Iterative Receiver Concept for TDMA Packet Data Systems*

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Abstract. In this paper we present an iterative receiver concept that improves the radio link performance of TDMA mobile communication systems. We consider sub-optimal receiver structures comprised of channel estimator, detector and channel decoder, where the performance is improved by iterative data processing among the receiver blocks. As a practical example we consider packet data transmission in GSM and EDGE (Enhanced Data rates for Global Evolution). Low-complexity soft-in-soft-out (SISO) equalizers for EDGE are introduced and its modifications suitable for iterative detection in EDGE are derived. Application of iterative detection and channel estimation techniques in GSM/EDGE shows a significant performance improvement. Furthermore, we show that retransmission schemes specified for EDGE also benefit from iterative data processing.

1 INTRODUCTION

A general problem of reliable data transmission over channels with intersymbol interference (ISI) includes joint estimation, detection and decoding, and as a whole it is non-tractable because of tremendous complexity involved. A suboptimal method to solve this problem is to split the processing into tractable sub-blocks, and then iteratively exchange locally processed information among sub-blocks. For example, decoding of celebrated turbo codes is based on iterative information update among relatively simple decoders [1]. The same principle applied to channel equalization leads to a turbo equalization scheme [2] that recently has gained a lot of interest [3]-[12]. This technique performs iterative ISI removal, where iterations proceed between a detector and a channel decoder relying on channel estimates that usually are obtained based on a known training sequence.

On the other hand, in practice channel estimates may have rather poor quality that in turn deteriorates the efficiency of equalization. This fact motivated us to consider a decision-directed channel estimation similar to [13], [14]. The principle of turbo equalization applied to channel estimation leads to iterative (turbo) estimation schemes [15]-[18]. Following [18] in this paper we extend this approach and include iterative channel estimation (ICE) into turbo detection loop. Furthermore, we show that iterative data processing applied to Incremental Redundancy (IR), the sophisticated retransmission scheme retains for EDGE, can favorably benefit from such a receiver.

In this paper we propose a novel generic TDMA receiver that performs iterative equalization and estimation in a joint/iterative fashion. Although this receiver design can be used for any TDMA system, it is particularly interesting for high order modulation such as 8-PSK adopted in EDGE standard [19].

The maximum-likelihood (ML) detection of 8 PSK modulated symbols in the presence of ISI currently is too

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complex for mobile communications, so to keep the overall receiver complexity low a sub-optimal equalizer has to be introduced. In this paper we introduce a low complexity suboptimal SISO equalizers based on the Decision Feedback Sequence Estimation (DFSE) [20],[21] that as such can be used in conventional receiver without iterative data processing. Next, we present its modifications suitable for turbo equalization, which as it is shown in [6], [12] brings a substantial gain in EDGE environment. To keep the receiver complexity low we also suggest a simple method of updating channel estimate that includes the decoder outputs into the iteration process and also may be coupled with iterative detection.

To study receiver performance-complexity trade-off we consider different iterative estimation-equalization scenarios for GSM/EDGE packet data services. In particular, we consider General Packet Radio System (GPRS) and Enhanced GPRS (EGPRS) where enhanced packet data services in GSM environment will be provided by 8PSK modulation used instead of binary GMSK modulation [19]. As it is shown in the paper, the iterative detection gain increases for high level modulation schemes.

The paper is organized as follows. In Section 2, we introduce notations and an overview of the conventional receivers. In Section 3, we discuss reduced complexity SISO equalizers. Notably, two different soft output computation methods are compared based on forward-backward recursion and forward-only recursion. Next, in Section 4, we consider turbo-detection for Q-ary modulation and propose a combined iterative estimation - equalization scheme. Application of turbo-equalization for retransmission schemes is addressed in the same section. Trade-off between performance gain and receiver complexity in (E)GPRS under different scenarios is addressed in Section 5, with conclusions following in Section 6.

2 SYSTEM MODEL

2.1 NOTATION

Let us consider a digital communication system with a block diagram depicted in figure 1. A data sequence $\mathbf{u} = (u_1, ..., u_K)^\top$ is encoded by a channel encoder C_o resulting in a coded sequence $\mathbf{c} = (c_1, ..., c_N)^\top = G\mathbf{u}$. Coded bits are interleaved by an interleaver Π, punctured to match data rates to a transmit format and then grouped into τ Q-ary symbols ($Q = 2^q$) forming N_b transmission bursts. Each burst consists of τ symbols $\mathbf{a} = (\mathbf{a}_1, ..., \mathbf{a}_\tau)^\top$, including known symbols for channel estimation and synchronization purposes together with tail and guard symbols. Let $\mathbf{m}$ denote a training sequence consisting of L preamble and P midamble symbols. The resulting data burst $\mathbf{a}$ of length τ is formed by sub-blocks, $\mathbf{a} = (\mathbf{d}_1^\top, \mathbf{m}^\top, \mathbf{d}_2^\top)^\top$, where $\mathbf{d}_1 = (\mathbf{a}_1, \cdots, \mathbf{a}_{\tau/2})^\top = \mathbf{a}_1^{\tau/2}, \mathbf{m} = \mathbf{a}_{\tau/2+1}^{\tau/2+L+P}, \mathbf{d}_2 = \mathbf{a}_{\tau/2+L+P+1}^{\tau}$. To each symbol $\mathbf{a}_n = (a_{n,1}, ..., a_{n,q})^\top$, a Q-ary signal mapper Ψ associates a complex-valued symbol z_n. The transmitter produces the complex base-band waveform:

$$w_x(t) = \sum_n z_n h_x(t - nT)$$

where $h_x(t)$ denotes the base-band complex impulse response of the low-pass equivalent transmitter filter and $\frac{1}{T}$ is the symbol rate. At reception, the received base-band signal is given by:

$$w_r(t) = \sum_n z_n g(t - nT) + \epsilon'(t)$$

where the complex impulse response $g(t)$ takes into account the transmitter and receiver filters, together with the dispersive channel. $\epsilon'(t)$ denotes the convolution of the complex zero-mean Gaussian noise $\epsilon(t)$ (of single-sided power spectral density N_0) with the receiver filter $h_r(t)$. The signal is then sampled at rate $\frac{1}{T}$ to yield the non-quantized sequence $\mathbf{y} = (y_1, ..., y_\tau)^\top$.

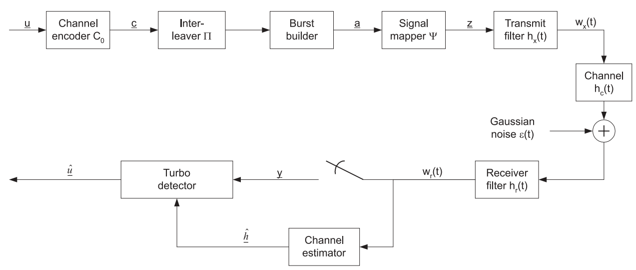

Figure 1: *Block diagram of communication system.*

2.2 THE EQUIVALENT DISCRETE-TIME CHANNEL MODEL

The equivalent discrete-time channel model is made of an encoder C_o with associated puncturing schemes, an interleaver Π, a signal mapper Ψ, and a transversal filter with $\mu + 1$ complex coefficient vector $\mathbf{h} = (h_0, h_1, ..., h_\mu)^\top$. At the output of the equivalent discrete-time channel (including transmit and receive filters), received samples are given by:

$$y_n = h_0 z_n + \sum_{k=1}^{\mu} h_k z_{n-k} + w_n$$

or in the matrix form

$$\mathbf{y} = \mathbf{Zh} + \mathbf{w} \tag{1}$$

where $\sum_{k=1}^{\mu} h_k z_{n-k}$ corresponds to the ISI introduced by the channel and w_n is a circularly complex Gaussian variable of variance $2\sigma^2$ considered independent identically distributed (i.i.d.) in the following. Here and below bold and capital bold letters are used for vectors and matrices, respectively.

Taking into account the i.i.d. assumption, the autocorrelation matrix $\mathbf{R}$ of the noise vector $\mathbf{w}$ is $\mathbf{R} = 2\sigma^2\mathbf{I}$ with $\mathbf{I}$ being the $(\tau \times \tau)$ dimensional unit matrix. As well known, the equivalent discrete-time ISI channel can be regarded as a non-recursive non-systematic rate-1 convolutional code with memory μ, whose single complex-valued generator polynomial may vary in time. The time progression of the states, as well as the possible transitions can be presented by a regular trellis diagram. We denote by S_n and B_n state and branch spaces at trellis section n, respectively. Due to time-invariant property, the state and the branch space complexities satisfy:

$$\begin{aligned} |\mathrm{S}_n| &= Q^{\mu}, \forall n \in [0, \tau] \\ |\mathrm{B}_n| &= Q^{\mu+1}, \forall n \in [1, \tau] \end{aligned} \tag{2}$$

The ML symbol by symbol detection may be performed by the BCJR algorithm [22], which operates on the full ISI channel trellis with complexity $O\left(|\mathrm{B}|\right)$.

2.3 Conventional receiver

We define $y_1^{N_b\ \tau}$ as the received samples of N_b consecutively transmitted bursts $\mathbf{a}_1^{N_b\ \tau}$, and $\mathbf{h}_1^{N_b}$ as the set of channel coefficient vectors corresponding to N_b bursts.

According to the ML criterion applied to minimize block error rate for data block $\mathbf{u}$, the optimal receiver is to find

$$\begin{aligned} [\hat{\mathbf{u}}, \hat{\mathbf{h}}_1^{N_b}] &= \arg\max_{\mathbf{h}_1^{N_b}, \mathbf{u}} \Pr(y_1^{N_b\ \tau} \left| \mathbf{h}_1^{N_b}, \mathbf{a}_1^{N_b\ \tau} \right.) \\ &= \arg\max_{\mathbf{h}_1^{N_b}, \mathbf{u}} \Pr(y_1^{N_b\ \tau} | \mathbf{h}_1^{N_b}, \mathbf{m}_1^{N_b}, G\mathbf{u}, \Pi) \end{aligned} \tag{3}$$

The optimal solution of (3) is prohibitively complex, and in practice (3) is split into several problems, which are then considered separately. Separating channel equalization (detection) and decoding, and taking into account that the training sequence is known, a suboptimal solution for (3) may be presented as

$$\begin{aligned} \hat{\mathbf{a}}_1^{N_b\ \tau} &= \arg\max_{\mathbf{h}_1^{N_b}, \mathbf{a}_1^{N_b\ \tau}} \Pr(y_1^{N_b\ \tau} | \mathbf{h}_1^{N_b}, \mathbf{a}_1^{N_b\ \tau}) \\ \Rightarrow \hat{\mathbf{c}} &= \arg\max_{\mathbf{h}_1^{N_b}, \mathbf{c}} \Pr(y_1^{N_b\ \tau} | \mathbf{h}_1^{N_b}, \mathbf{c}) \end{aligned} \tag{4}$$

$$\hat{\mathbf{u}} = \arg\max_{\mathbf{u}} \Pr(\hat{\mathbf{c}} | \mathbf{u}) \tag{5}$$

The optimal solution requires a search over all possible $\mathbf{c}$ and $\mathbf{h}_1^{N_b}$ that is impractical for realistic values of τ, N_b, μ. A typical suboptimal solution of (4) is to separate channel estimation and equalization and perform them burst by burst, i.e.

$$\hat{\mathbf{h}} = \arg\max_{\mathbf{h}} \Pr(\mathbf{y} | \mathbf{m}, \mathbf{h}) \tag{6}$$

$$\hat{\mathbf{a}} = \arg\max_{\mathbf{a}} \Pr(\mathbf{y} | \hat{\mathbf{h}}, \mathbf{a}) \tag{7}$$

In particular, assuming a linear channel with time-invariant channel impulse response (CIR) during a transmitted block, a received block can be presented as

$$\mathbf{y} = \mathbf{Z}\mathbf{h} + \mathbf{w} = \left[\mathbf{y}_1^\top\ \mathbf{y}_m^\top\ \mathbf{y}_2^\top\right]^\top$$

where $\mathbf{Z}$ is $\tau \times (L+1)$ block matrix formed by data $\mathbf{d}_1, \mathbf{d}_2$ and training sequence $\mathbf{m}$ mapped into Q-ary symbols; $\mathbf{Z} = \left[\mathbf{Z}_1^\top\ \mathbf{M}^\top\ \mathbf{Z}_2^\top\right]^\top$.

In a conventional receiver the channel estimation is made based on received symbols $\mathbf{y}_m = \mathbf{M}\mathbf{h} + \mathbf{w}$, and the ML channel estimate [23]

$$\begin{aligned} \hat{\mathbf{h}}_{ML} &= \arg\max_{\mathbf{h}} \Pr(\mathbf{y}_m | \mathbf{M}, \mathbf{h}) \\ &= \mathbf{C}(\hat{\mathbf{h}}_{ML}) \mathbf{M}^H \mathbf{R}^{-1} \mathbf{y}_m \end{aligned} \tag{8}$$

where $\mathbf{C}(\hat{\mathbf{h}}_{ML}) = (\mathbf{M}^H \mathbf{R}^{-1} \mathbf{M})^{-1}$ is a covariance matrix of the estimate.

Given $\hat{\mathbf{h}}$, one of equalization algorithms is applied to remove ISI and obtain $\hat{\mathbf{c}}$. Finally, a decoder recovers transmitted information $\hat{\mathbf{u}}$.

3 Low Complexity Q-ary SISO Equalizer

3.1 Decision Feedback Soft-In Soft-Out (DF-SISO) equalizer

Depending on the optimization criterion, the detection (7) may be implemented by BCJR algorithm [22] or Viterbi algorithm (VA) for max *a posteriori* probability (Max-APP) or MLSE criterion, respectively. Both algorithms operate on the full ISI channel trellis with complexity of $O\left(|\mathrm{B}|\right)$. For 8-PSK modulation adopted in EDGE and a typical GSM channels the optimal equalizer currently seems unacceptable for complexity reasons.

To reduce the equalizer complexity a number of suboptimal schemes have been proposed. Among the set of trellis-based Reduced-States Sequence Estimators (RSSE) [20], the Decision Feedback Sequence Estimators (DFSE) [21] seem to be the most suitable candidate providing acceptable performance at moderate complexity [8]. To provide soft decision outputs that are necessary for the decoding different methods may be applied. In particular, a soft output equalizer based on MMSE [7], and the DFSE where soft decisions are formed using neural networks approach [9] are recently introduced. In this paper we consider the trellis-based equalizers and present the DFSE modifications of different complexity to provide soft decision outputs.

The main idea of RSSE [20] and DFSE [21] is to operate on a reduced complexity trellis Γ, where only ν symbols (related to ν channel taps) form the trellis state space S, and the other $\mu - \nu$ symbols are used through the embedded decision-feedback structure.

Let's define S_n and B_n as state and branch spaces at n^{th} section (time instant) of a reduced trellis Γ. For a positive integer ν , we say that a trellis input sequence $\mathbf{a}_1^n$ ends at a sub-state $s \in S_n$ if $\mathbf{a}_1^n$ terminates with the substring $s = \mathbf{a}_{n-\nu+1}^n$. At any trellis depth n, the sub-state space S_n coincides with the full BCJR trellis state space if $\nu = \mu$. In a case where $\nu < \mu$, the state space S is reduced to a subset made of all possible sub-states s derived from full states trellis, so that:

$$\begin{aligned} |S_n| &= Q^{\nu}, \forall n \in [0, \tau] \\ |B_n| &= Q^{\nu+1}, \forall n \in [1, \tau] \end{aligned} \tag{9}$$

Any trellis branch $b_n \in B_n$ at depth n of a sub-trellis Γ is characterized by three fields, $b_n = \{s_{n-1}, s_n, \mathcal{A}_n\}$:

- a starting state $s_{n-1} \in S_{n-1}$;
- an arrival state $s_n \in S_n$;
- a label $\mathcal{A}_n$ representing an input symbol $\mathbf{a}_n$, where $\mathcal{A}_n = \{\mathcal{A}_{n,1}, \mathcal{A}_{n,2}, ...\mathcal{A}_{n,q}\}$ is the binary presentation of $\mathbf{a}_n$

The above formalism defines a reduced trellis $\Gamma(S, B)$ on which the DFSE algorithm proceeds. Each trellis path is a set of edges $\{b_1, b_2, ..., b_\tau\}$ starting from state δ at time $n = 0$ and terminating at state η at time $n = \tau$. At each trellis section $n \in [1, \tau]$ and for all bit indices $j \in [1, q]$, an optimal symbol by symbol algorithm is to compute the log *a posteriori* probability ratio (LAPPR):

$$\lambda\left(a_{n,j}\right) = \ln \frac{\Pr\left(a_{n,j} = 1 \middle| \mathbf{y}, \widehat{\mathbf{h}}\right)}{\Pr\left(a_{n,j} = 0 \middle| \mathbf{y}, \widehat{\mathbf{h}}\right)} \tag{10}$$

It is assumed that channel taps $\hat{\mathbf{h}}$ are estimated by some channel estimator using the training sequence $\mathbf{m}$ before the equalization process. In the following derivation the conditioning by $\hat{\mathbf{h}}$ is implicit and omitted for the ease of expressions.

Equation (10) can be rewritten as follows:

$$\lambda(a_{n,j}) = \ln \frac{\sum_{\mathbf{a}, a_{n,j}=1} p\left(\mathbf{a}, \mathbf{y}\right)}{\sum_{\mathbf{a}, a_{n,j}=0} p\left(\mathbf{a}, \mathbf{y}\right)} \tag{11}$$

where $p\left(\mathbf{a}, \mathbf{y}\right) = p(\mathbf{y} \left| \mathbf{a} \right.) \Pr(\mathbf{a})$.

Based on the approximation

$$\ln\left(\sum_i \exp\left(\Delta_i\right)\right) \approx \max_i \Delta_i, \tag{12}$$

the approximated soft decision output is defined as:

$$\begin{aligned} \lambda_{eq}(a_{n,j}) &= \max_{\mathbf{a}, a_{n,j}=1} \{\ln p\left(\mathbf{a}, \mathbf{y}\right)\} \\ &\quad - \max_{\mathbf{a}, a_{n,j}=0} \{\ln p\left(\mathbf{a}, \mathbf{y}\right)\} \end{aligned} \tag{13}$$

or equivalently:

$$\begin{aligned} \lambda_{eq}(a_{n,j}) &= \min_{\mathbf{a}, a_{n,j}=0} \{-\ln p\left(\mathbf{a}, \mathbf{y}\right)\} \\ &\quad - \min_{\mathbf{a}, a_{n,j}=1} \{-\ln p\left(\mathbf{a}, \mathbf{y}\right)\} \end{aligned} \tag{14}$$

where $\{-\ln p\left(\mathbf{a}, \mathbf{y}\right)\}$ corresponds to the trellis path metric associated with symbol sequence $\mathbf{a}$. In trellis terminology, $\lambda_{eq}(a_{n,j})$ is the algebraic difference at time instant n between the path metric associated with the best trellis path that decodes a bit 0 at position j and the path metric associated with the best trellis path that decodes a bit 1 at position j.

Let $\mu^{\leftrightarrow}(b_n)$ be the accumulated metric of the best path starting from state $s_0 = \delta$, terminating at state $s_\tau = \eta$, and passing by transition $b_n = \{s_{n-1}, s_n, \mathcal{A}_n\}$ at trellis section n. The DF-SISO equalizer soft output can be equivalently rewritten as:

$$\lambda_{eq}(a_{n,j}) = \min_{b_n \in B_n, \mathcal{A}_{n,j}=0} \mu^{\leftrightarrow}(b_n) - \min_{b \in B_n, \mathcal{A}_{n,j}=1} \mu^{\leftrightarrow}(b_n); \tag{15}$$

where $\mu^{\leftrightarrow}(b_n)$ may be presented in the form

$$\mu^{\leftrightarrow}(b_n) = \alpha(s_{n-1}) + \gamma(b_n) + \beta(s_n); \tag{16}$$

- $\alpha(s_{n-1})$ is the accumulated metric of the best subpath starting from state $s_0 = \delta$ and terminating at state s_{n-1};
- $\beta(s_n)$ is the accumulated metric of the best subpath starting from state s_n and terminating at state $s_\tau = \eta$;
- $\gamma(b_n)$ is the (approximated) edge metric associated with b_n. For any state $s_n \in S_n$ at time n the metric $\alpha(s_n)$ can be recursively computed using the forward recursion similar to [22]:

  $$\alpha(s_n) = \min_{b_n \in B_n^{(s_n)}} \{\alpha(s_{n-1}) + \gamma(b_n)\} \tag{17}$$

  with a boundary conditions $\alpha(s_0 = \delta) = 0$ and $\alpha(s_0 \neq \delta) = \infty$ at time $n = 0$, where $B_n^{(s_n)}$ denotes the conditional subset of branches B_n terminating at state s_n.

At each section (time instant n) of the trellis Γ and for all transitions, the edge metric computation involves a convolution of discrete-time CIR with a sequence of μ already estimated symbols. The first ν estimated symbols of that sequence are derived based on the current trellis branch b_n of the sub-trellis Γ. The remaining part is evaluated by per-survivor processing [21]. In particular, for the existing transitions the edge metric expression used in the DF-SISO is

$$\gamma\left(b_n\right) = \frac{1}{2\sigma^2} \left\| r_n - \widehat{h}_0 z_n - I_n^{(1)} - \widehat{I}_n^{(2)} \right\|^2 \tag{18}$$

where:

- $I_n^{(1)}$ is the first part of ISI contribution term

$$I_n^{(1)} = \sum_{k=1}^{\nu} \widehat{h}_k \Psi\left(\mathbf{a}_{n-k}\right); \tag{19}$$

which involves the sequence $\mathbf{a}_{n-\nu}^{n-1}$ of Q-ary symbols contained in the trellis state s_{n-1};

- $\widehat{I}_n^{(2)}$ is the second part of ISI contribution term

$$\widehat{I}_n^{(2)} = \sum_{k=\nu+1}^{\mu} \widehat{h}_k \Psi\left(\widehat{\mathbf{a}}_{n-k}^{(s_{n-1})}\right)); \tag{20}$$

which involves the sequence $\widehat{\mathbf{a}}_{n-\mu}^{n-\nu-1}$ of Q-ary symbols estimated by reading off the survivor path terminating at s_{n-1}.

Similarly, for any state $s_n \in S_n$ at time n the metric $\beta(s_n)$ can be recursively computed using the backward recursion:

$$\beta(s_n) = \min_{b_{n+1} \in B_{n+1}^{(s_n)}} \{\beta(s_{n+1}) + \gamma(b_{n+1})\} \tag{21}$$

with boundary condition $\beta(s_\tau = \eta) = 0$ and $\beta(s_\tau \neq \eta) = \infty$ at time, $n = \tau$.

Since only the first ν channel taps form the trellis structure, it is beneficial for decision feedback equalizers to have the minimum phase CIR [24] . To meet this requirement the usual practice is to put a pre-filter before the equalizer. That pre-filter provides the minimum phase CIR for the forward recursion, while it is not true for the backward recursion. To avoid this problem it is suggested to perform first the forward recursion and to keep all associated edge metrics $\gamma(b_n)$ calculated according to (18). Then the stored edge metrics are used for the backward recursion (21).

Finally, the LAPPR on bit $a_{n,j}$ at the DF-SISO equalizer output

$$\begin{aligned}\lambda_{eq}(a_{n,j}) = &\min_{b_n \in B_n, \mathcal{A}_{n,j}=0} \{\alpha(s_{n-1}) + \gamma(b_n) + \beta(s_n)\} \\ &- \min_{b_n \in B_n, \mathcal{A}_{n,j}=1} \{\alpha(s_{n-1}) + \gamma(b_n) + \beta(s_n)\}\end{aligned} \tag{22}$$

It must be emphasized that the above minimization operation, as well as the metric expressions are exact if and only if $\nu = \mu$. In that case, the DF-SISO becomes formally equivalent to the max-log-MAP algorithm applied for the full ISI channel trellis. For the reduced-state trellis the estimated sequences taken from the path history and involved in edge metric derivations inevitably introduce a degradation in performance due to a possible error propagation effect.

3.2 DF-SISO EQUALIZER WITH FORWARD RECURSION

Described above DF-SISO performs the forward-backward recursions to provide bit-wise soft decisions. To reduce the equalizer complexity we can exclude the backward recursion which results to the original DFSE performing VA with only forward recursion. Recall that VA provides MLSE solution (i.e. only hard decisions), and to form bit-wise soft decisions for the DFSE one may follow the SOVA approach [25]. Soft values in SOVA are obtained by comparing only two of the most likely sequences with different bits at a particular time instant. According to SOVA the soft decisions are calculated via recursive updating of soft values within the VA decision delay window D (typically D=5μ). Further complexity reduction may be provided by soft-output Viterbi equalizer (SOVE) [26], where only a few operations are added to the VA to form soft decisions. The simplification is obtained by omitting recursive update and shortening the VA decision delay ($D = \mu$) that results in a small degradation (0.2 dB) compared to the SOVA [27]. This degradation may be practically eliminated by expanding the decision delay $D = \mu + 2$ [28].

Specifically, soft decisions in SOVE may be obtained if we set $\beta(s_n) = 0$ in (16), skip the backward recursion (21) and perform only the forward recursion according to (17)-(22). Finally, the LAPPR on bit $a_{n-D,j}$ at time instant n delivered by DF-SISO with forward-only recursion is

$$\begin{aligned}\lambda_{eq}(a_{n-D,j}) = &\min_{b_n \in B_n, \widehat{a}_{n-D,j}^{(s_{n-1})}=0} \{\alpha(s_{n-1}) + \gamma(b_n)\} \\ &- \min_{b_n \in B_n, \widehat{a}_{n-D,j}^{(s_{n-1})}=1} \{\alpha(s_{n-1}) + \gamma(b_n)\}\end{aligned} \tag{23}$$

On the other hand, the original forward-only SOVA [25] may be easily modified to approach performance of max-log-MAP with only minor increase of complexity [29]. Hence, the forward only approach provides a number of low complexity solutions to approach max-log MAP performance.

To compare the forward-backward and the forward-only DF-SISO equalizers we considered their performance in static channels without prefiltering and with ideal channel estimates (see figure 2). We found that for the static channel CIR_1=$\{$0.5,0.71.0.5$\}$ the performance of the two considered DF-SISO equalizers with μ=2 is very close when comparing their hard decision outputs. To evaluate the quality of soft decisions for both algorithms we include the strongest EGPRS coding scheme MCS5 which is based on a rate 1/3 convolutional code with constraint length 7. Simulation results presented at figure 2 show that DF-SISO with forward-backward recursions provides some gain (0.3 dB at BER=10^{-3}) compared to the forward-only DF-SISO equalizer. On the other hand, this gain is not as visible (0.1 dB at BER=10^{-3}) for a minimum phase channel with CIR_2=$\{$0.77,0.55,0.33$\}$. Recall that a minimum phase prefilter is essential for decision-feedback equalizers, so to keep the receiver complexity low we consider below only the DF-SISO equalizer with forward-only recursion.

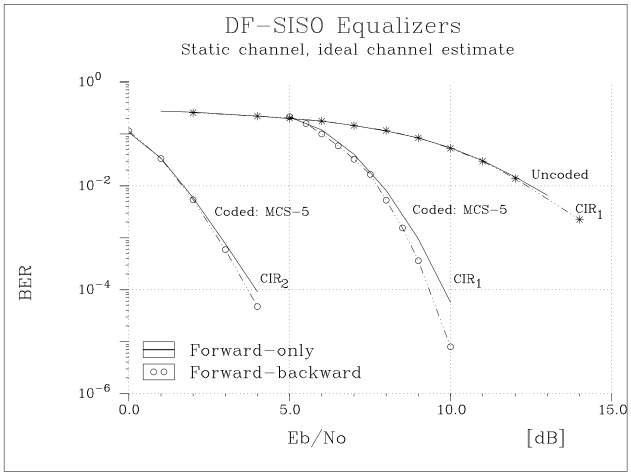

Figure 2: *DF-SISO performance in static channels with CIR_1 = 0.5, 0.71, 0.5 , CIR_2 = 0.77, 0.55, 0.33 .*

4 Iterative Receiver for EDGE

4.1 Turbo detection principle

One of the solutions related to DFSE is the method of turbo equalization [2]. This method is based on iterations between detection and decoding stages and attempts to find ML solution $\hat{\mathbf{u}}$ over a combined trellis formed by a multipath channel and encoder, i.e.

$$\hat{\mathbf{u}} = \arg\max_{\mathbf{u}} \Pr(y_1^{N_b\ \tau} | \hat{\mathbf{h}}_1^{N_b}, \mathbf{m}_1^{N_b}, G\mathbf{u}, \Pi) \tag{24}$$

An iterative receiver with turbo equalization is outlined at figure 3. The SISO equalizer delivers bit-wise LAPPR on bits $a_{n,j}$ that can be split into two (called intrinsic and extrinsic) parts

$$\lambda_{eq}(a_{n,j}) = \lambda^a\left(a_{n,j}\right) + \lambda_{eq}^{ext}(a_{n,j}). \tag{25}$$

After de-interleaving Π^{-1}, the sequence of extrinsic LAPPR, $\lambda_{eq}^{ext}(\mathbf{a}_1^{N_b\ \tau})$, becomes a sequence of log *a priori* probability ratios $\lambda^a\left(\mathbf{c}\right)$ on coded bits for the decoder. Similarly at the output of the SISO decoder, the LAPPR on coded bit $\lambda_d\left(c_{n,j}\right)$ can be split into an intrinsic and an extrinsic parts. The latter can be computed by bit-wise substraction of the *a priori* information $\lambda^a\left(c_{n,j}\right)$ at the input of the decoder from the corresponding LAPPR $\lambda_d\left(c_{n,j}\right)$ at the output,

$$\lambda_d^{ext}\left(c_{n,j}\right) = \lambda_d\left(c_{n,j}\right) - \lambda^a\left(c_{n,j}\right). \tag{26}$$

Sequence of extrinsic LAPPRs on coded bits $\lambda_d^{ext}\left(\mathbf{c}\right)$ is re-interleaved by Π and passed to the SISO detector as a new sequence of log *a priori* probability ratios $\lambda^a\left(\mathbf{a}\right)$ for the next detection attempt.

4.2 Q-ary turbo-detection with DF-SISO equalizers

The transition metric in the presence of independent *a priori* information on the transmitted symbol, $\Pr\left(\mathbf{a}_n = \mathcal{A}_n\right)$, for DF-SISO equalizers is given by

$$\begin{aligned}\gamma^a\left(b_n\right) &= \frac{1}{2\sigma^2}\left\| r_n - \widehat{h}_0 z_n - I_n^{(1)} - \widehat{I}_n^{(2)} \right\|^2 \\ &\quad - \ln\Pr\left(\mathbf{a}_n = \mathcal{A}_n\right)\end{aligned} \tag{27}$$

Assuming perfect decorrelation between symbols bits $\{a_{n,j}\}$, $j \in [1,q]$, $n \in [1,\tau]$ after re-interleaving Π of the re-encoded sequence

$$\Pr\left(\mathbf{a}_n = \mathcal{A}_n\right) = \prod_{j=1}^{q} \Pr\left(a_{n,j} = \mathcal{A}_{n,j}\right) \tag{28}$$

Since it is always possible to write:

$$\begin{aligned}\prod_{j=1}^{q} \Pr\left(a_{n,j} = \mathcal{A}_{n,j}\right) &= \Pr\left(a_{n,j} = \mathcal{A}_{n,j}\right) \\ &\quad \prod_{\substack{l=1\\ l\neq j}}^{q} \Pr\left(a_{n,l} = \mathcal{A}_{n,l}\right)\end{aligned} \tag{29}$$

then $-\ln Pr\left(a_{n,j} = \varepsilon|\mathbf{y}\right)$, $\varepsilon \in \{0,1\}$ at the equalizer output can be presented as

$$\min_{b_n \in B_n, \mathcal{A}_{n,j} = \varepsilon} \left\{\alpha(s_{n-1}) + \gamma^a\left(b_n\right) + \beta(s_n)\right\}$$

or

$$\min_{b_n \in B_n, \mathcal{A}_{n,j} = \varepsilon} \left\{\alpha(s_{n-1}) + \gamma^{ext}(b_n) + \beta(s_n)\right\} - \ln\Pr(a_{n,j} = \varepsilon) \tag{30}$$

where:

$$\begin{aligned}\gamma^{ext}\left(b_n\right) &= \frac{1}{2\sigma^2}\left\| r_n - \widehat{h}_0 z_n - I_n^{(1)} - \widehat{I}_n^{(2)} \right\|^2 \\ &\quad - \sum_{\substack{l=1\\ l\neq j}}^{q} \ln\Pr\left(a_{n,l} = \mathcal{A}_{n,l}\right)\end{aligned} \tag{31}$$

Finally, the LAPPR on the bit $a_{n,j}$, $\lambda_{eq}(a_{n,j})$, at the output of Q-ary SISO equalizer

$$\begin{aligned}\lambda_{eq}(a_{n,j}) &= \ln\frac{\Pr\left(a_{n,j} = 1 \,|\mathbf{y}\right)}{\Pr\left(a_{n,j} = 0 \,|\mathbf{y}\right)} \\ &= \lambda^a(a_{n,j}) + \lambda_{eq}^{ext}(a_{n,j})\end{aligned} \tag{32}$$

where $\lambda^a(a_{n,j})$ is the log *a priori* probability ratio on the bit $a_{n,j}$ provided by SISO decoder

$$\lambda^a(a_{n,j}) = \ln\frac{\Pr\left(a_{n,j} = 1\right)}{\Pr\left(a_{n,j} = 0\right)}; \tag{33}$$

and the incremental knowledge on bit $a_{n,j}$ brought by detection process (the extrinsic information) is

$$\lambda_{eq}^{ext}(a_{n,j}) = \min_{b_n \in B_n, \mathcal{A}_{n,j}=0} \left\{ \alpha(s_{n-1}) + \gamma^{ext}(b_n) + \beta(s_n) \right\} - \min_{b_n \in B_n, \mathcal{A}_{n,j}=1} \left\{ \alpha(s_{n-1}) + \gamma^{ext}(b_n) + \beta(s_n) \right\} \quad (34)$$

In the similar way, the extrinsic information for forward-only DF-SISO equalizer is formed as

$$\lambda_{eq}^{ext}(a_{n-D,j}) = \min_{b_n \in B_n, \widehat{a}_{n-D,j}^{(s_{n-1})}=0} \left\{ \alpha(s_{n-1}) + \gamma^{ext}(b_n) \right\} - \min_{b_n \in B_n, \widehat{a}_{n-D,j}^{(s_{n-1})}=1} \left\{ \alpha(s_{n-1}) + \gamma^{ext}(b_n) \right\} \quad (35)$$

Note that the effect of *a priori* information $\Pr(\mathbf{a}_n = \mathcal{A}_n)$ on Q-ary turbo-detection is twofold. First, it is accumulated during forward-backward recursions in $\alpha(s_n)$ and $\beta(s_n)$ due to the usage of $\gamma^a(b_n)$ instead of $\gamma(b_n)$ in (17),(21). Second, it explicitly presents in (31),(34) as $\sum_{\substack{l=1 \\ l\neq j}}^{q} \ln \Pr(a_{n,l} = \mathcal{A}_{n,l})$. In case of binary modulation (e.g., GMSK, q=1) the second term is not present. Hence, turbo-detection is expected to provide more gain for schemes with high level modulation. Simulation results presented below confirm this conjecture.

4.3 Iterative (turbo) channel estimation

In the turbo detection scheme (24) the iteration proceeds only between the signal detector and channel decoder assuming a known channel state $\hat{\mathbf{h}}$ during iterations. Given a known training sequence $\mathbf{m}$, the channel estimate may be obtained by (8) based on the data $\mathbf{y}_m$. However, in many cases the accuracy of channel estimate, which is based only on a relatively short training sequence $\mathbf{m}$, may be rather low. That in turn may cause a significant performance degradation at the receiver that cannot be fully compensated by the turbo detection. This fact motivated us to use a decision-directed adaptive channel estimation method during the iteration process similar to [13],[14]. The idea is to feed back the decoded symbols to the channel estimator and update previous channel estimates assuming that the whole burst is now known by the receiver (figure 3) [15]-[18]. Hence, the receiver relies on the hard decoded data symbols $\check{\mathbf{c}}_1^{\tau}$ and the known training sequence $\mathbf{m}$ to form a new channel estimate. In other words, the receiver iteratively updates the channel estimate based on an "extended" training sequence. In particular, after decoding procedure the data $\hat{\mathbf{u}}$ are re-encoded as $\check{\mathbf{c}} = G\hat{\mathbf{u}}$ interleaved and then combined with the training sequence $\mathbf{m}$, forming a new "extended" training sequence $\mathbf{a}$ of length τ. If we would use all available data $\mathbf{a}$ as the known training sequence, then the ML channel estimate in AWGN channel is

$$\widehat{\mathbf{h}}^{extend} = C(\widehat{\mathbf{h}}^{extend})\mathbf{Z}^H\mathbf{y} \quad (36)$$

where the covariance matrix of the new "extended" estimate is

$$\begin{aligned} C(\widehat{\mathbf{h}}^{extend}) &= (\mathbf{Z}^H\mathbf{Z})^{-1} \\ &= (\mathbf{Z}_1^H\mathbf{Z}_1 + \mathbf{M}^H\mathbf{M} + \mathbf{Z}_2^H\mathbf{Z}_2)^{-1} \end{aligned} \quad (37)$$

and matrix $\mathbf{Z}$ is formed by all data $\mathbf{a}$ [16].

The variance of the "extended" estimate $\widehat{h}^{extend}$ may be bounded by Cramer-Rao lower bound (CRLB) [23] (see appendix)

$$var(\hat{h}_i^{extend}) \geqslant \frac{\sigma^2}{P + \dfrac{(\tau - L - P)\sigma^2}{4p - 4p^2 + \sigma^2}} \quad (38)$$

where p is bit error probability for bits $\check{\mathbf{c}}$ forming the extension of the training sequence $\mathbf{m}$.

As an illustration the bounds (see (40),(44) in appendix) are visualized at figure 4 for parameters P=20 and N_d=58 accepted in GSM. As can be seen from (38), in case of minimum variance unbiased channel estimator the variance of the "extended" estimate is always lower than one calculated only from the training sequence, i.e.$var(\hat{h}_i^{extend}) < var(\hat{h}_i)$ for $p > 0$. It can be explained by an observation that by extending training sequence even with unreliable symbols, in average we make covariance matrix (37) more diagonal dominant, and that finally improves the channel estimate. Another point to mention is that the gain from the "extended" training sequence is mainly visible at low signal/noise ratios (SNR) and practically disappears at high SNR where the initial estimate is already rather accurate.

As it follows from (36)-(37), the channel re-estimation includes the inverse of matrix built for every transmitted block. To avoid heavy computation of the matrix inverse a suboptimal methods based on Toplitz presentation is proposed in [15]. However, this method seems still to be rather complex, and in this paper we suggest an adaptive algorithm to update the estimate. In particular, we applied stochastic adaptation of the estimate [18] based on the LMS algorithm [23]

$$\widehat{\mathbf{h}}^{(k+1)} = \widehat{\mathbf{h}}^{(k)} - \alpha(\hat{\mathbf{Z}}^{(k)})^H(\hat{\mathbf{Z}}^{(k)}\widehat{\mathbf{h}}^{(k)} - \mathbf{y}) \quad (39)$$

where $\widehat{\mathbf{h}}^{(k)}$ is a vector of channel coefficients estimated at k^{th} iteration, $\hat{\mathbf{Z}}^{(k)}$ is an estimated data matrix containing all (data+training) symbols known at k^{th} iteration, $\mathbf{y}$ is the received vector and α is a step size of the iterative algorithm. At the initial round the channel estimate could be based on some conventional method, e.g., one-shot ML estimate (8) which exploits only the known training sequence.

4.4 Combined iterative estimation-detection

Iteratively updating channel estimate and decoded symbols (turbo-estimation) on one hand, and detected and decoded symbols (turbo-detection) on the other hand, we actually attempt to find a solution to the general problem (3).

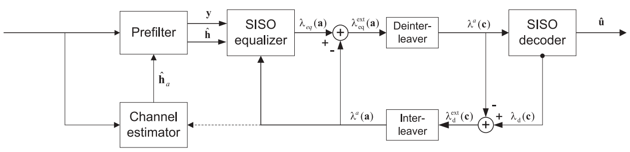

Figure 3: *Block diagram of iterative receiver.*

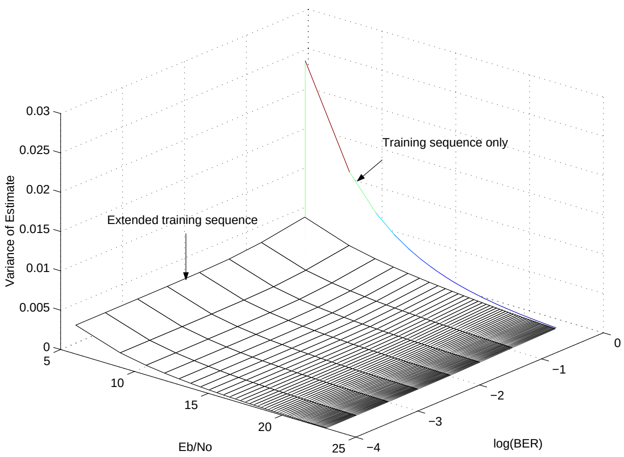

Figure 4: *Variance of estimate.*

The block diagram of the suggested iterative receiver is presented at figure 3. The proposed algorithm may be described as follows [18]:

Initialization (conventional receiver):

1. Make an initial channel estimate based on the known training sequence. The initial channel estimate could be based on some conventional method, e.g. one-shot estimate, $\hat{\mathbf{h}}_a^{(0)} = (\mathbf{M}^H\mathbf{M})^{-1}\mathbf{M}^H\mathbf{y}_m$

2. Calculate the prefilter coefficients and the resulting minimum phase channel $\widehat{\mathbf{h}}^{(0)}$, based on channel estimate $\hat{\mathbf{h}}_a^{(0)}$.

3. Given a channel estimate $\hat{\mathbf{h}}^{(0)}$, detect a sequence $\hat{\mathbf{c}}^{(0)}$ $\hat{\mathbf{c}}^{(0)} = \arg\max_{\mathbf{c}} \Pr(y_1^{N_b\ \tau}|\hat{\mathbf{h}}^{(0)}, \mathbf{c})$. Reliability for detected bits may be calculated according to (22) or (23).

4. Decode detected symbols $\hat{\mathbf{u}}^{(0)} = \arg\min_{\mathbf{u}} \left\|\hat{\mathbf{c}}^{(0)} - G\mathbf{u}\right\|^2 = \Omega(\hat{\mathbf{c}}^{(0)})$ In case of turbo-detection a SISO decoder should be used.

   Iterations (iterative receiver)

5. Based on decoded symbols make re-encoding operation, $\check{\mathbf{c}}^{(k)} = G\hat{\mathbf{u}}^{(k)} = G\Omega(\hat{\mathbf{c}}^{(k)})$ for the k^{th} iteration.

6. Rebuild the matrix $\hat{\mathbf{Z}}^{(k)}$ based on the updated $\check{\mathbf{c}}^{(k)}$ ($\hat{\mathbf{c}}^{(k)}$ for uncoded data).

7. Update channel estimate using some adaptation rule, e.g. the LMS:
$$\widehat{\mathbf{h}}_a^{(k+1)} = \widehat{\mathbf{h}}_a^{(k)} - \mu(\hat{\mathbf{Z}}^{(k)})^H(\hat{\mathbf{Z}}^{(k)}\widehat{\mathbf{h}}_a^{(k)} - \mathbf{y})$$

8. Recalculate the prefilter coefficients and the resulting minimum phase channel $\widehat{\mathbf{h}}^{(k+1)}$, based on channel estimate $\widehat{\mathbf{h}}_a^{(k+1)}$.

9. Given channel estimate $\widehat{\mathbf{h}}^{(k+1)}$, update detected sequence $\hat{\mathbf{c}}^{(k+1)}$ and its bit-wise likelihoods $\lambda_{eq}(a_{n,j})$ (e.g., according to (22),(23)). In case of turbo-detection the extrinsic information from the decoder $\lambda_d^{ext}(\check{\mathbf{c}}_n)$ is to be interleaved, $\lambda^a(\mathbf{a}_n) = \Pi\lambda_d^{ext}(\check{\mathbf{c}}_n)$, and then to be used as *a priori* information at the detection stage (27).

10. Update decoded symbols $\hat{\mathbf{u}}^{(k+1)} = \Omega(\hat{\mathbf{c}}^{(k+1)})$. In case of turbo-detection a SISO decoder is to provide the soft outputs $\lambda_d(\check{\mathbf{c}}_n)$.

11. Iterate between steps 5-10 (ICE) or/and between steps 9-10 (turbo-detection) as needed.

4.5 TURBO DETECTION FOR RETRANSMISSION SCHEMES

Efficiency of iterative data processing in schemes presented above clearly depends on the used channel coding scheme (CS). In modulation-coding schemes (MCS) where the coding rate is one (e.g., the coding rates for CS4/GPRS and MCS9/EGPRS [30]) a gain from turbo-equalization is not expected (in reality, the presence of a coded packet header brings some gain). To meet quality of service (QoS) requirements the (E)GPRS relies on retransmissions. This mechanism can be favorably exploited by an iterative receiver even in the case of uncoded transmission. A simple way to couple the turbo-equalization/turbo-estimation with retransmissions is presented below.

Let $\mathbf{c}^{(k)}$ denotes the k^{th} re-transmitted coded block. Retransmitted blocks may be repeated (e.g., GPRS) or differ in applied puncturing patterns (e.g., EGPRS). Let λ_{eq}^{ext,k,n_k} be a sequence of log extrinsic probability ratios (LEPRs) on a coded block $\mathbf{c}^{(k)}$ at the output of the equalizer, n_k is a number of turbo-equalization/estimation iterations performed for the k^{th} retransmitted block. If puncturing is applied then zeros are inserted instead of punctured bits. The proposed algorithm is the following

---

k=0
while QoS not satisfied
    k=k+1
    Request (re-)transmission
    for $i = 1..n_k$
        Perform channel (re-)estimation
        Perform SISO detection
        Update LEPR for decoder (bitwise sum)

$\lambda_{\text{det}}^{ext,k,i} \leftarrow \lambda_{\text{det}}^{ext,k,i} + \lambda_{\text{det}}^{ext,k-1,n_{k-1}}$
Perform SISO decoding
end for
Store $\lambda_{\text{det}}^{ext,k,n_k}$
Test QoS
end while

—

5 SIMULATION RESULTS

To find an efficient way to perform iterations described above it is necessary to study a trade-off between receiver complexity and performance gain provided by different iteration scenarios. Turbo detection method is based on soft decision decoder outputs that itself significantly (2-4 times) increases decoder complexity. On the other hand, iterative estimation method in the form presented above does not require soft decisions and hence, modifications of the decoder. A simple channel update based on an adaptation rule allows us to avoid complex calculations associated with matrix inverse.

As a practical testbed we considered performance of iterative receivers for (E)GPRS in typical mobile radio channels. In particular, we present results for schemes with the strongest channel coding CS1/GPRS and MCS5/EGPRS that employ a 1/2-rate and a 1/3-rate convolutional codes [30], respectively. Rectangular interleaving over 4 bursts is used in all cases. We use 5-taps estimator (8) with the LMS adaptation rule (39), the DF-SISO equalizer (ν=2) with the forward-only recursion (23) and max-log-MAP decoder to provide soft decision outputs. Quality of service in packet data transmission is characterized by block error rate (BLER).

We evaluated performance of different iterative scenarios in typical urban and hilly terrain channels with speed v=3km/h (TU3) and 100km/h (HT100), respectively. As it was expected, the iterative methods designed to compensate the residual ISI provide more gain in case of the HT channel with longer delay spread. The performance figures are presented for: (a) the conventional receiver (figures 5-7, solid lines); (b) separately after iterative channel estimate update (dashed lines at figures 5-6) and after turbo detection (lines with labels at figures 5-7); (c) after turbo detection iterations, where the channel estimate is no more changed (figure 6). Performance gain obtained from iterative data processing is summarized in Table 1 and Table 2. As it was expected, the turbo-detection gain is getting lower at higher coding rates, but still remains noticeable (Table 1). Even in case of MCS9 there is 0.7dB gain mainly due to the presence of a coded packet header in each transmitted block. Table 2 presents results obtained from only turbo-estimation (TE) and turbo detection (TD) after 2 and 4 iterations, where 1^{st} iteration corresponds to the conventional receiver. In case of the combined TE-TD we used only one turbo round for each operation. For example, in case of GPRS the gain achieved after one channel estimate update is around 1dB at BLER=10^{-2}, and the TD is able to provide only 0.2 dB extra gain on top of that (figure 5). The TE used alone shows also the same improvement in EGPRS environment (figure 6). However, now the TD provides 2-3 dB gain, and that justifies the increase of decoder complexity to provide soft decisions.

The EGPRS standard defines a sophisticated retransmission scheme known as Incremental Redundancy (IR) [30]. According to the IR scheme each MCS has three disjoint puncturing patterns which are taken cyclically for retransmissions. Simulation results for MCS9/EGPRS are shown at figure 7 for first, second and third retransmissions with number of iterations: $(n_1 = 1, n_2 = 0, n_3 = 0)$, $(n_1 = 1, n_2 = 1, n_3 = 0)$, $(n_1 = 1, n_2 = 1, n_3 = 1)$, $(n_1 = 1, n_2 = 2, n_3 = 1)$ and finally $(n_1 = 1, n_2 = 2, n_3 = 2)$. As one can see the proposed scheme provides 1.8dB gain at BLER 10^{-2} if we compare retransmissions without $(n_1 = 1, n_2 = 1, n_3 = 1)$ and with $(n_1 = 1, n_2 = 2, n_3 = 2)$ TE-TD.

Table 1: *EGPRS: Turbo-Detection (TD) gain in [dB] for MCS5-9 at BLER=10^{-1}*

Coding scheme	MCS5	MCS6	MCS7	MCS8	MCS9
Code rate	0.37	0.49	0.76	0.92	1.0
TE gain 2iter (4iter)	2.2 (3.2)	2.1 (3.1)	1.9 (2.4)	0.9 (1.7)	0.7 (1.4)

Table 2: *(E)GPRS: Performance improvement in [dB] due to TD and TE at BLER=10^{-2}*

Coding scheme	Code rate	Modulation	TD 2it (4it)	TE 2it (4it)
CS1	0.5	GMSK	0.2 (0.3)	1.0 (1.1)
MCS5	0.37	8PSK	2.2 (3.2)	1.0 (1.1)

Coding scheme	Code rate	Modulation	TE- TD one round each
CS1	0.5	GMSK	1.1
MCS5	0.37	8PSK	2.4

6 CONCLUSIONS

In this paper a receiver concept that includes channel estimation, detection, re-transmission and channel decoding into a common iterative process is presented. In order to keep the receiver complexity low (which is especially important for high level modulation) suboptimal DF-SISO equalizers and its modifications suitable for turbo equalization are derived. It is shown that turbo-detection provides more gain for high level modulation schemes such as 8-PSK. To further improve the radio-link performance we include decoder into the ICE procedure and suggest a simple method to update channel estimates. We also propose a simple way to embed the iterative detection into retransmission schemes. Evaluation of the combined iterative re-

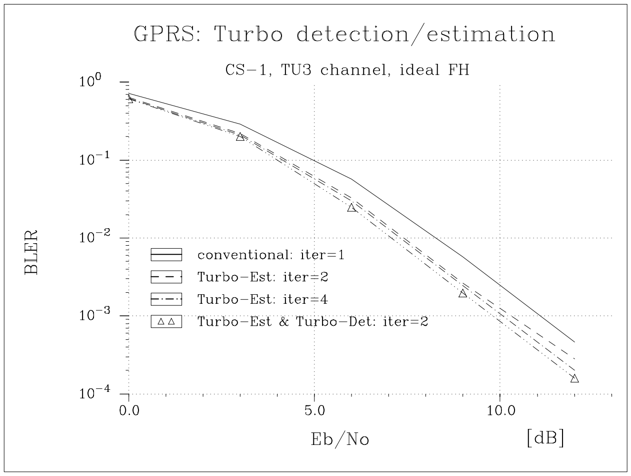

Figure 5: *Performance of GPRS iterative receiver.*

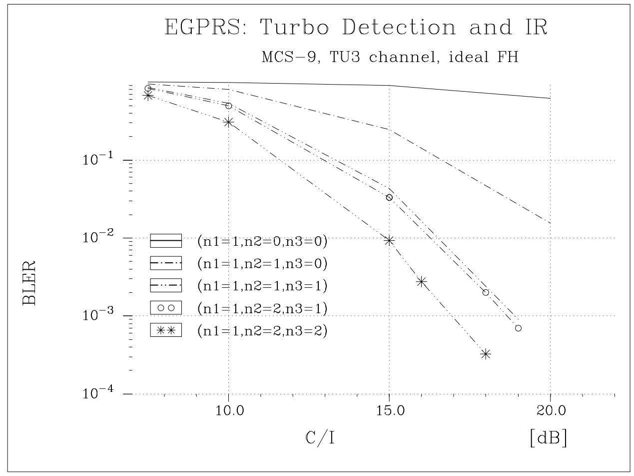

Figure 7: *Performance of turbo-equalization combined with incremental redundancy.*

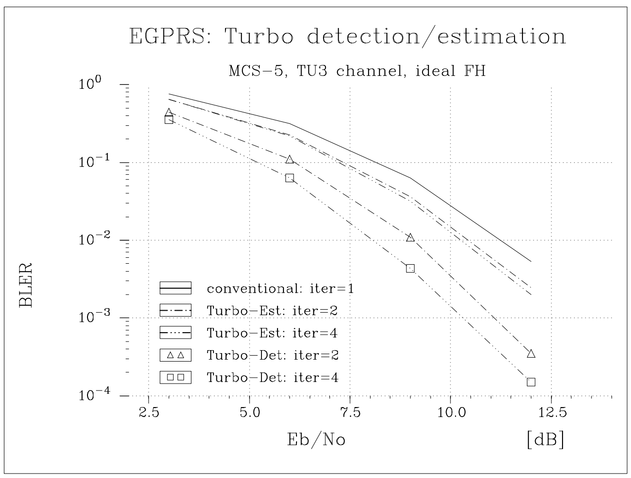

Figure 6: *Performance of EGPRS iterative receiver.*

ceiver in the context of GSM EDGE Radio Access Network (GERAN) third generation TDMA system shows that the suggested approach provides a significant performance improvement.

A APPENDIX

Let's consider AWGN channel with noise samples $w_n = \mathcal{N}(0, \sigma_n^2)$ and a constant variance σ^2 during the received block $\mathbf{y}_m$, i.e. $\sigma_n^2 = \sigma^2, n = 1...\tau$. The variance of the estimate $\hat{\mathbf{h}}$ based only on the training sequence $\mathbf{m}$ may be bounded by Cramer-Rao lower bound (CRLB) [23] as $var(\hat{h}_i) \geqslant [\mathbf{I}(\mathbf{h})]_{ii}^{-1}$, where $\mathbf{I}(\mathbf{h}) = \{I_{ij}\}$ is the $(\mu+1)\times(\mu+1)$ Fisher information matrix with elements $I_{ij} = -\dfrac{\partial^2 \ln \Pr(\mathbf{y}_m|\mathbf{m},\mathbf{h})}{\partial h_i \partial h_j}$.

In our case CRLB may be written as

$$var(\hat{h}_i) \geqslant [\mathbf{M}^H \mathbf{R}^{-1} \mathbf{M}]_{ii}^{-1} \approx \frac{1}{\sum_{n=1}^{P} \frac{1}{\sigma_n^2}} = \frac{\sigma^2}{P} \tag{40}$$

where i=0...μ.
For the "extended" training sequence $\mathbf{y}$ it gives

$$\begin{aligned} var(\hat{h}_i^{extend}) &\geqslant [\mathbf{Z}^H \mathbf{R}^{-1} \mathbf{Z}]_{ii}^{-1} \\ &\approx \frac{1}{\sum_{n=1}^{\tau} \frac{1}{\sigma_n^2}} \\ &= \frac{1}{\frac{P}{\sigma^2} + \frac{\tau - P - L}{\sigma_d^2}} \\ &= \frac{\sigma^2}{P + (\tau - P - L)\frac{\sigma^2}{\sigma_d^2}} \end{aligned} \tag{41}$$

where σ_d^2 is the variance associated with the "extended" data $\check{\mathbf{c}}$ provided by the decoder.
To evaluate (41) let's denote p_n as an error probability for n^{th} complex symbol $\check{z}_n$ after decoding/re-encoding operations, $\check{z}_n \in \check{\mathbf{z}}$, $\check{\mathbf{z}} = \Psi(\check{\mathbf{c}}_n)$. The probabilities p_n are calculated by averaging over a number of received blocks. Let's consider the antipodal signalling $(z_n = 1, \bar{z}_n = -1)$, and assume that data block with $z_n = 1$ is repeatedly transmitted. The generalization to Q-ary modulation is straight forward. We can treat symbols $\check{\mathbf{z}}$ forming the extension of training sequence as a set of random variables with the mean and the variance as follows

$$E[\check{z}_n] = (1 - p_n) z_n + p_n \bar{z}_n$$

$$\begin{aligned} var[\check{z}_n] &= E[|\check{z}_n|^2] - |E[\check{z}_n]|^2 \\ &= |z_n|^2 - p_n(|z_n|^2 - |\bar{z}_n|^2) - (z_n - p_n(z_n - \bar{z}_n))^2, \end{aligned}$$

where $\bar{z}_n \neq z_n$.
Assuming a time-invariant channel during the transmitted blocks (i.e. $p_n = p$, n=0,..., $\tau - P - L - 1$) for antipodal signalling it results in

$$var[\check{z}_n] = 4p(1-p) \tag{42}$$

The variance of the "extended" data is

$$\begin{aligned}\sigma_d^2 &= var[\check{z}_n + w_n] \\ &= var[\check{z}_n] + var[w_n] + 2cov[\check{z}_n w_n]\end{aligned} \tag{43}$$

Taking into account that more errors appear after decoding/re-encoding operations at high noise levels (i.e. larger σ^2 results in larger p, and hence, in larger $var[\check{z}_n]$), the correlation term in (43) $cov[\check{z}_n w_n] \geqslant 0$, therefore $\sigma_d^2 \geqslant var[\check{z}_n] + var[w_n]$. As can be seen from (43) the variance (41) is a function of $cov[\check{z}_n w_n]$, and it achieves its minimum $var(\hat{h}_i^{extend})$ if $cov[\check{z}_n w_n]$ =0. Based on (41) and (42) this minimum may be presented as

$$var(\hat{h}_i^{extend}) \geqslant \frac{\sigma^2}{P + \dfrac{(\tau - L - P)\sigma^2}{4p - 4p^2 + \sigma^2}} \tag{44}$$

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