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Laser-induced collective excitations in a two-component Fermi gas

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We consider the linear density response of a two-component (superfluid) Fermi gas of atoms when the perturbation is caused by laser light. We show that various types of laser excitation scheme can be transformed into linear density perturbations; however, a Bragg spectroscopy scheme is needed for transferring energy and momentum into a collective mode. This makes other types of laser probing schemes insensitive for collective excitations and therefore well suited for the detection of the superfluid order parameter. We show that, for the special case when laser light is coupled between the two components of the Fermi gas, density response is always absent in a homogeneous system.

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I. INTRODUCTION

Fermionic atoms have already been cooled well below the Fermi temperature [1]. Once the quantum regime has been achieved, the predicted BCS superfluid transition [2] for two different hyperfine states with attractive interactions between them would be the next goal to achieve. There are several methods proposed for probing the superconducting gap [3–6]. Some of them are based on the use of laser light. For instance, it has been proposed to measure the superfluid coherence and the existence of the superfluid gap by the absorption of almost on-resonance laser light coupled between the two hyperfine states [4] or between one of them and another atomic hyperfine state [3]. The use of off-resonant (or very low intensity) laser light [5] has also been proposed. There both hyperfine components are coupled to other excited states not involved in the pairing, and the excited state populations remain negligible.

In this paper we review and analyze the linear density response of the system to the different laser excitations in the collisionless regime where the lifetime of the quasiparticles is much longer than the period of the applied field. The motivation for studying the response is twofold. First, in the above-mentioned probing schemes for the gap a below-gap density response (e.g., Anderson-Bogoliubov phonons) would be undesirable because the schemes rely on the absence of below-gap excitations. Therefore it is of interest to clarify in what kind of laser probing schemes density reponses can be avoided. Second, the density response itself may be the object of study and the laser excitation schemes that can produce a response are of interest.

Off-resonant laser excitation caused by two intersecting waves which form a density grating in space and time (Bragg spectroscopy) has been used to induce collective excitations in atomic Bose-Einstein condenstates [7]. In the case of near-resonant laser excitations, we transform the nondiagonal atom-light interaction Hamiltonian into a form of a density perturbation. We show that the initial Hamiltonian of the interacting two-component Fermi gas is preserved by this transformation in case of a contact interaction and therefore a standard linear density response calculation for a BCS system can be applied. In general, an intensity grating in space and time is needed for providing momentum and energy transfer for density perturbations. We show that when the laser light is coupled between the two paired hyperfine components, phonons cannot be excited even when the light forms an intensity grating. We will also discuss the connection to other light-induced collective excitations such as polaritons [8].

In Sec. II, the Hamiltonian for a two-component Fermi gas is given and the density response is defined. The laser probing schemes considered in this article are summarized in Sec. III. In Sec. IV we transform the perturbation caused by the probing lasers into a density perturbation by appropiate rotation of the coupled states. The results are presented in detail in Sec. V and summarized in Sec. VI. The linear response calculation is presented in the Appendix.

II. THE SYSTEM

The Hamiltonian for the two-component Fermi gas, including only two-body interactions between the atoms reads

$$H_0=\int d\mathbf{r}\sum_{\alpha=\uparrow,\downarrow}\left[\psi_\alpha^\dagger(\mathbf{r})\left(-\frac{\nabla^2}{2m}+V_\alpha(\mathbf{r})-\mu_\alpha\right)\psi_\alpha(\mathbf{r})\right]+\frac{1}{2}\sum_{\alpha,\beta=\uparrow,\downarrow}\int d\mathbf{r}\int d\mathbf{r}' g_{\alpha\beta}(\mathbf{r}-\mathbf{r}')\times\psi_\alpha^\dagger(\mathbf{r})\psi_\beta^\dagger(\mathbf{r}')\psi_\beta(\mathbf{r}')\psi_\alpha(\mathbf{r}),\quad(1)$$

where the two hyperfine states trapped are denoted by $\uparrow$ and $\downarrow$. When the interaction $g_{\uparrow\downarrow}(\mathbf{r}-\mathbf{r}')$ is attractive, the system has been predicted to undergo a BCS transition [2] associated with the appearance of an order parameter. We assume the system to be below the critical temperature and the Hamiltonian H_0 is treated within the BCS approximation.

The density response of the system to perturbations, caused by, e.g., probe lasers or the trapping fields, is studied in the linear response regime [9]. The density perturbation is a linear function of an external potential $U(\mathbf{r},t)$ expressed through the density-density response function $\chi(\mathbf{r},t;\mathbf{r}',t')$:

$$\delta n(\mathbf{r},t) = \delta n_{\uparrow}(\mathbf{r},t) + \delta n_{\downarrow}(\mathbf{r},t) = \int d\mathbf{r}dt\chi(\mathbf{r},t;\mathbf{r}',t')U(\mathbf{r},t), \quad (2)$$

where the density operators are defined in the usual way $n_{\alpha=\uparrow,\downarrow}(\mathbf{r}) = \psi_{\alpha}^{\dagger}(\mathbf{r})\psi_{\alpha}(\mathbf{r})$. Fourier transforming the previous expression one obtains

$$\delta n(\mathbf{k},\omega) = \chi(\mathbf{k},\omega)U(\mathbf{k},\omega). \quad (3)$$

The poles of the reponse function χ give the collective modes of the system while its modulus gives the spectral weight of the modes.

It is known that a perturbation of the form $U(\mathbf{r},t) \times[\psi_{\uparrow}^{\dagger}(\mathbf{r})\psi_{\uparrow}(\mathbf{r}) + \psi_{\downarrow}^{\dagger}(\mathbf{r})\psi_{\downarrow}(\mathbf{r})]$ leads in the long wavelength limit to appearance of the Anderson-Bogoliubov phonon [10–12], that is, a collective mode with energy below the superconducting gap.

In this paper we study the reponse function $\chi(\mathbf{k},\omega)$ and potential $U(\mathbf{k},\omega)$ for a perturbation H' created by different laser probing schemes. Most of the schemes have been considered in the literature as proposed techniques for observing the superconducting gap.

III. PROBING SCHEMES

A. Laser coupling between two states

(a) Coupling the two paired states. Let us assume that the states $\uparrow$ and $\downarrow$ are coupled by light [4]. Using the rotating-wave approximation (RWA) [13], the interaction of the laser light with the matter fields can be described by a time-independent Hamiltonian in which the detuning δ plays the role of a chemical potential and a Rabi frequency $\Omega(\mathbf{r})$ characterizes the local strength of the interaction. The perturbation Hamiltonian is $H' = H_{\mu} + H_T$, where the transfer Hamiltonian (H_T) and H_{μ} are given by

$$H_T = \int d\mathbf{r}\Omega(\mathbf{r})\psi_{\uparrow}^{\dagger}(\mathbf{r})\psi_{\downarrow}(\mathbf{r}) + \Omega^*(\mathbf{r})\psi_{\downarrow}^{\dagger}(\mathbf{r})\psi_{\uparrow}(\mathbf{r}),$$

$$H_{\mu} = \frac{\delta}{2}\int d\mathbf{r}\psi_{\uparrow}^{\dagger}(\mathbf{r})\psi_{\uparrow}(\mathbf{r}) - \frac{\delta}{2}\int d\mathbf{r}\psi_{\downarrow}^{\dagger}(\mathbf{r})\psi_{\downarrow}(\mathbf{r}). \quad (4)$$

In principle, one can imagine a direct coupling of the states $\uparrow$ and $\downarrow$ by one field, in which case δ and Ω correspond directly to that field. In practice, for coupling of two hyperfine states which have an energy difference far from any laser frequency one may prefer a Raman transition. In that case the Rabi frequencies and δ depend on the parameters from both lasers, especially $\Omega \propto \Omega_1^* \Omega_2$.

(b) Coupling to a nonpaired state. When one of the states $\alpha = \{\uparrow \text{ or } \downarrow\}$ is coupled to some excited hyperfine state e [3], the perturbation reads

$$H_T = \int d\mathbf{r}\Omega(\mathbf{r})\psi_{\alpha}^{\dagger}(\mathbf{r})\psi_e(\mathbf{r}) + \Omega^*(\mathbf{r})\psi_e^{\dagger}(\mathbf{r})\psi_{\alpha}(\mathbf{r}),$$

$$H_{\mu} = \frac{\delta}{2}\int d\mathbf{r}\psi_{\alpha}^{\dagger}(\mathbf{r})\psi_{\alpha}(\mathbf{r}) - \frac{\delta}{2}\int d\mathbf{r}\psi_e^{\dagger}(\mathbf{r})\psi_e(\mathbf{r}). \quad (5)$$

(c) Far-off-resonant light. In [3,4] the coupled light was proposed to be almost on resonance, that is, atomic population is transferred in the excitation process. In [5] coupling of the paired states with some excited states was assumed to be done by far-off-resonant or very weak intensity light, meaning that the excited state population remains negligible and the excited state can be eliminated from the problem. Far-off-resonance coupling can be treated by starting with an initial Hamiltonian of the same form as Eq. (5); then after adiabatically eliminating the excited states one arrives at $(|\Omega|^2/\delta)\psi_{\alpha}^{\dagger}\psi_{\alpha}$ (cf. Sec. IV).

Polaritons are predicted to appear in thermal atomic gases and Bose condensates [8] whenever the wavelength λ of the exciting laser is large compared to the interparticle distance, $\lambda^3 n \gg 1$, and the process is nearly resonant, i.e., $\delta \leqslant nd^2 = \gamma n \lambda^3$. Here n is the density of the gas, d the dipole moment of the atomic transition ($\propto \Omega$) and γ the decay rate. In our case the condition $\lambda^3 n \gg 1$ is valid. However, in cases (a) and (b), the transfer of population is between two long-lived hyperfine states (rather than a transfer to some highly excited state). The transfer process is realized by a Raman (or microwave) transition where both of the participating laser fields are far detuned from the intermediate excited state. Thus the second condition is valid neither in cases (a),(b) nor in case (c). Existence of polaritons in a BCS-paired gas under suitable conditions would be an interesting topic of further study.

B. Space and time dependence of the perturbation

(d) Spatial variation of the intensity. The common way of creating intensity gratings used, e.g., for creating collective excitations in Bose-Einstein condensates [7] is to use two intersecting waves. When two intersecting waves with the same polarization but different wave vector and frequency are coupled to the same two-level system, the total Rabi frequency is $\Omega = \Omega_1 e^{i\mathbf{k}_1\cdot\mathbf{r} - i\omega_1 t} + \Omega_2 e^{i\mathbf{k}_2\cdot\mathbf{r} - i\omega_2 t}$. After making the RWA for the frequency ω_1, $\Omega = \Omega_1 e^{i\mathbf{k}_1\cdot\mathbf{r}} + \Omega_2 e^{i\mathbf{k}_2\cdot\mathbf{r} + i\omega_{12}t}$ where $\omega_{12} = \omega_1 - \omega_2$.

Also the beam profile may vary spatially on the cloud size scale. The Rabi frequency would then be, for instance, $\Omega(\mathbf{r}) \propto e^{-r^2/(2\sigma^2)}$ for a Gaussian beam shape.

As will be shown in the following section, the essential feature in considering a possible density response is whether $|\Omega|^2$ is time and space dependent. Clearly, for Bragg spectroscopy as considered above this is true. For the Gaussian beam profile, $|\Omega|^2$ is dependent on position but not on time. In contrast, for a single laser $\Omega = |\Omega| e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$ and for a Raman excitation $\Omega \propto |\Omega_1||\Omega_2| e^{i\mathbf{k}_1\cdot\mathbf{r} - i\omega_1 t} e^{-i\mathbf{k}_2\cdot\mathbf{r} + i\omega_2 t}$; thus in both cases $|\Omega|^2$ is a constant.

(e) Time dependence of the perturbation. As discussed above, Bragg scattering schemes lead to a time-varying intensity grating, that is, the energy ω_{12} is transferred to the system via a perturbation that is proportional to the intensity.

Another possible source of time dependence is the turning on of the perturbation. As is usually done in the linear re-

sponse theory [9], we consider smooth turning on of the perturbing potential.

Direct intensity modulation of the probing laser(s) would be one more source of time dependence.

IV. TRANSFORMING H_T INTO A DENSITY PERTURBATION

In order to use linear response theory we have to convert all the perturbing Hamiltonians of the previous section into a density perturbation.

(a) The perturbation term H_T involves products of the field operators of the different atomic states. In the mean field approach they correspond to the Fock terms $\langle \psi_\uparrow^\dagger(\mathbf{r})\psi_\downarrow(\mathbf{r})\rangle$, which are zero. Therefore no linear response theory can be directly applied.

A perturbation of the type (note that the two-component ψ here is *not* the same as that in the standard BCS theory, used, for instance, in the Appendix)

$$(\psi_\uparrow^\dagger(\mathbf{r})\psi_\downarrow^\dagger(\mathbf{r}))\begin{pmatrix}\delta/2 & \Omega(\mathbf{r})\\ \Omega^*(\mathbf{r}) & -\delta/2\end{pmatrix}\begin{pmatrix}\psi_\uparrow(\mathbf{r})\\ \psi_\downarrow(\mathbf{r})\end{pmatrix}\equiv\psi^\dagger(\mathbf{r})W\psi(\mathbf{r}) \tag{6}$$

can be diagonalized for any δ and Ω by an appropiate rotation in the space of the two states $\uparrow$ and $\downarrow$, where $\psi=\mathcal{U}\tilde{\psi}$ and $\mathcal{U}=R_z(\alpha)R_y(\beta)R_z(\gamma)$ with $R_j(\theta)=\exp(i\sigma_j\theta)$, $j=\{x,y,z\}$, and σ_j is the corresponding Pauli spin matrix. The transformation matrix is

$$\mathcal{U}=\begin{pmatrix}e^{i(\alpha+\gamma)/2}\cos\beta/2 & e^{i(\alpha-\gamma)/2}\sin\beta/2\\ -e^{-i(\alpha-\gamma)/2}\sin\beta/2 & e^{-i(\alpha+\gamma)/2}\cos\beta/2\end{pmatrix}. \tag{7}$$

The rotated perturbation matrix is $\mathcal{U}^\dagger W\mathcal{U}$. The off-diagonal terms are

$$e^{-i\gamma}[(\delta/2)\sin(\beta)-e^{i\alpha}\Omega^*\sin^2(\beta/2) + \Omega\cos^2(\beta/2)e^{-i\alpha}] \tag{8}$$

and the complex conjugate. Off-diagonal terms that are zero can be obtained by a rotation with the Euler angles $\{\gamma=0,\cos\alpha=(\Omega+\Omega^*)/2|\Omega|,\tan\beta=-2|\Omega|/\delta\}$ which yields

$$\begin{pmatrix}\sqrt{\left(\frac{\delta}{2}\right)^2+\Omega\Omega^*} & 0\\ 0 & -\sqrt{\Omega\Omega^*+\left(\frac{\delta}{2}\right)^2}\end{pmatrix} \tag{9}$$

for the perturbation matrix. This procedure is extensively used in describing the interaction of laser light with a two-level system. The form $|\Omega|^2/\delta$ for far-off-resonant light comes from Eq. (9) for $|\delta|\gg|\Omega|$.

One has to check whether the rotated states experience the same two-body interaction as the nonrotated ones, that is, whether the rotated states are still described by a Hamiltonian of the initial form which implies BCS pairing. Assuming, for simplicity, that $\psi_\uparrow$ and $\psi_\downarrow$ experience the same potential $V_\alpha-\mu_\alpha$, we obtain $\psi_\uparrow^\dagger(\mathbf{r})\psi_\uparrow(\mathbf{r})+\psi_\downarrow^\dagger(\mathbf{r})\psi_\downarrow(\mathbf{r}) \rightarrow \tilde{\psi}_\uparrow^\dagger(\mathbf{r})\tilde{\psi}_\uparrow(\mathbf{r})+\tilde{\psi}_\downarrow^\dagger(\mathbf{r})\tilde{\psi}_\downarrow(\mathbf{r})$. The kinetic energy term transforms as $\nabla\psi_\uparrow^\dagger(\mathbf{r})\nabla\psi_\uparrow(\mathbf{r})+\nabla\psi_\downarrow^\dagger(\mathbf{r})\nabla\psi_\downarrow(\mathbf{r}) \rightarrow \nabla\tilde{\psi}_\uparrow^\dagger(\mathbf{r})\nabla\tilde{\psi}_\uparrow(\mathbf{r})+\nabla\tilde{\psi}_\downarrow^\dagger(\mathbf{r})\nabla\tilde{\psi}_\downarrow(\mathbf{r})$ if the momentum k transferred by the laser is small. The extra terms coming into the transformation of the kinetic energy are $\sim k,k^2$. The momentum of the laser and the recoil energy are very small compared to the momentum of the atoms participating in the process (typically close to the Fermi surface, $\sim k_F$).

The interaction term in the Hamiltonian transforms $\psi_\uparrow^\dagger(\mathbf{r})\psi_\downarrow^\dagger(\mathbf{r}')\psi_\downarrow(\mathbf{r}')\psi_\uparrow(\mathbf{r})\rightarrow\tilde{\psi}_\uparrow^\dagger\tilde{\psi}_\downarrow^\dagger\tilde{\psi}_\downarrow\tilde{\psi}_\uparrow$ in the case of a contact interaction $g(\mathbf{r}-\mathbf{r}')\propto\delta(\mathbf{r}-\mathbf{r}')$. As $\tilde{\psi}_\alpha$ is a linear combination of the two species $\uparrow$ and $\downarrow$, one might expect interaction terms between the same species for the new matter fields (interactions between both $\tilde{\psi}_\uparrow$ and $\tilde{\psi}_\uparrow$, and between $\tilde{\psi}_\uparrow$ and $\tilde{\psi}_\downarrow$). But the interactions of the type $\tilde{\psi}_\alpha\tilde{\psi}_\alpha$ are forbidden by fermionic behavior (commutation rules are preserved by the rotation). The requirement of a contact interaction is obvious here: $\tilde{\psi}_\alpha(\mathbf{r})\tilde{\psi}_\alpha(\mathbf{r})=0$, but $\tilde{\psi}_\alpha(\mathbf{r})\tilde{\psi}_\alpha(\mathbf{r}')$ can be nonzero. As a summary, since the fermionic field commutator relations are preserved by the rotation, for contact interaction $\tilde{H}_0=H_0$.

We have thus shown that the system can be transformed in such a way that one can study the collective mode spectra by linear response theory for a BCS system for a perturbation of the type $U(n_\uparrow-n_\downarrow)$, where $U(\mathbf{k},\omega)=\mathcal{F}[\sqrt{\Omega(\mathbf{r},t)\Omega(\mathbf{r},t)^*+(\delta/2)^2}]$, and the rest of the Hamiltonian still corresponds to the BCS Hamiltonian. The main difference from the standard linear density response treatment for a BCS system is the minus sign between $n_\uparrow$ and $n_\downarrow$. We will show that this leads to zero overall response in the homogeneous case.

(b) When diagonalizing the interaction Hamiltonian of Eq. (5) in the same way as in case (a), the initial Hamiltonian H_0 is *not* preserved. We thus obtain a perturbation $U(n_\alpha-n_e)$ with $U(\mathbf{k},\omega)$ as given above, but with the initial Hamiltonian modified, $\tilde{H}_0\neq H_0$.

(c) In the case of coupling to an excited state by far-detuned light the rotation does not modify the initial Hamiltonian because at the limit $|\delta|\gg|\Omega|$ the transformed states are very close to the initial ones, $\psi_\alpha\sim\tilde{\psi}_\alpha$ and $\psi_e\sim\tilde{\psi}_e$; moreover, the excited state population is assumed to be negligible. Thus the light-atom interaction Hamiltonian is in the form of a density perturbation, and the initial Hamiltonian is preserved.

In all these cases, $U(\mathbf{k},\omega)$ can be either a constant $U(\mathbf{k},\omega)=U'$ or dependent on $\mathbf{k}$ and ω. The latter happens when the intensity $|\Omega|^2$ is space and time dependent. Note that the $\mathbf{k}$ dependence $\Omega e^{i\mathbf{k}\cdot\mathbf{r}}$ is not sufficient, that is, the laser momentum given in a single-laser or Raman two-level excitation (near-resonant or off-resonant) is not enough to transfer momentum through a density perturbation.

V. RESULTS

A. GRPA for the case of light coupled between two paired states

The response of the system to the perturbation $U(k,\omega)\times(n_\uparrow-n_\downarrow)$ in a homogenous system is calculated in the Ap-

pendix using the generalized random-phase approximation (GRPA) following the derivation in [11]. The result is that for the homogeneous case the responses of the two components are equal but with opposite signs and no collective modes are excited at any frequencies.

The response of one of the spin components is $L_{11}=(a+b)(1+g_0R)/(1-g_0R)(g_0+a+b)$, where we use the integrals a, b, c, d, and R as defined in [11]. The reponse of the spin component has the factor $(a+b)$ which at $T=0$ in the low-q,ω region is $a+b\approx -N(\epsilon_F)c_B^2q^2/(12\Delta^2)$ where $N(\epsilon_F)$ is the density of states at the Fermi surface and the sound velocity $c_B^2=v_F^2/3$. This factor is very small and it makes the response for one spin component negligible even without the cancellation effect. This is because the superfluid cannot participate in the relative motion of the two spin components [15]. Above T_c, b vanishes and the response of one of the spins is no longer negligible, as the integral a is some fraction of $N(\epsilon_F)$. However, due to the cancellation the overall response is zero.

We have considered the homogeneous case, that is, when the trapping potential treated within the local density approximation is sufficient. The strongly trapped case can be approached by using multipole expansions [16]: the overall response is not zero for spin-dipole excitations in the trapped case above T_c. The difference in our results is a consequence of the geometry, as in the homogeneous case we get the same spatially uniform response in opposite directions for both spin species. As $T\rightarrow 0$, one gets vanishing response also for the inhomogeneous case [16], at least at energies below the gap, because one needs to break pairs in order to have relative motion of the two components.

B. Density response for different probing schemes

(a) Coupling the paired states. The perturbation reads $U(n_\uparrow-n_\downarrow)$ for the rotated states, where $U(\mathbf{k},\omega)=\mathcal{F}[\sqrt{|\Omega(\mathbf{r},t)|^2+\delta^2}]$. When the light is far off resonant, one can take the limit $U(\mathbf{k},\omega)\sim\mathcal{F}[|\Omega(\mathbf{r},t)|^2/\delta]$. In the case of homogeneous laser intensity $\Omega=|\Omega|e^{i\mathbf{k}\cdot\mathbf{r}}$, $U\propto\delta(\omega)\delta(\mathbf{k})$. There is no density reponse simply because U does not provide the momentum and energy for a collective excitation.

A different behavior arises when using a Bragg scattering scheme to provide an intensity grating. Assuming $|\Omega_1|=|\Omega_2|$ [Ω_1,Ω_2 as introduced in Sec. III B case (d)] one obtains $|\Omega|^2=|\Omega_1|^2[2+2\cos(\omega_{12}t-\mathbf{k}_{12}\cdot\mathbf{r})]\sim|\Omega_1|^2 2\cos(\omega_{12}t-\mathbf{k}_{12}\cdot\mathbf{r})$ and the density response $\chi(\omega,\mathbf{k})$ has to be analyzed at $\chi(\omega_{12},\mathbf{k}_{12})$. The density response χ gives zero in the homogeneous case for all temperatures, as one component annihilates the response of the other, and for the inhomogeneous case well below T_c [16], as considered in the previous subsection.

For a general case where the beam profile of the laser light gives a spatially varying intensity of the form $|\Omega(\mathbf{r})|^2\sim e^{r^2/\sigma^2}$, the far-off-resonant perturbation reads $U(\mathbf{k},\omega)=|\Omega(k)|^2/\delta\sim e^{\sigma^2k^2/4}$. If σ is some fraction f of the trap size R_{TF}, one can estimate the momentum given by $p=\hbar/\sigma$. A collective mode with an energy of the order of the gap energy (e.g., half the gap) has momentum $q\sim\sqrt{3}\Delta_G/v_F$ [9,10] and the ratio between them is $p/q=\hbar\omega_T/f\Delta_G$. For typical parameters, this could be a non-negligible number; however, this alone is not enough to produce a density response because of the lack of time dependence, i.e., $U(\omega,\mathbf{k})\propto\delta(\omega)f(\mathbf{k})$.

(b) Coupling to an excited state. When diagonalizing the interaction Hamiltonian in this case the initial Hamiltonian was modified. Therefore the linear response calculation for a BCS system cannot be directly applied. One may guess, however, that the arguments about the perturbation U being time and space dependent will hold also in this case even when the response function χ has a different form. Therefore we do not expect a density response except when a Bragg spectroscopy scheme is used.

(c) Far-off-resonant coupling. When both paired states are coupled to some excited states and the light is far detuned, the density perturbation potential is proportional to $|\Omega|^2/\delta$. No collective mode of the system is excited if there is no intensity variation in space and time. When considering intensity modulations of the type $|\Omega|^2\sim|\Omega_1|^2 2\cos(\omega t-\mathbf{k}\cdot\mathbf{r})$, an Anderson-Bogoliubov phonon can be excited [10–12].

VI. CONCLUSIONS

We have reviewed the density response of some typical laser excitation schemes. We have shown that, since the BCS Hamiltonian for a contact interaction is preserved under a rotation, most of the considered laser excitations can be expressed in terms of a perturbation acting on the density. In this form, the perturbation potential is proportional to $|\Omega|^2$ where Ω is the (effective) Rabi frequency. Therefore, even when the laser light provides momentum and energy ($\Omega\propto e^{i\mathbf{k}_L\cdot\mathbf{r}}e^{i\omega_Lt}$), the transformed potential acting as a density perturbation is not time and space dependent. This leads to absence of a density response whenever $|\Omega|^2$ is a constant spatially and temporally. This makes many proposed laser-probing schemes well suited for observing the superconducting gap since they do not induce below-gap collective excitations.

For Bragg scattering, $|\Omega|^2\propto f(\mathbf{r},t)$. In this case Anderson-Bogoliubov phonons can be excited, in general. The exception is the case when the laser(s) couple between the two paired components of the gas. We have shown that, in the homogeneous case, the density response becomes zero because the contributions of the components cancel each other. In a harmonic trap, spin-dipole response is predicted for temperatures near T_c. Therefore, the presence or absence of low-energy collective excitations under a perturbation of the type $U(\omega,k)(n_\uparrow-n_\downarrow)$ could be used to observe whether the trapped system can be approximated by a homogeneous system (local density approximation) or whether the trapping effects are dominant.

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APPENDIX

The formalism used is based on the functional differentiation technique as described in [14] following the derivation in [11]. The response function χ is viewed as a functional derivative of the one-particle matrix Green's functions G with respect to the external field $U(\mathbf{r},t)$. To allow for pairing in the superconducting phase, it is convenient to work with a single-particle Green's function G given by a 2×2 matrix defined as

$$G(1,2)\equiv-\langle T\Psi(1)\Psi^{\dagger}(2)\rangle = \begin{pmatrix} G_{\uparrow}(1,2) & F(1,2) \\ F^{*}(1,2) & -G_{\downarrow}(2,1) \end{pmatrix}, \tag{A1}$$

where $\Psi=(\psi_{\uparrow}\psi_{\downarrow}^{\dagger})^{T}$ and $1\equiv(\mathbf{r}_1,\tau_1)$. Imaginary times (Matsubara formalism) are used so that one can deal with finite temperatures. In the absence of external fields, the equal time ($\tau_2=\tau_1^{+}$) single-particle Green's function components reduce to $F(\mathbf{r})\equiv\langle\psi_{\downarrow}(\mathbf{r})\psi_{\uparrow}(\mathbf{r})\rangle$, the s-wave order parameter, and $G_{\uparrow,\downarrow}(\mathbf{r})=\langle n_{\uparrow,\downarrow}(\mathbf{r})\rangle$.

From the equation of motion of the Green's function one gets the generalized Dyson equation for $G(1,2)$ in terms of G_0, the noninteracting single-particle Green's function, the matrix self-energy $\Sigma(1,2)$ which is evaluated in the pairing approximation (Hartree-Fock-Bogoliubov), and $W(1)$ which is the the external perturbing field matrix.

The density response matrix is obtained in the random phase approximation (RPA) by taking the functional derivative of the Green's function with respect to the external field U. One can define the three-point correlation function $L(1,2,5)\equiv-\sigma_3\delta G(1,2)/\delta U(5)$, whose limit $L(1,2)\equiv L(1,1^{+},2)$ will give the density-density response function $\chi(1,2)=L_{11}(1,2)+L_{22}(1,2)$. Here σ_3 is the third Pauli matrix.

When deriving the Dyson equation for $G(1,2)$ with respect to U in order to get the three-point correlation function L, both the self-energy matrix and the external field matrix W contribute. The lowest-order (single-bubble) result [11] for L is given by

$$L^{0}(1,2,5)=-\sigma_3\int d\bar{3}\int d\bar{4}G(1,\bar{3})\frac{\delta W(\bar{3},\bar{4})}{\delta U(5)}G(\bar{4},2). \tag{A2}$$

For the Anderson-Bogoliubov phonon (see [10,11]) the pertubation matrix $W(1)=U(1)\sigma_3$, and $L^{0}_{AB}(1,2,5)=\tilde{G}(1,5)\tilde{G}(5,2)$, where we have introduced $\tilde{G}\equiv\sigma_3 G$. In our case $L^{0}(1,2,5)=\tilde{G}(1,5)G(5,2)$ because the perturbation matrix $W(1)=U(1)\mathcal{I}$ due to the minus sign in $U(n_{\uparrow}-n_{\downarrow})$. From now on, we denote by the subindex AB the quantities that are calculated for the perturbation matrix $W(1)=U(1)\sigma_3$.

It is useful to rewrite the GRPA integral equations in terms of irreducible two-particle Green's functions $\bar{L}_{ij}(1,2,5)$, and for the homogeneous system, to Fourier transform in order to solve coupled equations. The result reads [11]

$$L_{ij}(\mathbf{q},i\omega_n)=\bar{L}_{ij}(\mathbf{q},i\omega_n)+\frac{\bar{L}_{ABij}(\mathbf{q},i\omega_n)g_{\uparrow\downarrow}(\mathbf{q})\bar{L}_{ll}(\mathbf{q},i\omega_n)}{1-g_{\uparrow\downarrow}(\mathbf{q})\bar{L}_{ABll}(\mathbf{q},i\omega_n)}, \tag{A3}$$

where equal indices mean summation over the possible values. We denote by the subindex AB the three-point correlation functions for the Anderson-Bogoliubov phonon type of perturbation.

The sum of diagonal terms reduces to

$$\chi(\mathbf{q},i\omega_n)=L_{11}(\mathbf{q},i\omega_n)+L_{22}(\mathbf{q},i\omega_n) =\frac{\bar{L}_{ii}(\mathbf{q},i\omega_n)}{1-g_{\uparrow\downarrow}(\mathbf{q})\bar{L}_{ABll}(\mathbf{q},i\omega_n)}. \tag{A4}$$

This shows that all linear perturbations have the same poles as the Anderson-Bogoliubov phonon (plus possibly some additional ones), but the spectral weight depends on the trace of the irreducible two-particle Green's functions $\bar{L}_{ii}(\mathbf{q},i\omega_n)$ of the specific perturbation.

For a contact interaction [$g_{\uparrow\downarrow}(\mathbf{q})=g_0$], the equation for the irreducible correlation function [11] reduces to a set of linear algebraic equations

$$\bar{L}_{ij}(\mathbf{q},i\omega_n)=L^{0}_{ij}(\mathbf{q},i\omega_n)-L^{0}_{ABiklj}(\mathbf{q},i\omega_n)g_0\bar{L}_{kl}(\mathbf{q},i\omega_n), \tag{A5}$$

where we have defined the four-index tensor L^{0}_{ABiklj}, indicating components of the two factor matrices. Defining column vectors $\bar{L}=(\bar{L}_{11}\,\bar{L}_{12}\,\bar{L}_{21}\,\bar{L}_{22})^{T}$ and a 4×4 matrix $L^{0}_{(AB)mn}$ as in [11], Eq. (A5) reduces to

$$\bar{L}=[I+g_0L^{0}_{AB}]^{-1}L^{0}. \tag{A6}$$

As discussed in [11] one can get L^{0}_{ijkl} by Matsubara frequency summations, [Eq. (4.27) in [11]] and using the symmetry properties they then reduce just to six independent elements. For the weak-coupling limit and contact interaction, the independent elements are reduced to four: a, b, c, and d, which are integrals defined in [11] [Eq. (A8)]. L^{0}_{m} can be calculated from the 4×4 matrix by $L^{0T}_{m}=(L^{0}_{1l1}\,L^{0}_{1l2}\,L^{0}_{2l1}\,L^{0}_{2l2})$.

Now we derive the 4×4 matrix for our case when $W(1)=U\mathcal{I}$ by reconsidering the symmetry properties of the matrix elements in order to derive the vector L^{0}_{m} that we insert in Eq. (A6). The sum of the first and last components of $\bar{L}_{m}$ contributes to Eq. (A4) and characterizes the spectral weight and perhaps some additional poles of the response function. Such a perturbation gives from Eq. (A2) the lowest-order correlation function $L^{0}(1,2,5)=\tilde{G}(1,5)G(5,2)$ [cf. $L^{0}_{AB}(1,2,5)=\tilde{G}(1,5)\tilde{G}(5,2)$]. We calculate the 4×4 matrix L^{0}_{mn} using the symmetry properties of the new L^{0}_{ijkl} elements, in the same fashion as in [11] (see Appendix A there), obtaining

$$L^0_{mn}=\begin{pmatrix} a & c & -c & b \\ c & -d & b & -c \\ c & -b & d & -c \\ -b & c & -c & -a \end{pmatrix}. \tag{A7}$$

This leads to $L^{0T}_m=(a+b\ 0\ 0\ -a-b)$. Multiplying L^0_m by $[I+g_0L^0_{ABmn}]^{-1}$ as in Eq. (A6) one gets $\bar{L}^T_m=((a+b)/(g_0+a+b)\ 0\ 0\ -(a+b)/(g_0+a+b))$. This means $\bar{L}_{11}=-\bar{L}_{22}$, and the spectral weight in Eq. (A4) vanishes and $\chi=0$.

Inserting the well-known result [11,10] $\bar{L}_{ABll}=2R/(1+g_0R)$ into Eq. (A4), one obtains the spin-density response $L_{11}=(a+b)(1+g_0R)/(1-g_0R)(g_0+a+b)$, where $R=A+4c^2/(1+g_0B)$ and $B\equiv b+d$, $A\equiv a-b$ are defined in [11].

In the case of a general perturbation

$$W=U\begin{pmatrix} 1 & 0 \\ 0 & c_u \end{pmatrix}, \tag{A8}$$

where c_u is any real number, the single-bubble result is

$$L^0_{ijkl}=\tilde{G}_{ij}\left(\begin{pmatrix} 1 & 0 \\ 0 & c_u \end{pmatrix}_{ks} G\right)_{sl}. \tag{A9}$$

By calculating the symmetry properties of the elements of L_{mn}, one obtains $L^{0T}_m=(a+c_ub\ c-c_uc\ c-c_uc\ -c_ua-b)$, which leads to

$$\bar{L}_{11}-\bar{L}_{22}=\frac{(c_u+1)(a+b)}{1+g_0(a+b)} \tag{A10}$$

and

$$\bar{L}_{11}+\bar{L}_{22}=\frac{(1-c_u)R}{1+g_0R}. \tag{A11}$$

The reponse for one spin would be

$$L_{11}=\frac{r_u}{(1+g_0R)(1+g_0B)} \tag{A12}$$

where $r_u=g_0AB+g_02c^2[(1-c_u)+2g_0(a+b)]/[1+g_0(a+b)]+\{(a+c_ub)+g_0[a^2+c_ub^2+db(1+c_u)]\}/[1+g_0(a+b)]$.

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