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Josephson effect in superfluid atomic Fermi gases

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We consider an analog of the internal Josephson effect in superfluid atomic Fermi gases. Four different hyperfine states of the atoms are assumed to be trapped and to form two superfluids via the BCS type of pairing. We show that Josephson oscillations can be realized by coupling the superfluids with two laser fields. Choosing the laser detunings in a suitable way leads to an asymmetric below-gap tunneling effect for which there exists no analog in the context of solid-state superconductivity.

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Cooling of trapped gases of Fermionic atoms well below the Fermi temperature [1–5] makes it reasonable to anticipate the achievement of the predicted BCS transition [6–9]. The existence of a gap in the excitation spectrum of the superfluid Fermi gas will be the first issue to address, and several methods for detecting it have already been proposed [10,11]. Trapped atomic Fermi gases will allow to study and test fermion-fermion pairing theories in a tunable, controlled manner. For example, the classic problem of the BCS-BEC crossover when the interparticle attraction varies [12] could be studied using the possibility of tuning the interatomic scattering length using Feshbach resonances [7,13]. Besides the standard superfluid phenomenology, gases of Cooper-paired atoms are expected to have properties that are specific to atomic gases only and not present, or not easily realizable, in metallic superconductors or helium. For instance, the trapping potential has a major effect on the characteristic lengths of the superfluid Fermi gas [14].

In this paper we propose a way to investigate the Josephson effect in trapped superfluids of fermionic atoms. We find a phenomenon that is unique to atomic Fermi gases, namely, an asymmetry in the Josephson currents corresponding to the "up" and "down" spin states. We assume that Fermionic atoms in four different hyperfine states (we label them $|g\rangle$, $|g'\rangle$, $|e\rangle$, and $|e'\rangle$) are trapped simultaneously in an optical trap—recently all-optical trapping and cooling below the degeneracy point of the two lowest hyperfine states of ^{6}Li has been demonstrated [4]. The *s*-wave scattering lengths are assumed to be large and negative between atoms in states $|g\rangle$ and $|g'\rangle$, as well as between those in $|e\rangle$ and $|e'\rangle$, and the chemical potentials $\mu_g \simeq \mu_{g'}$ and $\mu_e \simeq \mu_{e'}$. For all other combinations of two atoms in different states the scattering length is assumed to be small and/or the chemical potentials unequal. This leads to the existence of two superfluids, one consisting of Cooper pairs of atoms in the states $|g\rangle$ and $|g'\rangle$, and the other of $|e\rangle$–$|e'\rangle$ pairs. The configuration is experimentally challenging, but probably possible by the choice of right atoms and hyperfine states, adjusting the number of atoms, and tuning the scattering lengths in magnetic fields by using Feshbach resonances [7,13]. Adjusting the number of atoms is limited by technical issues, whereas the choice of hyperfine states and magnetic fields is restricted by spin-exchange collisions.

The two superfluids are coupled by driving laser-induced transitions between the states $|g\rangle$ and $|e\rangle$ with the laser Rabi frequency Ω and detuning δ, and between the states $|g'\rangle$ and $|e'\rangle$ with the Rabi frequency Ω' and detuning δ'. For a Raman process these are effective quantities. If several lasers are used then in order to be able to see the Josephson oscillations they should maintain their phase coherence for a time much longer than the inverse of the detunings. In the case of metals, the two superconductors are spatially separated and connected by a tunneling junction. In our scheme, the superfluids share the same spatial region and are connected by the laser coupling of the atoms' internal states; this resembles the internal Josephson effect in atomic Bose-Einstein condensates [15] or in superfluid ^{3}He-*A* [16].

For metallic superconductors the ac Josephson current is driven by applying a voltage over the junction—here the role of the voltage is played by the laser detunings. The difference is that the detunings can be different for the two states forming the pair; in the metallic superconductor analogy this would mean having a different voltage for the spin-up and spin-down electrons, a situation that has not been investigated in the context of metallic superconductors. There is an interesting connection to recent experiments on superconductor-ferromagnet proximity effects, where the chemical potentials of the spin-up and spin-down electrons are slightly different in the ferromagnet due to the exchange interaction [17].

We consider a system described by the standard BCS theory. The laser interaction is assumed to be a small perturbation and its effect is calculated using linear response theory. The observable of interest is the change in the number of particles in one of the states, say $|e\rangle$ or $|e'\rangle$.

In the rotating-wave approximation the interaction of the laser light with the matter fields can be described by a time-independent Hamiltonian in which the detunings δ and δ' play the role of an externally imposed difference in the

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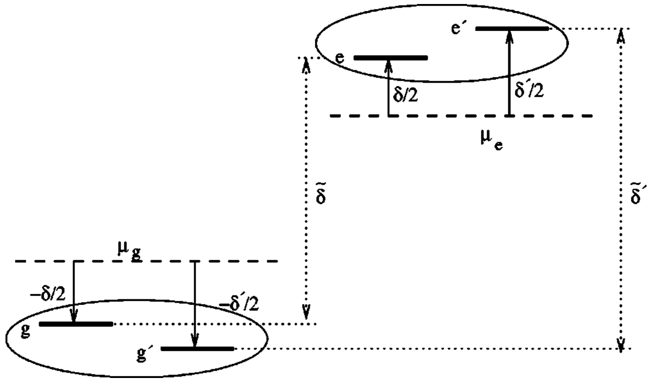

FIG. 1. The effect of the detunings δ and δ' of the two lasers. The initial chemical potentials (in the absence of laser couplings) were $\mu_g=\mu_{g'}$ and $\mu_e=\mu_{e'}$. The states (gg') and (ee') are Cooper paired.

chemical potential of the two states. The total Hamiltonian becomes then $\hat{H}=\hat{H}_0+\hat{H}_T$, where

$$\hat{H}_0=\hat{H}_{(gg')}+\hat{H}_{(ee')}+\left(\mu_e+\frac{\delta}{2}\right)\int d\vec{r}\hat{\psi}_e^\dagger(\vec{r})\hat{\psi}_e(\vec{r})+\left(\mu_g-\frac{\delta}{2}\right)\int d\vec{r}\hat{\psi}_g^\dagger(\vec{r})\hat{\psi}_g(\vec{r})+\left(\mu_e+\frac{\delta'}{2}\right)\int d\vec{r}\hat{\psi}_{e'}^\dagger(\vec{r})\hat{\psi}_{e'}(\vec{r})+\left(\mu_g-\frac{\delta'}{2}\right)\int d\vec{r}\hat{\psi}_{g'}^\dagger(\vec{r})\hat{\psi}_{g'}(\vec{r}).$$

Here μ_e and μ_g are the chemical potentials of the Fermi gases before the laser was turned on, $\mu_g=\mu_{g'}$ and $\mu_e=\mu_{e'}$ in order to allow standard BCS pairing. The Hamiltonians $\hat{H}_{(gg')}$ and $\hat{H}_{(ee')}$ are the interaction Hamiltonians corresponding to the two superconductors with the chemical potential included [18]. Figure 1 presents a schematic view of what happens in this case: the laser detunings shift the chemical potentials of the four hyperfine states.

The transfer Hamiltonian is given by

$$\hat{H}_T=\int d\vec{r}\Omega(\vec{r})\hat{\psi}_e^\dagger(\vec{r})\hat{\psi}_g(\vec{r})+\Omega^*(\vec{r})\hat{\psi}_g^\dagger(\vec{r})\hat{\psi}_e(\vec{r})+\int d\vec{r}\Omega'(\vec{r})\hat{\psi}_{e'}^\dagger(\vec{r})\hat{\psi}_{g'}(\vec{r})+\Omega'^*(\vec{r})\hat{\psi}_{g'}^\dagger(\vec{r})\hat{\psi}_{e'}(\vec{r}), \tag{1}$$

with $\Omega(\vec{r})$ and $\Omega'(\vec{r})$ characterizing the local strength of the matter-field interaction.

The main observable of interest, the rate of transferred atoms from, say, state $|g\rangle$ to state $|e\rangle$, is defined by

$$I_e=\frac{\partial}{\partial t}\int d\vec{r}\langle\Psi(t)|\hat{\psi}_e^\dagger(\vec{r})\hat{\psi}_e(\vec{r})|\Psi(t)\rangle \tag{2}$$

(the definition for $I_{e'}$ is similar) and can be further evaluated with the help of the Schrödinger equation $i\hbar(\partial/\partial t)|\Psi(t)\rangle=\hat{H}|\Psi(t)\rangle$ as

$$I_e=i\int d\vec{r}\langle\Psi(t)|\Omega^*(\vec{r})\hat{\psi}_g^\dagger(\vec{r})\hat{\psi}_e(\vec{r})-\Omega(\vec{r})\hat{\psi}_e^\dagger(\vec{r})\hat{\psi}_g(\vec{r})|\Psi(t)\rangle. \tag{3}$$

In the following we call I_e the current in analogy to metallic superconductors where the flux of electrons out of the superconductor constitutes the electrical current.

We introduce an interaction representation with respect to $\hat{H}_0$ and use linear response theory with respect to $\hat{H}_T$. Validity of the linear response theory requires that the laser intensity is small so that the transfer of atoms can be treated as a perturbation.

We split the result for the current I_e into a part that corresponds to the Josephson current I_{eJ} and to the part that describes normal single-particle current I_{eS}, $I_e=I_{eJ}+I_{eS}$. The single-particle current can be evaluated at a finite temperature using the standard techniques of superconducting Green's functions in the BCS approximation; the result at $T=0$ and positive detunings is

$$I_{eS}=-2\pi\sum_{n,m}\left|\int d\vec{r}\Omega(\vec{r})v_n^e(\vec{r})u_m^g(\vec{r})\right|^2\delta(\epsilon_n^e+\epsilon_m^g-\tilde{\delta}).$$

Here the triplet $(u_n,v_n);\epsilon_n$ is a solution of the (nonuniform) Bogoliubov–de Gennes equations for superconductors [18] and $\tilde{\delta}=\mu_e-\mu_g+\delta$. This is the standard Fermi golden rule result and very similar to the ones obtained in Ref. [11]. The current I_{eS} is zero when $\tilde{\delta}<\Delta+\Delta'$ since pair breaking is required for single-particle excitations. Next we analyze the Josephson current. We will not be interested in the dissipative cosine Josephson term that appears only above the sum of the gaps but we will concentrate, in the rest of the paper, on the usual sine Josephson current, which is the only contribution at zero temperature and below $\Delta+\Delta'$.

This current can be calculated as

$$I_{eJ}=-2\,\mathrm{Im}\left[e^{-i(\tilde{\delta}+\tilde{\delta}')t}\sum_{n,m}\int d\vec{r}d\vec{r}'\Omega^*(\vec{r})\Omega'^*(\vec{r}')\times u_n^{g*}(\vec{r})u_m^e(\vec{r}')v_m^{e*}(\vec{r})v_n^g(\vec{r}')\times\left(\frac{1}{\tilde{\delta}'+\epsilon_n^g+\epsilon_m^e+i\eta}-\frac{1}{\tilde{\delta}'-\epsilon_n^g-\epsilon_m^e+i\eta}\right)\right]. \tag{4}$$

The current $I_{e'J}$ is the same only that $\tilde{\delta}$ and $\tilde{\delta}'$ are interchanged. Note that the oscillating term is proportional to both of the detunings whereas the rest of the expression is proportional only to $\tilde{\delta}'$. The initial relative phase of the two superconductors (or, equivalently, the phases of the laser fields) is set to zero, since its contribution amounts to the same phase shift in the currents I_{eJ} and $I_{e'J}$. For the choice

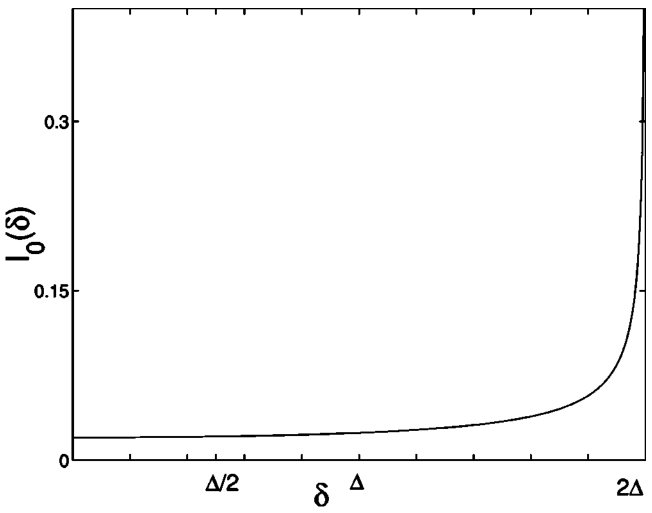

FIG. 2. The current I_0 (arbitrary units) as a function of detuning δ, given in units of Δ.

of a homogeneous system (large trap, local density approximation) and a constant laser profile the expression simplifies into

$$I_{eJ} = I_0(\tilde{\delta}')\sin[(\tilde{\delta}+\tilde{\delta}')t], \tag{5}$$

$$I_{e'J} = I_0(\tilde{\delta})\sin[(\tilde{\delta}+\tilde{\delta}')t]. \tag{6}$$

Both partners of the pair thus oscillate in phase, with the same frequency $\tilde{\delta}+\tilde{\delta}'$. But the amplitudes are different whenever the detunings $\tilde{\delta}$ and $\tilde{\delta}'$ differ. This means that more atoms are transferred, say, in the $|g\rangle - |e\rangle$ oscillation than in the $|g'\rangle - |e'\rangle$ one.

A simple expression for $I_0(\delta)$ can be derived when we assume identical superfluids, that is, $\Delta = \Delta'$ and $\mu_g = \mu_e \equiv \mu$,

$$I_0(\delta) = \frac{\sqrt{2m^3}V}{\pi^2}\Delta^2\Omega^2 \int_{-\mu}^{\infty} \frac{d\xi\sqrt{\mu+\xi}}{\sqrt{\xi^2+\Delta^2}(4\xi^2+4\Delta^2-\delta^2)},$$

where V is the volume of the sample and the variable ξ is the continuous version of $\xi_k = k^2/2m - \mu$. Since $\Delta \ll \mu$, the result can also be written as

$$I_0(\delta) = \frac{\sqrt{2m^3\mu}V}{\pi^2}\Delta^2\Omega^2 \int_{-\infty}^{\infty} \frac{d\xi}{\sqrt{\xi^2+\Delta^2}(4\xi^2+4\Delta^2-\delta^2)}.$$

A plot of the intensity I_0 as a function of the detuning δ is shown in Fig. 2. This result shows a divergence at $\delta = 2\Delta$, which reflects the divergence of the density of states for the two superconductors at the gap. For quite a large range of detunings, the amplitude I_0 of the Josephson current is approximately constant—thus no asymmetry effect will be visible. The asymmetry is most pronounced when the time scale of the oscillation, that is, $1/(\delta+\delta')$ is close to $1/(2\Delta)$. Note that $1/(2\Delta)$ can also be understood as the Cooper pair correlation time based on the uncertainty principle.

In the conventional intuitive picture of the Josephson effect, the particles for a Cooper-pair tunnel ‘‘together’’ through the junction. Therefore our result seems counterintuitive at first glance. The physics becomes, however, more transparent by a closer look at Eq. (4). For simplicity, we consider here the transfer process in one direction only, from the superfluid (gg') to (ee'), which corresponds to the first denominator in Eq. (4). In the initial state $|g\rangle$ is paired with $|g'\rangle$, in the final state $|e\rangle$ with $|e'\rangle$. The process has, however, an intermediate state as indicated by the second-order form of the observable, and the intermediate states corresponding to the observables I_{eJ} and $I_{e'J}$ are different: For I_{eJ}, $|g'\rangle$ has been transferred into $|e'\rangle$. Therefore its pairing partner $|g\rangle$ is left as an excitation in the superfluid (gg') with the energy ϵ_n^g and $|e'\rangle$ becomes an excitation in the superfluid (ee') with the energy $\epsilon_m^{e'} = \epsilon_m^e$. In contrast, for $I_{e'J}$, the atom in $|g'\rangle$ remains as a quasiparticle of the energy $\epsilon_n^{g'} = \epsilon_n^g$ in the superfluid (gg') and $|e\rangle$ becomes an excitation in the superfluid (ee'). For I_{eJ}, the initial energy of the Cooper pair was $(\mu_g - \delta/2) + (\mu_g - \delta'/2)$ and the energy of the intermediate state is $[(\mu_g - \delta/2) + \epsilon_n^g] + [(\mu_e + \delta'/2) + \epsilon_m^e]$ (for explanation, see Fig. 1). The relative energy of the intermediate state with respect to the initial state is $\epsilon_n^g + \epsilon_m^e + \tilde{\delta}'$, which is precisely the first denominator in Eq. (4). For $I_{e'J}$, the initial energy of the pair is again $(\mu_g - \delta/2) + (\mu_g - \delta'/2)$, but the intermediate state has an energy $[(\mu_g - \delta'/2) + \epsilon_n^g] + [(\mu_e + \delta/2) + \epsilon_m^e]$, or a relative energy $\epsilon_n^g + \epsilon_m^e + \tilde{\delta}$. In summary, the intermediate states of the transfer processes for ‘‘spin-up’’ and ‘‘spin-down’’ atoms have different energies and this results in different amplitudes for I_{eJ} and $I_{e'J}$.

The asymmetry in the currents implies the existence of excitations in the superfluids. A calculation of the many-body wave function of the system in the Schrödinger picture reveals that it contains excitations corresponding to the asymmetry and that the so-called Fermi surface polarization $(\langle N_e\rangle - \langle N_{e'}\rangle)/(\langle N_e\rangle + \langle N_{e'}\rangle) \simeq (\langle N_e\rangle - \langle N_{e'}\rangle)/(\langle N_e\rangle_0 + \langle N_{e'}\rangle_0)$ is nonzero and oscillates as $f(\tilde{\delta},\tilde{\delta}')\cos[(\tilde{\delta}+\tilde{\delta}')t]$ where $f(\tilde{\delta},\tilde{\delta}) = 0$. We also found that *time-independent* perturbation theory is insufficient to reveal the asymmetry in the amplitudes: e.g., the treatment of Ref. [19] cannot be applied to our system because the ansatz used does not take into account any excitations. This and the fact that the oscillation is most pronounced for time scales of the order of the Cooper-pair correlation time confirms that the asymmetry is related to the dynamics of the superfluid state.

To observe the Josephson effect one should be able to measure the number of particles in two of the states, e.g., $|e\rangle$ and $|e'\rangle$, at different stages of the oscillations, either destructively or nondestructively. The scale of the gap energy is for typical systems 1–100 kHz, which means that the highest time resolution needed should be just somewhat above 10 μs. Measuring the number of particles accurately is the more challenging part of the observation. In Ref. [11] we considered laser probing of the superfluid Fermi gas, where the laser was creating excitations in the BCS state. The number of particles transferred was directly reflected in the absorption of the light. Here one can use similar techniques to detect the Josephson oscillations. The use of perturbation theory restricts the number of atoms involved in the oscillations, $\langle N_e\rangle - \langle N_e\rangle_0 = I_e(\delta)(\delta+\delta')^{-1}[1-\cos(\delta+\delta')t] \sim I_e(\delta)/\Delta \ll N_e(0)$. Furthermore, to keep the BCS theory valid, the asymmetry in the currents I_e and $I_{e'}$ has to result, at any time, in small variations of the chemical potentials of e and e' with respect to the gap, $\mu_e - \mu_{e'} \ll \Delta$. For typical values such as a number 10^5–10^6 of particles and a gap $\Delta = 0.1\mu$ we find that our formalism can be applied to describe Josephson oscillations with equal detunings involving up to 10^4–10^5 particles, while for the asymmetry effect the difference in the number of atoms on the states e and e' could be at most 10^3–10^4. These numbers are within the present experimental detection limits and can be considerably larger in the case of strong coupling or resonance superfluidity.

In summary, we propose a method to realize Josephson oscillations in superfluid atomic Fermi gases. The coupling between two superfluids is provided by laser light, and the laser detuning plays the same role as voltage over metallic superconductor junctions. Detunings that affect the two atomic internal states involved in pairing can be chosen to be different—this would correspond to different voltage for spin-up and spin-down electrons. This leads to asymmetry in the oscillation amplitudes of the two states. The asymmetry is pronounced when the time scale of the oscillation is the same order of magnitude as the Cooper-pair correlation time. This is an effect unique to atomic Fermi gases in the superfluid state.

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