Helsinki University of Technology Department of Electrical and Communications Engineering Laboratory of Electromechanics
Teknillinen korkeakoulu Sähkö- ja tietoliikennetekniikan osasto Sähkömekaniikan laboratorio
Espoo 2003 Report 69

ELECTROMAGNETIC FORCES ACTING BETWEEN THE STATOR AND ECCENTRIC CAGE ROTOR

Doctoral thesis

Asmo Tenhunen

Dissertation for the degree of Doctor of Science in Technology to be presented with due permission of the Department of Electrical and Communications Engineering, for public examination and debate in Auditorium S4 at Helsinki University of Technology (Espoo, Finland), on the 22th of August, 2003, at 12 o’clock noon.

ESPOO 2003

Distribution
Helsinki University of Technology
Laboratory of Electromechanics
P.O. Box 3000
FIN-02015 HUT
Tel. +358 9 451 2384
Fax. +358 9 451 2991
E-mail: electromechanics@hut.fi

ISBN 951-22-6682-2
ISSN 1456-6001

Picaset Oy
Helsinki 2003

Tenhunen, A., ”**Electromagnetic forces acting between the stator and eccentric cage rotor**”, Doctoral Thesis, Helsinki University of Technology, Laboratory of Electromechanics, Report 69, Espoo 2003, 40 p.

Keywords: induction motor, electromagnetic force, finite element analysis, impulse method, eccentricity, whirling motion.

Abstract

Electromagnetic forces act between the rotor and stator when the rotor is performing eccentric motions with respect to the stator. The aim of this research was to study the charactarestics of the forces and develop the tools to calculate these forces accurately and as quickly as possible.

A new method, called the impulse method, is developed into the finite element analysis of the electromagnetic field to calculate the forces for a wide whirling frequency range by one simulation. The idea of the impulse method is to move the rotor from its central position for a short period of time. This displacement excitation disturbs the magnetic field and, by doing this, produces forces between the rotor and stator. Using spectral analysis techniques, the frequency response function of the forces is calculated using the excitation and response signals. The impulse method is based on the assumption of the spatial linearity of the force.

The impulse method is utilised in the analysis of the rotor eccentricity. The spatial linearity of the force and the effects of the circulating currents and saturation on the forces are studied herein.

The field of investigation is enlarged from the cylindrical whirling motion to the conical motions of the rotor. The modelling of the conical motion requires that the axial variations of the magnetic field be taken into account. This is done by multislice finite element analysis.

Preface

This work was carried out in the Laboratory of Electromechanics at Helsinki University of Technology. It is part of a research project concerning the electromechanical interaction in electrical machines. The purpose of the work has been to develop the methods and study the forces, acting between the stator and eccentric rotor.

I would like to express my gratitude to my supervising Professor Antero Arkkio, whose help and guidance made the start, development and completion of this work possible.

I owe a debt of gratitude to Emeritus Professor Tapani Jokinen, who has to carry on the responsibility on that I have even started my postgraduate studies. I would like to thank Timo Holopainen, Anouar Belachen, Tommaso Benedetti and Professor Asko Niemenmaa for their fruitful co-operation and discussions which helped me to overcome many obstacles.

I would like to thank all the members of the laboratory whose names are not mentioned here for making our working environment so pleasant and for their help with every day life.

Last but not least I would like to thank my loving wife Saara for her help, patience and support during this work.

The financial support of the Imatran Voima Foundation, Finnish Cultural Foundation and Tekniikan Edistämissäätiö is gratefully acknowledged.

Espoo, July 2003

Asmo Tenhunen

Contents

List of publications

The thesis consists of the overview and the following publications.

P1. Tenhunen, A. and Arkkio, A., “Modelling of induction machines with skewed rotor slots”, *IEE Proceedings Electric Power Applications*, Vol. 148, No. 1, January 2001, pp. 45-50.

P2. Tenhunen, A., “Finite-element calculation of unbalanced magnetic pull and circulating current between parallel windings in induction motor with non-uniform eccentric rotor”, *Proceedings of Electromotion'01*, June 19-20, 2001, Bologna Italy, pp. 19-24.

P3. Tenhunen, A., Holopainen, T. P. and Arkkio, A., “Impulse method to calculate the frequency response of the electromagnetic forces on whirling cage rotors”, *IEE Proceedings Electric Power Applications*, accepted to be published.

P4. Tenhunen, A., Holopainen, T. P. and Arkkio, A., ”Spatial linearity of unbalanced magnetic pull in induction motors during eccentric rotor motions”, *COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, Vol. 22, No. 4, 2003, accepted to be published.

P5. Tenhunen, A., Holopainen, T. P. and Arkkio, A., “Effects of saturation on the forces in induction motors with whirling cage rotor”, *Proceedings of Compumag'03*, Vol. II, July 13-18, 2003, Saratoga Springs, NY, USA, pp. 66-67.

P6. Tenhunen, A., Holopainen, T. P. and Arkkio, A., ”Effects of equalising currents on electromagnetic forces of whirling cage rotor”, *Proceedings of IEMDC'03*, Vol. 1, June 1-4, 2003, Madison, WI, USA, pp. 257-263.

P7. Tenhunen, A., Benedetti, T., Holopainen, T. P. and Arkkio, A., ” Electromagnetic forces of the cage rotor in conical whirling motion”, *IEE Proceedings Electric Power Applications*, accepted to be published.

P8. Tenhunen, A., Benedetti, T., Holopainen, T. P. and Arkkio, A., ”Electromagnetic forces in cage induction motors with rotor eccentricity”, *Proceedings of IEMDC'03*, Vol. 3, June 1-4, 2003, Madison, WI, USA, pp. 1616-1622.

List of principal symbols

$\boldsymbol{A}$	magnetic vector potential
$\boldsymbol{B}$	magnetic flux density
B_ϕ	tangential component of the flux density
B_r	radial component of the flux density
$\boldsymbol{D}$	electric flux density
$\boldsymbol{E}$	electric field strength
$\boldsymbol{e}_z$	unit vector in the z-direction
$\boldsymbol{F}$	force vector
F_1	forces acting on the magnetic bearing
F_2	forces acting on the magnetic bearing
$\boldsymbol{G}_{\mathrm{J}}$	Jacobian matrix
$\boldsymbol{H}$	magnetic field strength
i	imaginary unit
$\boldsymbol{J}$	electric current density
k_0	parameter of the force model
k_{p+1}	parameter of the force model
k_{p-1}	parameter of the force model
l_1, l_2	distance from the magnetic bearing to the centre of the rotor
M	moment of force
$\boldsymbol{n}$	unit normal vector
p	number of pole pairs
p_{c}	rotor displacement
r	radius
r_{r}	inner radius of the air gap
r_{s}	outer radius of the air gap
S	integration surface
s	slip
S_{ag}	cross sectional area of the air gap
t	time
U	voltage
W_{c}	coenergy functional
z_{p+1}	parameter of the force model
z_{p-1}	parameter of the force model

$\boldsymbol{\sigma}$	Maxwell's stress tensor
ϕ	electric scalar potential
ε	permittivity
μ_0	permeability of free space
μ_r	relative permeability
ν	reluctivity
ρ	electric charge density
σ	conductivity
φ	angle
Ω	integration area
ω_w	whirling frequency

Abbreviations

AMB	Active magnetic bearings
BEM	Boundary element method
FEA	Finite element analysis
FEM	Finite element method
FRF	Frequency response function
RNM	Reluctance network method

1. Introduction

1.1 Background of the study

Electrical machine designers and researchers in industry face increasing demands to consider many physical effects more accurately when they give birth to the next generation of electrical energy converters, such as electrical machines.

The electrical machines used in variable speed drives and generators are mounted with diesel motors on common flexible baseframes. Rotation speeds are increasing and the operation at critical speeds has become an important aspect. Simultaneously, the machines should be lighter, noiseless, and more reliable and user friendly.

When designing the construction of an electrical machine, the designer has to be aware of the forces acting on the rotor. In addition to the weight of the rotor and the forces caused by the load, the electrical machine itself may generate electromagnetic forces that have to be supported by the bearings.

The radial forces acting upon the surface of the rotor are very large but are cancelled out when the rotor is concentric with the stator. Similarly, the tangential forces are balanced so that only a torque on an axially rotating moment is produced. If the rotor becomes eccentric, unbalanced magnetic pull (UMP) occurs. The phenomenon can be described as an imbalance of the radial and tangential forces acting upon the rotor (or stator) surface so that a net radial force is developed. The imbalance of the forces can result in vibration and noise, particularly if exacerbated by a flexible shaft, culminating in the possibility of the stator and rotor touching. It can also lead to a reduction in the critical speed, which is an important aspect in high-speed machine design.

Induction machines, which usually have small air gaps, are especially vulnerable to slight variations in the dimensions of the stator, rotor, end rings and bearings which can lead to significant variation of the air gap between the stator and rotor at different angular positions. Taking into account the manufacturing tolerances, one can assume that the rotor is hardly ever perfectly concentric and aligned with respect to the stator, and some net force is acting between the rotor and stator.

The eccentric motions of the rotor, including stationary rotor displacement, can be described by whirling motions of the rotor. The cylindrical circular whirling motion means that the centreline of the rotor travels around the geometrical centreline of the stator in a circular orbit, with a certain frequency, known as the whirling frequency, and with a certain radius, known as the whirling radius. The often mentioned basic modes, static and dynamic eccentricity, are two special cases of whirling motion. The static eccentricity means that the rotor displacement is stationary with respect to the stator, i.e. the whirling frequency is zero. In dynamic eccentricity, the position of the minimum air gap rotates with the rotor, i.e. the whirling frequency is the same as the mechanical angular speed.

Knowledge of the electromagnetic forces at different whirling frequencies provides a good basis for designers and researchers with which to deal with some of the demands that exist for the next generation electrical machines.

1.2 Calculation of electromagnetic forces

The electromagnetic forces acting between the rotor and stator when the rotor is eccentric with respect to the stator have been the point of interest and mathematical models have been developed almost through the whole history of the electrical machines. The mathematical models can be applied in analytical calculations or numerical analysis. In this subchapter, these methods of analyses are introduced shortly. A deeper literature study is presented in Chapter 2.

1.2.1. Analytical methods

Analytical calculation of the electromagnetic forces has, until now, been the most widely used method for computing the electromagnetic forces. The basics of the theory of air gap permeance harmonics is more than 50 years old and it has been developed further and utilised because it has given suitable results. Another sometimes-used analytical method is the conformal transformation technique coupled to a winding impedance approach. The analysis of air gap permeance harmonics is a general method, which can be used to calculate the electromagnetic forces or the force distribution in the machine's air gap. The conformal transformation technique is a more specific tool developed to calculate the forces when the rotor is eccentric.

The advantage of the analytical methods is that they rapidly give a general conception of the forces and their sources. The calculation times are very short but unfortunately a number of simplifications have to be performed in problem settings. Because of the simplifications and the nature of the analytical methods, some parts of the results, such as the frequencies of the force components, can be calculated very accurately but respectively, the amplitudes of these force components are usually just indicative. Despite the drawbacks, analytical methods are useful when studying the theory behind the behaviour of electrical machines.

1.2.2. Numerical methods

Progressive improvement in the power and speed of computers has resulted in a situation in which time-stepping finite element analysis of electrical machines is used as both a research and industrial tool. For most types of electrical machines the modelling is two-dimensional. Their potentially more accurate three-dimensional counterparts require one or two orders of magnitude more of computer resources, so they are still beyond the bounds of economic viability.

Force calculation using numerical methods has been a popular research topic during the last few decades, but the numerical field calculation methods have only rarely been used for analysing eccentric rotors.

The finite element method (FEM) has been used in all the numerical analyses of eccentric rotors. Two-dimensional FEM has very good accuracy when calculating the forces. Unfortunately, the required simulation times are very long in transient analysis before the steady state and reliable results are reached.

The other numerical methods the boundary element method (BEM) and the reluctance network method (RNM) have not been used to analyse electrical machines with eccentric rotors. BEM has been of growing importance in recent years. Many scientific papers have dealt with the applications of this method. However, there is little sign that BEM can be applied more efficiently for modelling electrical machines than FEM recently, but the intensive research about boundary element method might offer this possibility in the future.

The reluctance network method has not been at the top of the list of research methods over the last years but it may have some advantages over FEM. In particular, in the modelling of two-pole machines, in which rotor eccentricity may cause a notable homopolar flux, RNM may be even a better tool than the FEM. The homopolar flux can be taken into account in the RNM, but in the widely used formulation of the two-dimensional FEM, it is almost impossible.

1.3 Aim of the work

The main task in this study was to calculate the electromagnetic forces in induction machines acting between the stator and whirling cage rotor during different kind of whirling motions.

The main part of the study was developing the calculation methods. Analytically, the calculation of the forces is quite quick but the drawback of the analytical methods is the poor level of accuracy. Numerically, respectively, the accuracy is very good but the calculation of the forces has really been time consuming. One goal of the work was to develop a fast finite element tool for calculating the net force acting between the rotor and stator, which has a level of accuracy that is competitive with the analysis, used before.

Another main goal of this study was to create a finite element model, which calculates the axial distribution of the radial forces between the rotor and stator when the rotor is not aligned with the stator.

1.4 The scientific contribution of this work

The most notable scientific contributions of this study are listed below.

1. Modeling the three-dimensional effects of the motor using time-stepping, multi-slice finite element analysis.
2. Developement of the impulse method to calculate the electromagnetic forces acting between the stator and whirling cage rotor (together with Timo Holopainen, details of contributions are given in Section 1.5 Structure of the work in Publication 3).
3. Calculation of the electromagnetic forces when the rotor is not aligned with the stator.
4. New knowledge of the effects of equalising currents and saturation on the electromagnetic forces when the rotor is performing eccentric motions with respect to the stator.

Each item from the list above is discussed in sections presented in the publications.

1.5 Structure of the work

The research work accomplished for this thesis can be divided into the following major steps:

1. A detailed literature review in Chapter 2 is given to present the state of the art of the methods used to analyse a motor with an eccentric rotor and to calculate the electromagnetic forces acting between the stator and rotor.

2. Overview of the new methods in the finite element analysis developed in this study is presented in Chapter 3.
3. Measurement of the electromagnetic forces on the whirling cage rotor is presented in Chapter 4.
4. Results of calculations with the comparison with the measured forces are given in Chapter 5.
5. The calculation methods used and results obtained are discussed in Chapter 6.

In the following chapters, these steps will be presented and explained in details as separate entities. Steps from 2 to 5 are based on the publications. The publications are reprinted in the Publications chapter at the end of the thesis.

Publication P1

Publication P1 presents the multi-slice finite element model. Some of the three-dimensional effects of the motor are taken into account by enlarging the two-dimensional finite element analysis by taking a set of cross sections of the motor and coupling them together. The model is originally developed to model the effects of skewing in induction motors but it is also used to model the conical motions of the eccentric rotors. The model is validated by measurements. The results of the measurements and calculations show very good agreement.

There is nothing new in the idea of modelling the axial variations of the magnetic field in the motor with a set of the coupled two-dimensional models. In the multi-slice time-stepping model proposed in this publication, the field and electric equations are discretized and solved together. The main contribution of the presented model is the use of rotor bar currents as a variable in the model. In some cases, a straight use of bar currents as a variable is not possible because of singular matrices of the rotor cage impedances. The method for solving the problem of matrix singularity is presented in this publication.

The paper has been written by Asmo Tenhunen. The multi-slice model presented in the publication is built by Asmo Tenhunen. Professor Antero Arkkio, the supervisor of the thesis, contributed with his insight to build the finite element model.

Publication P2

Publication P2 is one of the first studies in which the electromagnetic forces acting between the stator and diagonal eccentric rotor are studied. In this paper, the force calculation routine used in this thesis was implemented into the multi-slice model. The publication is focused on two special cases of whirling motion, static and dynamic eccentricity. In the time-stepping finite element analysis, the position of the centre point of the rotor in each slice is forced to move along a path at constant speed to model the eccentric motion of the rotor. Later on, this method for analysing the eccentric rotor is called the forced whirling method.

The electromagnetic forces are calculated as a function of slip for three different types of eccentricity; 1) rotor is aligned with stator, 2) one end of the rotor is eccentric and another one is concentric, and 3) eccentricity in opposite directions at each end. The effects of parallel-connected poles in stator winding on the forces are also studied. The results show that the type of eccentricity and the connection of the stator winding have a strong influence on the forces. The results also show that if the interbar currents are neglected, the circulating currents cannot be induced due to conical motion (case 3), and the forces are independent of the whirling frequency. In the paper, the influence of the closed rotor slots on the magnetic field and forces are also briefly discussed.

The paper has been written by Asmo Tenhunen. The paper acknowledges Professor Antero Arkkio for his valuable help during the work.

Publication P3

The huge calculation times and requirements for computers have limited enthusiasm for studying the electromagnetic force acting between the stator and the whirling rotor. Publication P3 presents a new and fast method called the impulse method, applied to the time-stepping finite element analysis to calculate the forces for a wide whirling frequency range by one simulation. The idea of this method is to move the rotor from its central position for a short period of time. The rotor displacement excitation produces harmonic fields into the air gap and, by doing this, creates forces between the rotor and stator. Using spectral analysis techniques, the frequency response function of the forces is calculated from the excitation and response signals. The method is based on the assumption of the spatial linearity of the force. If the system is linear and there are no hidden sources inside the system, the transfer function, defined by impulse excitation, is exactly the same as the frequency response obtained by harmonic excitation, i.e. by the forced whirling method.

In the publication, the forces calculated by the impulse method are compared with the results got from conventional calculation. The two methods yield almost identical results. The new method is computationally very efficient requiring less than 5 % of the computational time of the conventional method when studying the forces as a function of the whirling frequency. The advantage of the impulse method is that it gives the forces for a wide whirling frequency range, including special cases, the static and the dynamic eccentricity, in one simulation.

The paper has been written by Asmo Tenhunen. Timo Holopainen discovered the original idea of the impulse method and developed the first working analytical implementation on which the presented impulse method is based. Professor Antero Arkkio, who has been the co-author of the paper, contributed with his valuable comments and insight.

Publication P4

The use of the impulse method in the force calculation is based on the assumption of the spatial linearity property of the force. The publication presents analytically the background of the assumption of the spatial linearity property of the forces and the limits within which the assumption is valid. Then, the spatial linearity is studied numerically utilising the impulse method in the finite element analysis to calculate the frequency responses of the forces at different values of amplitude of the pulse, supply voltage and slip. The assumption of linearity is valid if the frequency response of the force is independent of the amplitude of the excitation pulse.

The calculations are done for two different induction motors, one with open and another with closed rotor slots. The results show that the assumption of spatial linearity is usually valid for small values of the relative rotor displacement, but the closed rotor slots may break the linearity property in some operating conditions. Despite the exception with closed rotor slots, the paper offers corrobatitive evidence for the use of the impulse method for the calculation of the forces between the stator and the eccentric rotor.

Asmo Tenhunen has written the paper. The co-authors Timo Holopainen and Antero Arkkio contributed to the paper with their valuable comments.

Publication P5

Magnetic saturation influences strongly on the magnetic fields in electrical machines. In Publication P5, the effects of magnetic saturation on the electromagnetic forces are studied when the rotor is performing cylindrical circular whirling motion. The impulse method in the finite element analysis is utilised to calculate the forces for a 15 kW induction motor. The forces are calculated at different supply voltages in order to find out the effects of saturation on the forces. The maximum radial force is found to be limited by saturation and the saturation also couples the eccentricity harmonics together.

The paper has been written by Asmo Tenhunen. The co-authors Timo Holopainen and Professor Antero Arkkio contributed to the paper with valuable comments.

Publication P6

In Publication P2, the effects of circulating currents in the parallel branches of the stator windings on the forces were studied in two special cases of whirling motion. The impulse method, presented in Publication P3, provided a possible method for studying the effects of the circulating currents in a wide whirling frequency range. In publication P6, the damping effects of the equalising currents in the rotor cage and in the stator windings on the forces are studied utilising the impulse method. Based on the results presented in Publication P2, the paper focuses on the cylindrical whirling motion of the rotor.

The contribution of the publication is that it shows how the damping effects of the circulating currents on the forces depend on the geometry of the motor and it also presents the dependence between the whirling frequency and circulating currents when the stator pole pairs are connected in parallel.

Asmo Tenhunen has written the paper. The co-authors Timo Holopainen and Professor Antero Arkkio contributed with valuable advises during the work.

Publication P7

Publication P7 deals with the electromagnetic forces in induction machines when the cage rotor is performing different kinds of whirling motions. The studied whirling motions are cylindrical, symmetric conical and a combination of these two whirling motions. The studied motions describe the eccentric motions of the rigid rotor. The axial variation of the force acting between the stator and rotor performing a conical motion is taken into account by calculating the forces using the impulse method in the multi-slice finite element analysis. The force distribution in the axial direction in the conical motion is described by the net force and the moment of the force about the centre point of the rotor. The net force and the moment were measured for an induction motor using active magnetic bearings. The measurement is described in the publication.

The results presented in the publication show that the superposition principle is valid, the forces of the different kinds of whirling motions can be combined and the result is the force of the combined motion. The publication presents a low order force model to describe the forces acting between the stator and eccentric rigid rotor.

Asmo Tenhunen has written the paper. Tommaso Benedetti performed the measurements. Timo Holopainen contributed with precious comments on enlarging the parametric force model. Professor Antero Arkkio contributed with his vast experience with force calculation and measurements.

Publication 8

Publication P8 widens the study contained in Publication P7, in which both the cylindrical and symmetric conical whirling motions had the same whirling frequencies in the combined motion. Publication P8 however, focuses more on the measurements than Publication P7 does.

The main contribution of the publication is to show that the superposition is valid when combining the forces of basic modes of the whirling motions with small values of eccentricity, despite the whirling frequencies.

The paper has been written by Asmo Tenhunen. Tommaso Benedetti performed the measurements. Timo Holopainen and Professor Antero Arkkio contributed with their valuable comments during the work.

2. Overview of the electromagnetic field and force calculation

A short summary about the basic theory behind the electromagnetic forces and the most significant analytical and numerical methods in concern with the topic are presented in this chapter. The purpose of the following literature review is to highlight the major lines of research and achievements concerning the topic.

Laws governing electromagnetic fields can be expressed very concisely by a single set of equations, namely those associated with the name of Maxwell. The basic variables are the following

Electric field strength	$\boldsymbol{E}$
Magnetic field strength	$\boldsymbol{H}$
Electric flux density	$\boldsymbol{D}$
Magnetic flux density	$\boldsymbol{B}$
Electric current density	$\boldsymbol{J}$
Electric charge density	ρ
Time	t

The Maxwell relations may be cast in differential form:

$$\nabla\times\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{2.1}$$

$$\nabla\times\boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \tag{2.2}$$

$$\nabla\cdot\boldsymbol{D} = \rho \tag{2.3}$$

$$\nabla\cdot\boldsymbol{B} = 0 \tag{2.4}$$

To these differential relations are added the constitutive relations:

$$\boldsymbol{D} = \varepsilon\boldsymbol{E} \tag{2.5}$$

$$\boldsymbol{H} = \nu\boldsymbol{B} \tag{2.6}$$

$$\boldsymbol{J} = \sigma\boldsymbol{E} \tag{2.7}$$

These describe the macroscopic properties of the medium dealt with in terms of permittivity ε, reluctivity ν and conductivity σ.

For electrical machines, the reluctivity ν of the laminated iron core is field dependent and might be highly non-linear (Silvester and Ferrari, 1990). The polarisation and displacement current are assumed to be small compared with the conductive currents in the conductors

$$\frac{\partial \boldsymbol{D}}{\partial t} \ll \boldsymbol{J} \tag{2.8}$$

So, the last term of Equation (2.2) can be ignored. At the frequencies encountered in electrical machines this is a good approximation.

In the finite element formulation of electromagnetic field problems, magnetic vector potential $\boldsymbol{A}$ can play an important role. It is commonly used in the solution of two-dimensional magnetic field, because in that case it reduces to a single component variable. To satisfy the non-divergence of the magnetic field, the vector potential is defined so that its curl is equal to the magnetic field density

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{2.9}$$

Accordingly $\nabla \cdot (\nabla \times \boldsymbol{A}) = 0$ is satisfied for any $\boldsymbol{A}$. However, to define the vector potential uniquely, Equation (2.9) alone is not enough. In magnetostatic field problems the Coulomb gauge is usually used to specify the magnetic vector potential

$$\nabla \cdot \boldsymbol{A} = 0 \tag{2.10}$$

Equations (2.1) and (2.2) can be formulated by the use of the magnetic vector potential and Equations (2.6) and (2.8)

$$\nabla \times \boldsymbol{E} = -\frac{\partial (\nabla \times \boldsymbol{A})}{\partial t} = -\nabla \times \frac{\partial \boldsymbol{A}}{\partial t} \tag{2.11}$$

$$\nabla \times (\nu \nabla \times \boldsymbol{A}) = \boldsymbol{J} \tag{2.12}$$

This expression can be written into (2.13) where the appearance of $-\nabla \phi$ is no surprise since $\nabla \times (\nabla \cdot \phi) \equiv 0$. The symbol ϕ stands for the reduced electric scalar potential

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla \phi \tag{2.13}$$

Substituting (2.13) into (2.7), we get the equation concerning the current density (2.14)

$$\boldsymbol{J} = -\sigma \frac{\partial \boldsymbol{A}}{\partial t} - \sigma \nabla \phi \tag{2.14}$$

It satisfies the continuity condition

$$\nabla \cdot \boldsymbol{J} = 0 \tag{2.15}$$

The equations for the vector and scalar potential are obtained by substituting Equation (2.12) in Equations (2.14) and (2.15)

$$\nabla \times (\nu \nabla \times \boldsymbol{A}) + \sigma \frac{\partial \boldsymbol{A}}{\partial t} + \sigma \nabla \phi = 0 \tag{2.16}$$

$$\nabla \cdot \left(\sigma \frac{\partial \boldsymbol{A}}{\partial t} \right) + \nabla \cdot (\sigma \nabla \phi) = 0 \tag{2.17}$$

Equation (2.10) is required for the uniqueness of the solution.

In a general three-dimensional case, there are four unknown quantities, the components of the magnetic vector potential and the electric scalar potential in Equations (2.16) and (2.17). The unknown quantities depend on three spatial co-ordinates and time. Because of the complicated geometry of the electrical machines and the non-linearity of iron, solving the three-dimensional fields requires huge computer resources and computation times. The solution of the field equations becomes much easier if it is possible to use a two-dimensional model in which the material quantities are independent of the z-coordinate and the magnetic vector potential and current density are given by

$$\boldsymbol{A} = A(x,y,t)\boldsymbol{e}_z \tag{2.18}$$

$$\boldsymbol{J} = J(x,y,t)\boldsymbol{e}_z \tag{2.19}$$

in which x and y are the Cartesian coordinates and $\boldsymbol{e}_z$ is the unit vector parallel to the z-axis. The expressions of the vector potential and current density in (2.18) and (2.19) give satisfactory approximations for the field and current density in the core region of the unskewed electrical machines.

2.1 Calculation of the forces

The calculation of the electromagnetic forces has been a very popular research topic during the last decades. There are two basic methods used to calculate the forces acting between the rotor and stator, namely methods based on the Maxwell's stress and methods based on the principle of the virtual work. The basic equations for these methods are presented in this section.

Maxwell stress tensor

The methods based on the Maxwell stress tensor are commonly used in the calculation of forces and torques in the finite element analysis of electric devices (Reichert, Freundl and Vogt, 1976). The electromagnetic force is obtained as a surface integral

$$\boldsymbol{F} = \oint_S \boldsymbol{\sigma} \cdot dS \tag{2.20}$$

$$\boldsymbol{F} = \oint_S \left[\frac{1}{\mu_0} (\boldsymbol{B} \cdot \boldsymbol{n}) \boldsymbol{B} - \frac{1}{2\mu_0} \boldsymbol{B}^2 \boldsymbol{n} \right] dS \tag{2.21}$$

where $\boldsymbol{\sigma}$ is Maxwell stress tensor, $\boldsymbol{n}$ is unit normal vector of the integration surface S and $\boldsymbol{B}$ is magnetic flux density. In a two-dimensional model, the surface integral is reduced to a line integral along the air gap. If a circle of radius r_i is taken as the integration path, the force is obtained from the equation

$$\boldsymbol{F} = \int_0^{2\pi} \left[\frac{1}{\mu_0} B_r B_\phi \boldsymbol{e}_\phi + \frac{1}{2\mu_0} \left(B_r^2 - B_\phi^2 \right) \boldsymbol{e}_r \right] r d\phi \tag{2.22}$$

where B_r, B_ϕ are the radial- and tangential-components of the flux density. If the solution were exact, the force would be independent on the integration radius r_i when r_i varies within the air gap. However, the calculated force depends greatly on the choice of the integration radius. More reliable results are obtained if the line integral in Equation (2.22) is transformed to a surface integral over the cross section of the air gap (Arkkio, 1987)

$$\boldsymbol{F} = \frac{1}{r_\mathrm{s} - r_\mathrm{r}} \int_{S_\mathrm{ag}} \left[\frac{1}{\mu_0} B_r B_\phi \boldsymbol{e}_\phi + \frac{1}{2\mu_0} \left(B_r^2 - B_\phi^2 \right) \boldsymbol{e}_r \right] dS \tag{2.23}$$

where r_s and r_r are the outer and inner radii of the air gap and S_ag is the cross sectional area of the air gap. The drawback of the above method is the assumption of the rotational symmetry. In case of the rotor eccentricity, the previous method is not valid. At small eccentricities, however, an approximate solution can be calculated by using approximation $\Delta r(\varphi) = r_\mathrm{s}(\varphi)-r_\mathrm{r}(\varphi)$ (Antila, 1998).

Methods based on Maxwell's stress are commonly used in analytical studies with some simplifications. Supposing that the permeability of the core material is infinite, the flux lines enter and leave the stator and rotor surfaces perpendicularly. Maxwell's stress tensor $\sigma(x,t)$ is then calculated from the radial component of the flux density distribution in the air gap $B_r(x,t)$ (Früchtenicht, Jordan and Seinsch, 1982)

$$\sigma(x,t) = \frac{1}{2\mu_0}\left(B_r(x,t)\right)^2 \tag{2.24}$$

The flux density distribution can be solved by harmonic analysis of the air gap fields or by using the conformal transformation technique. The force is then obtained by integrating the Maxwell's stress tensor around the rotor (Früchtenicht et al., 1982).

Virtual work

Coulomb (1983) presented a method, based on the principle of virtual work, for calculating the forces from a finite element solution. In this method, the force is calculated as a partial derivative of the coenergy functional with respect to virtual movement

$$W_\mathrm{c} = \int_V \left(\int_0^{\boldsymbol{H}} \boldsymbol{B} d\boldsymbol{H} \right) dV \tag{2.25}$$

$$\boldsymbol{F} = \frac{\partial W_\mathrm{c}}{\partial \boldsymbol{p}} = \left[\frac{\partial W_\mathrm{c}}{\partial x} \frac{\partial W_\mathrm{c}}{\partial y} \right]^\mathrm{T} \tag{2.26}$$

where W_c is the coenergy functional and $\boldsymbol{F}$ is the force vector. The force in the direction of x is calculated as follows

$$F_x = \sum_e \int_{\Omega_e} \left[-\left[B_x \; B_y \right] \boldsymbol{G}_\mathrm{J}^{-1} \frac{\partial \boldsymbol{G}_\mathrm{J}}{\partial x} \left[H_x \; H_y \right]^\mathrm{T} + \int_0^{\boldsymbol{H}} \boldsymbol{B} d\boldsymbol{H} \left| \boldsymbol{G}_\mathrm{J} \right|^{-1} \frac{\partial \boldsymbol{G}_\mathrm{J}}{\partial x} \right] d\Omega \tag{2.27}$$

where the summation is done over virtually distorted finite elements. When applying this method to electrical machines, the summation is over the air gap elements (Antila, Lantto and Arkkio, 1998). $\boldsymbol{G}_\mathrm{J}$ is the Jacobian matrix, which couples the local coordinates and the global ones.

2.2 Analytical methods

According to von Kaene (1963) and Dorrell (1993), the occurrence and the causes of net radial force, or as it is traditionally called unbalanced magnetic pull (UMP), have been discussed for more than one hundred years (Fisher-Hinnen, 1899). As early as 1918, a review of work already carried out was published (Gray and Perch, 1918). Early papers, such as that of Rosenburg (1918) used B – H curves in order to try to calculate the imbalance of flux in the air gap and hence quantify UMP. Robinson (1943), Crawford (1951) and Covo (1954) were still using this method, making little distinction between the different non-salient machines.

Summers (1955) began to develop the theory of electromagnetic forces by using rotating field components in a two-pole machine. Based on the theory of rotating fields, Freise and Jordan (1962) derived equations for the forces caused by eccentricity. In these equations they used damping factors for taking into account the force reduction caused by equalising currents. They also noticed that these currents change the direction of the force from the direction of the shortest air gap. In addition, they discussed the effect of eccentricity forces on the critical speed of a rotor. Meiler et al. (1973) compared the forces measured for several asynchronous motors with the corresponding analytical results. The agreement was often quite bad. The authors longed for better methods of modelling the effects of equalising currents, stator and rotor slotting, leakage flux and saturation. Some years later, Jaenicke and Jordan (1976) presented equations for taking these factors approximately into account. The theory of the rotating fields in the air gap has become the most common analytical method to calculate the forces acting between the rotor and stator and it is widely used in the studies of the noise and vibrations in electrical machines (Ellison and Yang, 1974), (Heller and Hamata, 1977), (Timar, 1989), (Dorrell, 1993), (Maliti, 2000).

Tereshonkov (1989) used the rotating field theory when he enlarged the study of varying air gap length by taking the core ovalities into account in his investigation. He presented a table of spatial orders and angular frequencies at which the static and dynamic eccentricity and rotor and stator ovality cause forces.

Freise and Jordan (1962) showed that two-pole machines are special cases where homopolar flux exists. This flux crosses the air gap only once and returns via the shaft and casing. The theory of the rotating fields shows that the homopolar flux can exist in any machine but it is most likely to be significant in two-pole machines (Kovacs, 1977), (Belmans, Vandenput and Geysen, 1987a), (Belmans, Vandenput and Geysen, 1987b).

Swann (1963) gave an interesting solution of a series connected machine with a blank rotor. He used a conformal transformation of two eccentric circles to two concentric circles, hence transforming a machine with eccentric rotor and symmetrical windings to one with a concentric rotor but asymmetrical windings. Once the field in the air gap is solved in the latter, it can be mapped back to the former. Later on, Dorrell and Smith (1994) developed this technique coupled to a winding impedance approach to analyse induction motors with eccentric rotors. The use of the coupling enables the modelling of any kind of stator winding connections, including parallel branches in the stator winding. The conformal transformation technique has not become a widely used tool to analyse the rotor eccentricity and the forces due to the eccentricity.

A technique for representing rotating fields as space vectors was put forward by Kovács and Rácz (1959) and continued by Stepina (1970). Another space vector method developed more thoroughly by Eastham and Williamson (1973) and furthered by Smith and Dorrell (1996) allowed the calculation of an impedance matrix defining all stator and rotor currents and fields. Dorrell (1995) and (2000) extends the analysis much further allowing the non-uniform eccentricity to be modelled.

The method, based on the coupled magnetic circuit approach has been developed to model the effects of eccentricity during the last years (Toliyat et al., 2000a) (Joksimović et al., 2000). This model is derived by means of winding functions. The effect of eccentricity is included in calculations of machine inductances. This kind of model allows all harmonics of magnetomotive force to be taken into account, so it can be used to study the effects of eccentricity on the converter supplied motor or sinusoidal supplied motor in transient or steady state operation.

Most of the references given above focus on the two special cases of whirling motion, i.e. static and dynamic eccentricity. Früchtenicht et al. (1982) derived the equations for the forces in induction machines in cylindrical circular whirling motion. They also considered the equations of the motion of the rotor, and showed that the tangential component of the force has a significant effect on the stability of the motion.

2.3 Numerical methods

The most convenient way to analyse the effects of rotor eccentricity on the electromagnetic forces is to perform the finite element analysis (FEA). The basic theory of the finite element method and the use of it to analyse electrical machines is presented in (Arkkio, 1987), (Silvester and Ferrari, 1990) and (Salon, DeBortoli and Palma, 1990). A solution of the complete, three-dimensional magnetic field of an induction machine is still too large a task for present day computers. This means that only the eccentric rotors, which are parallel with the stator, can be modelled. Usually, some simplifications also have to be done to keep the amount of the computation at a reasonable level. The magnetic field is assumed to be two-dimensional. End-region fields are modelled approximately by constant end-winding impedance in the circuit equations of the windings (Arkkio, 1987).

The assumption of the two-dimensional field makes it very difficult to model the homopolar flux. Thus, the homopolar flux is usually neglected in the finite element analysis (Silvester and Ferrari, 1990).

Computation times are usually quite long when computing eccentric fields, because the whole cross section of the motor has to be modelled and the eccentricity creates notable harmonics into the rotor currents, which are not included in the computation of the initial guess. The lacks of the initial guess lead to long simulation times (Maliti, 2000).

The long simulation times and the size of the problem may be the reason why numerical methods have only rarely been used for analysing eccentric rotors (Stoll, 1997). Nevertheless, as long ago as 1982 Benaragama (1982) used finite element analysis, although without the rotor eddy currents, to study unbalanced magnetic pull in a synchronous generator.

DeBortoli et al. (1993) and Salon et al. (1992) used a time-stepping finite element method for studying the equalising currents set up by an eccentric rotor in the parallel circuits of the stator windings. They also presented some results of the force calculation.

Arkkio and Lindgren (1994) also used time-stepping FEA to study the unbalanced magnetic pull in a high-speed induction motor. They validated their calculated forces by measurements, in which they used magnetic bearings. In References (1996) and (1997) Arkkio studied unbalanced magnetic pull for motors with different kinds of rotor asymmetry, including eccentricity. His study includes both static and dynamic eccentricity and the effects of equalising currents in stator windings. The rotor abnormalities are also studied in (Bangura and Demerdash, 2000a) and (Bangura and Demerdash, 2000b) where the effects of rotor eccentricity on the currents, torque and losses are discussed from the condition monitoring point of view.

Arkkio et al. (2000) also studied electromagnetic forces, mostly in the cylindrical circular whirling motions by the finite element analysis and validated their results by measurements. Based on the results of their calculations, they developed a low order force model (2.28) which presents the forces as a function of the whirling frequency ω_w assuming that the force F is a linear function of the displacement p_c

$$F\left(i\omega_\mathrm{w}\right)=\left(k_0+\frac{k_{p-1}}{i\omega_\mathrm{w}-z_{p-1}}+\frac{k_{p+1}}{i\omega_\mathrm{w}-z_{p+1}}\right)p_C\left(i\omega_\mathrm{w}\right) \tag{2.28}$$

in which i is imaginary unit and k_0, $k_{p\text{-}1}$, k_{p+1}, $z_{p\text{-}1}$ and z_{p+1} are the complex valued parameters of the force model. The subscripts p-1 and p+1 refer to the respective eccentricity harmonics of the electromagnetic fields. Because the determination of the parameters of the force model (2.28) is done by finite element analysis, the effects of equalising currents, saturation and stator and rotor slotting are included in the model.

Numerical analyses are going further quite fast. Coupling between the mechanical system and the electromagnetic system has already done in (Ha and Hong, 2001) and (Holopainen, Tenhunen and Arkkio, 2002) for two-dimensional problems. Studying the coupled problems gives new aspects to analyse the rotor eccentricity. Thus, it is possible to explain the interaction between the rotor vibration and its resulting change of magnetic field (Ha and Hong, 2001).

Analytical methods are quite often used to study a background of the problem and to explain the results of numerical analysis. An interesting direction for analysis is to fit the results of numerical analysis into the simple parametric model (Arkkio et al., 2000) and use the parametric model as a tool of investigation. Holopainen et al. (2002) studied the electromechanical interaction by coupling the parametric force model and simple mechanical model. The comparison between the results of the complete numerical model and the new parametric model show very good agreement.

From the numerical studies of the electromagnetic forces acting on an eccentric rotor, the method based on the principle of virtual work is used in (Arkkio, 1987), (Arkkio and Lindgren, 1994), (Arkkio, 1996), (Arkkio, 1997) and (Holopainen et al., 2002). The methods based on the Maxwell stress tensor are used in (Maliti, 2000), (DeBortoli et al., 1993), (Salon et al., 1992) and (Ha and Hong, 2001).

2.4 Conclusions

At the beginning of this chapter, the basic laws and equations of electromagnetic fields are presented. The state of the art of the calculation of electromagnetic forces between the stator and the eccentric rotor is also discussed from the analytical and numerical point of view.

The occurrence and causes of net radial force have been discussed and analytical methods have been developed virtually throughout the entire history of electrical machines. However, numerical methods have only rarely been used for analysing eccentric rotors, perhaps because of the long simulation times. During the last decade numerical methods have started to displace analytical methods as a research tool. Both of the methods have their own advantages and disadvantages. Analytical methods obtain the results quickly but especially the amplitudes of the forces are very difficult to calculate accurately. Knowledge of the sources of the forces and good accuracy with the frequencies are clear advantages of analytical methods. The numerical results have very good accuracy but the analysis is usually very time consuming when dealing with rotor eccentricity.

One would wish to have the speed and the simplicity of the analytical methods and the flexibility and relatively straightforward applicability of the finite element method in one package.

In this thesis, the electromagnetic forces acting between the stator and eccentric rotor are studied by numerical means. This way, the accuracy of the force calculation can be maintained. The impulse method in the finite element analysis is utilised in the force calculation in order to obtain acceptable calculation times. The main advantage of the impulse method is that one simulation of multi-slice time-stepping finite element analysis gives the electromagnetic forces for a wide whirling frequency range. The use of the multi-slice model allows the axial variations in the air gap length to be taken into account. The calculation method is presented, discussed and validated in the following chapters of this thesis.

3. New methods of analysis

This chapter presents the main developments achieved for finite element analysis in order to reach the goals of the study.

The axial variation in the magnetic field is a problem in the two-dimensional FE analysis. Rotor skewing is a well-known source of axial variations. Usually the numerical models, which can handle the axial variations, are done for modelling the rotor skewing. The conical whirling motion of the rotor is also a source for the axial variations of the magnetic field. In this thesis, the multi-slice finite element analysis is used to take the axial variations of the field into account when the rotor is performing conical whirling motion. The multi-slice model is presented in Publication P1 and it is briefly presented in Section 3.1. The first time, when the multislice model is applied to the analysis of the rotor eccentricity is Publication P2.

As was mentioned in the previous chapter, the problem of the required magnitude of computer resources and the huge calculation times have limited enthusiasm for using numerical methods as a research tool when studying rotor eccentricity. The impulse method is an approach that improves the usability of the time-stepping finite element analysis presented in this thesis. The impulse method is presented in Publication P3 and briefly in section 3.2. The use of the impulse method is based on the assumption of the spatial linearity property of the electromagnetic force. The spatial linearity is studied in Publication P4.

3.1 Multi-slice model

The used multi-slice model was originally developed for analysing the effects of rotor skew in induction machines (Tenhunen, 2000). If the rotor slots are skewed, the magnetic field is no longer constant in the axial direction and these changes in the fields have to be included into the model some way or another. The use of 3D FEM is one solution, but the size of the problem and computation times limit the use of 3D modelling. An alternative method is to use a multi-slice model, in which 3D effects are included into the 2D FEM by taking a set of cross sections of the motor perpendicular to the stator axis and coupling the slices together. The idea of the technique is illustrated in Figure 3.1.

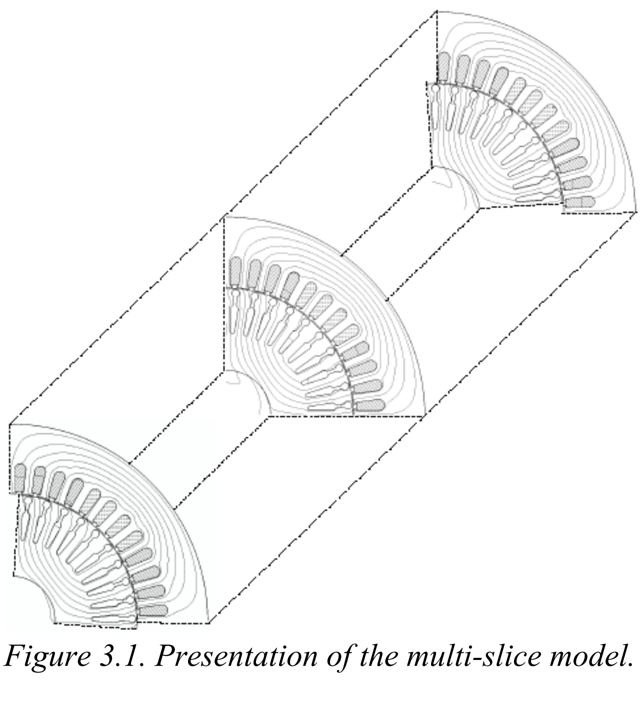


Figure 3.1. Presentation of the multi-slice model.

There are many variations of applying the multi-slice technique into the finite element analysis. Arkkio (1987) used a slice model with time-harmonic FEM to study skewed cage induction machines. Williamson (1990) used separate field and circuit models where the induced rotor bar currents were calculated using time-stepped circuit equations.

Gyselinck and Melkebeek (1995) and Boualem and Piriou (1998) solved the field and circuit equations together as a system of equations. They developed the model in magnetodynamic by coupling the magnetic and circuit equations for the stator and rotor. In the rotor side, the bar currents are unknowns but for a given bar, the current is the same in different machine sections. In these conditions, the coupling model implies the simultaneous solution of the coupled equations in the "*n*" disks. The unknowns are the nodal values of the vector potential, the slot currents in the stator side and bar voltages in the rotor side. These last ones are obtained by adding the electric scalar potentials calculated in each section. Ho, Fu and Wong (1997) built up their model almost in the same way, except that in their model, instead of the potential difference equations, the end winding circuit equations gave rise to the governing formulas in the rotor domain. Later on, they developed their model further by allowing the interbar currents to flow from bar to bar through the rotor package (Ho, Li and Fu, 1999).

In the present work, I propose a multislice time-stepped model, in which the field and circuit equations are solved together as a system of equations. The slices are coupled together by forcing the currents of the rotor bars and the stator winding to be continuous from slice to slice. The free variables of the system are the magnetic vector potential, rotor bar currents and stator currents. Normally in FEM calculations, stranded conductors have current excitation and solid conductors, such as rotor bars, have potential difference excitation. The continuity of the currents from slice to slice is included directly into the solved equations when the current excitation is used in the rotor bars. Unfortunately, the straight use of rotor bar currents as variables is not possible because of the singular matrices of the rotor cage impedances when the whole cross section of the motor is modelled. Publication P1 presents a method for avoiding the problem of matrix singularity.

Figure 3.2 shows how the conical motion of the rotor is modelled by the multi-slice technique. The motor is divided into the three slices with equal length. Each of these slices is modelled by a cross section taken from the middle of the slice. The centre point of the rotor in each of the slices is forced to move along a path defined by the whirling radius and frequency. The electromagnetic force acting between the stator and rotor can then be calculated as a slice force and the net force is the sum of the slice forces.

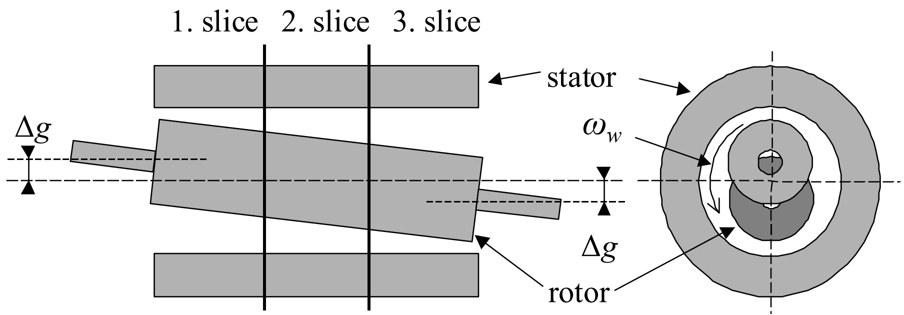


Figure 3.2. Conical motion of the rotor modelled by multi-slice technique. Δg is the whirling radius.

In the presented multi-slice model, the two-dimensional slices are coupled together by using the rotor bar currents as variables and forcing them to be continuous from slice to slice. This choice of variables prevents the inclusion of the interbar currents into the model.

3.2 Impulse method for force calculation

The background of the impulse method can be said to be the study made by Arkkio et al. (2000), in which the electromagnetic forces are calculated as a function of the whirling frequency. They modelled the whirling motion by forcing the centre point of the rotor to move along a circular path at a constant speed i.e. they used a harmonic excitation to create the forces. Because the harmonics of the rotor currents, caused by the eccentricity, are not taken into account in the initial guess of the magnetic field, long simulations are required before the steady state is reached and the forces are calculated accurately. Also, numerous simulations are needed before the forces, as a function of whirling frequency, i.e. the frequency response function (FRF) of the force, are determined.

In mechanical engineering, the vibration characteristics of a system are widely studied by defining the frequency response function using different kinds of excitation, including harmonic and impulse excitation. The transfer function $K(s)$ is a generalisation of the frequency response function $K(i\omega)$. If the system is linear and there are no hidden sources inside the system, the transfer function, defined by impulse excitation, is exactly the same as the frequency response, obtained by harmonic excitation (Ewins, 2000).

An impulse method is proposed to calculate the forces between the rotor and stator when the rotor is displaced from the centre point of the stator. For the force - displacement relation in electrical machines, the connection between the transfer function and frequency response function is presented in (Holopainen et al., 2003).

The basic idea of the approach is to move the rotor from its central position for a short period of time. This displacement excitation disturbs the flux density distribution and, by doing this, produces forces between the rotor and stator. Using spectral analysis techniques, the frequency response function is defined from the excitation and response signals. The force is then obtained from the frequency response function. Figure 3.3 illustrates the impulse method utilised in the force calculation.

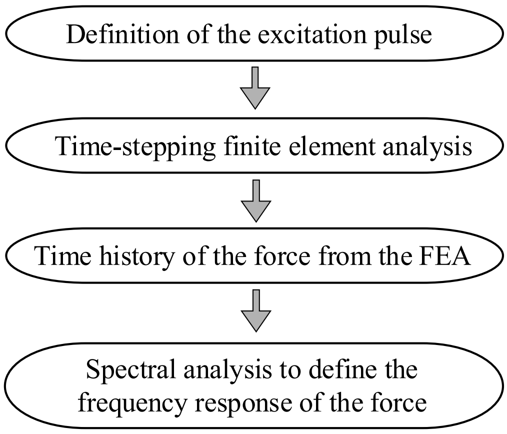


Figure 3.3. The flow chart of the force calculation by impulse method.

During the time-stepping finite element analysis, the rotor is moved from its central position for a short period of time. There are several possibilities to choose the type of excitation pulse. Regardless of the type of pulse, the parameters describing the pulse can be derived from the excitation requirements. First of all, it is required that all frequencies within the studied whirling frequency range are excited. This requirement defines the duration of the pulse. In the calculations, the length of the used excitation pulse was typically 0.01 s. The duration and amplitude of the pulse define the amount of energy which is given to the system. However, if the excitation pulse has too big amplitude, the forces are no longer linear functions of displacement. In this thesis, the amplitudes used for the excitation pulse have been 10-20 % of the air gap length.

The electromagnetic forces are calculated in the time-stepping finite element analysis at each time step. Despite having a goal of short calculation times, the required simulation time in the analysis is quite long. Typically, 1 s of simulation time was used in the analysis presented in this thesis. The advantage of the method is that one finite element simulation gives the forces for a wide whirling frequency range.

The frequency response of the force is calculated using spectral analysis technique as a ratio of response (time history of the forces) and excitation (excitation pulse in time domain), both transferred into the frequency domain. The frequency response function gives the electromagnetic forces per whirling radius as a function of the whirling frequency. Assuming the spatial linearity, the force for a specified whirling radius is easy to calculate from the frequency response function FRF by multiplying it by the whirling radius. The results of the verification of the impulse method are presented in Chapter 5.

Conclusions

In this Chapter, new methods of analysis are presented. The multi-slice model is originally developed to model the rotor skew. In this thesis it is used to model the conical motion of the rotor. In the multislice model, the 3D effects are included into the 2D FEM by taking a set of cross sections of the motor perpendicular to the stator axis and coupling the slices together.

An impulse method is developed to calculate the forces between the rotor and stator when the rotor is performing eccentric motions with respect to the stator. The advantage of the method is that from one finite element simulation the electromagnetic forces are obtained for a wide whirling frequency range.

4. Measurements

The developed methods of analysis are validated by measurements. In this chapter, the measurements of the electromagnetic forces on the whirling motion are briefly described. The measurement set-up and the procedure followed for the force measurement have been carried out by Arkkio et al. (2000). The measurements and the results of the measurements are presented in publications P7, P8 and discussed briefly in Chapter 5.

The test motor is a 15 kW four-pole cage induction motor. The main parameters of the motor are given in Table I and the quarter of the cross sectional geometry is shown in Figure 4.1. The test motor was equipped with active magnetic bearings (AMB) to measure the forces and to create the whirling motion of the rotor. Only radial bearings were installed. The electrical motor itself acts as an axial bearing. The radial bearings were ordinary eight-pole heteropolar bearings with bias-current linearisation. The magnetic-bearing operation and the parameters of this particular bearing type are listed by Lantto (1999).

TABLE I
THE MAIN PARAMETERS OF THE TEST MOTOR.

Parameter	
Number of poles	4
Number of phases	3
Number of parallel paths	1
Outer diameter of stator [mm]	235
Core length [mm]	195
Inner diameter of stator [mm]	145
Airgap length [mm]	0.45
Number of stator slots	36
Number of rotor slots	34
Connection	Delta
Skew	0
Rated voltage [V]	380
Rated frequency [Hz]	50
Rated current [A]	28
Rated power [kW]	15

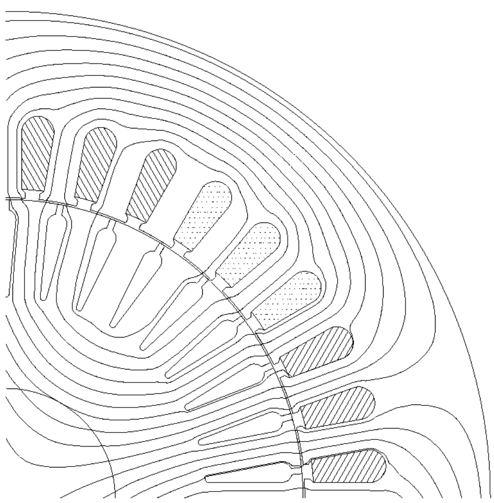


Figure 4.1. The Cross sectional geometry of the test motor.

Because of the mechanical tolerances, the test motor was not perfectly aligned with the magnetic position sensors in either the rotor or the stator. This misalignment is compensated for by defining the rotor trajectory by following the calibration procedure (Simon, 1999).

Before doing the force measurement for the conical rotor motions, one extra calibration was performed. The trajectory of the rotor was determined in such a way that the total force acting on the rotor was zero (i.e. $F_1 = -F_2$ in Figure 4.2). Then, the centre point of the rotor is concentric and the conical motion only causes the moment of the force about the centre point of the rotor.

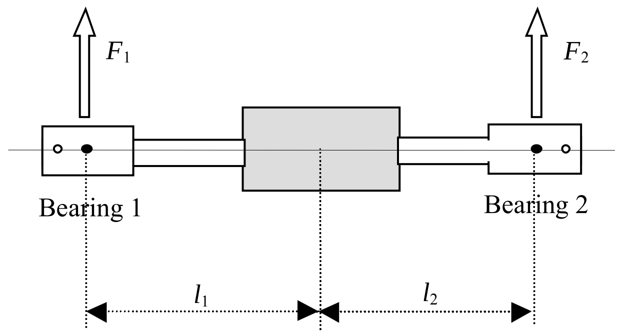


Figure 4.2. Axial rotor geometry of the test motor.
● *centres of the bearing magnets*
○ *centres of the position sensors*
⇨ *the forces acting on the magnetic bearings*

In Figure 4.2, F_1 and F_2 are the forces acting on the centres of the bearing magnets and l_1 = 0.2725 m and l_2 = 0.2475 m are the distances from the centres of the bearing magnets to the centre point of the rotor.

The effect of inertia forces was eliminated by subtracting the forces obtained in similar conditions without the voltage supply to the rotor from the forces obtained with the voltage supply.

According to the results presented in Reference (Arkkio et al., 2000), the measurement procedure seems to be accurate. As a result of the measurement, one gets the forces acting on both bearing magnets. In the measurement of the conical whirling motion, the force is assumed to act on the centres of the magnets.

Conclusions

In this chapter, the measurement of electromagnetic forces and the moment of the force about the centre point of the rotor is presented and discussed. The measurement is used for the validation of the methods of analysis and to verify the superposition principle of the forces.

The used measurement procedure seems to be accurate when the forces acting between the rotor and stator are measured. The use of active magnetic bearings gives the possibility of controlling the eccentric movements of the rotor accurately and the forces can also be measured at the same time.

5. Discussion of the results

In this chapter, the results of the calculations and measurements are introduced shortly and discussed. The chapter is based on Publications P2, P3, P4, P5, P6, P7 and P8. The verification results of the impulse method are presented at the beginning of this chapter. Then, the results of the analysis of the rotor eccentricity are discussed, focusing on the effects of the circulating currents and the magnetic saturation. Last but not least, in the final section in this chapter the results of the analysis when the conical motions of the rotor are studied.

Verification of the impulse method

The use of the impulse method in the calculation of electromagnetic forces, or in actuality the frequency response of the force, is verified by the conventional calculation in Publication P3. In the conventional calculation, called the forced whirling method, the centre point of the rotor is forced to move along a circular path with a certain radius and with a certain frequency (Arkkio et al., 2000). The method is really time consuming when the forces are studied as a function of the whirling frequency. The radial and tangential components of the electromagnetic force obtained by these two methods and presented in Figure 5.1 have excellent agreement. The forced whirling method required weeks of calculation but with the impulse method these results were obtained in a day. The details of the 15 kW induction motor for which the calculations were performed are given in the previous chapter.

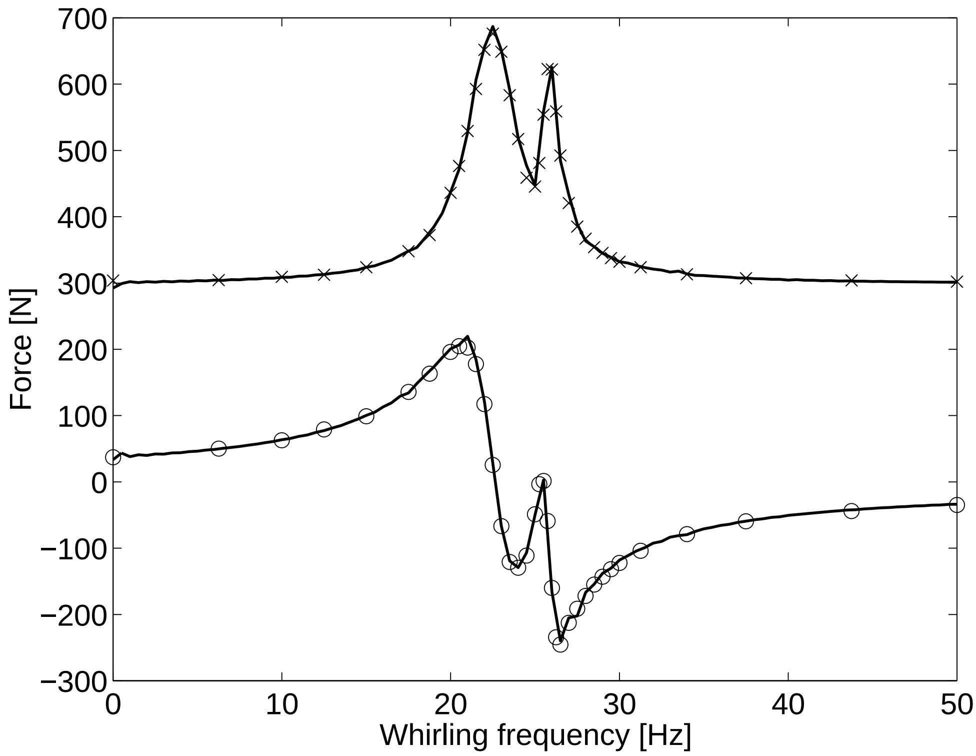


Figure 5.1. Electromagnetic forces calculated by a forced whirling and impulse method for a 15 kW induction motor at rated load (s = 3,2%). The results of the forced whirling method are marked by x for the radial and ○ for the tangential component. The continuous lines present the results of the impulse method.

The results obtained by calculations were validated by the measurements, presented in Publications P7 and P8, and also compared with the measured results presented by Arkkio et al. (2000). Agreement of the results is very good.

The used spectral analysis in the impulse method is quite simple. The length of the response signal is extended by adding zeroes to the end of the signal in order to increase the frequency resolution. Windowing or filtering is not used to improve the response signal. The exponential window might improve the accuracy, but the effect of windowing should be taken into account in the frequency response. The correction for the exponential windowing can only be done at a parametric force model level (Ewins, 2000). However, the results obtained without windowing already have very good accuracy, but with windowing, the required simulation times would be even shorter in reaching the same accuracy (Holopainen et al., 2003).

The impulse method is based on the assumption of the spatial linearity of the electromagnetic force. Spatial linearity is studied in Publication P4. The impulse method is very efficient in studying spatial linearity. If the force has the spatial linearity property, the frequency response of the force is independent of the amplitude of the excitation pulse. The results of the analysis presented in P4 show that spatial linearity is usually valid for small values of rotor displacement. However, the closed rotor slots may break spatial linearity in some operating conditions of the motor.

Equalising currents

An eccentric rotor creates an asymmetric flux distribution that causes the forces. The nonideal field may induce circulating currents in the rotor cage and parallel paths of the stator winding. These currents tend to equalise the flux distribution and they may significantly reduce the amplitudes of the radial forces. Publications P2 and P6 concentrate on these effects of equalising currents on the electromagnetic forces. Actually, these Publications concentrate on the effects of the circulating currents in the parallel connected stator pole pairs. The effects of circulating currents in the rotor cage on the forces have already been presented and discussed in Publication P3. According to the results presented in Publication P6, the circulating currents in the rotor cage have stronger damping effects on the amplitude of the force than the circulating currents in the stator windings.

In P2, it is shown that the equalising currents cannot be induced either in the rotor cage or in the stator windings if the rotor is performing a symmetric conical whirling motion and the interbar currents are neglected. This is the reason why Publication P6 focuses on the cylindrical whirling motion.

Figure 5.2 shows the frequency responses of the force for the series and parallel connected 15 kW induction motor at rated load (s = 3,2 %). The parallel connection is obtained by connecting the pole pairs in parallel. Figure shows that parallel connection clearly reduces the amplitude of the force and affects the direction of the force because the ratio of the radial and tangential components is changed. The damping effects of the circulating currents in the stator windings are strongest close to the whirling frequencies at which the currents in the rotor cage do not damp the forces.

In Publication P6, it is also shown that the geometry of the motor affects the circulating currents and the forces. The results show that the harmonic currents in the rotor cage have a notably stronger effect on the forces if the rotor has open slots than if the rotor slots are closed.

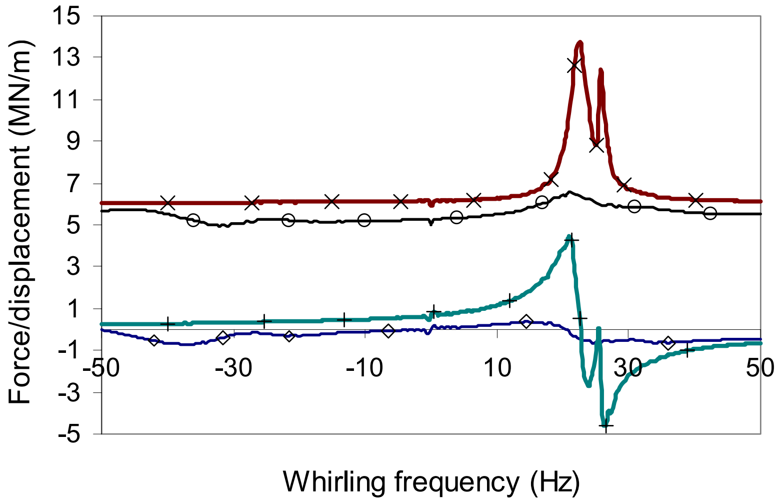
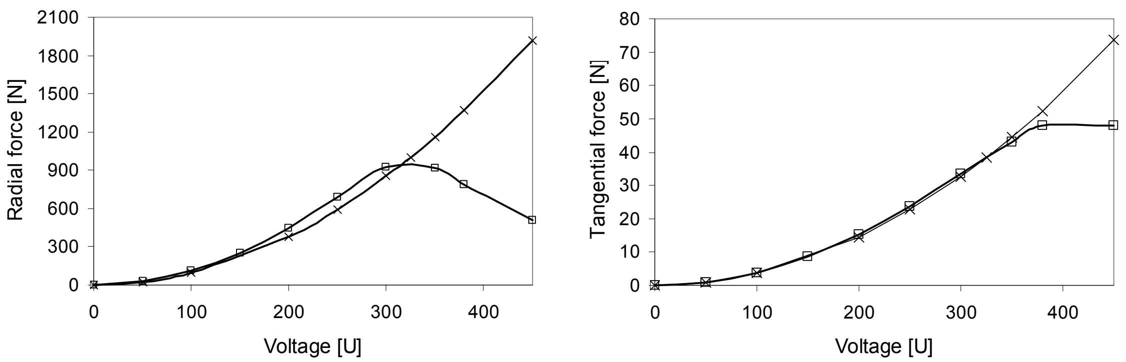


Fig 5.2. The radial and tangential components of the FRF of the force at rated load for series and parallel connected 15 kW motor.
—x— series radial component, —o— parallel radial component
—+— series tangential component, —◊— parallel tangential component

Magnetic saturation

Magnetic saturation strongly affects the magnetic field and because electromagnetic forces depend on the field, the saturation also affects the forces. In Publication P5, the saturation effects on the forces are studied numerically, using the developed method of analysis. According to the results, the radial force is found to be limited by the saturation and saturation also couples the eccentricity harmonics together.

Figure 5.3 shows a) the radial and b) the tangential component of the force at whirling frequency 25 Hz for the linearized and normal 15 kW induction motor at no load. For a linearized motor, the relative permeability of the iron core is constant $\mu_r = 1000$. The ratio of the tangential and radial force components changes with the voltage. This means that the saturation also affects the direction of the force.



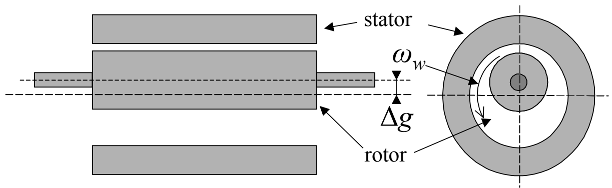
a) *b)*

Figure 5.3. The amplitudes of a) the radial and b) tangential component of the force as a function of supply voltage at no load at 25 Hz whirling frequency for the 15 kW induction motor at no-load condition. Marking: x – linearized, □ - normal.

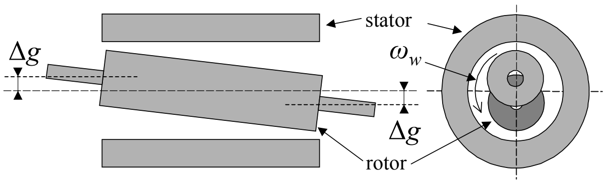
The effects of saturation are quite difficult to take into account analytically and the analytical models can only give an approximation of the saturation effects. If saturation is neglected in the analysis, the amplitude of the radial force is easily overestimated because of the behaviour of the force as a function of supply voltage, as is illustrated in Figure 5.3.

Forces in conical whirling motion

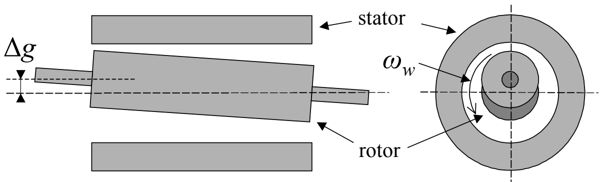
The eccentric motions of the rigid rotor are studied in Publications P2, P7 and P8. The cylindrical whirling motion, symmetric conical whirling motion or a combination of them describe the eccentric movements of the rigid rotor in induction machines. Figure 5.4 illustrates these eccentric motions of the rotor.



a) cylindrical whirling motion



b) symmetric conical whirling motion



c) a combination of the two whirling motions

Figure 5.4. The types of rigid rotor motions. Δg is the whirling radius of the rotor.

Publication P2 focuses on the static and dynamic eccentricities, but in Publications P7 and P8, the motions of the rotor are studied in more general whirling motion. The multi-slice model, presented in Publication P1, is utilised in the analysis of the conical motions of the rotor. The forces caused by these motions are also measured following the procedure presented in Chapter 4. The calculated and measured results are compared in Publications P7 and P8.

The cylindrical whirling motion causes net force acting between the rotor and stator as is presented in Sections above. When the rotor is performing a symmetric conical whirling motion, the net force between the rotor and stator is zero. The force distribution is described by a moment of force *M* about the centre point of the rotor. Figure 5.5 shows the calculated and measured moment *M* as a function of the whirling frequency for the 15 kW induction motor at no load and supplied by a 230 V voltage when the rotor is performing a symmetric conical whirling motion with a whirling radius 20 µm. The calculated moment is constant within the studied whirling frequency range but the measured moment depends on the frequency. The explanation for this behaviour may be the interbar currents. The test motor has an aluminium cast cage rotor. The resistance between the bars through the rotor sheet stack is not infinite and circulating currents can flow in the end parts of the rotor cage through the rotor stack from bar to bar. The used multi-slice model neglects the interbar currents.

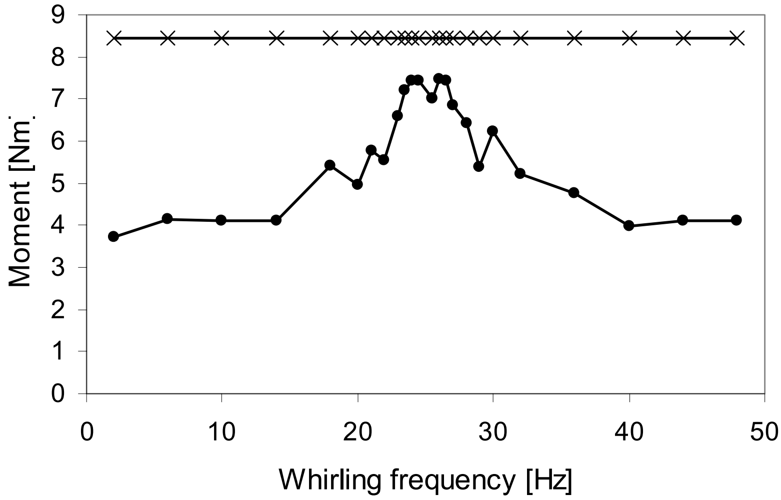


Figure 5.5. Moments in symmetric conical whirling motion with whirling radius 20 µm.
—●— *measured and* —x— *calculated*

One goal of the study of conical motions was to investigate the superposition for the forces when the rotor is performing a combined cylindrical and conical whirling motion. This means that the forces calculated for the cylindrical and conical motion can be superposed to obtain the force for the combined motion. Despite the difference between the measured and calculated moment of force, both the calculated and measured results show that the superposition is valid for combining the forces of different whirling motions. The results in Publication P8 also show that the superposition of the forces is independent of the type of whirling motion and whirling frequency. In fact, the result is to be expected because of the spatial linearity of the force.

Three slices are used in the multi-slice finite element analysis to model the conical motions of the rotor. A bigger number of slices would improve the accuracy but the size of the problem in the finite element analysis may become too large to solve.

Conclusions

In this Chapter, the main results of this thesis are presented and discussed briefly. The impulse method is verified by conventional calculation. The results show excellent agreement. The method is also validated by comparing the calculated results with the measured ones with very good agreement.

The impulse method is based on the assumption of spatial linearity of the force. Spatial linearity is studied by calculating the frequency responses of the forces with different excitation impulses. If the spatial linearity is valid, the frequency response is independent of the excitation pulse. It is shown that the forces are usually linear in proportion to the rotor displacement. However, the closed rotor slot may break the spatial linearity in some operating conditions of the motor.

The effects of equalising currents and saturation on the forces are studied. The equalising currents reduce the amplitude and change the direction of the force. The type of windings and the geometry of the machine affect the equalising currents. The saturation limits the maximum amplitude of the force and also affects the direction of the force. The saturation couples the eccentricity harmonics together.

The electromagnetic force in the symmetric conical whirling motion is presented by the moment of force about the centre point of the rotor. The electromagnetic forces of the different whirling motions can be superposed and the result is the force of the combined motion.

6. Summary

In the study, the electromagnetic forces acting between the stator and eccentric rotor are studied. The methods of analysis are developed in order to obtain accurate results for different whirling motions, including the conical motion, of the rotor. The multi-slice finite element analysis is used to take the axial variations of the magnetic field into account. The axial variations are considered by coupling a set of slices cut by planes perpendicular to the shaft. The slices are connected together by forcing the rotor-bar and stator-winding currents to be continuous from slice to slice.

The impulse method is developed to calculate the frequency response of the force. The impulse method is applied to the finite element analysis by moving the rotor from its central position for a short period of time. The displacement excitation disturbs the magnetic field and produces the forces between the rotor and stator. The frequency response of the force is calculated from the excitation and response signals. The advantage of the impulse method is that in one simulation the forces are obtained for a wide whirling frequency range. The forces calculated by the impulse method are compared with those calculated by conventional computation. The results show very good agreement.

The use of the impulse method is based on the assumption of the spatial linearity of the force. Spatial linearity is investigated by calculating the frequency responses of the force for different amplitudes of excitation pulses. If the system is linear, the response is independent of the excitation pulse. The results show that spatial linearity is usually valid for small values, up to 20 – 40 % of the air gap length depending on the motor geometry and loading condition, of displacement. However, the closed rotor slots may break the linearity in some operating conditions of the motor.

The developed methods of analysis are used to analyse the rotor eccentricity. The effects of equalising currents and saturation on the forces are studied. The equalising currents reduce the amplitude of the force and change the direction of the force. The type of stator windings and motor geometry affect the equalising currents and forces. The saturation limits the maximum radial force and also couples the eccentricity harmonics together.

The multi-slice model is exploited in modelling the conical motions of the rotor. The forces are measured for the test motor equipped with the active magnetic bearings. The active magnetic bearings are used to generate the eccentric motions and also to measure the electromagnetic forces. The computed and measured forces show relatively good agreement. Both the measured and calculated results show that the forces of different whirling motions can be superposed and the result is the force for the combined motion.

References

Antila, M. 1998. Electromechanical properties of radial active magnetic bearings. *Acta Polytech. Scand. Electr. Eng. Ser.*, 1998, No. 92, Helsinki, 96 p. <<http://lib.hut.fi/Diss/199X/isbn9512256029/>>.

Antila, M., Lantto, E., Arkkio, A. 1998. Determination of forces and linearized parameters of radial active magnetic bearings by finite element technique. *IEEE Trans. on Magnetics*, 1998, Vol. 34, No. 3, pp. 684-694.

Arkkio, A. 1987. Analysis of induction motors based on the numerical solution of the magnetic field and circuit equations. *Acta Polytech. Scand. Electr. Eng. Ser.*, 1987, No. 59, Helsinki, 97 p. <<http://lib.hut.fi/Diss/198X/isbn951226076X/>>.

Arkkio, A. 1996. Unbalanced Magnetic Pull in Cage Induction Motors – Dynamic and Static Eccentricity. *Proceedings of ICEM'96*, September 10-12, 1996, Vigo, Spain, pp. 192-197.

Arkkio, A. 1997. Unbalanced Magnetic Pull in Cage Induction Motors with Asymmetry in Rotor Structures. *IEE Conference Publication Proceedings of the 1997 8^{th} International Conference on Electrical Machines and Drives*, September 1-3, 1997, No. 444, pp. 36-40.

Arkkio, A., Antila, M., Pokki, K., Simon, A., Lantto, E. 2000. Electromagnetic force on a whirling cage rotor. *IEE Proc. Electr. Power Appl.*, 2000, Vol. 147, No. 5, pp. 353-358.

Arkkio, A., Lindgren, O. 1994. Unbalanced magnetic pull in a high-speed induction motor with an eccentric rotor. *Proceedings of ICEM'94*, September 5-8, 1994, Paris, France, pp. 53-58.

Bangura, J. F., Demerdash, N. A. 2000a. Effects of broken bars/end-ring connectors and airgap eccentricities on ohmic and core losses of induction motors in ASD's using a coupled finite element-state space method. *IEEE Trans. on Energy Conversion*, 2000, Vol. 15, No. 1, pp. 40-47.

Bangura, J. F., Demerdash, N. A. 2000b. Comparison between characterization and diagnosis of broken bars/end-ring connectors and airgap eccentricities of induction motors in ASD's using a coupled finite element-state space method. *IEEE Trans. on Energy Conversion*, 2000, Vol. 15, No. 1, pp. 48-56.

Belmans, R., Vandenput, A., Geysen, W. 1987a. Influence of unbalanced magnetic pull on the radial stability of flexible-shaft induction machines. *Proc. IEE*, 1987, Vol. 134, No. 2, pp. 101-109.

Belmans, R., Vandenput, A., Geysen, W. 1987b. Calculation of the flux density and the unbalanced pull in two pole induction machines. *Archiv für Elektrotechnik*, 1987, Vol. 70, pp. 151-161.

Benaragama, D. S., Ward, D. M., Tavner, P. J. 1982. Unbalanced magnetic pull on the eccentric rotor of a turbo-generator. *Proceedings of ICEM 1982*, September 5-9, 1982, Budapest, Hungary, pp. 922-925.

Boualem, B., Piriou, F. 1998. Numerical models for rotor cage induction machines using finite element method. *IEEE Transactions on Magnetics*, 1998, Vol. 34, No. 5, pp. 3202-3205.

Coulomb, J. L. 1983. A methodology for the determination of global electromechanical quantities from a finite element analysis and its application to the evaluation of magnetic forces, torques, and stiffness. *IEEE Trans. on Magnetics*, 1983, Vol. 19, No. 6, pp. 2514-2519.

Covo, A. 1954. Unbalanced magnetic pull in induction motors with eccentric rotors. *Trans. Am. I.E.E.*, 1954, Vol. 73, pp. 1421-1425.

Crawford, W. G. 1951. Unbalanced magnetic pull and the mechanical stability of rotating electrical machines. *Engineering*, April, 1951, pp. 504-505.

DeBortoli, M. J., Salon, S. J., Burow, D. W., Slavik, C. J. 1993. Effects of rotor eccentricity and parallel windings on induction machine behavior: a study using finite element analysis. *IEEE Trans. on magnetics*, 1993, Vol. 29, No. 2, pp. 1676-1682.

Dorrell, D. 2000. Modelling of non-uniform rotor eccentricity and calculation of unbalanced magnetic pull in a 3-phase cage induction motors. *Proceeding of ICEM 2000*, August 28-30, 2000, Espoo, Finland, pp. 1820-1824.

Dorrell, D. G. 1993. Calculation of unbalanced magnetic pull in cage induction machines. Doctoral thesis, University of Cambridge, Cambridge, United Kingdom, 1993, 169 p.

Dorrell, D. G. 1995. Modelling rotor eccentricity in cage induction motors with axial variation of eccentricity. *Proceedings of the Universities Power Engineering Conference*, UPEC 1995, September 1995, London, UK, pp. 1-4.

Dorrell, D. G., Smith, A. C. 1994. Calculation of u.m.p in induction motors with series or parallel winding connections. *IEEE Trans. On Energy Conversion*, 1994, Vol. 9, No. 2, pp. 304-310.

Eastham, J. F., Williamson, S. 1973. Generalised theory of induction motors with asymmetrical airgaps and primary windings. *Proc. I.E.E.*, 1973, Vol. 120, No. 7, pp. 767-775.

Ellison, A. J., Yang, S. J. 1974. Effects of rotor eccentricity on acoustic noise from induction machines. *Proc. IEE*, 1974, Vol. 118, No. 1, pp. 174-184.

Ewins, D. J. 2000. Modal testing: theory, practice and application, 2nd edition. Research studies press Ltd., Hertfordshire, England, 2000.

Fisher-Hinnen, J. 1899. Dynamo Design. Van Nostrand, 1899.

Freise, W., Jordan, H. 1962. Einseitige magnetische Zugkräfte in Drehstrommaschinen. *ETZ-A*, 1962, Vol. 83, No. 9, pp 299-303.

Früchtenicht, J., Jordan H., Seinsch, H. O. 1982. Exzentrizitätsfelder als Ursache von Laufinstabilitäten bei Asynchronmaschinen. Teil I und II. *Arch. Elektrotech.*, 1982, Vol. 65, pp. 271-292.

Gray, A., Pertsh, J. G. 1918. Critical review of the bibliografy on unbalanced magnetic pull in dynamo-electric machines. *Trans. Am. I.E.E.*, 1918, Part 2, Vol. 37, pp. 1417-1424.

Gyselinck, J., Melkebeek, J. 1995. Modelling of electric machines with skewed slots using the two dimensional finite element method: an efficient solving technique. *Sys. Anal. Modelling and Simulation*, 1995, Vols. 18-19, pp. 559-562.

Ha, K. H., Hong, J. P. 2001. Dynamic rotor eccentricity analysis by coupling electromagnetic and structural time stepping FEM. *IEEE Trans. on Magnetics*, 2001, Vol 37, No 5, pp. 3452-3455.

Heller, B., Hamata, V. 1977. Harmonic field effects in induction machines. Amsterdam, Elsevier Scientific Publishing Company, 1977, 330 p.

Ho, S. L., Fu, W. N., Wong, H. C. 1997. Solution of 3-D complex finite element model of skewed rotor induction motors using an iterative method. *Proceedings of International Electric Machines and Drives Conference, IEMDC'97*, May 18-21, 1997, Milwaukee, WI, USA, Pages: WB3/2.1 – WB3/2.3.

Ho, S. L., Li, H. L., Fu, W. N. 1999. Inclusion of interbar currents in a network-field coupled time-stepping finite-element model of skewed rotor induction motors. *IEEE Transactions on Magnetics*, 1999, Vol. 35, No. 5, pp. 4218-4225.

Holopainen, T. P., Tenhunen, A., Lantto, E., Arkkio, A. 2003. Numerical identification of electromagnetic force parameters for linearized rotordynamic model of cage induction motors.

Proceedings of ASME International 19th Biennial Conference on Mechanical Vibration and Noise, September, 2-6, 2003, Chicago, IL, USA, accepted to be published.

Holopainen, T., Tenhunen, A., Arkkio, A. 2002. Electromechanical Interaction in Rotor Vibrations of Electric Machines. *Proceedings of the Fifth World Congress on Computational Mechanics (WCCM V*), July 7-12, 2002, Vienna, Austria, Editors: Mang, H.A.; Rammerstorfer, F.G.; Eberhardsteiner, J., Publisher: Vienna University of Technology, Austria, ISBN 3-9501554-0-6, <<http://wccm.tuwien.ac.at>>.

Jaenicke, P., Jordan, H. 1976. Berechnung der electromagnetischen Radialkraft bei Drehstromasynchron-maschinen mit exzentrischer Verlagerung des Läufers. *Siemens Forch. & Entwickl.ber.*, 1976, Vol. 5, No. 5, pp. 249-256.

Joksimović, G. M., Durović, M. D., Penman, J., Arthur, N. 2000. Dynamic simulation of dynamic eccentricity in induction machines – winding function approach. *IEEE Trans. on Energy Conversion*, 2000, Vol. 15, No. 2, pp. 143-148.

von Kaene, P. 1963. Unbalanced magnetic pull in rotating electrical machines. Survey of Published Work, *E.R.A. Report Ref. Z/T 142*, 1963, 30 p.

Kovacs, K. P. 1977. Two-pole induction-motor vibrations caused by homopolar alternating fluxes. *Trans. IEEE Trans. on Power Apparatus and Systems*, 1977, Vol. PAS-96, No. 4, pp. 1105-1108.

Kovács, K. P., Rács, I. 1959. Transiente Vorgänge in Wechselstrommaschinen. Budapest, Verlag der Ungarischen Akademie der Wissenschafte, 1959. Band I 519 p., Band II 271 p.

Lantto, E. 1999. Robust control of magnetic bearings in subcritical machines. *Acta Polytech. Scand. Electr. Eng. Ser.,* 1999, No. 94, Helsinki, 96 p. <<http://lib.hut.fi/Diss/199X/isbn9512255758/>>.

Maliti, K. C. 2000. Modelling and analysis of magnetic noise in squirrel-cage induction motors. Doctoral thesis, Royal Institute of Technology, Department of Electrical Power Engineering, Stockholm, 2000, 197 p.

Meiler, G., Sperling, P. – G., Tikvicki, M. 1973. Ermittlung einseitiger Zugkräfte in Drehstrom-Asynchronmaschinen. *Siemens-Z*, 1973, Vol. 47, No. 9, pp. 680-685.

Reichert, K., Freundl, H., Vogt, W. 1976. The calculation of forces and torques within numerical magnetic field calculation methods. *Proceedings of Compumag*, Oxford, UK, 1976, pp. 64-73.

Robinson, R. C. 1943. The calculation of unbalanced magnetic pull in synchronous and induction machines. *Electrical Engineering*, October 1943, pp. 620-623.

Rosenburg, E. 1918. Magnetic pull in electrical machines. *Trans. Am. Inst. Elect. Eng.*, 1918, Part 2, Vol. 37, pp. 1425-1469.

Salon, S. J., DeBortoli, M. J., Burow, D. W., Slavik, C. J. 1993. Calculation of circulating current between parallel windings in induction motors with eccentric rotors by the finite element method. *Proceedings of ICEM'92*, September 15-17, 1992, Manchester, UK, pp. 371-375.

Salon, S. J., DeBortoli, M. J., Palma, R. 1990. Coupling of transient fields, circuits, and motion using finite element analysis. *Journal of Electromagnetic Waves and Applications*, 1990, Vol. 4, No. 11, pp. 1077-1106.

Silvester, P. P., Ferrari, R. L. 1990. Finite elements for electrical engineers. Second Edition, Cambridge University Press, 1990, 344 p.

Simon, A. 1999. Use of magnetic bearings to measure the forces caused by an eccentric rotor in an induction motor. Master of Science thesis, Technical University of Budapest, Faculty of Electrical Engineering and Informatics, 1999, 80 p.

Smith, A. C., Dorrell, D. G. 1996. Calculation and measurement of unbalanced magnetic pull in cage induction motors with eccentric rotors. Part 1: Analytical model. *IEE Proc. Electr. Power Appl.*, 1996, Vol. 143, No. 3, pp. 193-201.

Stepina, J. 1970. "Space vector representation of radial force waves in asynchronous machines", ETZ-A, 1970, Vol. 91, No. 9, pp. 503-505.

Stoll, R. L. 1997. Simple computational model for calculating the unbalanced magnetic pull on a two-pole turbogenerator due to eccentricity. *IEE Proc. Elect. Power Appl.*, 1997, Vol. 144, No. 4, pp. 263-270.

Summers, E. W. 1955. Vibration in 2-pole induction motors related to slip frequency. *Trans. Am. I.E.E.*, 1955, Vol. 74, pp. 69-72.

Swann, S. A. 1963. Effect of rotor eccentricity on the magnetic field in the air-gap of a non-salient-pole machine. *Proc. I.E.E*, 1963, Vol. 110, No. 5, pp. 903-915.

Tenhunen, A. 2000. Modelling skewed rotor slots within two-dimensional finite element analysis of induction machines. *Acta Polytech. Scand. Electr. Eng. Ser.*, 2000, No. 102, Helsinki, 70 p.

Tereshonkov, V. A. 1989. Magnetic forces in electric machines with air gap eccentricities and core ovalities. *Elektrotekhnika*, 1989, Vol. 60, No. 9, pp. 50-53.

Timar, P. L. 1989. Noise and vibrations of electrical machines. Elsevier Science Publishers, Amsterdam – Oxford – New York – Tokyo, 1989, 339 p., ISBN 0-444-98896-3.

Toliyat, H. A., Arafeen, M. S., Parlos, A. G. 1996. A method for dynamic simulation of air-gap eccentricity in induction machines. *IEEE Trans. on Industry Applications*, 1996, Vol. 32, No. 4, pp. 910-917.

Williamson, S., Lim, L. H., Robinson, M. J. 1990. Finite element models for cage induction motor analysis. *IEEE Trans. On Industry Applications*, Vol. 26, No. 6, pp. 1007-1017.