$$

where

$$\bar{\lambda}_p = \sqrt{\frac{f_{yb}}{\sigma_{cr}}} = 1.052 \frac{b_p}{t} \sqrt{\frac{f_{yb}}{E \cdot k_\sigma}}\,, \qquad \textbf{( 2.1.12 )}$$

The reduction factor, $\chi$, for the thickness of the stiffener can be determined according to the following formula

$$\chi = \frac{1}{\phi + (\phi^2 - \bar{\lambda}^2)^{0.5}}\,, \qquad \textbf{( 2.1.13 )}$$

where $\phi = 0.5 \cdot \left[1 + \alpha \cdot (\bar{\lambda} - 0.2) + \bar{\lambda}^2\right]$, $\alpha = 0.13$, $\bar{\lambda} = \sqrt{\frac{f_{yb}}{\sigma_{cr,s}}}$. If $\chi < 1$, it may optionally be refined iteratively and the iteration will be continued until $\chi$ is approximately equal to, but not more than, the previous value.

### 2.1.3 Design of the purlin considering free flange behavior

The bending moment in the free flange due to lateral load, $q_{h,Fd}$ is determined by

$$M_{fz,sd} = \beta_R \cdot M_{0,fz,Sd}\,, \qquad \textbf{( 2.1.14 )}$$

where $M_{0,fz,Sd}$ is the initial lateral bending moment in the free flange without any spring support and determined by transferring the vertical uniform load on the beam to the lateral load on the free flange using

$$q_{h,Fd} = k_h \cdot q_{Fd}\,, \qquad \textbf{( 2.1.15 )}$$

where $k_h$ is a coefficient and is given by

$$k_h = \begin{cases} \dfrac{b^2 \cdot h \cdot t}{4 \cdot I_u}, & \text{for Z - shape purlins,} \\ \dfrac{e}{h}, & \text{for } \Sigma \text{ - shape purlins,} \end{cases} \tag{2.1.16}$$

where $I_u$ is the moment inertia of the cross-section for bending about $u-u$ axis that is perpendicular to the web height; and $e$ is the shear center of the cross-section.

The correction factor, $\beta_R$, for the relevant location and boundary condition is given in Table 10.1 in Eurocode 3, Part 1.3, using the value of coefficient R of the spring support given by

$$R = \frac{KL_a^{\ 4}}{\pi^4 \cdot E \cdot I_{fz}}, \tag{2.1.17}$$

in which $I_{fz}$ is the second moment of area of the gross cross-section of the free flange plus $1/6$ of the web height for bending about z-z axis; $L_a$ is the distance between anti-sag bars, or if none are present, the span $L$ of the purlin is used; and $K$ is the lateral spring stiffness per unit length and is determined from

$$\frac{1}{K} = \frac{1}{K_A} + \frac{1}{K_B}, \tag{2.1.18}$$

where $K_A$ is the lateral stiffness corresponding to the rotational stiffness of the connection between the sheeting and the purlin and is determined by

$$K_A = \frac{E \cdot t^3}{4 \cdot (1-\nu^2) \cdot h^2 \cdot (h_d + e)}, \tag{2.1.19}$$

$K_B$ is the lateral stiffness due to distortion of the cross-section of the purlin and is determined by

$$K_B = \frac{C_D}{h^2}, \tag{2.1.20}$$

where $C_D$ is the rotational stiffness, which is composed of the rotational stiffness of the connection between the sheeting and purlin, and the rotational stiffness corresponding to the flexural stiffness of the sheeting.

## 2.2 Modified Eurocode 3 Method for Effective Cross-Section Properties

When designing the cold-formed purlin based on Eurocode 3, Part 1.3, the calculation of effective cross-section properties can be replaced by the modified Eurocode 3 method. The main idea of this method is to integrate elastic local and distortional buckling stresses calculated based on a numerical method, such as Finite Strip Method (FSM), into the design procedure described in Eurocode 3. In this process, the effective width of the plane element $i$ for the cross-section, $b_{eff,i}$ is calculated using

$$b_{eff,i} = \rho \cdot b_{p,i}, \tag{2.2.1}$$

where the reduction factor, $\rho$, is calculated from

$$\begin{array}{ll} \bar{\lambda}_p \le 0.673 & \rho = 1.0\text{, and} \\ \bar{\lambda}_p > 0.673 & \rho = (1.0 - 0.22/\bar{\lambda}_p)/\bar{\lambda}_p, \end{array} \tag{2.2.2}$$

where $\bar{\lambda}_p = \sqrt{\dfrac{f_{yb}}{\sigma_{cr}}}$ is calculated using the numerical elastic buckling stress.

Similarly, the reduction factor, $\chi$, for the thickness of the stiffener can be determined according to the following formula

$$\chi = \frac{1}{\phi + (\phi^2 - \overline{\lambda}^2)^{0.5}} \tag{2.2.3}$$

where $\phi = 0.5 \cdot \left[1 + \alpha \cdot (\overline{\lambda} - 0.2) + \overline{\lambda}^2\right]$, $\alpha = 0.13$, $\overline{\lambda} = \sqrt{\frac{f_{yb}}{\sigma_{cr,s}}}$. The distortional buckling stress is calculated using a numerical method instead of using the formula given in Eurocode 3. In this way the interaction among the lip, flange and the web are taken into account due to treating the cross-section as a whole. The elastic local and distortional buckling stresses are calculated using the computer program CUFSM, which has been developed by Schafer (2002) and can be downloaded from the website.

# 3 GENETIC ALGORITHM: PRINCIPLES, OPERATORS AND ALGORITHMS

## 3.1 Overview

Holland's GA is a method for moving from one population of "chromosomes" to a new population by using a kind of "natural selection" together with the genetics-inspired operators of crossover (recombination), mutation and inversion (Mitchell, 1996). The structure of the GA is shown in Figure 3.1.1.

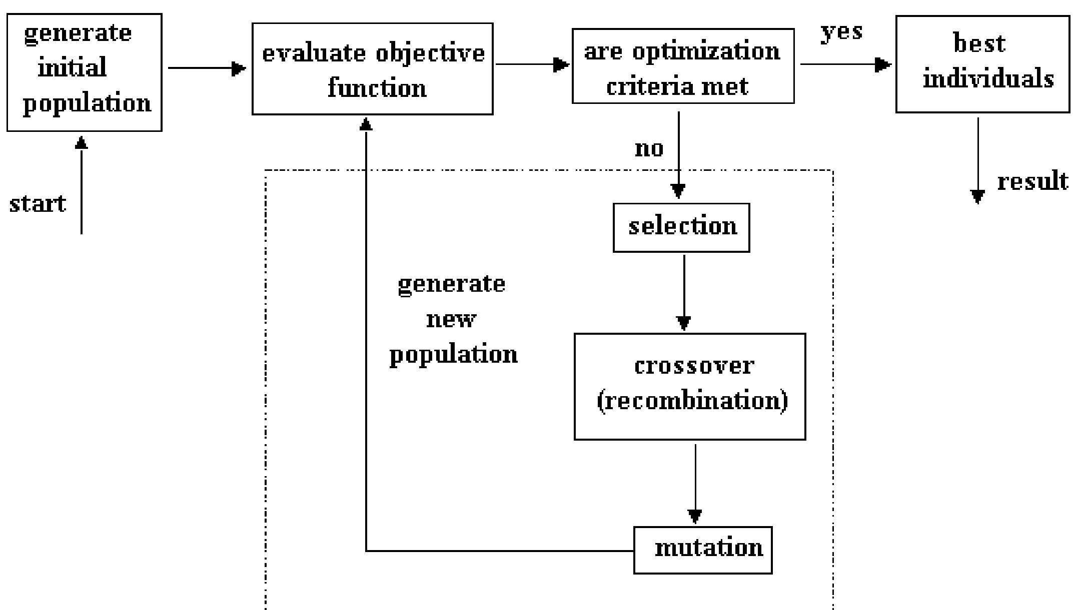



**Figure 3.1.1** The structure of the GA (Pohlheim, 1994-1997)

At the beginning of the computation, a number of individuals are randomly initialized. The fitness function is then evaluated for these individuals. If the optimization criteria are not met, the creation of new generation starts. Individuals are selected according to their fitness, recombination (crossover) and mutation are carried out with a certain probability. The offspring is inserted into the population and the new generation is created. This process is repeated until the stopping criteria are reached.

The two major steps in applying genetic algorithm to a particular problem are the specification of the representation and the fitness function. These two items form the bridge between the original problem context and the problem-solving framework (Eiben, Hinterding and Michalewicz, 1999). When defining the GA, we need to choose its components such as selection, mutation and crossover (recombination). Each of these components may have parameters like the probability of the recombination and mutation, the selection rule and the population size. These values greatly determine whether the algorithm will find a near optimum solution and whether to find such a solution efficiently.

## 3.2 Encoding and Decoding

GA starts with a set of population represented by chromosomes. The encoding of a chromosome is problem dependent. Since any possible data structure can be used as encoding for the creation of a search space to represent a given problem, there are several ways to encode the design variables, such as the binary encoding, gray codes, direct encoding and tree based encoding.

In binary encoding, every chromosome is a string of bits 0 and 1, as shown in Figure 3.2.1. Since the encoding in computers is binary, the binary encoding of a chromosome is natural. However, one problem with the binary encoding is that small changes in the encoding do not always result in small changes in the real design values (Sharpe, 2000).

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Chromosome A | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| Chromosome B | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |

**Figure 3.2.1** Chromosome with binary encoding

Gray codes are such binary encoding that single bit flip in a string leads to a unit change in the encoded value of the string. The difference from the binary encoding is that they are represented as strings of zeros and ones instead of as numbers written in base two. With this encoding, the search space is much smoother.

Direct value encoding can be used in problems where some complicated value such as real numbers are used. In this encoding, every chromosome is a string of real numbers. However, for this encoding it is often necessary to develop some new crossover and mutation operators.

In tree encoding, each chromosome is a tree of some objects, such as functions or commands in programming languages. This encoding is more suitable for genetic programming put forward by Koza (Koza, 1992).

In order to evaluate the fitness of each chromosome, each chromosome should be decoded. The decoding rule depends on what kind of encoding rule is used.

## 3.3 Fitness Evaluation

The evaluation of the fitness is problem dependent as well. For every individual in the population, a level of fitness must be assigned to it. The fitness of an individual is an indicator of how well an individual is suited to its current environment. Fitness is established by means of a function. Developing this function can be very simple or may be very complex involving a simulation.

GA is suitable for solving unconstrained problems. However, the optimization problems in structural engineering are all constrained problems. Therefore, it is necessary to transform a constrained problem to an unconstrained problem by introducing penalty functions into the objective function if the constraints are violated. Several types of penalties have been presented (Chen and Rajan, 1995, Rajeev and Krishnamoorthy, 1992).

## 3.4 Selection Rule

Evolution is predicted based on the concept of "survival of fittest". Therefore, it makes sense that the more fit an individual is, the more likely it will be chosen as a parent to the next generation. The selection method determines how individuals are chosen for mating. If the selection method only picks up the best individual, the population will quickly converge to that individual. So the selector should be biased toward better individuals, but also pick up some that are not quite good ones (Wall).

Proportional selection is a selection method, in which the probability of the selection is proportional to the individual fitness. The probability of the selection is defined as

$$p_i = \frac{f_i}{\sum f_i} = \frac{f_i}{\bar{f} \cdot N}, \tag{3.4.1}$$

where $p_i$ is the selection probability for each individual; $f_i$ is the fitness for each individual; $\bar{f}$ is the average fitness for the whole population and $N$ is the number of the individuals in the whole population. Since this selection method works with the explicit values of the fitness, it is easy to make the GA be premature convergence. The solution for this problem is to use the fitness scaling techniques (Poli, 1996).

Ranking selection is a method where all individuals in a population are sorted from the best to the worst and probability of the selection is fixed for the whole evolution process. The distribution of the probability can be linear or non-linear (Poli, 1996). The advantage of this method is that there is no premature convergence and the explicit fitness is not needed. However, this method requires the sorting operation carry out in advance.

Using the selection method mentioned above, the individuals in the population are sampled using two following rules: roulette wheel sampling and stochastic universal sampling. By roulette wheel sampling, each individual is assigned a slice of circular roulette wheel, and the size of the slice is proportional to the individual's fitness. The wheel is spun N times, where N is the number of individuals in the population. With each spin, the individual under the wheel's marker is selected to be in the mating pool as the parents for the next generation. Rather than spinning the roulette wheel N times to select N parents, with stochastic universal sampling, the wheel is spun once but with N equally spaced pointers, which are used to select the N parents.

Tournament selection is a method, in which some numbers of individuals (usually two) compete for selection to the next generation. This competition step is repeated a number of times equal to the size of the population. The essence of this selection is ranking selection with some noise. The advantage of this method is that there is no premature convergence, no global sorting is required and no explicit fitness is needed.

## 3.5 Crossover

Crossover or recombination is one of the basic operators in GA. With this operator, the genes of the parents are passed to the offspring. It is this operator that gives the GA its exploration ability. There are three basic types of crossover: one-point crossover, two-point crossover and uniform crossover. Whichever method is used, crossover is applied to the individuals of a population with a constant probability, and normally the value of the probability is from 0.5 to 0.8.

One-point crossover

One-point crossover involves cutting the chromosomes of the parents at a randomly chosen common point and exchanging the right-hand side sub-string from the selected point. The scheme of one-point crossover is shown in Figure 3.5.1.

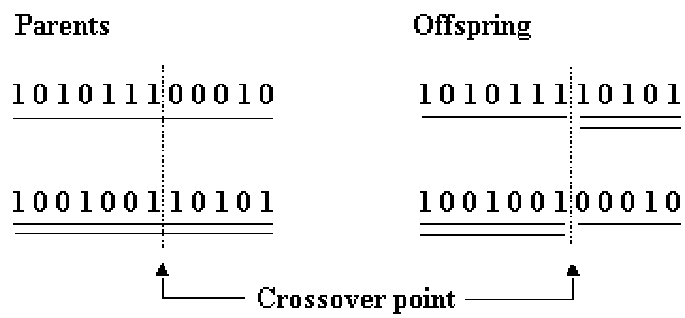



**Figure 3.5.1** One-point crossover

Multi-point crossover

The multi-point crossover is carried out at multiple randomly selected points rather than one point and the crossover procedure is similar to that of the one-point crossover, i.e. the sub-strings of two parents are exchanged at the crossover points. Figure 3.5.2 shows the scheme of two-point crossover.

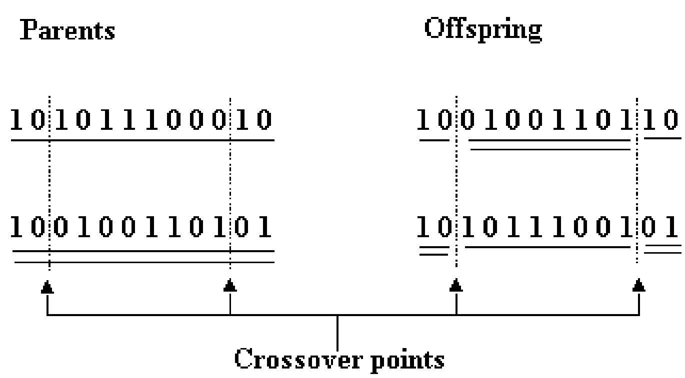



**Figure 3.5.2** Two-point crossover

Uniform crossover

In uniform crossover, each gene of the offspring is selected randomly from the corresponding genes of the parents and each gene is the crossover point. This means that each gene is inherited independently from any other gene and there is no linkage among genes. The one point and two point crossover produce two offspring, whilst uniform crossover produce one offspring.

## 3.6 Mutation

Mutation is considered as a method to recover lost genetic material rather than to search for best solutions. With mutation involved in GA, it can prevent all solutions of the solved problem from falling into a local minimum. Mutation is performed by randomly inverting one or more genes in the chromosome as shown in Figure 3.6.1.

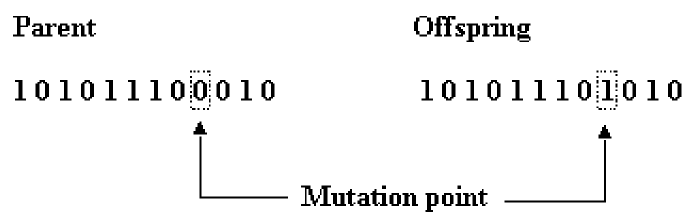



**Figure 3.6.1** Mutation

One of the key issues regarding mutation is the rate at which it is applied. Usually, the probability of the mutation is very small, e.g. 0.001 (Mitchell, 1996). Using the variable mutation rate throughout the run of GA is also possible (Fogarty, 1989; Tuson, 1995).

## 3.7 Elitism

When creating a new population by crossover and mutation, it is very possible to lose the best individuals. Using elitism can avoid this problem. Elitism is a method that copies the best chromosome or a few best chromosomes to the new population. With this method, the convergence of the algorithm is guaranteed.

# 4 GA-BASED DESIGN OF COLD-FORMED STEEL PURLINS

## 4.1 Description of the Optimization Problem

As described in Chapter 2, when the purlin is continuous over two spans, it should satisfy the criteria in the formula (2.1.1) to (2.1.3) for cross-section resistance.

The objective of the optimization is to maximize the load efficiency, which is defined as the distributed load, $q$, over the gross area of the cross-section, $A_g$, when the material reaches its yield strength. The bending moments in the above formulas are functions of distributed load $q$. Therefore, the optimization problem is defined as:

$$\text{Maximize} \qquad \frac{q}{A_g} \tag{4.1.1}$$

subjected to the following geometrical constraints, which are specified in Eurocode 3:

$$\frac{h}{t} \leq 500, \quad \frac{b}{t} \leq 60, \quad 0.2 \leq \frac{c}{b} \leq 0.6, \text{ and } 0.1 \leq \frac{d}{b} \leq 0.3, \tag{4.1.2}$$

where $b$, $c$, and $h$ are the overall width of a flange, a lip and a web, respectively, measured to the face of the material. If the cross-section has a second fold along the lip, the value of $d$ is the width of the second edge stiffener.

In addition, there might exist some types of fabrication constraints in practical applications. For instance, the length of the strip, $L_{strip}$, which is used to form the desired shape of the cross-section, should be in the range of values that the manufacturer provides. In this analysis, the following condition should be satisfied

$$200 \le L_{strip} \le 625\,. \tag{4.1.3}$$

Since in the further analysis, the span of the purlin is set to a fixed value, the span of the purlin is not included in the objective function. The design variables are the dimensions of the cross-section. These values are varied from cross-section to cross-section and will be given in the following chapters, when the detailed cross-sections are presented.

Because a GA is directly used for solving an unconstrained optimization problem, the constrained optimization problem mentioned above should be transformed into an unconstrained problem by including a penalty function (Rajan, S. D., 1995). In this analysis, a quadratic penalty function is used, and the corresponding unconstrained optimization problem becomes

$$Maximize \quad F = \begin{cases} \dfrac{q}{A_g} - KK \cdot n \cdot CC & \text{when } \dfrac{q}{A_g} > KK \cdot n \cdot CC, \text{and} \\ 0 & \text{otherwise,} \end{cases} \tag{4.1.4}$$

where $CC = \sum c_i$ is the sum of constraint violations, $c_i$, which is given by

$$c_i = \begin{cases} 0 & \text{if } \alpha_i = 0, \text{and} \\ {\alpha_i}^2 & \text{otherwise,} \end{cases} \tag{4.1.5}$$

where $\alpha_i$ are the normalized constraints provided by

$$\begin{aligned} &\alpha_1 = {}^{h/t}\!/_{500} - 1, \quad \alpha_2 = {}^{b/t}\!/_{60} - 1, \quad \alpha_3 = {}^{c/b}\!/_{0.6} - 1, \quad \alpha_4 = 1 - {}^{c/b}\!/_{0.2} \\ &\alpha_5 = {}^{d/b}\!/_{0.3} - 1, \quad \alpha_6 = 1 - {}^{d/b}\!/_{0.1}, \quad \alpha_7 = {}^{L_{strip}}\!/_{625} - 1, \quad \alpha_8 = 1 - {}^{L_{strip}}\!/_{200}. \end{aligned} \tag{4.1.6}$$

Moreover, $n$ is the coefficient that makes the values of $q/A_g$ and $CC$ be of the same order so as to avoid one value dominating over the other. In this analysis, the value of $n$ is defined as $10^{L_f - 1 - L_c}$ so as to keep the order of $CC$ one degree lower than that of $q/A_g$, in which $L_f$ and $L_c$ are the orders of $q/A_g$ and $CC$, respectively. Moreover, $KK \geq 0$ is a coefficient and the solution of the penalty problem can be made arbitrarily close to the solution of the original problem by choosing $KK$ to be sufficiently large (Bazaraa, Sherali and Shetty, 1993). In GA terminology, equation (4.1.7) is called a fitness function that is used in the reproduction phase in order to guide the genetic search. Moreover, when the value of $q/A_g$ is not larger than $KK \cdot n \cdot CC$, the fitness is set to zero so that the strings corresponding to this case are discarded with no chance to enter the mating pool.

## 4.2 Outline of the Design Process

Figure 4.2.1 shows how the purlin design is integrated into the GA optimization process. The design process is described as follows:

- GA-based design starts by randomly generating an initial population that is composed of candidate solutions to the problem. Each individual in the population is a bit string of fixed length.

- After decoding, these individuals that represents the dimensions of the purlins are sent to the purlin design program.

- The constraints are checked and if the constraints are violated, the penalty is applied.

- By combining the fittest individuals in the previous population, the new generation is created using such operators as selection, crossover and mutation. In order to keep the best individuals in each generation, elitism may also be used.

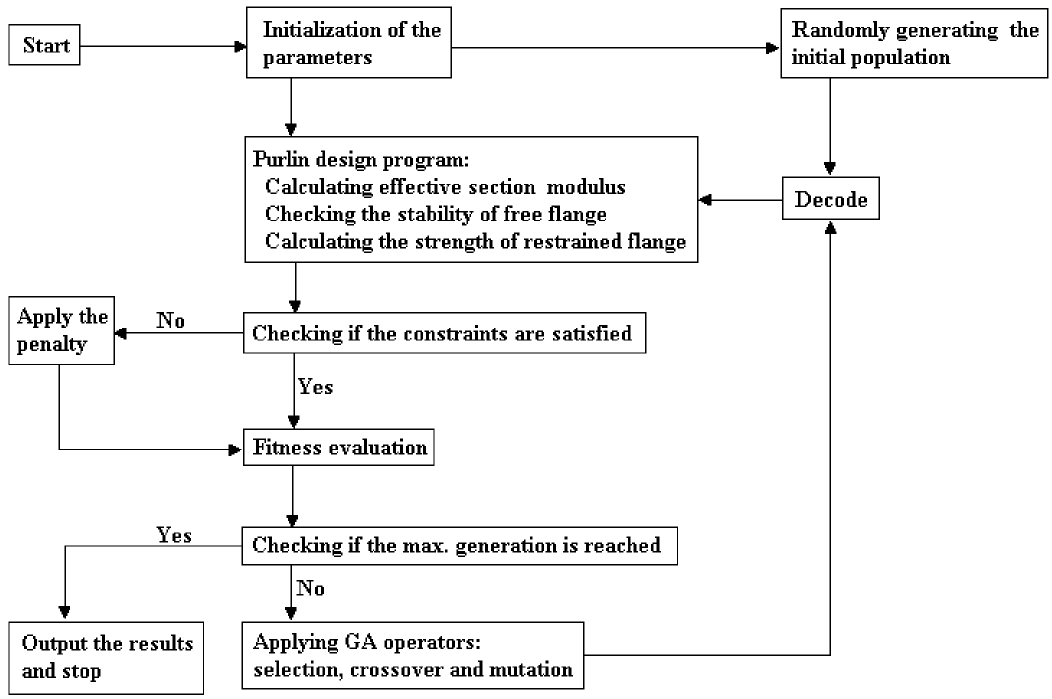



**Figure 4.2.1** Integration of purlin design into GA optimization

- This process is continued until the specified stopping criteria are satisfied.

## 4.3 Computer Implementation of GA-Based Design of Cold-Formed Steel Purlins

A computer program named ODSP-GA is developed for **O**ptimizing the **D**imensions of Z-shaped and $\Sigma$-shaped cold-formed **S**teel **P**urlins using **G**enetic **A**lgorithms. This program is coded using MATLAB language. In order to obtain the optimum dimension in this analysis, it is necessary to integrate the purlin analysis program into the GA program. However, it is possible that the analysis programs for the engineering optimization problems are not available or the source code of the commercial analysis program cannot be accessed. In some cases, the program for GA and the analysis program for the engineering problem itself are written in two different kinds of computer languages, which might be difficult to combine in the application process. Moreover, by developing this computer program, other analysis software, such as finite strip and finite element analysis software can be easily integrated into the GA-based optimization process. In this study, with ODSP-GA, further research using modified Eurocode 3 method on the optimum dimensions of the cross-section can be carried out.

The program ODSP-GA is mainly composed of two parts: the optimization part using genetic algorithms and the design part for cold-formed steel purlins. The structure and the function of each part are described in the following sections.

### 4.3.1 Structures of computer program for optimization using GA

The structure of the computer program for optimization using GA is shown in Figure 4.3.1 and the function of each module is described as follows:

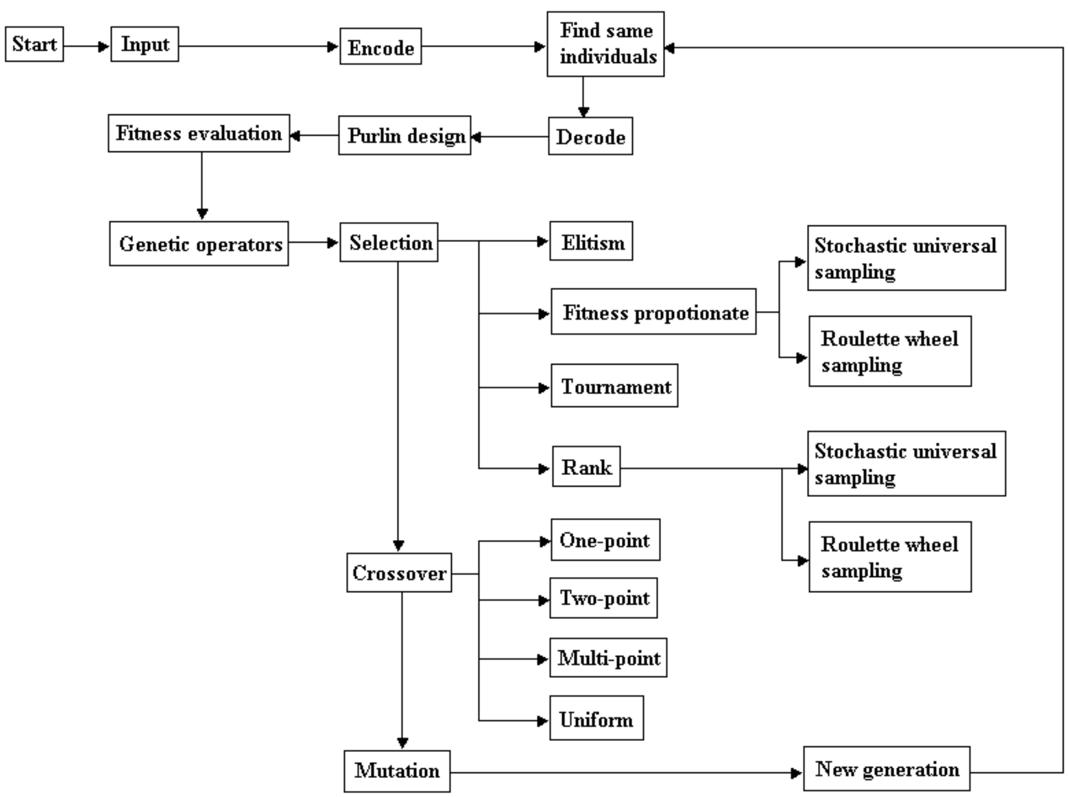



**Figure 4.3.1** Structure of optimization part using GA

- Input module: with this module, the parameters required by the GA, such as the size of the population, the crossover rate, the mutation rate, the generation required to get the solution and the types of the genetic operators are input.

- Encode module: with this module, a number of population is randomly generated. Currently, only the binary encoding strategy is available.

- Find same individuals: normally, it is quite time consuming to access the analysis program. In the process of the evolution, the same individuals exist. Thus, with this module, the repeated calculation with the same design variables can be avoided.

- Decode module: with this module, the design variables are calculated from the encoded individuals and these design variables are sent to the purlin design program.

- Fitness evaluation module: the distributed load and section properties calculated from the purlin design program are sent to this module. The fitness function including the effect of the penalty violation is calculated here.

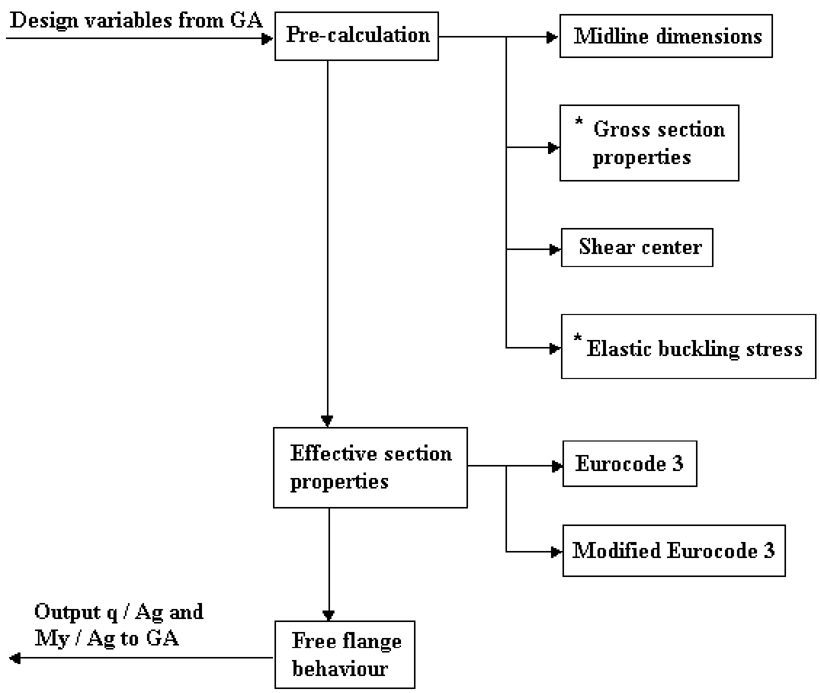
- Selection module: the selection principal based on fitness-proportionate selection, rank selection and tournament selection are available in this module. The user can choose these schemes from the input module. Also, the elitism can be either included or not included in the selection process. Two sampling methods are included in this module, roulette wheel sampling and stochastic universal sampling.

- Crossover module: using one of the following schemes, such as one-point crossover, two-point crossover, multi-point crossover and uniform crossover, the crossover is carried out in this module. The position of the crossover point is randomly generated. The crossover rate is provided by the input module.

- Mutation module: with a given mutation rate, the mutation for each bit of the individuals is performed in this module.

### 4.3.2 Structure of computer program for design of cold-formed steel purlins

The structure of the computer program for the design of cold-formed steel purlins is shown in Figure 4.3.2.




*Carried out using computer software named CUFSM, made by Schafer (2002)

**Figure 4.3.2** Structure of computer program for purlin design

The modules in the program are described as follows

- Pre-calculation module: in this module, the mid-line dimension of the cross-section is calculated. Furthermore, the gross section properties, such as the position of the neutral axis, the section modulus and moment of inertia are calculated. The computer program CUFSM made by Schafer (2002) was taken for this part of calculation. However, there is no calculation for the shear center in CUFSM. Therefore, this part of the calculation is added. In addition, the calculation of elastic local and distortional buckling stresses is carried out using CUFSM in this module.

- Effective section properties module: in this module, the effective section properties such as effective section modulus and effective moment of inertia are calculated. These calculations are both based on Eurocode 3 and modified Eurocode 3 method.

- Free flange behavior module: in this module, the design of the purlin taking the free flange behavior into account based on Eurocode 3 is carried out. The outputs from this module are sent to the fitness calculation modules in GA.

### 4.3.3 Evaluation of the computer program

Normally, the calculation of the fitness function is different from the engineering optimization itself. However, the genetic operations such as the selection, the crossover and the mutation can be used in general. Therefore, the program for optimization using GA can be slightly modified to apply to other optimization problems. Furthermore, this part of the program provides several choices for each genetic operator to the users. In addition, it is possible that in each generation there might exist several occurrences of the same individuals. Especially, after several generations from the beginning of running GA, the numbers of the same individuals are increased. It is quite time consuming to repeatedly evaluate this kind of individuals. Thus, the individuals are classified before the evaluation is started in this part of the program.

As far as the computer program for designing the cold-formed steel purlin is concerned, it can currently be used to design Z-shaped and $\Sigma$-shaped purlins. However, with slight modifications, the program can be used to design purlins of any other shape. Since the design rules for cold-formed purlins are quite complex, engineers will experience difficulties if they try to obtain results by hand calculation. In particular, the calculation of effective cross-section properties can be very tedious and time consuming. This part of the program can be used independently from the optimization so as to provide a design aid for structural designers.

However, there do exist some needs to improve this program. For instance, a friendly Graphic User Interface (GUI) should be added. Moreover, the design of Z-shaped and $\Sigma$-shaped purlins is carried out separately.

# 5 BEHAVIOR OF Z-SHAPED PURLIN

## 5.1 Introduction

The Z-shape purlin is assumed to be connected to trapezoidal sheeting at the wider flange. The loading, the span and the dimensions of the cross-section are shown in Figure 5.1.1. In the figure, $b_1$ is the width of top (wider) flange; $b_2$ is the width of the bottom flange and $c$ is the depth of the lip; $t$ is the thickness of the cross-section; $h$ is the height of the cross-section and $L$ is the span of the purlin. The width of top flange is assumed to be 6 mm wider than that of the bottom flange so as to overlap easily when being transported to the construction site.

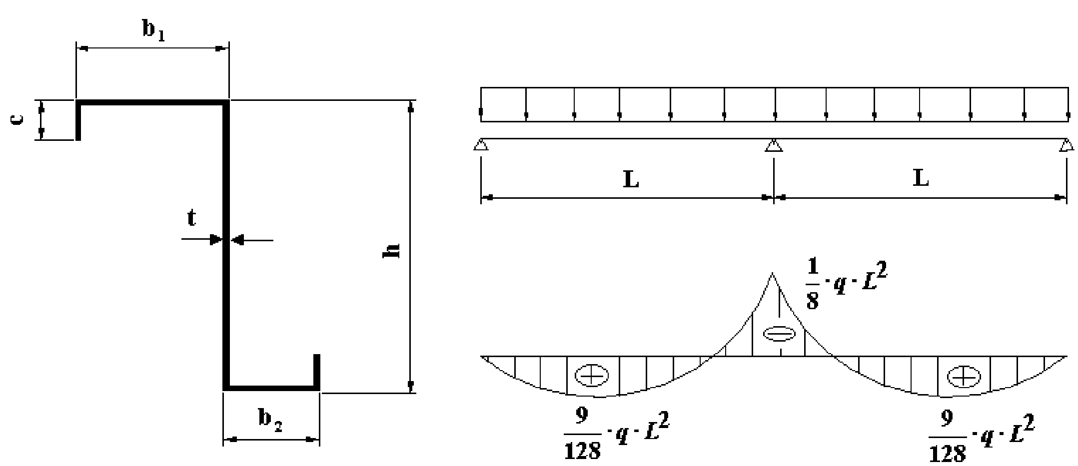



**Figure 5.1.1** Dimensions of Z-shaped purlin continuous over two spans under gravity load

In this chapter, the effects of the dimensions on the effective section properties and on the load resistance considering the free flange behavior are investigated firstly in order to obtain the basic information for the possible optimum dimensions. These investigations are based on the Eurocode 3 method. In order to integrate the numerical elastic buckling stress into the modified Eurocode 3 method, the elastic buckling stresses are calculated using computer software named CUFSM developed by Schafer (Schafer, B., 2002) based on Finite Strip Method (FSM). These results are compared with those calculated using the Eurocode 3 method. Finally, the moment capacity and load efficiency calculated based on the modified Eurocode 3 method are compared with those calculated based on Eurocode 3 method.

The calculations in this chapter are carried out for the cross-sections with the width of the flange being varied from 40 mm to 100 mm with step of 10 mm; the ratio of $c$ to $b_2$ being varied from 0.2 to 0.6 with step of 0.2; and the height of being varied from 100 mm to 300 mm with step of 100 mm. The thickness of the cross-section is assumed to be 2 mm and the span of the purlin is assumed to be 4 m. The yield strength is 350 MPa and Young's modulus is 210 GPa.

## 5.2 Purlin Design Based on Eurocode 3 Method

In this section, the effects of the dimensions of the cross-section on the resistance of the cross-section and on the load resistance considering the free flange behavior are investigated. The calculations are based on Eurocode 3, Part 1.3. The symbol, like h100_02, shown in the figures, means that the height of the cross-section is 100 mm and the ratio of $c$ to $b_2$ is 0.2.

### 5.2.1 Effects of the dimensions on the cross-section resistance

Figure 5.2.1 shows the effects of the dimensions on the moment capacity of the cross-section. As for the moment capacity, its value is increased with the increase of the width of the flange. This is due to the fact that the effective width for the wider flange is larger than the fully effective width for the narrower flange. For instance, the effective width of flange of 100 mm is higher than the fully effective width of flange of 40 mm. However, the slope of increase is changed when the width of the top flange is more than 70 mm because the top flange is not fully effective after this value. Similarly, the value of moment capacity is increased with the increase of the ratio of $c$ to $b_2$ and the height of the web.

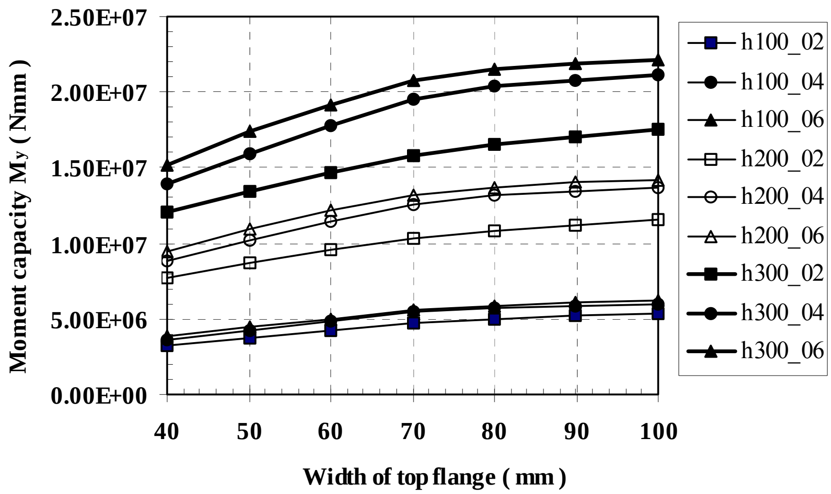



**Figure 5.2.1** Effects of the dimensions on the moment capacity

Figure 5.2.2 shows the effects of the dimensions on the moment efficiency, which is defined as the ratio of the moment capacity to the gross area of the cross-section. As far as the moment efficiency is concerned, its value is decreased with the increase of the gross area of the cross-section. Thus, the maximum value of moment efficiency does not correspond to the widest flange and largest ratio of $c$ to $b_2$. This is shown in Figure 5.2.2 (b). The maximum value of moment efficiency is reached when the width of the top flange is 70 mm and the ratio of $c$ to $b_2$ is 0.4.

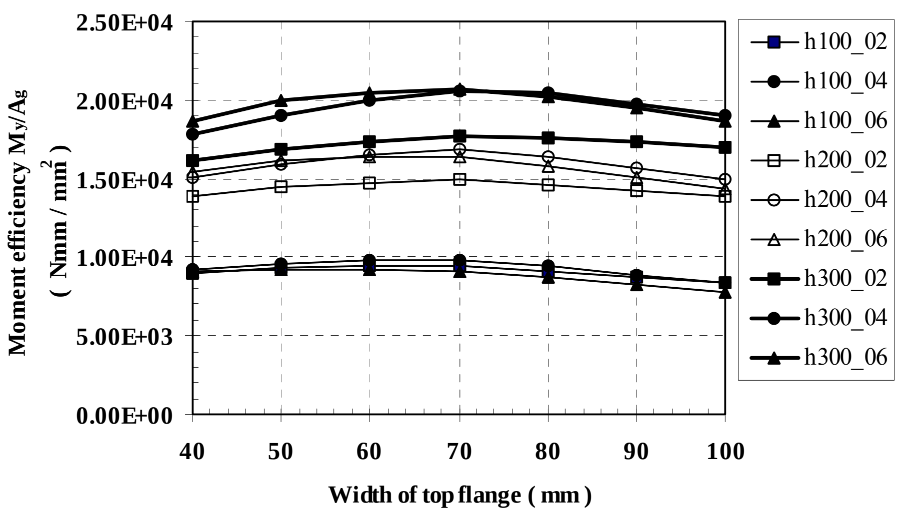



**Figure 5.2.2** Effects of the dimensions on the moment efficiency

### 5.2.2 Purlin design considering the free flange behavior based on Eurocode 3 method

Figure 5.2.3 shows the effects of the dimensions of the cross-section on the load resistance applied to the purlin. This load resistance is increased when the ratio of $c$ to $b_2$ is increased from 0.2 to 0.6 and the width of the flange is increased from 40 mm to 100 mm. The load resistances with a height of 200 mm are higher than those with a height of 100 mm. However, when the height of the cross-section is increased to 300 mm, the load resistances with $c/b_2 = 0.2$ are lower than those with $c/b_2 = 0.4$ or $c/b_2 = 0.6$ and a height of 200 mm. This is due to the fact that the gross section modulus of the free flange is decreased for the cross-section with a height of 300 mm when comparing to those with $c/b_2 = 0.4$ or $c/b_2 = 0.6$, and a height of 200 mm.

Figure 5.2.4 shows the effects of the dimensions on the load efficiency, which is defined as the ratio of the load resistance to the gross area of the cross-section. The load efficiency is not only affected by the load resistance but also by the area of the cross-section. Thus, when the height of the cross-section is increased from 100 mm to 200 mm, the load efficiency is increased. However, for the sections with a height of 300 mm, the load efficiency is decreased for some sections. Besides, the load efficiency is decreased for cross-sections with longer flanges and deeper lips although the load resistance is higher for these sections.

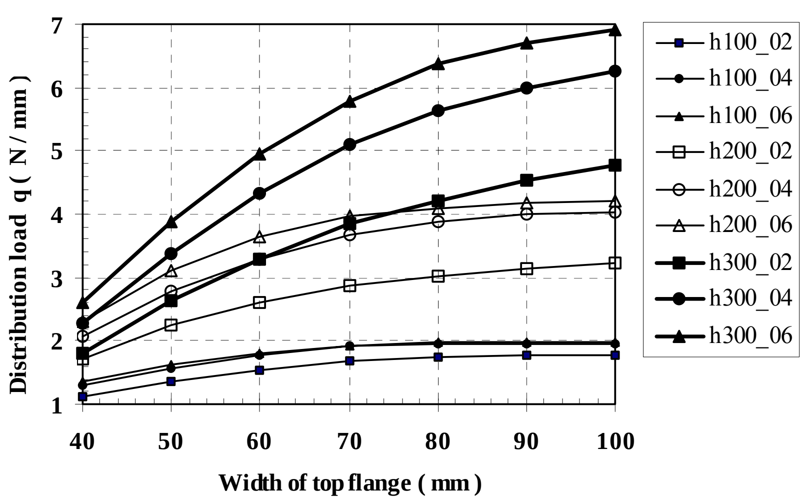



**Figure 5.2.3** Effects of the dimensions on the load resistance

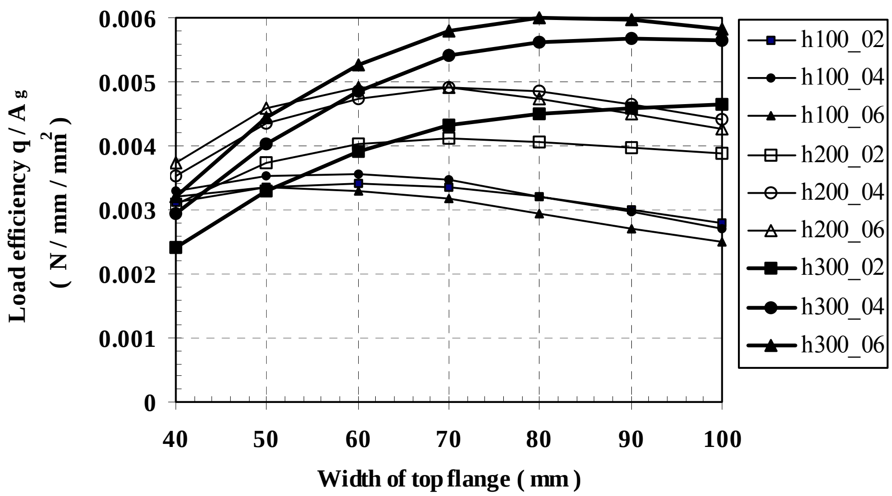



**Figure 5.2.4** Effects of the dimensions on load efficiency

In summary, the moment capacity is increased with the increase of such dimensions as the width of the flange, the depth of the lip and the height of the web. This is due to the fact that effective widths of the longer elements are larger than those of the shorter fully effective elements. As far as the load resistances are concerned, their values depend not only on the moment capacity but also on the gross section modulus of the free flange plus $1/6$ of the web height. Thus, it is possible that the value of the load resistance for the cross-section with longer web height is lower than that with shorter web height. Moreover, as for the moment efficiency or the load resistance efficiency, the maximum value does not occur at the largest dimensions due to the higher area of such cross-section.

## 5.3 Effects of the Dimensions of the Cross-Section on the Elastic Buckling Stress based on Finite Strip Method

The buckling modes of cold-formed steel sections can be classified as local, distortional and flexural-torsional buckling modes. Local buckling is characterized by internal buckling of the elements of the cross-sections in which there is no relative movement of the nodes. Flexural-torsional buckling is a rigid-body translation of a cross-section without any change in the cross-section shape. Distortional buckling involves the movement of the nodes relative to one another and occurs at half-wavelength between local and flexural-torsional buckling.

### 5.3.1 Elastic buckling stress calculated based on FSM

Some typical buckling curves calculated using CUFSM is shown in Figure 5.3.1 in which the symbols like 'h200_b50_02' represents the cross-section with a height of 200 mm, a width of the flange of 50 mm and $c/b_2 = 0.2$.

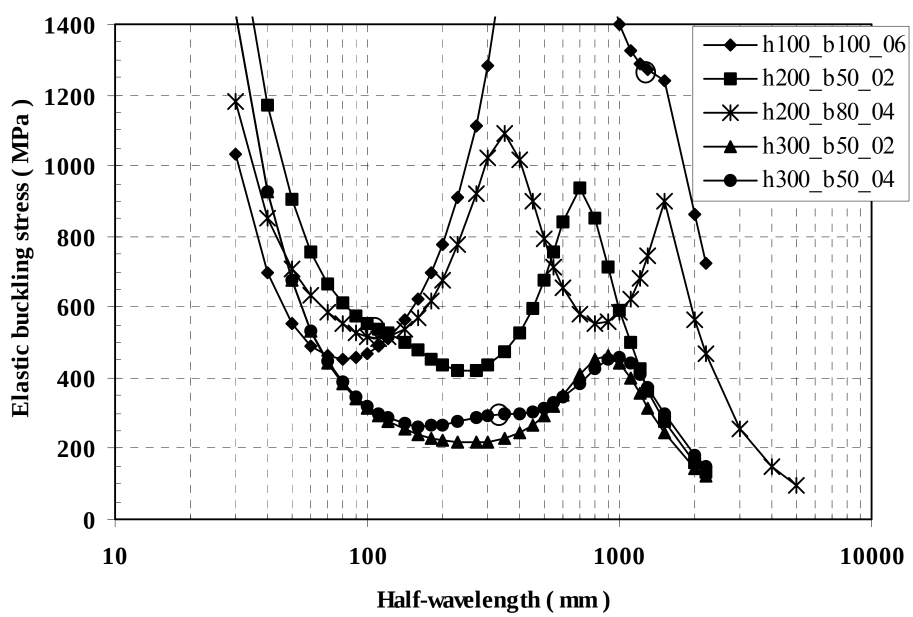
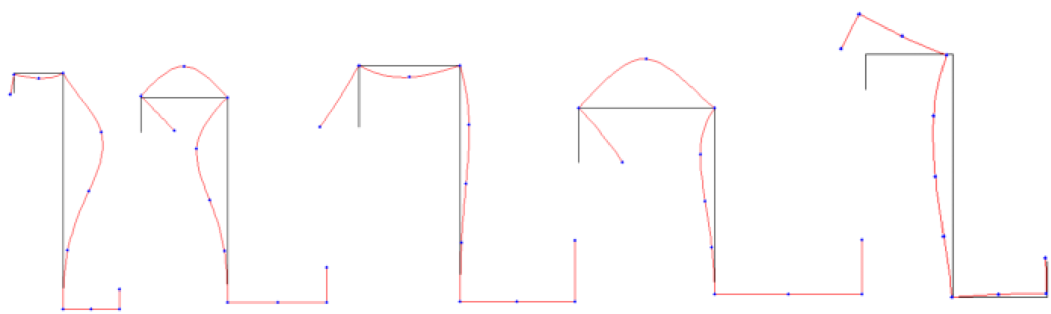



**Figure 5.3.1** Elastic buckling curves ( $t = 2.0$ mm )

The curve, 'h200_b80_04', is a 'standard' buckling curve. The minima of this curve indicate the critical elastic buckling stresses, i.e. the first minimum is the critical local buckling stress and the second minimum is the critical distortional buckling stress. However, for some sections there might exist an indistinct distortional buckling mode (h100_b100_06 and h300_b50_04) and local buckling mode (h200_b50_02), respectively, as shown in Figure 5.3.1. These indistinct buckling modes make the determination of the elastic buckling stress difficult when integrating the finite strip method into the design procedure of Eurocode 3. According to Schafer (2002), the elastic buckling stress in this case can be determined as the minimum value before the current buckling mode is transformed into another buckling mode.

These values are highlighted with circles. Besides, for some sections there is no local buckling mode at all (h300_b50_02). Thus, for this kind of cross-section, the elastic buckling stress is calculated using the Eurocode 3 method in the modified Eurocode 3 design procedure.

Figure 5.3.2 shows the typical local and distortional buckling modes. The main local buckling mode is the combination of the lip, flange and web buckling. However, due to the difference of the width for each element, the buckling is initiated by the longest element amongst the lip (Figure 5.3.2 (c)), the flange (Figure 5.3.2 (d)) and the compression part of the web (Figure 5.3.2 (a)). When the length of each element is similar, the buckling mode is shown in Figure 5.3.2 (b), i.e. the buckling of each element occurs at the same time. In addition, when the lengths of the lip and the flange are not large enough, there exists no local buckling mode, i.e. the distortional buckling mode is the only buckling mode. The typical distortional buckling mode is shown in Figure 5.3.2 (e).



(a) (b) (c) (d) (e)

**Figure 5.3.2** Elastic local and distortional buckling modes

Figure 5.3.3 and Figure 5.3.4 show effects of variations of the dimensions on the elastic local and distortional buckling stresses. As for the cross-section with a height of 100 mm, when $c/b_2 = 0.4$, the local buckling stress is decreased with the increase of the width of the flange. This is due to the fact that the local buckling mode is transformed from mode (a) via mode (b) to mode (d). When $c/b_2 = 0.6$, the variation of the local buckling stress has a similar trend to that with $c/b_2 = 0.4$. However, when the width of the flange varied from 40 mm to 70 mm, the local buckling stress with $c/b_2 = 0.6$ is lower than that with $c/b_2 = 0.4$. This is because of the buckling mode (c) for $c/b_2 = 0.6$ rather than the buckling mode (b). In addition, when $c/b_2 = 0.2$ and the width of the flange is 40 mm or 50 mm, no local buckling mode exists, i.e. the distortional buckling mode is the only buckling mode.

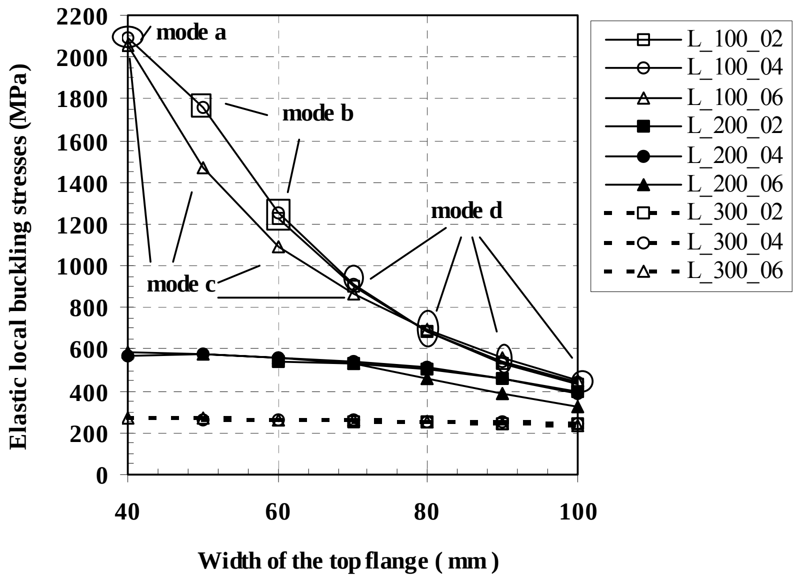



**Figure 5.3.3** Elastic local buckling stresses

As far as the cross-sections with a height of 200 mm are concerned, the local buckling modes are transformed from mode (a) (the width of the flange is less than 80 mm) to mode (b) when $c/b_2 = 0.2$ or $c/b_2 = 0.4$. However, when $c/b_2 = 0.6$, the buckling mode is transformed from mode (a) to mode (c) with the same variation of the width of the flange. If the height of the cross-section is increased to 300 mm, mode (a) is the only critical buckling mode and the variations of the local buckling stresses are almost the same when the width of the flange is increased from 40 mm to 100 mm and the ratio of $c$ to $b_2$ is increased from 0.2 to 0.6.

In general, the differences between the width of the flange and the compression part of the web mainly determine the elastic buckling stress. When the width of the flange is larger than that of the compression part of the web, the critical buckling mode is flange buckling. The elastic local buckling stress can be increased by the increase of the lip. However, the increase is not so large. On the other hand, when the width of the flange is less than that of the compression part of the web, the critical buckling mode is web buckling. When the width of the flange and the depth of the lip are not large enough, there exists only the distortional buckling mode instead of the local buckling mode. Moreover, when lengths of these two elements are similar, the critical buckling mode is mode (b). The elastic bucking stress is increased when the depth of the lip is increased, though the increase is not so large. However, if the depth of the lip is increased further, the critical buckling mode is transformed from buckling mode (b) to buckling mode (c). Besides, when the height is increased from 100 mm to 300 mm, the elastic buckling stresses are decreased with a constant value of width of the flange and depth of the lip. However, when the width of the flange is increased to 80 mm, the differences of the local buckling stresses are not so large due to the critical buckling mode of the flange.

When the height of the cross-section and the width of the flange are fixed, the elastic distortional buckling stress is increased with the depth of the lip. When the width of the flange and the depth of the lip are fixed, the elastic distortional buckling stress decreases with the increase of the height of the web. However, the effects of the width of the flange on the elastic distortional buckling stress are varied with the critical local buckling mode. If mode (a) is the critical buckling mode, the elastic distortional buckling stress is increased with the increase of the width of the flange (cross-sections with a height of 300 mm). If the critical buckling mode is mode (b) or mode (d), the elastic distortional buckling stress is decreased with the increase of the width of the flange (cross-sections with a height of 100 mm).

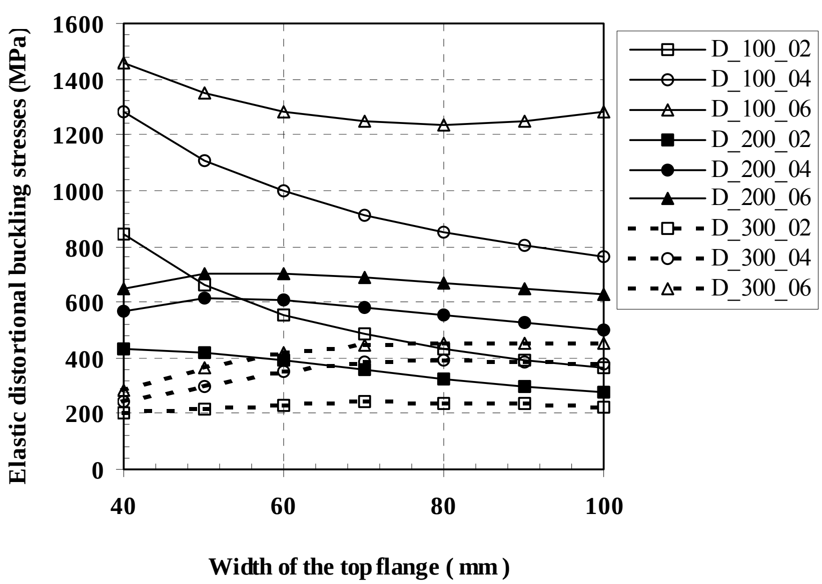
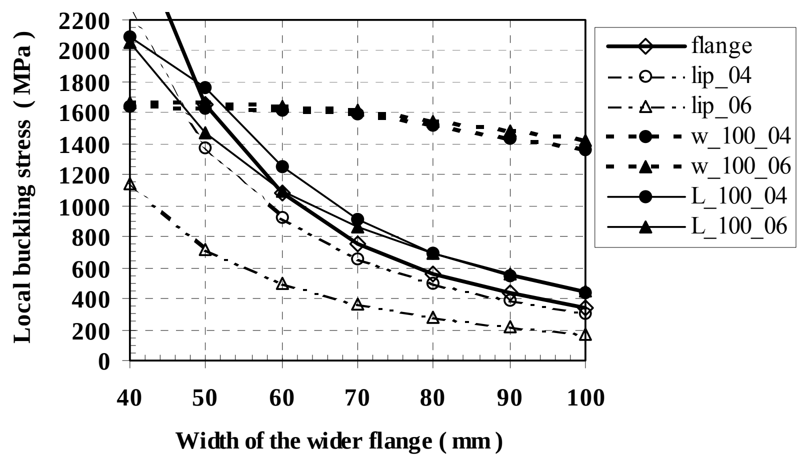


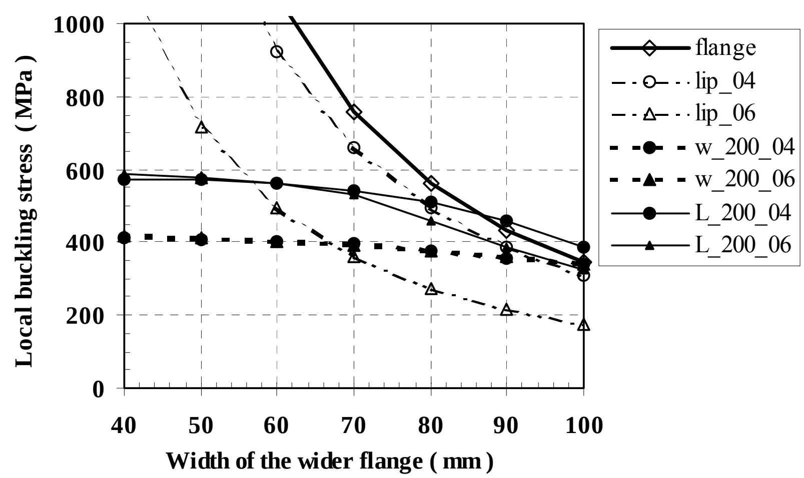

**Figure 5.3.4** Elastic distortional buckling stress

### 5.3.2 Comparison of elastic buckling stress calculated using Eurocode 3 method and finite strip method

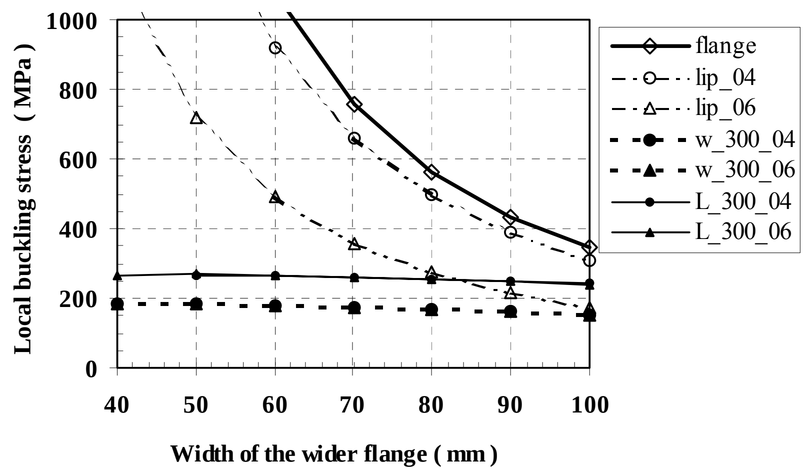
Figure 5.3.5 shows the comparisons of elastic local buckling stresses calculated using the Eurocode 3 method. Such symbols as 'lip_04', 'w_100_04' and 'flange' represent the elastic buckling stress of lip, web and flange, respectively, calculated using the Eurocode 3 method. The symbols such as 'L_100_04' represent the local buckling stress calculated using FSM.




**(a)** h = 100 mm




**(b)** h = 200 mm

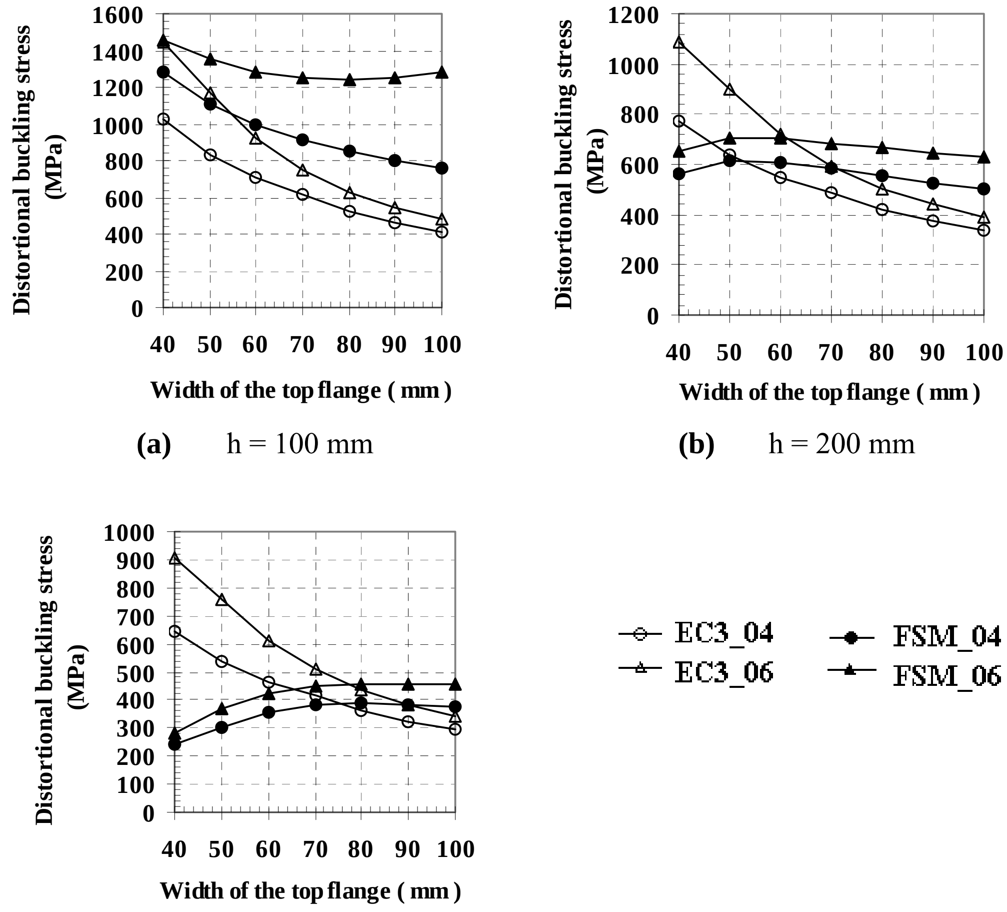



**(c)** h =300 mm

**Figure 5.3.5** Comparisons of elastic local buckling stresses calculated using Eurocode 3 method with those calculated by FSM

When web buckling is the critical local buckling mode, the elastic local buckling stresses calculated using FSM are higher than web buckling stresses calculated using the Eurocode 3 method as shown in Figure 5.3.5 (b) and (c). This is due to the fact that in Eurocode 3, the web element is assumed as simply supported at both ends; while in FSM, the interaction of the flange and web is included. For a similar reason, the elastic buckling stresses calculated using FSM are higher than corresponding elastic buckling stresses calculated using the Eurocode 3 method when the lip-buckling (Figure 5.3.5 (a) if the width of the flange is less than 70 mm and $c/b_2 = 0.6$) and flange buckling (Figure 5.3.5 (a) if the width of the flange is larger than 70 mm and $c/b_2 = 0.4$) are critical, respectively.

Figure 5.3.6 shows the comparisons of elastic distortional buckling stresses calculated using the Eurocode 3 and FSM. For most cross-sections, the values of the elastic distortional buckling stresses calculated using FSM are higher than those calculated using the Eurocode 3 method. However, when the critical local buckling mode is the web-buckling mode, the distortional buckling stress calculated using FSM is less than those calculated using the Eurocode 3 method. This is due to the fact that the effect of the reduction in the flexural restraints provided by the buckling web is not considered in Eurocode 3.




(c) h = 300 mm

**Figure 5.3.6 Comparisons of elastic distortional buckling stresses calculated by EC3 with those calculated by FSM**

## 5.4 Design of Z-Shape Purlins Based on Modified Eurocode 3 Method

The typical buckling modes for Z-shape purlins are local buckling of flange, the lip and the compression part of the web and the distortional buckling mode as shown in Figure 5.3.2. When the effective widths of the flange, the lip and the compression part of the web are determined, the reduction factor, $\rho$, is calculated using the elastic buckling stress corresponding to the first type of buckling mode. The effective width for the flange is

$$b_{eff,1} = 0.5 \cdot \rho \cdot b, \text{ and}$$
$$b_{eff,2} = 0.5 \cdot \rho \cdot b. \qquad \textbf{(5.4.1)}$$

The effective width of the lip is

$$c_{eff} = \rho \cdot c \,. \qquad \textbf{(5.4.2)}$$

and the effective width of the compression elements is

$$h_{ceff,1} = 0.4 \cdot \rho \cdot h_c, \text{ and}$$
$$h_{ceff,2} = 0.6 \cdot \rho \cdot h_c, \qquad \textbf{(5.4.3)}$$

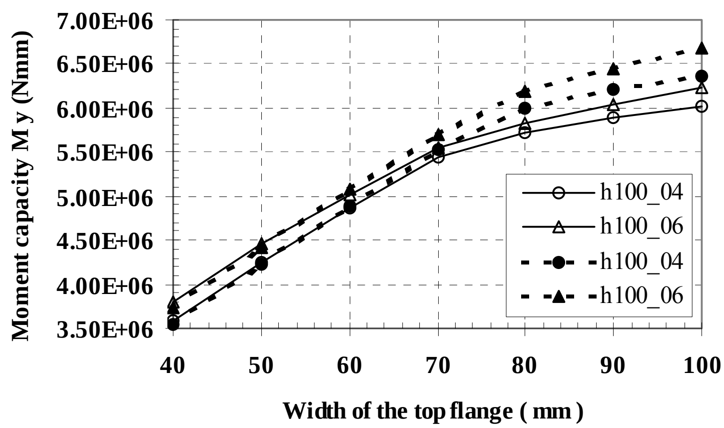
in which $h_c$ is the length of the compression part. When the reduced thickness for the edge stiffener is determined, the reduction factor, $\chi$, is calculated using the distortional buckling stress corresponding to the distortional buckling mode.

## 5.5 Comparison of Eurocode 3 Method and Modified Eurocode 3 Method

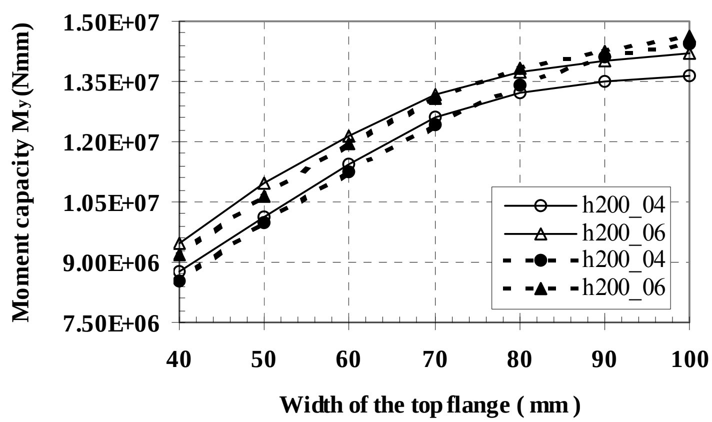
A symbol in the figures shown in this section such as 'EC3_04' means that the calculation is based on the Eurocode 3 method with $c/b_2 = 0.4$; and 'MEC3_04' represents the calculation is based on Eurocode 3 method with $c/b_2 = 0.4$.

### 5.5.1 Comparisons of moment capacity

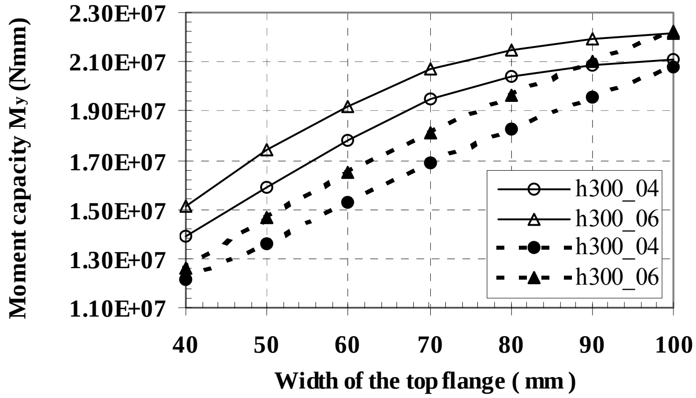
Figure 5.5.1 shows comparisons of the moment capacities of the cross-section calculated using the Eurocode 3 method to those calculated using the modified Eurocode 3 method. If the flange buckling is the critical buckling mode, the moment capacity calculated using the modified Eurocode 3 method is higher than that calculated using the Eurocode 3 method (Figure 5.5.1 (a)) due to the fact that the elastic buckling stresses calculated using FSM are higher than those calculated using the Eurocode 3 method. If the web buckling is the critical buckling mode, the moment capacity calculated using the modified Eurocode 3 method is lower than that calculated using the Eurocode 3 method (Figure 5.5.1 (b) and (c)). Two reasons lead to this result; one is that the effect of the reduction in the flexural restraints provided by the buckling web is not considered in Eurocode 3. The other is that in the modified Eurocode 3 method, the same elastic buckling stress is applied to determine the effective width for the flange, lip and web. When the height of the web is 200 mm, the difference is not so large. However, when the height of the web is 300 mm, the difference is greater.




**(a)** h = 100 mm




**(b)** h = 200 mm

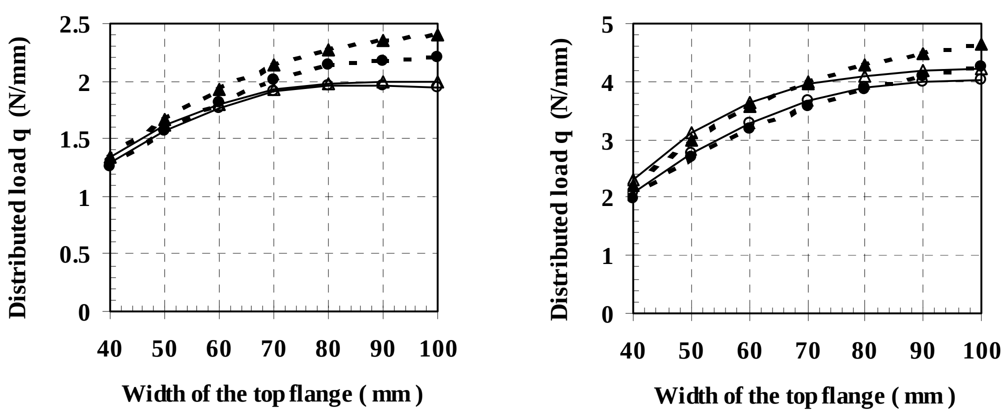



**(c)** h = 300 mm

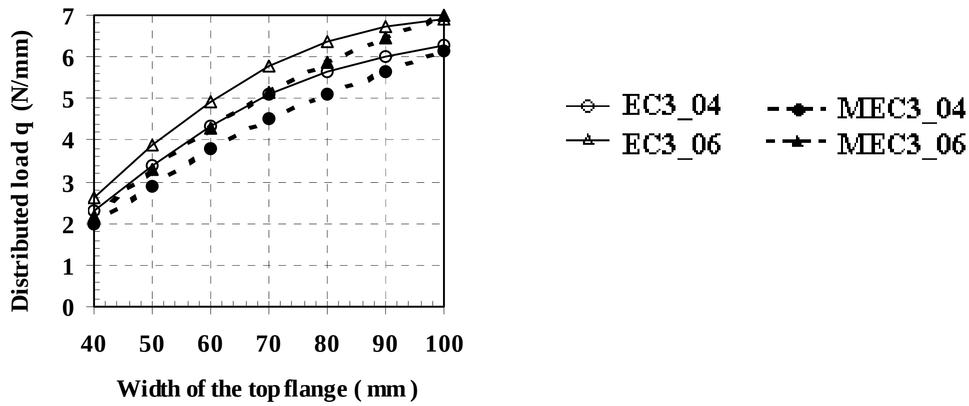
**Figure 5.5.1** Comparison of moment capacities calculated using the Eurocode 3 method to those using the modified Eurocode 3 method

### 5.5.2 Comparisons of load distribution and load efficiency

A comparisons of the load resistances calculated using the Eurocode 3 method to those calculated using the modified Eurocode 3 method are shown in Figure 5.5.2. When comparing to Figure 5.5.1, it can be seen that the variations of the load resistances have similar trends to those of moment capacity since the in-plane bending is dominating instead of bending due to the lateral load.




**(a)** h =100 mm **(b)** h =200 mm




**(c)** h = 300 mm

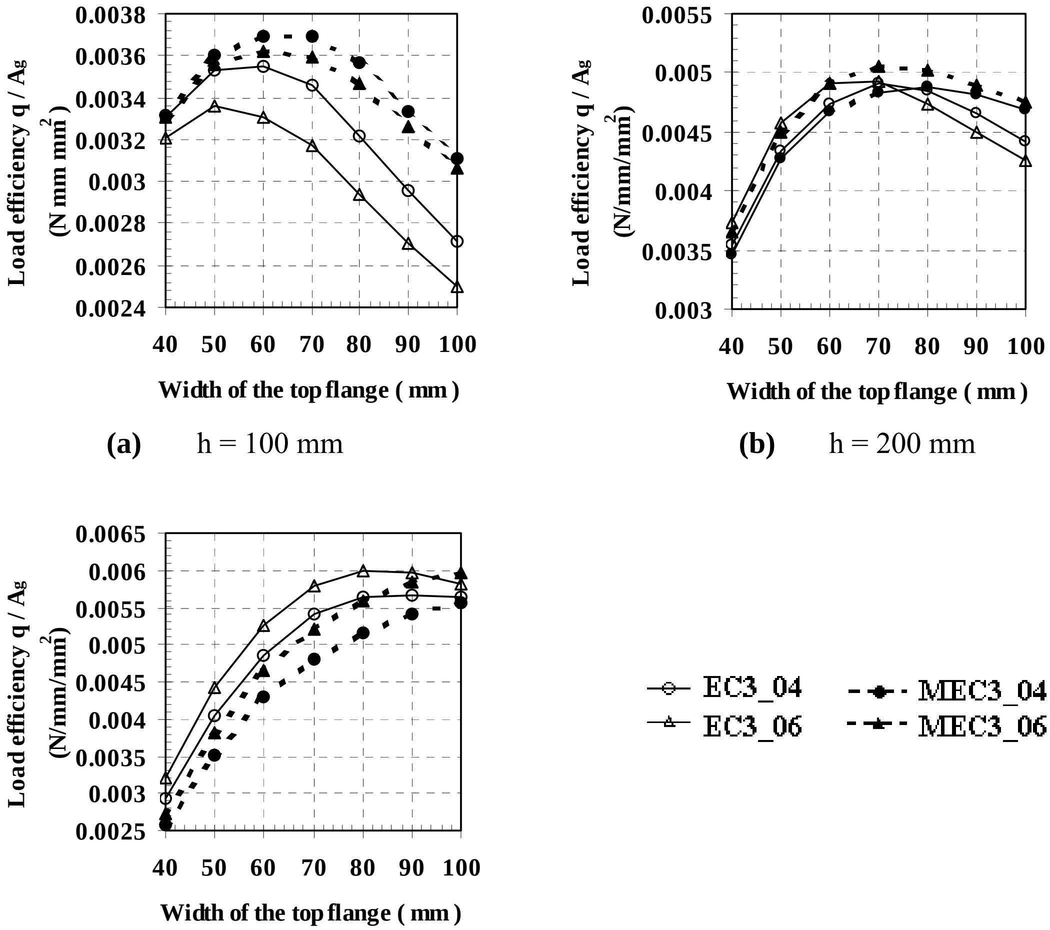
**Figure 5.5.2** Comparisons of load resistances calculated using the Eurocode 3 method to those calculated using the Modified Eurocode 3 method

Figure 5.5.3 shows the comparisons of load efficiencies calculated using the Eurocode 3 method to those calculated using the modified Eurocode 3 method. The optimum dimensions obtained using these two methods are not same due to the different values of the load resistance and the area of the cross-section.

### 5.5.3 Discussion

In the current Eurocode 3 method, the calculation of bending moment in the free flange due to the lateral load for a Z-shape purlin can be only applied to the cross-section with an equal width of the two flanges. In this thesis, the width of the top flange is 6 mm wider than that of the bottom flange. However, the calculation is still based on the rules specified in Eurocode 3, Part 1.3. Thus, the influence of the position of the shear center and the widths of the two flanges on the behavior of the Z-shape purlins should be investigated further.

Besides, the purlin is attached to the profiled sheeting using screws. This sheeting can provide lateral and rotational restraints to the purlin. The calculation of the local buckling stress using FSM did not consider the effects of these restraints.




**(a)** h = 100 mm

**(b)** h = 200 mm

**(c)** h = 300 mm

**Figure 5.5.3 Comparisons of load efficiency calculated using the Eurocode 3 method to those calculated using the modified Eurocode 3 method**

# 6 OPTIMIZATION OF Z-SHAPE PURLINS

In this chapter, the dimensions of the Z-shape purlins, which have been investigated in Chapter 5, are optimized using the computer program, **ODSP-GA**, developed in Chapter 4.

## 6.1 Descriptions of Problems

The design variables are the width of the top (wider) flange, $b_1$, and the depth of the lip, $c$. Other parameters, such as the thickness of the cross-section, $t$, the height of the cross section, $h$, and the span of the purlin, $L$, are the given variables.

According to Eurocode 3, the ratio of $c$ to $b_2$ should satisfy the formula

$$0.2 \le \frac{c}{b_2} \le 0.6 . \qquad \textbf{( 6.1.1 )}$$

Assuming $\alpha = \frac{c}{b_2}$, the depth of the lip, $c$, can be replaced by $\alpha$. The width of top flange is assumed to be 6 mm greater than that of the bottom flange. The possible values for the design variables are given in Table 6.1.1.

**Table 6.1.1** Values for the design and given variables

| Design variables | Possible values |
|---|---|
| Width of the top flange $b_1$ | Varied from 40 mm to 100 mm with step 1 mm |
| Ratio of $c$ to $b_2$, $\alpha$ | Varied from 0.2 to 0.6 with step 0.01 |

The optimization problem is defined as

$$Maximize \qquad \frac{q}{A_g}, \tag{6.1.2}$$

subjected to the following geometrical constraints specified in Eurocode 3 and fabrication constraints:

$$\frac{h}{t} \le 500, \quad \frac{b_1}{t} \le 60, \quad \text{and} \quad 200 \le L_{stip} \le 625, \tag{6.1.3}$$

where the length of the strip is calculated as:

$$L_{strip} = b_1 + b_2 + 2 \cdot c + h. \tag{6.1.4}$$

By integrating the penalty function into the fitness evaluation described in Chapter 4, the constrained problem is transformed into the following unconstrained problem:

$$Maximize \quad F = \begin{cases} \dfrac{q}{A_g} - KK \cdot n \cdot CC & \text{when } \dfrac{q}{A_g} > KK \cdot n \cdot CC, \\ 0 & \text{otherwise.} \end{cases} \tag{6.1.5}$$

## 6.2 Verification Problem

In order to demonstrate the capability and the optimization process of the GA, an example is presented. In the following calculation, the height of the cross-section is set to 150 mm, the thickness of the cross section to 1.5 mm and the length for single span is 4500 mm. Besides, the yield strength is 350 MPa and Young's modulus is 210000 MPa.

### 6.2.1 Optimization process

Encoding the design variables into fixed-length bit strings is the first step of GA. Binary encoding is used here. The optimization problem described here is a multi-parameter problem. The multi-parameter can be encoded to a single string by simply concatenating the design variables end by end. For example, two design variables are defined in this optimization problem, $b_1$ and $\alpha$. The possible values of $b_1$ are given in Table 6.1.1, which can be represented using six bits. Similarly, the values of $\alpha$ can also be coded using six bits. By concatenating these two 6-bit strings, a single 12-bit string is formed as shown in Figure 6.2.1.

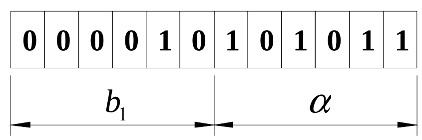



**Figure 6.2.1** Individual string with binary encoding

The actual value of a design variable is calculated from each corresponding binary string. For instance, if the coded binary string is '000010' for $b_1$, the value corresponding to this string can be calculated as $0\times2^5+0\times2^4+0\times2^3+0\times2^2+1\times2+0\times2^0=2$. Therefore, the width of the top flange is selected as 41 mm, which is the second value in the range of 40 mm to 100 mm with step 1 mm. However, it is possible that the decoded value is larger than the possible number of each design variable. In this case, some values of the design variables are selected twice.

The details for generating the first and the second generation using GA are shown in Table 6.2.1 and Table 6.2.2. The GA starts from randomly generating a group of binary strings (individuals), which are listed in column (2). For demonstration purposes, only eight individuals are listed here. After decoding, the values of the design variables are listed in columns (3) to (4). Then, the constraints are evaluated with results listed in columns (6) to (8) and the calculated penalty (9) is integrated into the computation of the fitness function (10). Using the tournament selection method, the better individuals are selected and sent to the mating pool (12). Using a two-point crossover operator with probability of 80 % (13) and a one-point mutation operator with probability of 0.001 (14), the first generation (15) is generated. In column (15), two better individuals from previous generation, which are called elitisms, are kept. This first generation is treated at the initial population and the procedure in forming the first generation is repeated until the maximum number of generation provided is reached.

### 6.2.2 Selection of parameters

There are several options for each genetic operator as given in Chapter 3, such as the fitness proportional selection method, the rank selection method and the tournament selection method, the probability of crossover (crossover rate), the probability of mutation (mutation rate) and the size of the population. In this section, the effects of these options on the behavior of GA are investigated. The investigation is based on the crossover rate of 0.8, the population size of 24 and the mutation rate of 0.001 if the parameters are not specified. The curve in the figures for each given parameter is chosen as the maximum value of fitness for five runs of the GA, in which one run is defined as the complete run of GA.

Figure 6.2.2 shows the effects of such selection methods as the fitness proportional method, the rank selection method and the tournament selection method on the value of fitness. It can be seen that each method leads to a convergence of the calculation. However, the speed of convergence for the rank selection is slower than those for the other two methods. Moreover, there is no big difference when using either the tournament selection method or the proportional method in this case. Thus, the tournament selection method is used for further analysis. In addition, the values of the fitness show no large variations from generation of 30 to 80 in the process of convergence based on tournament selection. Therefore, the maximum generation number for stopping running the GA is chosen as 30.

**Table 6.2.1** The details of computation: generation 1

| No (1) | Population (2) | $b_1$ (3) | $\alpha$ (4) | c (5) | Total length (6) | $\frac{h}{t}$ (7) | $\frac{b_1}{t}$ (8) | Penalty (9) | Fitness (10) | Selected times (11) | Mating pool (12) | Cross-over position (13) | Mutation position (14) | New generation (15) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 110111101110 | 95 | 0.25 | 23.75 | 378.50 | 102.74 | 65.07 | 0.007136 | 0 | 0 | 000101110100 | ------- | ------- | 000101110100 |
| 2 | 101001000110 | 81 | 0.26 | 21.06 | 345.00 | 102.74 | 55.47 | 0 | 0.002737 | 0 | 100110001100 | | ------- | 100110001100 |
| 3 | 111010011001 | 98 | 0.45 | 44.10 | 422.80 | 102.74 | 67.12 | 0.04753 | 0.000667 | 1 | 111010011001 | 11, 12 | ------- | 111010011000 |
| 4 | 000101110100 | 45 | 0.31 | 13.95 | 258.18 | 102.74 | 30.82 | 0 | 0.002974 | 2 | 100110001100 | | ------- | 100110001101 |
| 5 | 100110001100 | 78 | 0.32 | 24.96 | 346.08 | 102.74 | 53.42 | 0 | 0.002888 | 2 | 101101001100 | 4, 6 | ------- | 101101001100 |
| 6 | 010111000011 | 63 | 0.23 | 14.49 | 296.22 | 102.74 | 43.15 | 0 | 0.002896 | 1 | 000101110100 | | ------- | 000101110100 |
| 7 | 100101000000 | 77 | 0.20 | 15.40 | 326.40 | 102.74 | 52.74 | 0 | 0.002620 | 0 | 101101001100 | 7, 7 | ------- | 101101000011 |
| 8 | 101101001100 | 85 | 0.32 | 27.20 | 364.56 | 102.74 | 58.22 | 0 | 0.002749 | 2 | 010111000011 | | ------- | 010111001100 |

**Table 6.2.2** The details of computation: generation 2

| No (1) | Population (2) | $b_1$ (3) | $\alpha$ (4) | c (5) | Total length (6) | $\frac{h}{t}$ (7) | $\frac{b_1}{t}$ (8) | Penalty (9) | Fitness (10) | Selected times (11) | Mating pool (12) | Cross-over position (13) | Mutation position (14) | New generation (15) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 000101110100 | 45 | 0.31 | 13.95 | 258.18 | 102.74 | 30.82 | 0 | 0.002974 | 3 | 010111001100 | 7, 8 | ------- | 010111001100 |
| 2 | 100110001100 | 78 | 0.32 | 24.96 | 346.08 | 102.74 | 53.45 | 0 | 0.002888 | 0 | 100110001101 | | ------- | 100110001101 |
| 3 | 111010011000 | 98 | 0.44 | 43.12 | 420.96 | 102.74 | 67.12 | 0.03861 | 0.001022 | 0 | 000101110100 | -------- | ------- | 000101110100 |
| 4 | 100110001101 | 78 | 0.33 | 25.74 | 347.52 | 102.74 | 53.42 | 0 | 0.002893 | 3 | 100110001101 | | ------- | 100110001101 |
| 5 | 101101001100 | 85 | 0.32 | 27.20 | 364.56 | 102.74 | 58.22 | 0 | 0.002749 | 0 | 000101110100 | 3, 11 | ------- | 000110001100 |
| 6 | 000101110100 | 45 | 0.31 | 13.95 | 258.18 | 102.74 | 30.82 | 0 | 0.002974 | 0 | 100110001101 | | ------- | 100101110101 |
| 7 | 101101000011 | 85 | 0.23 | 19.55 | 350.34 | 102.74 | 58.22 | 0 | 0.002594 | 0 | 000101110100 | 5, 6 | ------- | 000101110100 |
| 8 | 010111001100 | 63 | 0.32 | 20.16 | 306.48 | 102.74 | 43.15 | 0 | 0.003143 | 2 | 010111001100 | | ------- | 010111001100 |

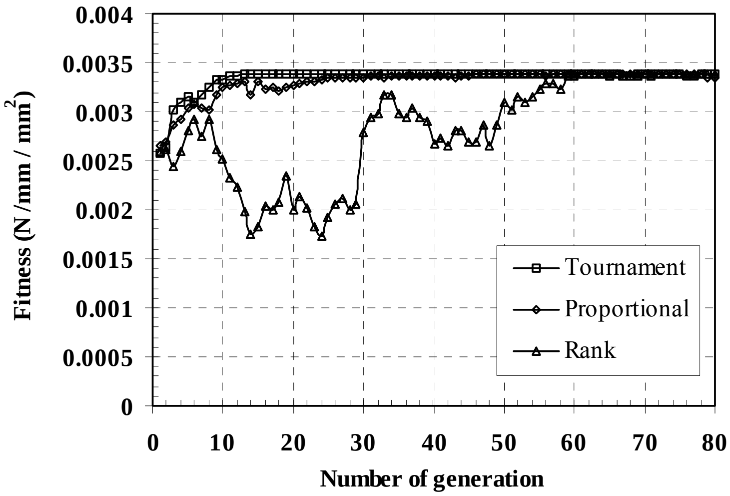



**Figure 6.2.2** Effects of selection methods on the value of $q/A_g$

Such parameters as the size of the population, the crossover rate and the mutation rate have the influences on the behavior of the GA. The size of the population is at least equal to the length of the binary string so as to make all the combinations possible. Figure 6.2.3 shows the effects of the population size on the values of the fitness. The size of the population is set to one, one and a half, two, and two and a half times the length of each individual. In this analysis, the length of the individual is 12, thus, the population sizes are 12, 18, 24 and 30. The size of the population has no big influence in this case and a population size of 24 is chosen for further analysis.

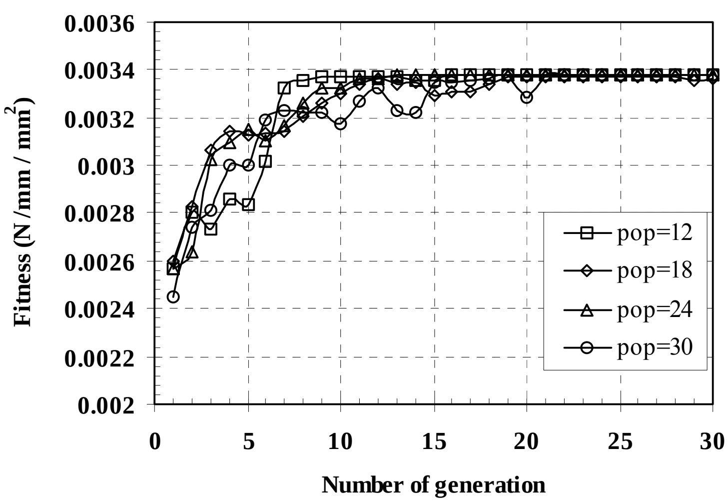



**Figure 6.2.3** Effect of the population size on the performance of GA

The crossover rate describes how often crossover occurs and the suggested value is from 0.6 to 0.9 according to Mitchell (1996). Figure 6.2.4 shows the effects of crossover rate on the performance of GA as the rate of crossover is varied from 0.6 to 0.9 with the population size being set to 30 and mutation rate being set to 0.001. In this case, the values of the crossover rate have no big influence on the performance of GA and the value of 0.8 is chosen for further analysis.

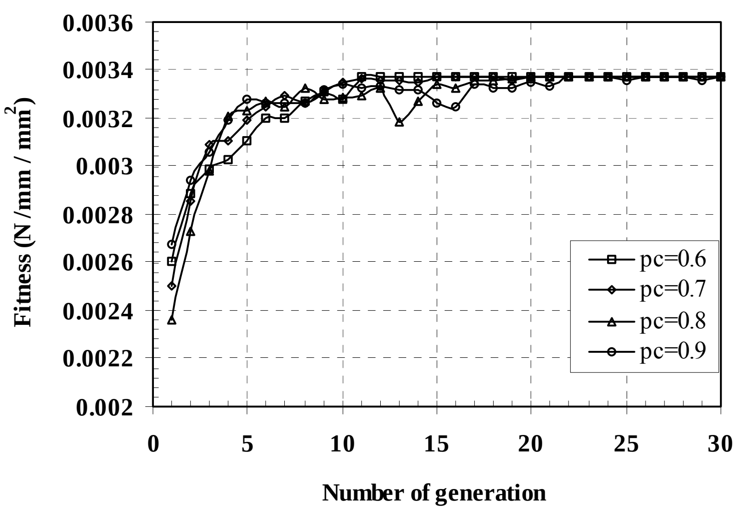



**Figure 6.2.4** Effects of crossover rate on the performance of the GA

Mutation is used to exploit new information during the running of GA. Figure 6.2.5 shows the effects of the mutation rate on the performance of the GA. When the rate of mutation is varied from 0.0001 and 0.01, the GA shows the better behavior than that when the rate of mutation is 0.1. The value of 0.001 is chosen in the further calculation. This analysis is carried out with the population size of 30 and crossover rate of 0.8.

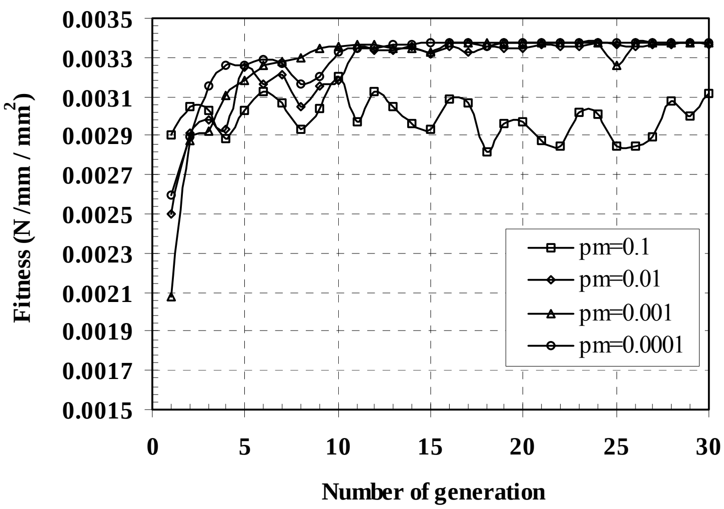
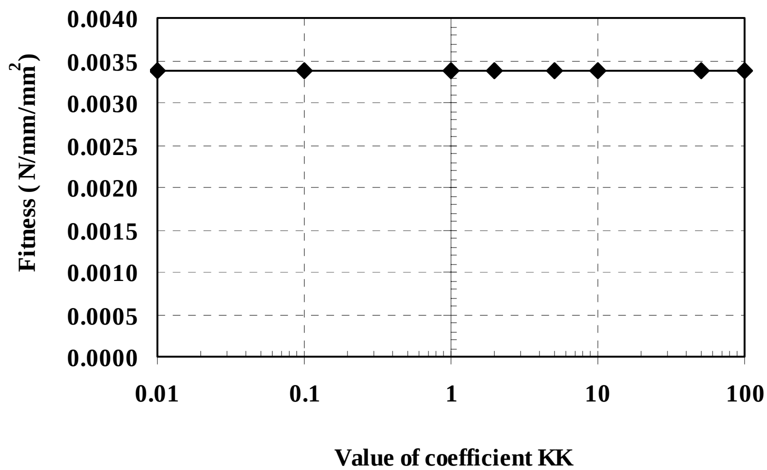



**Figure 6.2.5** Effects of mutation rate on the performance of the GA

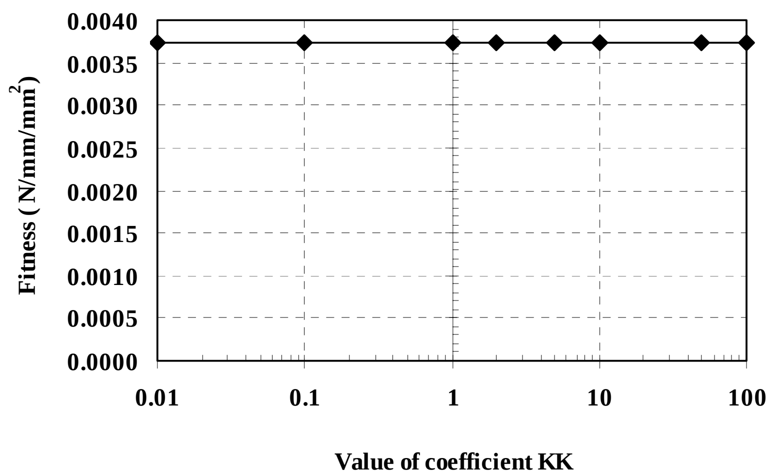
In general, the speed of convergence for the rank selection method is slower than those for two other methods, i.e. the tournament selection method and the fitness proportional method. The calculations in this thesis are all based on the tournament selection method. Moreover, the values of the population size, the crossover rate and the mutation rate, which are given in the process of the investigation, all lead GA to convergence except the case with a mutation rate of 0.1. Particularly, the difference of the fitness for each compared case is not so large. In principle, each reasonable investigated value can be chosen in the running of GA. However, in this thesis, a GA is running with the size of the population being two times of the length of the encoded binary string; a crossover rate of 0.8 and a mutation rate of 0.001.

### 6.2.3 Sensitivity analysis of the fitness function

The sensitivity analyses of fitness to the values of $KK$ given in the objective function are shown in Figure 6.2.6 (a) and (b). The calculation is carried out for the cross-section with a height of 350 mm. Since with this height, the fabrication constraint, i.e. the length of the strip, is more likely to be violated than that with a height of 150 mm.




**(a)** *h=150 mm*, *t=1.5 mm* and *L=4500 mm*




**(b)** *h=350 mm*, *t=2.0 mm* and *L=5000 mm*

**Figure 6.2.6** Effects of value KK to the value of the fitness in 10 runs

Figure 6.2.6 (a) indicates that when the value of $KK$ is varied from 0.01 to 100, the maximum values of the fitness in 10 runs are varied from 0.003376 to 0.003375 N/mm/mm$^2$ with the dimension $b_1 = 54$ mm, $c = 25.92$ mm and $b_1 = 54$ mm, $c = 26.88$ mm, respectively. In addition, Figure 6.2.6 (b) indicates that when the value of $KK$ is varied from 0.01 to 100, the maximum values of the fitness in 10 runs are the same. Therefore, we may conclude that for Z-shape cross-section, the fitness is not sensitive to the value of $KK$. In the further analysis, the value of $KK$ is set to 0.1.

### 6.2.4 Optimization results

Using the above selected parameters, the GA-based design has been running for 10 times and the optimization results are shown in Table 6.2.3. For each run, the GA starts from a different initial population. Thus, it provides the possibility to find the global optimum instead of being stuck to the local optimum. Table 6.2.3 shows that the optimum cross-section has a width of the top flange equal to 54 mm and a depth of the lip equal to 25.92 mm, i.e. the ratio of $c$ to $b_2$ is 0.48. The corresponding value of $q/A_g$ is 0.003376 N/mm/mm$^2$. The optimum dimensions are selected as those with the maximum value of fitness in the 10 runs.

**Table 6.2.3** Optimization results for 10 runs

| No. of runs | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 53 | 25.38 | 0.003375 | 12515.32 | 1.45 | 430.58 |
| 2 | 56 | 26.50 | 0.003370 | 12428.26 | 1.49 | 442.61 |
| 3 | 53 | 23.97 | 0.003373 | 12551.13 | 1.44 | 426.47 |
| 4 | 54 | 24.48 | 0.003375 | 12527.59 | 1.45 | 430.88 |
| 5 | 54 | 25.44 | 0.003376 | 12503.55 | 1.46 | 433.68 |
| 6 | 56 | 25.50 | 0.003369 | 12454.37 | 1.48 | 439.69 |
| 7 | 53 | 23.97 | 0.003373 | 12551.13 | 1.44 | 426.47 |
| 8 | 53 | 25.38 | 0.003375 | 12515.32 | 1.45 | 430.58 |
| **9** | **54** | **25.92** | **0.003376** | **12489.93** | **1.47** | **435.08** |
| 10 | 54 | 24.48 | 0.003375 | 12527.59 | 1.45 | 430.88 |

In order to verify the results obtained here, another kind of optimization process is carried out by simply investigating the effects of the variations of the width of the flange and the depth of the lip on the value of $q/A_g$ for the same purlin. The width of the wider flange is varied from 40 mm to 100 mm with step of 1 mm and the ratio of the depth of the lip to the width of the bottom flange is varied from 0.2 to 0.6 with step of 0.01. Both the geometrical constraints and fabrication constraints are not violated. The optimum dimension is chosen as the one that make the value of $q/A_g$ maximum. The comparisons of the optimum dimensions for these two calculations are shown in Table 6.2.4.

In the table, 'Opt-GA' is the optimization procedure of GA-based design, and 'Opt-Cal' is the other optimization procedure. The maximum value of $q/Ag$ using GA-based design is the same as that via investigating the variation of the parameter. The corresponding optimum dimensions are the same as well. Thus, the computer program for optimization and selected parameters for GA are verified and can be used for further analysis.

**Table 6.2.4** Comparison of optimization results by calculation to that by GA

| Calculation methods | $b_1$ (mm) | $b_2$ (mm) | $\alpha$ | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) |
|---|---|---|---|---|---|
| **Opt-Cal** | **54** | 48 | **0.54** | 25.92 | **0.003376** |
| **Opt-GA** | **54** | 48 | **0.54** | 25.92 | **0.003376** |

### 6.2.5 Analysis of Results

Figure 6.2.7 shows the comparison of other results in 10 runs with the optimum one listed in Table 6.2.3. In the figure, such symbol as '44-051' represents the cross-section with the width of the flange being 54 mm and the ratio of the lip to the bottom flange being 0.54.

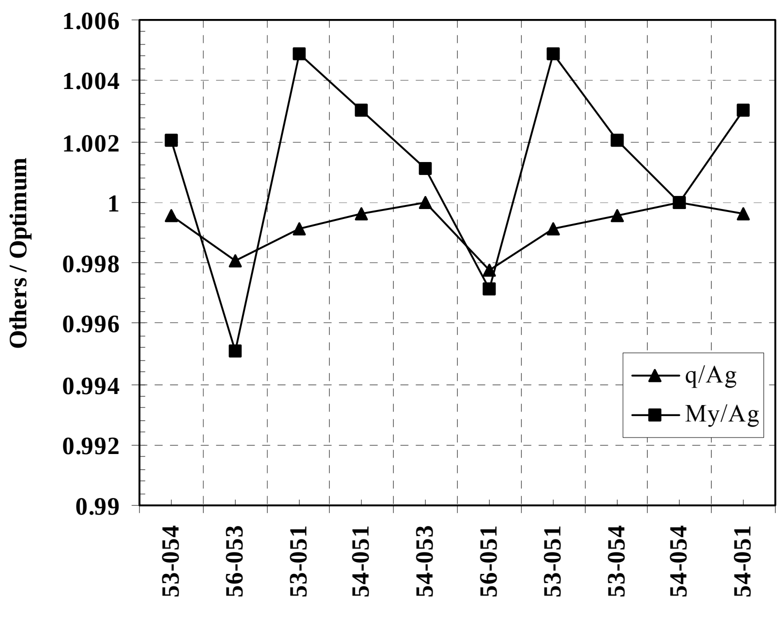



**Figure 6.2.7** Comparison of other results to the optimum in 10 runs of GA

Figure 6.2.7 shows that both the values of $q/A_g$ and design variables are narrowed down to a relatively small range. This is due to the characteristics of GA, i.e. finding the near optimum. In addition, the purpose of this research is to find the nominal dimension of the cross-section. Thus, the final optimum dimension can be modified based on the results in 10 runs since Figure 6.2.7 shows that a small change of the width of the flange or the depth of the lip does not change the value of $q/A_g$ considerably.

Figure 6.2.7 shows the comparison of value $M_y/A_g$ as well. The differences among the values of $M_y/A_g$ are larger than those among $q/A_g$. However, this difference is quite small from the point of view of engineering design.

## 6.3 Optimization Results based on Eurocode 3 Method

### 6.3.1 Optimization results

With the selected parameters and the design procedures defined in Section 6.2, the GA-based design is applied to Z-shape purlins continuous over two spans subjected to a gravity load with given heights, thicknesses and spans. The design of the purlins is based on Eurocode 3, Part 1.3.

In order to simplify the design process, two formulas, in which the relation of the width of the top flange and the depth of the lip are defined, are provided. The first one is expressed as:

$$c = \frac{11}{30} \cdot (b_1 - 40) + 13 \,. \qquad \textbf{(6.3.1)}$$

This formula is derived according to the linear relationship of $b_1$ and $c$, i.e. the depth of the lip is 13 mm when the width of the flange is 40 mm; and the depth of the lip is 35 mm when the width of the flange is 100 mm. The other values of $c$ can be interpolated with this formula. For the comparisons that will be carried out in the following section, this case is called 'Choice 1'. Another relation is that the depth of the lip is 15 mm when the width of the flange is 40 mm, and the depth of the lip is 40 mm when the width of the flange is 90 mm. This relation can be formulated as:

$$c = \frac{1}{2} \cdot (b_1 - 40) + 15 . \qquad \textbf{(6.3.2)}$$

Similarly, this case is called 'Choice 2'. With these two formulas, the optimum dimensions together with the corresponding values of $q/A_g$ for a variety of heights, thicknesses and spans are shown in Table 6.3.1 and Table 6.3.2, respectively.

When compared with equation (6.3.1) and equation (6.3.2), the case with no other limits for the values of $b_2$ and $c$ except those listed in Eurocode 3, part 1.3 is called 'Choice 3'. The optimum dimensions together with the corresponding values of $q/A_g$ for a variety of heights, thicknesses and spans for this case are shown in Table 6.3.3.

Moreover, the results for these three cases are obtained by choosing the dimension corresponding to the maximum value of $q/A_g$ from 10 runs of GA for each given height, thickness and span. In addition, together with the values of $q/A_g$, the values of $M_y/A_g$ and the areas of cross-sections corresponding to the optimum dimensions are listed in Table 6.3.3, Table 6.3.1 and Table 6.3.2 as well. The detailed results are shown in Appendix A.

**Table 6.3.1** Results for Choice 1

| $h$ (mm) | $L$ (mm) | $t$ (mm) | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 100 | 1400 | 1.0 | **41** | **13.37** | 0.018349 | 8102.564 | 190.94 |
| 100 | 1600 | 1.2 | **44** | **14.47** | 0.015151 | 8842.566 | 239.30 |
| 100 | 1900 | 1.5 | **53** | **17.77** | 0.011658 | 9518.614 | 335.35 |
| 100 | 2200 | 2.0 | **60** | **20.33** | 0.009159 | 9821.831 | 483.78 |
| 150 | 1700 | 1.0 | **48** | **15.93** | 0.015386 | 9564.788 | 257.30 |
| 150 | 2000 | 1.2 | **51** | **17.03** | 0.012168 | 10901.82 | 319.49 |
| 150 | 2500 | 1.5 | **59** | **19.97** | 0.008775 | 12295.31 | 432.30 |
| 150 | 3000 | 2.0 | **69** | **23.63** | 0.006961 | 13945.02 | 630.00 |
| 200 | 3000 | 1.5 | **63** | **21.43** | 0.006970 | 14138.01 | 521.26 |
| 200 | 3800 | 2.0 | **74** | **25.47** | 0.005201 | 16536.56 | 754.78 |
| 200 | 4500 | 2.5 | **85** | **29.50** | 0.004221 | 18236.01 | 1016.37 |
| 250 | 3200 | 1.5 | **77** | **26.57** | 0.006706 | 14754.75 | 650.13 |
| 250 | 4300 | 2.0 | **81** | **28.03** | 0.004475 | 18444.75 | 890.28 |
| 250 | 5100 | 2.5 | **91** | **31.70** | 0.003680 | 20744.53 | 1179.72 |
| 300 | 4500 | 2.0 | **90** | **31.33** | 0.004323 | 19574.76 | 1036.50 |
| 300 | 5500 | 2.5 | **99** | **34.63** | 0.003397 | 22696.80 | 1356.51 |
| 300 | 6000 | 3.0 | **100** | **35.00** | 0.003152 | 24953.42 | 1634.39 |
| 350 | 5000 | 2.0 | **100** | **35.00** | 0.003674 | 20331.16 | 1188.07 |
| 350 | 6000 | 2.5 | **100** | **35.00** | 0.003000 | 24413.12 | 1486.23 |
| 350 | 7000 | 3.0 | **100** | **35.00** | 0.002489 | 26992.67 | 1782.39 |
| 350 | 7500 | 3.5 | **100** | **35.00** | 0.002309 | 28494.64 | 2076.55 |

**Table 6.3.2** Results for Choice 2

| $h$ (mm) | $L$ (mm) | $t$ (mm) | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 100 | 1400 | 1.0 | **40** | **15.0** | 0.018954 | 8187.290 | 192.15 |
| 100 | 1600 | 1.2 | **44** | **17.0** | 0.015718 | 8926.625 | 245.18 |
| 100 | 1900 | 1.5 | **52** | **21.0** | 0.012056 | 9516.639 | 341.87 |
| 100 | 2200 | 2.0 | **60** | **25.0** | 0.009416 | 9743.894 | 502.07 |
| 150 | 1700 | 1.0 | **48** | **19.0** | 0.016022 | 9650.098 | 263.19 |
| 150 | 2000 | 1.2 | **51** | **20.5** | 0.012667 | 10977.07 | 327.54 |
| 150 | 2500 | 1.5 | **56** | **23.0** | 0.009147 | 12496.60 | 432.39 |
| 150 | 3000 | 2.0 | **68** | **29.0** | 0.007238 | 14009.90 | 647.11 |
| 200 | 3000 | 1.5 | **63** | **26.5** | 0.007273 | 14265.40 | 536.05 |
| 200 | 3800 | 2.0 | **71** | **30.5** | 0.005432 | 16832.61 | 762.75 |
| 200 | 4500 | 2.5 | **85** | **37.5** | 0.004380 | 18362.45 | 1055.73 |
| 250 | 3200 | 1.5 | **76** | **33.0** | 0.007040 | 14977.32 | 665.99 |
| 250 | 4300 | 2.0 | **77** | **33.5** | 0.004673 | 18842.47 | 896.03 |
| 250 | 5100 | 2.5 | **86** | **38.0** | 0.003845 | 21161.99 | 1186.11 |
| 300 | 4500 | 2.0 | **90** | **40.0** | 0.004535 | 19785.73 | 1070.47 |
| 300 | 5500 | 2.5 | **90** | **40.0** | 0.003543 | 23321.97 | 1338.63 |
| 300 | 6000 | 3.0 | **90** | **40.0** | 0.003220 | 25090.94 | 1604.79 |
| 350 | 5000 | 2.0 | **90** | **40.0** | 0.003829 | 21327.71 | 1168.47 |
| 350 | 6000 | 2.5 | **90** | **40.0** | 0.003101 | 25226.00 | 1461.63 |
| 350 | 7000 | 3.0 | **90** | **40.0** | 0.002510 | 27255.40 | 1752.79 |
| 350 | 7500 | 3.5 | **90** | **40.0** | 0.002294 | 28533.48 | 2041.95 |

**Table 6.3.3** Results for Choice 3

| $h$ (mm) | $L$ (mm) | $t$ (mm) | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 100 | 1400 | 1.0 | **40** | **20.06** | 0.019821 | 8069.29 | 201.87 |
| 100 | 1600 | 1.2 | **44** | **22.80** | 0.016312 | 8723.04 | 258.63 |
| 100 | 1900 | 1.5 | **48** | **25.20** | 0.012355 | 9238.82 | 342.46 |
| 100 | 2200 | 2.0 | **56** | **30.00** | 0.009596 | 9448.36 | 505.99 |
| 150 | 1700 | 1.0 | **48** | **25.20** | 0.016807 | 9587.67 | 275.10 |
| 150 | 2000 | 1.2 | **49** | **25.80** | 0.013234 | 11006.92 | 335.19 |
| 150 | 2500 | 1.5 | **56** | **30.00** | 0.009449 | 12305.44 | 452.83 |
| 150 | 3000 | 2.0 | **66** | **36.00** | 0.007382 | 13665.38 | 666.71 |
| 200 | 3000 | 1.5 | **62** | **33.60** | 0.007532 | 14217.23 | 553.86 |
| 200 | 3800 | 2.0 | **70** | **37.76** | 0.005559 | 16655.55 | 787.29 |
| 200 | 4500 | 2.5 | **84** | **46.80** | 0.004436 | 17949.06 | 1096.57 |
| 250 | 3200 | 1.5 | **73** | **40.20** | 0.007302 | 15123.96 | 678.26 |
| 250 | 4300 | 2.0 | **76** | **42.00** | 0.004803 | 18738.68 | 925.43 |
| 250 | 5100 | 2.5 | **86** | **48.00** | 0.003918 | 20865.63 | 1235.31 |
| 300 | 4500 | 2.0 | **87** | **48.60** | 0.004675 | 19878.90 | 1092.43 |
| 300 | 5500 | 2.5 | **94** | **52.80** | 0.003635 | 22940.90 | 1421.29 |
| 300 | 6000 | 3.0 | **100** | **56.40** | 0.003375 | 25115.28 | 1761.08 |
| 350 | 5000 | 2.0 | **89** | **49.80** | 0.003952 | 21361.67 | 1202.97 |
| 350 | 6000 | 2.5 | **91** | **49.30** | 0.003187 | 25175.61 | 1512.31 |
| 350 | 7000 | 3.0 | **95** | **45.39** | 0.002601 | 27657.43 | 1814.30 |
| 350 | 7500 | 3.5 | **95** | **45.39** | 0.002384 | 28952.33 | 2113.85 |

### 6.3.2 Comparisons of the results

The comparisons of the values of $q/A_g$ for the three cases mentioned above are shown in Figure 6.3.1. This comparison is carried out based on the ratio of the values of $q/A_g$ for Choice 1 and Choice 2 to those for Choice 3. It can be seen that the values of $q/A_g$ for Choice 3 is about 1.8 % to 4.4 % higher than those for Choice 2 and is about 3.2 % to 8.5 % higher than those for Choice 1. In addition, most of the values of $q/A_g$ for Choice 2 are higher than those for Choice 1 except for the cross-section of 'h350_L75_t35'. This symbol represents the cross-section with a height of 350 mm, a span of 7500 mm and a thickness of 3.5 mm. Other symbols in the figure can be interpreted in a similar way.

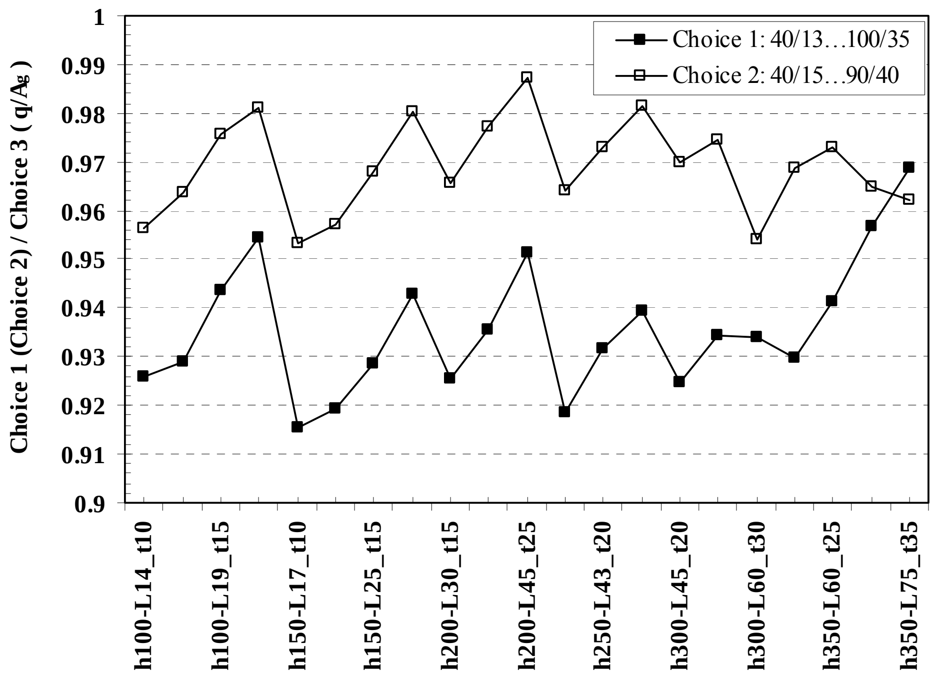



**Figure 6.3.1** Comparison of the values of $q/A_g$ for three cases

Similarly, Figure 6.3.2 shows the comparisons of values of $M_y/A_g$ for the three cases mentioned above. It can be seen that the values of $M_y/A_g$ for Choice 3 are not always higher than those for Choice 1 and Choice 2. The variation of the ratio, which is defined as the value of $M_y/A_g$ for Choice 1 or Choice 2 over that for Choice 3, is in the range of –5 % to 4 %. In addition, most of the values of $M_y/A_g$ for Choice 2 are higher than those for Choice 1 except the cross-sections labeled 'h100_L19_t15' and 'h100_L22_t20'.

Figure 6.3.3 shows the variations of the ratio of the depth of the lip to the width of the top flange. As for the free optimum case, most values of this ratio are varied from 0.5 to 0.56 except for the two cross-sections of 'h350_L70_t30' and 'h350_L75_t35', whose values are both 0.48. This is due to the fact that for the cross-sections with a height of 350 mm, the fabrication constraint is likely to be violated with the higher value of width of the flange and depth of the lip. As far as the two simplified cases are concerned, the variations of this ratio are quite smooth. For Choice 1, this ratio is in the range of 0.33 to 0.35; while for Choice 2, the ratio is in the range of 0.38 to 0.44.

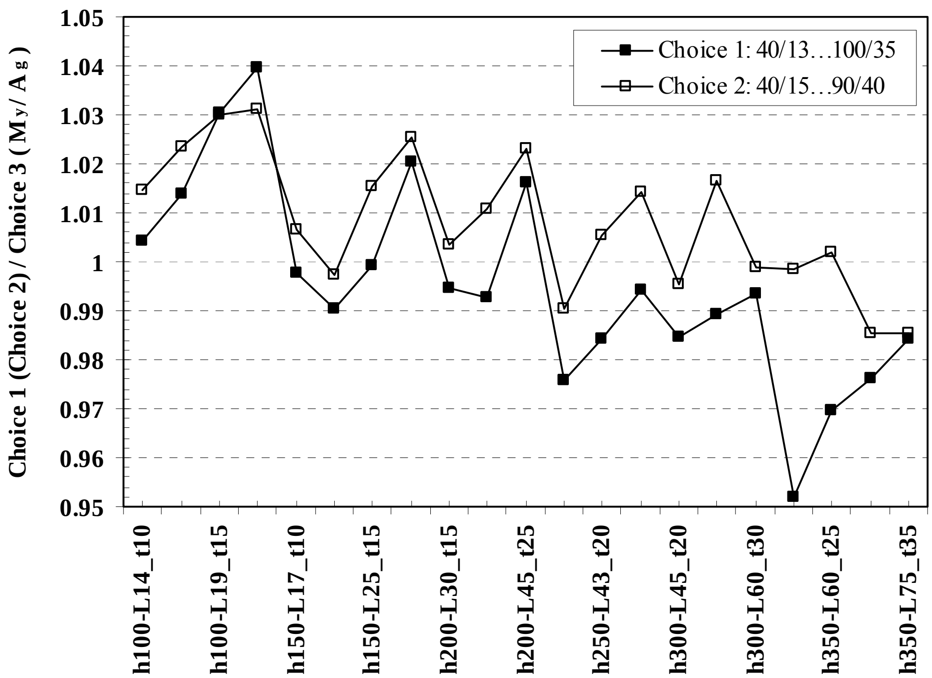



**Figure 6.3.2** Comparison of the values of $M_y / A_g$ for three cases

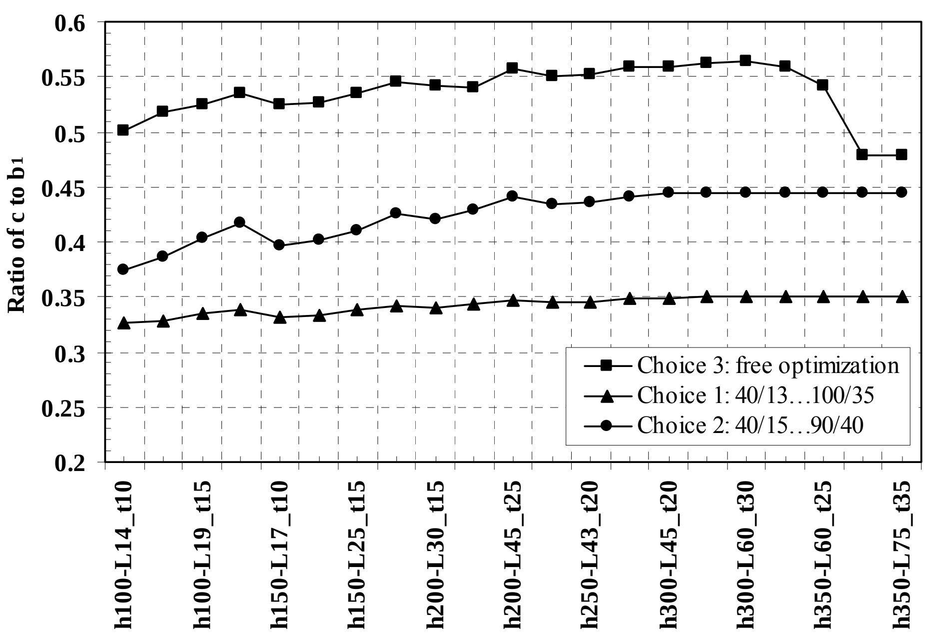



**Figure 6.3.3** Ratio of depth of the lip to the width of the flange

Figure 6.3.4 shows the comparison of optimum dimension with a given height, a thickness and a span to the values of $q/A_g$ calculated by varying the width of the top flange from 40 mm to 100 mm with step 10 mm and the ratio of $c/b_2$ from 0.2 to 0.6 with step 0.2. The optimum results using GA are represented by two diamonds, in which 'h200_54' represents the cross-section with a height of 200 mm and a ratio of $c/b_2$ equal to 0.54. It can be seen that the optimum dimensions obtained using GA fit the curve well. However, the corresponding values of $q/A_g$ are different since the span of the purlin optimized using GA is 4500 mm and that from a previous analysis is 4000 mm. It seems that the span has no great influence on the optimum dimensions for the cases listed here.

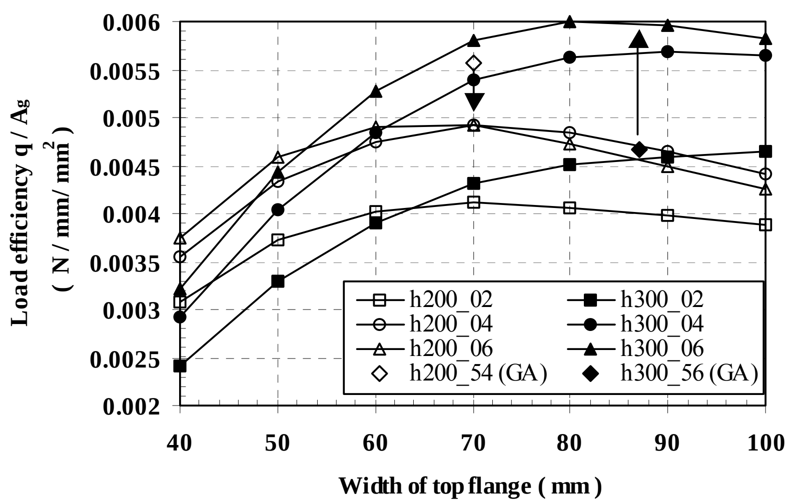



**Figure 6.3.4** Comparisons of optimization results using GA with previous analysis

### 6.3.3 Optimum dimensions

Analyses from 6.3.2 indicate that, in general, Choice 3 shows the higher values of $q/A_g$ than those for Choice 1 and Choice 2. Furthermore, Choice 2 shows better behavior than that for Choice 1 when both the values of $q/A_g$ and $M_y/A_g$ are compared. However, in order to provide multiple choices to structural designers and steel manufactures, three kinds of optimum dimensions for the Z-shape purlin continuous over two spans under gravity load using GA are proposed in Table 6.3.4 corresponding to the three optimization cases mentioned above.

**Table 6.3.4** Proposed optimum dimensions for Z-shape purlin

| | | Choice 1 | | Choice 2 | | Choice 3 | |
|---|---|---|---|---|---|---|---|
| $h$ | $t$ | $b_1$ | $c$ | $b_1$ | $c$ | $b_1$ | $c$ |
| 100 | 1.0 | **40** | **13.0** | **40** | **15.0** | **40** | **20** |
| 100 | 1.2 | **45** | **14.8** | **45** | **17.5** | **45** | **20** |
| 100 | 1.5 | **50** | **16.7** | **50** | **20.0** | **50** | **25** |
| 100 | 2.0 | **60** | **20.3** | **60** | **25.0** | **50** | **20** |
| 150 | 1.0 | **50** | **16.7** | **50** | **20.0** | **50** | **25** |
| 150 | 1.2 | **50** | **16.7** | **50** | **20.0** | **50** | **25** |
| 150 | 1.5 | **60** | **20.3** | **55** | **22.5** | **55** | **30** |
| 150 | 2.0 | **70** | **24.0** | **70** | **30.0** | **65** | **35** |
| 200 | 1.5 | **65** | **22.1** | **65** | **27.5** | **65** | **35** |
| 200 | 2.0 | **75** | **25.9** | **70** | **30.0** | **70** | **35** |
| 200 | 2.5 | **85** | **29.6** | **85** | **37.5** | **85** | **45** |
| 250 | 1.5 | **80** | **27.7** | **80** | **35.0** | **75** | **40** |
| 250 | 2.0 | **80** | **27.7** | **80** | **35.0** | **75** | **40** |
| 250 | 2.5 | **90** | **31.3** | **90** | **40.0** | **85** | **45** |
| 300 | 2.0 | **90** | **31.3** | **90** | **40.0** | **90** | **50** |
| 300 | 2.5 | **100** | **35.0** | **90** | **40.0** | **95** | **55** |
| 300 | 3.0 | **100** | **35.0** | **90** | **40.0** | **100** | **55** |
| 350 | 2.0 | **100** | **35.0** | **90** | **40.0** | **90** | **50** |
| 350 | 2.5 | **100** | **35.0** | **90** | **40.0** | **90** | **50** |
| 350 | 3.0 | **100** | **35.0** | **90** | **40.0** | **95** | **45** |
| 350 | 3.5 | **100** | **35.0** | **90** | **40.0** | **95** | **45** |

For Choice 1 and Choice 2, the width of the top flange is determined firstly by rounding the optimum value to the number with ‘0’ or ‘5’ at last position; and the depth of the lip is recalculated according to formula of (6.3.1) or (6.3.2). For instance, for the cross-section with a height of 100 mm and a thickness of 1.5 mm in Choice 1, the optimum width of the flange is 41 mm. The proposed width of the flange is 40 mm and the depth of the lip is recalculated using equation (6.3.1) and is given as 13.0 mm.

For Choice 3, the proposed width of the flange and depth of the lip is determined by rounding the optimum value to the number with ‘0’ or ‘5’ at last position. For instance, the calculated optimum dimensions for the cross-section with a height of 100 mm and a thickness of 1.5 mm are the width of the flange being 48 mm and the depth of the lip being 25.20 mm. Therefore, the proposed optimum dimensions are the width of the top flange being 50 mm and the depth of the lip being 25 mm.

## 6.4 Integrating modified Eurocode 3 method into the optimization process

In order to show that the numerical analysis program such as CUFSM can be integrated into the GA optimization process, the modified Eurocode 3 method is integrated into GA-based design. The optimum dimensions together with the corresponding values of $q/A_g$ and $M_y/A_g$ are shown in Table 6.4.1. In addition, the optimization results based on the Eurocode 3 method are also listed in the same table. In the table, 'EC3' represents the Eurocode 3 method and 'EC3_M' represents the modified Eurocode 3 method.

Table 6.4.1 shows that the optimum dimensions obtained by integrating the Eurocode 3 method and modified Eurocode 3 method into the GA-based design procedure can be similar (cross-sections with a height of 100 mm and a thickness of 1.0 mm; and with a height of 350 mm and a thickness of 2.0 mm) or be different (other cross-sections listed in the table) depending on the case. However, the difference of the corresponding values of $q/A_g$ is not so large. For instance, the variation is from – 4.1% to 3.8 %.

**Table 6.4.1 Optimization results based on different design method**

| $h$ (mm) | $L$ (mm) | $t$ (mm) | Eurocode 3 | | | | Modified Eurocode 3 | | | | EC3_M / EC3 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ | $M_y/A_g$ |
| 100 | 1400 | 1.0 | **40** | **20.1** | 0.019821 | 8069.29 | **41** | **20.6** | 0.019864 | 8059.78 | **1.002** | **0.999** |
| 150 | 1700 | 1.0 | **48** | **25.2** | 0.016807 | 9587.67 | **56** | **30.0** | 0.016710 | 9147.820 | **0.994** | **0.954** |
| 200 | 3000 | 1.5 | **62** | **33.6** | 0.007532 | 14217.23 | **73** | **39.5** | 0.007497 | 13601.85 | **0.995** | **0.957** |
| 250 | 3200 | 1.5 | **76** | **33.0** | 0.007040 | 14977.32 | **87** | **48.6** | 0.007310 | 14301.50 | **1.038** | **0.955** |
| 300 | 4500 | 2.0 | **90** | **40.0** | 0.004535 | 19785.73 | **100** | **56.4** | 0.004654 | 18937.30 | **1.026** | **0.957** |
| 350 | 5000 | 2.0 | **90** | **40.0** | 0.003829 | 21327.71 | **91** | **49.3** | 0.003671 | 19350.79 | **0.959** | **0.907** |

# 7 BEHAVIOR OF $\Sigma$-SHAPE PURLINS

## 7.1 Introduction

In this chapter, the behavior of $\Sigma$-shape purlins with one axis of symmetry is investigated. The purlins are connected to profiled sheeting on one flange and are continuous over two spans subjected to a gravity load. The dimensions of the cross-section are shown in Figure 7.1.1. In the figure, $b$ is the width of the flange; $c$ is the depth of the lip; $h$ is the height of the cross-section; $h_1$ and $h_5$ are the distances of the web stiffeners to the top and bottom flange, respectively; $h_2$ and $h_4$ are the depths of the web stiffener in the direction of the web height; $d_s$ is the depth of the web stiffener perpendicular to the web height; and $h_3$ is the distance between the stiffeners along the web.

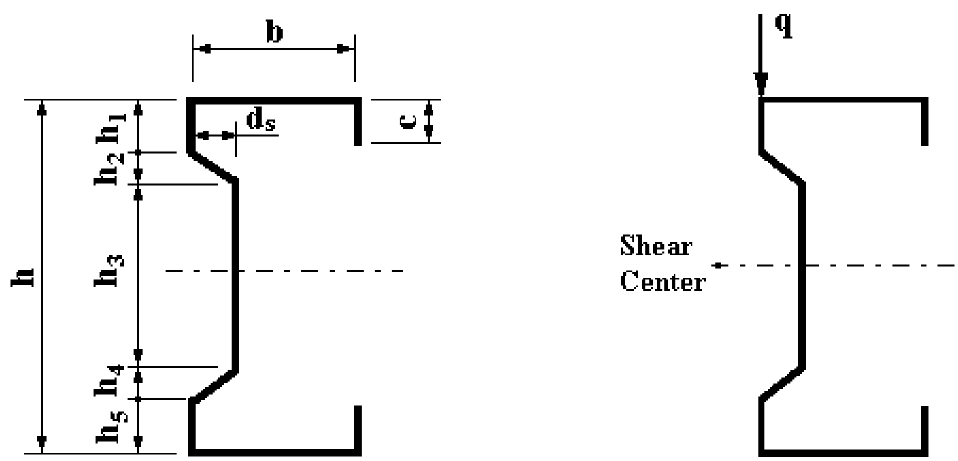



**Figure 7.1.1 Dimensions of the cross-section of the $\Sigma$-shape purlin**

The investigation is concentrated on the effects of the position and the size of the web stiffener on the cross-sectional properties and the load efficiency calculated using Eurocode 3 firstly. After the buckling modes are classified using the computer program CUFSM based on the FSM, the modified Eurocode 3 method for $\Sigma$-shape purlins is put forward. The load efficiency and moment efficiency calculated using this modified Eurocode 3 method are compared with those using the Eurocode 3 method.

## 7.2 Effects of Web Stiffeners on the Section Properties and Load efficiency Based on Eurocode 3

Such effects as the position and the size of the web stiffeners on the effective section modulus and load resistance of a $\Sigma$-shape purlin are investigated in this section. The design of the purlin is based on Eurocode 3, Part 1.3.

### 7.2.1 Effects of the position of web stiffeners

Figure 7.2.1 and Figure 7.2.2 indicate the effects of the position of the web stiffeners on the values of $W/A_g$ and $q/A_g$, respectively. These calculations are carried out using the cross-section with $b = 80$ mm, $c = 36$ mm, $h_2 = 25$ mm and $d_s = 32$ mm. The value of $h_1$ is varied from 35 mm to 100 mm with a step of 10 mm.

Figure 7.2.1 indicates that for the cross-section with a thickness of 2.0 mm and a height of 350 mm, the value of $W/A_g$ is increased when the height of $h_1$ is varied from 35 mm to 70 mm. If the value of $h_1$ is increased further, the value of $W/A_g$ starts to decrease. This is due to the fact that the fully effective element is transferred from $h_1$ to $h_3$ after the length of is $h_1$ more than 70 mm.

If the thickness of the cross-section is increased to 2.5 mm, the value of $W/A_g$ is increased when the length of $h_1$ is varied from 35 mm to 45 mm. This is due to the fact that with this variation, the length of $h_3$ is increased to be fully effective. When the length of $h_1$ is increased further to 91 mm, the value of $W/A_g$ starts to decrease although the lengths of $h_1$

and $h_3$ are both fully effective. This decrease is mainly due to the reduced thickness of the stiffener by taking the distortional buckling behavior into account. If the length of $h_1$ is increased further, the length of $h_1$ is not fully effective and the value of $W/A_g$ decreases further.

As for the cross-section with a thickness of 3.0 mm, the value of $W/A_g$ is decreased when the length of $h_1$ is varied from 35 mm to 100 mm. This is due to the fact that the lengths of $h_1$ and $h_3$ are both fully effective at the beginning of the variation of $h_1$ and the reduced thickness of the stiffener is decreased with the increase of $h_1$.

Moreover, the value of $W/A_g$ is increased more when the thickness is increased from 2.0 mm to 2.5 mm than when the thickness is increased from 2.5 mm to 3.0 mm. This is due to the fact that the width of the flange is partially effective when the thickness is less than 2.5 mm.

Figure 7.2.1 also shows that the value of $W/A_g$ for the cross-section with a height of 350 mm is larger than that with a height of 300 mm. This is due to the fact that with the same variation of the length of $h_1$, the length of $h_3$ is shorter when the height is 300 mm than when the height is 350 mm.

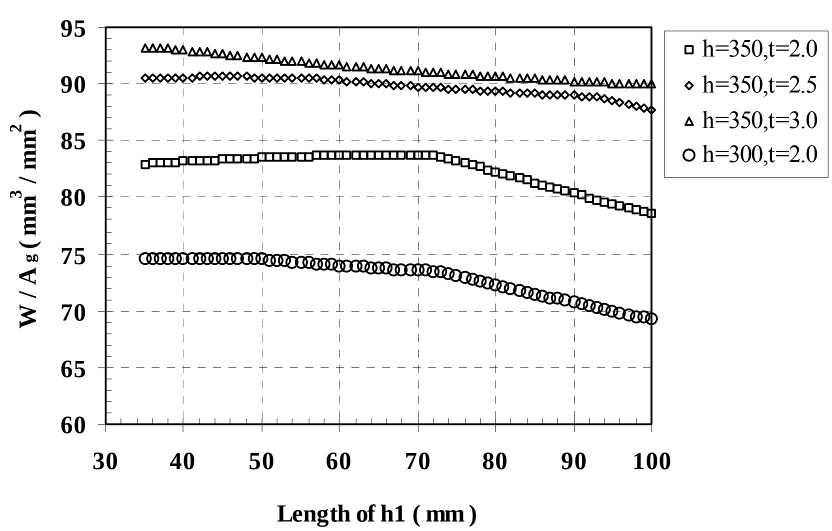



**Figure 7.2.1** Effects of the position of the web stiffeners on the value of $W/A_g$

Figure 7.2.2 indicates that for the cross-section with a thickness of 2.0 mm and a height of 350 mm, the value of $q/A_g$ is increased when the height of $h_1$ is varied from 35 mm to 80 mm. When the length of $h_1$ is increased further, the value of $q/A_g$ starts to decrease. Since the shear center of the cross-section is transferred from a point between the two flanges to a point outside the cross-section with the increase of the length of $h_1$. The effects of the length of $h_1$ on the position of the shear center are shown in Figure 7.2.3. The origin of the coordinate system is assumed to be at the bottom left corner of the cross-section, i.e. at the intersection of the bottom flange to the web. The negative values mean that the position of the shear center is outside the cross-section to the left.

Figure 7.2.2 also shows that for cross-sections with the same height, the values of $q/A_g$ are increased when the thickness is varied from 2.0 mm to 2.5 mm. When the thickness of the cross-section is varied from 2.5 mm to 3.0 mm, there is no great increase of the value of $q/A_g$. This is due to the fact that the section modulus is increased sharply when the thickness is varied from 2.0 mm to 2.5 mm when compared to the case where the thickness is varied from 2.5 mm to 3.0 mm.

For a cross-section with a thickness of 2.5 mm and a height of 350 mm, the shear center starts to transfer when the length of $h_1$ is 78 mm, i.e. from the inside of the cross-section if $h_1 < 80$ mm to the outside of the cross-section if $h_1 > 80$. The transfer point of the shear center for the cross-section with a thickness of 3.0 mm and of 2.0 mm is near that with a thickness of 2.5 mm. Thus, the variation of the thickness has no big influence on the position of the shear center for this studied cross-section.

In addition, when the height of the cross-section is decreased from 350 mm to 300 mm, the transfer point for the shear center is moved from around $h_1 = 80$ mm to $h_1 = 60$ mm. Thus, the maximum value of $q/A_g$ occurred at $h_1 = 60$ mm for the cross-section with a height of 300 mm. This can also be seen from Figure 7.2.2 and Figure 7.2.3.

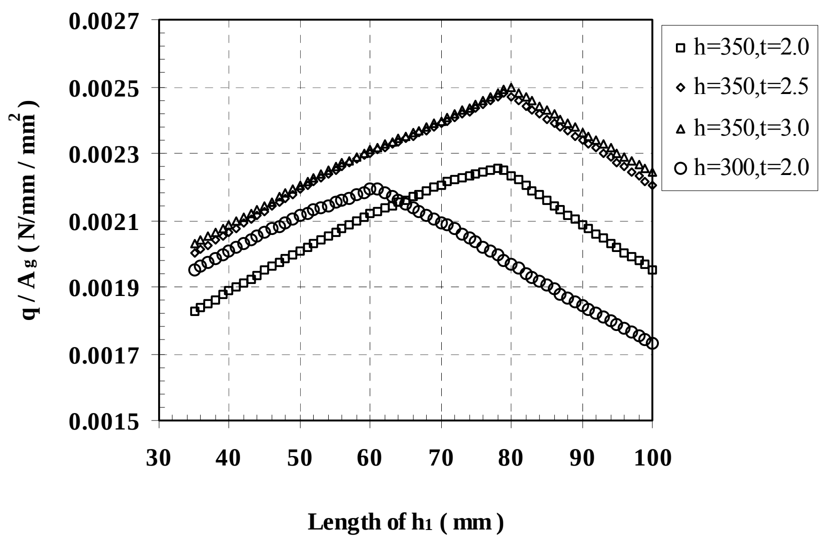



**Figure 7.2.2 Effects of the position of web stiffeners on the value of $q/A_g$**

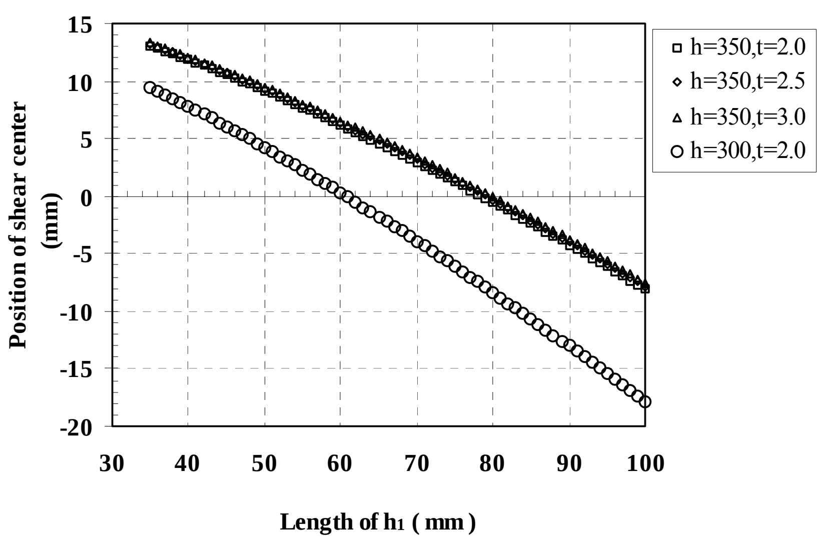



**Figure 7.2.3 Position of the shear center**

### 7.2.2 Effects of the size of the web stiffeners

Effects of the lengths of $h_2$ and $d_s$ on the effective section modulus, $W$, the value of $W/A_g$, the load resistance, $q$, and the value of $q/A_g$ are investigated in this section. These calculations are carried out for cross-section with $h = 350$ mm, $b = 80$ mm, $c = 36$ mm and

$h_1 = 80$ mm. The lengths of $h_2$ and $d_s$ are, in turn, varied from 5 mm to 40 mm with a step of 1 mm.

For the same length of $h_2$, the values of the effective modulus are increased when the value of $d_s$ is varied from 5 mm to 40 mm. When the length of $h_2$ is varied from 10 mm to 40 mm with the length of $d_s$ being fixed, the effective section modulus in decreased. These effects are shown in Figure 7.2.4

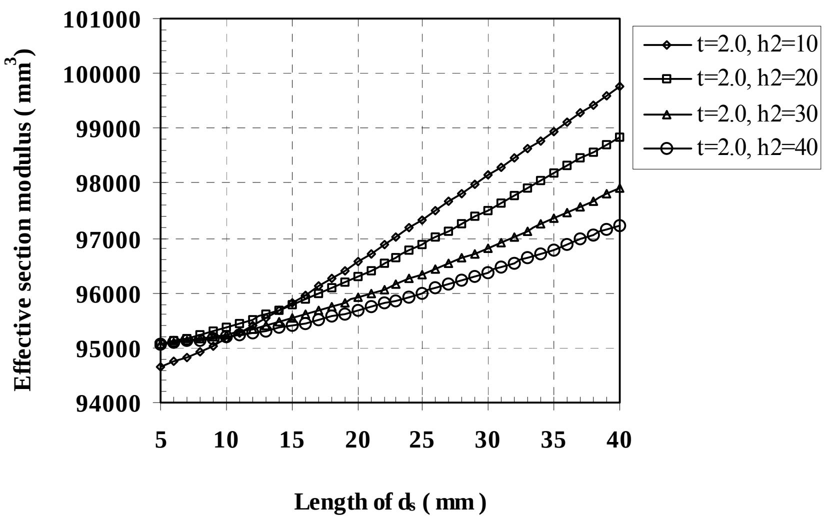



**Figure 7.2.4** Effects of the size of the stiffeners on the effective section modulus

Figure 7.2.5 illustrates the effects of the length of $h_2$ and $d_s$ on the value of $W/A_g$. With the value of $h_2$ being fixed, the effective section modulus is increased with the increasing of the length of $d_s$. However, the amount of increase is not so large. Thus, when the length of $d_s$ is varied from 5 mm to 40 mm, the value of $W/A_g$ is decreased. In addition, the value of $W/A_g$ is increased with the increase of the length of $h_2$. This is due to the fact that the increase of the length $h_2$ at the same time decreases the length of $h_3$, which in turn decreases the area of the cross-section.

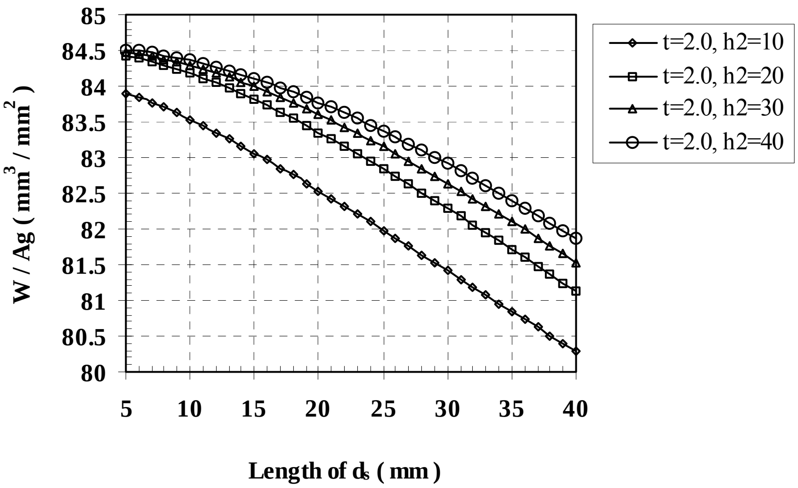



**Figure 7.2.5** Effects of size of the web stiffeners on the value of $W/A_g$

As shown in Section 7.2.1, the value of load resistance, $q$ and the value of $q/A_g$ depend on the position of the shear center. The effects of the length of $h_2$ and $d_s$ on the position of the shear center are shown in Figure 7.2.6. The shear center is transferred from outside of the cross-section to the position between the two flanges when the length of $d_s$ is varied from 30 mm to 35 mm.

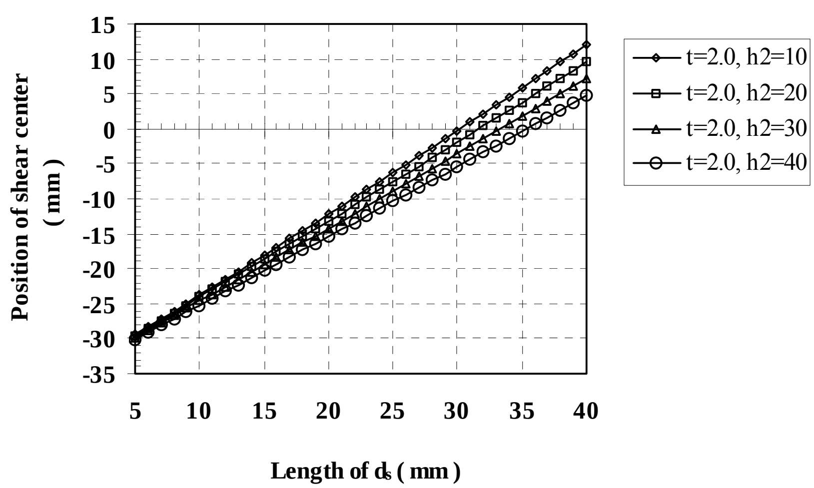



**Figure 7.2.6** Effects of the length of $h_2$ and $d_s$ on the position of the shear center

Figure 7.2.7 and Figure 7.2.8 shows the effects of the length of $h_2$ and $d_s$ on the distributed load and the value of $q/A_g$ . It can be seen that the maximum values of $q$ and $q/A_g$ are reached at the point where the applied load passes through the shear center. This case occurs when the length of $d_s$ is varied from 30 mm to 35 mm and the length of $h_2$ is varied from 10 mm to 40 mm.

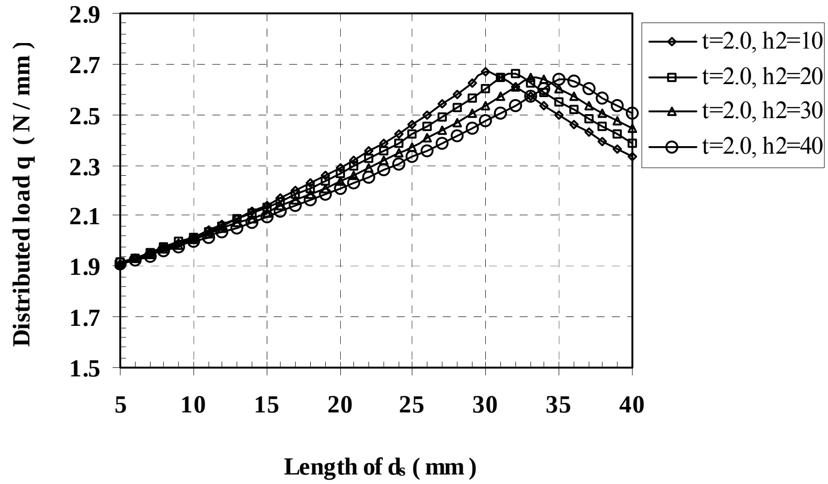



**Figure 7.2.7** Effects of the size of the stiffeners on the load distribution

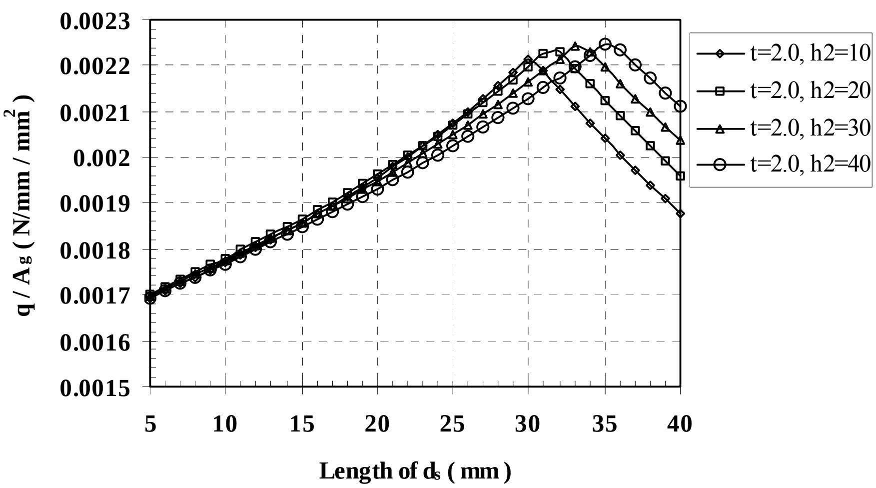



**Figure 7.2.8** Effects of the size of the web stiffener on the value of $q/A_g$

In general, with fixed width of the flange and depth of the lip, the value of $W/A_g$ depends on the lengths of $h_1$ and $h_3$. The best position of the web stiffener is at the position where both $h_1$ and $h_3$ are starting to be fully effective. Moreover, the longer the length of $h_2$ and the shorter the length of $d_s$, the higher the value of $W/A_g$ . The load efficiency depends on the position of the shear center. When the position and the size of the web stiffener make the shear center pass through the shear center, the load efficiency reaches its maximum value.

### 7.2.3 Discussions

The current Eurocode 3, Part 1.3 (1996) assumes that the load is applied on the top of the web and the shear center of the cross-section is to the left of the applied load as shown in Figure 7.1.1. However, the analyses being carried out in Section 7.2 indicate that the shear center can be located at a point between the top and the bottom flange. In this case, the applied load actually moves to the point on the top flange passing through the shear center. (This is based on a personal discussion with Professor Torsten Höglund from Royal Institute in Sweden.)

In the current Eurocode 3, Part 1.3, the lateral load, $q_{h,Fd}$, acting on the free flange due to torsion and lateral bending is calculated using

$$q_{h,Fd} = k_h \cdot q_{Fd} , \qquad \textbf{( 7.2.1 )}$$

where the coefficient $k_h$ for Σ-shape cross-section is calculated as:

$$k_h = \frac{e}{h} , \qquad \textbf{( 7.2.2 )}$$

in which $e$ is the distance between the applied load and the shear center. When the shear center is at a point inside the cross-section, the value of $e$ is zero. This makes the value of $k_h$ be zero.

Figure 7.2.9 shows the value of $q/A_g$ that is calculated based on the above-mentioned criteria, i.e. the value of $k_h$ is zero when the shear center is at a point between two flanges.

These calculations are carried out for the cross-section with $b = 80$ mm, $c = 36$ mm, $h_2 = 25$ mm and $d_s = 32$ mm. The value of $h_1$ is varied from 35 mm to 100 mm with a step of 1 mm.

When the shear centers are inside the cross-section, the values of $q/A_g$ calculated using $e = 0$ are higher than those calculated using the Eurocode 3 method. In addition, when $e = 0$, the variation of the value of $q/A_g$ mainly depends on the effective section modulus, $W$ and the reduction factor for flexural buckling of the free flange, $\chi$. The reduction factor is increased when the length of $h_1$ is varied from 35 mm to 60 mm with a height of 350 mm. When the length of $h_1$ is increased further, the reduction factor stays constant. This is shown in Figure 7.2.10. Therefore, the maximum value of $q/A_g$ is reached when the shear center is at a point inside the cross-section instead of the load passing through the shear center.

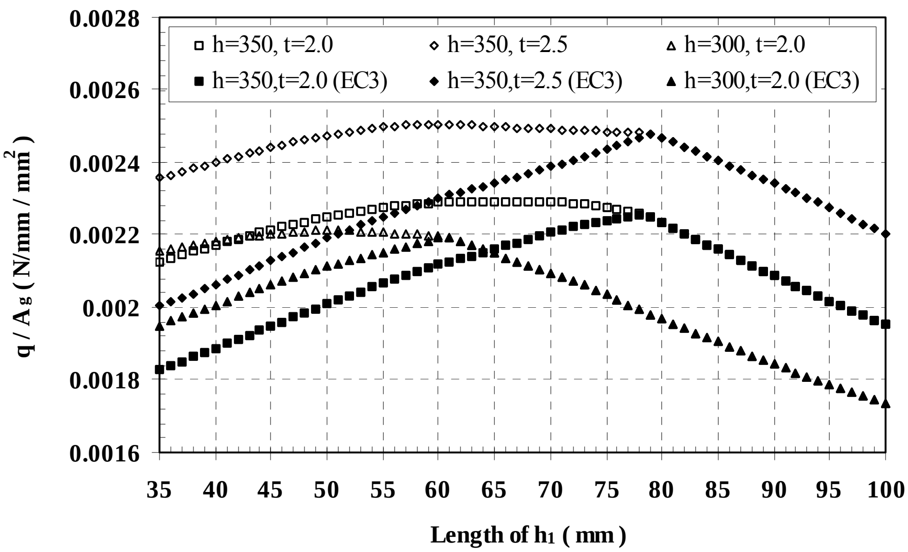



**Figure 7.2.9** Comparisons of the values of $q/A_g$ calculated based on $e = 0$ to those based on $e \neq 0$

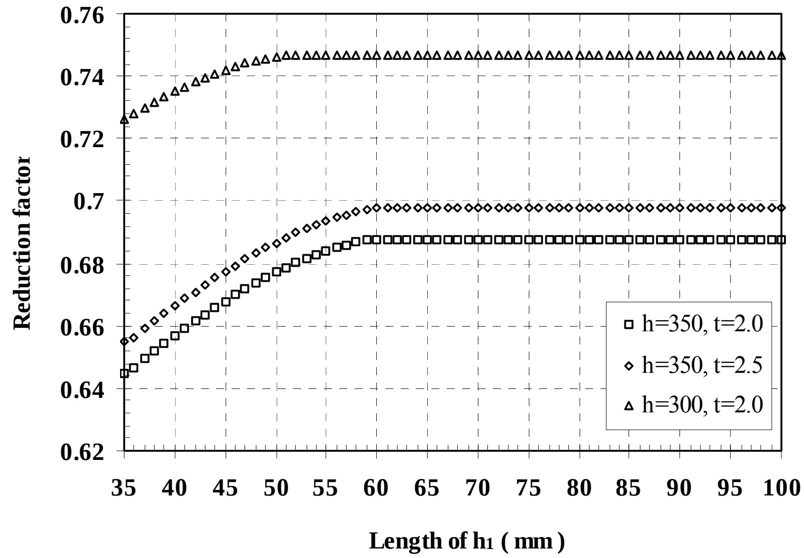



**Figure 7.2.10** Effects of the length of $h_1$ on the reduction factor for flexural buckling of the free flange

## 7.3 Effects of the Web Stiffener on the Elastic Buckling Stress Calculated Based on Finite Strip Method

As mentioned in Chapter 3, the main idea in the modified Eurocode 3 method is to integrate the numerical elastic buckling stress into the design process based on Eurocode 3. In this section, the effects of position and size of the web stiffeners on the elastic buckling stresses are investigated. These calculations are carried out for a cross-section with $h = 350$ mm, $t = 2.0$ mm and the ratio of $c/b = 0.45$.

### 7.3.1 Effects of the position of the stiffener

Figure 7.3.1 shows the effects of the value of $h_1$ on the elastic buckling stress when the width of the flange, $b$, is 60 mm. In this calculation, other given dimensions are $h_2 = 25$ mm and $d_s = 32$ mm. The typical buckling modes for this case are shown in Figure 7.3.2.

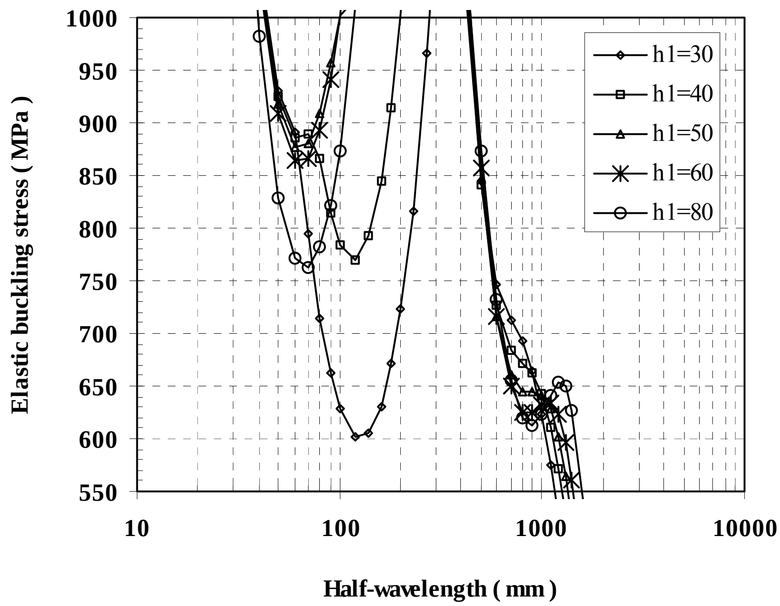
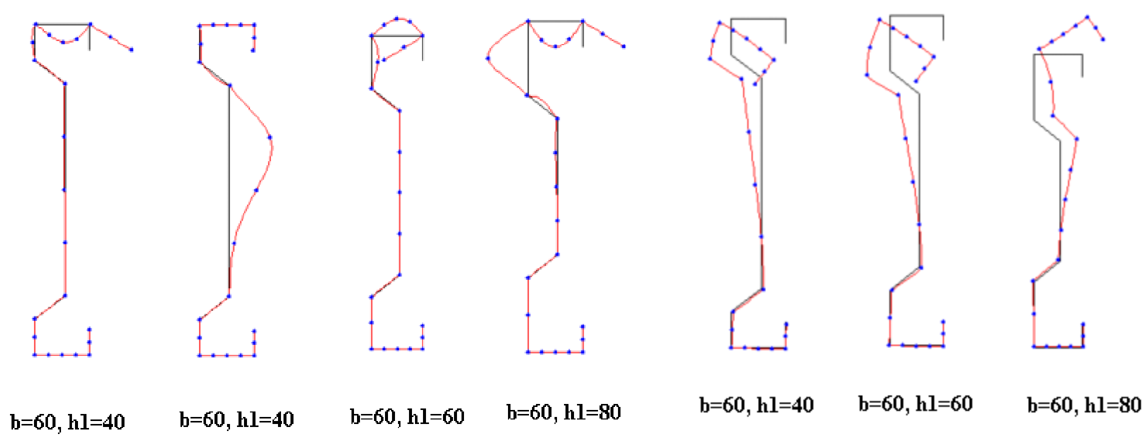



**Figure 7.3.1** Effects of the length of $h_1$ on the elastic buckling stress ( $b = 60$ mm )




**(a)** Local buckling modes **(b)** Distortional buckling modes

**Figure 7.3.2** Buckling modes ( $b = 60$ mm )

When the value of $h_1$ is varied from 30 mm to 50 mm, there are two types of the local buckling mode: one is the local buckling mode of the flange and the other is the local buckling mode of $h_3$. With the same variation of the value of $h_1$, the elastic local buckling stresses corresponding to the flange local buckling mode are decreased and those corresponding to the local bucking mode of $h_3$ are increased. When the value of $h_1$ is increased further, the only local buckling mode is the buckling of $h_1$ and the elastic local buckling stresses corresponding to this mode decrease further. The distortional buckling modes are the same when the length of $h_1$ is varied from 30 mm to 80 mm and the elastic distortional buckling stresses are decreased with the same variation of $h_1$.

If the width of the flange is 80 mm, only one local buckling mode exists with variations of $h_1$ from 40 mm to 100 mm. However, the element that initiates the buckling is transformed from flange to element of $h_1$. The elastic local buckling stresses are decreased with the same variations of $h_1$. Moreover, the elastic distortional buckling stress is decreased when the value of $h_1$ is varied from 40 mm to 80 mm and is increased when the value of $h_1$ is varied from 80 mm to 100 mm. The buckling curves and the typical buckling modes are shown in Figure 7.3.3 and Figure 7.3.4, respectively.

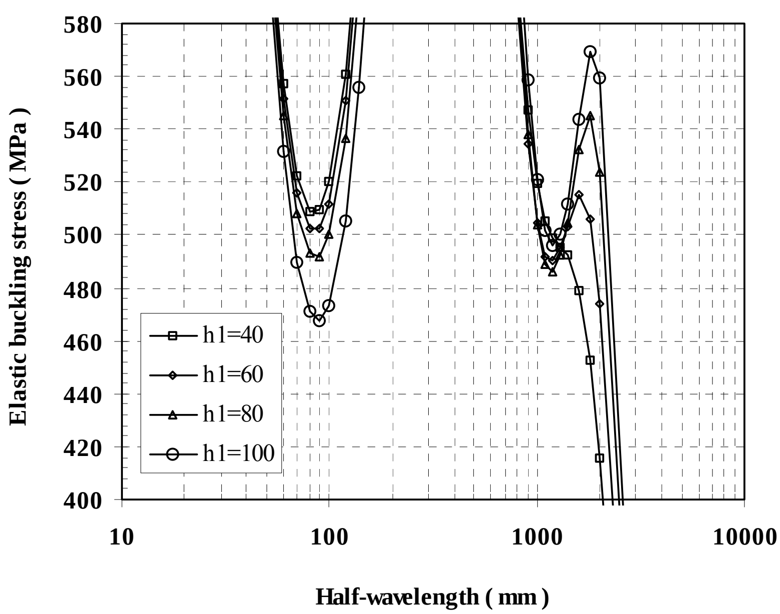
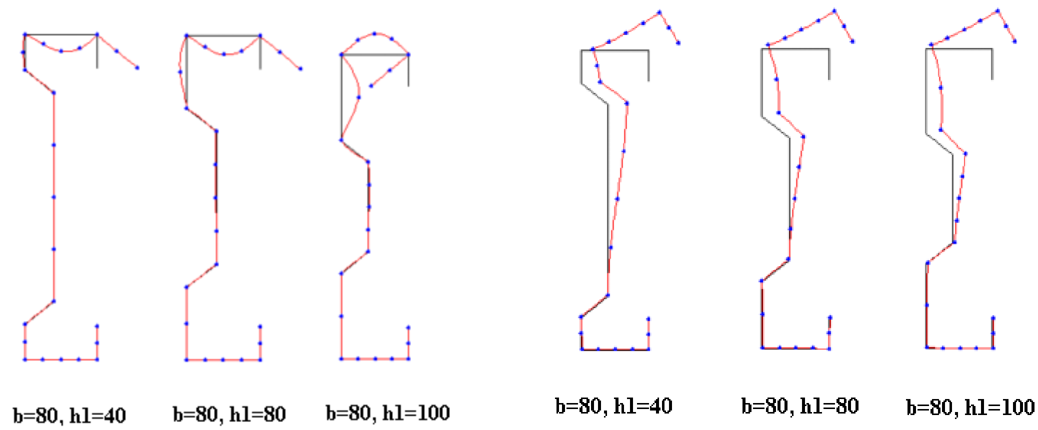



**Figure 7.3.3** Effects of the value of $h_1$ on the elastic buckling stress ( $b = 80$ mm )




**(a)** Local buckling modes **(b)** Distortional buckling modes

**Figure 7.3.4** Buckling modes ( $b = 80$ mm )

In general, with the same width of the flange and depth of the lip, the shorter the length of $h_1$, the higher the values of elastic buckling stresses. As for the elastic distortional buckling stresses, their values depend on the relation of the width of the flange to the length of $h_1$. If the value of $h_1$ is increased from a value less than the width of the flange to the value equal to the width of the flange, the distortional buckling stress decreases with the increase of $h_1$. If the value of $h_1$ is increased from a value equal to the width of the flange to a value larger than the width of the flange, the distortional buckling stress increases with the increase of $h_1$.

### 7.3.2 Effects of the value of $d_s$

The effects of the length of $d_s$ on the elastic buckling stresses with $h_2 = 10$ mm are shown in Figure 7.3.5. In the figure, such symbol as '$h_1 = 40, d_s = 5$' represents a cross-section with $h_1 = 40$ mm and $d_s = 5$ mm.

When the value of $d_s$ is 5 mm and the value of $h_1$ is varied from 40 mm to 80 mm, three types of critical buckling modes exist: the local buckling mode (Figure 7.3.6 (a)), the web distortional buckling mode (Figure 7.3.6 (b)), in which the web stiffener moves with other elements on the web, and the conventional distortional buckling mode (Figure 7.3.6 (c)).

When the value of $d_s$ is increased to 35 mm, the type of local buckling mode depends on the value of $h_1$. When the value of $h_1$ is 60 mm, there exist two local buckling modes: one is the buckling of the elements above the web stiffener and the other is the buckling of the element under the web stiffener. When the value of $h_1$ is increased to 80 mm, only the first buckling mode exists because the value of $h_3$ is decreased with the increase of the value of $h_1$. The distortional buckling mode for both of these two cases is the conventional distortional buckling mode.

The elastic buckling stresses of the indistinct buckling modes can be determined as the minimum value before the current buckling mode is transformed to another buckling mode similar to those of Z-shape purlin described in Chapter 5. For instance, when the half-wavelength is less than 60 mm, the local buckling mode is the flange-lip buckling mode and after that the buckling mode starts to become the web distortional buckling mode. Thus, the elastic local buckling is assumed to occur at half-wavelength of 60 mm.

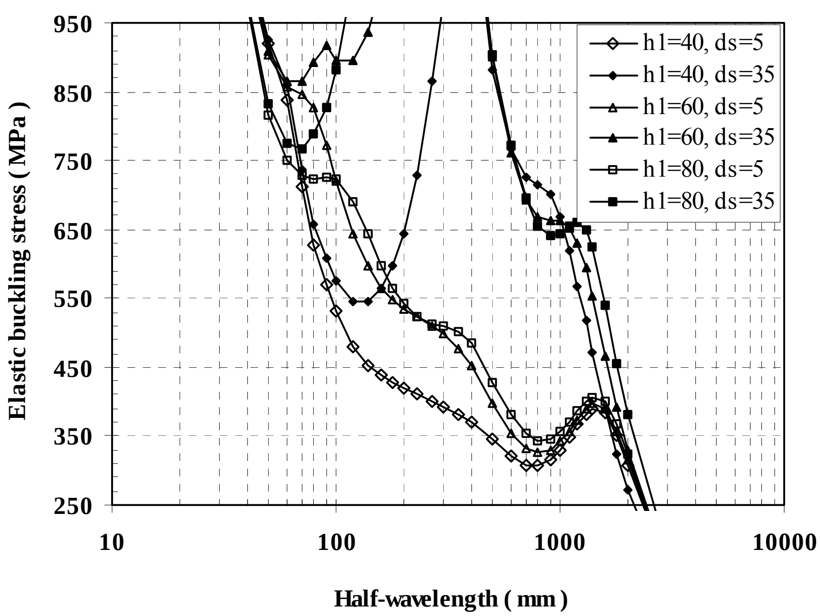



**Figure 7.3.5** Effects of values of $d_s$ on elastic buckling stresses

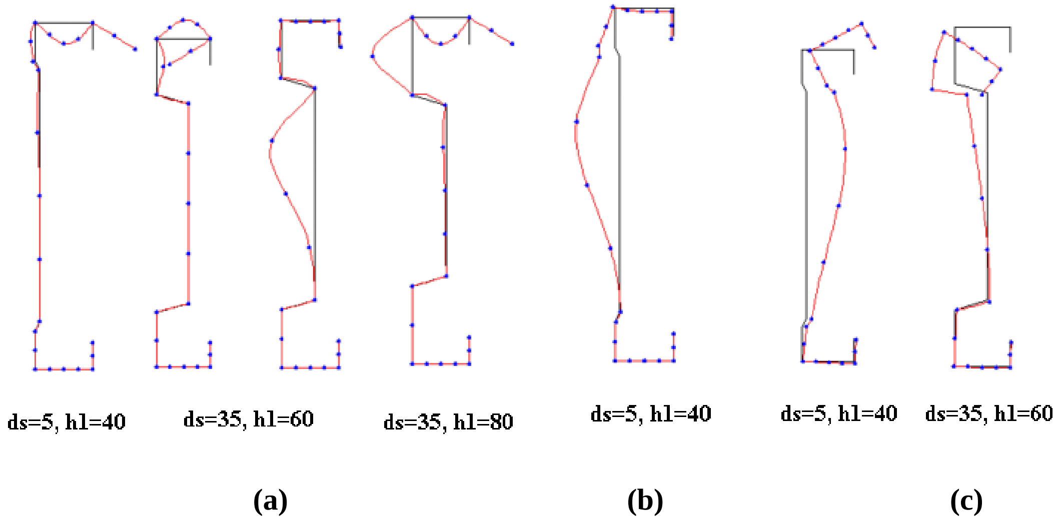



**Figure 7.3.6** Buckling modes (a) Local buckling modes (b) Web distortional buckling mode (c) Distortional buckling mode

In summary, the length of $d_s$ should be large enough to avoid the web distortional buckling mode. The conventional elastic distortional buckling stress is also lower when the web distortional buckling mode occurs.

### 7.3.3 Effects of the value of $h_2$

The effects of the length of $h_2$ on elastic buckling stresses for the cross-section with $d_s = 10$ mm are shown in Figure 7.3.7. In the figure, such symbol as '$h_1 = 40, h_2 = 5$' represents the cross-section with $h_1 = 40$ mm and $h_2 = 5$.

When the length of $h_1$ is 40 mm and the length of $h_2$ is varied from 5 mm to 35 mm; or when the length of $h_1$ is 60 mm and the length of $h_2$ is 5 mm, there exist three buckling modes, i.e. the local buckling mode of the flange, lip and the element above the web stiffener, the local buckling of the element under the web stiffener and the conventional distortional buckling mode. For the other three cases listed in Figure 7.3.7, the local buckling mode of the element under the web stiffener is replaced by the web distortional buckling mode. The typical buckling modes are shown in Figure 7.3.8. Similar to the length of $d_s$, the length of $h_2$ should be large enough to avoid the web distortional buckling mode.

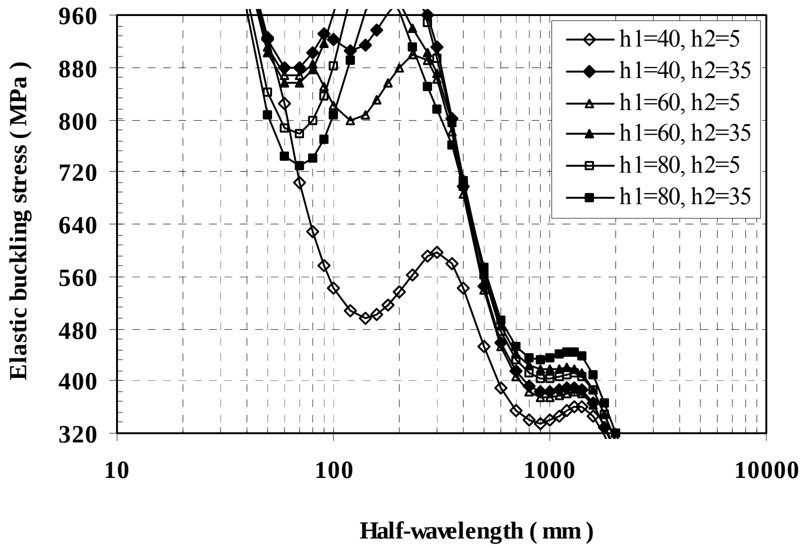



**Figure 7.3.7** Effects of value of $h_2$ on elastic buckling stresses

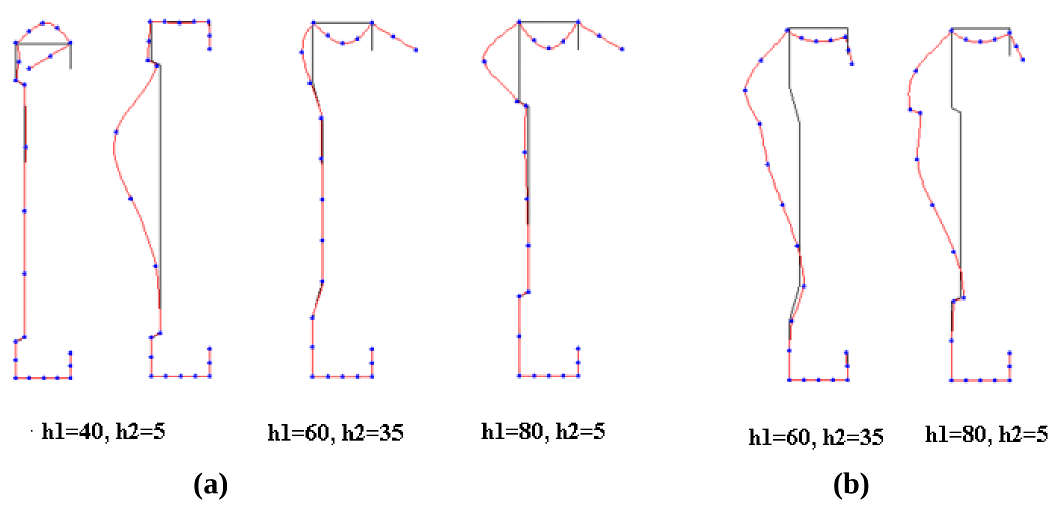
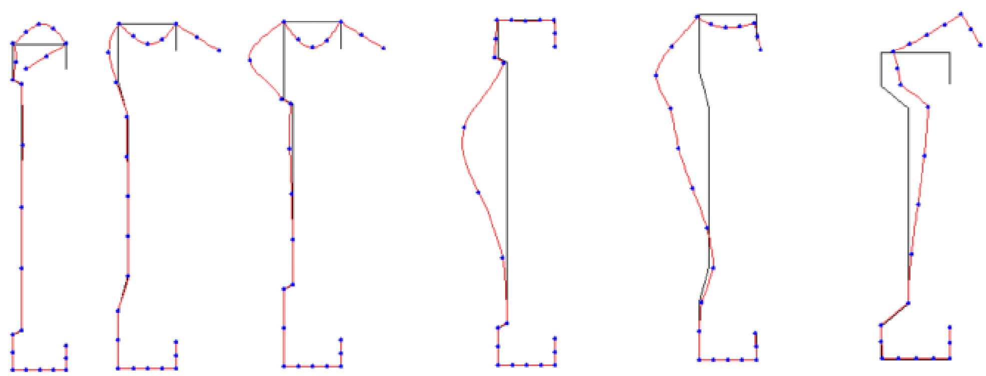



**Figure 7.3.8** Buckling modes (a) Local buckling mode (b) Web distortional buckling mode

## 7.4 Design of Σ-Shape Purlins Based on the Modified Eurocode 3 Method

The typical buckling modes for Σ-shape purlins from the analyses in Section 7.3 are shown in Figure 7.4.1. These buckling modes can be classified into four types. The first type is the local buckling of the flange, the lip and the element above the web stiffener (Figure 7.4.1 (a)). In this mode, the buckling may be initiated by any one of the above-mentioned elements. The second type is the local buckling of the compression part under the web stiffener (Figure 7.4.1 (b)). The third type is the web distortional buckling mode (Figure 7.4.1 (c)), in which the web stiffener is moved with other elements along the web. The last type is the distortional buckling mode (Figure 7.4.1 (d)).



**(a)** **(b)** **(c)** **(d)**

**Figure 7.4.1** Typical buckling modes (a) Local buckling of flange, lip and element above the web stiffener; (b) Local buckling of the element under the web stiffener; (c) Web distortional buckling mode; (d) Conventional distortional buckling mode

When the effective widths of the flange, the lip and the compression element above the web stiffener are determined, the reduction factor, $\rho$, is calculated using the elastic buckling stress corresponding to the first type buckling mode. The effective width for the flange is

$$
\begin{aligned}
b_{eff,1} &= 0.5 \cdot \rho \cdot b, \text{ and} \\
b_{eff,2} &= 0.5 \cdot \rho \cdot b
\end{aligned}
\qquad \textbf{(7.4.1)}
$$

the effective width of the lip is

$$c_{eff} = \rho \cdot c\,, \tag{7.4.2}$$

and the effective width of the compression elements is

$$\begin{aligned} h_{1eff,1} &= 0.4 \cdot \rho \cdot h_1\text{, and} \\ h_{1eff,2} &= 0.6 \cdot \rho \cdot h_1 \end{aligned} \tag{7.4.3}$$

The calculation of the effective width for the compression part under the web stiffener depends on the actual buckling mode. If the second type of buckling modes is dominated, the effective width of the element is calculated using the elastic local buckling stress corresponding to this buckling mode. The effective width of this element is

$$\begin{aligned} h_{3eff,1} &= 0.4 \cdot \rho \cdot h_3\text{, and} \\ h_{3eff,2} &= 0.6 \cdot \rho \cdot h_3 \end{aligned} \tag{7.4.4}$$

If the web distortional buckling mode is critical, the compression parts above and under the web stiffener are designed as a whole element, i.e. the reduction factor is calculated using this buckling stress for the element with a length of $h_1 + h_2 + h_3$. The effective widths are

$$\begin{aligned} h_{1eff,1} &= 0.4 \cdot \rho \cdot h_c\,, \\ h_{1eff,2} &= 0, \\ h_{3eff,1} &= 0\text{, and} \\ h_{3eff,2} &= 0.6 \cdot \rho \cdot h_c\,. \end{aligned} \tag{7.4.5}$$

where $h_c$ is defined as $h_c = h_1 + h_2 + h_3$. For other cases, the effective width of the element is calculated using the method described in Eurocode 3.

When the reduced thickness for the edge stiffener is determined, the reduction factor, $\chi$, is calculated using the distortional buckling stress corresponding to the fourth buckling mode.

As far as the web stiffener is concerned, if the web distortional buckling mode is one of the critical buckling modes, the compressed parts above and under the web stiffener are designed as a single element and the reduced thickness is zero. If the local buckling mode of the compression element under the stiffener is critical, the thickness of the web stiffener is assumed fully effective. For other cases, the reduced thickness is determined using the Eurocode 3 procedure.

## 7.5 Comparisons of Eurocode 3 Method and Modified Eurocode 3 Method

The comparisons of the values of $q/A_g$ and $M_y/A_g$ calculated based on the Eurocode 3 method and the modified Eurocode 3 method are given in Table 7.5.1. These calculations are carried out for a cross-section with $h = 350$ mm, $t = 2.0$ mm and $L = 6000$ mm. The unit for $q/A_g$ is $N/mm/mm^2$ and for $M_y/A_g$ is $N \cdot mm/mm^2$.

Table 7.5.1 shows that for the case when the local buckling mode of the element under the web stiffener is critical, the values of $q/A_g$ and $M_y/A_g$ calculated based on the modified Eurocode 3 method are 5.1 % and 5.8 % higher than those calculated based on the Eurocode 3 method. When the web distortional buckling mode is one of the local buckling modes, the values of $q/A_g$ and $M_y/A_g$ calculated based on modified Eurocode 3 method are 4.8 % and 6.2 % lower than those calculate using the Eurocode 3 method since the elastic distortional buckling stress is lower than those calculated using the Eurocode 3 method. Thus, the values of $q/A_g$ and $M_y/A_g$ are lower than those calculated using the Eurocode 3 method. However, the differences are not significant since the differences of the distortional buckling stress are not significant. For other cases listed in the table, the differences between the values of these two methods are not very large. The reason is that the elastic local buckling stresses and distortional buckling stresses calculated using the Eurocode 3 method and FSM are not significantly different.

**Table 7.5.1** Comparison of Eurocode 3 method (EC3) and modified Eurocode 3 method (MEC3)

| $c$ | $h_1$ | $h_2$ | $d_s$ | **Buckling modes** | **MEC3** | | **EC3** | | **MEC3 / EC3** | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $q/A_g$ | $M_y/A_g$ | $q/A_g$ | $M_y/A_g$ | $q/A_g$ | $M_y/A_g$ |
| $b = 60$ mm | | | | | | | | | | |
| 27 | 40 | 25 | 32 | **(a) (b) (d)** | 0.002191 | 28067.45 | 0.002084 | 26540.30 | **1.051** | **1.058** |
| 27 | 40 | 10 | 5 | **(a) (c) (d)** | 0.002341 | 24312.19 | 0.002459 | 25921.65 | **0.952** | **0.938** |
| 27 | 60 | 20 | 10 | **(a) (d)** | 0.002814 | 26875.24 | 0.002825 | 27005.02 | **0.996** | **0.995** |
| 36 | 40 | 20 | 10 | **(a) (d)** | 0.002697 | 25909.62 | 0.002776 | 26827.80 | **0.972** | **0.966** |
| 36 | 60 | 20 | 10 | **(a) (d)** | 0.002747 | 26594.38 | 0.002816 | 27423.62 | **0.975** | **0.970** |
| $b = 80$ mm | | | | | | | | | | |
| 36 | 40 | 25 | 32 | **(a) (d)** | 0.003243 | 26503.23 | 0.003238 | 26458.14 | **1.002** | **1.002** |
| 36 | 40 | 20 | 10 | **(a) (d)** | 0.002897 | 25318.87 | 0.002978 | 26303.41 | **0.973** | **0.962** |
| 36 | 60 | 20 | 10 | **(a) (d)** | 0.002907 | 25804.15 | 0.002994 | 26898.88 | **0.971** | **0.959** |

The elastic buckling stresses for each element calculated using Eurocode 3 method and based on finite strip method are shown in Table 7.5.2. In the table, the 'web_1' represents the element above the web stiffener and 'web_2' represents the compression part under the web stiffener.

**Table 7.5.2 Elastic buckling stress calculated using Eurocode 3 method and finite strip method**

| Dimensions ( mm ) | | | | | methods | Elastic local buckling stress ( MPa ) | | | | Elastic distortional buckling stress (MPa) | Partially effective elements |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $b$ | $c$ | $h_1$ | $h_2$ | $d_s$ | | flange | lip | web_1 | web_2 | | |
| 60 | 27 | 40 | 25 | 32 | EC3 | 871.87 | 721.22 | 2250.13 | 353.56 | 419.77 | web_2 |
| | | | | | FSM | | 884.86 | | 769.32 | 671.00 | -------- |
| 60 | 27 | 40 | 10 | 5 | EC3 | 871.87 | 721.22 | 2227.61 | 276.04 | 419.77 | web_2 |
| | | | | | FSM | | 920.64 | | ------- | 306.26 | compression parts on the web (392.83 MPa) |
| 60 | 27 | 60 | 20 | 10 | EC3 | 871.87 | 721.22 | 1026.12 | 476.01 | 419.77 | web_2 |
| | | | | | FSM | | 861.26 | | 476.01 | 405.43 | web_2 |
| 60 | 36 | 40 | 20 | 10 | EC3 | 871.87 | 485.11 | 2227.61 | 325.58 | 536.30 | web_2 |
| | | | | | FSM | | 601.29 | | 325.58 | 349.89 | flange, web_1, web_2 |
| 60 | 36 | 60 | 20 | 10 | EC3 | 871.87 | 485.11 | 1026.12 | 476.01 | 536.30 | web_2 |
| | | | | | FSM | | 598.58 | | 476.01 | 379.95 | flange, web_1, web_2 |
| 80 | 36 | 40 | 25 | 32 | EC3 | 481.39 | 399.54 | 2250.13 | 353.85 | 358.04 | flange, web_2 |
| | | | | | FSM | | 509.14 | | 353.85 | 495.36 | flange, web_1, web_2 |
| 80 | 36 | 40 | 20 | 10 | EC3 | 481.39 | 399.54 | 2227.61 | 325.85 | 358.04 | flange, web_2 |
| | | | | | FSM | | 505.90 | | 325.85 | 350.94 | flange, web_1, web_2 |
| 80 | 36 | 60 | 20 | 10 | EC3 | 481.39 | 399.54 | 1026.12 | 476.40 | 358.08 | flange, web_2 |
| | | | | | FSM | | 500.56 | | 476.40 | 368.03 | flange, web_1, web_2 |

# 8 OPTIMIZATION OF Σ-SHAPE PURLINS

## 8.1 Introduction

The computer program, ODSP-GA, developed in Chapter 4 is used to optimize the dimensions of Σ-shape purlins, whose behaviors have been investigated in Chapter 7. The purlin is assumed to be continuous over two spans subject to gravity loading. Two types of the cross-sections are mainly considered, one with a single edge stiffener and the other with double edge stiffeners. The dimensions of the cross-section are shown in Figure 8.1.1.

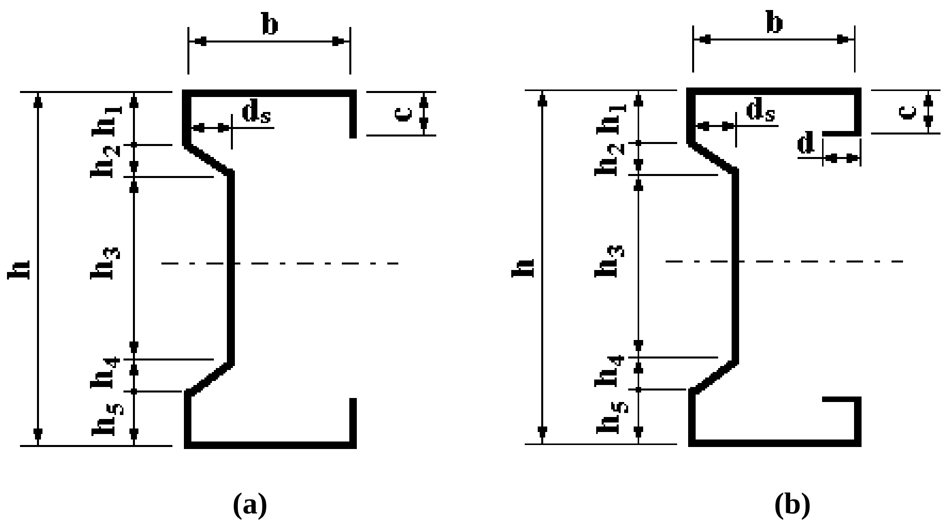



**Figure 8.1.1** Dimensions of Σ-shape purlins: (a) cross-section with double edge stiffeners; (b) cross-section with single edge stiffener

In the Figure 8.1.1, $b$ is the width of the flange; $c$ is the depth of the lip; $h$ is the height of the cross-section; $h_1$ and $h_5$ are distances of the web stiffeners to the top and bottom flange, respectively; $h_2$ and $h_4$ are depth of the web stiffener in the direction of the web height; $d_s$ is the depth of the web stiffener perpendicular to the web height; $h_3$ is the distance between the stiffeners along the web; and $d$ is the width of the second edge stiffener.

The optimization is carried out for a set of cross-sections with a given height, thickness and span. The purlins are designed based on two methods: the Eurocode 3 method and the modified Eurocode 3 method. For purlins designed based on the Eurocode 3 method, the optimization is carried out for the cases with freely varied design variables first and then for the simplified cases. In addition, an example is demonstrated in this chapter to show the integration of the modified Eurocode 3 method into the GA optimization process. Moreover, the optimization is also performed with the objective to maximize the moment resistance.

## 8.2 Problem Formulation and Sensitivity Analysis

### 8.2.1 Design variables

The design variables are such dimensions of the cross-section as $b, c, d, h_1, h_2$ and $d_s$, and the height, the thickness and the span are preassigned variables. According to Eurocode 3, the ratio of $c$ to $b$ should in the range of 0.2 and 0.6, i.e. $0.2 \le c/b \le 0.6$. By assuming $\alpha = c/b$, one of design variables, $c$, can be replaced by this ratio, $\alpha$. Similarly, the ratio of the depth of the second lip to the length of the flange, $\beta$, whose value is between 0.1 and 0.3, i.e. $0.1 \le \beta = d/b \le 0.3$, can be chosen as one of the design variables instead of the depth of the second lip. Moreover, the ratio of the value of $h_1$ to the length of the flange is chosen as a design variable instead of the value of $h_1$. The design variables are discrete values, and their possible values are listed in Table 6.1.1.

**Table 8.2.1** Values of the design variables

| **Design variables** | **Possible values** |
|---|---|
| Length of the flange $b$ | from 40 mm to 100 mm with step 1 mm |
| Ratio of $c$ to $b$ | from 0.2 to 0.6 with step 0.01 |
| Ratio of $d$ to $b$ | from 0.1 to 0.3 with step 0.01 |
| Ratio of $h_1$ to $b$ | from 0.45 to 1 with step 0.01 |
| Values of $h_2$ | from 5 to 40 mm with step 1 mm |
| Values of $d_s$ | from 5 to 40 mm with step 1 mm |

### 8.2.2 Objective function and constraints

The objective of the optimization is to maximize the load resistance over the gross area of the cross-section, i.e.

$$\textit{Maximize} \qquad \frac{q}{A_g} \tag{8.2.1}$$

subjected to the following constraints according to Eurocode 3, Part 1.3:

$$\frac{h}{t} \leq 500, \text{ and } \frac{b}{t} \leq 60. \tag{8.2.2}$$

From the manufacturer's point of view, the sum of flanges, lips and the web should be in the range of the given width of the strip, i.e.

$$200 \text{ mm} \leq 2 \cdot b + 2 \cdot c + 2 \cdot d + 2 \cdot h_1 + 2 \cdot h_3 + 2 \cdot \sqrt{h_2^2 + d_s{}^2} \leq 625 \text{ mm}, \tag{8.2.3}$$

and the height of the web part above the web stiffener should not be less than 35 mm, i.e.

$$h_1 \geq 35 \text{ mm}. \tag{8.2.4}$$

In addition, due to the shape requirement, the following constraint should be satisfied

$$d_s < b\,. \tag{8.2.5}$$

The constrained problem defined above is transformed into an unconstrained problem by using a quadratic penalty function as described in Chapter 4. Therefore, the optimization problem becomes:

$$Maximize \quad F = \begin{cases} \dfrac{q}{A_g} - KK \cdot n \cdot CC, & \text{when } \dfrac{q}{A_g} > KK \cdot n \cdot CC, \\ 0, & \text{otherwise.} \end{cases} \tag{8.2.6}$$

### 8.2.3 Sensitivity analysis of the fitness function

Sensitivity analyses of the fitness to the coefficient, $KK$, given in the objective function are shown in Table 8.2.2 and Figure 6.2.6. These analyses are calculated for the cross-section with $h = 350$ mm, $t = 2.5$ mm and $L = 8000$ mm. And the optimization is carried out in 10 runs for each case, in which one run is defined as a complete run of GA.

Table 8.2.2 indicates that when the value of $KK$ is increased from 0.5 to 100, the maximum value of fitness in each of the 10 runs is 0.002573 N/mm/mm$^2$, and the optimum dimensions are the same for each case. Thus, the variation of $KK$ has no influence on finding the maximum values for these cases.

In addition, Figure 8.1.1 shows that the value of $KK$ has effects on finding the number of results in feasible regions. For instance, when the value of $KK$ is 0.5, only five of 10 runs are found in the feasible regions. When the value of $KK$ is increased further, the results found are all in the feasible regions.

**Table 8.2.2 Effects of value KK to the maximum value of $q/A_g$ in 10 runs**

| $KK$ | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| **0.5** | 91 | 33.67 | 43.68 | 22 | 27 | **0.002573** | 28214.39 | 1508.86 |
| **1** | 91 | 33.67 | 43.68 | 22 | 27 | **0.002573** | 28214.39 | 1508.86 |
| **2** | 91 | 33.67 | 43.68 | 22 | 27 | **0.002573** | 28214.39 | 1508.86 |
| **5** | 91 | 33.67 | 43.68 | 22 | 27 | **0.002573** | 28214.39 | 1508.86 |
| **10** | 91 | 33.67 | 43.68 | 22 | 27 | **0.002573** | 28214.39 | 1508.86 |
| **50** | 91 | 33.67 | 43.68 | 22 | 27 | **0.002573** | 28214.39 | 1508.86 |
| **100** | 91 | 33.67 | 43.68 | 22 | 27 | **0.002573** | 28214.39 | 1508.86 |

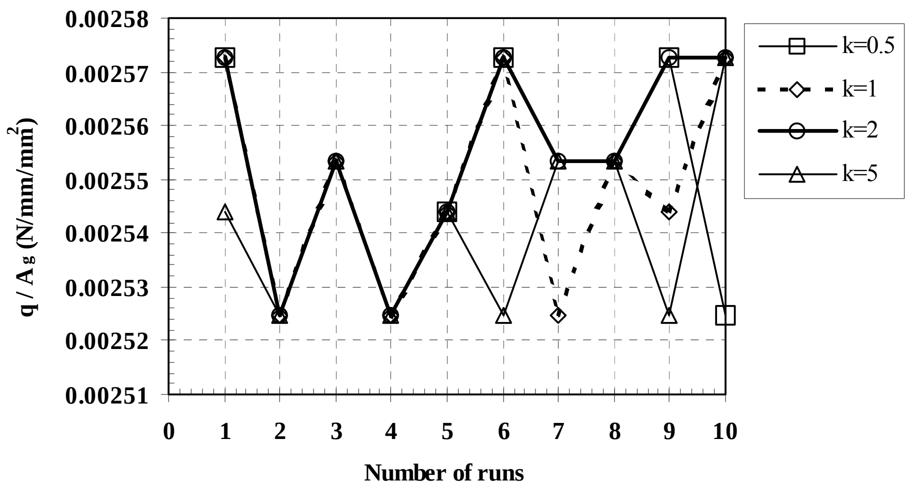



**Figure 8.2.1** Effects of value KK to the value of $q/A_g$ in 10 runs

From the above analysis, we conclude that the coefficient, $KK$, does have an influence on finding the optimum dimensions. But this influence is not so great. Thus, we set $KK = 2$. If one of the results in 10 runs is not in the feasible domain, the value of $KK$ is increased until all the results in 10 runs are in the feasible domain.

Moreover, from the parametric analysis in Chapter 6, it can be seen that when the values of the crossover rate are varied from 0.6 to 0.9 and the mutation rate from 0.01 to 0.0001, the performance of the GA does not change very much. Thus, in the further analysis, similar parameters to those of optimizing Z-shape purlins are selected, i.e. with a crossover rate of 0.8 and a mutation rate of 0.001. The size of the population is set equal to twice the length of the encoded binary string and the tournament selection method is used.

## 8.3 Optimization Results Based on the Eurocode 3 Method

The optimization is carried out for the Σ-shape purlins designed based on the Eurocode 3 method in this section. The design variables are freely varied with values provided in Table 6.1.1 and this process is named as 'Optimization with freely varied parameters' in the following analysis. According to the optimization results from the optimization with freely varied parameters, some values of the design variables are fixed and transformed to preassigned variables so as to simplify the optimization process.

### 8.3.1 Optimization with freely varied parameters

The optimization results with freely varied parameters for the cross-sections with a single edge stiffener are shown in Table 8.3.1, and those for the cross-sections with double edge stiffeners are shown in Table 8.3.2. In the table, the optimum dimensions are chosen as the ones corresponding to the largest value of $q/A_g$ in the 10 runs. The detailed results in 10 runs for each case are given in Appendix B.

Figure 8.3.1 shows the comparisons of the values of $q/A_g$ and $M_y/A_g$ for the cross-section with a single edge stiffener to those with double lip stiffeners. As far as the values of $q/A_g$ are concerned, most of the cross-sections with a single lip stiffener show higher values. As for the value of $M_y/A_g$, most of the cross-sections with double lip stiffeners show higher values. However, the improvements are not significant, and the maximum improvement is about 5 %. Thus, the cross-section with a single lip stiffener is selected for further analysis.

**Table 8.3.1 Optimization results for the cross-section with single lip stiffener (with freely varied parameters)**

| $h$ (mm) | $L$ (mm) | $t$ (mm) | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 250 | 4000 | 1.5 | **79** | **36.34** | **38.71** | **31** | **31** | 0.007428 | 18210.46 |
| 250 | 5500 | 2.5 | **81** | **40.50** | **42.93** | **35** | **35** | 0.004723 | 21775.39 |
| 300 | 5000 | 2.0 | **92** | **46.00** | **46.00** | **29** | **35** | 0.005990 | 23082.50 |
| 300 | 6500 | 3.0 | **92** | **46.00** | **46.00** | **30** | **35** | 0.003995 | 26373.00 |
| 350 | 6000 | 2.0 | **83** | **33.20** | **68.06** | **28** | **30** | 0.004241 | 26451.04 |
| 350 | 7500 | 3.0 | **91** | **33.67** | **43.68** | **22** | **27** | 0.003094 | 29654.31 |
| 400 | 6000 | 2.0 | **78** | **28.08** | **46.80** | **38** | **21** | 0.003982 | 28808.56 |
| 400 | 8000 | 3.0 | **79** | **27.65** | **49.77** | **36** | **20** | 0.002398 | 31972.39 |

**Table 8.3.2** Optimization results for the cross-section with double lip stiffeners (with freely varied parameters)

| $h$ (mm) | $L$ (mm) | $t$ (mm) | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 250 | 4000 | 1.5 | **76** | **36.48** | **18.24** | **37.24** | **33** | **33** | 0.007537 | 18950.84 |
| 250 | 5500 | 2.5 | **77** | **32.34** | **15.40** | **40.04** | **32** | **32** | 0.004572 | 21870.06 |
| 300 | 5000 | 2.0 | **81** | **38.88** | **19.44** | **49.41** | **25** | **33** | 0.005921 | 24183.59 |
| 300 | 6500 | 3.0 | **81** | **37.26** | **18.63** | **47.79** | **21** | **31** | 0.003674 | 26024.20 |
| 350 | 6000 | 2.0 | **76** | **31.92** | **15.20** | **74.48** | **31** | **32** | 0.004108 | 27258.13 |
| 350 | 7500 | 3.0 | **77** | **31.57** | **15.40** | **73.15** | **29** | **31** | 0.002762 | 29088.70 |
| 400 | 6000 | 2.0 | **73** | **20.44** | **9.49** | **59.86** | **39** | **18** | 0.003566 | 29001.89 |
| 400 | 8000 | 3.0 | **77** | **21.56** | **10.01** | **63.14** | **40** | **18** | 0.002203 | 31714.06 |

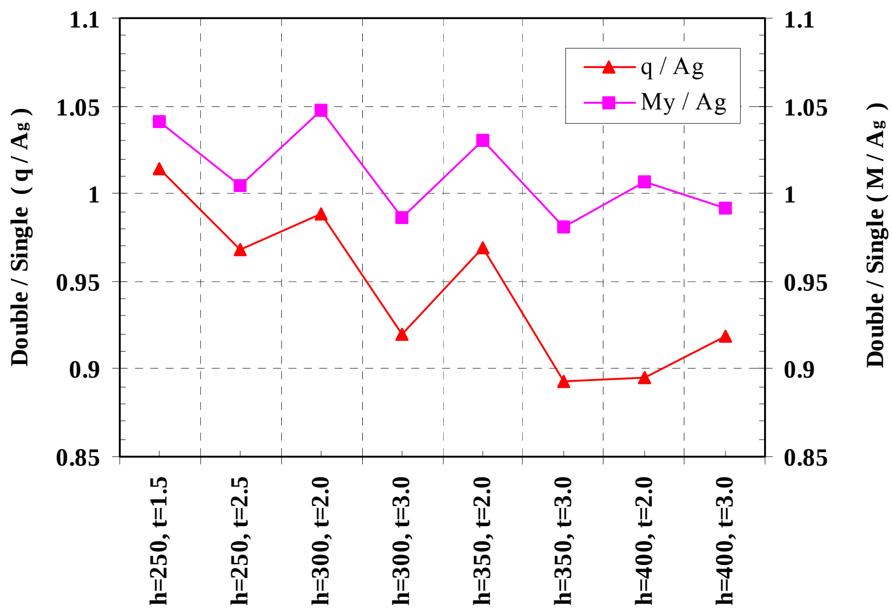



**Figure 8.3.1** Comparison of the values of $q/A_g$ and $M_y/A_g$ for the cross-section with single lip stiffener to those with double lip stiffeners

In addition, from the results listed in Table 8.3.1, it can be seen that for the cross-sections with the same height, but with different thicknesses and spans, the optimum dimensions are similar except for the cross-section with a height of 350 mm. In order to verify the results for the two cross-sections with a height of 350 mm listed in the table, two extra results in 10 runs are given in Table 8.3.3 together with that of the optimum dimension (No.1) for the cross-section with $t = 3.0$ mm, and $L = 7500$ mm.

**Table 8.3.3** Comparisons of optimum dimension with two other cases for the cross-section with $H = 350$ mm, $t = 3.0$ mm, and $L = 7500$ mm

| No. | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 91 | 33.67 | 43.68 | 22 | 27 | 0.003094 | 29654.31 | 1808.544 |
| 2 | 83 | 32.37 | 68.06 | 25 | 29 | 0.002941 | 28966.28 | 1756.449 |
| 3 | 83 | 33.20 | 68.06 | 28 | 30 | 0.002926 | 29021.34 | 1760.186 |

When compared to the optimum dimension for the cross-section with a thickness of 2 mm (Table 8.3.1), the dimensions of No. 2 in Table 8.3.3 are quite similar to the optimum dimensions with a thickness of 2 mm; and the dimensions of No. 3 are the same as those with a thickness of 2.0 mm. However, the values of $q/A_g$ corresponding to these two cross-sections are both lower than those of No. 1 listed in Table 8.3.3. Therefore, the dimensions of No. 1, which has not similar dimensions to those with a thickness of 2 mm, are the optimum dimensions of the cross-section with a thickness of 3 mm. This dissimilarity is possibly due to the transformation of the shear center when the thickness is increased to 3.0 mm.

### 8.3.2 Optimization with some parameters fixed

In order to simplify the optimization process, such dimensions as $d_s$ and $h_2$ are treated as preassigned variables taken from the analysis in Section 8.3.1. According to the results shown in Table 8.3.1, the values of $d_s$ and $h_2$ are set to 32 mm and 25 mm, respectively. Also the value of $h_1$ is redefined as not less than 45 mm, i.e. $h_1 \geq 45$. Therefore, the objective of the optimization is to find the optimum values of $b$, $c$, and $h_1$ so as to maximize the value of $q/A_g$. This optimization process is named as Choice 1.

For further simplification of the optimization process, two other cases are defined according to the relation of the width of the flange to the depth of the lip similar to Z-shape purlin. One is called Choice 2, i.e. the relation between $c$ and $b$ is defined as:

$$c = \frac{11}{30} \cdot (b - 40) + 13, \qquad \textbf{( 8.3.1 )}$$

and the other is called Choice 3, i.e. the relation between $c$ and $b$ is defined as:

$$c = \frac{1}{2} \cdot (b - 40) + 15. \qquad \textbf{( 8.3.2 )}$$

The optimization results are shown in Table 8.3.4, Table 8.3.5, and Table 8.3.6. The results for these three cases are obtained by choosing the dimensions corresponding to the maximum value of $q/A_g$ from 10 runs of GA for each given height, thickness and span. The detailed results are shown in Appendix C. Besides, together with $q/A_g$, the values of $M_y/A_g$ and the areas of the cross-sections corresponding to the optimum dimensions are listed in these three tables.

As for the cross-section with a height of 400 mm, the optimum value of $d_s$ and $h_2$ shown in Table 8.3.1 are 20 mm or 21 mm, and 36 mm or 38 mm, respectively, due to the limitation of strip length. Therefore, with the given values of $d_s$ and $h_2$, i.e. $d_s = 32$ mm and $h_2 = 20$ mm, the corresponding values of $q/A_g$ are lower than those with the optimum value of $d_s$ and $h_2$. Thus, the values of $d_s$ and $h_2$ are modified to $d_s = 20$ mm and $h_2 = 36$ mm. In order to reduce the variation of the dimensions, an extra calculation is carried out for the cross-section $d_s = 20$ and $h_2 = 25$. The optimization results for these cases are also listed in Table 8.3.4, Table 8.3.5 and Table 8.3.6.

**Table 8.3.4** Optimization results for Choice 1

| $d_s = 32, h_2 = 25$, and $h_1 \geq 45$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| $h$ (mm) | $L$ (mm) | $t$ (mm) | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) |
| 250 | 4000 | 1.5 | **82** | **30.34** | **45.10** | 0.007343 | 17786.29 |
| 250 | 5500 | 2.5 | **82** | **31.98** | **45.10** | 0.004713 | 21629.42 |
| 300 | 5000 | 2.0 | **92** | **34.96** | **46.00** | 0.005946 | 22941.95 |
| 300 | 6500 | 3.0 | **92** | **36.80** | **46.00** | 0.003983 | 26201.57 |
| 350 | 6000 | 2.0 | **84** | **37.80** | **70.56** | 0.004248 | 26458.07 |
| 350 | 7500 | 3.0 | **97** | **24.25** | **66.93** | 0.003056 | 27787.72 |
| 400 | 6000 | 2.0 | **71** | **25.56** | **66.74** | 0.002769 | 28720.87 |
| 400 | 8000 | 3.0 | **71** | **25.56** | **66.74** | 0.001733 | 31143.81 |
| $d_s = 20, h_2 = 36$, and $h_1 \geq 40$ | | | | | | | |
| 400 | 6000 | 2.0 | **77** | **30.03** | **44.66** | 0.003881 | 29029.81 |
| 400 | 8000 | 3.0 | **80** | **27.20** | **53.60** | 0.002400 | 31915.50 |
| $d_s = 20, h_2 = 25$, and $h_1 = 50$ | | | | | | | |
| 400 | 6000 | 2.0 | **75** | **30.00** | **50.00** | 0.003843 | 28902.46 |
| 400 | 8000 | 3.0 | **85** | **22.40** | **50.00** | 0.002391 | 30440.39 |

**Table 8.3.5** Optimization results for Choice 2

| $d_s = 32, h_2 = 25,$ and $h_1 \geq 45$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| $h$ (mm) | $L$ (mm) | $t$ (mm) | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) |
| 250 | 4000 | 1.5 | **82** | **28.40** | **45.92** | 0.007251 | 17677.34 |
| 250 | 5500 | 2.5 | **83** | **28.77** | **47.31** | 0.004631 | 21342.53 |
| 300 | 5000 | 2.0 | **92** | **32.07** | **47.84** | 0.005870 | 22759.06 |
| 300 | 6500 | 3.0 | **93** | **32.43** | **49.29** | 0.003936 | 25863.51 |
| 350 | 6000 | 2.0 | **85** | **29.50** | **76.50** | 0.004123 | 25627.49 |
| 350 | 7500 | 3.0 | **86** | **29.87** | **78.26** | 0.002938 | 28535.15 |
| 400 | 6000 | 2.0 | **71** | **24.37** | **68.16** | 0.002743 | 28540.75 |
| 400 | 8000 | 3.0 | **71** | **24.37** | **68.16** | 0.001721 | 30961.31 |
| $d_s = 20, h_2 = 36,$ and $h_1 \geq 40$ | | | | | | | |
| 400 | 6000 | 2.0 | **77** | **26.57** | **44.66** | 0.003868 | 28488.79 |
| 400 | 8000 | 3.0 | **79** | **27.30** | **49.77** | 0.002397 | 31924.56 |
| $d_s = 20, h_2 = 25,$ and $h_1 = 50$ | | | | | | | |
| 400 | 6000 | 2.0 | **76** | **26.20** | **50.00** | 0.003827 | 28245.20 |
| 400 | 8000 | 3.0 | **78** | **26.93** | **50.00** | 0.002362 | 31649.54 |

**Table 8.3.6** Optimization results for Choice 3

| $d_s = 32, h_2 = 25,$ and $h_1 \geq 45$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| $h$ (mm) | $L$ (mm) | $t$ (mm) | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) |
| 250 | 4000 | 1.5 | **82** | **36.00** | **45.10** | 0.007132 | 17895.76 |
| 250 | 5500 | 2.5 | **82** | **36.00** | **45.10** | 0.004640 | 21783.30 |
| 300 | 5000 | 2.0 | **81** | **35.50** | **59.94** | 0.005842 | 23480.35 |
| 300 | 6500 | 3.0 | **92** | **41.00** | **45.08** | 0.003943 | 26386.94 |
| 350 | 6000 | 2.0 | **84** | **37.00** | **72.24** | 0.004253 | 26397.66 |
| 350 | 7500 | 3.0 | **84** | **37.00** | **72.24** | 0.002967 | 29143.63 |
| 400 | 6000 | 2.0 | **67** | **28.50** | **53.60** | 0.002442 | 29134.13 |
| 400 | 8000 | 3.0 | **67** | **28.50** | **53.60** | 0.001516 | 31399.44 |
| $d_s = 20, h_2 = 36,$ and $h_1 \geq 40$ | | | | | | | |
| 400 | 6000 | 2.0 | **69** | **29.50** | **60.03** | 0.003836 | 29813.52 |
| 400 | 8000 | 3.0 | **71** | **30.50** | **66.03** | 0.002212 | 31931.37 |
| $d_s = 20, h_2 = 25,$ and $h_1 = 50$ | | | | | | | |
| 400 | 6000 | 2.0 | **73** | **31.50** | **50.00** | 0.003835 | 29158.77 |
| 400 | 8000 | 3.0 | **73** | **31.50** | **50.00** | 0.002270 | 31994.46 |

### 8.3.3 Analysis of the results

Figure 8.3.2 and Figure 8.3.3 show the comparisons of the values of $q/A_g$ and $M_y/A_g$, respectively, for the three cases mentioned above. This comparison is carried out based on the ratio of the values of $q/A_g$ or $M_y/A_g$ for Choice 2 or Choice 3 to those for Choice 1. In these two figures, the filled squares and triangles represent the results for the cross-section with $d_s = 32$ and $h_2 = 25$; and unfilled squares and triangles represent the cross-section with $d_s = 20$, $h_2 = 36$ when the height is 400 mm.

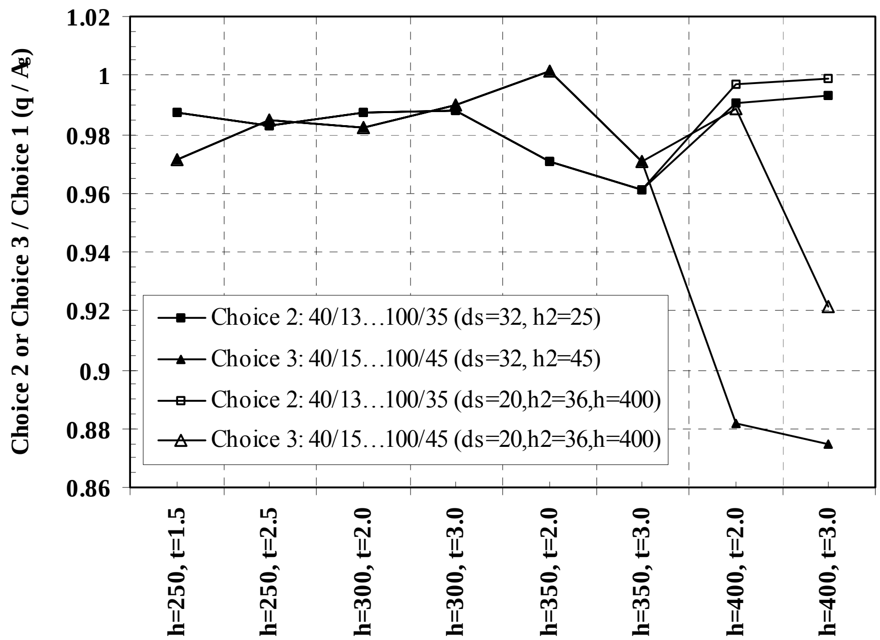



**Figure 8.3.2** Comparison of the values of $q/A_g$ amongst three cases

With $d_s = 32$ mm and $h_2 = 25$ mm, the values of $q/A_g$ for the three cases mentioned above have no big difference when the heights are 250 mm and 300 mm, respectively. When the height is increased to 350 mm, Choice 2 shows the lowest value. But the difference is not significant. When the height is 400 mm, Choice 3 gives lowest value and the difference is much larger than the cases either Choice 1 or Choice 2.

When the cross-sections with $d_s = 20$ mm and $h_2 = 36$ mm are considered, the values of $q/A_g$ are higher than those with $d_s = 32$ mm and $h_2 = 25$ mm when the height of the web is

400 mm for all the three cases mentioned above. However, if compared to either Choice 1 or Choice 2, the values for Choice 3 are still lower. For the cross-sections with $d_s = 20$ mm and $h_2 = 25$ mm, the values of $q/A_g$ are decreased in comparison to the case with $d_s = 20$ mm and $h_2 = 36$ mm. But they are much higher than those with $d_s = 32$ mm and $h_2 = 25$ mm. Thus, the fixed dimension can be set to $h_2 = 25$ mm and $d = 20$ mm for the cross-section with the height of the web of 400 mm, and $h_2 = 25$ and $d = 32$ for other types of cross-sections.

Figure 8.3.3 shows the comparisons of the values of $M_y/A_g$ amongst the three cases mentioned above. For most of the cross-sections, Choice 3 shows higher values of $M_y/A_g$ than those for Choice 1, whereas Choice 2 shows lower values than those for Choice 1. This is due to the fact that the objective is to maximize $q/A_g$ instead of $M_y/A_g$. However, the differences are not great.

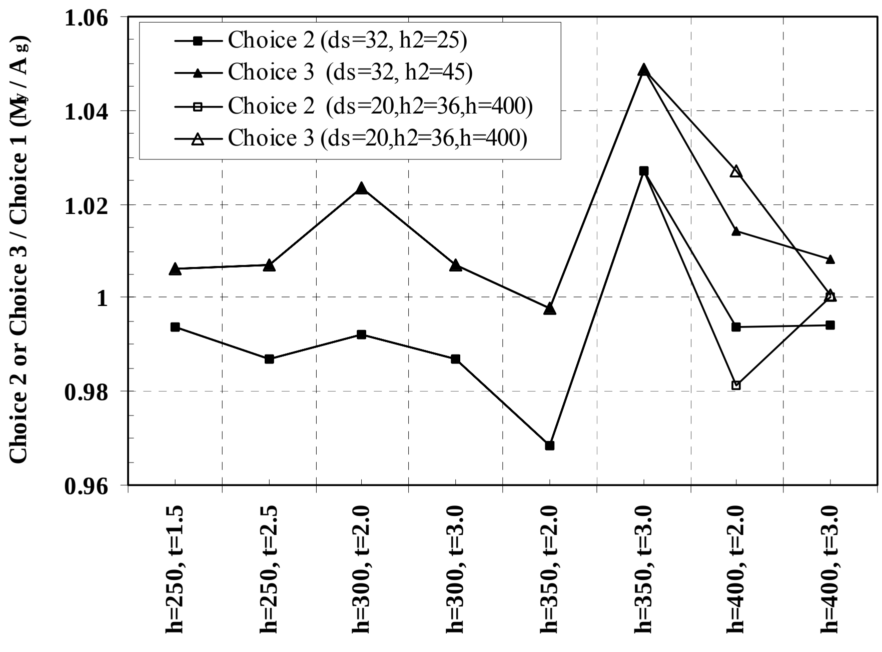



**Figure 8.3.3** Comparison of the values of $M_y/A_g$ amongst the three cases

### 8.3.4 Optimum dimensions

The analyses described in Section 8.3.3 indicate that Choice 1 gives the higher values of $q/A_g$ than the two simplified cases. Further Choice 2 shows better behavior than that for Choice 3 when the values of $q/A_g$ are compared. However, in order to provide multiple choices to the structural designers and steel manufacturers, three kinds of optimum dimensions for the $\Sigma$-shape purlin continuous over two spans under gravity load using GA are proposed in Table 8.3.7.

**Table 8.3.7** Proposed optimum dimensions for $\Sigma$-shaped purlin

| Sections | | Choice 1 | | | | | Choice 2 | | | | | Choice 3 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $h$ | $t$ | $b$ | $c$ | $h_1$ | $h_2$ | $d_s$ | $b$ | $c$ | $h_1$ | $h_2$ | $d_s$ | $b$ | $c$ | $h_1$ | $h_2$ | $d_s$ |
| 250 | 1.5 | **80** | **30** | **50** | **25** | **32** | **80** | **27.7** | **50** | **25** | **32** | **80** | **35.0** | **50** | **25** | **32** |
| 250 | 2.5 | **80** | **30** | **50** | **25** | **32** | **80** | **27.7** | **50** | **25** | **32** | **80** | **35.0** | **50** | **25** | **32** |
| 300 | 2.0 | **90** | **35** | **50** | **25** | **32** | **90** | **31.3** | **50** | **25** | **32** | **80** | **35.0** | **50** | **25** | **32** |
| 300 | 3.0 | **90** | **35** | **50** | **25** | **32** | **90** | **31.3** | **50** | **25** | **32** | **90** | **40.0** | **50** | **25** | **32** |
| 350 | 2.0 | **85** | **35** | **70** | **25** | **32** | **85** | **29.5** | **70** | **25** | **32** | **85** | **37.5** | **70** | **25** | **32** |
| 350 | 3.0 | **95** | **25** | **70** | **25** | **32** | **85** | **29.5** | **70** | **25** | **32** | **85** | **37.5** | **70** | **25** | **32** |
| 400 | 2.0 | **75** | **30** | **50** | **25** | **20** | **75** | **25.8** | **50** | **25** | **20** | **75** | **32.5** | **50** | **25** | **20** |
| 400 | 3.0 | **85** | **25** | **50** | **25** | **20** | **80** | **27.7** | **50** | **25** | **20** | **75** | **32.5** | **50** | **25** | **20** |

## 8.4 Σ-Shaped Purlins without Considering the Free Flange Behavior

In this section, the optimization is carried out to maximize the value of $M_y/A_g$ instead of $q/A_g$. The optimization is carried out for the freely varied design variables for the cross-section with a single edge stiffener and with double edge stiffeners.

### 8.4.1 Objective function and constraints

The optimization problem in this section is defined as

$$\text{Maximize} \quad \frac{M_y}{A_g} \tag{8.4.1}$$

subjected to the similar constraints described in Section 8.2.2. Therefore, the constrained optimization problem is transformed into the following unconstrained problem:

$$\text{Maximize} \quad F = \begin{cases} \dfrac{M_y}{A_g} - KK \cdot n \cdot CC, & \text{when } \dfrac{q}{A_g} > KK \cdot n \cdot CC, \\ 0, & \text{otherwise.} \end{cases} \tag{8.4.2}$$

### 8.4.2 Sensitivity analyses of the fitness function to the coefficient *KK*

The sensitivity analyses of the value of $M_y/A_g$ to the coefficient, $KK$, are shown in Table 8.4.1. These analyses are carried out for a cross-section with $h = 250$ mm, $t = 1.5$ mm and $L = 4000$ mm.

**Table 8.4.1** Sensitivity analyses of the value of $M/A_g$ to the coefficient $KK$

| $KK$ | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 55 | 26.4 | 35.75 | 33 | 33 | 19535.18 | 0.004347 | 632.71 |
| 1 | 55 | 26.4 | 35.75 | 33 | 33 | 19535.18 | 0.004347 | 632.71 |
| 2 | 55 | 26.4 | 35.75 | 33 | 33 | 19535.18 | 0.004347 | 632.71 |
| 5 | 55 | 26.4 | 35.75 | 33 | 33 | 19535.18 | 0.004347 | 632.71 |
| 10 | 55 | 26.4 | 35.75 | 33 | 33 | 19535.18 | 0.004347 | 632.71 |
| 50 | 55 | 26.4 | 35.75 | 33 | 33 | 19535.18 | 0.004347 | 632.71 |
| 100 | 55 | 26.4 | 35.75 | 33 | 33 | 19535.18 | 0.004347 | 632.71 |

Table 8.4.1 indicates that the value of $KK$ has no influence on finding the optimum dimensions for this special cross-section. Thus, we set $KK = 2$. If one of the results in 10 runs is not in the feasible regions, the value of $KK$ is increased until all the results in 10 runs are in the feasible regions.

### 8.4.3 Optimization results

The optimization is carried out for a cross-section with a single edge stiffener and a cross-section with double edge stiffeners. All parameters are freely varied except $h_1 \geq 35$ mm. The results for the cross-section with a single edge stiffener are given in Table 8.4.2 and in Table 8.3.6 for the cross-section with double edge stiffeners.

The comparisons of the values of $q/A_g$ and $M_y/A_g$ for the cross-sections with a single edge stiffener to those for the cross-section with double lip stiffeners are shown in Figure 8.4.1. It can be seen that the values of $q/A_g$ for the cross-section with double edge stiffener are higher than those for the cross-section with a single edge stiffener when the thickness is increased from the lower value to the higher value. However, the values of $M_y/A_g$ are almost the same for these two cases.

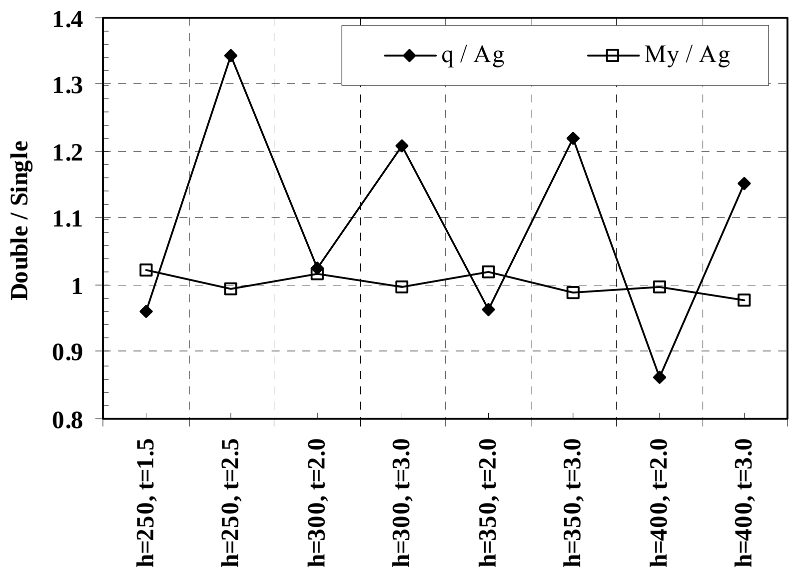



**Figure 8.4.1** Comparisons of the values of $q/A_g$ and $M_y/A_g$ for the cross-section with single edge lip to that with double edge lips

**Table 8.4.2** Optimization results for the free optimization with single edge stiffener (Objective function being $M_y/A_g$ )

| $h$ (mm) | $L$ (mm) | $t$ (mm) | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 250 | 4000 | 1.5 | **55** | **26.40** | **35.75** | **33** | **33** | 19535.18 | 0.004347 |
| 250 | 5500 | 2.5 | **79** | **44.24** | **39.50** | **5** | **5** | 22271.84 | 0.002993 |
| 300 | 5000 | 2.0 | **69** | **38.64** | **60.03** | **14** | **5** | 24695.43 | 0.003604 |
| 300 | 6500 | 3.0 | **96** | **53.76** | **64.32** | **14** | **5** | 26744.68 | 0.002518 |
| 350 | 6000 | 2.0 | **71** | **42.60** | **67.45** | **22** | **9** | 27641.35 | 0.002897 |
| 350 | 7500 | 3.0 | **87** | **48.72** | **84.39** | **14** | **5** | 30225.33 | 0.002094 |
| 400 | 6000 | 2.0 | **69** | **41.40** | **60.03** | **22** | **9** | 30003.48 | 0.002949 |
| 400 | 8000 | 3.0 | **71** | **39.76** | **67.45** | **14** | **5** | 32605.93 | 0.001756 |

**Table 8.4.3** Optimization results for the free optimization with double edge stiffeners (Objective function being $M_y/A_g$ )

| $h$ (mm) | $L$ (mm) | $t$ (mm) | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 250 | 4000 | 1.5 | **49** | **23.03** | **11.27** | **39.20** | **32** | **32** | 19961.07 | 0.004177 |
| 250 | 5500 | 2.5 | **80** | **30.40** | **15.20** | **35.20** | **23** | **23** | 22174.70 | 0.004020 |
| 300 | 5000 | 2.0 | **66** | **39.60** | **19.80** | **49.50** | **22** | **9** | 25115.03 | 0.003700 |
| 300 | 6500 | 3.0 | **97** | **35.89** | **17.46** | **48.50** | **40** | **22** | 26660.30 | 0.003045 |
| 350 | 6000 | 2.0 | **68** | **40.80** | **20.40** | **51.00** | **22** | **9** | 28158.44 | 0.002790 |
| 350 | 7500 | 3.0 | **89** | **29.37** | **14.24** | **40.94** | **31** | **18** | 29888.14 | 0.002554 |
| 400 | 6000 | 2.0 | **58** | **34.80** | **17.40** | **43.50** | **22** | **9** | 29948.13 | 0.002545 |
| 400 | 8000 | 3.0 | **70** | **24.50** | **11.90** | **35.00** | **36** | **20** | 31838.95 | 0.002022 |

Moreover, the values of $q/A_g$ and $M_y/A_g$ are compared for two cases, one with $q/A_g$ as the objective function and the other with $M_y/A_g$ as the objective function. The results are shown in Figure 8.4.2.

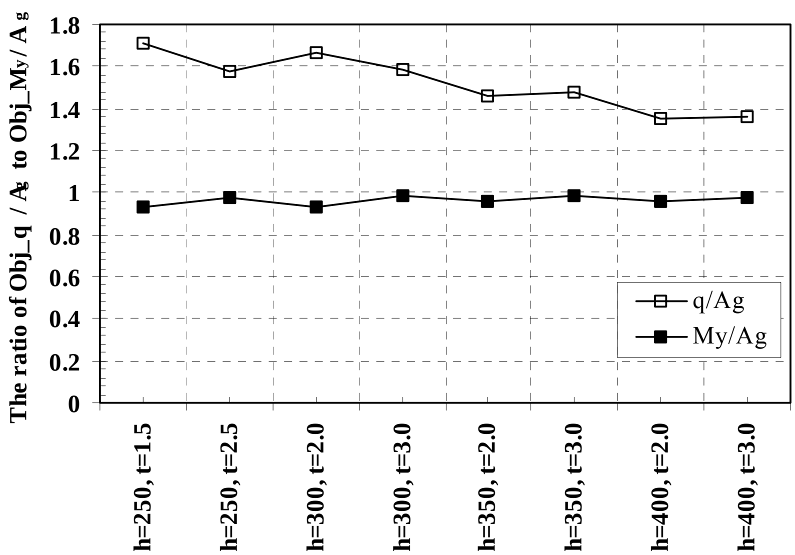



**Figure 8.4.2** Comparisons of the cases with $q/A_g$ as the objective function and with $M_y/A_g$ as the objective function

Figure 8.4.2 shows that when the objective function is $q/A_g$, the values of $q/A_g$ are higher than those of cross-sections with $M_y/A_g$ as the objective function. However, the values of $M_y/A_g$ with $q/A_g$ as the objective function are lower than those with $M_y/A_g$ as the objective function. However, the difference is not too large.

When comparing the optimum dimensions listed in Table 8.4.2 to those listed in Table 8.3.1, it can be seen that the optimum dimensions differs a lot. This can explained with the investigation in Chapter 7. Chapter 7 shows that the maximum value of $q/A_g$ is reached when the applied load passing through the shear center. However, the maximum value of $M_y/A_g$ occurs when the lengths of $h_1$ and $h_3$ are starting to be fully effective; the length of $d_s$ becomes shorter; and the value of $h_2$ becomes larger.

## 8.5 Integration of Modified Eurocode 3 Method into the Optimization

A demonstration example is used to show how the modified Eurocode 3 method can be integrated into the GA optimization process for Σ-shape purlins. The objective of the optimization is to maximize the value of $q/A_g$ for a cross-section with $h = 350$ mm, $t = 2.0$ mm and $L = 6000$ mm. The optimization results for 10 runs are shown in Table 8.5.1. The optimum dimensions for the cross-section are $b = 83$ mm, $c = 33.20$ mm, $h_1 = 68.06$ mm, $h_2 = 28$ mm and $d_s = 30$ mm.

**Table 8.5.1** Optimization results for 10 runs using the modified Eurocode 3 method

| No. | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 81 | 28.35 | 59.94 | 17 | 25 | 0.004188 | 26622.36 | 1147.992 |
| 2 | **83** | **33.20** | **68.06** | **28** | **30** | **0.004236** | **26421.06** | **1174.677** |
| 3 | 80 | 27.20 | 56.00 | 16 | 24 | 0.004145 | 26698.31 | 1138.002 |
| 4 | 82 | 30.34 | 63.96 | 21 | 27 | 0.004215 | 26549.78 | 1159.877 |
| 5 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004236 | 26421.06 | 1174.677 |
| 6 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004236 | 26421.06 | 1174.677 |
| 7 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004236 | 26421.06 | 1174.677 |
| 8 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004236 | 26421.06 | 1174.677 |
| 9 | 91 | 33.67 | 43.68 | 22 | 27 | 0.004206 | 25915.39 | 1206.821 |
| 10 | 82 | 30.34 | 63.96 | 21 | 27 | 0.004215 | 26549.78 | 1159.877 |

Table 8.5.2 lists the comparisons of optimization results designed based on the Eurocode 3 method (EC3) to those based on the modified Eurocode 3 method (MEC3). Table 8.5.2 indicates that for this specific cross-section, the optimal dimensions based on 'MEC3' are the same as those based on 'EC3'; and the values of $q/A_g$ and $M_y/A_g$ calculated using 'MEC3' are about 0.12 % and 0.13 % lower than those calculated using 'EC3', respectively.

**Table 8.5.2** Comparisons of optimization results based on the Modified Eurocode 3 method (MEC3) with those based on the Eurocode 3 method (EC3)

| Methods | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| EC3 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004241 | 26451.04 | 1174.68 |
| MEC3 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004236 | 26421.06 | 1174.68 |

There are two types of buckling modes in the modified Eurocode 3 method for this specific cross-section, i.e. the local buckling modes of the flange, lip and the element above the web stiffener; and the conventional distortional buckling mode. The buckling mode of the element under the web stiffener is neither local buckling of this element nor the web distortional buckling. Therefore, this element is designed according to the Eurocode 3 method and the elastic local buckling stress for this element is the same as that calculated using Eurocode 3 method. The other elastic buckling stresses calculated using these two methods are shown in Table 8.5.3. In the table, 'web_1' refers to the element above the web stiffener and 'web_2' refers to the compression part under the web stiffener.

**Table 8.5.3** Elastic buckling stress calculated using EC3 and MEC3 (MPa)

| Methods | flange | lip | web_1 | web_2 | Distortional buckling |
|---|---|---|---|---|---|
| EC3 | 446.32 | 424.92 | 817.68 | 682.47 | 314.99 |
| MEC3 | 454.44 | | | 682.47 | 454.44 |

The elastic local buckling stress of 'web_1' calculated using the modified Eurocode 3 method is lower than that calculated using the Eurocode 3 method. Thus, this element is fully effective when designed based on Eurocode 3 and is not fully effective when using the modified Eurocode 3 method. However, the elastic distortional buckling stress calculated using 'MEC3' is higher than that calculated using 'EC3'. Thus, the values of $q/A_g$ and $M_y/A_g$ calculated using these two methods are almost the same.

# 9 CONCLUSIONS AND FUTURE RESEARCH

An important advantage of cold-formed steel is the great flexibility of cross-sectional shapes and sizes available to the structural steel designer. However, the lack of standard optimized shapes makes the selection of the most economical shape very difficult. This task is further complicated by the complex and highly nonlinear nature of the rules that govern their designs. A survey of the literature shows that only relatively little work has been done in this area. In this thesis, a relatively new method is introduced into the field of cold-formed steel, i.e. Genetic Algorithms (GAs). With GAs, the optimization is carried out for the cold-formed steel purlins that are continuous over two spans subjected to a gravity load. The following sections summarize the conclusions and suggest further research from the studies in this thesis.

## 9.1 Methodology for GA-Based Design

The main methodology to perform the GA-based design for the cold-formed steel purlin is via computer programs. Therefore, a computer program named **ODSP-GA** is developed firstly. It is composed of two parts: the first part is the optimization with GA, and the second part is the design of cold-formed steel purlins.

The first part of the program provides several choices for the users, such as three kinds of selection methods with two types of the sampling methods and four kinds of crossover operators. In order to avoid the repeated evaluation of fitness for the same individuals appearing in one generation, the different individuals in one generation are first picked out so as to improve the efficiency of the calculation. Although the calculation of the fitness function is varied with the engineering optimization itself, the genetic operations such as the selection, the crossover and the mutation can be used in general. Therefore, the program for optimization using GA can be slightly modified to allow its application to other optimization problems.

As far as the computer program for the design of cold-formed steel purlins is concerned, it can currently be used to design Z-shaped and $\Sigma$-shaped purlins. However, with the slight modifications, the program can be used for purlin design with other shapes. The design rules for cold-formed purlins are complex and engineers might experience difficulty in hand calculation. In particular, the calculation of effective cross-section properties can be tedious and time consuming. Thus, this part of the program may be used independently from the optimization part so as to provide a design aid for structural designers and a tool for researchers.

## 9.2 GA-Based Design for Z-Shape Cold-Formed Steel Purlin

The behavior of Z-shape cold-formed steel purlins is investigated in Chapter 5 using the second part of the computer program **ODSP-GA**. The investigations are carried out based on the Eurocode 3 method and the modified Eurocode 3 method, and the following conclusions are made:

(1) When the purlin is designed based on Eurocode 3, the moment capacity of the cross-section is increased with the increase of such dimensions as the width of the flange, the depth of the lip and the height of the web. This is due to the fact that the effective widths of the longer elements are larger than those of the shorter fully effective elements. As far as the distributed loads are concerned, their values depend not only on the moment capacity but also on the gross section modulus of the free flange plus $1/6$ of the web height. Thus, it is possible that the value of the load resistance for the cross-section with greater web height is lower than that with smaller web height. Moreover, as for the moment efficiency or the load efficiency, the maximum value does not occur at the largest dimensions due to the larger area of such a cross-section.

(2) The main local buckling mode is the combination of the lip, flange and web buckling. However, due to the difference of the length for each element, buckling might be

initiated by the longest element amongst the lip, the flange and the compression part of the web. When the length of each element is similar, the buckling of each element occurs at the same time. In addition, when the widths of the lip and the flange are not great enough, there exists no local buckling mode, i.e. the distortional buckling mode is the only buckling mode.

(3) The difference between the width of the flange and the width of the compression part of the web mainly determines the elastic buckling stress. When the width of the flange is larger than that of the compression part of the web, the critical buckling mode is flange buckling. The elastic local buckling stress can be increased by the increase of the lip. On the other hand, when the width of the flange is less than that of the compression part of the web, the critical buckling mode is web buckling. When the width of the flange and the depth of the lip are not large enough, there exists only distortional buckling mode and no local buckling mode. Moreover, when the lengths of these two elements are similar, the elastic bucking stress is increased when the depth of the lip is increased, though the increase is not so large. However, if the depth of the lip is increased further, lip buckling mode becomes the critical buckling mode.

(4) The elastic distortional buckling stress increases with the increase of the depth of the lip and decreases with the increase of the height of the web. However, the effects of the width of the flange on the elastic distortional buckling stress vary with the critical local buckling mode. If web buckling is the critical buckling mode, the elastic distortional buckling stress is increased with the increase of the width of the flange. If the critical buckling mode is flange buckling, the elastic distortional buckling stress is decreased with the increase of the width of the flange.

(5) The elastic buckling stresses calculated using FSM are higher than those calculated using the Eurocode 3 method due to the interaction of the elements. For most cross-sections, the values of the elastic distortional buckling stresses calculated using FSM are higher than those calculated using the Eurocode 3 method. However, when web-buckling mode is the critical local buckling mode, the distortional buckling stresses calculated using FSM are smaller than those calculated using the Eurocode 3 method. This is due to the fact that the effect of the reduction in the flexural restraints provided by the buckling web is not considered in Eurocode 3.

(6) If the flange buckling is the critical buckling mode, the moment capacity calculated using the modified Eurocode 3 method is higher than that calculated using the Eurocode 3 method due to the fact that the elastic buckling stresses calculated using FSM are higher than those calculated using the Eurocode 3 method. If the web buckling is the critical buckling mode, the moment capacity calculated using the modified Eurocode 3 method is lower than those calculated using the Eurocode 3 method. Two reasons lead to this result; one is that the effect of the reduction in the flexural restraints provided by the buckling web is not considered in Eurocode 3. The other is that in the modified Eurocode 3 method, the same elastic buckling stress is applied to determine the effective width for the flange, lip and web. The larger the height, the larger the difference.

(7) The comparisons of load efficiencies calculated using two given methods show that the optimum dimensions are not the same due to the different values of the load resistance and the area of the cross-section.

In Chapter 6, such parameters as the selection method, the crossover rate and the mutation rate are selected before the GA-based design is carried out. The following conclusions are drawn:

(1) The behaviors of the three selection methods are mainly investigated, i.e. rank selection method, tournament selection method and fitness proportional method. In general, the speed of convergence for the rank selection method is slower than those for the other two methods: the tournament selection method and the fitness proportional method. The calculations in this thesis are all based on the tournament selection method.

(2) Moreover, the values of the population size, the crossover rate and the mutation rate, which are given in the process of the investigation, all lead GA to convergence except the case with high mutation rate of 0.1. In particular, the difference among the fitness for the compared cases is not large. In principle, each reasonable investigated value can be chosen in the running of GA. However, in this thesis, a GA is running with the

size of the population being two times the length of the encoded binary string, the crossover rate being 0.8 and the mutation rate being 0.001.

(3) The sensitivity analyses of fitness to the values of $KK$ given in the objective function indicate that the variation of $KK$ has no influence on finding the maximum value of the fitness, but has effects on the number of results found in feasible regions. Thus, in this thesis, the value of $KK$ is set to an initial value in the beginning. If one of the results in 10 runs is not in the feasible domain, the value of $KK$ is increased until all the results in 10 runs are in the feasible domain.

With the selected parameters, the GA-based design is applied to Z-shape purlins continuous over two spans subjected to a gravity load with given heights, thicknesses and spans. The following conclusions are made:

(1) The running of the GA shows that both the values of $q/A_g$ and design variables are narrowed down to a relatively small range. This is due to the characteristics of GA, i.e. finding the near optimum. In addition, the purpose of this research was to find the nominal dimension of the cross-section. Thus, the final optimum dimension can be modified based on the results in 10 runs since a small change of the width of the flange or the depth of the lip does not change the value of $q/Ag$ so much. Therefore, the detailed results are provided in the Appendixes.

(2) The optimization is carried out for three cases, Choice 1, Choice 2 and Choice 3, which are distinguished by the relation of the width of the top flange and the depth of the lip. Analyses indicate that, in general, Choice 3 shows higher values of $q/A_g$ than those for the two simplified cases. Furthermore, Choice 2 shows better behavior than that for Choice 1 when both the values of $q/A_g$ and $M/A_g$ are compared.

(3) In order to provide multiple choices to structural designers and steel manufactures, three kinds of optimum dimensions for the Z-shape purlin continuous over two spans under gravity load using GA are proposed. For the two simplified cases, the width of the top flange is determined firstly by rounding the optimum value to the number with ‘0’ or ‘5’ at last position. The depth of the lip is then recalculated according to the

relation of the width of the top flange and the depth of the lip. For the 'free optimization case', the proposed width of the flange and the depth of the lip is determined by rounding the optimum value to the number with '0' or '5' at last position.

(4) In order to show that a numerical analysis program such as CUFSM can be integrated into the GA optimization process, the modified Eurocode 3 method is integrated into GA-based design. Analyses show that the optimal dimensions obtained by integrating the Eurocode 3 method and the modified Eurocode 3 method into the GA-based design procedure can be similar or be different.

## 9.3 GA-Based Design for Σ-Shape Cold-Formed Steel Purlins

The behavior of Σ-shaped cold-formed steel purlins is investigated in Chapter 7. The investigation is concentrated on the effects of the position and the size of the web stiffener on the cross-sectional properties and the load efficiency based on the Eurocode 3 method and the modified Eurocode 3 method, and the following conclusions are made:

(1) When design is based on the Eurocode 3 method, the best position of the web stiffener for gaining the higher value of $W/A_g$ is at the point where both $h_1$ and $h_3$ are starting to become fully effective. Moreover, the longer the length of $h_2$ and the shorter the length of $d_s$, the higher the value of $W/A_g$. The load efficiency depends on the position of the shear center. When the position and the size of the web stiffener make the applied load passing through the shear center, the load efficiency reaches its maximum value.

(2) When the width of the flange and the depth of the lip are fixed, the shorter the length of $h_1$ is, the higher the values of elastic local buckling stresses are. As for the elastic distortional buckling stresses, their values depend on the relation of the width of the flange to the length of $h_1$. If the value of $h_1$ is increased from a value less than the width of the flange to a value equal to the width of the flange, the distortional buckling

stress decreases with the increase of $h_1$. If the value of $h_1$ is increased from a value equal to the width of the flange to a value larger than the width of the flange, the distortional buckling stress increases with the increase of $h_1$.

(3) The length of $d_s$ should be large enough to avoid the web distortional buckling mode. The conventional elastic distortional buckling stress is decreased also when the web distortional buckling mode occurs.

(4) The buckling modes can be classified into four types. The first type is the local buckling of the flange, the lip and the element above the web stiffener. In this mode, the buckling may be initiated by any one of the mentioned above elements. The second type is the local buckling of the compression part under the web stiffener. The third type is the web distortional buckling mode, in which the web stiffener is moved with other elements along the web. The last type is the distortional buckling mode.

(5) When the critical local buckling mode is the local buckling of the element under the web stiffener, the values of $q/A_g$ and $M_y/A_g$ calculated based on the modified Eurocode 3 method are higher than those calculated based on the Eurocode 3 method. When the web distortional buckling mode is one of the local buckling modes, the values of $q/A_g$ and $M_y/A_g$ calculated based on the modified Eurocode 3 method are lower than those calculated using the Eurocode 3 method since the elastic distortional buckling stress is lower than those calculated using the Eurocode 3 method.

When Σ-shape purlins, which are continuous over two spans and subjected to a gravity load with given heights, thicknesses and spans, are optimized with GAs, the following conclusions are drawn:

(1) The optimization is firstly carried out for cross-sections with a single edge stiffener and with double edge stiffeners. As far as the values of $q/A_g$ are concerned, most of the cross-sections with a single lip stiffener show higher values. As for the value of $M_y/A_g$, most of the cross-sections with double lip stiffeners show higher values.

However, the improvements are not large, and the maximum improvement is about 5%.

(2) In order to simplify the optimization process, such dimensions as $d_s$ and $h_2$ are treated as preassigned variables and set to 32 mm and 25 mm, respectively. Similarly to Z-shape purlins, the optimization is carried out for three cases, Choice 1, Choice 2 and Choice 3, which is distinguished by the relation of the width of the top flange and the depth of the lip. Analyses indicate that the case with freely varied $b$, $c$ and $h_1$ shows higher values of $q/A_g$ than the two simplified cases. Furthermore, Choice 2 shows better behavior than for Choice 3 when the values of $q/A_g$ are compared.

(3) In order to provide multiple choices to structural designers and steel manufacturers, three kinds of optimum dimensions for $\Sigma$-shape purlins continuous over two spans under gravity load are proposed using GA.

(4) The optimization is also carried out to maximize the value of $M_y/A_g$ instead of $q/A_g$. Analyses show that when the objective function is $q/A_g$, the values of $q/A_g$ are higher than those for which the objective function is $M_y/A_g$. In addition, the values of $M_y/A_g$ with $q/A_g$ as objective function are lower than those of with $M_y/A_g$ as objective function. However, the difference is not very large. When comparing the optimum dimensions for these two cases, it can be seen that the optimum dimensions differ a lot. This can be explained by the investigation in Chapter 7. Chapter 7 shows that the maximum value of $q/A_g$ is reached when the applied load passes through the shear center. However, the maximum value of $M_y/A_g$ occurs when the lengths of $h_1$ and $h_3$ are starting to be fully effective, the length of $d_s$ becomes shorter; and the value of $h_2$ becomes larger.

(5) A demonstration example is used to show how the modified Eurocode 3 method is integrated into the GA optimization process for Σ-shape purlins. The comparisons of optimization results based on the Eurocode 3 method to those based on the modified

Eurocode 3 method indicate that for this specific cross-section, the optimal dimensions are the same using these two methods.

## 9.4 Discussion and Future Research

### 9.4.1 Discussion

The current Eurocode 3, Part 1.3 (1996) assumes that the load is applied on the top of the web and the shear center of the cross-section is to the left of the applied load for the Σ-shape purlin. In the current Eurocode 3, Part 1.3, the lateral load, $q_{h,Fd}$, acting on the free flange due to torsion and lateral bending is calculated using the distance between the applied load and the shear center, which is not zero. However, the analyses being carried out in Section 7.2 indicate that the shear center can be located at a point inside the cross-section between the top and bottom flanges. In this case, the applied load moves to the point on the top flange passing through the shear center. The lateral load is actually zero.

When the shear center is at a point inside the cross-section between two flanges, the values of $q/A_g$ calculated based on zero distance between the applied load and the shear center are higher than those calculated using the Eurocode 3 method. In addition, the variation of $q/A_g$ mainly depends on the effective section modulus, $W$, and the reduction factor for flexural buckling of the free flange, $\chi$. Therefore, the maximum value of $q/A_g$ is reached when the shear center is at a point inside the cross-section between two flanges and not when the load passes through the shear center.

### 9.4.2 Future research

In the current Eurocode 3 method, the calculation for bending moment in the free flange due to the lateral load for Z-shape purlins can be only applied to cross-sections with equal width of flanges. In this thesis, the width of the top flange is 6 mm wider than that of the bottom flange. However, the calculation is still based on the rules specified in Eurocode 3, Part 1.3. Thus, the influence of the position of the shear center and the widths of the two flanges on the behavior of the Z-shape purlins should be investigated further.

The purlin is attached to the profiled sheeting using some connections. This sheeting can provide lateral and rotational restraints to the purlin. The calculation of the elastic buckling stresses using FSM did not consider the effects of these restraints.

There do exist some needs to improve the developed computer program, ODSP_GA, for example, a friendly Graphic User Interface (GUI) should be added. Right now, the program is only suitable for binary encoding. Moreover, the design of Z-shaped and $\Sigma$-shaped purlins is carried out separately.

The modified Eurocode 3 method is only applied in this thesis to some specific cases for demonstration purpose. However, in the process of the integration of the elastic buckling stresses into the modified Eurocode 3 method for general cases, the elastic buckling stresses and buckling modes can be very different for different section. It might be time consuming to do the calculation. Neural networks could be used in determination of the elastic buckling stresses and buckling modes (Lu et al., 2000 and Lu, 2000).

The GA-based design concept in this thesis is applied to the optimization of cold-formed steel purlins. However, it could find more applications in the field of cold-formed steel design for other members, especially those with multiple stiffeners along the flange and web.

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# APPENDIX A

# OPTIMIZATION RESULTS FOR Z-SHAPE PURLINS

## Choice 1 ( 40/13…100/35 )

*h=100 mm, L=1400 mm and t=1.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **41** | **13.37** | **0.018349** | **8102.564** | **3.503527** | **190.9376** |
| 2 | 41 | 13.37 | 0.018349 | 8102.564 | 3.503527 | 190.9376 |
| 3 | 44 | 14.47 | 0.018217 | 7929.123 | 3.621669 | 198.8096 |
| 4 | 41 | 13.37 | 0.018349 | 8102.564 | 3.503527 | 190.9376 |
| 5 | 42 | 13.73 | 0.018329 | 8058.936 | 3.547803 | 193.5616 |
| 6 | 42 | 13.73 | 0.018329 | 8058.936 | 3.547803 | 193.5616 |
| 7 | 42 | 13.73 | 0.018329 | 8058.936 | 3.547803 | 193.5616 |
| 8 | 40 | 13.00 | 0.018326 | 8119.137 | 3.451048 | 188.3136 |
| 9 | 40 | 13.00 | 0.018326 | 8119.137 | 3.451048 | 188.3136 |
| 10 | 42 | 13.73 | 0.018329 | 8058.936 | 3.547803 | 193.5616 |

*h=100 mm, L=1600 mm and t=1.2 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **44** | **14.47** | **0.015151** | **8842.566** | **3.625647** | **239.3003** |
| 2 | 44 | 14.47 | 0.015151 | 8842.566 | 3.625647 | 239.3003 |
| 3 | 44 | 14.47 | 0.015151 | 8842.566 | 3.625647 | 239.3003 |
| 4 | 46 | 15.20 | 0.015144 | 8811.810 | 3.720051 | 245.6416 |
| 5 | 43 | 14.10 | 0.015127 | 8835.233 | 3.571870 | 236.1296 |
| 6 | 44 | 14.47 | 0.015151 | 8842.566 | 3.625647 | 239.3003 |
| 7 | 44 | 14.47 | 0.015151 | 8842.566 | 3.625647 | 239.3003 |
| 8 | 44 | 14.47 | 0.015151 | 8842.566 | 3.625647 | 239.3003 |
| 9 | 44 | 14.47 | 0.015151 | 8842.566 | 3.625647 | 239.3003 |
| 10 | 44 | 14.47 | 0.015151 | 8842.566 | 3.625647 | 239.3003 |

*h=100 mm, L=1900 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **53** | **17.77** | **0.011658** | **9518.614** | **3.909658** | **335.3523** |
| 2 | 53 | 17.77 | 0.011658 | 9518.614 | 3.909658 | 335.3523 |
| 3 | 54 | 18.13 | 0.011645 | 9510.555 | 3.951490 | 339.3429 |
| 4 | 53 | 17.77 | 0.011658 | 9518.614 | 3.909658 | 335.3523 |
| 5 | 51 | 17.03 | 0.011646 | 9485.110 | 3.812648 | 327.3709 |
| 6 | 53 | 17.77 | 0.011658 | 9518.614 | 3.909658 | 335.3523 |
| 7 | 51 | 17.03 | 0.011646 | 9485.110 | 3.812648 | 327.3709 |
| 8 | 52 | 17.40 | 0.011653 | 9502.609 | 3.861471 | 331.3616 |
| 9 | 53 | 17.77 | 0.011658 | 9518.614 | 3.909658 | 335.3523 |
| 10 | 53 | 17.77 | 0.011658 | 9518.614 | 3.909658 | 335.3523 |

*h=100 mm, L=2200 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **60** | **20.33** | **0.009159** | **9821.831** | **4.430757** | **483.7803** |
| 2 | 60 | 20.33 | 0.009159 | 9821.831 | 4.430757 | 483.7803 |
| 3 | 59 | 19.97 | 0.009158 | 9806.486 | 4.381605 | 478.4229 |
| 4 | 60 | 20.33 | 0.009159 | 9821.831 | 4.430757 | 483.7803 |
| 5 | 59 | 19.97 | 0.009158 | 9806.486 | 4.381605 | 478.4229 |
| 6 | 59 | 19.97 | 0.009158 | 9806.486 | 4.381605 | 478.4229 |
| 7 | 60 | 20.33 | 0.009159 | 9821.831 | 4.430757 | 483.7803 |
| 8 | 60 | 20.33 | 0.009159 | 9821.831 | 4.430757 | 483.7803 |
| 9 | 59 | 19.97 | 0.009158 | 9806.486 | 4.381605 | 478.4229 |
| 10 | 59 | 19.97 | 0.009158 | 9806.486 | 4.381605 | 478.4229 |

*h=150 mm, L=1700 mm and t=1.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 47 | 15.57 | 0.015382 | 9632.352 | 3.917446 | 254.6816 |
| 2 | **48** | **15.93** | **0.015386** | **9564.788** | **3.958989** | **257.3056** |
| 3 | 47 | 15.57 | 0.015382 | 9632.352 | 3.917446 | 254.6816 |
| 4 | 48 | 15.93 | 0.015386 | 9564.788 | 3.958989 | 257.3056 |
| 5 | 48 | 15.93 | 0.015386 | 9564.788 | 3.958989 | 257.3056 |
| 6 | 48 | 15.93 | 0.015386 | 9564.788 | 3.958989 | 257.3056 |
| 7 | 48 | 15.93 | 0.015386 | 9564.788 | 3.958989 | 257.3056 |
| 8 | 52 | 17.40 | 0.015336 | 9283.575 | 4.106898 | 267.8016 |
| 9 | 48 | 15.93 | 0.015386 | 9564.788 | 3.958989 | 257.3056 |
| 10 | 48 | 15.93 | 0.015386 | 9564.788 | 3.958989 | 257.3056 |

# APPENDIX A



*h=150 mm, L=2000 mm and t=1.2 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 52 | 17.40 | 0.012163 | 10837.82 | 3.924543 | 322.6656 |
| 2 | **51** | **17.03** | **0.012168** | **10901.82** | **3.887639** | **319.4949** |
| 3 | 54 | 18.13 | 0.012140 | 10704.61 | 3.994305 | 329.0069 |
| 4 | 51 | 17.03 | 0.012168 | 10901.82 | 3.887639 | 319.4949 |
| 5 | 51 | 17.03 | 0.012168 | 10901.82 | 3.887639 | 319.4949 |
| 6 | 51 | 17.03 | 0.012168 | 10901.82 | 3.887639 | 319.4949 |
| 7 | 52 | 17.40 | 0.012163 | 10837.82 | 3.924543 | 322.6656 |
| 8 | 52 | 17.40 | 0.012163 | 10837.82 | 3.924543 | 322.6656 |
| 9 | 51 | 17.03 | 0.012168 | 10901.82 | 3.887639 | 319.4949 |
| 10 | 50 | 16.67 | 0.012167 | 10961.14 | 3.848651 | 316.3243 |

*h=150 mm, L=2500 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 60 | 20.33 | 0.008774 | 12273.13 | 3.827844 | 436.2869 |
| 2 | **59** | **19.97** | **0.008775** | **12295.31** | **3.793539** | **432.2963** |
| 3 | 59 | 19.97 | 0.008775 | 12295.31 | 3.793539 | 432.2963 |
| 4 | 58 | 19.60 | 0.008774 | 12312.83 | 3.757898 | 428.3056 |
| 5 | 59 | 19.97 | 0.008775 | 12295.31 | 3.793539 | 432.2963 |
| 6 | 59 | 19.97 | 0.008775 | 12295.31 | 3.793539 | 432.2963 |
| 7 | 60 | 20.33 | 0.008774 | 12273.13 | 3.827844 | 436.2869 |
| 8 | 60 | 20.33 | 0.008774 | 12273.13 | 3.827844 | 436.2869 |
| 9 | 59 | 19.97 | 0.008775 | 12295.31 | 3.793539 | 432.2963 |
| 10 | 59 | 19.97 | 0.008775 | 12295.31 | 3.793539 | 432.2963 |

*h=150 mm, L=3000 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **69** | **23.63** | **0.006961** | **13945.02** | **4.385648** | **629.9963** |
| 2 | 69 | 23.63 | 0.006961 | 13945.02 | 4.385648 | 629.9963 |
| 3 | 70 | 24.00 | 0.006959 | 13943.05 | 4.421236 | 635.3536 |
| 4 | 69 | 23.63 | 0.006961 | 13945.02 | 4.385648 | 629.9963 |
| 5 | 70 | 24.00 | 0.006959 | 13943.05 | 4.421236 | 635.3536 |
| 6 | 73 | 25.10 | 0.006933 | 13886.92 | 4.516087 | 651.4256 |
| 7 | 70 | 24.00 | 0.006959 | 13943.05 | 4.421236 | 635.3536 |
| 8 | 72 | 24.73 | 0.006942 | 13907.11 | 4.485058 | 646.0683 |
| 9 | 70 | 24.00 | 0.006959 | 13943.05 | 4.421236 | 635.3536 |
| 10 | 69 | 23.63 | 0.006961 | 13945.02 | 4.385648 | 629.9963 |

*h=200 mm, L=3000 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 64 | 21.80 | 0.006969 | 14074.83 | 3.660312 | 525.2496 |
| 2 | 64 | 21.80 | 0.006969 | 14074.83 | 3.660312 | 525.2496 |
| 3 | **63** | **21.43** | **0.006970** | **14138.01** | **3.633212** | **521.2589** |
| 4 | 64 | 21.80 | 0.006969 | 14074.83 | 3.660312 | 525.2496 |
| 5 | 64 | 21.80 | 0.006969 | 14074.83 | 3.660312 | 525.2496 |
| 6 | 65 | 22.17 | 0.006966 | 14010.20 | 3.686554 | 529.2403 |
| 7 | 63 | 21.43 | 0.006970 | 14138.01 | 3.633212 | 521.2589 |
| 8 | 62 | 21.07 | 0.006968 | 14195.13 | 3.604409 | 517.2683 |
| 9 | 64 | 21.80 | 0.006969 | 14074.83 | 3.660312 | 525.2496 |
| 10 | 63 | 21.43 | 0.006970 | 14138.01 | 3.633212 | 521.2589 |

*h=200 mm, L=3800 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **74** | **25.47** | **0.005201** | **16536.56** | **3.925249** | **754.7829** |
| 2 | 73 | 25.10 | 0.005200 | 16549.64 | 3.896807 | 749.4256 |
| 3 | 74 | 25.47 | 0.005201 | 16536.56 | 3.925249 | 754.7829 |
| 4 | 76 | 26.20 | 0.005200 | 16505.36 | 3.980265 | 765.4976 |
| 5 | 74 | 25.47 | 0.005201 | 16536.56 | 3.925249 | 754.7829 |
| 6 | 74 | 25.47 | 0.005201 | 16536.56 | 3.925249 | 754.7829 |
| 7 | 71 | 24.37 | 0.005195 | 16570.33 | 3.837869 | 738.7109 |
| 8 | 74 | 25.47 | 0.005201 | 16536.56 | 3.925249 | 754.7829 |
| 9 | 74 | 25.47 | 0.005201 | 16536.56 | 3.925249 | 754.7829 |
| 10 | 74 | 25.47 | 0.005201 | 16536.56 | 3.925249 | 754.7829 |

*h=200 mm, L=4500 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 88 | 30.60 | 0.004212 | 18211.18 | 4.366066 | 1036.546 |
| 2 | 88 | 30.60 | 0.004212 | 18211.18 | 4.366066 | 1036.546 |
| 3 | 86 | 29.87 | 0.004220 | 18242.52 | 4.317801 | 1023.098 |
| 4 | 86 | 29.87 | 0.004220 | 18242.52 | 4.317801 | 1023.098 |
| 5 | 86 | 29.87 | 0.004220 | 18242.52 | 4.317801 | 1023.098 |
| 6 | 86 | 29.87 | 0.004220 | 18242.52 | 4.317801 | 1023.098 |
| 7 | **85** | **29.50** | **0.004221** | **18236.01** | **4.289704** | **1016.374** |
| 8 | 85 | 29.50 | 0.004221 | 18236.01 | 4.289704 | 1016.374 |
| 9 | 86 | 29.87 | 0.004220 | 18242.52 | 4.317801 | 1023.098 |
| 10 | 85 | 29.50 | 0.004221 | 18236.01 | 4.289704 | 1016.374 |

# APPENDIX A

*h=250 mm, L=3200 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 78 | 26.93 | 0.006705 | 14684.05 | 4.386163 | 654.1189 |
| 2 | **77** | **26.57** | **0.006706** | **14754.75** | **4.359573** | **650.1283** |
| 3 | 77 | 26.57 | 0.006706 | 14754.75 | 4.359573 | 650.1283 |
| 4 | 80 | 27.67 | 0.006702 | 14541.89 | 4.437290 | 662.1003 |
| 5 | 78 | 26.93 | 0.006705 | 14684.05 | 4.386163 | 654.1189 |
| 6 | 77 | 26.57 | 0.006706 | 14754.75 | 4.359573 | 650.1283 |
| 7 | 78 | 26.93 | 0.006705 | 14684.05 | 4.386163 | 654.1189 |
| 8 | 76 | 26.20 | 0.006705 | 14825.10 | 4.332262 | 646.1376 |
| 9 | 78 | 26.93 | 0.006705 | 14684.05 | 4.386163 | 654.1189 |
| 10 | 77 | 26.57 | 0.006706 | 14754.75 | 4.359573 | 650.1283 |

*h=250 mm, L=4300 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 79 | 27.30 | 0.004468 | 18491.08 | 3.930305 | 879.5696 |
| 2 | **81** | **28.03** | **0.004475** | **18444.75** | **3.984420** | **890.2843** |
| 3 | 80 | 27.67 | 0.004474 | 18476.05 | 3.958761 | 884.9269 |
| 4 | 81 | 28.03 | 0.004475 | 18444.75 | 3.984420 | 890.2843 |
| 5 | 81 | 28.03 | 0.004475 | 18444.75 | 3.984420 | 890.2843 |
| 6 | 82 | 28.40 | 0.004472 | 18383.63 | 4.005212 | 895.6416 |
| 7 | 79 | 27.30 | 0.004468 | 18491.08 | 3.930305 | 879.5696 |
| 8 | 81 | 28.03 | 0.004475 | 18444.75 | 3.984420 | 890.2843 |
| 9 | 81 | 28.03 | 0.004475 | 18444.75 | 3.984420 | 890.2843 |
| 10 | 79 | 27.30 | 0.004468 | 18491.08 | 3.930305 | 879.5696 |

*h=250 mm, L=5100 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **91** | **31.70** | **0.00368** | **20744.53** | **4.341746** | **1179.718** |
| 2 | 90 | 31.33 | 0.00368 | 20757.54 | 4.316864 | 1172.994 |
| 3 | 91 | 31.70 | 0.00368 | 20744.53 | 4.341746 | 1179.718 |
| 4 | 92 | 32.07 | 0.00368 | 20730.13 | 4.366127 | 1186.442 |
| 5 | 91 | 31.70 | 0.00368 | 20744.53 | 4.341746 | 1179.718 |
| 6 | 91 | 31.70 | 0.00368 | 20744.53 | 4.341746 | 1179.718 |
| 7 | 92 | 32.07 | 0.00368 | 20730.13 | 4.366127 | 1186.442 |
| 8 | 92 | 32.07 | 0.00368 | 20730.13 | 4.366127 | 1186.442 |
| 9 | 91 | 31.70 | 0.00368 | 20744.53 | 4.341746 | 1179.718 |
| 10 | 91 | 31.70 | 0.00368 | 20744.53 | 4.341746 | 1179.718 |

*h=300 mm, L=4500 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **90** | **31.33** | **0.004323** | **19574.76** | **4.480671** | **1036.500** |
| 2 | 90 | 31.33 | 0.004323 | 19574.76 | 4.480671 | 1036.500 |
| 3 | 90 | 31.33 | 0.004323 | 19574.76 | 4.480671 | 1036.500 |
| 4 | 88 | 30.60 | 0.004322 | 19707.64 | 4.433351 | 1025.786 |
| 5 | 90 | 31.33 | 0.004323 | 19574.76 | 4.480671 | 1036.500 |
| 6 | 90 | 31.33 | 0.004323 | 19574.76 | 4.480671 | 1036.500 |
| 7 | 88 | 30.60 | 0.004322 | 19707.64 | 4.433351 | 1025.786 |
| 8 | 92 | 32.07 | 0.004322 | 19438.70 | 4.525886 | 1047.215 |
| 9 | 90 | 31.33 | 0.004323 | 19574.76 | 4.480671 | 1036.500 |
| 10 | 90 | 31.33 | 0.004323 | 19574.76 | 4.480671 | 1036.500 |

*h=300 mm, L=5500 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 95 | 33.17 | 0.003389 | 22763.75 | 4.505500 | 1329.614 |
| 2 | 100 | 35.00 | 0.003397 | 22660.46 | 4.630338 | 1363.234 |
| 3 | **99** | **34.63** | **0.003397** | **22696.80** | **4.608168** | **1356.510** |
| 4 | 100 | 35.00 | 0.003397 | 22660.46 | 4.630338 | 1363.234 |
| 5 | 100 | 35.00 | 0.003397 | 22660.46 | 4.630338 | 1363.234 |
| 6 | 100 | 35.00 | 0.003397 | 22660.46 | 4.630338 | 1363.234 |
| 7 | 100 | 35.00 | 0.003397 | 22660.46 | 4.630338 | 1363.234 |
| 8 | 100 | 35.00 | 0.003397 | 22660.46 | 4.630338 | 1363.234 |
| 9 | 99 | 34.63 | 0.003397 | 22696.80 | 4.608168 | 1356.510 |
| 10 | 99 | 34.63 | 0.003397 | 22696.80 | 4.608168 | 1356.510 |

*h=300 mm, L=6000 mm and t=3.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 95 | 33.17 | 0.003116 | 24768.31 | 4.967417 | 1593.940 |
| 2 | **100** | **35.00** | **0.003152** | **24953.42** | **5.151420** | **1634.394** |
| 3 | 99 | 34.63 | 0.003146 | 24918.52 | 5.115656 | 1626.303 |
| 4 | 100 | 35.00 | 0.003152 | 24953.42 | 5.151420 | 1634.394 |
| 5 | 100 | 35.00 | 0.003152 | 24953.42 | 5.151420 | 1634.394 |
| 6 | 100 | 35.00 | 0.003152 | 24953.42 | 5.151420 | 1634.394 |
| 7 | 100 | 35.00 | 0.003152 | 24953.42 | 5.151420 | 1634.394 |
| 8 | 94 | 32.80 | 0.003108 | 24728.00 | 4.928958 | 1585.850 |
| 9 | 99 | 34.63 | 0.003146 | 24918.52 | 5.115656 | 1626.303 |
| 10 | 99 | 34.63 | 0.003146 | 24918.52 | 5.115656 | 1626.303 |

# APPENDIX A

*h=350 mm, L=5000 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 95 | 33.17 | 0.003662 | 20671.56 | 4.252164 | 1161.287 |
| 2 | **100** | **35.00** | **0.003674** | **20331.16** | **4.364854** | **1188.074** |
| 3 | 99 | 34.63 | 0.003672 | 20400.13 | 4.343162 | 1182.716 |
| 4 | 100 | 35.00 | 0.003674 | 20331.16 | 4.364854 | 1188.074 |
| 5 | 100 | 35.00 | 0.003674 | 20331.16 | 4.364854 | 1188.074 |
| 6 | 100 | 35.00 | 0.003674 | 20331.16 | 4.364854 | 1188.074 |
| 7 | 100 | 35.00 | 0.003674 | 20331.16 | 4.364854 | 1188.074 |
| 8 | 94 | 32.80 | 0.003658 | 20738.05 | 4.228273 | 1155.930 |
| 9 | 99 | 34.63 | 0.003672 | 20400.13 | 4.343162 | 1182.716 |
| 10 | 99 | 34.63 | 0.003672 | 20400.13 | 4.343162 | 1182.716 |

*h=350 mm, L=6000 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 95 | 33.17 | 0.002971 | 24462.92 | 4.316084 | 1452.614 |
| 2 | **100** | **35.00** | **0.003000** | **24413.12** | **4.458823** | **1486.234** |
| 3 | 99 | 34.63 | 0.002995 | 24425.70 | 4.431332 | 1479.510 |
| 4 | 100 | 35.00 | 0.003000 | 24413.12 | 4.458823 | 1486.234 |
| 5 | 100 | 35.00 | 0.003000 | 24413.12 | 4.458823 | 1486.234 |
| 6 | 100 | 35.00 | 0.003000 | 24413.12 | 4.458823 | 1486.234 |
| 7 | 100 | 35.00 | 0.003000 | 24413.12 | 4.458823 | 1486.234 |
| 8 | 94 | 32.80 | 0.002964 | 24468.71 | 4.285839 | 1445.890 |
| 9 | 99 | 34.63 | 0.002995 | 24425.70 | 4.431332 | 1479.510 |
| 10 | 99 | 34.63 | 0.002995 | 24425.70 | 4.431332 | 1479.510 |

*h=350 mm, L=7000 mm and t=3.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 95 | 33.17 | 0.002444 | 26758.93 | 4.257824 | 1741.940 |
| 2 | **100** | **35.00** | **0.002489** | **26992.67** | **4.436163** | **1782.394** |
| 3 | 99 | 34.63 | 0.002481 | 26948.22 | 4.401664 | 1774.303 |
| 4 | 100 | 35.00 | 0.002489 | 26992.67 | 4.436163 | 1782.394 |
| 5 | 100 | 35.00 | 0.002489 | 26992.67 | 4.436163 | 1782.394 |
| 6 | 100 | 35.00 | 0.002489 | 26992.67 | 4.436163 | 1782.394 |
| 7 | 100 | 35.00 | 0.002489 | 26992.67 | 4.436163 | 1782.394 |
| 8 | 94 | 32.80 | 0.002434 | 26708.64 | 4.220302 | 1733.850 |
| 9 | 99 | 34.63 | 0.002481 | 26948.22 | 4.401664 | 1774.303 |
| 10 | 99 | 34.63 | 0.002481 | 26948.22 | 4.401664 | 1774.303 |

*h=350 mm, L=7500 mm and t=3.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 95 | 33.17 | 0.002260 | 28230.23 | 4.586305 | 2029.267 |
| 2 | **100** | **35.00** | **0.002309** | **28494.64** | **4.794569** | **2076.554** |
| 3 | 99 | 34.63 | 0.002300 | 28444.05 | 4.754325 | 2067.096 |
| 4 | 100 | 35.00 | 0.002309 | 28494.64 | 4.794569 | 2076.554 |
| 5 | 100 | 35.00 | 0.002309 | 28494.64 | 4.794569 | 2076.554 |
| 6 | 100 | 35.00 | 0.002309 | 28494.64 | 4.794569 | 2076.554 |
| 7 | 100 | 35.00 | 0.002309 | 28494.64 | 4.794569 | 2076.554 |
| 8 | 94 | 32.80 | 0.002249 | 28173.83 | 4.542432 | 2019.810 |
| 9 | 99 | 34.63 | 0.002300 | 28444.05 | 4.754325 | 2067.096 |
| 10 | 99 | 34.63 | 0.002300 | 28444.05 | 4.754325 | 2067.096 |

## Choice 2 ($40/15 \cdots 90/40$)

*h=100 mm, L=1400 mm and t=1.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **40** | **15** | **0.018954** | **8187.290** | **3.642009** | **192.1536** |
| 2 | 40 | 15 | 0.018954 | 8187.290 | 3.642009 | 192.1536 |
| 3 | 44 | 17 | 0.018809 | 7934.279 | 3.830834 | 203.6736 |
| 4 | 40 | 15 | 0.018954 | 8187.290 | 3.642009 | 192.1536 |
| 5 | 40 | 15 | 0.018954 | 8187.290 | 3.642009 | 192.1536 |
| 6 | 40 | 15 | 0.018954 | 8187.290 | 3.642009 | 192.1536 |
| 7 | 40 | 15 | 0.018954 | 8187.290 | 3.642009 | 192.1536 |
| 8 | 40 | 15 | 0.018954 | 8187.290 | 3.642009 | 192.1536 |
| 9 | 40 | 15 | 0.018954 | 8187.290 | 3.642009 | 192.1536 |
| 10 | 40 | 15 | 0.018954 | 8187.290 | 3.642009 | 192.1536 |

*h=100 mm, L=1600 mm and t=1.2 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **44** | **17.0** | **0.015718** | **8926.625** | **3.853692** | **245.1776** |
| 2 | 44 | 17.0 | 0.015718 | 8926.625 | 3.853692 | 245.1776 |
| 3 | 44 | 17.0 | 0.015718 | 8926.625 | 3.853692 | 245.1776 |
| 4 | 43 | 16.5 | 0.015688 | 8927.192 | 3.791698 | 241.6976 |
| 5 | 43 | 16.5 | 0.015688 | 8927.192 | 3.791698 | 241.6976 |
| 6 | 44 | 17.0 | 0.015718 | 8926.625 | 3.853692 | 245.1776 |
| 7 | 44 | 17.0 | 0.015718 | 8926.625 | 3.853692 | 245.1776 |
| 8 | 44 | 17.0 | 0.015718 | 8926.625 | 3.853692 | 245.1776 |
| 9 | 44 | 17.0 | 0.015718 | 8926.625 | 3.853692 | 245.1776 |
| 10 | 44 | 17.0 | 0.015718 | 8926.625 | 3.853692 | 245.1776 |

# APPENDIX A

*h=100 mm, L=1900 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **52** | **21.0** | **0.012056** | **9516.639** | **4.121481** | **341.8736** |
| 2 | 52 | 21.0 | 0.012056 | 9516.639 | 4.121481 | 341.8736 |
| 3 | 54 | 22.0 | 0.012031 | 9488.423 | 4.218348 | 350.6336 |
| 4 | 52 | 21.0 | 0.012056 | 9516.639 | 4.121481 | 341.8736 |
| 5 | 51 | 20.5 | 0.012043 | 9501.273 | 4.064270 | 337.4936 |
| 6 | 52 | 21.0 | 0.012056 | 9516.639 | 4.121481 | 341.8736 |
| 7 | 51 | 20.5 | 0.012043 | 9501.273 | 4.064270 | 337.4936 |
| 8 | 52 | 21.0 | 0.012056 | 9516.639 | 4.121481 | 341.8736 |
| 9 | 52 | 21.0 | 0.012056 | 9516.639 | 4.121481 | 341.8736 |
| 10 | 52 | 21.0 | 0.012056 | 9516.639 | 4.121481 | 341.8736 |

*h=100 mm, L=2200 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **60** | **25.0** | **0.009416** | **9743.894** | **4.727291** | **502.0736** |
| 2 | 60 | 25.0 | 0.009416 | 9743.894 | 4.727291 | 502.0736 |
| 3 | 61 | 25.5 | 0.009415 | 9753.278 | 4.782617 | 507.9536 |
| 4 | 60 | 25.0 | 0.009416 | 9743.894 | 4.727291 | 502.0736 |
| 5 | 59 | 24.5 | 0.009414 | 9733.492 | 4.671313 | 496.1936 |
| 6 | 59 | 24.5 | 0.009414 | 9733.492 | 4.671313 | 496.1936 |
| 7 | 60 | 25.0 | 0.009416 | 9743.894 | 4.727291 | 502.0736 |
| 8 | 60 | 25.0 | 0.009416 | 9743.894 | 4.727291 | 502.0736 |
| 9 | 59 | 24.5 | 0.009414 | 9733.492 | 4.671313 | 496.1936 |
| 10 | 60 | 25.0 | 0.009416 | 9743.894 | 4.727291 | 502.0736 |

*h=150 mm, L=1700 mm and t=1.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 47 | 18.5 | 0.016015 | 9722.324 | 4.168990 | 260.3136 |
| 2 | **48** | **19.0** | **0.016022** | **9650.098** | **4.216983** | **263.1936** |
| 3 | 47 | 18.5 | 0.016015 | 9722.324 | 4.168990 | 260.3136 |
| 4 | 48 | 19.0 | 0.016022 | 9650.098 | 4.216983 | 263.1936 |
| 5 | 48 | 19.0 | 0.016022 | 9650.098 | 4.216983 | 263.1936 |
| 6 | 48 | 19.0 | 0.016022 | 9650.098 | 4.216983 | 263.1936 |
| 7 | 48 | 19.0 | 0.016022 | 9650.098 | 4.216983 | 263.1936 |
| 8 | 52 | 21.0 | 0.015969 | 9348.799 | 4.386950 | 274.7136 |
| 9 | 48 | 19.0 | 0.016022 | 9650.098 | 4.216983 | 263.1936 |
| 10 | 49 | 19.5 | 0.016020 | 9576.360 | 4.262618 | 266.0736 |

*h=150 mm, L=2000 mm and t=1.2 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 52 | 21.0 | 0.012664 | 10908.15 | 4.192083 | 331.0176 |
| 2 | **51** | **20.5** | **0.012667** | **10977.07** | **4.148957** | **327.5376** |
| 3 | 54 | 22.0 | 0.012644 | 10764.98 | 4.273449 | 337.9776 |
| 4 | 51 | 20.5 | 0.012667 | 10977.07 | 4.148957 | 327.5376 |
| 5 | 51 | 20.5 | 0.012667 | 10977.07 | 4.148957 | 327.5376 |
| 6 | 51 | 20.5 | 0.012667 | 10977.07 | 4.148957 | 327.5376 |
| 7 | 51 | 20.5 | 0.012667 | 10977.07 | 4.148957 | 327.5376 |
| 8 | 52 | 21.0 | 0.012664 | 10908.15 | 4.192083 | 331.0176 |
| 9 | 51 | 20.5 | 0.012667 | 10977.07 | 4.148957 | 327.5376 |
| 10 | 50 | 20.0 | 0.012664 | 11043.52 | 4.103978 | 324.0576 |

*h=150 mm, L=2500 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 55 | 22.5 | 0.009146 | 12532.48 | 3.914720 | 428.0136 |
| 2 | **56** | **23.0** | **0.009147** | **12496.60** | **3.955225** | **432.3936** |
| 3 | 55 | 22.5 | 0.009146 | 12532.48 | 3.914720 | 428.0136 |
| 4 | 56 | 23.0 | 0.009147 | 12496.60 | 3.955225 | 432.3936 |
| 5 | 56 | 23.0 | 0.009147 | 12496.60 | 3.955225 | 432.3936 |
| 6 | 55 | 22.5 | 0.009146 | 12532.48 | 3.914720 | 428.0136 |
| 7 | 55 | 22.5 | 0.009146 | 12532.48 | 3.914720 | 428.0136 |
| 8 | 56 | 23.0 | 0.009147 | 12496.60 | 3.955225 | 432.3936 |
| 9 | 55 | 22.5 | 0.009146 | 12532.48 | 3.914720 | 428.0136 |
| 10 | 56 | 23.0 | 0.009147 | 12496.60 | 3.955225 | 432.3936 |

*h=150 mm, L=3000 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **68** | **29.0** | **0.007238** | **14009.90** | **4.683779** | **647.1136** |
| 2 | 68 | 29.0 | 0.007238 | 14009.90 | 4.683779 | 647.1136 |
| 3 | 68 | 29.0 | 0.007238 | 14009.90 | 4.683779 | 647.1136 |
| 4 | 68 | 29.0 | 0.007238 | 14009.90 | 4.683779 | 647.1136 |
| 5 | 68 | 29.0 | 0.007238 | 14009.90 | 4.683779 | 647.1136 |
| 6 | 73 | 31.5 | 0.007188 | 13863.86 | 4.862900 | 676.5136 |
| 7 | 70 | 30.0 | 0.007230 | 13983.21 | 4.763804 | 658.8736 |
| 8 | 69 | 29.5 | 0.007238 | 14006.57 | 4.726223 | 652.9936 |
| 9 | 67 | 28.5 | 0.007237 | 14011.00 | 4.640460 | 641.2336 |
| 10 | 68 | 29.0 | 0.007238 | 14009.90 | 4.683779 | 647.1136 |

# APPENDIX A

*h=200 mm, L=3000 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **63** | **26.5** | **0.007273** | **14265.40** | **3.898750** | **536.0536** |
| 2 | 64 | 27.0 | 0.007272 | 14196.25 | 3.930172 | 540.4336 |
| 3 | 63 | 26.5 | 0.007273 | 14265.40 | 3.898750 | 536.0536 |
| 4 | 64 | 27.0 | 0.007272 | 14196.25 | 3.930172 | 540.4336 |
| 5 | 64 | 27.0 | 0.007272 | 14196.25 | 3.930172 | 540.4336 |
| 6 | 59 | 24.5 | 0.007250 | 14512.55 | 3.759580 | 518.5336 |
| 7 | 63 | 26.5 | 0.007273 | 14265.40 | 3.898750 | 536.0536 |
| 8 | 62 | 26.0 | 0.007272 | 14332.94 | 3.866264 | 531.6736 |
| 9 | 64 | 27.0 | 0.007272 | 14196.25 | 3.930172 | 540.4336 |
| 10 | 63 | 26.5 | 0.007273 | 14265.40 | 3.898750 | 536.0536 |

*h=200 mm, L=3800 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 72 | 31.0 | 0.005431 | 16797.89 | 4.174639 | 768.6336 |
| 2 | 72 | 31.0 | 0.005431 | 16797.89 | 4.174639 | 768.6336 |
| 3 | **71** | **30.5** | **0.005432** | **16832.61** | **4.143437** | **762.7536** |
| 4 | 70 | 30.0 | 0.005432 | 16864.94 | 4.111276 | 756.8736 |
| 5 | 71 | 30.5 | 0.005432 | 16832.61 | 4.143437 | 762.7536 |
| 6 | 73 | 31.5 | 0.005429 | 16760.89 | 4.204928 | 774.5136 |
| 7 | 71 | 30.5 | 0.005432 | 16832.61 | 4.143437 | 762.7536 |
| 8 | 72 | 31.0 | 0.005431 | 16797.89 | 4.174639 | 768.6336 |
| 9 | 71 | 30.5 | 0.005432 | 16832.61 | 4.143437 | 762.7536 |
| 10 | 71 | 30.5 | 0.005432 | 16832.61 | 4.143437 | 762.7536 |

*h=200 mm, L=4500 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 88 | 39.0 | 0.004364 | 18270.50 | 4.703733 | 1077.874 |
| 2 | 88 | 39.0 | 0.004364 | 18270.50 | 4.703733 | 1077.874 |
| 3 | **85** | **37.5** | **0.004380** | **18362.45** | **4.624477** | **1055.734** |
| 4 | 82 | 36.0 | 0.004379 | 18363.64 | 4.526132 | 1033.594 |
| 5 | 83 | 36.5 | 0.004380 | 18364.53 | 4.559483 | 1040.974 |
| 6 | 82 | 36.0 | 0.004379 | 18363.64 | 4.526132 | 1033.594 |
| 7 | 84 | 37.0 | 0.004380 | 18364.12 | 4.592256 | 1048.354 |
| 8 | 84 | 37.0 | 0.004380 | 18364.12 | 4.592256 | 1048.354 |
| 9 | 83 | 36.5 | 0.004380 | 18364.53 | 4.559483 | 1040.974 |
| 10 | 84 | 37.0 | 0.004380 | 18364.12 | 4.592256 | 1048.354 |

*h=250 mm, L=3200 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **76** | **33.0** | **0.007040** | **14977.32** | **4.688484** | **665.9936** |
| 2 | 76 | 33.0 | 0.007040 | 14977.32 | 4.688484 | 665.9936 |
| 3 | 76 | 33.0 | 0.007040 | 14977.32 | 4.688484 | 665.9936 |
| 4 | 75 | 32.5 | 0.007039 | 15054.65 | 4.657244 | 661.6136 |
| 5 | 75 | 32.5 | 0.007039 | 15054.65 | 4.657244 | 661.6136 |
| 6 | 76 | 33.0 | 0.007040 | 14977.32 | 4.688484 | 665.9936 |
| 7 | 74 | 32.0 | 0.007037 | 15131.49 | 4.625060 | 657.2336 |
| 8 | 74 | 32.0 | 0.007037 | 15131.49 | 4.625060 | 657.2336 |
| 9 | 75 | 32.5 | 0.007039 | 15054.65 | 4.657244 | 661.6136 |
| 10 | 75 | 32.5 | 0.007039 | 15054.65 | 4.657244 | 661.6136 |

*h=250 mm, L=4300 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 78 | 34.0 | 0.004673 | 18782.38 | 4.214851 | 901.9136 |
| 2 | **77** | **33.5** | **0.004673** | **18842.47** | **4.187470** | **896.0336** |
| 3 | 77 | 33.5 | 0.004673 | 18842.47 | 4.187470 | 896.0336 |
| 4 | 78 | 34.0 | 0.004673 | 18782.38 | 4.214851 | 901.9136 |
| 5 | 78 | 34.0 | 0.004673 | 18782.38 | 4.214851 | 901.9136 |
| 6 | 77 | 33.5 | 0.004673 | 18842.47 | 4.187470 | 896.0336 |
| 7 | 78 | 34.0 | 0.004673 | 18782.38 | 4.214851 | 901.9136 |
| 8 | 80 | 35.0 | 0.004669 | 18651.44 | 4.266200 | 913.6736 |
| 9 | 78 | 34.0 | 0.004673 | 18782.38 | 4.214851 | 901.9136 |
| 10 | 77 | 33.5 | 0.004673 | 18842.47 | 4.187470 | 896.0336 |

*h=250 mm, L=5100 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 88 | 39 | 0.003844 | 21095.94 | 4.616241 | 1200.874 |
| 2 | 88 | 39 | 0.003844 | 21095.94 | 4.616241 | 1200.874 |
| 3 | **86** | **38** | **0.003845** | **21161.99** | **4.560300** | **1186.114** |
| 4 | 86 | 38 | 0.003845 | 21161.99 | 4.560300 | 1186.114 |
| 5 | 88 | 39 | 0.003844 | 21095.94 | 4.616241 | 1200.874 |
| 6 | 86 | 38 | 0.003845 | 21161.99 | 4.560300 | 1186.114 |
| 7 | 86 | 38 | 0.003845 | 21161.99 | 4.560300 | 1186.114 |
| 8 | 86 | 38 | 0.003845 | 21161.99 | 4.560300 | 1186.114 |
| 9 | 86 | 38 | 0.003845 | 21161.99 | 4.560300 | 1186.114 |
| 10 | 86 | 38 | 0.003845 | 21161.99 | 4.560300 | 1186.114 |

# APPENDIX A

*h=300 mm, L=4500 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **90** | **40.0** | **0.004535** | **19785.73** | **4.854450** | **1070.474** |
| 2 | 90 | 40.0 | 0.004535 | 19785.73 | 4.854450 | 1070.474 |
| 3 | 90 | 40.0 | 0.004535 | 19785.73 | 4.854450 | 1070.474 |
| 4 | 88 | 39.0 | 0.004535 | 19934.10 | 4.800740 | 1058.714 |
| 5 | 89 | 39.5 | 0.004535 | 19860.34 | 4.827917 | 1064.594 |
| 6 | 90 | 40.0 | 0.004535 | 19785.73 | 4.854450 | 1070.474 |
| 7 | 88 | 39.0 | 0.004535 | 19934.10 | 4.800740 | 1058.714 |
| 8 | 90 | 40.0 | 0.004535 | 19785.73 | 4.854450 | 1070.474 |
| 9 | 90 | 40.0 | 0.004535 | 19785.73 | 4.854450 | 1070.474 |
| 10 | 89 | 39.5 | 0.004535 | 19860.34 | 4.827917 | 1064.594 |

*h=300 mm, L=5500 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 87 | 38.5 | 0.003529 | 23413.27 | 4.645736 | 1316.494 |
| 2 | **90** | **40.0** | **0.003543** | **23321.97** | **4.742144** | **1338.634** |
| 3 | 89 | 39.5 | 0.003539 | 23354.56 | 4.710856 | 1331.254 |
| 4 | 89 | 39.5 | 0.003539 | 23354.56 | 4.710856 | 1331.254 |
| 5 | 89 | 39.5 | 0.003539 | 23354.56 | 4.710856 | 1331.254 |
| 6 | 90 | 40.0 | 0.003543 | 23321.97 | 4.742144 | 1338.634 |
| 7 | 87 | 38.5 | 0.003529 | 23413.27 | 4.645736 | 1316.494 |
| 8 | 90 | 40.0 | 0.003543 | 23321.97 | 4.742144 | 1338.634 |
| 9 | 90 | 40.0 | 0.003543 | 23321.97 | 4.742144 | 1338.634 |
| 10 | 90 | 40.0 | 0.003543 | 23321.97 | 4.742144 | 1338.634 |

*h=300 mm, L=6000 mm and t=3.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 87 | 38.5 | 0.003180 | 24938.63 | 5.018493 | 1578.154 |
| 2 | **90** | **40.0** | **0.003220** | **25090.94** | **5.166782** | **1604.794** |
| 3 | 89 | 39.5 | 0.003207 | 25041.46 | 5.118124 | 1595.914 |
| 4 | 89 | 39.5 | 0.003207 | 25041.46 | 5.118124 | 1595.914 |
| 5 | 89 | 39.5 | 0.003207 | 25041.46 | 5.118124 | 1595.914 |
| 6 | 90 | 40.0 | 0.003220 | 25090.94 | 5.166782 | 1604.794 |
| 7 | 87 | 38.5 | 0.003180 | 24938.63 | 5.018493 | 1578.154 |
| 8 | 90 | 40.0 | 0.003220 | 25090.94 | 5.166782 | 1604.794 |
| 9 | 90 | 40.0 | 0.003220 | 25090.94 | 5.166782 | 1604.794 |
| 10 | 90 | 40.0 | 0.003220 | 25090.94 | 5.166782 | 1604.794 |

*h=350 mm, L=5000 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 87 | 38.5 | 0.003808 | 21538.42 | 4.381884 | 1150.834 |
| 2 | **90** | **40.0** | **0.003829** | **21327.71** | **4.473592** | **1168.474** |
| 3 | 89 | 39.5 | 0.003822 | 21398.99 | 4.443725 | 1162.594 |
| 4 | 89 | 39.5 | 0.003822 | 21398.99 | 4.443725 | 1162.594 |
| 5 | 89 | 39.5 | 0.003822 | 21398.99 | 4.443725 | 1162.594 |
| 6 | 90 | 40.0 | 0.003829 | 21327.71 | 4.473592 | 1168.474 |
| 7 | 87 | 38.5 | 0.003808 | 21538.42 | 4.381884 | 1150.834 |
| 8 | 90 | 40.0 | 0.003829 | 21327.71 | 4.473592 | 1168.474 |
| 9 | 90 | 40.0 | 0.003829 | 21327.71 | 4.473592 | 1168.474 |
| 10 | 90 | 40.0 | 0.003829 | 21327.71 | 4.473592 | 1168.474 |

*h=350 mm, L=6000 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 87 | 38.5 | 0.003073 | 25300.75 | 4.423748 | 1439.494 |
| 2 | **90** | **40.0** | **0.003101** | **25226.00** | **4.531871** | **1461.634** |
| 3 | 89 | 39.5 | 0.003092 | 25253.29 | 4.496764 | 1454.254 |
| 4 | 89 | 39.5 | 0.003092 | 25253.29 | 4.496764 | 1454.254 |
| 5 | 89 | 39.5 | 0.003092 | 25253.29 | 4.496764 | 1454.254 |
| 6 | 90 | 40.0 | 0.003101 | 25226.00 | 4.531871 | 1461.634 |
| 7 | 87 | 38.5 | 0.003073 | 25300.75 | 4.423748 | 1439.494 |
| 8 | 90 | 40.0 | 0.003101 | 25226.00 | 4.531871 | 1461.634 |
| 9 | 90 | 40.0 | 0.003101 | 25226.00 | 4.531871 | 1461.634 |
| 10 | 90 | 40.0 | 0.003101 | 25226.00 | 4.531871 | 1461.634 |

*h=350 mm, L=7000 mm and t=3.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 87 | 38.5 | 0.002465 | 27061.03 | 4.254442 | 1726.154 |
| 2 | **90** | **40.0** | **0.002510** | **27255.40** | **4.399293** | **1752.794** |
| 3 | 89 | 39.5 | 0.002495 | 27192.03 | 4.351814 | 1743.914 |
| 4 | 89 | 39.5 | 0.002495 | 27192.03 | 4.351814 | 1743.914 |
| 5 | 89 | 39.5 | 0.002495 | 27192.03 | 4.351814 | 1743.914 |
| 6 | 90 | 40.0 | 0.002510 | 27255.40 | 4.399293 | 1752.794 |
| 7 | 87 | 38.5 | 0.002465 | 27061.03 | 4.254442 | 1726.154 |
| 8 | 90 | 40.0 | 0.002510 | 27255.40 | 4.399293 | 1752.794 |
| 9 | 90 | 40.0 | 0.002510 | 27255.40 | 4.399293 | 1752.794 |
| 10 | 90 | 40.0 | 0.002510 | 27255.40 | 4.399293 | 1752.794 |

*h=350 mm, L=7500 mm and t=3.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 87 | 38.5 | 0.002247 | 28330.80 | 4.519038 | 2010.814 |
| 2 | **90** | **40.0** | **0.002294** | **28533.48** | **4.684835** | **2041.954** |
| 3 | 89 | 39.5 | 0.002279 | 28467.29 | 4.630489 | 2031.574 |
| 4 | 89 | 39.5 | 0.002279 | 28467.29 | 4.630489 | 2031.574 |
| 5 | 89 | 39.5 | 0.002279 | 28467.29 | 4.630489 | 2031.574 |
| 6 | 90 | 40.0 | 0.002294 | 28533.48 | 4.684835 | 2041.954 |
| 7 | 87 | 38.5 | 0.002247 | 28330.80 | 4.519038 | 2010.814 |
| 8 | 90 | 40.0 | 0.002294 | 28533.48 | 4.684835 | 2041.954 |
| 9 | 90 | 40.0 | 0.002294 | 28533.48 | 4.684835 | 2041.954 |
| 10 | 90 | 40.0 | 0.002294 | 28533.48 | 4.684835 | 2041.954 |

## Choice 3 (free optimization)

*h=100 mm, L=1400 mm and t=1.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| **1** | **40** | **20.06** | **0.019821** | **8069.294** | **4.001341** | **201.8688** |
| 2 | 40 | 15.98 | 0.019179 | 8181.224 | 3.721451 | 194.0352 |
| 3 | 56 | 23.50 | 0.017822 | 7023.375 | 4.262833 | 239.1936 |
| 4 | 41 | 20.65 | 0.019763 | 8000.179 | 4.049832 | 204.9216 |
| 5 | 40 | 20.06 | 0.019821 | 8069.294 | 4.001341 | 201.8688 |
| 6 | 46 | 22.80 | 0.019281 | 7657.916 | 4.215812 | 218.6496 |
| 7 | 40 | 20.06 | 0.019821 | 8069.294 | 4.001341 | 201.8688 |
| 8 | 40 | 20.06 | 0.019821 | 8069.294 | 4.001341 | 201.8688 |
| 9 | 41 | 20.65 | 0.019617 | 7855.583 | 4.139724 | 211.0272 |
| 10 | 44 | 22.42 | 0.019532 | 7780.891 | 4.181474 | 214.0800 |

*h=100 mm, L=1600 mm and t=1.2 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 43 | 21.83 | 0.016293 | 8763.954 | 4.139416 | 254.0632 |
| 2 | 44 | 22.04 | 0.016265 | 8759.222 | 4.178035 | 256.8704 |
| 3 | **44** | **22.80** | **0.016312** | **8723.037** | **4.218877** | **258.6336** |
| 4 | 43 | 21.83 | 0.016293 | 8763.954 | 4.139416 | 254.0632 |
| 5 | 44 | 22.42 | 0.016291 | 8742.151 | 4.198943 | 257.7520 |
| 6 | 43 | 21.83 | 0.016293 | 8763.954 | 4.139416 | 254.0632 |
| 7 | 43 | 21.83 | 0.016293 | 8763.954 | 4.139416 | 254.0632 |
| 8 | 44 | 22.04 | 0.016265 | 8759.222 | 4.178035 | 256.8704 |
| 9 | 42 | 21.24 | 0.016270 | 8768.581 | 4.073471 | 250.3744 |
| 10 | 44 | 22.42 | 0.016291 | 8742.151 | 4.198943 | 257.7520 |

*h=100 mm, L=1900 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 48 | 24.78 | 0.012345 | 9262.166 | 4.212605 | 341.2312 |
| 2 | 48 | 24.78 | 0.012345 | 9262.166 | 4.212605 | 341.2312 |
| 3 | 47 | 24.60 | 0.012353 | 9242.310 | 4.172693 | 337.7856 |
| 4 | 48 | 24.78 | 0.012345 | 9262.166 | 4.212605 | 341.2312 |
| 5 | 47 | 24.19 | 0.012343 | 9264.970 | 4.154517 | 336.5884 |
| 6 | 56 | 29.50 | 0.012203 | 9121.224 | 4.617239 | 378.3736 |
| 7 | **48** | **25.20** | **0.012355** | **9238.816** | **4.231034** | **342.4576** |
| 8 | 48 | 25.20 | 0.012355 | 9238.816 | 4.231034 | 342.4576 |
| 9 | 48 | 25.20 | 0.012355 | 9238.816 | 4.231034 | 342.4576 |
| 10 | 49 | 25.80 | 0.012354 | 9234.027 | 4.288272 | 347.1296 |

*h=100 mm, L=2200 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 55 | 28.91 | 0.009588 | 9462.814 | 4.772774 | 497.8008 |
| 2 | 56 | 29.50 | 0.009590 | 9470.044 | 4.833476 | 504.0336 |
| 3 | 58 | 29.64 | 0.009574 | 9526.179 | 4.906058 | 512.4224 |
| 4 | 60 | 31.86 | 0.009560 | 9458.214 | 5.056934 | 528.9648 |
| 5 | 56 | 29.50 | 0.009590 | 9470.044 | 4.833476 | 504.0336 |
| 6 | 56 | 29.50 | 0.009590 | 9470.044 | 4.833476 | 504.0336 |
| 7 | **56** | **30.00** | **0.009596** | **9448.362** | **4.855401** | **505.9936** |
| 8 | 56 | 29.50 | 0.009590 | 9470.044 | 4.833476 | 504.0336 |
| 9 | 55 | 28.91 | 0.009588 | 9462.814 | 4.772774 | 497.8008 |
| 10 | 55 | 24.99 | 0.009495 | 9610.722 | 4.580845 | 482.4344 |

*h=150 mm, L=1700 mm and t=1.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 47 | 20.50 | 0.016343 | 9729.276 | 4.317151 | 264.1536 |
| 2 | 45 | 23.01 | 0.016796 | 9836.653 | 4.453240 | 265.1328 |
| 3 | 48 | 24.78 | 0.016772 | 9599.288 | 4.600280 | 274.2912 |
| 4 | 46 | 23.60 | 0.016799 | 9759.061 | 4.505134 | 268.1856 |
| 5 | 46 | 22.80 | 0.016712 | 9774.276 | 4.456348 | 266.6496 |
| 6 | 46 | 23.60 | 0.016799 | 9759.061 | 4.505134 | 268.1856 |
| 7 | 48 | 24.36 | 0.016732 | 9608.147 | 4.575875 | 273.4848 |
| 8 | 41 | 17.85 | 0.016209 | 10138.81 | 4.012402 | 247.5456 |
| 9 | **48** | **25.20** | **0.016807** | **9587.674** | **4.623444** | **275.0976** |
| 10 | 43 | 21.83 | 0.016753 | 9985.479 | 4.339607 | 259.0272 |

# APPENDIX A

*h=150 mm, L=2000 mm and t=1.2 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **49** | **25.80** | **0.013234** | **11006.92** | **4.435810** | **335.1936** |
| 2 | 50 | 25.08 | 0.013145 | 10974.46 | 4.414595 | 335.8432 |
| 3 | 49 | 25.37 | 0.013208 | 11022.05 | 4.414102 | 334.1960 |
| 4 | 49 | 25.37 | 0.013208 | 11022.05 | 4.414102 | 334.1960 |
| 5 | 52 | 23.46 | 0.012911 | 10887.01 | 4.347523 | 336.7248 |
| 6 | 49 | 25.37 | 0.013208 | 11022.05 | 4.414102 | 334.1960 |
| 7 | 49 | 25.37 | 0.013208 | 11022.05 | 4.414102 | 334.1960 |
| 8 | 48 | 24.78 | 0.013206 | 11091.73 | 4.364733 | 330.5072 |
| 9 | 49 | 25.37 | 0.013208 | 11022.05 | 4.414102 | 334.1960 |
| 10 | 49 | 24.51 | 0.013148 | 11045.48 | 4.367909 | 332.2008 |

*h=150 mm, L=2500 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 58 | 30.68 | 0.009411 | 12220.07 | 4.335335 | 460.6592 |
| 2 | 57 | 30.09 | 0.009430 | 12283.25 | 4.300174 | 456.0164 |
| 3 | 54 | 28.32 | 0.009441 | 12408.74 | 4.173935 | 442.0880 |
| 4 | 56 | 25.50 | 0.009286 | 12454.37 | 4.082953 | 439.6936 |
| 5 | 57 | 30.09 | 0.009430 | 12283.25 | 4.300174 | 456.0164 |
| 6 | 55 | 24.99 | 0.009288 | 12492.40 | 4.043049 | 435.2844 |
| 7 | 54 | 28.32 | 0.009441 | 12408.74 | 4.173935 | 442.0880 |
| 8 | 56 | 29.50 | 0.009437 | 12327.45 | 4.259496 | 451.3736 |
| 9 | **56** | **30.00** | **0.009449** | **12305.44** | **4.278843** | **452.8336** |
| 10 | 54 | 28.32 | 0.009441 | 12408.74 | 4.173935 | 442.0880 |

*h=150 mm, L=3000 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **66** | **36.00** | **0.007382** | **13665.38** | **4.921713** | **666.7136** |
| 2 | 72 | 38.94 | 0.007330 | 13559.01 | 5.143616 | 701.7584 |
| 3 | 65 | 34.81 | 0.007377 | 13705.10 | 4.854889 | 658.1288 |
| 4 | 66 | 36.00 | 0.007382 | 13665.38 | 4.921713 | 666.7136 |
| 5 | 67 | 31.11 | 0.007303 | 13917.31 | 4.757497 | 651.4648 |
| 6 | 67 | 31.11 | 0.007303 | 13917.31 | 4.757497 | 651.4648 |
| 7 | 67 | 31.11 | 0.007303 | 13917.31 | 4.757497 | 651.4648 |
| 8 | 66 | 36.00 | 0.007382 | 13665.38 | 4.921713 | 666.7136 |
| 9 | 65 | 35.40 | 0.007382 | 13672.60 | 4.875169 | 660.4416 |
| 10 | 66 | 35.40 | 0.007377 | 13698.51 | 4.901210 | 664.3616 |

*h=200 mm, L=3000 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 61 | 32.45 | 0.007520 | 14308.80 | 4.118050 | 547.5876 |
| 2 | 61 | 32.45 | 0.007520 | 14308.80 | 4.118050 | 547.5876 |
| 3 | 72 | 38.94 | 0.007416 | 13441.11 | 4.439540 | 598.6584 |
| 4 | 61 | 32.45 | 0.007491 | 14335.81 | 4.078013 | 544.3756 |
| 5 | 61 | 32.45 | 0.007520 | 14308.80 | 4.118050 | 547.5876 |
| 6 | 60 | 32.40 | 0.007531 | 14362.48 | 4.100591 | 544.5216 |
| 7 | 61 | 31.35 | 0.007491 | 14335.81 | 4.078013 | 544.3756 |
| 8 | **62** | **33.60** | **0.007532** | **14217.23** | **4.171536** | **553.8656** |
| 9 | 63 | 33.63 | 0.007517 | 14161.09 | 4.186011 | 556.8732 |
| 10 | 62 | 33.60 | 0.007532 | 14217.23 | 4.171536 | 553.8656 |

*h=200 mm, L=3800 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **70** | **37.76** | **0.005559** | **16655.55** | **4.376418** | **787.2928** |
| 2 | 71 | 38.35 | 0.005558 | 16616.86 | 4.410110 | 793.5256 |
| 3 | 72 | 38.94 | 0.005555 | 16576.24 | 4.442811 | 799.7584 |
| 4 | 70 | 37.76 | 0.005545 | 16703.12 | 4.337722 | 782.2752 |
| 5 | 71 | 38.35 | 0.005558 | 16616.86 | 4.410110 | 793.5256 |
| 6 | 68 | 37.20 | 0.005550 | 16650.05 | 4.314096 | 777.2576 |
| 7 | 72 | 37.62 | 0.005542 | 16625.88 | 4.403319 | 794.5840 |
| 8 | 70 | 37.76 | 0.005559 | 16655.55 | 4.376418 | 787.2928 |
| 9 | 73 | 39.53 | 0.005552 | 16533.80 | 4.474568 | 805.9912 |
| 10 | 72 | 38.94 | 0.005555 | 16576.24 | 4.442811 | 799.7584 |

*h=200 mm, L=4500 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 84 | 46.02 | 0.004435 | 17991.23 | 4.845901 | 1092.732 |
| 2 | **84** | **46.80** | **0.004436** | **17949.06** | **4.863996** | **1096.570** |
| 3 | 84 | 46.02 | 0.004435 | 17991.23 | 4.845901 | 1092.732 |
| 4 | 83 | 45.43 | 0.004435 | 17998.64 | 4.811646 | 1084.909 |
| 5 | 83 | 46.20 | 0.004436 | 17957.07 | 4.829570 | 1088.698 |
| 6 | 88 | 49.20 | 0.004416 | 17832.80 | 4.981103 | 1128.058 |
| 7 | 83 | 43.89 | 0.004431 | 18072.90 | 4.773161 | 1077.332 |
| 8 | 88 | 41.82 | 0.004387 | 18170.39 | 4.789457 | 1091.748 |
| 9 | 75 | 40.71 | 0.004416 | 18022.20 | 4.514945 | 1022.327 |
| 10 | 83 | 45.43 | 0.004435 | 17998.64 | 4.811646 | 1084.909 |

# APPENDIX A

*h=250 mm, L=3200 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 71 | 38.35 | 0.007285 | 15305.49 | 4.859272 | 667.0156 |
| 2 | **73** | **40.20** | **0.007302** | **15123.96** | **4.952915** | **678.2576** |
| 3 | 73 | 39.53 | 0.007289 | 15140.61 | 4.929224 | 676.3012 |
| 4 | 73 | 39.53 | 0.007289 | 15140.61 | 4.929224 | 676.3012 |
| 5 | 71 | 38.35 | 0.007285 | 15305.49 | 4.859272 | 667.0156 |
| 6 | 70 | 38.40 | 0.007295 | 15371.63 | 4.845775 | 664.2416 |
| 7 | 74 | 38.76 | 0.007255 | 15081.98 | 4.911652 | 676.9728 |
| 8 | 71 | 39.00 | 0.007299 | 15289.70 | 4.882716 | 668.9136 |
| 9 | 73 | 39.53 | 0.007289 | 15140.61 | 4.929224 | 676.3012 |
| 10 | 72 | 38.94 | 0.007288 | 15223.33 | 4.894839 | 671.6584 |

*h=250 mm, L=4300 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 76 | 41.30 | 0.004797 | 18764.01 | 4.426172 | 922.6896 |
| 2 | **76** | **42.00** | **0.004803** | **18738.68** | **4.444690** | **925.4336** |
| 3 | 75 | 40.71 | 0.004794 | 18816.54 | 4.393829 | 916.4568 |
| 4 | 75 | 40.71 | 0.004794 | 18816.54 | 4.393829 | 916.4568 |
| 5 | 77 | 42.60 | 0.004802 | 18669.61 | 4.473677 | 931.7056 |
| 6 | 73 | 40.20 | 0.004788 | 18876.25 | 4.341239 | 906.6176 |
| 7 | 76 | 39.90 | 0.004782 | 18803.06 | 4.386324 | 917.2016 |
| 8 | 78 | 42.48 | 0.004794 | 18625.37 | 4.482893 | 935.1552 |
| 9 | 75 | 40.71 | 0.004794 | 18816.54 | 4.393829 | 916.4568 |
| 10 | 76 | 41.30 | 0.004797 | 18764.01 | 4.426172 | 922.6896 |

*h=250 mm, L=5100 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 86 | 47.20 | 0.003916 | 20901.80 | 4.821621 | 1231.378 |
| 2 | **86** | **48.00** | **0.003918** | **20865.63** | **4.840345** | **1235.314** |
| 3 | 86 | 47.20 | 0.003916 | 20901.80 | 4.821621 | 1231.378 |
| 4 | 86 | 47.20 | 0.003916 | 20901.80 | 4.821621 | 1231.378 |
| 5 | 85 | 47.40 | 0.003915 | 20882.09 | 4.806018 | 1227.442 |
| 6 | 88 | 49.20 | 0.003916 | 20786.50 | 4.899504 | 1251.058 |
| 7 | 86 | 48.00 | 0.003918 | 20865.63 | 4.840345 | 1235.314 |
| 8 | 88 | 48.38 | 0.003914 | 20823.72 | 4.880450 | 1247.023 |
| 9 | 85 | 39.50 | 0.003863 | 21136.43 | 4.591390 | 1188.574 |
| 10 | 85 | 46.61 | 0.003913 | 20917.62 | 4.787478 | 1223.555 |

*h=300 mm, L=4500 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 87 | 47.79 | 0.004668 | 19902.23 | 5.084555 | 1089.250 |
| 2 | 86 | 48.00 | 0.004675 | 19957.95 | 5.077571 | 1086.154 |
| 3 | **87** | **48.60** | **0.004675** | **19878.90** | **5.107264** | **1092.426** |
| 4 | 87 | 47.79 | 0.004668 | 19902.23 | 5.084555 | 1089.250 |
| 5 | 87 | 48.60 | 0.004675 | 19878.90 | 5.107264 | 1092.426 |
| 6 | 88 | 49.20 | 0.004675 | 19799.02 | 5.136190 | 1098.698 |
| 7 | 85 | 47.40 | 0.004674 | 20036.09 | 5.047076 | 1079.882 |
| 8 | 82 | 45.60 | 0.004666 | 20264.14 | 4.950417 | 1061.066 |
| 9 | 89 | 48.97 | 0.004667 | 19742.60 | 5.141385 | 1101.716 |
| 10 | 85 | 47.40 | 0.004674 | 20036.09 | 5.047076 | 1079.882 |

*h=300 mm, L=5500 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 87 | 48.60 | 0.003618 | 23249.78 | 4.942488 | 1366.186 |
| 2 | 93 | 51.33 | 0.003633 | 23040.00 | 5.118765 | 1409.137 |
| 3 | **94** | **52.80** | **0.003635** | **22940.90** | **5.166227** | **1421.290** |
| 4 | 93 | 51.33 | 0.003633 | 23040.00 | 5.118765 | 1409.137 |
| 5 | 94 | 51.92 | 0.003631 | 22975.15 | 5.145415 | 1416.960 |
| 6 | 96 | 54.00 | 0.003630 | 22799.84 | 5.216501 | 1437.034 |
| 7 | 94 | 52.80 | 0.003635 | 22940.90 | 5.166227 | 1421.290 |
| 8 | 96 | 54.00 | 0.003630 | 22799.84 | 5.216501 | 1437.034 |
| 9 | 97 | 45.50 | 0.003568 | 22971.62 | 4.995863 | 1400.134 |
| 10 | 97 | 53.69 | 0.003624 | 22763.36 | 5.219409 | 1440.428 |

*h=300 mm, L=6000 mm and t=3.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 99 | 47.43 | 0.003328 | 25389.85 | 5.665236 | 1702.059 |
| 2 | 94 | 51.92 | 0.003338 | 25136.48 | 5.671384 | 1699.040 |
| 3 | 99 | 55.80 | 0.003370 | 25115.28 | 5.903286 | 1751.610 |
| 4 | 100 | 55.46 | 0.003372 | 25157.64 | 5.919868 | 1755.517 |
| 5 | 99 | 55.80 | 0.003370 | 25115.28 | 5.903286 | 1751.610 |
| 6 | 97 | 54.60 | 0.003360 | 25112.08 | 5.821417 | 1732.666 |
| 7 | **100** | **56.40** | **0.003375** | **25115.28** | **5.943270** | **1761.082** |
| 8 | 100 | 56.40 | 0.003375 | 25115.28 | 5.943270 | 1761.082 |
| 9 | 100 | 47.94 | 0.003333 | 25396.74 | 5.703137 | 1710.998 |
| 10 | 87 | 47.79 | 0.003274 | 25055.58 | 5.347084 | 1633.150 |

# APPENDIX A

*h=350 mm, L=5000 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 87 | 47.79 | 0.003932 | 21534.74 | 4.668858 | 1187.250 |
| 2 | 86 | 48.00 | 0.003932 | 21595.16 | 4.656158 | 1184.154 |
| 3 | 90 | 48.72 | 0.003943 | 21311.69 | 4.741524 | 1202.656 |
| 4 | 91 | 49.30 | 0.003947 | 21232.37 | 4.771669 | 1208.850 |
| 5 | **89** | **49.80** | **0.003952** | **21361.67** | **4.754359** | **1202.970** |
| 6 | 88 | 49.20 | 0.003946 | 21440.45 | 4.722420 | 1196.698 |
| 7 | 91 | 48.45 | 0.003939 | 21243.14 | 4.748322 | 1205.518 |
| 8 | 91 | 48.45 | 0.003939 | 21243.14 | 4.748322 | 1205.518 |
| 9 | 96 | 43.20 | 0.003857 | 20884.14 | 4.646258 | 1204.538 |
| 10 | 87 | 47.79 | 0.003932 | 21534.74 | 4.668858 | 1187.250 |

*h=350 mm, L=6000 mm and t=2.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **91** | **49.30** | **0.003187** | **25175.61** | **4.819386** | **1512.310** |
| 2 | 91 | 48.45 | 0.003181 | 25188.38 | 4.797614 | 1508.128 |
| 3 | 93 | 46.98 | 0.003175 | 25141.25 | 4.796546 | 1510.735 |
| 4 | 91 | 49.30 | 0.003187 | 25175.61 | 4.819386 | 1512.310 |
| 5 | 95 | 45.39 | 0.003163 | 25072.4 | 4.784147 | 1512.752 |
| 6 | 91 | 49.30 | 0.003187 | 25175.61 | 4.819386 | 1512.310 |
| 7 | 94 | 45.76 | 0.003166 | 25111.31 | 4.779073 | 1509.653 |
| 8 | 91 | 48.45 | 0.003181 | 25188.38 | 4.797614 | 1508.128 |
| 9 | 96 | 44.10 | 0.003149 | 25020.31 | 4.758767 | 1511.326 |
| 10 | 87 | 47.79 | 0.003158 | 25294.54 | 4.690115 | 1485.200 |

*h=350 mm, L=7000 mm and t=3.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 90 | 50.40 | 0.002584 | 27424.63 | 4.687586 | 1814.362 |
| 2 | 97 | 42.77 | 0.002588 | 27619.07 | 4.685120 | 1810.632 |
| 3 | 93 | 46.98 | 0.002596 | 27589.12 | 4.703785 | 1811.875 |
| 4 | **95** | **45.39** | **0.002601** | **27657.43** | **4.718544** | **1814.302** |
| 5 | 95 | 45.39 | 0.002601 | 27657.43 | 4.718544 | 1814.302 |
| 6 | 91 | 49.30 | 0.002590 | 27488.70 | 4.697169 | 1813.770 |
| 7 | 96 | 44.10 | 0.002597 | 27667.07 | 4.706999 | 1812.586 |
| 8 | 96 | 44.10 | 0.002597 | 27667.07 | 4.706999 | 1812.586 |
| 9 | 96 | 44.10 | 0.002597 | 27667.07 | 4.706999 | 1812.586 |
| 10 | 89 | 47.31 | 0.002563 | 27456.35 | 4.587514 | 1790.149 |

*h=350 mm, L=7500 mm and t=3.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 100 | 40.42 | 0.002373 | 28902.73 | 5.015918 | 2114.060 |
| 2 | 97 | 42.77 | 0.002378 | 28929.41 | 5.016098 | 2109.562 |
| 3 | 93 | 46.98 | 0.002377 | 28912.37 | 5.018875 | 2111.015 |
| 4 | 98 | 42.32 | 0.002380 | 28943.07 | 5.030187 | 2113.368 |
| 5 | 96 | 44.10 | 0.002382 | 28946.90 | 5.030269 | 2111.846 |
| 6 | 100 | 40.42 | 0.002373 | 28902.73 | 5.015918 | 2114.060 |
| 7 | 102 | 38.40 | 0.002359 | 28815.29 | 4.986332 | 2113.922 |
| 8 | 97 | 42.77 | 0.002378 | 28929.41 | 5.016098 | 2109.562 |
| 9 | 96 | 44.10 | 0.002382 | 28946.90 | 5.030269 | 2111.846 |
| 10 | **95** | **45.39** | **0.002384** | **28952.33** | **5.039681** | **2113.852** |

# APPENDIX B

## Results for Σ-Shape Purlin with Objective Function $q/A_g$ and with Freely Varied Parameters

### Single Edge Stiffener

*h=250 mm, L=4000 mm and t=1.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 81 | 42.12 | 43.74 | 37 | 37 | 0.007359 | 17867.07 | 5.588404 | 759.3749 |
| 2 | **79** | **36.34** | **38.71** | **31** | **31** | **0.007428** | **18210.46** | **5.417857** | **729.4002** |
| 3 | 78 | 34.32 | 36.66 | 29 | 29 | 0.007399 | 18281.58 | 5.313578 | 718.1628 |
| 4 | 81 | 41.31 | 42.93 | 36 | 36 | 0.007380 | 17908.07 | 5.577774 | 755.8002 |
| 5 | 79 | 36.34 | 38.71 | 31 | 31 | 0.007428 | 18210.46 | 5.417857 | 729.4002 |
| 6 | 79 | 31.60 | 39.50 | 30 | 30 | 0.007417 | 18158.14 | 5.298046 | 714.3499 |
| 7 | 79 | 31.60 | 39.50 | 30 | 30 | 0.007417 | 18158.14 | 5.298046 | 714.3499 |
| 8 | 79 | 31.60 | 39.50 | 30 | 30 | 0.007417 | 18158.14 | 5.298046 | 714.3499 |
| 9 | 80 | 37.60 | 40.80 | 32 | 32 | 0.007280 | 18098.08 | 5.367056 | 737.2089 |
| 10 | 80 | 33.60 | 41.60 | 32 | 32 | 0.007395 | 18065.90 | 5.364968 | 725.5289 |

# APPENDIX B

*h=250 mm, L=5500 mm and t=2.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 81 | 42.12 | 43.74 | 37 | 37 | 0.004635 | 21682.98 | 5.877090 | 1268.084 |
| 2 | 80 | 37.60 | 40.80 | 32 | 32 | 0.004703 | 21923.87 | 5.787881 | 1230.736 |
| 3 | 80 | 32.80 | 41.60 | 31 | 31 | 0.004697 | 21831.05 | 5.660604 | 1205.082 |
| 4 | **81** | **40.50** | **42.93** | **35** | **35** | **0.004723** | **21775.39** | **5.931933** | **1256.038** |
| 5 | 81 | 40.50 | 42.93 | 35 | 35 | 0.004723 | 21775.39 | 5.931933 | 1256.038 |
| 6 | 81 | 40.50 | 42.93 | 35 | 35 | 0.004723 | 21775.39 | 5.931933 | 1256.038 |
| 7 | 80 | 32.80 | 41.60 | 31 | 31 | 0.004697 | 21831.05 | 5.660604 | 1205.082 |
| 8 | 82 | 43.46 | 45.92 | 38 | 38 | 0.004650 | 21586.22 | 5.959551 | 1281.635 |
| 9 | 80 | 37.60 | 40.80 | 32 | 32 | 0.004703 | 21923.87 | 5.787881 | 1230.736 |
| 10 | 80 | 37.60 | 40.80 | 32 | 32 | 0.004703 | 21923.87 | 5.787881 | 1230.736 |

*h=300 mm, L=5000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 91 | 42.77 | 41.86 | 24 | 32 | 0.005955 | 23146.90 | 6.888479 | 1156.766 |
| 2 | **92** | **46.00** | **46.00** | **29** | **35** | **0.005990** | **23082.50** | **7.040185** | **1175.316** |
| 3 | 80 | 30.40 | 56.00 | 23 | 28 | 0.005688 | 23493.29 | 5.998255 | 1054.494 |
| 4 | 90 | 36.00 | 39.60 | 27 | 30 | 0.005925 | 23178.33 | 6.613975 | 1116.302 |
| 5 | 92 | 46.00 | 46.00 | 29 | 35 | 0.005990 | 23082.50 | 7.040185 | 1175.316 |
| 6 | 91 | 38.22 | 43.68 | 31 | 32 | 0.005910 | 23196.99 | 6.676683 | 1129.802 |
| 7 | 92 | 46.00 | 46.00 | 29 | 35 | 0.005990 | 23082.50 | 7.040185 | 1175.316 |
| 8 | 91 | 42.77 | 41.86 | 24 | 32 | 0.005955 | 23146.90 | 6.888479 | 1156.766 |
| 9 | 90 | 40.50 | 37.80 | 20 | 30 | 0.005864 | 23143.48 | 6.708126 | 1143.926 |
| 10 | 81 | 38.07 | 58.32 | 24 | 32 | 0.005855 | 23540.13 | 6.435609 | 1099.142 |

# APPENDIX B

*h=300 mm, L=6500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 91 | 38.22 | 43.68 | 31 | 32 | 0.003964 | 26295.59 | 6.709734 | 1692.464 |
| 2 | **92** | **46.00** | **46.00** | **30** | **35** | **0.003995** | **26342.64** | **7.026430** | **1759.026** |
| 3 | 92 | 46.00 | 46.00 | 30 | 35 | 0.003995 | 26342.64 | 7.026430 | 1759.026 |
| 4 | 92 | 46.00 | 46.00 | 30 | 35 | 0.003995 | 26342.64 | 7.026430 | 1759.026 |
| 5 | 92 | 46.00 | 46.00 | 30 | 35 | 0.003995 | 26342.64 | 7.026430 | 1759.026 |
| 6 | 91 | 38.22 | 43.68 | 31 | 32 | 0.003964 | 26295.59 | 6.709734 | 1692.464 |
| 7 | 92 | 46.00 | 46.00 | 30 | 35 | 0.003995 | 26342.64 | 7.026430 | 1759.026 |
| 8 | 92 | 45.08 | 46.00 | 27 | 34 | 0.003995 | 26373.00 | 7.011604 | 1755.143 |
| 9 | 91 | 42.77 | 41.86 | 24 | 32 | 0.003973 | 26477.74 | 6.884105 | 1732.828 |
| 10 | 92 | 46.00 | 46.00 | 30 | 35 | 0.003995 | 26342.64 | 7.026430 | 1759.026 |

*h=350 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 83 | 36.52 | 66.40 | 18 | 29 | 0.004216 | 26379.73 | 5.054609 | 1199.033 |
| 2 | 83 | 34.03 | 68.06 | 30 | 31 | 0.004200 | 26511.13 | 4.948987 | 1178.393 |
| 3 | 83 | 36.52 | 66.40 | 17 | 29 | 0.004222 | 26350.62 | 5.069989 | 1200.806 |
| 4 | **83** | **33.20** | **68.06** | **28** | **30** | **0.004241** | **26451.04** | **4.981339** | **1174.677** |
| 5 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004241 | 26451.04 | 4.981339 | 1174.677 |
| 6 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004241 | 26451.04 | 4.981339 | 1174.677 |
| 7 | 83 | 34.03 | 68.06 | 30 | 31 | 0.004200 | 26511.13 | 4.948987 | 1178.393 |
| 8 | 83 | 36.52 | 66.40 | 17 | 29 | 0.004222 | 26350.62 | 5.069989 | 1200.806 |
| 9 | 84 | 35.28 | 72.24 | 31 | 32 | 0.004213 | 26414.51 | 5.008149 | 1188.838 |
| 10 | 82 | 30.34 | 63.96 | 21 | 27 | 0.004160 | 26202.89 | 4.825355 | 1159.877 |

# APPENDIX B

*h=350 mm, L=7500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 91 | 33.67 | 43.68 | 22 | 27 | 0.003094 | 29654.31 | 5.595716 | 1808.544 |
| 2 | 83 | 32.37 | 68.06 | 25 | 29 | 0.002941 | 28966.28 | 5.165940 | 1756.449 |
| 3 | 89 | 31.15 | 35.60 | 18 | 25 | 0.002967 | 29404.21 | 5.284973 | 1781.102 |
| 4 | 83 | 32.37 | 68.06 | 25 | 29 | 0.002941 | 28966.28 | 5.165940 | 1756.449 |
| 5 | **91** | **33.67** | **43.68** | **22** | **27** | **0.003094** | **29654.31** | **5.595716** | **1808.544** |
| 6 | 84 | 35.28 | 72.24 | 31 | 32 | 0.002975 | 29096.90 | 5.299895 | 1781.619 |
| 7 | 90 | 32.40 | 39.60 | 20 | 26 | 0.003037 | 29572.83 | 5.450091 | 1794.702 |
| 8 | 89 | 31.15 | 35.60 | 18 | 25 | 0.002967 | 29404.21 | 5.284973 | 1781.102 |
| 9 | 90 | 32.40 | 39.60 | 20 | 26 | 0.003037 | 29572.83 | 5.450091 | 1794.702 |
| 10 | 91 | 33.67 | 43.68 | 22 | 27 | 0.003094 | 29654.31 | 5.595716 | 1808.544 |

*h=400 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **78** | **28.08** | **46.80** | **38** | **21** | **0.003982** | **28801.56** | **4.793574** | **1203.870** |
| 2 | 78 | 27.30 | 46.02 | 36 | 20 | 0.003867 | 28585.49 | 4.639892 | 1199.891 |
| 3 | 78 | 27.30 | 46.02 | 36 | 20 | 0.003867 | 28585.49 | 4.639892 | 1199.891 |
| 4 | 78 | 27.30 | 46.02 | 36 | 20 | 0.003867 | 28585.49 | 4.639892 | 1199.891 |
| 5 | 77 | 26.95 | 42.35 | 36 | 20 | 0.003867 | 28477.95 | 4.620091 | 1194.599 |
| 6 | 78 | 28.08 | 46.80 | 38 | 21 | 0.003982 | 28801.56 | 4.793574 | 1203.870 |
| 7 | 77 | 27.72 | 43.12 | 38 | 21 | 0.003902 | 28702.46 | 4.676307 | 1198.538 |
| 8 | 71 | 24.85 | 66.03 | 35 | 20 | 0.003794 | 29101.59 | 4.414218 | 1163.335 |
| 9 | 77 | 21.56 | 44.66 | 40 | 18 | 0.003628 | 27710.81 | 4.238897 | 1168.400 |
| 10 | 77 | 26.95 | 42.35 | 36 | 20 | 0.003867 | 28477.95 | 4.620091 | 1194.599 |

# APPENDIX B

*h=400 mm, L=8000 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 79 | 22.12 | 52.14 | 39 | 18 | 0.002271 | 31075.32 | 4.012551 | 1767.013 |
| 2 | **79** | **27.65** | **49.77** | **36** | **20** | **0.002398** | **31972.39** | **4.332544** | **1806.820** |
| 3 | 78 | 27.30 | 46.02 | 36 | 20 | 0.002385 | 31867.55 | 4.290817 | 1798.828 |
| 4 | 81 | 26.73 | 57.51 | 32 | 18 | 0.002313 | 31855.91 | 4.186682 | 1810.429 |
| 5 | 79 | 27.65 | 49.77 | 36 | 20 | 0.002398 | 31972.39 | 4.332544 | 1806.820 |
| 6 | 80 | 22.40 | 56.00 | 39 | 18 | 0.002273 | 31107.54 | 4.033326 | 1774.591 |
| 7 | 81 | 22.68 | 59.94 | 39 | 18 | 0.002271 | 31140.26 | 4.047509 | 1782.169 |
| 8 | 81 | 21.06 | 59.13 | 35 | 16 | 0.002183 | 30826.85 | 3.863942 | 1769.806 |
| 9 | 79 | 27.65 | 49.77 | 36 | 20 | 0.002398 | 31972.39 | 4.332544 | 1806.820 |
| 10 | 79 | 27.65 | 49.77 | 36 | 20 | 0.002398 | 31972.39 | 4.332544 | 1806.820 |

## Double Edge Stiffeners

*h=250 mm, L=4000 mm and t=1.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **76** | **36.48** | **18.24** | **37.24** | **33** | **33** | **0.007537** | **18950.84** | **5.831786** | **773.7143** |
| 2 | 78 | 39.78 | 19.50 | 39.00 | 36 | 36 | 0.007477 | 18715.39 | 5.955103 | 796.4980 |
| 3 | 76 | 36.48 | 18.24 | 37.24 | 33 | 33 | 0.007537 | 18950.84 | 5.831786 | 773.7143 |
| 4 | 78 | 39.78 | 19.50 | 39.00 | 36 | 36 | 0.007477 | 18715.39 | 5.955103 | 796.4980 |
| 5 | 75 | 30.75 | 15.00 | 39.00 | 31 | 31 | 0.007519 | 18937.47 | 5.580537 | 742.1829 |
| 6 | 68 | 25.84 | 12.24 | 43.52 | 28 | 28 | 0.007327 | 19173.67 | 5.097669 | 695.7180 |
| 7 | 76 | 36.48 | 18.24 | 37.24 | 33 | 33 | 0.007537 | 18950.84 | 5.831786 | 773.7143 |
| 8 | 78 | 40.56 | 20.28 | 39.78 | 37 | 37 | 0.007463 | 18649.29 | 5.987334 | 802.2627 |
| 9 | 75 | 30.75 | 15.00 | 39.00 | 31 | 31 | 0.007519 | 18937.47 | 5.580537 | 742.1829 |
| 10 | 76 | 35.72 | 17.48 | 36.48 | 32 | 32 | 0.007506 | 18987.62 | 5.765347 | 768.0664 |

# APPENDIX B

*h=250 mm, L=5500 mm and t=2.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 76 | 35.72 | 17.48 | 36.48 | 32 | 32 | 0.004538 | 21875.88 | 5.804947 | 1279.250 |
| 2 | 78 | 39.00 | 19.50 | 39.00 | 35 | 35 | 0.004556 | 21750.23 | 6.019403 | 1321.280 |
| 3 | 75 | 30.00 | 14.25 | 38.25 | 30 | 30 | 0.004553 | 21873.43 | 5.582433 | 1226.220 |
| 4 | 78 | 39.00 | 19.50 | 39.00 | 35 | 35 | 0.004556 | 21750.23 | 6.019403 | 1321.280 |
| 5 | 75 | 30.00 | 14.25 | 38.25 | 30 | 30 | 0.004553 | 21873.43 | 5.582433 | 1226.220 |
| 6 | **77** | **32.34** | **15.40** | **40.04** | **32** | **32** | **0.004572** | **21870.06** | **5.747873** | **1257.307** |
| 7 | 76 | 35.72 | 17.48 | 36.48 | 32 | 32 | 0.004538 | 21875.88 | 5.804947 | 1279.250 |
| 8 | 75 | 30.00 | 14.25 | 38.25 | 30 | 30 | 0.004553 | 21873.43 | 5.582433 | 1226.220 |
| 9 | 75 | 30.00 | 14.25 | 38.25 | 30 | 30 | 0.004553 | 21873.43 | 5.582433 | 1226.220 |
| 10 | 76 | 35.72 | 17.48 | 36.48 | 32 | 32 | 0.004538 | 21875.88 | 5.804947 | 1279.250 |

*h=300 mm, L=5000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 79 | 34.76 | 17.38 | 45.03 | 18 | 29 | 0.005843 | 24187.05 | 6.668271 | 1141.151 |
| 2 | **81** | **38.88** | **19.44** | **49.41** | **25** | **33** | **0.005921** | **24183.59** | **6.955722** | **1174.679** |
| 3 | 81 | 38.88 | 19.44 | 49.41 | 25 | 33 | 0.005921 | 24183.59 | 6.955722 | 1174.679 |
| 4 | 81 | 38.88 | 19.44 | 49.41 | 25 | 33 | 0.005921 | 24183.59 | 6.955722 | 1174.679 |
| 5 | 81 | 38.88 | 19.44 | 49.41 | 25 | 33 | 0.005921 | 24183.59 | 6.955722 | 1174.679 |
| **6** | 80 | 37.60 | 18.40 | 49.60 | 24 | 32 | 0.005905 | 24228.60 | 6.850539 | 1160.075 |
| 7 | 79 | 34.76 | 17.38 | 45.03 | 18 | 29 | 0.005843 | 24187.05 | 6.668271 | 1141.151 |
| 8 | 77 | 32.34 | 16.17 | 42.35 | 14 | 27 | 0.005649 | 24152.73 | 6.325669 | 1119.797 |
| 9 | 81 | 38.88 | 19.44 | 49.41 | 25 | 33 | 0.005921 | 24183.59 | 6.955722 | 1174.679 |
| 10 | 79 | 34.76 | 17.38 | 45.03 | 18 | 29 | 0.005843 | 24187.05 | 6.668271 | 1141.151 |

# APPENDIX B

*h=300 mm, L=6500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 85 | 39.95 | 19.55 | 51.00 | 23 | 32 | 0.003597 | 26070.94 | 6.425111 | 1786.181 |
| 2 | 83 | 40.67 | 19.92 | 51.46 | 27 | 34 | 0.003668 | 25940.07 | 6.533517 | 1781.292 |
| 3 | **81** | **37.26** | **18.63** | **47.79** | **21** | **31** | **0.003674** | **26024.20** | **6.397051** | **1741.061** |
| 4 | 82 | 40.18 | 19.68 | 52.48 | 27 | 34 | 0.003655 | 25888.44 | 6.473071 | 1771.050 |
| 5 | 81 | 38.07 | 18.63 | 48.60 | 24 | 32 | 0.003667 | 25986.87 | 6.394807 | 1743.703 |
| **6** | 81 | 38.07 | 18.63 | 48.60 | 24 | 32 | 0.003667 | 25986.87 | 6.394807 | 1743.703 |
| 7 | 79 | 34.76 | 17.38 | 45.03 | 18 | 29 | 0.003589 | 26047.56 | 6.117681 | 1704.758 |
| 8 | 84 | 41.16 | 20.16 | 53.76 | 27 | 34 | 0.003595 | 25929.03 | 6.440299 | 1791.533 |
| 9 | 83 | 40.67 | 19.92 | 51.46 | 27 | 34 | 0.003668 | 25940.07 | 6.533517 | 1781.292 |
| 10 | 81 | 38.07 | 18.63 | 48.60 | 24 | 32 | 0.003667 | 25986.87 | 6.394807 | 1743.703 |

*h=350 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 74 | 27.38 | 13.32 | 68.82 | 22 | 27 | 0.004033 | 27156.69 | 4.687849 | 1162.305 |
| 2 | **76** | **31.92** | **15.20** | **74.48** | **31** | **32** | **0.004108** | **27258.13** | **4.923711** | **1198.458** |
| 3 | 76 | 31.92 | 15.20 | 74.48 | 31 | 32 | 0.004108 | 27258.13 | 4.923711 | 1198.458 |
| 4 | 76 | 31.92 | 15.20 | 74.48 | 31 | 32 | 0.004108 | 27258.13 | 4.923711 | 1198.458 |
| 5 | 73 | 24.82 | 11.68 | 64.24 | 15 | 24 | 0.003917 | 26719.88 | 4.477622 | 1143.243 |
| 6 | 76 | 31.92 | 15.20 | 74.48 | 31 | 32 | 0.004108 | 27258.13 | 4.923711 | 1198.458 |
| 7 | 75 | 28.50 | 13.50 | 69.00 | 24 | 28 | 0.004080 | 27218.73 | 4.780477 | 1171.611 |
| 8 | 76 | 31.92 | 15.20 | 74.48 | 31 | 32 | 0.004108 | 27258.13 | 4.923711 | 1198.458 |
| 9 | 75 | 28.50 | 13.50 | 69.00 | 24 | 28 | 0.004080 | 27218.73 | 4.780477 | 1171.611 |
| 10 | 77 | 30.80 | 14.63 | 72.38 | 27 | 30 | 0.004072 | 27258.51 | 4.865644 | 1194.875 |

# APPENDIX B

*h=350 mm, L=7500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 77 | 30.80 | 14.63 | 72.38 | 27 | 30 | 0.002755 | 29068.61 | 4.920998 | 1786.456 |
| 2 | **77** | **31.57** | **15.40** | **73.15** | **29** | **31** | **0.002762** | **29088.71** | **4.960405** | **1796.250** |
| 3 | 77 | 31.57 | 15.40 | 73.15 | 29 | 31 | 0.002762 | 29088.71 | 4.960405 | 1796.250 |
| 4 | 76 | 30.40 | 14.44 | 72.96 | 28 | 30 | 0.002734 | 29009.74 | 4.853568 | 1775.264 |
| 5 | 77 | 30.80 | 14.63 | 72.38 | 27 | 30 | 0.002755 | 29068.61 | 4.920998 | 1786.456 |
| 6 | 77 | 30.80 | 14.63 | 72.38 | 27 | 30 | 0.002755 | 29068.61 | 4.920998 | 1786.456 |
| 7 | 77 | 29.26 | 13.86 | 70.84 | 23 | 28 | 0.002724 | 29030.42 | 4.825316 | 1771.662 |
| 8 | 77 | 31.57 | 15.40 | 73.15 | 29 | 31 | 0.002762 | 29088.71 | 4.960405 | 1796.250 |
| 9 | 77 | 30.80 | 14.63 | 72.38 | 28 | 30 | 0.002749 | 29077.83 | 4.906695 | 1784.677 |
| 10 | 77 | 31.57 | 15.40 | 73.15 | 29 | 31 | 0.002762 | 29088.71 | 4.960405 | 1796.250 |

*h=400 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 70 | 24.50 | 11.90 | 35.00 | 36 | 20 | 0.003486 | 29141.57 | 4.178717 | 1198.771 |
| 2 | 71 | 19.88 | 9.23 | 58.22 | 39 | 18 | 0.003565 | 28968.39 | 4.169073 | 1169.387 |
| 3 | 75 | 20.25 | 9.75 | 60.75 | 37 | 17 | 0.003477 | 28821.73 | 4.129624 | 1187.636 |
| 4 | 70 | 24.50 | 11.90 | 35.00 | 36 | 20 | 0.003486 | 29141.57 | 4.178717 | 1198.771 |
| 5 | 75 | 19.50 | 9 | 60.00 | 36 | 16 | 0.003395 | 28588.23 | 4.008165 | 1180.502 |
| 6 | 73 | 20.44 | 9.49 | 59.86 | 39 | 18 | 0.003566 | 29001.89 | 4.209878 | 1180.441 |
| 7 | **73** | **20.44** | **9.49** | **59.86** | **39** | **18** | **0.003566** | **29001.89** | **4.209878** | **1180.441** |
| 8 | 73 | 20.44 | 9.49 | 59.86 | 39 | 18 | 0.003566 | 29001.89 | 4.209878 | 1180.441 |
| 9 | 75 | 20.25 | 9.75 | 60.75 | 37 | 17 | 0.003477 | 28821.73 | 4.129624 | 1187.636 |
| 10 | 73 | 20.44 | 9.49 | 59.86 | 39 | 18 | 0.003566 | 29001.89 | 4.209878 | 1180.441 |

# APPENDIX B

*h=400 mm, L=8000 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 70 | 24.50 | 11.90 | 35.00 | 36 | 20 | 0.002022 | 31838.95 | 3.625143 | 1792.949 |
| 2 | 71 | 19.88 | 9.23 | 58.22 | 39 | 18 | 0.002117 | 31319.06 | 3.702551 | 1748.643 |
| 3 | **77** | **21.56** | **10.01** | **63.14** | **40** | **18** | **0.002203** | **31714.06** | **3.961724** | **1798.216** |
| 4 | 81 | 19.44 | 8.91 | 63.18 | 31 | 14 | 0.002102 | 31308.43 | 3.778921 | 1797.806 |
| 5 | 79 | 19.75 | 9.48 | 62.41 | 33 | 15 | 0.002119 | 31409.69 | 3.797846 | 1792.558 |
| 6 | 75 | 21.00 | 9.75 | 61.50 | 39 | 18 | 0.002181 | 31587.80 | 3.885999 | 1782.032 |
| 7 | 77 | 21.56 | 10.01 | 63.14 | 40 | 18 | 0.002203 | 31714.06 | 3.961724 | 1798.216 |
| 8 | 77 | 21.56 | 10.01 | 63.14 | 40 | 18 | 0.002203 | 31714.06 | 3.961724 | 1798.216 |
| 9 | 76 | 21.28 | 9.88 | 63.84 | 39 | 18 | 0.002188 | 31644.15 | 3.916920 | 1790.379 |
| 10 | 77 | 21.56 | 10.01 | 63.14 | 40 | 18 | 0.002203 | 31714.06 | 3.961724 | 1798.216 |

# APPENDIX C

# Results for Σ-Shape Purlin with $d_2$ and $h_2$ fixed

## Choice 1 (free optimization)

***Case 1*** $d_s = 32\,mm,\ h_2 = 25\,mm,\ h_1 \geq 45\,mm$

*h=250 mm, L=4000 mm and t=1.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **82** | **30.34** | **45.1** | **0.007343** | **17786.29** | **5.348977** | **728.4802** |
| 2 | 82 | 31.16 | 45.1 | 0.007324 | 17814.10 | 5.353052 | 730.8746 |
| 3 | 82 | 29.52 | 45.92 | 0.007312 | 17737.72 | 5.309157 | 726.0858 |
| 4 | 82 | 29.52 | 45.92 | 0.007312 | 17737.72 | 5.309157 | 726.0858 |
| 5 | 84 | 21.84 | 49.56 | 0.007045 | 17089.73 | 4.998658 | 709.5002 |
| 6 | 83 | 24.90 | 48.14 | 0.007197 | 17440.19 | 5.149737 | 715.5154 |
| 7 | 82 | 30.34 | 45.92 | 0.007283 | 17771.84 | 5.305602 | 728.4802 |
| 8 | 82 | 30.34 | 45.1 | 0.007343 | 17786.29 | 5.348977 | 728.4802 |
| 9 | 83 | 25.73 | 47.31 | 0.007235 | 17495.28 | 5.194086 | 717.9390 |
| 10 | 82 | 27.88 | 45.92 | 0.007213 | 17663.00 | 5.203012 | 721.2970 |

*h=250 mm, L=5500 mm and t=2.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 82 | 31.16 | 45.92 | 0.004697 | 21566.54 | 5.729378 | 1219.786 |
| 2 | 83 | 27.39 | 48.14 | 0.004606 | 21206.85 | 5.555949 | 1206.157 |
| 3 | 82 | 33.62 | 45.10 | 0.004691 | 21709.40 | 5.778872 | 1231.889 |
| 4 | 82 | 32.80 | 45.10 | 0.004707 | 21671.51 | 5.779499 | 1227.854 |
| 5 | 84 | 23.52 | 49.56 | 0.004524 | 20696.21 | 5.392686 | 1192.037 |
| 6 | 83 | 28.22 | 47.31 | 0.004638 | 21296.96 | 5.612686 | 1210.241 |
| 7 | 83 | 29.88 | 47.31 | 0.004616 | 21427.56 | 5.623671 | 1218.408 |
| 8 | 82 | 32.80 | 45.10 | 0.004707 | 21671.51 | 5.779499 | 1227.854 |
| 9 | 82 | 32.80 | 45.10 | 0.004707 | 21671.51 | 5.779499 | 1227.854 |
| 10 | **82** | **31.98** | **45.10** | **0.004713** | **21629.42** | **5.768460** | **1223.820** |

# APPENDIX C

*h=300 mm, L=5000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **92** | **34.96** | **46.00** | **0.005946** | **22941.95** | **6.710315** | **1128.614** |
| 2 | 92 | 34.04 | 47.84 | 0.005913 | 22892.95 | 6.652094 | 1125.008 |
| 3 | 92 | 34.96 | 46.00 | 0.005946 | 22941.95 | 6.710315 | 1128.614 |
| 4 | 92 | 34.96 | 46.00 | 0.005946 | 22941.95 | 6.710315 | 1128.614 |
| 5 | 93 | 29.76 | 49.29 | 0.005873 | 22571.34 | 6.531582 | 1112.150 |
| 6 | 80 | 43.20 | 55.20 | 0.005842 | 23632.83 | 6.507086 | 1113.875 |
| 7 | 93 | 29.76 | 49.29 | 0.005873 | 22571.34 | 6.531582 | 1112.150 |
| 8 | 80 | 43.20 | 55.20 | 0.005842 | 23632.83 | 6.507086 | 1113.875 |
| 9 | 92 | 34.96 | 46.00 | 0.005946 | 22941.95 | 6.710315 | 1128.614 |
| 10 | 92 | 36.80 | 46.00 | 0.005916 | 23009.74 | 6.719811 | 1135.827 |

*h=300 mm, L=6500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **92** | **36.80** | **46.00** | **0.003983** | **26201.57** | **6.775930** | **1701.267** |
| 2 | 92 | 34.96 | 47.84 | 0.003961 | 26061.78 | 6.695544 | 1690.374 |
| 3 | 92 | 36.80 | 46.00 | 0.003983 | 26201.57 | 6.775930 | 1701.267 |
| 4 | 92 | 36.80 | 46.00 | 0.003983 | 26201.57 | 6.775930 | 1701.267 |
| 5 | 92 | 35.88 | 47.84 | 0.003949 | 26115.45 | 6.697014 | 1695.820 |
| 6 | 93 | 31.62 | 49.29 | 0.003943 | 25794.92 | 6.611212 | 1676.521 |
| 7 | 92 | 36.80 | 46.00 | 0.003983 | 26201.57 | 6.775930 | 1701.267 |
| 8 | 92 | 38.64 | 46.00 | 0.003957 | 26284.92 | 6.774268 | 1712.159 |
| 9 | 92 | 36.80 | 46.00 | 0.003983 | 26201.57 | 6.775930 | 1701.267 |
| 10 | 92 | 35.88 | 46.00 | 0.003965 | 26153.02 | 6.724748 | 1695.820 |

*h=350 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 84 | 35.28 | 72.24 | 0.004210 | 26331.27 | 5.037382 | 1196.508 |
| 2 | 84 | 35.28 | 72.24 | 0.004210 | 26331.27 | 5.037382 | 1196.508 |
| 3 | 84 | 36.12 | 70.56 | 0.004197 | 26402.75 | 5.035382 | 1199.801 |
| 4 | 85 | 31.45 | 76.50 | 0.004157 | 25815.40 | 4.927804 | 1185.415 |
| 5 | **84** | **37.8** | **70.56** | **0.004248** | **26458.07** | **5.124950** | **1206.387** |
| 6 | 84 | 35.28 | 72.24 | 0.004210 | 26331.27 | 5.037382 | 1196.508 |
| 7 | 85 | 32.30 | 74.80 | 0.004210 | 25976.09 | 5.004405 | 1188.747 |
| 8 | 86 | 27.52 | 78.26 | 0.004093 | 25237.84 | 4.805309 | 1173.929 |
| 9 | 84 | 37.80 | 70.56 | 0.004248 | 26458.07 | 5.124950 | 1206.387 |
| 10 | 96 | 24.96 | 64.32 | 0.004111 | 24512.76 | 4.946397 | 1203.094 |

*h=350 mm, L=7500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **97** | **24.25** | **66.93** | **0.003056** | **27787.72** | **5.514900** | **1804.571** |
| 2 | 97 | 24.25 | 66.93 | 0.003056 | 27787.72 | 5.514900 | 1804.571 |
| 3 | 86 | 29.24 | 78.26 | 0.002940 | 28465.52 | 5.200820 | 1768.991 |
| 4 | 96 | 24.96 | 62.40 | 0.002988 | 28042.53 | 5.387775 | 1802.854 |
| 5 | 85 | 33.15 | 76.50 | 0.002955 | 28849.48 | 5.278335 | 1786.219 |
| 6 | 85 | 32.30 | 76.50 | 0.002960 | 28778.79 | 5.272010 | 1781.187 |
| 7 | 85 | 35.70 | 74.80 | 0.002968 | 29057.38 | 5.347043 | 1801.315 |
| 8 | 86 | 29.24 | 78.26 | 0.002940 | 28465.52 | 5.200820 | 1768.991 |
| 9 | 85 | 34.00 | 74.80 | 0.002980 | 28939.40 | 5.338236 | 1791.251 |
| 10 | 96 | 24.96 | 64.32 | 0.003015 | 28009.89 | 5.434750 | 1802.854 |

*h=400 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 70 | 26.60 | 64.40 | 0.002720 | 28905.37 | 3.279124 | 1205.603 |
| 2 | **71** | **25.56** | **66.74** | **0.002769** | **28720.87** | **3.33744** | **1205.446** |
| 3 | 71 | 23.43 | 68.16 | 0.002705 | 28376.11 | 3.237665 | 1197.096 |
| 4 | 70 | 26.60 | 64.40 | 0.002720 | 28905.37 | 3.279124 | 1205.603 |
| 5 | 70 | 26.60 | 63.00 | 0.002696 | 28883.84 | 3.249903 | 1205.603 |
| 6 | 71 | 24.14 | 68.16 | 0.002734 | 28500.91 | 3.279942 | 1199.880 |
| 7 | 69 | 26.22 | 60.72 | 0.002593 | 28841.83 | 3.111544 | 1200.193 |
| 8 | 70 | 24.50 | 64.40 | 0.002667 | 28683.75 | 3.201110 | 1200.115 |
| 9 | 70 | 26.60 | 64.40 | 0.002720 | 28905.37 | 3.279124 | 1205.603 |
| 10 | 71 | 24.85 | 66.03 | 0.002729 | 28592.13 | 3.282367 | 1202.663 |

*h=400 mm, L=8000 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 80 | 16.80 | 53.60 | 0.001699 | 29596.98 | 3.072130 | 1807.827 |
| 2 | **71** | **25.56** | **66.74** | **0.001733** | **31143.81** | **3.130396** | **1806.406** |
| 3 | 71 | 24.14 | 68.16 | 0.001717 | 30929.98 | 3.086889 | 1797.999 |
| 4 | 71 | 25.56 | 66.74 | 0.001733 | 31143.81 | 3.130396 | 1806.406 |
| 5 | 71 | 19.17 | 66.03 | 0.001596 | 30112.47 | 2.823360 | 1768.577 |
| 6 | 71 | 24.85 | 66.03 | 0.001714 | 31066.71 | 3.089611 | 1802.203 |
| 7 | 69 | 26.91 | 59.34 | 0.001619 | 31326.10 | 2.919056 | 1802.558 |
| 8 | 70 | 24.50 | 64.40 | 0.001672 | 31112.45 | 3.006640 | 1798.355 |
| 9 | 70 | 26.60 | 63.00 | 0.001684 | 31304.41 | 3.043060 | 1806.643 |
| 10 | 71 | 24.85 | 66.03 | 0.001714 | 31066.71 | 3.089611 | 1802.203 |

# APPENDIX C

**Case 2** $d_s = 20\,mm,\ h_2 = 36\,mm,\ h_1 \geq 40\,mm$

*h=400 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 77 | 29.26 | 44.66 | 0.003879 | 28919.23 | 4.669192 | 1203.655 |
| 2 | 77 | 29.26 | 44.66 | 0.003879 | 28919.23 | 4.669192 | 1203.655 |
| 3 | 77 | 30.03 | 43.12 | 0.003879 | 28980.82 | 4.681020 | 1206.673 |
| 4 | 78 | 28.86 | 46.80 | 0.003874 | 28858.29 | 4.671474 | 1206.007 |
| 5 | **77** | **30.03** | **44.66** | **0.003881** | **29029.81** | **4.682865** | **1206.673** |
| 6 | 77 | 30.03 | 44.66 | 0.003881 | 29029.81 | 4.682865 | 1206.673 |
| 7 | 77 | 30.03 | 43.12 | 0.003879 | 28980.82 | 4.681020 | 1206.673 |
| 8 | 76 | 31.16 | 41.04 | 0.003875 | 28806.02 | 4.643660 | 1198.245 |
| 9 | 77 | 30.03 | 43.12 | 0.003879 | 28980.82 | 4.681020 | 1206.673 |
| 10 | 78 | 28.86 | 48.36 | 0.003872 | 28904.83 | 4.670140 | 1206.007 |

*h=400 mm, L=8000 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 80 | 26.4 | 53.60 | 0.002398 | 31800.14 | 4.329013 | 1805.340 |
| 2 | 79 | 27.65 | 52.14 | 0.002393 | 31972.20 | 4.324488 | 1806.820 |
| 3 | 79 | 27.65 | 49.77 | 0.002398 | 31972.39 | 4.332544 | 1806.820 |
| 4 | 79 | 27.65 | 52.14 | 0.002393 | 31972.20 | 4.324488 | 1806.820 |
| 5 | 79 | 27.65 | 49.77 | 0.002398 | 31972.39 | 4.332544 | 1806.820 |
| 6 | **80** | **27.20** | **53.60** | **0.002400** | **31915.50** | **4.344432** | **1810.076** |
| 7 | 80 | 27.20 | 53.60 | 0.002400 | 31915.50 | 4.344432 | 1810.076 |
| 8 | 78 | 28.86 | 48.36 | 0.002387 | 32086.94 | 4.315609 | 1808.063 |
| 9 | 78 | 27.30 | 46.02 | 0.002398 | 31972.39 | 4.332544 | 1806.820 |
| 10 | 79 | 26.07 | 49.77 | 0.002394 | 31749.60 | 4.302870 | 1797.466 |

**Case 3** $d_s = 20mm,\ h_2 = 25mm,\ h_1 = 50mm$

$h = 400mm, L = 6000mm, t = 2.0mm$

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 76 | 28.88 | 50.00 | 0.003839 | 28677.33 | 4.626483 | 1205.116 |
| 2 | **75** | **30.00** | **50.00** | **0.003843** | **28902.46** | **4.633455** | **1205.586** |
| 3 | 80 | 21.60 | 50.00 | 0.003766 | 27119.82 | 4.489689 | 1192.258 |
| 4 | 75 | 28.50 | 50.00 | 0.003840 | 28681.96 | 4.606492 | 1199.706 |
| 5 | 77 | 27.72 | 50.00 | 0.003831 | 28430.73 | 4.614547 | 1204.489 |
| 6 | 80 | 20.80 | 50.00 | 0.003758 | 26961.98 | 4.468304 | 1189.122 |
| 7 | 73 | 30.66 | 50.00 | 0.003839 | 29082.74 | 4.607976 | 1200.334 |
| 8 | 76 | 26.60 | 50.00 | 0.003829 | 28312.81 | 4.580606 | 1196.178 |
| 9 | 80 | 24.80 | 50.00 | 0.003794 | 27721.58 | 4.571194 | 1204.802 |
| 10 | 75 | 30.00 | 50.00 | 0.003843 | 28902.46 | 4.633455 | 1205.586 |

$h = 400mm, L = 8000mm, t = 3.0mm$

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 84 | 21.00 | 50.00 | 0.002390 | 30589.32 | 4.319252 | 1807.186 |
| 2 | 81 | 24.30 | 50.00 | 0.002389 | 31269.58 | 4.321483 | 1808.962 |
| 3 | 84 | 21.00 | 50.00 | 0.002390 | 30589.32 | 4.319252 | 1807.186 |
| 4 | 79 | 26.07 | 50.00 | 0.002373 | 31543.55 | 4.289957 | 1807.600 |
| 5 | 82 | 22.96 | 50.00 | 0.002390 | 31018.36 | 4.318997 | 1806.949 |
| 6 | **85** | **20.40** | **50.00** | **0.002391** | **30440.39** | **4.326330** | **1809.554** |
| 7 | 83 | 21.58 | 50.00 | 0.002388 | 30726.92 | 4.309165 | 1804.700 |
| 8 | 85 | 20.40 | 50.00 | 0.002391 | 30440.39 | 4.326330 | 1809.554 |
| 9 | 81 | 23.49 | 50.00 | 0.002384 | 31121.00 | 4.300988 | 1804.167 |
| 10 | 75 | 29.25 | 50.00 | 0.002316 | 31855.05 | 4.174639 | 1802.746 |

## Choice 2 (40/13…100/35)

### *Case 1* $d_s = 32\,mm,\ h_2 = 25\,mm,\ h_1 \geq 45\,mm$

*h=250 mm, L=4000 mm and t=1.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **82** | **28.40** | **45.92** | **0.007251** | **17677.34** | **5.240819** | **722.8154** |
| 2 | 82 | 28.40 | 45.92 | 0.007251 | 17677.34 | 5.240819 | 722.8154 |
| 3 | 82 | 28.40 | 45.92 | 0.007251 | 17677.34 | 5.240819 | 722.8154 |
| 4 | 82 | 28.40 | 45.92 | 0.007251 | 17677.34 | 5.240819 | 722.8154 |
| 5 | 83 | 28.77 | 47.31 | 0.007190 | 17691.89 | 5.197209 | 722.8154 |
| 6 | 82 | 28.40 | 45.92 | 0.007251 | 17677.34 | 5.240819 | 722.8154 |
| 7 | 82 | 28.40 | 45.92 | 0.007251 | 17677.34 | 5.240819 | 722.8154 |
| 8 | 82 | 28.40 | 45.92 | 0.007251 | 17677.34 | 5.240819 | 722.8154 |
| 9 | 84 | 29.13 | 49.56 | 0.006853 | 17512.22 | 5.008504 | 730.7967 |
| 10 | 82 | 28.40 | 45.92 | 0.007251 | 17677.34 | 5.240819 | 722.8154 |

*h=250 mm, L=5500 mm and t=2.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **83** | **28.77** | **47.31** | **0.004631** | **21342.53** | **5.616745** | **1212.930** |
| 2 | 84 | 29.13 | 49.56 | 0.004485 | 21296.09 | 5.470710 | 1219.654 |
| 3 | 84 | 29.13 | 49.56 | 0.004485 | 21296.09 | 5.470710 | 1219.654 |
| 4 | 82 | 28.40 | 45.92 | 0.004585 | 21372.90 | 5.530148 | 1206.206 |
| 5 | 83 | 28.77 | 48.14 | 0.004590 | 21326.75 | 5.567772 | 1212.930 |
| 6 | 84 | 29.13 | 49.56 | 0.004485 | 21296.09 | 5.470710 | 1219.654 |
| 7 | 83 | 28.77 | 47.31 | 0.004631 | 21342.53 | 5.616745 | 1212.930 |
| 8 | 84 | 29.13 | 50.4 | 0.004485 | 21296.09 | 5.470710 | 1219.654 |
| 9 | 83 | 28.77 | 47.31 | 0.004631 | 21342.53 | 5.616745 | 1212.930 |
| 10 | 83 | 28.77 | 47.31 | 0.004631 | 21342.53 | 5.616745 | 1212.930 |

*h=300 mm, L=5000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **92** | **32.07** | **47.84** | **0.005870** | **22759.06** | **6.558143** | **1117.272** |
| 2 | 93 | 32.43 | 49.29 | 0.005824 | 22703.06 | 6.538021 | 1122.629 |
| 3 | 92 | 32.07 | 47.84 | 0.005870 | 22759.06 | 6.558143 | 1117.272 |
| 4 | 92 | 32.07 | 47.84 | 0.005870 | 22759.06 | 6.558143 | 1117.272 |
| 5 | 92 | 32.07 | 47.84 | 0.005870 | 22759.06 | 6.558143 | 1117.272 |
| 6 | 82 | 28.40 | 63.96 | 0.005726 | 22968.91 | 6.091139 | 1063.699 |
| 7 | 92 | 32.07 | 47.84 | 0.005870 | 22759.06 | 6.558143 | 1117.272 |
| 8 | 92 | 32.07 | 47.84 | 0.005870 | 22759.06 | 6.558143 | 1117.272 |
| 9 | 92 | 32.07 | 47.84 | 0.005870 | 22759.06 | 6.558143 | 1117.272 |
| 10 | 93 | 32.43 | 49.29 | 0.005824 | 22703.06 | 6.538021 | 1122.629 |

*h=300 mm, L=6500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 92 | 32.07 | 47.84 | 0.003887 | 25856.33 | 6.503212 | 1673.245 |
| 2 | **93** | **32.43** | **49.29** | **0.003936** | **25863.51** | **6.616924** | **1681.336** |
| 3 | 93 | 32.43 | 51.15 | 0.003886 | 25827.36 | 6.533428 | 1681.336 |
| 4 | 92 | 32.07 | 45.08 | 0.003818 | 25913.70 | 6.387904 | 1673.245 |
| 5 | 93 | 32.43 | 49.29 | 0.003936 | 25863.51 | 6.616924 | 1681.336 |
| 6 | 83 | 28.77 | 65.57 | 0.003628 | 25157.38 | 5.806381 | 1600.429 |
| 7 | 93 | 32.43 | 49.29 | 0.003936 | 25863.51 | 6.616924 | 1681.336 |
| 8 | 92 | 32.07 | 47.84 | 0.003887 | 25856.33 | 6.503212 | 1673.245 |
| 9 | 93 | 32.43 | 49.29 | 0.003936 | 25863.51 | 6.616924 | 1681.336 |
| 10 | 93 | 32.43 | 49.29 | 0.003936 | 25863.51 | 6.616924 | 1681.336 |

*h=350 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 86 | 29.87 | 78.26 | 0.004076 | 25480.00 | 4.822036 | 1183.128 |
| 2 | **85** | **29.50** | **76.50** | **0.004123** | **25627.49** | **4.855876** | **1177.771** |
| 3 | 85 | 29.50 | 76.50 | 0.004123 | 25627.49 | 4.855876 | 1177.771 |
| 4 | 85 | 29.50 | 76.50 | 0.004123 | 25627.49 | 4.855876 | 1177.771 |
| 5 | 85 | 29.50 | 76.50 | 0.004123 | 25627.49 | 4.855876 | 1177.771 |
| 6 | 85 | 29.50 | 76.50 | 0.004123 | 25627.49 | 4.855876 | 1177.771 |
| 7 | 85 | 29.50 | 76.50 | 0.004123 | 25627.49 | 4.855876 | 1177.771 |
| 8 | 86 | 29.87 | 78.26 | 0.004076 | 25480.00 | 4.822036 | 1183.128 |
| 9 | 85 | 29.50 | 76.50 | 0.004123 | 25627.49 | 4.855876 | 1177.771 |
| 10 | 85 | 29.50 | 75.65 | 0.004107 | 25678.69 | 4.836639 | 1177.771 |

*h=350 mm, L=7500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **86** | **29.87** | **78.26** | **0.002938** | **28535.15** | **5.208493** | **1772.701** |
| 2 | 86 | 29.87 | 78.26 | 0.002938 | 28535.15 | 5.208493 | 1772.701 |
| 3 | 85 | 29.50 | 76.5 | 0.002900 | 28506.94 | 5.116889 | 1764.611 |
| 4 | 86 | 29.87 | 78.26 | 0.002938 | 28535.15 | 5.208493 | 1772.701 |
| 5 | 86 | 29.87 | 78.26 | 0.002938 | 28535.15 | 5.208493 | 1772.701 |
| 6 | 86 | 29.87 | 78.26 | 0.002938 | 28535.15 | 5.208493 | 1772.701 |
| 7 | 86 | 29.87 | 79.12 | 0.002923 | 28523.52 | 5.180908 | 1772.701 |
| 8 | 86 | 29.87 | 78.26 | 0.002938 | 28535.15 | 5.208493 | 1772.701 |
| 9 | 86 | 29.87 | 78.26 | 0.002938 | 28535.15 | 5.208493 | 1772.701 |
| 10 | 86 | 29.87 | 78.26 | 0.002938 | 28535.15 | 5.208493 | 1772.701 |

*h=400 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **71** | **24.37** | **68.16** | **0.002743** | **28540.75** | **3.293440** | **1200.768** |
| 2 | 71 | 24.37 | 68.16 | 0.002743 | 28540.75 | 3.293440 | 1200.768 |
| 3 | 71 | 24.37 | 68.16 | 0.002743 | 28540.75 | 3.293440 | 1200.768 |
| 4 | 71 | 24.37 | 68.16 | 0.002743 | 28540.75 | 3.293440 | 1200.768 |
| 5 | 71 | 24.37 | 68.16 | 0.002743 | 28540.75 | 3.293440 | 1200.768 |
| 6 | 71 | 24.37 | 68.16 | 0.002743 | 28540.75 | 3.293440 | 1200.768 |
| 7 | 71 | 24.37 | 68.16 | 0.002743 | 28540.75 | 3.293440 | 1200.768 |
| 8 | 71 | 24.37 | 68.16 | 0.002743 | 28540.75 | 3.293440 | 1200.768 |
| 9 | 71 | 24.37 | 68.16 | 0.002743 | 28540.75 | 3.293440 | 1200.768 |
| 10 | 70 | 24.00 | 64.40 | 0.002621 | 28481.46 | 3.133216 | 1195.411 |

*h=400 mm, L=8000 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **71** | **24.37** | **68.16** | **0.001721** | **30961.31** | **3.097019** | **1799.341** |
| 2 | 71 | 24.37 | 68.16 | 0.001721 | 30961.31 | 3.097019 | 1799.341 |
| 3 | 71 | 24.37 | 68.16 | 0.001721 | 30961.31 | 3.097019 | 1799.341 |
| 4 | 71 | 24.37 | 68.16 | 0.001721 | 30961.31 | 3.097019 | 1799.341 |
| 5 | 71 | 24.37 | 68.16 | 0.001721 | 30961.31 | 3.097019 | 1799.341 |
| 6 | 71 | 24.37 | 68.16 | 0.001721 | 30961.31 | 3.097019 | 1799.341 |
| 7 | 71 | 24.37 | 68.16 | 0.001721 | 30961.31 | 3.097019 | 1799.341 |
| 8 | 71 | 24.37 | 68.16 | 0.001721 | 30961.31 | 3.097019 | 1799.341 |
| 9 | 71 | 24.37 | 68.16 | 0.001721 | 30961.31 | 3.097019 | 1799.341 |
| 10 | 71 | 24.37 | 68.16 | 0.001721 | 30961.31 | 3.097019 | 1799.341 |

**Case 2** $d_s = 20\,mm,\ h_2 = 36\,mm,\ h_1 \geq 40\,mm$

*h=400 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 78 | 26.93 | 46.02 | 0.003865 | 28524.68 | 4.632127 | 1198.454 |
| 2 | **77** | **26.57** | **44.66** | **0.003868** | **28488.79** | **4.615393** | **1193.097** |
| 3 | 78 | 26.93 | 46.02 | 0.003865 | 28524.68 | 4.632127 | 1198.454 |
| 4 | 77 | 26.57 | 44.66 | 0.003868 | 28488.79 | 4.615393 | 1193.097 |
| 5 | 78 | 26.93 | 46.02 | 0.003865 | 28524.68 | 4.632127 | 1198.454 |
| 6 | 77 | 26.57 | 44.66 | 0.003868 | 28488.79 | 4.615393 | 1193.097 |
| 7 | 77 | 26.57 | 44.66 | 0.003868 | 28488.79 | 4.615393 | 1193.097 |
| 8 | 77 | 26.57 | 44.66 | 0.003868 | 28488.79 | 4.615393 | 1193.097 |
| 9 | 78 | 26.93 | 46.02 | 0.003865 | 28524.68 | 4.632127 | 1198.454 |
| 10 | 71 | 24.37 | 67.45 | 0.003776 | 29072.69 | 4.383865 | 1160.953 |

*h=400 mm, L=8000 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **79** | **27.30** | **49.77** | **0.002397** | **31924.56** | **4.326179** | **1804.748** |
| 2 | 79 | 27.30 | 49.77 | 0.002397 | 31924.56 | 4.326179 | 1804.748 |
| 3 | 79 | 27.30 | 49.77 | 0.002397 | 31924.56 | 4.326179 | 1804.748 |
| 4 | 79 | 27.30 | 49.77 | 0.002397 | 31924.56 | 4.326179 | 1804.748 |
| 5 | 79 | 27.30 | 49.77 | 0.002397 | 31924.56 | 4.326179 | 1804.748 |
| 6 | 79 | 27.30 | 50.56 | 0.002396 | 31926.66 | 4.323995 | 1804.748 |
| 7 | 79 | 27.30 | 50.56 | 0.002396 | 31926.66 | 4.323995 | 1804.748 |
| 8 | 79 | 27.30 | 49.77 | 0.002397 | 31924.56 | 4.326179 | 1804.748 |
| 9 | 79 | 27.30 | 49.77 | 0.002397 | 31924.56 | 4.326179 | 1804.748 |
| 10 | 71 | 24.37 | 67.45 | 0.002200 | 31240.22 | 3.828707 | 1740.023 |

**Case 3** $d_s = 20mm,\ h_2 = 25mm,\ h_1 = 50mm$

$h = 400mm, L = 6000mm, t = 2.0mm$

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 78 | 26.93 | 50.00 | 0.003822 | 28234.07 | 4.607084 | 1205.325 |
| 2 | **76** | **26.20** | **50.00** | **0.003827** | **28245.20** | **4.572064** | **1194.610** |
| 3 | 76 | 26.20 | 50.00 | 0.003827 | 28245.20 | 4.572064 | 1194.610 |
| 4 | 76 | 26.20 | 50.00 | 0.003827 | 28245.20 | 4.572064 | 1194.610 |
| 5 | 76 | 26.20 | 50.00 | 0.003827 | 28245.20 | 4.572064 | 1194.610 |
| 6 | 76 | 26.20 | 50.00 | 0.003827 | 28245.20 | 4.572064 | 1194.610 |
| 7 | 76 | 26.20 | 50.00 | 0.003827 | 28245.20 | 4.572064 | 1194.610 |
| 8 | 76 | 26.20 | 50.00 | 0.003827 | 28245.20 | 4.572064 | 1194.610 |
| 9 | 76 | 26.20 | 50.00 | 0.003827 | 28245.20 | 4.572064 | 1194.610 |
| 10 | 71 | 24.37 | 50.00 | 0.003411 | 28243.61 | 3.983170 | 1167.824 |

$h = 400mm, L = 8000mm, t = 3.0mm$

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **78** | **26.93** | **50.00** | **0.002362** | **31649.54** | **4.267478** | **1806.791** |
| 2 | 78 | 26.93 | 50.00 | 0.002362 | 31649.54 | 4.267478 | 1806.791 |
| 3 | 78 | 26.93 | 50.00 | 0.002362 | 31649.54 | 4.267478 | 1806.791 |
| 4 | 78 | 26.93 | 50.00 | 0.002362 | 31649.54 | 4.267478 | 1806.791 |
| 5 | 78 | 26.93 | 50.00 | 0.002362 | 31649.54 | 4.267478 | 1806.791 |
| 6 | 77 | 26.57 | 50.00 | 0.002346 | 31583.43 | 4.220068 | 1798.701 |
| 7 | 77 | 26.57 | 50.00 | 0.002346 | 31583.43 | 4.220068 | 1798.701 |
| 8 | 78 | 26.93 | 50.00 | 0.002362 | 31649.54 | 4.267478 | 1806.791 |
| 9 | 78 | 26.93 | 50.00 | 0.002362 | 31649.54 | 4.267478 | 1806.791 |
| 10 | 71 | 24.37 | 50.00 | 0.002029 | 31140.72 | 3.550959 | 1750.157 |

## Choice 3 (40/15…100/45)

### *Case 1* $d_s = 32\ mm,\ h_2 = 25\ mm,\ h_1 \geq 45\ mm$

*h=250 mm, L=4000 mm and t=1.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **82** | **36.00** | **45.10** | **0.007132** | **17895.76** | **5.313072** | **745.0074** |
| 2 | 82 | 36.00 | 45.10 | 0.007132 | 17895.76 | 5.313072 | 745.0074 |
| 3 | 82 | 36.00 | 45.10 | 0.007132 | 17895.76 | 5.313072 | 745.0074 |
| 4 | 82 | 36.00 | 45.10 | 0.007132 | 17895.76 | 5.313072 | 745.0074 |
| 5 | 83 | 36.50 | 47.31 | 0.007132 | 17895.76 | 5.313072 | 745.0074 |
| 6 | 82 | 36.00 | 45.10 | 0.007132 | 17895.76 | 5.313072 | 745.0074 |
| 7 | 82 | 36.00 | 45.10 | 0.007132 | 17895.76 | 5.313072 | 745.0074 |
| 8 | 82 | 36.00 | 45.10 | 0.007132 | 17895.76 | 5.313072 | 745.0074 |
| 9 | 84 | 37.00 | 49.56 | 0.006637 | 17703.69 | 5.003036 | 753.7674 |
| 10 | 82 | 36.00 | 45.10 | 0.007132 | 17895.76 | 5.313072 | 745.0074 |

*h=250 mm, L=5500 mm and t=2.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **82** | **36.00** | **45.10** | **0.004640** | **21783.30** | **5.770677** | **1243.598** |
| 2 | 84 | 37.00 | 49.56 | 0.004370 | 21669.85 | 5.498469 | 1258.358 |
| 3 | 84 | 37.00 | 49.56 | 0.004370 | 21669.85 | 5.498469 | 1258.358 |
| 4 | 82 | 36.00 | 45.10 | 0.004640 | 21783.30 | 5.770677 | 1243.598 |
| 5 | 82 | 36.00 | 45.92 | 0.004603 | 21767.23 | 5.723891 | 1243.598 |
| 6 | 84 | 37.00 | 49.56 | 0.004370 | 21669.85 | 5.498469 | 1258.358 |
| 7 | 82 | 36.00 | 45.10 | 0.004640 | 21783.30 | 5.770677 | 1243.598 |
| 8 | 84 | 37.00 | 50.40 | 0.004370 | 21669.85 | 5.498469 | 1258.358 |
| 9 | 84 | 37.00 | 49.56 | 0.004370 | 21669.85 | 5.498469 | 1258.358 |
| 10 | 82 | 36.00 | 45.10 | 0.004640 | 21783.30 | 5.770677 | 1243.598 |

## APPENDIX C



*h=300 mm, L=5000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 92 | 41.00 | 45.08 | 0.005830 | 23090.33 | 6.717781 | 1152.291 |
| 2 | **81** | **35.50** | **59.94** | **0.005842** | **23480.35** | **6.353880** | **1087.611** |
| 3 | 81 | 35.50 | 59.94 | 0.005842 | 23480.35 | 6.353880 | 1087.611 |
| 4 | 81 | 35.50 | 59.94 | 0.005842 | 23480.35 | 6.353880 | 1087.611 |
| 5 | 81 | 35.50 | 59.94 | 0.005842 | 23480.35 | 6.353880 | 1087.611 |
| 6 | 81 | 35.50 | 59.13 | 0.005812 | 23493.17 | 6.321630 | 1087.611 |
| 7 | 81 | 35.50 | 59.94 | 0.005842 | 23480.35 | 6.353880 | 1087.611 |
| 8 | 92 | 41.00 | 45.08 | 0.005830 | 23090.33 | 6.717781 | 1152.291 |
| 9 | 81 | 35.50 | 59.94 | 0.005842 | 23480.35 | 6.353880 | 1087.611 |
| 10 | 81 | 35.50 | 59.94 | 0.005842 | 23480.35 | 6.353880 | 1087.611 |

*h=300 mm, L=6500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **92** | **41.00** | **45.08** | **0.003943** | **26386.94** | **6.806465** | **1726.131** |
| 2 | 92 | 41.00 | 45.08 | 0.003943 | 26386.94 | 6.806465 | 1726.131 |
| 3 | 92 | 41.00 | 45.08 | 0.003943 | 26386.94 | 6.806465 | 1726.131 |
| 4 | 92 | 41.00 | 45.08 | 0.003943 | 26386.94 | 6.806465 | 1726.131 |
| 5 | 92 | 41.00 | 45.08 | 0.003943 | 26386.94 | 6.806465 | 1726.131 |
| 6 | 82 | 36.00 | 61.50 | 0.003669 | 25633.34 | 6.006847 | 1637.331 |
| 7 | 92 | 41.00 | 45.08 | 0.003943 | 26386.94 | 6.806465 | 1726.131 |
| 8 | 92 | 41.00 | 45.08 | 0.003943 | 26386.94 | 6.806465 | 1726.131 |
| 9 | 92 | 41.00 | 45.08 | 0.003943 | 26386.94 | 6.806465 | 1726.131 |
| 10 | 92 | 41.00 | 45.08 | 0.003943 | 26386.94 | 6.806465 | 1726.131 |

*h=350 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 83 | 36.50 | 68.06 | 0.004089 | 26482.74 | 4.896334 | 1197.371 |
| 2 | 83 | 36.50 | 68.06 | 0.004089 | 26482.74 | 4.896334 | 1197.371 |
| 3 | 83 | 36.50 | 66.40 | 0.004047 | 26480.23 | 4.846002 | 1197.371 |
| 4 | **84** | **37.00** | **72.24** | **0.004253** | **26397.66** | **5.117790** | **1203.251** |
| 5 | 83 | 36.50 | 68.06 | 0.004089 | 26482.74 | 4.896334 | 1197.371 |
| 6 | 84 | 37.00 | 72.24 | 0.004253 | 26397.66 | 5.117790 | 1203.251 |
| 7 | 84 | 37.00 | 72.24 | 0.004253 | 26397.66 | 5.117790 | 1203.251 |
| 8 | 71 | 30.50 | 68.16 | 0.003248 | 26853.68 | 3.660196 | 1126.811 |
| 9 | 83 | 36.50 | 68.06 | 0.004089 | 26482.74 | 4.896334 | 1197.371 |
| 10 | 71 | 30.50 | 68.16 | 0.003248 | 26853.68 | 3.660196 | 1126.811 |

*h=350 mm, L=7500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 83 | 36.50 | 68.06 | 0.002854 | 29149.16 | 5.120167 | 1794.211 |
| 2 | 83 | 36.50 | 68.06 | 0.002854 | 29149.16 | 5.120167 | 1794.211 |
| 3 | 83 | 36.50 | 66.40 | 0.002833 | 29176.40 | 5.082615 | 1794.211 |
| 4 | 83 | 36.50 | 68.06 | 0.002854 | 29149.16 | 5.120167 | 1794.211 |
| 5 | 83 | 36.50 | 68.06 | 0.002854 | 29149.16 | 5.120167 | 1794.211 |
| 6 | **84** | **37.00** | **72.24** | **0.002967** | **29143.63** | **5.350646** | **1803.091** |
| 7 | 84 | 37.00 | 72.24 | 0.002967 | 29143.63 | 5.350646 | 1803.091 |
| 8 | 71 | 30.50 | 68.16 | 0.002193 | 28320.39 | 3.701658 | 1687.651 |
| 9 | 83 | 36.50 | 68.06 | 0.002854 | 29149.16 | 5.120167 | 1794.211 |
| 10 | 71 | 30.50 | 68.16 | 0.002193 | 28320.39 | 3.701658 | 1687.651 |

*h=400 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **67** | **28.50** | **53.60** | **0.002442** | **29134.13** | **2.933732** | **1201.291** |
| 2 | 67 | 28.50 | 53.60 | 0.002442 | 29134.13 | 2.933732 | 1201.291 |
| 3 | 67 | 28.50 | 53.60 | 0.002442 | 29134.13 | 2.933732 | 1201.291 |
| 4 | 67 | 28.50 | 53.60 | 0.002442 | 29134.13 | 2.933732 | 1201.291 |
| 5 | 67 | 28.50 | 53.60 | 0.002442 | 29134.13 | 2.933732 | 1201.291 |
| 6 | 67 | 28.50 | 53.60 | 0.002442 | 29134.13 | 2.933732 | 1201.291 |
| 7 | 67 | 28.50 | 53.60 | 0.002442 | 29134.13 | 2.933732 | 1201.291 |
| 8 | 66 | 28.00 | 50.16 | 0.002319 | 29036.75 | 2.772563 | 1195.411 |
| 9 | 67 | 28.50 | 53.60 | 0.002442 | 29134.13 | 2.933732 | 1201.291 |
| 10 | 67 | 28.50 | 53.60 | 0.002442 | 29134.13 | 2.933732 | 1201.291 |

*h=400 mm, L=8000 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **67** | **28.50** | **53.60** | **0.001516** | **31399.44** | **2.729545** | **1800.131** |
| 2 | 67 | 28.50 | 53.60 | 0.001516 | 31399.44 | 2.729545 | 1800.131 |
| 3 | 67 | 28.50 | 53.60 | 0.001516 | 31399.44 | 2.729545 | 1800.131 |
| 4 | 67 | 28.50 | 53.60 | 0.001516 | 31399.44 | 2.729545 | 1800.131 |
| 5 | 67 | 28.50 | 53.60 | 0.001516 | 31399.44 | 2.729545 | 1800.131 |
| 6 | 67 | 28.50 | 53.60 | 0.001516 | 31399.44 | 2.729545 | 1800.131 |
| 7 | 67 | 28.50 | 53.60 | 0.001516 | 31399.44 | 2.729545 | 1800.131 |
| 8 | 67 | 28.50 | 53.60 | 0.001516 | 31399.44 | 2.729545 | 1800.131 |
| 9 | 67 | 28.50 | 53.60 | 0.001516 | 31399.44 | 2.729545 | 1800.131 |
| 10 | 67 | 28.50 | 53.60 | 0.001516 | 31399.44 | 2.729545 | 1800.131 |

**Case 2** $d_s = 20\,mm,\ h_2 = 36\,mm,\ h_1 \geq 40\,mm$

*h=400 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | **69** | **29.50** | **59.34** | **0.003840** | **29797.89** | **4.504687** | **1173.235** |
| 2 | 69 | 29.50 | 59.34 | 0.003840 | 29797.89 | 4.504687 | 1173.235 |
| 3 | 69 | 29.50 | 59.34 | 0.003840 | 29797.89 | 4.504687 | 1173.235 |
| 4 | 70 | 30.00 | 62.30 | 0.003826 | 29882.60 | 4.511836 | 1179.115 |
| 5 | 69 | 29.50 | 59.34 | 0.003840 | 29797.89 | 4.504687 | 1173.235 |
| 6 | 69 | 29.50 | 59.34 | 0.003840 | 29797.89 | 4.504687 | 1173.235 |
| 7 | 70 | 30.00 | 63.00 | 0.003821 | 29897.36 | 4.505953 | 1179.115 |
| 8 | 70 | 30.00 | 62.3 | 0.003826 | 29882.60 | 4.511836 | 1179.115 |
| 9 | 69 | 29.50 | 59.34 | 0.003840 | 29797.89 | 4.504687 | 1173.235 |
| 10 | 69 | 29.50 | 59.34 | 0.003840 | 29797.89 | 4.504687 | 1173.235 |

*h=400 mm, L=8000 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 69 | 29.50 | 60.72 | 0.002170 | 31785.49 | 3.816199 | 1758.572 |
| 2 | **71** | **30.50** | **66.03** | **0.002212** | **31931.37** | **3.928931** | **1776.332** |
| 3 | 71 | 30.50 | 66.03 | 0.002212 | 31931.37 | 3.928931 | 1776.332 |
| 4 | 71 | 30.50 | 66.03 | 0.002212 | 31931.37 | 3.928931 | 1776.332 |
| 5 | 71 | 30.50 | 66.03 | 0.002212 | 31931.37 | 3.928931 | 1776.332 |
| 6 | 71 | 30.50 | 66.03 | 0.002212 | 31931.37 | 3.928931 | 1776.332 |
| 7 | 71 | 30.50 | 66.74 | 0.002209 | 31927.13 | 3.923069 | 1776.332 |
| 8 | 70 | 30.00 | 62.30 | 0.002206 | 31865.74 | 3.899260 | 1767.452 |
| 9 | 71 | 30.50 | 66.03 | 0.002212 | 31931.37 | 3.928931 | 1776.332 |
| 10 | 70 | 30.00 | 62.30 | 0.002206 | 31865.74 | 3.899260 | 1767.452 |

**Case 3** $d_s = 20mm,\ h_2 = 25mm,\ h_1 = 50mm$

$h = 400mm, L = 6000mm, t = 2.0mm$

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 69 | 29.50 | 50.00 | 0.003507 | 29150.91 | 4.138172 | 1180.106 |
| 2 | **73** | **31.50** | **50.00** | **0.003835** | **29158.77** | **4.615307** | **1203.626** |
| 3 | 71 | 30.50 | 50.00 | 0.003693 | 29170.35 | 4.401574 | 1191.866 |
| 4 | 73 | 31.50 | 50.00 | 0.003835 | 29158.77 | 4.615307 | 1203.626 |
| 5 | 73 | 31.50 | 50.00 | 0.003835 | 29158.77 | 4.615307 | 1203.626 |
| 6 | 71 | 30.50 | 50.00 | 0.003693 | 29170.35 | 4.401574 | 1191.866 |
| 7 | 72 | 31.00 | 50.00 | 0.003786 | 29165.71 | 4.534304 | 1197.746 |
| 8 | 72 | 31.00 | 50.00 | 0.003786 | 29165.71 | 4.534304 | 1197.746 |
| 9 | 71 | 30.50 | 50.00 | 0.003693 | 29170.35 | 4.401574 | 1191.866 |
| 10 | 71 | 30.50 | 50.00 | 0.003693 | 29170.35 | 4.401574 | 1191.866 |

$h = 400mm, L = 8000mm, t = 3.0mm$

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|
| 1 | 69 | 29.50 | 50.00 | 0.002032 | 31642.52 | 3.593472 | 1768.706 |
| 2 | **73** | **31.50** | **50.00** | **0.002270** | **31994.46** | **4.095378** | **1804.226** |
| 3 | 71 | 30.50 | 50.00 | 0.002150 | 31821.48 | 3.840255 | 1786.466 |
| 4 | 73 | 31.50 | 50.00 | 0.002270 | 31994.46 | 4.095378 | 1804.226 |
| 5 | 73 | 31.50 | 50.00 | 0.002270 | 31994.46 | 4.095378 | 1804.226 |
| 6 | 71 | 30.50 | 50.00 | 0.002150 | 31821.48 | 3.840255 | 1786.466 |
| 7 | 72 | 31.00 | 50.00 | 0.002209 | 31908.29 | 3.966727 | 1795.346 |
| 8 | 72 | 31.00 | 50.00 | 0.002209 | 31908.29 | 3.966727 | 1795.346 |
| 9 | 71 | 30.50 | 50.00 | 0.002150 | 31821.48 | 3.840255 | 1786.466 |
| 10 | 71 | 30.50 | 50.00 | 0.002150 | 31821.48 | 3.840255 | 1786.466 |

# APPENDIX D

## Results for Σ-Shape Purlin with Objective Function $M_y/A_g$

### Single Edge Stiffener

*h=250 mm, L=4000 mm and t=1.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| 1 | **55** | **26.40** | **35.75** | **33** | **33** | **19535.18** | **0.004647** | **632.714** |
| 2 | 56 | 26.32 | 37.52 | 32 | 32 | 19472.65 | 0.004893 | 634.191 |
| 3 | 55 | 26.40 | 35.75 | 33 | 33 | 19535.18 | 0.004647 | 632.714 |
| 4 | 55 | 26.40 | 35.75 | 33 | 33 | 19535.18 | 0.004647 | 632.714 |
| 5 | 55 | 26.40 | 35.75 | 33 | 33 | 19535.18 | 0.004647 | 632.714 |
| 6 | 55 | 25.85 | 35.75 | 32 | 32 | 19531.89 | 0.004721 | 629.899 |
| 7 | 55 | 27.50 | 35.75 | 35 | 35 | 19518.55 | 0.004502 | 638.346 |
| 8 | 55 | 25.85 | 35.75 | 32 | 32 | 19531.89 | 0.004721 | 629.899 |
| 9 | 56 | 26.88 | 37.52 | 33 | 33 | 19465.42 | 0.004815 | 637.036 |
| 10 | 55 | 26.40 | 35.75 | 33 | 33 | 19535.18 | 0.004647 | 632.714 |

# APPENDIX D

*h=250 mm, L=5500 mm and t=2.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 78 | 33.54 | 36.66 | 28 | 28 | 21984.72 | 0.004648 | 1192.769 |
| 2 | **79** | **44.24** | **39.50** | **5** | **5** | **22271.84** | **0.002993** | **1203.461** |
| 3 | 80 | 44.80 | 41.60 | 5 | 5 | 22269.02 | 0.002991 | 1211.136 |
| 4 | 80 | 35.20 | 40.80 | 29 | 29 | 21981.92 | 0.004517 | 1212.814 |
| 5 | 79 | 44.24 | 39.50 | 5 | 5 | 22271.84 | 0.002993 | 1203.461 |
| 6 | 79 | 44.24 | 39.50 | 5 | 5 | 22271.84 | 0.002993 | 1203.461 |
| 7 | 79 | 44.24 | 39.50 | 5 | 5 | 22271.84 | 0.002993 | 1203.461 |
| 8 | 80 | 44.80 | 41.60 | 5 | 5 | 22269.02 | 0.002991 | 1211.136 |
| 9 | 79 | 44.24 | 39.50 | 5 | 5 | 22271.84 | 0.002993 | 1203.461 |
| 10 | 80 | 44.80 | 41.60 | 5 | 5 | 22269.02 | 0.002991 | 1211.136 |

*h=300 mm, L=5000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 67 | 39.53 | 52.93 | 20 | 8 | 24677.39 | 0.003796 | 994.5754 |
| 2 | **69** | **38.64** | **60.03** | **14** | **5** | **24695.43** | **0.003604** | **996.3098** |
| 3 | 69 | 38.64 | 60.03 | 14 | 5 | 24695.43 | 0.003604 | 996.3098 |
| 4 | 69 | 38.64 | 60.03 | 14 | 5 | 24695.43 | 0.003604 | 996.3098 |
| 5 | 69 | 38.64 | 60.03 | 14 | 5 | 24695.43 | 0.003604 | 996.3098 |
| 6 | 63 | 32.13 | 39.06 | 32 | 36 | 24337.79 | 0.003555 | 1006.268 |
| 7 | 69 | 38.64 | 60.03 | 14 | 5 | 24695.43 | 0.003604 | 996.3098 |
| 8 | 67 | 39.53 | 52.93 | 20 | 8 | 24677.39 | 0.003796 | 994.5754 |
| 9 | 69 | 38.64 | 60.03 | 14 | 5 | 24695.43 | 0.003604 | 996.3098 |
| 10 | 67 | 39.53 | 52.93 | 20 | 8 | 24677.39 | 0.003796 | 994.5754 |

# APPENDIX D

*h=300 mm, L=6500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 95 | 53.20 | 59.85 | 14 | 5 | 26737.21 | 0.002524 | 1731.621 |
| 2 | 95 | 53.20 | 59.85 | 14 | 5 | 26737.21 | 0.002524 | 1731.621 |
| 3 | 95 | 53.20 | 59.85 | 13 | 5 | 26735.36 | 0.002524 | 1731.951 |
| 4 | **96** | **53.76** | **64.32** | **14** | **5** | **26744.68** | **0.002518** | **1740.857** |
| 5 | 95 | 53.20 | 59.85 | 14 | 5 | 26737.21 | 0.002524 | 1731.621 |
| 6 | 91 | 50.96 | 42.77 | 13 | 5 | 26713.29 | 0.002539 | 1695.011 |
| 7 | 95 | 53.20 | 59.85 | 14 | 5 | 26737.21 | 0.002524 | 1731.621 |
| 8 | 95 | 53.20 | 59.85 | 14 | 5 | 26737.21 | 0.002524 | 1731.621 |
| 9 | 95 | 53.20 | 59.85 | 14 | 5 | 26737.21 | 0.002524 | 1731.621 |
| 10 | 95 | 53.20 | 59.85 | 14 | 5 | 26737.21 | 0.002524 | 1731.621 |

*h=350 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 71 | 41.89 | 67.45 | 20 | 8 | 27630.56 | 0.002856 | 1117.507 |
| 2 | 71 | 41.89 | 67.45 | 20 | 8 | 27630.56 | 0.002856 | 1117.507 |
| 3 | 71 | 41.89 | 67.45 | 19 | 8 | 27608.05 | 0.002855 | 1117.785 |
| 4 | 71 | 41.89 | 67.45 | 20 | 8 | 27630.56 | 0.002856 | 1117.507 |
| 5 | **71** | **42.60** | **67.45** | **22** | **9** | **27641.35** | **0.002897** | **1121.181** |
| 6 | 67 | 34.17 | 52.26 | 32 | 36 | 27198.22 | 0.002830 | 1127.945 |
| 7 | 71 | 41.89 | 67.45 | 20 | 8 | 27630.56 | 0.002856 | 1117.507 |
| 8 | 71 | 41.89 | 67.45 | 20 | 8 | 27630.56 | 0.002856 | 1117.507 |
| 9 | 71 | 42.60 | 67.45 | 22 | 9 | 27641.35 | 0.002897 | 1121.181 |
| 10 | 71 | 41.89 | 67.45 | 20 | 8 | 27630.56 | 0.002856 | 1117.507 |

# APPENDIX D

*h=350 mm, L=7500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 83 | 48.14 | 67.23 | 18 | 7 | 30072.10 | 0.002129 | 1781.228 |
| 2 | 95 | 35.15 | 58.9 | 40 | 22 | 29720.36 | 0.002657 | 1800.726 |
| 3 | 87 | 48.72 | 84.39 | 13 | 5 | 30221.97 | 0.002093 | 1806.070 |
| 4 | **87** | **48.72** | **84.39** | **14** | **5** | **30225.33** | **0.002094** | **1805.740** |
| 5 | 87 | 48.72 | 84.39 | 14 | 5 | 30225.33 | 0.002094 | 1805.740 |
| 6 | 89 | 34.71 | 35.60 | 26 | 29 | 29823.22 | 0.002817 | 1803.969 |
| 7 | 87 | 48.72 | 84.39 | 14 | 5 | 30225.33 | 0.002094 | 1805.740 |
| 8 | 87 | 48.72 | 84.39 | 14 | 5 | 30225.33 | 0.002094 | 1805.740 |
| 9 | 87 | 48.72 | 84.39 | 14 | 5 | 30225.33 | 0.002094 | 1805.740 |
| 10 | 87 | 48.72 | 84.39 | 14 | 5 | 30225.33 | 0.002094 | 1805.740 |

*h=400 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 67 | 39.53 | 52.93 | 20 | 8 | 29677.73 | 0.002839 | 1190.575 |
| 2 | 70 | 40.60 | 63.70 | 18 | 7 | 29979.13 | 0.002858 | 1205.646 |
| 3 | 69 | 40.71 | 60.03 | 19 | 8 | 29897.20 | 0.002889 | 1203.319 |
| 4 | 71 | 39.76 | 67.45 | 14 | 5 | 29937.75 | 0.002770 | 1204.540 |
| 5 | 55 | 33.00 | 53.35 | 22 | 9 | 29319.89 | 0.002569 | 1118.829 |
| 6 | **69** | **41.40** | **60.03** | **22** | **9** | **30003.48** | **0.002949** | **1206.637** |
| 7 | 69 | 41.40 | 60.03 | 22 | 9 | 30003.48 | 0.002949 | 1206.637 |
| 8 | 70 | 40.60 | 63.70 | 18 | 7 | 29979.13 | 0.002858 | 1205.646 |
| 9 | 69 | 41.40 | 60.03 | 22 | 9 | 30003.48 | 0.002949 | 1206.637 |
| 10 | 71 | 24.85 | 66.03 | 36 | 20 | 29128.80 | 0.003792 | 1162.847 |

# APPENDIX D

*h=400 mm, L=8000 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 67 | 39.53 | 52.93 | 20 | 8 | 32169.42 | 0.001741 | 1784.866 |
| 2 | 70 | 40.60 | 63.70 | 18 | 7 | 32580.77 | 0.001789 | 1807.631 |
| 3 | 69 | 40.71 | 60.03 | 20 | 8 | 32489.70 | 0.001793 | 1803.691 |
| 4 | **71** | **39.76** | **67.45** | **14** | **5** | **32605.93** | **0.001756** | **1805.977** |
| 5 | 82 | 25.42 | 61.50 | 28 | 16 | 31654.96 | 0.002226 | 1805.813 |
| 6 | 77 | 28.49 | 43.12 | 40 | 22 | 32018.93 | 0.002306 | 1802.739 |
| 7 | 71 | 39.76 | 67.45 | 14 | 5 | 32605.93 | 0.001756 | 1805.977 |
| 8 | 70 | 40.60 | 63.70 | 18 | 7 | 32580.77 | 0.001789 | 1807.631 |
| 9 | 69 | 41.40 | 60.03 | 22 | 9 | 32551.28 | 0.001821 | 1809.118 |
| 10 | 69 | 41.40 | 60.03 | 22 | 9 | 32551.28 | 0.001821 | 1809.118 |

## Double Edge Stiffeners

*h=250 mm, L=4000 mm and t=1.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 47 | 22.09 | 10.81 | 37.6 | 32 | 32 | 19945.40 | 0.00406 | 624.110 |
| 2 | **49** | **23.03** | **11.27** | **39.2** | **32** | **32** | **19961.07** | **0.004347** | **634.038** |
| 3 | 49 | 23.03 | 11.27 | 39.2 | 32 | 32 | 19961.07 | 0.004347 | 634.038 |
| 4 | 49 | 23.03 | 11.27 | 39.2 | 32 | 32 | 19961.07 | 0.004347 | 634.038 |
| 5 | 49 | 22.05 | 10.78 | 38.71 | 30 | 30 | 19952.00 | 0.004482 | 627.327 |
| 6 | 47 | 22.09 | 10.81 | 37.6 | 32 | 32 | 19945.40 | 0.004060 | 624.110 |
| 7 | 47 | 22.09 | 10.81 | 37.6 | 32 | 32 | 19945.40 | 0.004060 | 624.110 |
| 8 | 47 | 22.09 | 10.81 | 37.6 | 32 | 32 | 19945.40 | 0.004060 | 624.110 |
| 9 | 49 | 23.03 | 11.27 | 39.2 | 32 | 32 | 19961.07 | 0.004347 | 634.038 |
| 10 | 49 | 23.03 | 11.27 | 39.2 | 32 | 32 | 19961.07 | 0.004347 | 634.038 |

# APPENDIX D

*h=250 mm, L=5500 mm and t=2.5 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 82 | 30.34 | 14.76 | 35.26 | 22 | 22 | 22159.53 | 0.003928 | 1248.539 |
| 2 | 78 | 31.98 | 15.60 | 35.10 | 26 | 26 | 22102.43 | 0.004241 | 1249.212 |
| 3 | 80 | 32.00 | 16.00 | 36.00 | 25 | 25 | 22145.30 | 0.004115 | 1259.080 |
| 4 | 82 | 30.34 | 14.76 | 35.26 | 22 | 22 | 22159.53 | 0.003928 | 1248.539 |
| 5 | 80 | 32.00 | 16.00 | 36.00 | 25 | 25 | 22145.30 | 0.004115 | 1259.080 |
| 6 | 82 | 30.34 | 14.76 | 35.26 | 22 | 22 | 22159.53 | 0.003928 | 1248.539 |
| 7 | **80** | **30.40** | **15.20** | **35.20** | **23** | **23** | **22174.70** | **0.004020** | **1243.197** |
| 8 | 84 | 31.08 | 15.12 | 36.12 | 22 | 22 | 22125.25 | 0.003874 | 1263.791 |
| 9 | 78 | 31.20 | 15.60 | 35.10 | 25 | 25 | 22113.66 | 0.004174 | 1243.336 |
| 10 | 82 | 30.34 | 14.76 | 35.26 | 22 | 22 | 22159.53 | 0.003928 | 1248.539 |

*h=300 mm, L=5000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 68 | 40.12 | 19.72 | 50.32 | 20 | 8 | 25096.02 | 0.003662 | 1072.678 |
| 2 | 68 | 40.80 | 20.4 | 51.00 | 22 | 9 | 25112.88 | 0.003706 | 1078.901 |
| 3 | 66 | 38.94 | 19.14 | 48.84 | 20 | 8 | 25092.07 | 0.003653 | 1057.939 |
| 4 | 68 | 40.80 | 20.4 | 51.00 | 22 | 9 | 25112.88 | 0.003706 | 1078.901 |
| 5 | **66** | **39.60** | **19.80** | **49.50** | **22** | **9** | **25115.03** | **0.003700** | **1064.005** |
| 6 | 68 | 40.12 | 19.72 | 50.32 | 20 | 8 | 25096.02 | 0.003662 | 1072.678 |
| 7 | 66 | 38.94 | 19.14 | 48.84 | 20 | 8 | 25092.07 | 0.003653 | 1057.939 |
| 8 | 68 | 40.12 | 19.72 | 50.32 | 20 | 8 | 25096.02 | 0.003662 | 1072.678 |
| 9 | 66 | 39.60 | 19.80 | 49.50 | 22 | 9 | 25115.03 | 0.003700 | 1064.005 |
| 10 | 68 | 40.12 | 19.72 | 50.32 | 20 | 8 | 25096.02 | 0.003662 | 1072.678 |

# APPENDIX D

*h=300 mm, L=6500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **97** | **35.89** | **17.46** | **48.50** | **40** | **22** | **26660.30** | **0.003045** | **1759.919** |
| 2 | 97 | 35.89 | 17.46 | 48.50 | 40 | 22 | 26660.30 | 0.003045 | 1759.919 |
| 3 | 95 | 35.15 | 17.10 | 47.50 | 40 | 22 | 26644.31 | 0.003069 | 1741.567 |
| 4 | 96 | 35.52 | 17.28 | 49.92 | 40 | 22 | 26656.76 | 0.003039 | 1750.743 |
| 5 | 95 | 35.15 | 17.10 | 47.50 | 40 | 22 | 26644.31 | 0.003069 | 1741.567 |
| 6 | 96 | 33.6 | 16.32 | 48.00 | 36 | 20 | 26637.37 | 0.003009 | 1730.907 |
| 7 | 97 | 35.89 | 17.46 | 48.50 | 40 | 22 | 26660.30 | 0.003045 | 1759.919 |
| 8 | 97 | 35.89 | 17.46 | 48.50 | 40 | 22 | 26660.30 | 0.003045 | 1759.919 |
| 9 | 95 | 35.15 | 17.10 | 47.50 | 40 | 22 | 26644.31 | 0.003069 | 1741.567 |
| 10 | 97 | 35.89 | 17.46 | 48.50 | 40 | 22 | 26660.30 | 0.003045 | 1759.919 |

*h=350 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 70 | 41.30 | 20.30 | 51.80 | 20 | 8 | 28084.09 | 0.002770 | 1185.417 |
| 2 | **68** | **40.80** | **20.40** | **51.00** | **22** | **9** | **28158.44** | **0.002790** | **1176.901** |
| 3 | 70 | 40.60 | 20.30 | 51.10 | 18 | 7 | 28003.40 | 0.002725 | 1181.789 |
| 4 | 70 | 41.30 | 20.30 | 51.80 | 20 | 8 | 28084.09 | 0.002770 | 1185.417 |
| 5 | 68 | 40.80 | 20.40 | 51.00 | 22 | 9 | 28158.44 | 0.002790 | 1176.901 |
| 6 | 68 | 40.80 | 20.40 | 51.00 | 22 | 9 | 28158.44 | 0.002790 | 1176.901 |
| 7 | 72 | 42.48 | 20.88 | 53.28 | 20 | 8 | 28055.94 | 0.002784 | 1200.156 |
| 8 | 68 | 40.80 | 20.40 | 51.00 | 22 | 9 | 28158.44 | 0.002790 | 1176.901 |
| 9 | 68 | 40.12 | 19.72 | 50.32 | 20 | 8 | 28079.76 | 0.002751 | 1170.678 |
| 10 | 68 | 40.80 | 20.40 | 51.00 | 22 | 9 | 28158.44 | 0.002790 | 1176.901 |

# APPENDIX D

*h=350 mm, L=7500 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 91 | 28.21 | 13.65 | 40.04 | 27 | 16 | 29824.60 | 0.002486 | 1796.775 |
| 2 | 93 | 26.97 | 13.02 | 39.06 | 24 | 14 | 29709.87 | 0.002415 | 1794.018 |
| 3 | 93 | 26.97 | 13.02 | 39.06 | 24 | 14 | 29709.87 | 0.002415 | 1794.018 |
| 4 | 90 | 27.9 | 13.50 | 41.40 | 27 | 16 | 29773.24 | 0.002481 | 1788.132 |
| 5 | **89** | **29.37** | **14.24** | **40.94** | **31** | **18** | **29888.14** | **0.002554** | **1798.062** |
| 6 | 84 | 31.08 | 15.12 | 43.68 | 39 | 22 | 29841.98 | 0.002670 | 1789.345 |
| 7 | 84 | 31.08 | 15.12 | 43.68 | 39 | 22 | 29841.98 | 0.002670 | 1789.345 |
| 8 | 87 | 29.58 | 14.79 | 40.89 | 33 | 19 | 29872.43 | 0.002583 | 1792.107 |
| 9 | 71 | 39.76 | 19.88 | 48.99 | 13 | 5 | 29659.96 | 0.001872 | 1763.605 |
| 10 | 74 | 41.44 | 20.72 | 52.54 | 13 | 5 | 29808.03 | 0.001915 | 1796.284 |

*h=400 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 61 | 25.62 | 12.20 | 58.56 | 32 | 32 | 29912.31 | 0.002589 | 1200.082 |
| 2 | 59 | 34.81 | 17.11 | 42.48 | 20 | 8 | 29839.33 | 0.002523 | 1202.351 |
| 3 | 61 | 25.62 | 12.20 | 58.56 | 32 | 32 | 29912.31 | 0.002589 | 1200.082 |
| 4 | 60 | 25.20 | 12.00 | 58.80 | 32 | 32 | 29839.62 | 0.002530 | 1193.731 |
| 5 | **58** | **34.80** | **17.40** | **43.50** | **22** | **9** | **29948.13** | **0.002545** | **1200.421** |
| 6 | 61 | 25.62 | 12.20 | 58.56 | 32 | 32 | 29912.31 | 0.002589 | 1200.082 |
| 7 | 61 | 25.62 | 12.20 | 58.56 | 32 | 32 | 29912.31 | 0.002589 | 1200.082 |
| 8 | 58 | 34.80 | 17.40 | 43.50 | 22 | 9 | 29948.13 | 0.002545 | 1200.421 |
| 9 | 57 | 34.20 | 17.10 | 41.61 | 22 | 9 | 29776.17 | 0.002500 | 1192.973 |
| 10 | 58 | 34.80 | 17.40 | 43.50 | 22 | 9 | 29948.13 | 0.002545 | 1200.421 |

# APPENDIX D

*h=400 mm, L=8000 mm and t=3.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $d$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $M_y/A_g$ (Nmm/mm$^2$) | $q/A_g$ (N/mm/mm$^2$) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **70** | **24.50** | **11.90** | **35.00** | **36** | **20** | **31838.95** | **0.002009** | **1792.949** |
| 2 | 59 | 34.81 | 17.11 | 42.48 | 20 | 8 | 31674.46 | 0.001459 | 1798.464 |
| 3 | 77 | 21.56 | 10.01 | 63.14 | 40 | 18 | 31714.06 | 0.002203 | 1798.216 |
| 4 | 58 | 34.22 | 16.82 | 42.92 | 20 | 8 | 31592.11 | 0.001433 | 1787.334 |
| 5 | 58 | 34.80 | 17.40 | 43.50 | 22 | 9 | 31700.71 | 0.001452 | 1795.543 |
| 6 | 75 | 21.00 | 9.75 | 61.50 | 40 | 18 | 31588.28 | 0.002179 | 1781.521 |
| 7 | 77 | 21.56 | 10.01 | 63.14 | 40 | 18 | 31714.06 | 0.002203 | 1798.216 |
| 8 | 58 | 34.80 | 17.40 | 43.50 | 22 | 9 | 31700.71 | 0.001452 | 1795.543 |
| 9 | 58 | 34.80 | 17.40 | 43.50 | 22 | 9 | 31700.71 | 0.001452 | 1795.543 |
| 10 | 57 | 34.20 | 17.10 | 41.61 | 22 | 9 | 31550.35 | 0.001419 | 1784.295 |

# APPENDIX E

# Results using the Modified Eurocode 3 Method

## Z-Shape Purlin

*h=100 mm, L=1400 mm and t=1.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 44 | 22.04 | 0.019772 | 7954.697 | 4.218377 | 213.3504 |
| 2 | 40 | 20.06 | 0.019850 | 8085.967 | 4.007060 | 201.8688 |
| 3 | 44 | 22.42 | 0.019771 | 7919.435 | 4.232648 | 214.0800 |
| 4 | 42 | 21.24 | 0.019854 | 8022.121 | 4.129104 | 207.9744 |
| 5 | 40 | 20.06 | 0.019850 | 8085.967 | 4.007060 | 201.8688 |
| 6 | 42 | 20.52 | 0.019820 | 8074.155 | 4.094598 | 206.5920 |
| 7 | 44 | 22.8 | 0.019767 | 7883.179 | 4.246084 | 214.8096 |
| 8 | 45 | 19.89 | 0.019586 | 8098.025 | 4.135413 | 211.1424 |
| 9 | **41** | **20.65** | **0.019864** | **8059.779** | **4.070652** | **204.9216** |
| 10 | 41 | 20.65 | 0.019864 | 8059.779 | 4.070652 | 204.9216 |

*h=150 mm, L=1700 mm and t=1.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 55 | 28.91 | 0.016682 | 9196.292 | 4.932186 | 295.6608 |
| 2 | 56 | 29.00 | 0.016671 | 9189.853 | 4.963914 | 297.7536 |
| 3 | 55 | 28.91 | 0.016682 | 9196.292 | 4.932186 | 295.6608 |
| 4 | 57 | 30.09 | 0.016654 | 9181.520 | 4.993062 | 299.8080 |
| 5 | 57 | 30.60 | 0.016708 | 9114.495 | 5.058132 | 302.7456 |
| 6 | 57 | 29.07 | 0.016654 | 9181.520 | 4.993062 | 299.8080 |
| 7 | **56** | **30.00** | **0.016710** | **9147.820** | **5.007456** | **299.6736** |
| 8 | 53 | 28.20 | 0.016650 | 9213.074 | 4.836034 | 290.4576 |
| 9 | 57 | 26.01 | 0.016422 | 9248.449 | 4.827078 | 293.9328 |
| 10 | 55 | 24.99 | 0.016352 | 9263.601 | 4.711489 | 288.1344 |

# APPENDIX E

*h=200 mm, L=3000 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | **73** | **39.53** | **0.007497** | **13601.85** | **4.523062** | **603.3012** |
| 2 | 73 | 39.53 | 0.007497 | 13601.85 | 4.523062 | 603.3012 |
| 3 | 76 | 41.30 | 0.007494 | 13481.21 | 4.625308 | 617.2296 |
| 4 | 73 | 39.53 | 0.007497 | 13601.85 | 4.523062 | 603.3012 |
| 5 | 76 | 41.30 | 0.007494 | 13481.21 | 4.625308 | 617.2296 |
| 6 | 71 | 39.00 | 0.007491 | 13623.09 | 4.464286 | 595.9136 |
| 7 | 76 | 39.90 | 0.007488 | 13565.89 | 4.590936 | 613.1416 |
| 8 | 76 | 41.30 | 0.007494 | 13481.21 | 4.625308 | 617.2296 |
| 9 | 73 | 39.53 | 0.007497 | 13601.85 | 4.523062 | 603.3012 |
| 10 | 76 | 41.30 | 0.007494 | 13481.21 | 4.625308 | 617.2296 |

*h=250 mm, L=3200 mm and t=1.5 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 86 | 47.20 | 0.007292 | 14347.99 | 5.371779 | 736.6576 |
| 2 | **87** | **48.60** | **0.007310** | **14301.50** | **5.436406** | **743.6656** |
| 3 | 87 | 47.79 | 0.007301 | 14327.93 | 5.412500 | 741.3004 |
| 4 | 87 | 47.79 | 0.007301 | 14327.93 | 5.412500 | 741.3004 |
| 5 | 87 | 48.60 | 0.007310 | 14301.50 | 5.436406 | 743.6656 |
| 6 | 81 | 42.75 | 0.007196 | 14450.44 | 5.102153 | 709.0636 |
| 7 | 87 | 48.60 | 0.007310 | 14301.50 | 5.436406 | 743.6656 |
| 8 | 87 | 47.79 | 0.007301 | 14327.93 | 5.412500 | 741.3004 |
| 9 | 87 | 41.31 | 0.007180 | 14458.94 | 5.186465 | 722.3788 |
| 10 | 87 | 47.79 | 0.007301 | 14327.93 | 5.412500 | 741.3004 |

*h=300 mm, L=4500 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 99 | 54.87 | 0.004637 | 18960.59 | 5.397618 | 1164.044 |
| 2 | 94 | 51.92 | 0.004584 | 18953.95 | 5.193528 | 1132.880 |
| 3 | 99 | 55.80 | 0.004645 | 18940.99 | 5.423606 | 1167.690 |
| 4 | 95 | 52.51 | 0.004596 | 18957.08 | 5.235151 | 1139.113 |
| 5 | 99 | 54.87 | 0.004637 | 18960.59 | 5.397618 | 1164.044 |
| 6 | 97 | 54.60 | 0.004625 | 18943.64 | 5.343064 | 1155.146 |
| 7 | **100** | **56.40** | **0.004654** | **18937.30** | **5.463066** | **1173.962** |
| 8 | 100 | 56.40 | 0.004654 | 18937.30 | 5.463066 | 1173.962 |
| 9 | 100 | 47.94 | 0.004558 | 19039.10 | 5.199932 | 1140.798 |
| 10 | 87 | 47.79 | 0.004491 | 18916.57 | 4.891877 | 1089.250 |

*h=350 mm, L=5000 mm and t=2.0 mm*

| No. of run | $b_1$ (mm) | $c$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|
| 1 | 90 | 49.56 | 0.003660 | 19329.34 | 4.414114 | 1205.949 |
| 2 | 97 | 42.77 | 0.003646 | 19388.20 | 4.400476 | 1206.772 |
| 3 | 93 | 46.98 | 0.003668 | 19380.38 | 4.429246 | 1207.595 |
| 4 | 95 | 45.39 | 0.003667 | 19396.75 | 4.434333 | 1209.202 |
| 5 | 95 | 45.39 | 0.003667 | 19396.75 | 4.434333 | 1209.202 |
| 6 | **91** | **49.30** | **0.003671** | **19350.79** | **4.437142** | **1208.850** |
| 7 | 95 | 43.61 | 0.003644 | 19383.38 | 4.380345 | 1202.225 |
| 8 | 91 | 49.30 | 0.003671 | 19350.79 | 4.437142 | 1208.850 |
| 9 | 96 | 44.10 | 0.003658 | 19395.27 | 4.419214 | 1208.066 |
| 10 | 93 | 46.98 | 0.003668 | 19380.38 | 4.429246 | 1207.595 |

# APPENDIX E

## Σ-Shape Purlin with Single Edge Stiffener (Free Optimized Case)

*h=350 mm, L=6000 mm and t=2.0 mm*

| No. of run | $b$ (mm) | $c$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $d_s$ (mm) | $q/A_g$ (N/mm/mm$^2$) | $M_y/A_g$ (Nmm/mm$^2$) | $q$ (N/mm) | $A_g$ (mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 81 | 28.35 | 59.94 | 17 | 25 | 0.004188 | 26622.36 | 4.808137 | 1147.992 |
| 2 | **83** | **33.20** | **68.06** | **28** | **30** | **0.004236** | **26421.06** | **4.975697** | **1174.677** |
| 3 | 80 | 27.20 | 56.00 | 16 | 24 | 0.004145 | 26698.31 | 4.717307 | 1138.002 |
| 4 | 82 | 30.34 | 63.96 | 21 | 27 | 0.004215 | 26549.78 | 4.889052 | 1159.877 |
| 5 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004236 | 26421.06 | 4.975697 | 1174.677 |
| 6 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004236 | 26421.06 | 4.975697 | 1174.677 |
| 7 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004236 | 26421.06 | 4.975697 | 1174.677 |
| 8 | 83 | 33.20 | 68.06 | 28 | 30 | 0.004236 | 26421.06 | 4.975697 | 1174.677 |
| 9 | 91 | 33.67 | 43.68 | 22 | 27 | 0.004206 | 25915.39 | 5.075303 | 1206.821 |
| 10 | 82 | 30.34 | 63.96 | 21 | 27 | 0.004215 | 26549.78 | 4.889052 | 1159.877 |

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