Bloch oscillations in Fermi gases

M. Rodríguez[1] and P. Törmä[2]

[1] *Laboratory of Computational Engineering, P.O.Box 9203, FIN-02015 Helsinki University of Technology, Finland*

[2] *Department of Physics, University of Jyväskylä, P.O.Box 35, FIN-40014 Jyväskylä, Finland*

The possibility of Bloch oscillations for a degenerate and superfluid Fermi gas of atoms in an optical lattice is considered. For a one-component degenerate gas the oscillations are suppressed for high temperatures and band fillings. For a two-component gas, Landau criterion is used for specifying the regime where robust Bloch oscillations of the superfluid may be observed. We show how the amplitude of Bloch oscillations varies along the BCS-BEC crossover.

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The experimental realization of optical lattices for bosonic atoms has led to several landmark experiments [1–3]. Very recently similar potentials have become available for trapping the *fermionic* isotopes as well [4, 5]. An increase in the superfluid transition temperature when using potentials created by standing light waves has been predicted [6]. For trapped cold atoms, the famous BCS-BEC crossover problem [7, 8] could be studied by tuning the interaction strength between the atoms using Feshbach resonances [5, 9]. In optical lattices the whole BCS-BEC crossover could be scanned experimentally also in an even simpler way by modulating the light intensity. We consider Bloch oscillations in these systems and show that they can be used as a tool for studying the crossover.

Bloch oscillations are a pure quantum phenomenon occuring in a periodic potential. They have never been observed in a natural lattice for electrons as predicted in [10] because the scattering time of the electrons by lattice defects or impurities is much shorter than the Bloch period. However, Bloch oscillations have recently been observed in semiconductor superlattices [11], for quasiparticles penetrating the cores of a vortex lattice in a cuprate superconductor [12], and for periodic optical systems such as waveguide arrays [13]. Also cold bosonic atoms and superfluids in optical lattices have been shown to be clean and controllable systems well suited for the observation of Bloch oscillations [2, 3].

Several novel aspects of the physics of Bloch oscillations arise for fermionic atoms in optical lattices. i) Impurity scattering can be made negligible, and the particle number controlled at will to produce any band filling. Even when Bloch oscillations were originally proposed for fermions, the effect of the Fermi sea has not played a major role. Due to impurity and defect scattering, the studies of transport in presence of a constant force have focused on drift velocities rather than oscillations. In this paper we generalize the semiclassical single particle description of Bloch oscillations to arbitrary band fillings. ii) The possibility of an oscillating fermionic superfluid becomes relevant. We use the Landau criterion for the optical lattice imposing the Cooper pair size to be of the order or smaller than the lattice spacing. For solid state systems, the Cooper pair radius is usually much larger than the lattice spacing and periodicity irrelevant for the superfluid, therefore the system is treated as homogeneous when calculating supercurrents. We calculate the superfluid velocity in the *periodic* potential and show that pairing, leading to smoothening of the Fermi edge, suppresses Bloch oscillations.

Using six counter-propagating laser beams of wavelength λ, an isotropic 3D simple cubic lattice potential can be created which is of the form

$$V(\mathbf{r}) = V_0\left[\cos^2\left(\frac{\pi x}{a}\right) + \cos^2\left(\frac{\pi y}{a}\right) + \cos^2\left(\frac{\pi z}{a}\right)\right], \quad (1)$$

where V_0 is proportional to the laser intensity and $a = \lambda/2$. With the Bloch ansatz the Schrödinger equation leads to a band structure in the energy spectrum $\varepsilon_n(\mathbf{k})$. One-component degenerate Fermi gas at low temperatures can be considered as *non-interacting* since p-wave scattering is negligible and s-wave scattering suppressed by Fermi statistics. We are interested in high enough values of V_0 such that tunneling is small and tight binding approximation can be applied. The dispersion relation for the lowest band becomes $\varepsilon(\mathbf{k}) = J[3 - \cos(k_x a) - \cos(k_y a) - \cos(k_z a)]$, where the band width $J = \frac{2}{\sqrt{\pi}} E_R\,(V_0/E_R)^{3/4} \exp\left(-2\sqrt{V_0/E_R}\right)$ is obtained using the WKB-approximation and $E_R = h^2/(8ma^2)$ is the recoil energy of the lattice [6].

In a two-component Fermi gas, atoms in two different hyperfine states (" $\downarrow, \uparrow$ ") may interact with each other. The interaction can be assumed pointlike, characterized by a scattering length a_S. The system Hamiltonian $\hat{H} = \sum_\alpha \int d^3\mathbf{r}\hat{\psi}^\dagger_\alpha(\mathbf{r})(T+V)\hat{\psi}_\alpha(\mathbf{r}) - |g|\int d^3\mathbf{r}\hat{\psi}^\dagger_\uparrow\hat{\psi}^\dagger_\downarrow\hat{\psi}_\downarrow\hat{\psi}_\uparrow$, where $g = 4\pi\hbar^2 a_S/m$ can then be mapped to the attractive Hubbard model $\hat{H} = J\sum_{\langle i,j\rangle\sigma}\hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} - U\sum_j \hat{c}^\dagger_{j\uparrow}\hat{c}^\dagger_{j\downarrow}\hat{c}_{j\downarrow}\hat{c}_{j\uparrow}$, where $U = E_R\sqrt{8\pi}|a_s|/a\,(V_0/E_R)^{3/4}$. Note that the BCS ($J \gg U$) to BEC ($U \gg J$) cross-over can be controlled by V_0 alone. One-band description is used in the Hubbard model also in the case of strong interactions [14]. We define the limits of the one-band approximation for the physical potential Eq.(1) by demanding the lowest band gap to be bigger than the effective interaction U (note that $U > |g|$ for the parameters of interest). The band gap can be estimated by approximating the cosine potential well by a quadratic one. Demanding the corresponding harmonic oscillator energy to be greater than U

gives the condition $V_0/E_R < \frac{1}{4\pi^2}(a/|a_s|)^4$. Since $a > |a_S|$ imposed by considering on-site interactions only, the condition is easily valid in general, and for the parameters of Fig. 2 in particular. Estimates made using exact numerical band gaps in 1D support this argument. One-band approximation is sufficient because larger V_0 means steeper optical potential wells which not only increase the effective interaction U but also the band gaps.

Bloch oscillations for a single atom can be characterized considering the mean velocity of a particle in a Bloch state $\mathbf{v}(n,\mathbf{k}) = \langle n,\mathbf{k}|\dot{\mathbf{r}}|n,\mathbf{k}\rangle$ given by

$$\mathbf{v}(n,\mathbf{k}) = \frac{1}{\hbar}\nabla_{\mathbf{k}}\varepsilon_n(\mathbf{k}). \tag{2}$$

When a particle in the Bloch state $|n,\mathbf{k}_0\rangle$ is adiabatically affected by a constant external force $\mathbf{F} = F_x\hat{\mathbf{x}}$ weak enough not to induce interband transitions, it evolves up to a phase factor into the state $|n,\mathbf{k}(t)\rangle$ according to $\mathbf{k}(t) = \mathbf{k}_0 + \mathbf{F}t/\hbar$. The time evolution has a period $\tau_B = h/(|F_x|a)$, corresponding to the time required for the quasimomentum to scan the whole Brillouin zone. If the force is applied adiabatically, it provides momentum to the system but not energy because the effective mass (given by $m(\varepsilon)^{-1} = \frac{1}{\hbar^2}\frac{\partial^2\varepsilon}{\partial k^2}$) is not always positive. For optical lattices the force (or tilt: $V = -\mathbf{F}\cdot\mathbf{r}$ term in the Hamiltonian) can be realized by accelerating the lattice [2, 3]. Using the tight-binding dispersion relation the velocity of an atom oscillates like

$$v_x(t) = \frac{Ja}{\hbar}\sin(k_{0x}a + \frac{F_x ta}{\hbar}). \tag{3}$$

For cold bosonic atoms and condensates [2, 3] nearly all of the population is in the lowest mode of the optical potential, Eq.(3) therefore describes the oscillation of the whole gas. We generalize the result for the case when many momentum states of the band (at $T = 0$, the states with wave vector $|\mathbf{k}| \le k_F$) are occupied. We calculate the velocity of the whole gas as the average over the normalized temperature-dependent distribution function (the Fermi distribution f) of the particles:

$$\langle v_x(t)\rangle = \frac{1}{\hbar}\sum_{\mathbf{k}_0} f(\mathbf{k_0})\nabla_{k_{0x}}\varepsilon(\mathbf{k}_0 + \frac{\mathbf{F}t}{\hbar}). \tag{4}$$

Using the tight-binding dispersion relation for the Bloch energies we obtain the oscillations shown in Fig. 1. At $T = 0$, Eq.(4) reduces to

$$\langle v_x(t)\rangle = \frac{Ja}{\hbar}\frac{\sin(k_{xF}a)}{k_{xF}a}\sin\left(\frac{F_x ta}{\hbar}\right). \tag{5}$$

This shows that the macroscopic coherent oscillation can still be observed if the band is not full, but the amplitude is suppressed by the band filling $k_{xF}a$. The effect of the temperature can be seen in Fig. 1: the amplitude starts to decrease at temperatures of the order $T \ge 0.1J$ but is still non-negligible at half J. These results are valid for a one-component degenerate Fermi gas at low temperatures. In a two-component

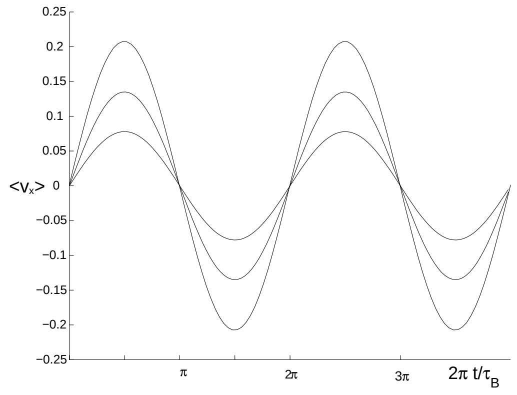

FIG. 1: The average velocity in $Ja/\hbar$ units as a function of time for a half filled band. The plotted lines correspond to atoms in the normal state at different temperatures $T = J$, $0.5J$ and $\le 0.1J$. Larger amplitudes correspond to lower temperatures.

Fermi gas, atoms in the different hyperfine states interact with each other which may lead to a superfluid state. Above T_c, weak interactions can be described by a mean field shift in the chemical potential, leading to no qualitative changes in Bloch oscillations. Inelastic scattering and consequent damping of Bloch oscillations can be described e.g. by balance equations [15]. In the following we consider the superfluid case where qualitative changes are expected.

In order to observe robust Bloch oscillations of a superfluid Fermi gas, the critical velocity of the superfluid should not be reached before the edge of the Brillouin zone in the presence of momentum changing collisions. When the superfluid breaks, one has to use a normal state description. A BCS-superconductor can carry a persistent current q until a critical velocity, $v_c = \Delta/p_F$. For higher current values, even at $T = 0$, it is energetically favorable to break Cooper pairs and create a pair of quasiparticles [16]. This costs 2Δ in binding energy and decreases the Bloch energy ($\xi = \varepsilon - \mu$, where μ is the chemical potential) by $|\xi_{k_F+q} - \xi_{k_F-q}| \equiv 2|E_D|$. Therefore, for the current to be stable $|E_D| < \Delta$. This is the Landau criterion of superfluidity. For the tight-binding dispersion relation, we rewrite the condition as $J\sin(qa)\sin k_F a < \Delta$. To complete a Bloch oscillation, $\sin(qa)$ should achieve its maximum value 1, i.e.

$$\sin k_F a < \Delta/J. \tag{6}$$

For weak coupling, Δ/J is given by the BCS theory, and in the attractive Hubbard model in the strong coupling limit the gap at $T = 0$ is given by $\Delta = \frac{1}{2}U$ for half filling [14]. Using these estimates, we show in Fig. 2 the relation (6) for a gas of ^{6}Li atoms together with the transition temperature. To relate the criterion to the Cooper pair size, we rewrite Eq.(6) in terms of the BCS coherence length $\xi_0 = \hbar v_F/\pi\Delta$ and insert $J\sin(k_F a) = \hbar v_F/a$ which yields $\xi_0 < a/\pi$. The observation of robust Bloch oscillations is thus restricted to superfluids with BCS coherence length smaller than the lattice periodicity. This is the intermediate – strong coupling regime. The

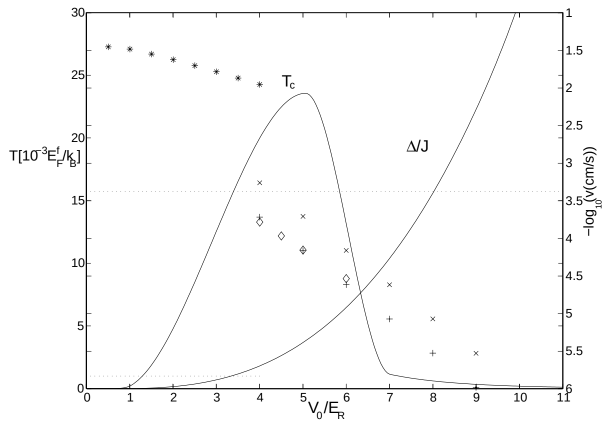

FIG. 2: The transition temperature, Landau criterion at $T = 0$ and the amplitude of the velocity Bloch oscillations for ^{6}Li atoms in hyperfine states with scattering length $a_s = -2.5\cdot 10^3 a_0$ for a half filled 3D CO_2 laser lattice ($a = 10^5 a_0$) as a function of the lattice depth. The amplitudes of the oscillations at $T = \frac{2}{3}T_c^{max}$ (long dashed line) are denoted by $*$ for the normal state oscillations, $\diamond$ for the superfluid velocity at $T = 0$ Eq.(12) and in the boson limit Eq.(13) pair size $l = a/3$ by $\times$ and $+$ for pair size $l = a/4$. The Landau criterion condition Eq.(6) requires $\Delta/J > 1$ (1 marked by short dashed line) for the half filled band. Here E_R is the recoil energy and E_F^f is the Fermi energy for free fermions with the same density.

length argument can be also understood by thinking that the pairs have to be smaller than the lattice sites in order to see it as a periodic potential.

For calculating the superfluid velocity a space dependent description of the superfluid has to be used. We combine the BCS ansatz with the Bloch ansatz for the lattice potential using the Bogoliubov – de Gennes (BdG) equations [17]. As given by the Landau criterion above, the interesting regime is the intermediate – strong coupling one. Note that even in the strong-coupling limit, the algebra of the BCS theory can be applied to all coupling strengths [8, 18] together with an extra definition for the chemical potential which in the weak coupling limit is given just by the Fermi energy of the non-interacting gas. The BdG equations are:

$$\begin{pmatrix} H(\mathbf{r})-\mu & \Delta(\mathbf{r}) \\ \Delta(\mathbf{r})^* & -[H(\mathbf{r})-\mu] \end{pmatrix}\begin{pmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{pmatrix} = E\begin{pmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{pmatrix}. \quad (7)$$

When the external potential is periodic one can use the Bloch ansatz for u and v because by self-consistency the Hartree and pairing fields are also periodic. We obtain

$$u_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}\tilde{u}_{\mathbf{k}}\phi_{\mathbf{k}}(\mathbf{r}) \quad ; \quad v_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}\tilde{v}_{\mathbf{k}}\phi_{\mathbf{k}}(\mathbf{r}) \qquad (8)$$

$$\Delta(\mathbf{r}) = \sum_{\mathbf{k}} |g|[1-2f(E_{\mathbf{k}})]u_{\mathbf{k}}(\mathbf{r})v_{\mathbf{k}}^*(\mathbf{r}), \qquad (9)$$

where $\phi_{\mathbf{k}}$ are the fully periodic part of the Bloch functions, such that $[H(\mathbf{r})-\mu]\phi_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}} = \xi_{\mathbf{k}}\phi_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}$.

To describe Bloch oscillations we impose the adiabatic condition, that is, momenta evolve according to $\mathbf{k} \longrightarrow \mathbf{k} + \mathbf{F}t/\hbar \equiv \mathbf{k}+\mathbf{q}$, i.e. we consider BCS state with a drift (again only in x-direction). The solutions of the BdG equations take the form

$$u_{\mathbf{k}}^{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}e^{i\mathbf{q}\cdot\mathbf{r}}\tilde{u}_{\mathbf{k}}^{\mathbf{q}}\phi_{\mathbf{k}+\mathbf{q}}(\mathbf{r});\ v_{\mathbf{k}}^{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}e^{-i\mathbf{q}\cdot\mathbf{r}}\tilde{v}_{\mathbf{k}}^{\mathbf{q}}\phi_{\mathbf{k}-\mathbf{q}}(\mathbf{r})$$

$$\Delta^{\mathbf{q}}(\mathbf{r}) = \sum_{\mathbf{k}} |g|[1-2f(E_{\mathbf{k}}^{+\mathbf{q}})]u_{\mathbf{k}}^{\mathbf{q}}(\mathbf{r})v_{\mathbf{k}}^{\mathbf{q}*}(\mathbf{r}), \qquad (10)$$

$E_{\mathbf{k}}^{\mathbf{q}} = (\xi_{\mathbf{k}+\mathbf{q}} - \xi_{\mathbf{q}-\mathbf{k}})/2 \pm \sqrt{(\xi_{\mathbf{k}+\mathbf{q}}+\xi_{\mathbf{q}-\mathbf{k}})^2/4 + |\Delta^{\mathbf{q}}|^2} \equiv E_D \pm \sqrt{E_A^2 + |\Delta^{\mathbf{q}}|^2}$, where $2E_D$ is the energy difference and E_A the average energy. The $\pm$ holds for the particle and hole branch, respectively, and the particle branch eigenfunctions are $|\tilde{u}_{\mathbf{k}}^{\mathbf{q}}|^2$, $|\tilde{v}_{\mathbf{k}}^{\mathbf{q}}|^2 = (1 \pm E_A/\sqrt{E_A^2+|\Delta^{\mathbf{q}}|^2})/2$. The Hamiltonian transformed under the Bogoliubov transformation leading to (7) has to be positive definite. This means that one should use the solutions for which $E_{\mathbf{k}}^{+\mathbf{q}} > 0$, i.e. $\min(\sqrt{E_A^2+|\Delta^{\mathbf{q}}|^2}) = |\Delta^{\mathbf{q}}| > |E_D(\mathbf{k}')|$, where $\mathbf{k}'$ minimizes E_A^2. Remarkably, this condition is closely related to the Landau criterion $|\Delta| > E_D(\mathbf{k_F})$.

In the BCS ansatz, a common momentum $\mathbf{q}$ can be added to all particles, leading to correlations of the type $\langle c^\dagger_{\mathbf{k}+\mathbf{q}}c^\dagger_{-\mathbf{k}+\mathbf{q}}\rangle$. The momentum per pair becomes $2\mathbf{q}$. One can formally calculate this obvious result also by using the plane wave ansatz $u_{\mathbf{k}} = |u_{\mathbf{k}}|e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}}$, $v_{\mathbf{k}} = |v_{\mathbf{k}}|e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}}$ [17] (Eq.(10) with $\phi = 1$) and introducing an (unnormalized) order-parameter wave function $\Delta^{\mathbf{q}}(\mathbf{r}) = e^{i2\mathbf{q}\cdot\mathbf{r}}C$, where C is given by Eq.(10) to be a constant in $\mathbf{r}$. Expectation values like momentum ($\mathbf{p} = -i\partial/\partial\mathbf{r}$) can be calculated: $\langle\mathbf{p}\rangle = \langle\Delta^{\mathbf{q}}(\mathbf{r})| - i\partial/\partial\mathbf{r}|\Delta^{\mathbf{q}}(\mathbf{r})\rangle/\langle\Delta^{\mathbf{q}}(\mathbf{r})|\Delta^{\mathbf{q}}(\mathbf{r})\rangle = 2\mathbf{q}$. The order-parameter wave function is defined in the spirit of (but not with a one-to-one correspondence to) the Ginzburg-Landau theory with a space dependent wave function whose absolute value equals the gap. In case of Fermionic atoms the Ginzburg-Landau approach has been used to describe harmonic confinement [19] and vortices [20]. For the periodic potential we introduce the order parameter wave function in the form $\Delta^{\mathbf{q}}(\mathbf{r}) = \sum_{\mathbf{k}}\Delta_{\mathbf{k}}^{\mathbf{q}}(\mathbf{r})$, where using Eq.(10),

$$\Delta_{\mathbf{k}}^{\mathbf{q}}(\mathbf{r}) = F(\mathbf{k},\mathbf{q})\phi_{\mathbf{k}+\mathbf{q}}e^{i(\mathbf{k}+\mathbf{q})\mathbf{r}}\phi^\dagger_{\mathbf{k}-\mathbf{q}}e^{-i(\mathbf{k}-\mathbf{q})\mathbf{r}} \qquad (11)$$

and $F(\mathbf{k},\mathbf{q}) = |g|[1-2f(E_{\mathbf{k}}^{+\mathbf{q}})]\tilde{u}_{\mathbf{k}}^{\mathbf{q}}\tilde{v}_{\mathbf{k}}^{\mathbf{q}*}$. We calculate the superfluid velocity using $\langle\mathbf{v}_S\rangle = \mathcal{N}\langle\Delta^{\mathbf{q}}(\mathbf{r})|\dot{\mathbf{r}}|\Delta^{\mathbf{q}}(\mathbf{r})\rangle$, where $\mathcal{N} = \langle\Delta^{\mathbf{q}}(\mathbf{r})|\Delta^{\mathbf{q}}(\mathbf{r})\rangle^{-1}$. Using $\langle\dot{\mathbf{r}}\rangle_{\phi^\dagger_{\mathbf{k}-\mathbf{q}}e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}}} = -\frac{1}{\hbar}\frac{d}{d\mathbf{k}}\xi_{\mathbf{k}-\mathbf{q}}$ and the tight-binding energy dispersion relation the superfluid velocity becomes

$$\langle v_{xS}\rangle = \mathcal{N}\sum_{\mathbf{k}}|F(\mathbf{k},\mathbf{q})|^2\frac{Ja}{\hbar}\cos k_x a\sin qa \qquad (12)$$
$$= \frac{Ja}{\hbar}\sin(qa)\mathcal{N}\sum_{\mathbf{k}}\left|\frac{\left[1-2f(E_{\mathbf{k}}^{\mathbf{q}})\right]\Delta^{\mathbf{q}}}{\sqrt{E_A^2+|\Delta^{\mathbf{q}}|^2}}\right|^2\cos k_x a.$$

The superfluid velocity for selected parameters is shown in Fig. 2. We have also calculated the thermal quasiparticle contribution but it turns out to be negligible for half filling.

In the composite boson limit, one could describe the center-of-mass movement of the composite particle by defining $J^* = J(m \to 2m)$. In order to give a simple estimate for the effect of the Fermi statistics, we interpret $|F(\mathbf{k},\mathbf{q})|^2 \sim |F(\mathbf{k})|^2$ in Eq.(12) as reflecting the internal wavefunction of the pair in the composite boson limit, c.f. [7, 8]. The average velocity for the bosons becomes $\langle v_{xB}\rangle \propto \frac{J^*a}{\hbar}\sin qa \sum_{\mathbf{k}} |F(\mathbf{k})|^2 \cos k_x a$. If the pairs were extremely strongly bound, the internal wave function in real space is a delta-function, corresponding to a constant in k-space. This means $\langle v_B\rangle = 0$ since the cosine integration in Eq.(12) would extend to the whole k-space with equal weight, i.e. there are no empty states in the Brillouin zone as required for Bloch oscillations. For on-site pairs, we use $|F(r)|^2 \propto \exp(-r^2/l^2)$ leading to $|F(k)|^2 \propto \mathcal{N}\exp(-l^2k^2/4)$, therefore the suppression factor for the Bloch oscillations becomes $S \sim \mathcal{N}\int dk \exp(-l^2k^2/4)\cos ka$, where l is the pair size. As a rough estimate for the average velocity we thus obtain

$$\langle v_{xB}\rangle \sim S\frac{J^*a}{\hbar}\sin\left(\frac{F_x ta}{\hbar}\right). \qquad (13)$$

This is shown in Fig. 2 for pair sizes $l = a/3$ and $l = a/4$. It gives an order-of-magnitude estimate, approaching the results given by the BCS algebra.

Another way of treating the composite boson limit is to derive a Gross-Pitaevskii type of equation for the composite bosons with $M = 2m$ and with a repulsive non-linear interaction term $n_B U_B = n_B 4\pi\hbar^2 a_B/M$, $a_B = 2a_s$ where a_s is the renormalized s-wave scattering length [21]. If the non-linear term is small compared to the Bloch energy $E_B = h^2/(Ma^2)$, the nonlinearity leads only to a change in the band width J [3]. Therefore, composite bosons oscillate but with a modified amplitude. Large non-linearity would not allow Bloch oscillations, corresponding to a large suppression factor in the above discussion. Note that the Landau criterion for a superfluid Bose gas gives the critical velocity $v_{\rm sound} = \sqrt{U_B n_B/M}$ which is orders of magnitude bigger than Eq.(13) for half filling and parameters in Fig. 2. Problems arise only in the extremely empty lattice limit.

In summary, we have defined a set of tools for qualitative and quantitative description of Bloch oscillations for the BCS-BEC crossover regime. The amplitude of the oscillations decreases when the crossover is scanned, in general due to the shrinking of the bandwidth. However, the change from the normal to the superfluid state description leads to a drastic change in the amplitude. This is due to smoothening of the Fermi-edge by pairing. Bloch oscillations could be used for exploring pairing correlations since any localization in space (pair size) leads to broadening in momentum which suppresses the amplitude in the same way as band filling in the non-interacting gas. Achievement of superfluidity is still a great challenge, but even at $T \gg T_c$, the effect of collisions on Bloch oscillations can be studied producing information useful for applications of Bloch oscillations such as production of Terahertz radiation [15, 22]. Observation of oscillating fermionic atoms in optical lattices would contribute to the quest for a steadily driven fermionic Bloch oscillator.

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