# Space-Time Trellis Coding with Turbo Equalisation for the EGPRS System

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Abstract - Transmit diversity techniques like delay diversity (DD) or space-time trellis coding (STTC) utilise several transmission antennas and thereby protect mobile radio systems against fast fading. The more complex STTC can recover bursts with a few errors, but on the other hand it increases bit errors for bad bursts. Therefore the concatenation of an outer convolutional code and STTC is not superior to the simple DD scheme. The outer coding is commonly used in mobile packet data applications to detect and correct transmission errors. In this paper we are interested in further improving STTC performance by iterative turbo equalisation (TE) technique, which performs space-time decoding-equalisation and outer decoding in an iterative fashion. Our objective is to analyse whether TE can exploit the more complex STTC structure by comparing the iterative gains of STTC and DD. The performance evaluation is done by simulations in the Enhanced General Packet Radio System (EGPRS) platform.

#### I. INTRODUCTION

A major harm for wireless mobile radio systems is fast fading phenomenon, which could be diminished by a number of techniques. One of them is transmit diversity, which exploits several uncorrelated transmitter antennas and hence, the transmission reliability is not dependent on a single fading channel. Significant gain in performance is achieved, as the probability of all transmission channels being in a fading dip simultaneously is very low.

DD is the simplest diversity scheme as it employs only repetition code with a fixed delay in the different antennas. STTC is more sophisticated method with a certain coding structure for the transmitted signals. STTC is able to recover a small number of errors within a burst, but when bursty errors occur, it generates even more errors [1]. Hence, it may improve block error rate (BLER), but not bit error rate (BER) of an uncoded system. As a consequence concatenating outer code with STTC may cause degradation in performance as shown for the EDGE system in [3].

TE is an iterative method resembling the celebrated turbo codes [4], but exploiting the equaliser and channel decoder iteratively [5,6]. TE provides significant performance gain for the EDGE system with 8-PSK modulation as shown by several authors [7-10]. However, those papers consider only single antenna transmission.

TE is a suboptimum algorithm to perform joint detection and decoding and in this paper we investigate whether TE can help concatenation of STTC and EDGE convolutional codes that are used in the actual coding schemes MCS-5 and MCS-8. We combine the DD/STTC system with the TE technique to study if TE benefits from the trellis structure of STTC. We evaluate the performance of the DD and STTC systems in the EGPRS platform using MCS-5 and MCS-8 schemes.

The paper is organised as follows. The next chapter presents the signal model with multiple transmit antennas. Then we describe the DD and STTC schemes in detail in Section III and after that the TE technique is discussed in Section IV. Then follows the performance evaluation with the simulation results and finally conclusions are drawn.

#### II. TRANSMIT DIVERSITY SYSTEM

In this paper, we use the EGPRS platform, which is modelled in Fig. 1. In the transmitter side a block of data bits **u** is protected by a convolutional encoder and punctured to an appropriate data rate. The coded bits **c** are interleaved over four successive transmission bursts to overcome fast fading phenomenon. After interleaving the bits are merged into 8-PSK symbols, i.e., every three successive bits form one symbol  $\mathbf{a}_k = (a_{k,1}, a_{k,2}, a_{k,3})$ . These symbols are organised as transmission bursts denoted by  $\mathbf{a} = (\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_K)^T$ . For transmit diversity a space-time (ST) encoder is used to map vector **a** to N separate complex-valued symbol vectors,  $\mathbf{d}_n = \mathbf{d}_n(\mathbf{a}) = (d_{n,1}, d_{n,2}, ..., d_{n,K})^T$ , where n = 1, 2, ..., N is the antenna index and N is the number of transmit antennas.

The ST coded signal is transmitted over frequency selective fading channels using N uncorrelated antennas. Thermal noise at the receiver is modelled as additive white Gaussian noise (AWGN).

The received signal  $\mathbf{r}$  that is sampled at the symbol rate can be given by the equation

$$\mathbf{r} = \sum_{n=1}^{N} \mathbf{D}_n \mathbf{h}_n + \mathbf{w} \quad , \tag{1}$$



Fig. 1. Transmission system model with 2 transmit antennas.

where  $\mathbf{D}_n$  is the matrix containing ST coded symbols  $d_{n,k}$  transmitted from antenna *n* as follows

$$\mathbf{D}_{n} = \begin{vmatrix} d_{n,L+1} & d_{n,L} & \cdots & d_{n,1} \\ d_{n,L+2} & d_{n,L+1} & & d_{n,2} \\ \vdots & & \ddots & \vdots \\ d_{n,K+L} & d_{n,K+L-1} & \cdots & d_{n,K} \end{vmatrix} .$$
(2)

The channel impulse response from the antenna *n* to the receiver antenna is described by the vector  $\mathbf{h}_n = (h_{n,0}, h_{n,1}, ..., h_{n,L})^T$ , which consists of symbol-spaced complex-valued channel taps. The white Gaussian noise samples are denoted by **w** with the noise variance of  $\sigma^2 = N_0/2$ .

The equaliser needs to estimate the *N* channel impulse responses by using the known training symbols in the middle of the burst. The joint maximum-likelihood (ML) channel estimate  $\hat{\mathbf{h}} = (\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2, \dots, \hat{\mathbf{h}}_N)^T$  for *N* channels in the presence of white Gaussian noise is given by [11]

$$\hat{\mathbf{h}} = \left(\mathbf{M}^H \mathbf{M}\right)^{-1} \mathbf{M}^H \mathbf{r} \quad , \tag{3}$$

where  $\mathbf{M} = (\mathbf{M}_1, \mathbf{M}_2, \dots, \mathbf{M}_N)$  contains the training sequence matrices, each of which formed from the midamble symbols according to (2).

#### **III. SPACE-TIME TRELLIS CODING**

In space-time (ST) trellis coding, the space-time encoder generates a symbol stream for each transmit antenna given a stream of input symbols according to a specific trellis diagram. Each new input symbol defines a transition from the current state and the state transition label defines the symbols to be transmitted from each antenna. The symbols are transmitted in bursts with a proper midamble assigned to each transmitting antenna.



Fig. 2. An 8-state STTC for 8-PSK and 2 transmit antennas (also the symbol constellation is shown).

#### A. Delay Diversity

A simple ST trellis code is DD code using two transmit antennas. In DD transmission, the second antenna transmits the same signal as the first one but with a delay. When antennas have uncorrelated channels, the channel taps are fading at different times, which can be advantageously utilised by the receiver. Since, effectively, this approach just increases the length of the channel impulse response, the receiver need not to know the presence of DD transmission as long as the *same* midamble is allocated for the both transmit antennas. A practical value for the delay offset between the antennas is one symbol interval.

## B. Optimised Space-Time Trellis Codes

When actual optimised ST trellis codes [1] are applied, the symbol streams assigned to the antennas are not just delayed versions of each other. Therefore the receiver also requires the knowledge of the used ST trellis structure to decode the data symbols. Moreover, the channel from each transmit antenna has to be estimated by using the different midambles transmitted from the antennas. Fig. 2 shows the trellis of an 8-state code for 8-PSK modulation and two transmit antennas [1], where the labels of the state transitions defining the symbols to be transmitted from the antennas are shown in the table beside the trellis.

Due to complexity restrictions it is mandatory to combine space-time decoding and equalisation at the receiver. Since the received signal is a superposition of N 8-PSK signals, the joint equaliser would have  $8^{NL}$  states, if the ST code structure were not taken into account. In practise, this dependency on the number of transmit antennas N is intolerable, because of which we employ joint space-time decoding and equalisation following the approach given in [2]. The well-known ML solution

$$\hat{\mathbf{a}} = \arg\min_{\mathbf{a}} \left\| \mathbf{r} - \sum_{n=1}^{N} \mathbf{D}_{n}(\mathbf{a}) \hat{\mathbf{h}}_{n} \right\|^{2} , \qquad (4)$$

where the ST coded symbol matrix  $\mathbf{D}_n$  is a function of the candidate data vector  $\mathbf{a}$ , is solved using the Viterbi algorithm.

Due to the multilevel 8-PSK symbols, the complexity of Maximum Likelihood Sequence Estimation (MLSE) is too extensive to be implemented. This is why a reduced-state equaliser like Decision Feedback Sequence Estimation (DFSE) has to be used. It incorporates the previous symbol decisions in the metrics calculation and therefore the number of states is significantly reduced [12]. A problem for multichannel equalisation techniques like STTC is to build a minimum phase prefilter in the front of DFSE. In this paper we have circumvented the problem by increasing the number of trellis states from 8 to 64 and using non-minimum phase channel in simulations.

### IV. TURBO EQUALISATION

## A. Turbo principle

Let us consider in more detail the iterative receiver structure, which is presented in Fig.3. The space-time equaliser calculates log-likelihood values  $\lambda_{eq}(\mathbf{a})$  exploiting the a priori values  $\lambda^a(\mathbf{a})$  coming from the previous iteration. At the very first iteration there is no a priori available, so  $\lambda^a(\mathbf{a}) \equiv 0$  during the initial iteration.

During the metrics calculation the a priori probabilities  $Pr(\mathbf{a}_k)$  for each transition are extracted from the corresponding log-likelihood ratios

$$\lambda^{a}(a_{k,j}) = \ln \frac{\Pr(a_{k,j} = 0)}{\Pr(a_{k,j} = 1)}.$$
(5)

The equaliser output  $\lambda_{eq}(\mathbf{a})$  consists of intrinsic and extrinsic information. The latter is the incremental information obtained in the equalisation and it is extracted from the output by bitwise subtraction as follows [5,6]

$$\lambda_{eq}^{ext}(a_{k,j}) = \lambda_{eq}(a_{k,j}) - \lambda^a(a_{k,j}).$$
(6)

The extrinsic information is then deinterleaved to achieve a priori information  $\lambda^a(\mathbf{c})$  on the coded data. These values are provided for the Soft-in-Soft-out (SISO) channel decoder, which calculates soft outputs  $\lambda_d(\mathbf{c})$  for the coded bits. However, the feedback into the ST equaliser only contains the incremental information obtained from the surrounding bits in the channel decoding. This extrinsic information is obtained as follows [5,6]

$$\lambda_d^{ext}(c_{k,j}) = \lambda_d(c_{k,j}) - \lambda^a(c_{k,j}) .$$
<sup>(7)</sup>



Fig. 3. Turbo equaliser structure.

The turbo equalisation technique is based on the utilisation of extrinsic information at the next iteration round [5]. Hence, it is interleaved and provided for the equaliser as a priori information  $\lambda^a(\mathbf{a})$  on the bit reliabilities. By exploiting the new information available in the detection, more reliable decisions are achieved.

The iterative processing may continue several rounds if there is processing power available. Nevertheless, it is useless to iterate without further improvement. At the final iteration round the conventional hard decoder is used instead of SISO to obtain information bits  $\hat{\mathbf{u}}$ .

## B. Soft-in-soft-out decoding

The turbo iteration is based on the feedback of soft information from the channel decoder to the equaliser, which requires SISO decoder. A suitable decoding algorithm is BCJRmax-log-MAP, which provides the a posteriori probability for each bit [13,14]. The algorithm exploits the following probability functions for the trellis state s at time k

$$\boldsymbol{\alpha}_{k}(s) = p(s, \mathbf{r}_{i \le k}) \tag{8}$$

$$\boldsymbol{\beta}_{k}(s) = p\left(\mathbf{r}_{j>k} \middle| s\right) \quad . \tag{9}$$

Furthermore, the log-probability of the transition to the state s given a starting state s' is as follows

$$\ln \gamma_k(s^{\prime},s) = \sum_{j=1}^M \lambda^a (c_{k,j}) c_{k,j} , \qquad (10)$$

if the coding rate is 1/M.

Using the previous definitions BCJR-max-log-MAP provides the following output [14]

$$\lambda_{d}(c_{k,j}) = \max_{\substack{(s',s)\\c_{k,j}=+1\\}} \{\ln \alpha_{k-1}(s') + \ln \gamma_{k}(s',s) + \ln \beta_{k}(s)\}$$

$$- \max_{\substack{(s,s)\\c_{k,j}=-1\\}} \{\ln \alpha_{k-1}(s') + \ln \gamma_{k}(s',s) + \ln \beta_{k}(s)\}$$
(11)

and the forward and backward recursions are computed by

$$\ln \alpha_k(s) = \max\left\{\ln \gamma_k(s', s) + \ln \alpha_{k-1}(s')\right\}$$
(12)

$$\ln \beta_{k}(s') = \max \{ \ln \gamma_{k+1}(s', s) + \ln \beta_{k+1}(s) \}.$$
(13)

#### **IV. PERFORMANCE RESULTS**

We evaluate the performance of transmit diversity techniques combined with TE iterations in the EGPRS platform. Hence, 8-PSK modulation is used and 1/3-rate outer code, which is punctured to the rate 0.37 (MCS-5) or 0.92 (MCS-8). 8-state STTC with two transmit antennas is considered and compared to the DD system with one symbol delay between the transmit antennas. The STTC equaliser has 64 trellis states and for a fair comparison we use 64-state equaliser for DD, too. The multipath channel profile is Typical Urban with the mobile speed of 3 km/h (TU3). No frequency hopping is used in any simulations. During one TE iteration both ST-equalisation and outer decoding are performed once. Moreover, the first iteration stands for the conventional performance without using any feedback information yet.

Fig. 4 shows the MCS-8 performance in terms of block error rate (BLER) for the first, second and fourth TE iteration. The performance of the single antenna transmission is given as a reference. The iterative improvement for the both STTC and DD systems is 1 dB after second and 1.5 dB after the fourth iteration. Hence, the TE method does not benefit from the more complex STTC structure, but provides equal improvement for the simple repetition code of DD. The benefit of STTC shows already in the first iteration and during the further iterations TE only makes the channel equalisation more reliable regardless of the ST trellis structure that is used. Transmit diversity as such provides clear gain compared to the single antenna transmission. DD gives 1.5 dB gain and STTC almost 2 dB at BLER 10<sup>-1</sup>. Furthermore, STTC has a steeper slope, so the difference is even larger for good signal quality.

Fig. 5 presents the performance in the stronger MCS-5 coding scheme. As expected, the first iteration for STTC is worse than for DD with the strong convolutional code [3]. Moreover, we can observe that TE provides more iterative gain for DD. After the second iteration STTC improves less than 2 dB, whereas DD achieves 2.5 dB gain at BLER 10<sup>-1</sup>. After the fourth iteration we observe 3 dB improvement for STTC, but almost 4 dB gain for DD. Thus these results also imply that TE provides equivalent or even smaller performance gain for STTC than for DD.



Fig. 4. BLER performance for MCS-8.



Fig. 5. BLER performance for MCS-5.

#### V. CONCLUSIONS

The performance of the STTC technique when concatenated with an outer code is not very good. In this paper we consider the iterative TE method in the receiver to enhance the performance. TE utilises the ST equaliser and the outer decoder in an iterative fashion to achieve more reliable decisions.

To analyse if TE is able to exploit the trellis structure of STTC we combine also the DD system with the TE receiver. DD is based on the simple repetition code. We evaluate the iterative gains of STTC and DD using EGPRS with an outer convolutional code as a simulation platform.

The simulations show an equivalent or lower iterative gain for STTC than for DD. Hence, TE is not able to utilise the more complex structure of STTC, but merely improves the channel equalisation reliability for the both systems. The achieved performance gains are 1.0-1.5 dB at BLER 10<sup>-1</sup> for the MCS-8 coding scheme. The difference between DD and STTC is observed already in the first iteration and it remains the same during the further iterations. When concatenated with the stronger outer code of the MCS-5 scheme DD performs better already in the first iteration and improves even more by the iterative data processing than STTC.

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