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Effects of equalizing currents on electromagnetic forces of whirling cage rotor

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Abstract - **The electromagnetic force acts between the rotor and stator of an induction motor, when the rotor is performing cylindrical circular whirling motion with respect to the stator. The non-symmetric flux distribution due to the eccentricity induces circulating currents in the rotor cage and parallel branches of the stator winding. These currents tend to equalize the flux distribution, and by doing this, reduce the electromagnetic force and change the direction of the force from the direction of the shortest air gap.**

Impulse method is utilized in the finite element analysis to calculate the frequency response of the force and the eccentricity harmonics of the flux density in the air gap and the circulating currents. The frequency responses were calculated for the test motors with different stator connections and with and without rotor cage.

The frequency response function of the force is calculated for two induction motors with different geometry, one with open and another one with closed rotor slots. The results show that the equalizing currents in both parallel branches in the stator winding and rotor cage damp strongly the amplitude of the force. The damping effects depend on the whirling frequency, loading condition and the geometry of the machine.

I. INTRODUCTION

If the rotor of an electrical machine is mechanically centered within the stator, the net electromagnetic force will create a rotating moment about its axis. However, this is an unstable point and any mechanical deviation from this position will create a net radial force pulling the rotor even further out of alignment [1]. This force is usually called unbalanced magnetic pull (UMP).

In induction machines with small air gaps, small variations in the dimensions due to the manufacturing tolerances or rotor whirl near to critical speeds can lead to significant variation of the air gap between the stator and rotor at different angular positions. During operation of the motor, the non-uniform air gap distorts the flux density distribution, giving rise to additional harmonics of the flux density. These harmonics may induce circulating currents in the rotor cage and parallel branches of the stator winding. The circulating currents tend to equalize the flux distribution, and by doing this, they may significantly reduce the net radial force.

In a previous paper [2], these phenomena were studied in two special cases of whirling motion, static and dynamic eccentricity, by time-stepping finite element analysis. The present paper deals with the effects of the circulating currents on the electromagnetic forces when the rotor is performing cylindrical circular whirling motion with certain frequency, called whirling frequency and with certain radius, called whirling radius. The influence of the circulating currents in the parallel connected stator pole pairs and in the rotor cage on the radial forces is studied by calculating the frequency response function (FRF) between the electromagnetic force and the rotor displacement from the impulse response.

The FRF is calculated for series and parallel connected motors with and without rotor cage. The material of the aluminum rotor cage was changed to be air in order to obtain the forces without the damping effects due to the equalizing currents in the rotor cage at no-load condition.

In [2], it is shown that the equalizing currents cannot be induced neither in the rotor cage nor the parallel paths of stator winding if the rotor is performing symmetric conical eccentric motion and the interbar currents are neglected. In the symmetric conical motion, the center point of the rotor is concentric but both the ends of the rotor are performing circular whirling motion with 180 degrees phase shift. The present study is focused on the cylindrical whirling motion of the rotor.

The occurrence and effect of an eccentric rotor in electrical machines has already been discussed more than one hundred years [3] and the beneficial effects of parallel windings in reducing the UMP have been discussed almost as long [4].

Later on, the analytical methods have been applied widely to the problems associated with eccentricity. Freise and Jordan [5] derived equations for the forces caused by eccentricity using analysis of the air gap permeance harmonics. They used damping factors for taking into account the force reduction caused by equalizing currents of the rotor cage. They also noticed that these currents change the direction of the total force from the direction of the shortest air gap.

Smith and Dorrell [6] took the equalizing currents in the stator windings into account in their air gap permeance harmonic approach by resolving the stator windings into harmonic conductor density distributions. They also used a conformal transformation technique coupled to a winding impedance approach to investigate the effects of the different winding connections on UMP [7].

However, the effects of saturation and stator and rotor slottings are difficult to model by analytical means. Accurate analytical modeling of the equalizing currents is also difficult. Despite of the lacks of the analytical models, the numerical field-calculation methods have only rarely been used for analyzing eccentric rotors and the effects of equalizing currents on the forces. DeBortoli et al. [8] used a time-stepping method to study the equalizing currents set up by an eccentric rotor in the parallel circuits of stator windings. They also presented some results of force calculation. Arkkio [9] investigate the effects of the equalizing currents on the radial forces in his study.

The references given above focus on the two special cases of whirling motion, the static and dynamic eccentricity. Früchtenicht et al. [10] developed analytical tools to study cage induction motors in a cylindrical whirling motion. Arkkio et al. [11] studied the electromagnetic forces and the damping effects of the rotor cage in cylindrical whirling motion by time-stepping analysis. They also validated the numerical results by measurements.

II. Description of the Analysis

The calculation of the electromagnetic force acting in the induction motor is based on time-stepping, finite element analysis of the magnetic field. The details of the method have been presented in [12]. The magnetic field in the core region of the motor is assumed to be two-dimensional, and the two-dimensional field equation is discretized by the finite element method. The effects of end region fields are taken into account approximately by constant end-winding impedances in the circuit equations of the windings. The field and the circuit equations are solved together as a system of equations. The time-dependence of the field is modeled by Crank-Nicholson method. The magnetic field, the currents and the potential differences of the windings are obtained in the solution of the coupled field and circuit equations.

The forces are calculated by a method, based on the principle of the virtual work [13]. If the force is divided into a radial component in the direction of the shortest air gap and a tangential component perpendicular to the radial one, the components are almost independent of time (Fig. 1).

The impulse method is utilized in the finite element analysis to calculate the force between the stator and the rotor. The details of the impulse method are presented in [14].

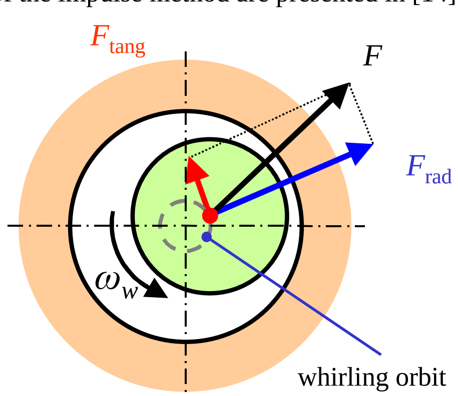

Fig. 1. Radial and tangential components of the force.

The basic idea of the impulse method is to move the rotor from its central position for a short period of time to one direction. This displacement excitation disturbs the flux density distribution, and by doing this, produces forces between the rotor and stator. Using spectral analysis techniques the frequency response function of the force is determined from the excitation and response signals.

The length of the rectangular displacement pulse in the simulation was 0.005 s. The amplitude of the static pulse was 15 % of the air gap length. Total simulation time was 1.0 s with constant time-step of 0.05 ms. To increase the spectral resolution, the sample size was extended to be 2 s by adding zeros to the end of the sample. This leads to frequency resolution of 0.5 Hz. The discrete excitation and force signals were transformed into the frequency domain by the FFT without filtering or windowing. The number of sample points used in FFT was 8192.

The frequency response function presents the electromagnetic forces per whirling radius as a function of whirling frequency. The assumption of the spatial linearity of the forces, which is shown to be valid for small values of rotor displacement in [10,11,15], is used in this study. The advantage of the impulse method is that from the results of one finite element analysis, the forces are obtained for a wide whirling frequency range.

The motion of the rotor is obtained by changing the one-layer finite element mesh in the air gap. Second order isoparametric, triangular elements were used. Several simplifications have been made to keep the amount of computation to a reasonable level. The magnetic field is assumed to be two-dimensional. The laminated iron core is treated as a non-conducting magnetically non-linear medium, and the non-linearity is modeled by a singe-valued magnetization curve. The homopolar flux, which may be associated with rotor eccentricity, is neglected.

III. Results

Two four-pole cage induction machines were chosen for test motors to study the damping effects of the equalizing currents on the forces acting between the rotor and the stator. The quarters of the cross-sectional geometry of the motors are shown in Fig. 2 and 3 and the main parameters of the motors are given in Table I. The main difference between these motors is that the 15 kW motor has open and the 37 kW motor closed rotor slots.

A. *Results of 15 kW motor*

Figure 4 shows the frequency response function (FRF) of the force for the series connected 15 kW induction motor without any damping effects. The material of the rotor cage is changed to be air in order to avoid the damping effects of the equalizing currents in rotor cage and for series connection the equalizing currents in stator side are neglible. The motor is running at no-load and supplied by rated sinusoidal voltage. The radial component of the FRF is constant and the tangential component of the FRF is almost zero, i.e. the direction of the force is the same as the direction of the shortest air gap at the studied whirling frequency range.

TABLE I

MAIN PARAMETERS OF THE MOTORS.

Parameter	15 kW	37 kW
Number of poles	4	4
Number of phases	3	3
Number of parallel paths	1	1
Outer diameter of stator [mm]	235	310
Core length [mm]	195	249
Inner diameter of stator [mm]	145	200
Airgap length [mm]	0.45	0.8
Number of stator slots	36	48
Number of rotor slots	34	40
Connection	Delta	Star
Rated voltage [V]	380	400
Rated frequency [Hz]	50	50
Rated current [A]	28	69
Rated power [kW]	15	37

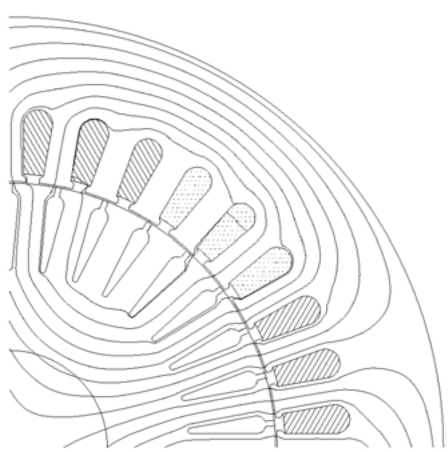

Fig. 2. Cross sectional geometry of the 15 kW motor.

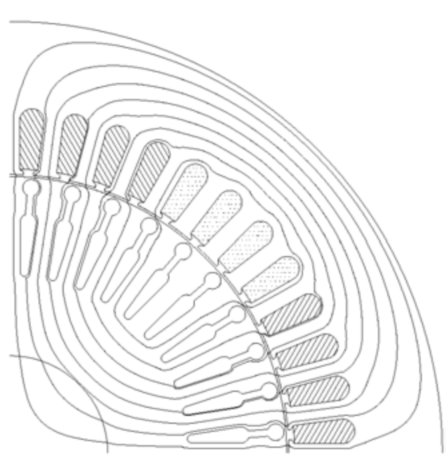

Fig. 3. Cross sectional geometry of the 37 kW motor.

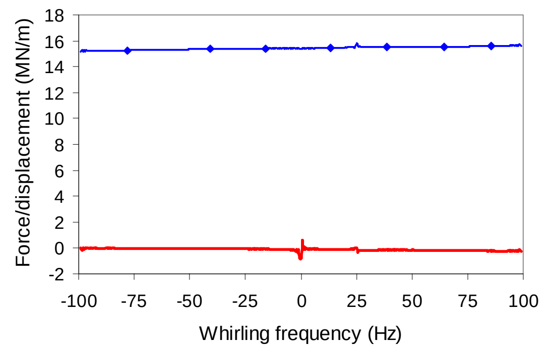

Figure 4. FRF of the force. The material of the rotor cage is air and stator is series connected.

radial component and tangential component

Figure 5 shows the damping effects of the parallel connected subwindings in the stator on the FRF of the force. The two parallel branches were obtained by connecting the subwindings on the two pole pairs in parallel. The equalizing currents that are now allowed to flow in the parallel branches clearly reduce the amplitude of the FRF. The radial component has two maximums at whirling frequencies –50 Hz and 50 Hz. The rotor eccentricity produces two additional harmonics into the flux density distribution in the air gap. At each maximum, another of these harmonics has a zero speed with respect to the stator and no equalizing currents are induced into the parallel branches of the stator winding. The direction of the force is deviates from – 50 to 35 degrees from the direction of the shortest air gap.

The damping effects of the rotor cage on the FRF of the forces when the stator is series connected are already studied in [11,14]. Figure 6 shows the FRF of the forces when the stator is series connected and the motor is running at no-load and supplied by the rated voltage. The harmonic rotor currents of order $p \pm 1$ (p is number of pole pairs of the motor) are zero at the same whirling frequency (25 Hz), no damping effects occur and the amplitude of the force reaches its maximum. At other whirling frequencies, the amplitude of the force is reduced up to 75 % from its maximum. The direction of the force deviates from – 65 to 65 degrees from the direction of the shortest air gap.

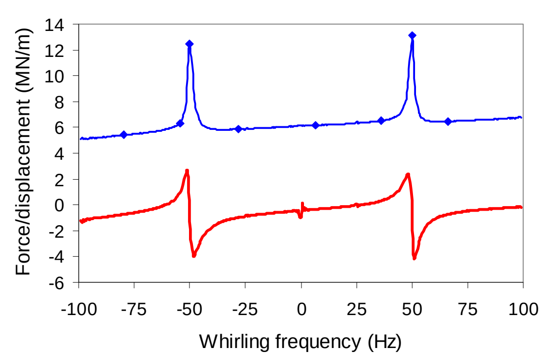

Figure 5. FRF of the force. The material of the rotor cage is air and stator is parallel connected.

radial component and tangential component

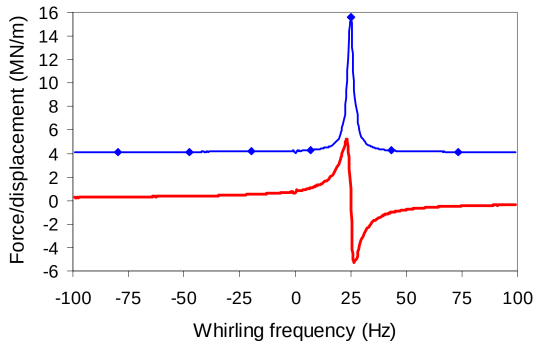

Figure 6. FRF of the force. The stator is series connected and the equalizing currents can flow in the rotor cage.
—♦— radial component and —— tangential component

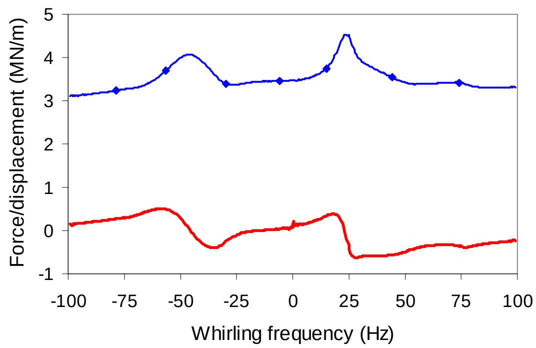

Figure 7. FRF of the force. The stator is parallel connected and the equalizing currents can also flow in the rotor cage.
—♦— radial component and —— tangential component

Comparison shows that the damping due to the equalizing currents flowing in the rotor cage (Fig. 6) is stronger than the corresponding effects of the parallel branches in the stator (Fig. 5).

If there are parallel-connected poles in the stator and a cage in the rotor, the equalizing currents can flow on both the stator and the rotor side. The FRF of the force in this case is presented in Fig. 7. The amplitude of the radial component is notably reduced and the tangential component varies close to zero level. The maximum deviation of the force from the direction of the shortest air gap is about 20 degrees.

The damping effects of the equalizing currents on the force can be investigated by studying the harmonics of the flux density and the circulating currents. They were also calculated by the impulse method as a function of whirling frequency. At each time step, the harmonics of order $p\pm1$ were calculated from the flux density and current distributions. The FRF's of the harmonics were calculated from the time history of the signals using spectral analysis technique, taking the coordinate systems of different harmonics into account.

The $p\pm1$ harmonic components of the flux density distribution in the air gap, which cause the force with the fundamental component of the flux density, are shown as a function of whirling frequency for a 50 µm whirling radius at no load in Fig 8 and 9 for the series and parallel connected motors.

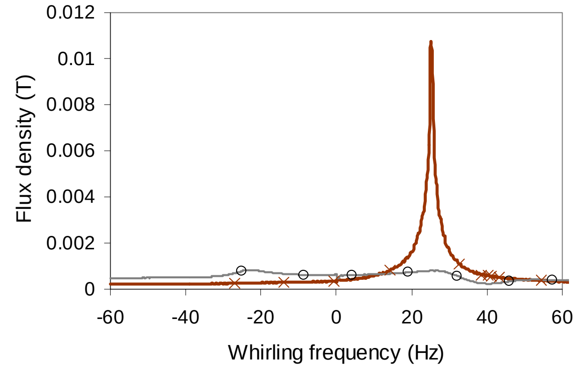

Fig. 8. Flux density harmonic of order p-1 as a function of whirling frequency.
—x— series and —o— parallel connection

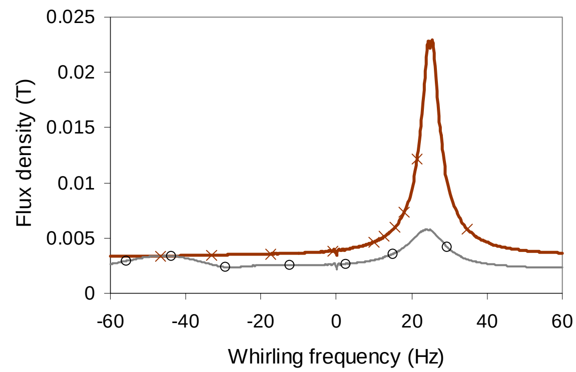

Fig. 9. Flux density harmonic of order p+1 as a function of whirling frequency.
—x— series and —o— parallel connection

The equalizing currents in parallel branches cause notable damping near to the synchronous frequency, at which equalizing currents in rotor do not damp the flux density harmonics.

Fig. 10 shows the p-1 harmonics and Fig. 11 the p+1 harmonics of the rotor bar currents for the series and parallel connected motors for a 50 µm whirling radius at no load condition and voltage 380 V.

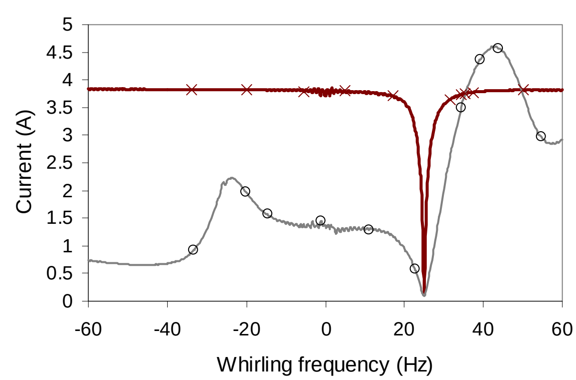

Fig. 10. Rotor current harmonic of order p-1 as a function of whirling frequency.
—x— series and —o— parallel connection

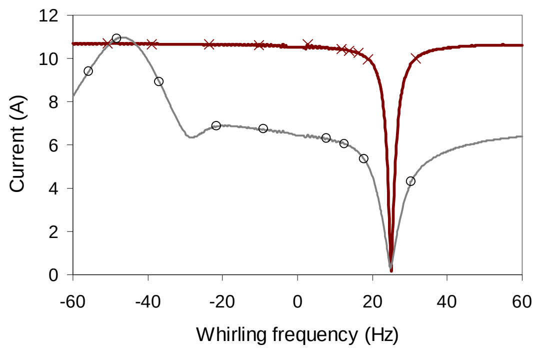

Fig. 11. Rotor current harmonic of order p+1 as a function of whirling frequency for 15 kW motor.
—x— series and —o— parallel connection

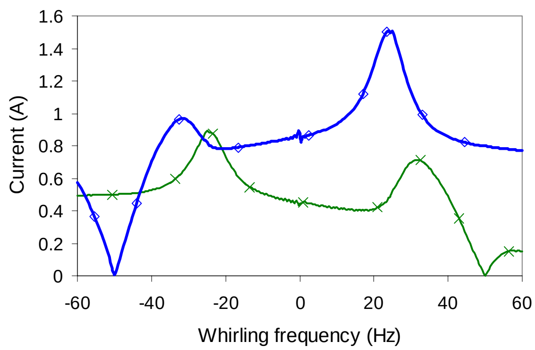

Fig. 12. $p\pm1$ harmonics of the stator current as a function of whirling frequency at no load condition.
—◊— p+1 and —x— p-1 harmonics

The equalizing currents $p\pm1$ of the stator side, at no load and supplied by 380 voltage, are presented as a function of whirling frequency in Fig. 12. The equalizing currents on the rotor and stator side are related. Near to 25 Hz whirling frequency, at which the slips of the $p\pm1$ harmonics in the air gap have zero slip with respect to the rotor and the rotor currents are then zero, the harmonic currents in the stator side have local maxims. Respectively, at the frequencies at which no equalizing currents are induced on the stator side, the rotor currents reach high amplitudes. The p-1 harmonic stator currents have interesting behavior around the whirling frequencies –25 Hz and 35 Hz. The reason for this behavior is not clear.

The effect of loading on the forces and damping effects is presented in Fig. 13, in which the forces are calculated at the rated load (slip s = 3,2%) for the series and parallel connected motors. Now, the rotor current harmonics are zero at different whirling frequencies [11] and for the series connected motor the FRF of the force has two maxims. The parallel branches in the stator windings reduce the force in the same way as it does for the no load condition. In general, the amplitude of the radial component of the force is higher for the loaded motor than at no-load, except for the series connection at no load when the maximum radial force is reached at the synchronous speed.

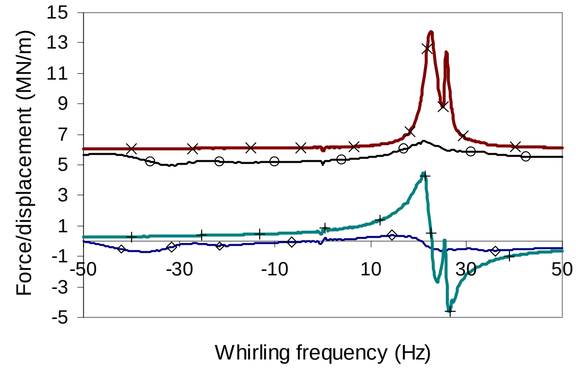

Fig 13. Radial and tangential components of the FRF of the forces at rated load for series and parallel connected 15 kW motor
—x— series radial component
—o— parallel radial component
—+— series tangential component
—◊— parallel tangential component

For the 15 kW motor, the maximum force is reached at the synchronous speed at no load condition if the motor has series connection. If the motor has parallel branches in the stator windings, in which the equalizing currents can flow, the maximum force acting between the stator and the whirling cage rotor depends on the load being near to the whirling frequency at which p+1 harmonic field is not damped by the rotor currents.

B. *Results of 37 kW motor*

The second test motor, 37 kW induction motor, differs from the first one by having closed rotor slots. The FRF of the force is calculated for this motor at the rated voltage (U = 400 V) at no load and rated load.

When the material of the rotor cage is changed to be air and the stator is series connected, the radial component of the FRF of the force is constant 10.2 MN/m. The tangential component is zero, as it was for the 15 kW motor. Fig. 14 shows the damping effects of the parallel connected stator on the FRF of the force when the material of the rotor cage is changed to be air for the 37 kW motor at no load.

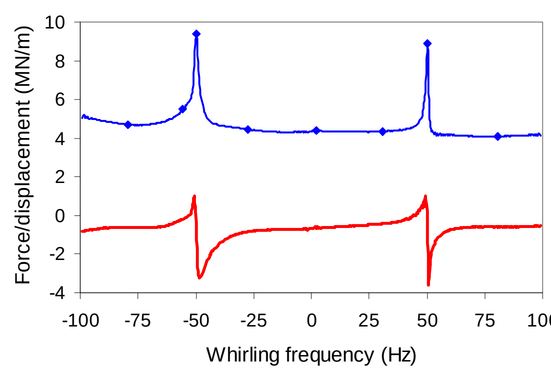

Figure 14. FRF of the force for 37 kW motor. The material of the rotor cage is air and stator is parallel connected.
—♦— radial component and —— tangential component

Fig 15 shows the FRF of the force for the series connected 37 kW motor with the normal aluminum-cast cage rotor. The damping effects of the rotor harmonic currents on the force are not as strong as they are for the 15 kW motor because the amplitude of the force is reduced only by 50 % from its maximum.

The FRF of the force for the 37 kW motor with parallel connection and normal rotor cage is shown in Fig. 16. The highest amplitude of the radial component is now reached at whirling frequency –50 Hz, at which the p+1 harmonic field does not induce currents into the stator windings. The peak in the radial component of the FRF due to the rotor side (whirling frequency 25 Hz) is much smaller than the peaks due to the stator current harmonics (±50 Hz).

The direction of the force with respect to the smallest air gap depends on the connection of the motor. When the motor is series connected and the material of the rotor cage is air, the direction is toward the minimum air gap. With the normal aluminum cast cage rotor, the direction of the force varies between –35 and 37 degrees from the minimum air gap. For the parallel connected motor, the phase angle between the minimum air gap and the direction of the force varies from –42 to 10 degrees for the normal motor and from –60 to 20 degrees for the motor without a rotor cage.

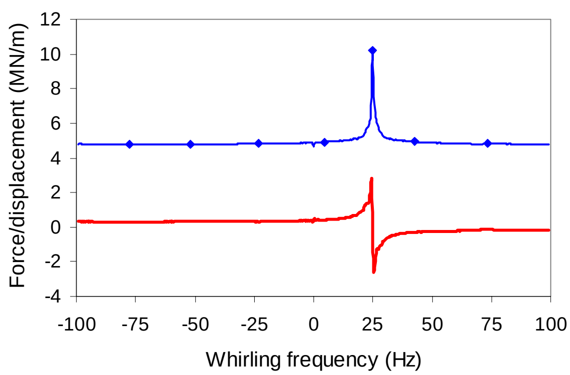

Figure 15. FRF of the force for 37 kW motor. The stator is series connected and the equalizing currents can flow in the rotor cage.
radial component and tangential component

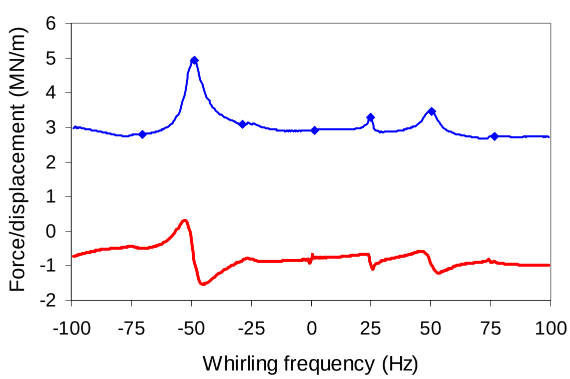

Figure 16. FRF of the force for 37 kW motor. The stator is parallel connected and the equalizing currents flows in the rotor cage.
radial component and tangential component

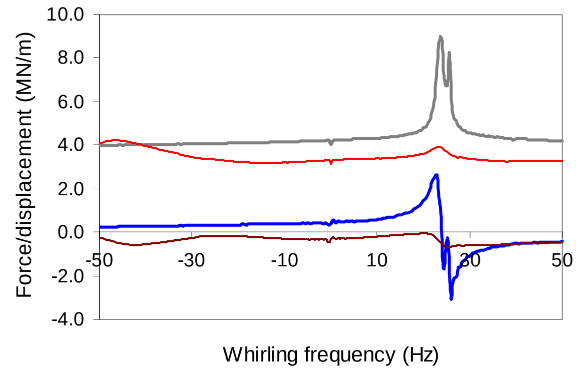

Fig 17. Radial and tangential components of the FRF of the forces at rated load for series and parallel connected 37 kW motor
series radial component
parallel radial component
series tangential component
parallel tangential component

The effect of loading on the FRF of the force is presented in Fig. 17, in which the forces are calculated at rated load (slip s = 1,6%) for the series and parallel connected 37 kW motors. The loading causes again two maxims on the FRF of the force for the series connected motor. According to the results presented in Fig. 15, 16 and 17, the amplitude of the force drops a bit with loading. The parallel branches in the stator winding reduce the amplitude of the force in the same way as at the no load condition.

C. Discussion of the results

The results agree well with the analytical theory [5,10] and numerical studies [11,14], which show that the amplitude of the force is reduced and the direction of the force is changed from the direction of the shortest air gap because of the equalizing currents.

The use of impulse method to calculate the harmonic currents in the rotor cage and in the stator windings may cause some error in the results. Especially, when calculating the harmonic components of the stator current, the accuracy in the definition of the different harmonic components with very small values from six independent variables (three-phase winding with two parallel subwindings) may cause some error on the results. However, the calculation method used is verified by comparing the forces, harmonic rotor currents and flux densities on the corresponding results calculated by conventional way and presented in [11]. The comparison shows very good agreement.

The comparison between the results of the test motors show that the harmonic currents in the rotor cage have notably stronger effects on the forces if the rotor has open slots than if the rotor slots are closed. The effects of the equalizing currents on the stator side are of about the same magnitude for both motors.

IV. CONCLUSION

The effects of equalizing currents on the electromagnetic force acting between the stator and whirling cage rotor are studied. The impulse method in the finite element analysis is used to calculate the frequency responses of the force, equalizing currents of the rotor cage and stator windings and magnetic fields in the air gap. The calculations are done for two induction motors, one with open and another one with closed rotor slots.

The results show clearly that the equalizing currents reduce the amplitude of the force and change the direction of the force from the direction of the minimum of the air gap. The stator connection and the motor geometry, especially the rotor slots influence strongly on the amplitude and the direction of the force acting between the stator and eccentric rotor.

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