Degree of Polarization in Near Fields of Thermal Sources: Effects of Surface Waves

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We introduce the concept of degree of polarization for electromagnetic near fields. The approach is based on the generalized Stokes parameters that appear as expansion coefficients of the 3×3 coherence matrix in terms of the Gell-Mann matrices. The formalism is applied to optical near fields of thermally fluctuating half-space sources with particular interest in fields that are strongly polarized owing to resonant surface plasmons or phonons. This novel method is particularly useful when assessing the full vectorial characteristics of random evanescent fields, e.g., for near-field spectroscopy and polarization microscopy.

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The polarization properties of classical and quantized electromagnetic fields show an extraordinary richness in their mathematical structure and in the fundamental physical phenomena, as evidenced, e.g., by the Poincaré-sphere construction, the Berry-Pancharatnam phase, and the optical EPR paradox. Normally, the polarization statistics of light fields are described for well-collimated, uniform beams in terms of a 2×2 equal-time coherence matrix or the Stokes parameters [1]. Besides beams, this approach applies to radiated wide-angle far fields, and it has been demonstrated that unpolarized sources may generate highly polarized far fields [2]. For nonuniform beams, methods based on the Wigner matrix and the beam coherence-polarization matrix are employed [3]. It is known that, in analogy with the correlation-induced spectral changes [4] and the changes in spatial coherence, the degree of polarization of a partially coherent, partially polarized beam may vary on propagation [5]. However, the two-dimensional techniques are inadequate to describe the partial polarization of arbitrary fields. In particular, fluctuating optical near fields, which are characterized by evanescent waves and which manifestly are not beamlike, must be represented by the full electromagnetic cross-spectral tensors [6].

Though frequently regarded as incoherent, the fields emitted by thermal sources have a certain degree of correlations; indeed, blackbody radiation is a classic example of a partially coherent, unpolarized field within both the classical and the quantized theories [6]. The spectral and spatial-coherence properties of optical near fields produced by thermal sources occupying an infinite half-space have recently been shown to exhibit interesting phenomena [7,8]. For example, the spectrum of the near field differs from the corresponding source and far-field values. The near-field (transverse) spatial correlation length may, contrary to common belief, be much shorter than the wavelength, or it may extend over several tens of wavelengths when resonant surface waves such as surface-plasmon or surface-phonon polaritons are excited. Related to these findings, another fundamental quantity of an electromagnetic near field is the degree of polarization, which provides information about the correlations between the three orthogonal electric-field components in a given point.

On extending the 2×2 Stokes formalism to 3×3 coherence matrices [1,9], we introduce in this Letter the degree of polarization for arbitrary electromagnetic fields and investigate its properties in optical near fields, in particular when surface waves contribute strongly to the field. New results are obtained concerning the vectorial characteristics of thermal near fields that are important in applications such as near-field imaging and spectroscopy, and polarization diffraction microscopy.

We consider a thermal source filling the half-space $z < 0$ and separated from a vacuum by a sharp boundary in the plane $z = 0$. The source, which is assumed to be in local thermodynamic equilibrium at a uniform temperature T, consists of a homogeneous, isotropic, and nonmagnetic lossy material whose dielectric properties, at frequency ω, are given by the complex dielectric constant $\epsilon(\omega)$. The spatial correlations of the Fourier components of a stationary thermal current $\mathbf{j}(\mathbf{r}, \omega)$, at points $\mathbf{r}_1$ and $\mathbf{r}_2$ in the region $z < 0$, are described by the fluctuation-dissipation theorem as [7,8]

$$\langle j_m^*(\mathbf{r}_1, \omega) j_n(\mathbf{r}_2, \omega') \rangle = \frac{\omega}{\pi} \epsilon_0 \epsilon''(\omega) \Theta(\omega, T) \delta(\mathbf{r}_1 - \mathbf{r}_2) \times \delta_{m,n} \delta(\omega - \omega'), \quad (1)$$

where the brackets represent an ensemble average and the superscript $*$ denotes complex conjugation. In Eq. (1), ϵ_0 is the vacuum permittivity, and $\epsilon''(\omega)$ is the imaginary part of the dielectric constant. The factor $\Theta(\omega, T) = \hbar\omega/[\exp(\hbar\omega/k_B T) - 1]$ is the thermal energy of a quantum oscillator at temperature T, and $\hbar$ and k_B are Planck's constant divided by 2π and Boltzmann's constant, respectively. The Kronecker delta $\delta_{m,n}$, with subscripts $(m, n) = (x, y, z)$, and the Dirac delta function in space are consequences of isotropy, homogeneity, and locality, while the delta function in frequency, consistent with the Wiener-Khintchine theorem [6], follows from the stationarity of the current fluctuations.

The current fluctuations within the half-space $z < 0$ emit an electromagnetic field that propagates to the vacuum side $z > 0$. We may consider a single frequency component of the stationary polychromatic field. The electric field is calculated from the equation,

$$E_k(\mathbf{r}, \omega) = i\mu_0\omega \int_V G_{k,m}(\mathbf{r}, \mathbf{r}', \omega) j_m(\mathbf{r}', \omega)\, d^3 r', \quad (2)$$

where μ_0 is the vacuum permeability, and volume V stands for the half-space $z' < 0$. The Green tensor $G(\mathbf{r}, \mathbf{r}', \omega)$, which gives the field components emanating from the vectorial point currents, is conveniently expressed as a superposition of plane waves [7,10]. Refractions and reflections of the plane waves at the boundary are taken into account through appropriate Fresnel coefficients. Consequently, the (nonradiating) evanescent waves resulting from total internal reflections at the interface are fully incorporated in the theory. In addition, since the Green tensor is explicitly equipped with the Fresnel coefficients, all plasmon effects are also naturally manifested, as they correspond to the presence of a pole in the transmission coefficient for p-polarized light.

The elements of the 3×3 coherence matrix that entirely specify the state of polarization of an arbitrary electromagnetic field, at frequency ω, can be written as [1,6]

$$\langle E_k^*(\mathbf{r}, \omega) E_l(\mathbf{r}, \omega')\rangle = \Phi_{k,l}(\mathbf{r}, \omega)\delta(\omega - \omega'), \quad (3)$$

where $\Phi_{k,l}$ is a Hermitian, non-negative definite matrix. The values of its components are obtained by substituting Eqs. (1) and (2) into Eq. (3). After some algebra, the off-diagonal elements can be shown to vanish, and each diagonal element of matrix $\Phi_{k,l}$ can be expressed as an infinite integral over the magnitude of the lateral wave vector. For more details on the calculation, we refer to Ref. [11], where the full electric cross-spectral density matrix, at points $\mathbf{r}_1$ and $\mathbf{r}_2$, has been calculated in the same geometry. The coherence matrix $\Phi_{k,l}$ is obtained from that by taking the limit $\mathbf{r}_2 \to \mathbf{r}_1$.

In the two-dimensional coherence-matrix formalism, it is customary to express the degree of polarization by writing it with the help of the Stokes parameters. These quantities, which are related to the Poincaré sphere, completely characterize the polarization state of an electromagnetic field [1]. The Stokes parameters are the coefficients in the expansion of the 2×2 coherence matrix in terms of the 2×2 unit matrix and the Pauli matrices, the generators of the special unitary symmetry group SU(2). However, the two-dimensional formalism is not adequate for the description of the polarization statistics of arbitrary electromagnetic waves, such as optical near fields, since three orthogonal components of the vectorial field are present. Consequently, the degree of polarization of the near fields must be extracted directly from the 3×3 coherence matrix. In analogy with the two-dimensional case, we expand the three-dimensional coherence matrix in terms of proper basis matrices, for which we choose the 3×3 unit matrix and the Gell-Mann matrices, the eight generators of the SU(3) symmetry group [12]. We write the expansion in the form [1]

$$\Phi(\mathbf{r}, \omega) = \frac{1}{3}\sum_{j=0}^{8} \Lambda_j(\mathbf{r}, \omega)\lambda_j, \quad (4)$$

where λ_0 is the unit matrix, and λ_j ($j = 1 \ldots 8$) are the Gell-Mann matrices. The coefficients $\Lambda_j(\mathbf{r}, \omega)$ are interpreted as the Stokes parameters, at frequency ω, of an arbitrary electromagnetic field at a point specified by $\mathbf{r}$. The basis matrices are Hermitian, linearly independent, and trace orthogonal. For them the following equation holds:

$$\mathrm{tr}(\lambda_j\lambda_k) = \begin{cases} 3, & \text{when } j = k = 0 \\ 2\delta_{j,k}, & \text{otherwise.} \end{cases} \quad (5)$$

On multiplying both sides of Eq. (4) by λ_k, taking the trace, and making use of Eq. (5), we can write the Stokes parameters in the form

$$\begin{aligned} \Lambda_0(\mathbf{r}, \omega) &= \mathrm{tr}[\Phi(\mathbf{r}, \omega)], \\ \Lambda_j(\mathbf{r}, \omega) &= \tfrac{3}{2}\,\mathrm{tr}[\lambda_j\Phi(\mathbf{r}, \omega)], \qquad (j > 0). \end{aligned} \quad (6)$$

It should be noted that we could have chosen other complete sets of 3×3 matrices for the basis and then identified the expansion coefficients as Stokes parameters [13,14]. However, the definitions adopted above conveniently lead to the first Stokes parameter $\Lambda_0(\mathbf{r}, \omega)$ being equal to the spectral density of the field, as well as the other parameters having physical interpretations analogous to those in the two-dimensional case.

We write the degree of polarization P of an arbitrary field in the form

$$P^2(\mathbf{r}, \omega) = \frac{1}{3}\left(\sum_{j=1}^{8} \Lambda_j^2(\mathbf{r}, \omega)\right) \Big/ \Lambda_0^2(\mathbf{r}, \omega). \quad (7)$$

By substituting the Stokes parameters from Eq. (6) into this equation, we find that [15]

$$P^2(\mathbf{r}, \omega) = \frac{3}{2}\left[\frac{\mathrm{tr}[\Phi^2(\mathbf{r}, \omega)]}{\mathrm{tr}^2[\Phi(\mathbf{r}, \omega)]} - \frac{1}{3}\right]. \quad (8)$$

The degree of polarization is invariant under unitary transformations, because only traces of different powers of the 3×3 coherence matrix are involved. Moreover, since $\Phi(\mathbf{r}, \omega)$ is a Hermitian matrix, it can be diagonalized by a unitary transformation, and we may readily express the degree of polarization with the eigenvalues of the coherence matrix. By doing so and noting that all eigenvalues are non-negative, since $\Phi(\mathbf{r}, \omega)$ is non-negative definite, we see that P is bounded to the interval $0 \le P(\mathbf{r}, \omega) \le 1$.

We remark that Eq. (8), for the degree of polarization had, in fact, been put forward in the literature already a couple of decades ago [14,16–18]. However, in those papers, the emphasis was on identifying appropriate scalar invariants of the coherence matrix to be employed as measures for the degree of polarization. Except for Ref. [17],

the connection of the degree of polarization to the Stokes parameters acquired a somewhat lesser role. We note that the degree of polarization as defined in Eq. (8) assumes the value $P = 1$ of full polarization for fields which are fully polarized in the two-dimensional formalism as well. However, fields which are fully unpolarized in the 2D formalism are not that in the 3D formalism. This is intuitively clear, since the oscillations in the two-dimensional field are restricted to a single plane. Obviously, such a field cannot be fully unpolarized in the 3D formalism. In the three-dimensional formalism, the field is fully unpolarized, if its intensity in the x, y, and z directions is the same and no correlations exist between any of the three orthogonal field components. This isotropic and unpolarized field corresponds to that of blackbody radiation.

Let us now apply the general formalism to electromagnetic fields emitted by some thermal half-space sources. In Fig. 1 we illustrate the degree of polarization as a function of the distance from the surface at wavelength $\lambda = 620$ nm for gold and silver, at $\lambda = 500$ nm for lossy glass and tungsten, at $\lambda = 400$ nm for gold, and at wavelengths $\lambda = 11.36$ μm and $\lambda = 9.1$ μm for silicon carbide (SiC). We first note that glass does not support surface plasmons or phonons and, consequently, the degree of polarization in the near field for glass decays monotonically within a wavelength and settles down to a constant value. This indicates that the evanescent waves, which are strong only within $z < \lambda$, have a clear effect

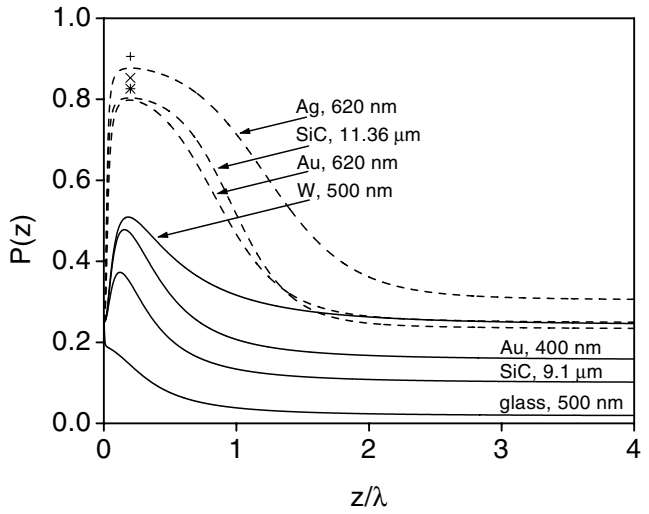

FIG. 1. Behavior of the degree of polarization as a function of distance z ($0 < z < 4\lambda$) from the surface for some materials at $T = 300$ K. Au at $\lambda = 620$ nm ($\epsilon = -9.1 + i1.2$) and at $\lambda = 400$ nm ($\epsilon = -1.1 + i6.5$), Ag at $\lambda = 620$ nm ($\epsilon = -15.0 + i1.0$), W at $\lambda = 500$ nm ($\epsilon = 4.2 + i18.1$), lossy glass at $\lambda = 500$ nm ($\epsilon = 2.25 + i0.001$), and SiC at wavelengths $\lambda = 11.36$ μm ($\epsilon = -7.6 + i0.4$) and $\lambda = 9.1$ μm ($\epsilon = 1.8 + i4.0$). Dielectric constants are from Ref. [19]. The symbols + (Ag), $\times$ (Au), and $*$ (SiC) denote the approximate values of P in the case of strong surface-wave effects, as discussed in the text.

on the near-field polarization. It can be shown that very close to the surface the degree of polarization approaches the value $1/4$, regardless of the material [11].

At the wavelength $\lambda = 620$ nm, both gold and silver have a plasmon resonance as their dielectric constants satisfy the relation $\mathrm{Re}(\epsilon) < -1$ [20]. Surface plasmons are confined electromagnetic modes due to collective oscillations of the free electrons in the metal. The oscillations propagate along the material surface, but decay exponentially in the direction perpendicular to it. The plasmon waves are known to be highly polarized in the plane spanned by their direction of propagation and the surface normal. Consequently, they strongly polarize the near field, which indeed is revealed in our analysis. The near-field degrees of polarization for both gold and silver have values as high as 0.80 and 0.88 at the distance of $z \approx 0.2\lambda$ from the surface, respectively. These values reflect the strength of the plasmon field in the vicinity of the gold and silver surface. We can also analytically obtain approximate values for the degree of polarization in the case when the surface-plasmon effects are strong [11]. In short, the approximation consists of taking all slowly varying terms at the plasmon pole value out of the integrals and noting that the relative values of the diagonal elements of the coherence matrix are related as $\Phi_{x,x}(\mathbf{r}, \omega) = \Phi_{y,y}(\mathbf{r}, \omega) = \Phi_{z,z}(\mathbf{r}, \omega)/2|\epsilon|$. The approximate values are plotted as symbols + and $\times$ in Fig. 1 for silver and gold, respectively. Although they do not exactly match the correct values, they are better the stronger the plasmon field. The difference between the exact and the approximate values follows from the fact that the exact coherence matrix has also an isotropic contribution due to the skin-layer currents [11]. On the other hand, for gold at wavelength $\lambda = 400$ nm for which $\mathrm{Re}(\epsilon) = -1.1$ the plasmon effect is greatly reduced. Similarly, in the case of tungsten at $\lambda = 500$ nm, for which $\mathrm{Re}(\epsilon) > -1$ and no surface plasmons exist, the peak in the near field is substantially smaller. Regarding the abrupt reduction of the degree of polarization immediately above the surface in the very near field, we point out that very close to the surface the so-called quasistatic field, which depends on the distance as $1/(kz)^3$, starts to dominate over the plasmon or any other effects [11]. Again, the degree of polarization approaches the value of $1/4$ in the limit $z \to 0$.

Finally, we analyze the near-field polarization of a sample of SiC at two wavelengths, $\lambda = 11.35$ and 9.1 μm. At the longer wavelength, SiC supports resonant collective lattice vibrations known as surface-phonon polaritons, whereas such a resonance does not exist at the 9.1 μm wavelength. We see that, as for gold, the near field is more polarized when a surface wave, in this case the surface phonon, is excited. The approximate value for the degree of polarization in the case of the strong phonon effect, calculated in the same way as previously for the strong plasmon effects, is plotted with the symbol $*$ in Fig. 1. It fits better to the exact value than the approximate

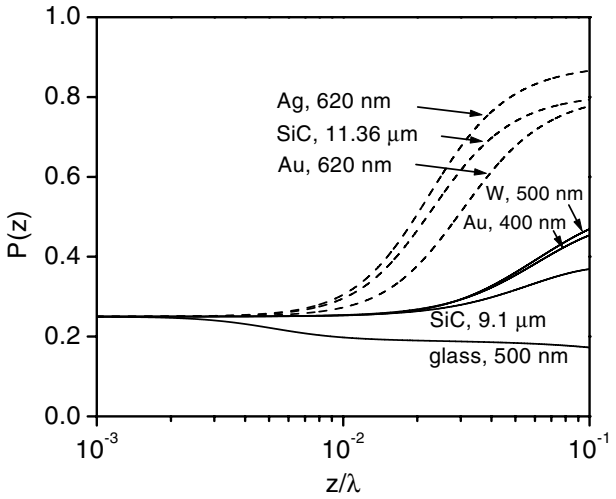

FIG. 2. The same as Fig. 1, but for the interval $10^{-3}\lambda < z < 10^{-1}\lambda$.

values for gold and silver, because for SiC the skin-layer currents are weaker.

Figure 2 illustrates in logarithmic z scale the behavior of the degree of polarization in the very near field, i.e., in the quasistatic region, for the materials studied above. We again see that for all materials the polarization degree smoothly approaches the value $P = 1/4$. This value can be analytically extracted by approximating the integrals in the coherence matrix for the case when the high spatial-frequency components dominate. For the mathematical details, we again refer to Ref. [11]. In the quasistatic region, each diagonal element of the coherence matrix turns out to depend on z as $1/(kz)^3$, as is expected for the intensity of the field in this region. Although the intensities in the x, y, and z directions depend on the material, their relative values do not. They are related as $\Phi_{x,x}(\mathbf{r}, \omega) = \Phi_{y,y}(\mathbf{r}, \omega) = \Phi_{z,z}(\mathbf{r}, \omega)/2$, which directly gives $P = 1/4$ in Eq. (8). As regards the far-zone values of the degree of polarization for the different materials, we note that, if the whole space would be filled with a thermal material, the radiation in it would be isotropic. However, the boundary surface breaks the isotropy, and the way it is broken depends on the materials that constitute the boundary. Thus the far-zone degree of polarization must be material dependent.

In summary, we have analyzed the polarization properties of electromagnetic near fields by extending the concept of the degree of polarization in 2D to three-dimensional fields. Our formula for the degree of polarization of an arbitrary electromagnetic field is consistent with the results put forward in the literature, and the numerical values are found to reflect the physical polarization behavior of near fields when surface waves are excited. Understanding of the polarization properties of optical near fields should greatly improve if the near-field Stokes parameters could be measured. For the two-dimensional fields, characterization of the state of polarization in terms of the four Stokes parameters is straightforward [1,6,21]. We hope our analysis inspires related studies of arbitrary electromagnetic fields, and optical near fields in particular.

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