Helsinki University of Technology Department of Electrical and Communications Engineering Laboratory of Electromechanics

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ANALYTICAL PREDICTION OF THE ELECTROMAGNETIC TORQUES IN SINGLE-PHASE AND TWO-PHASE AC MOTORS

Doctoral thesis

Mircea Popescu

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Helsinki University of Technology
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Abstract

The single-phase and two-phase versions of AC motors can be modelled by means of the two-axis (d-q) theory with sufficient accuracy when the equivalent circuit parameters are correctly estimated. This work attempts to present a unified approach to the analytical prediction of the electromagnetic torque of these machines.

Classical d-q axes formulation requires that the reference frame should be fixed on the frame where the asymmetries arise, i.e. the stator and rotor. The asynchronous torques that characterize the induction motors are modelled in a stationary reference frame, where the d-q axes coincide with the physical magnetic axes of the stator windings. For the permanent magnet motors, that may exhibit asymmetries on both stator and rotor, the proposed solution includes: a series of frame transformations, followed by symmetrical components decomposition. As in single-phase and two-phase systems the homopolar component is zero; -each symmetrical component - negative and positive - is further analysed using d-q axes theory. The superposition principle is employed to consider the magnets and rotor cage effects.

The developed models account for the most important asymmetries of the motor configuration. These are, from the stator point of view, different distribution, conductors' dimensions and number of effective turns, non-orthogonal magnetic axes windings and from the rotor point of view, asymmetrical rotor cage, variable reluctance, and permanent magnets effect. The time and space harmonics effect is ignored. Test data are compared with the computed data in order to observe how the simplifying assumptions affect the level of accuracy.

The analytical prediction methods make possible torque computation according to the nature of the torque being computed, namely, induction, reluctance and excitation (permanent magnet). The results are available for quasi steady-state, steady-state (rated or synchronous speed) and dynamic analyses. All the developed mathematical models can be used in preliminary design for further optimisation and accurate estimation in complex numerical models. Another important feature of the analytical models for single-phase and two-phase AC motors, is that they can be directly implemented in any suitable electrical drives control strategy.

Keywords single-phase, two-phase, induction motors, permanent magnet motors, electromagnetic torque, special winding	
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Preface

This work was carried out partly in the Laboratory of Electromechanics at Helsinki University of Technology and partly in the SPEED Laboratory at Glasgow University. It is the result of several years' work in the field of analytical modelling of electrical machines. The purpose of the study was to develop original models and methods with which to study the electromagnetic torques in single-phase and two-phase AC motors.

I would like to express my gratitude to Emeritus Professor Tapani Jokinen, Professor Antero Arkkio and Professor Asko Niemenmaa whose help and guidance made the start, initial development and completion of this work possible.

I also owe a great debt of gratitude to Professor TJE Miller for his fruitful co-operation and discussions that helped me overcome the many obstacles I encountered and enabled me to make substantial progress in my research. I would particularly like to acknowledge the beneficial discussions and help of Dr. Dan Ionel, Dr. Marius Rosu, Malcolm McGilp and Marian Negrea.

The special co-operation of staff at Zanussi Elettromeccanica Compressors, Italy made possible the completion of this work. In this connection my special thanks go to Giovanni Strappazzon. Funding for this work was provided in part by SPEED Consortium companies and in part by Helsinki University of Technology, and is gratefully acknowledged. I am obliged to Ruth Vilmi for the revision of the language.

Last but not least, I would like to thank my wife Madalina for her help, patience and support during this work and my daughter Miruna-Ioana for the smile she kept on my face.

Espoo, June 1999 – Glasgow, April 2004

Mircea Popescu

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P1. M. Popescu, A. Arkkio, E. Demeter, D. Micu, V. Navrapescu - "Development of an inverter fed two-phase variable speed induction motor drive" - in Conference Records of PEMC'98, pp.4-132 - 4-136 Sept. 1998, Prague, Czech Republic– ISBN – 80-01-01832-6

P2. M. Popescu, T. Jokinen, E. Demeter, V. Navrapescu - "Modeling and Analysis of a Two-Phase Induction Machine with Non-Orthogonal Stator Windings" – in Conference Records of IEEE IEMDC'99, May 1999, Seattle, USA, pp.389-391 – ISBN – 0-7803-5293-9/99

P3. M. Popescu, P. Lampola, T. Jokinen, A. Arkkio, V. Navrapescu, E. Demeter "Analysis of the Electrical Shift Angle Influence over a Variable Speed Two-Phase Induction Motor Drive" in Conference Records EPE'99, Sept. 1999, Lausanne, Switzerland

P4. M. Popescu, V. Navrapescu – "Modelling in stationary frame reference of single and two-phase induction machines including the effect of iron loss and magnetising flux saturation" – Conference Records of ICEM 2000, 28-30 August, Espoo, Finland, Vol. I, pp. 407-411 – ISBN – 951-22-5097-7

P5. M. Popescu, T.J.E. Miller, M.I. McGilp, G. Strappazzon, N. Trivillin, R. Santarossa. – "Asynchronous performance analysis of a single-phase capacitor-start, capacitor-run permanent magnet motor"- IEEE – Transactions on Energy Conversion – accepted for publication

P6. M. Popescu, T.J.E. Miller, M.I. McGilp, G. Strappazzon, N. Trivillin, R. Santarossa. –"Line-start permanent magnet motor: single-phase starting performance analysis" IEEE Transactions on Industry Applications, Vol. 39, No. 4, pp. 1021-1030, July/August 2003

P7. M. Popescu, D.M. Ionel, S. Dellinger, T.J.E. Miller, M.I. McGilp –"Analysis and design of a two-speed single-phase induction motor with 2 and 18 pole Special Windings"- IEEE – Transactions on Energy Conversion – accepted for publication

P8. M. Popescu, T.J.E. Miller, M.I. McGilp, G. Strappazzon, N. Trivillin, R. Santarossa. –"Torque behaviour of 1-phase permanent magnet AC motor"- Conference Record IEEE IEMDC'03, June 2003, Madison, USA, pp 820-825 ISBN–0-7803-7817-2/03

List of principal symbols

A	magnetic vector potential
B	magnetic flux-density vector
H	magnetic field strength vector
M	magnetization vector
B_θ	magnetic flux-density, tangential component
B_r	magnetic flux-density, radial component
B_m	semi-sum of flux-density value between two consecutive FE solutions
B_d	semi-difference flux-density value between two consecutive FE solutions
E_0	no-load induced voltage
E_a	induced voltage in auxiliary winding
E_m	induced voltage in main winding
H	magnetic field strength amplitude
H_m	mean magnetic field strength value between two consecutive FE solutions
H_d	average difference magnetic field strength value between two consecutive FE solutions
$\underline{I}_{as}$, $\underline{I}_{bs}$	complex phasor of stator currents
$\underline{I}'_{as}$, $\underline{I}'_{br}$	complex phasor of rotor currents referred to stator
$\underline{I}_{DS}$, $\underline{I}_{QS}$	complex phasor of stator currents in *d*-*q* axes
$\underline{I}'_{DR}$, $\underline{I}'_{QR}$	complex phasor of rotor currents in *d*-*q* axes referred to stator
i_{as}, i_{bs}	instantaneous stator currents
i'_{br}, I'_{ar}	instantaneous rotor currents referred to the stator
i_{DS}, i_{QS}	instantaneous stator currents in stationary *d*-*q* reference frame
i'_{DR}, i'_{QR}	instantaneous rotor currents in stationary *d*-*q* reference frame
$\underline{I}_d$, $\underline{I}_q$	complex *d*-*q* axis stator current components in rotor reference frame
i_d, i_q	instantaneous *d*-*q* axis stator current components in rotor reference frame
i_{dr}, i_{qr}	instantaneous *d*-*q* axis rotor current components in rotor reference frame
i_e	equivalent instantaneous excitation current in rotor reference frame
i_{dfe}, i_{qfe}	instantaneous *d*-*q* axis stator core loss currents in stator reference frame
$\underline{I}_{d\pm}$, $\underline{I}_{q\pm}$	positive (+) and negative (–) complex *d*-*q* axis stator current components in rotor reference frame
$I_{dm,qm}$	average *d*-*q* axes stator induced currents by rotor excited movement (rotor reference frame)
k	turns ratio (aux/main) between stator windings
$L_{mds,\ mqs}$	magnetising inductance in *d*-*q* axes (stationary reference frame)
$L_{las,\ lbs}$	leakage inductances of the *as* winding (*bs* winding)
$L_{mas,mbs}$	magnetizating inductances of the *as* winding (*bs* winding)
$L_{lar.lbr}$	rotor leakage inductances referred to the stator windings (*as, bs*)
$L_{sar,sbr}$	mutual inductance between the *as* winding (*bs* winding) and the rotor windings
M	phase number
N_m	number of turns on main stator winding
P	poles number
p	time derivative operator (d/dt)
R	radius in the air-gap
R_C	total iron loss equivalent resistance
R_{Cap}	resistance of the capacitive impedance in a capacitor motor
R_s, R_a, R_m	stator winding resistance: equivalent/auxiliary/main
R_{rd}, R_{rq}	rotor resistance for *d*-*q* axis
R_r	rotor resistance referred to stator main winding
s	slip
T_e	instantaneous electromagnetic torque
T_m	average (quasi steady-state) magnet braking torque

T_{pls}	pulsating torque in a permanent magnet motor – zero to peak amplitude
$T_{\mathrm{e(avg)}}$	average (steady-state) electromagnetic torque
$T_{\mathrm{e(pls)}}$	average (steady-state) pulsating torque
$T_{\mathrm{(avg)}\pm}$	positive (+) and negative (-) average (quasi steady-state) cage torque
U_{n}	rated voltage (rms)
$\underline{V}_{\mathrm{as}}$, $\underline{V}_{\mathrm{bs}}$	complex phasor of stator voltages
V_{a}	terminals voltage for auxiliary winding
V_{m}	terminals voltage for main winding
$v_{\mathrm{d,q}}$	instantaneous *d*-*q* axes stator voltages in rotor reference frame
W_{en}	magnetic stored energy
W_{coen}	magnetic coenergy
X_{Cap}	reactance of the capacitive impedance in a capacitor motor
X_{m}	main winding magnetization reactance
X_{ss}	main winding self-reactance
X'_{rr}	rotor self-reactance referred to stator
X_{d}, X_{q}	synchronous reactance for *d*-*q* axis
$\underline{X}_{\mathrm{d}\pm}$, $\underline{X}_{\mathrm{q}\pm}$	complex positive/negative asynchronous reactance for *d*-*q* axis
X_{lrd}, X_{lrq}	rotor leakage reactance for *d*-*q* axis
X_{lr}	rotor leakage reactance referred to stator main winding
X_{md}, X_{mq}	magnetising reactance for *d*-*q* axis
X_{ls} X_{la}, X_{lm}	stator leakage reactance: equivalent/auxiliary/main
$\underline{Z}_{\mathrm{m}}$	main phase impedance for locked-rotor conditions
$\underline{Z}_{\mathrm{a}}$	auxiliary phase impedance for locked-rotor conditions
$\underline{Z}_{\mathrm{C}}$	capacitive impedance
β	turns ratio (main/aux) between stator windings
φ_{a}	shift electrical angle between stator windings not in quadrature
$\phi_{m,a}$	diameter of the main/auxiliary winding
ω	synchronous angular velocity
ω_{b}	base angular velocity
$\underline{\psi}_{\mathrm{d}\pm,\ \mathrm{q}\pm}$	positive (+) and negative (-) sequence complex *d*-*q* axes stator flux-linkages in rotor reference frame
$\psi_{\mathrm{dm,qm}}$	average *d*-*q* axes stator flux-linkages due to induced currents by rotor excited movement in rotor reference frame
$\lambda_{\mathrm{DR,QR}}$	instantaneous *d*-*q* axes rotor flux-linkages in stationary reference frame
μ_{0}	air magnetic permeability
θ	actual angular displacement of the rotor
θ_{r}	electrical angular displacement of the rotor
ζ	shift electrical angle between stator windings magnetic axes

Abbreviations

AC, DC	alternative current, direct current
BEM	boundary element method
d-*q*	direct, quadrature axes
DSP	digital signal processor
EMF	electromotive force
FDM	finite difference method
FEA	finite element analysis
FEM	finite element method
FFT	fast Fourier transformation
LSPMM	line-start permanent magnet motor
m.m.f	magnetomotive force
PWM	pulse width modulation
SPIM	single-phase induction motor
TPIM	two-phase induction motor

1 INTRODUCTION

1.1 Background of the study

Single-phase and two-phase AC motors are the most-used converters in home appliances. The terms single or two-phase configuration refer to the supply voltage systems applied to the winding terminals. Single-phase AC motors can be constructed as induction motors, permanent magnet motors or synchronous reluctance motors. The line-start permanent magnet motor is a hybrid version that starts as an induction motor and operates synchronously as a permanent magnet motor. Naturally, a single-phase AC motor will exhibit just a pulsating-in-time air-gap field and consequently a pulsating torque. Thus, an auxiliary field has to be present in order to obtain a revolving air-gap field and an electromagnetic torque with an average value different from zero. The solutions employed to create the auxiliary field assume either unsymmetrical stator windings or, supplementary impedances that can be fixed values or electronically controlled.

A two-phase AC motor can have a configuration identical to the single-phase version of the motor, but the voltages applied to the windings terminals are independently controlled so that we have a two-phase supply voltage system. As the AC mains are available as single or three-phase systems, a two-phase voltage system is realized through the use of inverters with different control strategies.

There are available several analytical methods for the performance estimation of single- and two-phase AC motors, namely: double-revolving field, symmetrical components, cross-fields or two axis (*d*-*q*) theory. Numerical methods have also been successfully implemented to predict the performance of these motors.

Usually, the comparison between analytical and numerical methods emphasises the advantage of faster computation time versus accuracy level. However, the analytical methods are based on measurable physical parameters and permit the inclusion of non-linear effects. Therefore, many electrical machinery designers address new prototypes development through the use of analytical tools.

This work is an attempt to realize a unified approach to the electromagnetic torque prediction for single- and two-phase AC motors using the analytical methods.

1.2 Prediction of the electromagnetic torque in AC motors

The electromagnetic torque represents the main performance criterion for rotating electrical motors. With its multitude of aspects torque estimation is a challenging task when analyzing an existing motor or designing a new AC motor prototype. Various electromagnetic torque components - average, pulsating, instantaneous, ripple, cogging, harmonic - have been identified and analysed in AC motors operation. We can also define cage torque, alignment torque, reluctance torque according to their cause. The torque analysis can be further detailed by considering the torque created by the space mmf harmonics, voltage time harmonics or the parasitic synchronous torques created by the interaction between stator and rotor teeth harmonics.

Multiple mathematical models have been developed for each of the torque components and aspects. The mathematical models can be applied in analytical modelling or numerical modelling. In this subchapter, these methods of analysis are briefly introduced. A more complete literature study is presented in Chapter 2.

1.2.1 Analytical modelling

The analytical modelling of AC motors has been constantly developed from the beginning of electrical machinery history. The actual heritage of the theoretical analysis of electric motors includes contributions from big names in science and technology such as: Faraday, Maxwell, Tesla, Steinmetz, Richter.

Generically, the analytical modelling of the torque behaviour in an AC motor relies on the generalized forces theorem. In a simplified form this theorem states that the electromagnetic torque (or force) is given by the system coenergy or energy variation with incremental rotor displacement, if the currents and flux-linkages are constant during the elemental rotor movement. As the system is an electrical motor, the energy or coenergy can be expressed as a function of products between flux-linkages and currents. Furthermore, the flux-linkages are expressed as products between reactances and currents. Consequently, all the developed analytical models that compute the electromagnetic torques in AC motors are relying on the equivalent circuits parameters (resistances and reactances) that can have fixed or variable (linear or non-linear) values. Thus, the precision of any analytical model that estimates the electromagnetic torque in a rotating AC motor depends on the accuracy level that characterizes the motor parameters.

The main advantage of analytical modelling is that it gives an important starting point to any preliminary design and analysis of an electrical motor. However, there are still physical phenomena that occur in an AC motor (e.g., stray load losses) and cannot yet be mathematically modelled. Several simplifying assumptions are necessary for any analytical motor model.

One can state that analytical modelling of AC motors has to be a trade-off between computation time (very fast) and accuracy level. The accuracy can be significantly improved if the most important non-linear effects (i.e., saturation, core loss, windage and friction loss, and harmonics) are modelled through a sufficiently high number of elements in the mathematical model of the motor.

1.2.2 Numerical modelling

The numerical modelling of electric motors has its basics in the electromagnetic fields theory. There are several mathematical approaches such as finite-element method (FEM), finite-difference method (FDM), boundary element method (BEM) to solving the system equations in numerical methods. Regardless of the mathematics of these methods, the electromagnetic torque is estimated using either the Maxwell stress theory, virtual work (energy variation) or Laplace method (magnetizing currents). The employment of numerical methods in electrical machines analysis has been introduced relatively recently by Silvester and Chari, 1970.

Progressive improvement in the power and speed of computers has resulted in a situation in which the numerical analysis of electrical machines is successfully used as both a research and design tool. In rotating machines, the model most widely employed is two-dimensional. This approach ignores the end-effects and the three-dimensional eddy currents effect. Even though potentially more accurate, three-dimensional numerical models require one or two orders of magnitude more of computer resources, so they are still beyond the bounds of economic viability, especially in the electrical machinery industry where tens of design versions for only one motor might be requested in one day. If the problem settings are correctly formulated, the numerical modelling of AC motors will usually lead to a higher accuracy level for the estimated results than the analytical modelling.

In modern practise, a combination of analytical modelling and numerical modelling is required. A preliminary design optimised through analytical modelling represents the best initial solution for a further numerical model. There are also very well established combined numerical and analytical models that will numerically simulate the electrical motor and analytically simulate the external circuits from the drive system (inverters, connections, etc.)

1.3 Aim of the work

The main task of this work was to compute analytically the electromagnetic torque estimation in single- and two-phase AC motors taking into account all the possible motor asymmetries.

The first part of the study involved the development and testing of analytical models for single- and two-phase induction motors. Beside the faster computation time, the accuracy level was improved by taking into account the most important non-linear effects in the motor, i.e., saturation, core losses or mutual cross-coupling due to electrically non-orthogonal stator windings.

Another important goal of the study presented in the second part this thesis was to create analytical models that calculate the electromagnetic torque components for one of the most complex single- or two-phase AC motor configurations, the line-start permanent magnet motor. This motor type exhibits multiple possible structural asymmetries and usually operates in unbalanced conditions.

Finally, all the proposed analytical models are validated against test data where possible finite element analysis results were employed to check the analytical data.

1.4 Scientific contribution

The most notable scientific contributions of this study are:

1. Modelling the single-phase induction motor with non-orthogonal stator windings using the *d-q* axes theory;

2. Modelling the single-phase induction motor for vector control purposes with core loss and saturation effect included;

3. Computation of torque components for a single-phase line-start permanent magnet motor;

4. Comparative analysis of the single- and two-phase AC motors control strategies: line supply versus PWM inverter supply;

5. Modelling the single-phase induction motor with two special windings using the symmetrical components and double revolving field theory.

Each of these items is discussed in respective sections of the publications listed above.

1.5 Structure of the work

The structure of the research work is reflected in the divisions of the thesis:

1. A detailed literature review in Chapter 2 presents the state of the art of the methods used to analyse a single- or two-phase AC motor with *d-q* axes theory.

2. An overview of the new equivalent circuits and analytical computation methods in this study used in Chapter 3.

3. The measurement of the electromagnetic torques of single- and two-phase AC motors is presented in Chapter 4.

4. Chapter 5 reports the results of calculations and the comparison of these with the measured torques.

5. Finally, the calculation methods used and results obtained are discussed in Chapter 6.

In the following chapters, these steps will be presented and explained in detail as separate entities. Steps 2 to 5 are based on the publications. The publications are reprinted in the Publications chapter at the end of the thesis.

Publication P1

M. Popescu, A. Arkkio, E. Demeter, D. Micu, V. Navrapescu - "Development of an inverter fed two-phase variable speed induction motor drive" - in Conference Records of PEMC'98, pp.4-132 - 4-136 Sept. 1998, Prague, Czech Republic– ISBN – 80-01-01832-6

The two-phase induction motor (TPIM) is used in many low-power applications where three-phase supply is not readily available. The TPIM is usually a single-speed device operated from a fixed-frequency supply. Speed adjustments for two- and single-phase induction motors are normally realized by using tapped windings or voltage amplitude control, increasing the slip reduces the rotational speed. However as efficiency is a function of slip, speed reductions lead to increased losses in the induction machine. Also, a limited speed range is obtained because steady-state operation is restricted to the stable zone of the torque-speed characteristic.
For speed-control of a TPIM, a pulse width modulation (PWM) inverter with four outputs connected to the special distributed windings of the motor was used. A wide speed range in both directions was achieved. A lead-acid battery or a bridge rectifier connected to the AC supply voltage is used as the supply. A comparison between a two-phase drive with the induction motor supplied from a sinusoidal variable voltage amplitude and the same electric drive supplied from the PWM inverter was drawn. The first was measured while the latter was simulated.

The inverter-fed two-phase induction motor is a workable replacement for the capacitor-run single-phase induction motor.

The simulation and the experiments showed the following:

The inverter fed system gives direct control of the motor speed by using the combined strategy of making the constant voltage -frequency ratio and auxiliary-main amplitude voltage ratio equal to the windings turns ratio.

The instantaneous torque of a two-phase induction motor connected to an inverter has a smaller pulsating component than the capacitor-run motor.
The inverter-fed system can be adapted to every type of single- and two-phase induction motor (split-phase, capacitor-start, capacitor-run motors or servomotors).

Mircea Popescu wrote the paper and performed the experiments. He also realised the computations and structured the paper. The co-authors Antero Arkkio, Elek Demeter, Dan Micu and Valentin Navrapescu contributed to the paper with valuable comments and observations. Dan Micu developed the two-phase inverter and made available some experimental data.

Publication P2

M. Popescu, T. Jokinen, E. Demeter, V. Navrapescu - "Modeling and Analysis of a Two-Phase Induction Machine with Non-Orthogonal Stator Windings" – in Conference Records of IEEE IEMDC'99, May 1999, Seattle, USA, pp.389-391 – ISBN – 0-7803-5293-9/99

This work applies the stationary reference frame to the modelling and analysis of the torque behaviour of a two-phase unsymmetrical induction machine with a shifted auxiliary winding. By comparing the outcome of computer simulation with experimental results, the influence of the electrical auxiliary phase stator shift over the torque and currents of the machine is studied. The analysis is made for steady-state and transient-state operation. The torque-slip characteristics can be manipulated by shifting the auxiliary winding from the quadrature position.

By including a time-domain equivalent circuit, symmetrical components, or multiple reference frames, the analysis of the single- or two-phase induction machine with a shifted auxiliary winding was made. The first two methods are acceptable for analyzing the steady-state performance of the machine, while the third method involves a double number of equations and variables in order to describe the dynamic performance of the TPIM.

The mutual flux-linkage corresponding to the stator windings, due to the electrical shift angle, determines different variations of the currents. When the shift-angle is negative, the current variation shape is more elliptical. This phenomenon explains the higher pulsating torque and magnetic noise. The positive value of the shift-angle determines the evolution of the two axis currents toward a quasi-balanced operation of the TPIM.

The dynamic behaviour of a TPIM with non-orthogonal stator windings, supplied from variable voltage-variable frequency devices, may be analysed as well. The same conclusions are valid for this situation: 1) higher break-down torque and torque at rated speed, lower pulsating and starting torque are achieved for some positive values of the electrical shifted angle; 2) higher starting and pulsating torque, lower torque at rated speed and break-down torque are achieved for some negative values of the electrical shifted angle.

Mircea Popescu wrote this paper on the basis based on his original ideas on modelling this motor configuration. He also performed the computations and experiments. The co-authors, Professor Tapani

Jokinen, Elek Demeter, and Valentin Navrapescu contributed to the paper with valuable observations and corrections.

Publication P3

M. Popescu, P. Lampola, T. Jokinen, A. Arkkio, V. Navrapescu, E. Demeter "Analysis of the Electrical Shift Angle Influence over a Variable Speed Two-Phase Induction Motor Drive" in Conference Records EPE’99, Sept. 1999, Lausanne, Switzerland

There are several options, including the spatial placement of the auxiliary winding, available for the development of the two-phase induction machine (TPIM). The torque-slip characteristics can be manipulated by shifting the auxiliary winding from the quadrature position. For the analysed model, the machine stator windings are realized for three different values of the electrical shifted angle. A two-phase pulse width modulation (PWM) inverter with full logic control is used as a variable frequency supply. A new strategy for the controller ratio of the PWM inverter is proposed. The analysis is made for steady-state and transient-state operation. A two-phase induction motor supplied from a PWM inverter, can show improved performance when a) the system of the non-orthogonal stator windings is used; b) a new strategy relation for the controller ratio of the PWM inverter is implemented.

The influence of the sense and value of the electrical shift angle between the stator windings, over the two-phase induction machine’s torque behaviour supplied from variable voltage-variable frequency devices (type PWM inverter), is described by the following remarks:

1) higher starting, break-down and rated torque, lower pulsating torque if the positive value of the electrical shifted angle between the stator windings (opposite to the rotation sense) is realised;
2) higher pulsating torque, lower starting, rated and break-down torque if negative value of the electrical shifted angle (toward the rotation sense) is used;
3) the auxiliary voltage amplitude is modified by phase difference angle control;
4) the phase voltage amplitude ratio has to be identical with the winding turns ratio, and the phase-difference angle value will depend on the electrical shift angle between the stator windings.

Mircea Popescu has written the paper on the basis of his original ideas gained through studying this motor configuration. He also performed the computations and experiments. The co-authors Professor Tapani Jokinen, Petri Lampola, Antero Arkkio, Elek Demeter, and Professor Valentin Navrapescu contributed to the paper with valuable comments and suggestions during the work.

Publication P4

M. Popescu, V. Navrapescu – "Modelling in stationary frame reference of single and two-phase induction machines including the effect of iron loss and magnetising flux saturation" – Conference Records of ICEM 2000, 28-30 August, Espoo, Finland, Vol. I, pp. 407-411 – ISBN – 951-22-5097-7

A general model for simulating the operation of single- and two-phase induction machines is developed. The objective of this model is to predict motor performance parameters such as torque, and motor speed. The model includes representation of both main and auxiliary winding in a stationary reference frame and also the effect of core losses and the saturation of the main flux paths. A notable feature of the model is the technique used for representing the different level of saturation in the both axes of magnetisation. The developed model is suitable for simulating and modelling the steady-state and transient operation of the single and two-phase induction machines more accurately.

An equivalent circuit for the single- and two-phase induction machines including simultaneously, but as independent phenomena the iron loss and main flux saturation, is proposed. This model permits an improvement in operation of vector-controlled single and two-phase induction machines. The vector control structure suitable for unsymmetrical two-phase induction machine may be chosen from rotor or stator flux oriented control. The model can be implemented for simulations by using either experimentally determined parameters or FEM calculated parameters.

The stator field oriented system can be readily implemented on the same DSP board with the rotor field control version, as the same parameters are used. It is expected that the stator flux control would prove more suitable for low-speed applications; the rotor flux control system is useful for high-speed applications.

Mircea Popescu has written the paper on the basis of his original ideas on modelling this motor configuration. He also performed the computations and experiments. The co-author Professor Valentin Navrapescu suggested the DSP configuration and contributed to the paper with valuable advice during the work.

Publication P5

M. Popescu, T.J.E. Miller, M.I. McGilp, G. Strappazzon, N. Trivillin, R. Santarossa. –"Asynchronous performance analysis of a single-phase capacitor-start, capacitor-run permanent magnet motor"- IEEE – Transactions on Energy Conversion – accepted for publication

A detailed analysis of the asynchronous torque components (average cage, magnet braking torque and pulsating) for a single-phase capacitor-start, capacitor-run permanent magnet motor is presented. The computed envelope of pulsating torque superimposed over the average electromagnetic torque leads to an accurate prediction of starting torque. The developed approach is realized by means of a combination of symmetrical components and *d-q* axes theory and can be extended for any *m*-phase AC motor – induction, synchronous reluctance or synchronous permanent magnet. The resultant average electromagnetic torque is determined by superimposing the asynchronous torques and magnet braking torque effects.

The asynchronous performance prediction for a line start permanent magnet motor can be made, assuming that asynchronous cage torques and magnet braking torque effects can be superimposed. Obviously, the superposition principle will neglect the circuit cross-couplings. Important information about the motor torque capability is obtained through the study of different torque components. The deduced torque expressions may be extended for the general case of the *m*-phase AC motor, supplied with unbalanced stator voltage.

Mircea Popescu has written the paper on the basis of his original ideas on modelling this motor configuration. He also performed all the computations. The co-author Professor TJE Miller suggested the symmetrical components approach and contributed to the paper with his valuable advice during the work. Giovanni Strappazzon, Nicola Trivillin and Roberto Santarossa provided the test data and contributed with important suggestions during the work. Malcolm McGilp contributed to the paper with his valuable comments and help in performing the computations.

Publication P6

M. Popescu, T.J.E. Miller, M.I. McGilp, G. Strappazzon, N. Trivillin, R. Santarossa. –"Line-start permanent magnet motor: single-phase starting performance analysis" IEEE Transactions on Industry Applications, Vol. 39, No. 4, pp. 1021-1030, July/August 2003

Permanent magnet motors, equipped with a rotor-cage, may represent a higher-efficiency alternative to induction motors. Generally defined as line-start permanent magnet motors (LSPM), they may be supplied from a three- or single-phase voltage system.

LSPM motors run synchronously so that the cage rotor losses are minimized at nominal load. The capacitor-start, capacitor-run permanent magnet motor is the single-phase version of the LSPM. This special electric motor is suited for applications in home appliances, such as refrigerator compressors.

Beneficially for steady-state operation, permanent magnets considerably affect the starting capabilities of such motors. The torque oscillations, during the starting transient, are much higher than for an induction motor.

A detailed calculation of different torque components (average and pulsating) for a single-phase capacitor-run permanent magnet motor permits a correct estimation of motor performance. It extends the existing analysis made for a single-phase unsymmetrical or three-phase symmetrical permanent magnet motor. The analysis focuses on a single-phase capacitor-start, capacitor-run, 50 Hz two-pole motor with concentric windings

The starting performance prediction for a line-start permanent magnet motor can be made using a quasi steady-state analysis. The motor torque behaviour during asynchronous operation can be calculated through the study of different torque components such as cage torques, magnet braking torque, pulsating torques. The deduced torque expressions may be extended for the general case of the *m*-phase AC motor, supplied with unbalanced stator voltage.

Mircea Popescu has written the paper on the basis of his original ideas on modelling this motor configuration. He also performed the computations. The co-author Professor TJE Miller suggested the symmetrical components approach and contributed to the paper with valuable advice during the work. Giovanni Strappazzon, Nicola Trivillin and Roberto Santarossa provided the test data and contributed with important suggestions during the work. Malcolm McGilp contributed to the paper with his valuable comments and help in performing the computations.

Publication P7

M. Popescu, D.M. Ionel, S. Dellinger, T.J.E. Miller, M.I. McGilp – "Analysis and design of a two-speed single-phase induction motor with 2 and 18 pole Special Windings"- IEEE – Transactions on Energy Conversion – accepted for publication

The motor presented employs multiple independent windings for operation with two very different pole numbers. The 18-pole field is produced with a symmetrical three-phase winding connected in a Steinmetz arrangement to a single-phase supply. A unified analysis method has been developed and used to demonstrate the equivalence of a Steinmetz delta or star connection with a main and auxiliary winding of a single-phase motor. The method has been experimentally validated and some specific motor design considerations are also included.

The method developed for motor analysis allows the unified treatment of induction motors with a delta or star Steinmetz connection supplied from a single-phase voltage source. The overall agreement between calculated and measured performance can be considered as satisfactory for a first test of the theory developed. On the basis of the mathematically proven equivalence between the Steinmetz connection and a main and an auxiliary winding, the method can be implemented straightforwardly into existent induction motor design software; the motor can be optimised using single-phase motor procedures. Some of the most important peculiarities of a motor with a Steinmetz connection have also been discussed and some design recommendations have been given.

Mircea Popescu and Dan Ionel have written the paper on the basis of their ideas of analysing this motor configuration. Mircea Popescu also performed all the computations. Steve Dellinger provided the test data and contributed with important suggestions during the work. The co-author Professor TJE Miller suggested the symmetrical components approach and contributed to the paper with valuable advice during the work. Malcolm McGilp contributed to the paper with valuable comments and help in performing the computations.

Publication P8

M. Popescu, T.J.E. Miller, M.I. McGilp, G. Strappazzon, N. Trivillin, R. Santarossa. –"Torque behaviour of 1-phase permanent magnet AC motor"- Conference Record IEEE IEMDC'03, June 2003, Madison, USA, pp. 820-825-ISBN –0-7803-7817-2/03

A detailed comparative study of two starting and running methods for a single-phase permanent magnet synchronous motor, equipped with a squirrel-cage rotor is presented. The analysis of the motor performance is performed for a pulse-width modulated (PWM) inverter-fed motor and for a capacitor-start, capacitor-run motor. The developed approach may be extended to any 1-phase AC motor – induction, synchronous reluctance or synchronous permanent magnet.

This paper investigates two starting configurations for a single-phase permanent magnet AC motor: a capacitor-start, capacitor-run motor and a PWM inverter supplied motor.

The subject of the analysis for the single-phase starting performance is a single-phase permanent magnet AC motor, 50 Hz, two-pole motor with concentric windings. The rotor consists of an aluminium rotor cage, with interior ferrite magnets.

The torque behaviour prediction for a single-phase permanent magnet AC motor can be made for asynchronous operation using the quasi steady-state plus the dynamic analysis and, for synchronous operation, using the steady-state analysis. Measurements and simulations demonstrate that both starting methods have advantages and drawbacks.

The electromagnetic torque per current during asynchronous operation exhibits similar values in both analysed cases. However, for a high-load torque application, the capacitor motor solution is preferable, while, for a low-load torque application, the PWM inverter-fed solution seems to be more advantageous.

At synchronous-speed operation, the PWM inverter-fed solution leads to a torque vs. current ratio value that is not achievable with a capacitor motor. Torque oscillations are minimised for the PWM inverter-fed motor solution as the machine always runs at synchronous speed as a 2-phase balanced motor.

Mircea Popescu has written the paper on the basis of his original ideas on modelling this motor configuration. He also performed the computations. The co-author Professor TJE Miller suggested the symmetrical components approach and contributed to the paper with valuable advice during the work. Giovanni Strappazzon, Nicola Trivillin and Roberto Santarossa provided the test data and contributed with important suggestions during the work. Malcolm McGilp contributed to the paper with valuable comments and help in performing the computations.

2 OVERVIEW OF THE ELECTROMAGNETIC TORQUE PREDICTION IN SINGLE-PHASE AND TWO-PHASE AC MOTORS

A short summary of the basic theory behind the electromagnetic torque and analytical and numerical methods most significant to the topic are presented in this chapter.
The purpose of the following literature review is to highlight the major lines of research and achievements concerning the topic.

2.1 Analytical modelling

General model for single- and two-phase induction motor

The general expression for the electromagnetic torque in a single-phase induction motor is based on the following simplifying assumptions:

(a) Magnetic axes of the stator windings with sinusoidal spatial distribution are electrically orthogonal. This assumption is usual with respect to these motor configurations. The two-phase system can actually be considered as a part of a four-phase system. The 90 electrical degree shift-angle between magnetic axes of the windings ensures that there is no coupling effect between windings; consequently no mutual inductance between stator windings will occur. The effect of the non-orthogonal magnetic axes of the stator windings on motor performance is analysed in P2 and P3 of this study.

(b) Only the fundamental space harmonic component of the air-gap flux distribution is considered. This is a reasonable assumption as, in practice the single-phase induction motors are generally equipped with sinusoidal distributed windings on the stator (e.g., refrigerator compressor motors, washing machine motors). The main purpose is to minimise the space harmonics effect that could worsen motor performance (torque capability, vibration, noise).

(c) Magnetic-saturation effects, core-loss and stray load losses are negligible. Single- and two-phase induction motors may operate under strong saturation conditions, with stator teeth and yoke flux-density peak values over 2 T. A simple and efficient way to include the saturation effect is to use the saturated values of the leakage and magnetization inductances. From the loss point of view, the tested motors for this study showed that the losses were dominated by the copper losses in the stator winding and cage rotor.

(d) Skin-effect (i.e., deep-bar) in the rotor will be ignored. This assumption is typically valid in small induction machines. It is further justified by the fact that under general conditions, the single-phase induction motor will be operating at low slips; hence the rotor currents will vary with frequencies sufficiently low that skin-effects are insignificant.

(e) In steady-state operation, the voltages and currents are sinusoidal. This assumption allows the use of the phasors and space vectors for the motor analysis. Note that when the motor is PWM inverter fed a harmonic analysis is recommended as in P1.

(f) The machine rotor is described by two identical magnetically orthogonal windings. As the cage rotor is realised with bars that exhibit identical area and shape, the rotor resistances in two-axis models can be taken as equal to the phase rotor resistance referred to the stator windings. Obviously, this simplification does not allow the diagnosis of any rotor fault (e.g., broken rotor bars).

The electromagnetic torque of the single-phase induction machine can be determined from the generalised forces law

$$T_{\mathrm{e}} = -\left.\frac{\partial W_{\mathrm{en.}}}{\partial \theta}\right|_{\phi=\mathrm{ct}} = \left.\frac{\partial W_{\mathrm{coen}}}{\partial \theta}\right|_{i=\mathrm{ct}} \tag{2.1}$$

To account for a P pole machine we have to convert the actual angular rotor displacement (θ) to electrical angular displacement (θ_{r}):

$$T_{\mathrm{e}} = -\left(\frac{P}{2}\right)\left.\frac{\partial W_{\mathrm{en.}}}{\partial \theta_{\mathrm{r}}}\right|_{\phi=\mathrm{ct}} = \left(\frac{P}{2}\right)\left.\frac{\partial W_{\mathrm{coen}}}{\partial \theta_{\mathrm{r}}}\right|_{i=\mathrm{ct}} \tag{2.2}$$

Evaluation of the energy stored in the coupling field can be expressed as the sum of products between the self-inductances and the currents in the two windings coupled by the mutual inductance. Thus, the energy stored in the coupling field may be written:

$$\begin{aligned} W_{\mathrm{en}} &= \frac{1}{2}\left(\mathbf{i}_{\mathrm{abs}}\right)^{\mathrm{T}}\left(\mathbf{L}_{\mathrm{s}} - \mathbf{L}_{\mathrm{ls}}\right)\mathbf{i}_{\mathrm{abs}} + \left(\mathbf{i}_{\mathrm{abs}}\right)^{\mathrm{T}}\mathbf{L}_{\mathrm{sr}}\mathbf{i}_{\mathrm{abr}} \\ &\quad + \frac{1}{2}\left(\mathbf{i}_{\mathrm{abr}}\right)^{\mathrm{T}}\left(\mathbf{L}_{\mathrm{r}} - L_{\mathrm{lr}}\mathbf{I}\right)\mathbf{i}_{\mathrm{abr}} \end{aligned} \tag{2.3}$$

where **I** is the unity matrix and

$$\mathbf{L}_{\mathrm{s}} = \begin{bmatrix} L_{\mathrm{las}} + L_{\mathrm{mas}} & 0 \\ 0 & L_{\mathrm{lbs}} + L_{\mathrm{mbs}} \end{bmatrix} \tag{2.4}$$

$$\mathbf{L}_{\mathrm{r}} = \begin{bmatrix} L_{\mathrm{lar}} + L_{\mathrm{mar}} & 0 \\ 0 & L_{\mathrm{lbr}} + L_{\mathrm{mbr}} \end{bmatrix} \tag{2.5}$$

$$\mathbf{L}_{\mathrm{sr}} = \begin{bmatrix} L_{\mathrm{sar}}\cos\theta_{\mathrm{r}} & -L_{\mathrm{sar}}\sin\theta_{\mathrm{r}} \\ L_{\mathrm{sbr}}\sin\theta_{\mathrm{r}} & L_{\mathrm{sbr}}\cos\theta_{\mathrm{r}} \end{bmatrix} \tag{2.6}$$

$$\mathbf{L}_{\mathrm{ls}} = \begin{bmatrix} L_{\mathrm{la}} & 0 \\ 0 & L_{\mathrm{lb}} \end{bmatrix} \tag{2.7}$$

In (2.4), L_{las} and L_{mas} (L_{lbs} and L_{mbs}) are, respectively, the leakage and magnetizating inductances of the *as* winding (*bs* winding). In the case of the identical rotor bars, L_{lar} and L_{lbr} are the rotor leakage inductances referred to the stator windings (*as*, *bs*). In (2.6), L_{sar}, L_{sbr} are the amplitude of the mutual inductance between the *as* winding (*bs* winding) and the rotor windings.

The expression for electromagnetic torque may be obtained as:

$$T_{\mathrm{e}} = \left(\frac{P}{2}\right)\left(\mathbf{i}_{\mathrm{abr}}\right)^{\mathrm{T}}\frac{\partial}{\partial \theta_{\mathrm{r}}}\left[\left(\mathbf{L}_{\mathrm{sr}}\right)^{\mathrm{T}}\right]\mathbf{i}_{\mathrm{abs}} \tag{2.8}$$

In expanded form, (2.8), which is positive for motor action becomes:

$$T_{\mathrm{e}} = \frac{P}{2}\left[L_{\mathrm{sar}} i_{\mathrm{as}} \left(-i_{\mathrm{ar}} \sin\theta_{\mathrm{r}} - i_{\mathrm{br}} \cos\theta_{\mathrm{r}}\right) + L_{\mathrm{sbr}} i_{\mathrm{bs}} \left(i_{\mathrm{ar}} \cos\theta_{\mathrm{r}} - i_{\mathrm{br}} \sin\theta_{\mathrm{r}}\right)\right] \tag{2.9}$$

The electromagnetic torque relation can be simplified and presented in the following form, which depends on the motor currents and magnetizing inductance if we assume an identical stator winding distribution. Thus, the ratio between the magnetizing inductances in *as* and *bs* windings is equal to the square value of the turns ratio. Consequently, if we use rotor variables referred to the stator windings:

$$\begin{aligned} L'_{\mathrm{sar}} &= L_{\mathrm{mas}} \\ L'_{\mathrm{sbr}} &= L_{\mathrm{mbs}} = k^2 L_{\mathrm{mas}} \end{aligned} \tag{2.10}$$

and

$$T_{\mathrm{e}} = -\left(\frac{P}{2}\right) L_{\mathrm{mas}} \left[\left(i_{\mathrm{as}} i'_{\mathrm{ar}} + i_{\mathrm{bs}} i'_{\mathrm{br}}\right) \sin\theta_{\mathrm{r}} + \left(i_{\mathrm{as}} i'_{\mathrm{br}} - i_{\mathrm{bs}} i'_{\mathrm{ar}}\right) \cos\theta_{\mathrm{r}}\right] \tag{2.11}$$

where

i_{as}, i_{bs}	instantaneous stator currents in phases *as*, *bs*
i'_{br}, i'_{ar}	instantaneous rotor currents referred to the stator in phases *ar'*, *br'*

If the instantaneous currents are considered to have a sinusoidal variation in time, this relation shows that there are two components of the instantaneous electromagnetic torque: an average component with constant value for a given value of the rotor speed and a pulsating component with a frequency double the currents frequency (ω_{e}). The pulsating torque component determines an important magnetic noise for the single-phase induction machine compared to the three-phase induction machine.

The mechanical equation that links the torque and the rotor speed is:

$$T_{\mathrm{e}} = J\frac{2}{P}\frac{d\omega_{\mathrm{r}}}{dt} + B_{\mathrm{m}}\frac{2}{P}\omega_{\mathrm{r}} + T_{\mathrm{L}}, \tag{2.12}$$

where J is the rotor inertia, B the viscous friction coefficient associated with the rotational system of the machine and with the mechanical load, and P the number of poles of the analysed machine.

Analysis of the single-phase induction motor in a stationary reference frame

In order to eliminate the time dependence of the voltage and flux equations terms, a transformation of variables into a new reference frame is necessary. Reference frames linked to stator, rotor or arbitrarily fixed can be chosen for the poly-phase induction machine. As for the single-phase induction machine, the stator windings are usually not identical; the only transformation that maintains the winding parameters (resistance, inductance) unchanged is the stationary reference frame transformation.

In reference frame systems theory, we have to note that the parameters do not depend on the relative position between the stator and the rotor. It is important to observe that stator windings of the single-phase induction machine physically represent the *d-q* co-ordinates windings, as distinct from the three-phase induction machine where the two-axis co-ordinate equivalent windings are fictitious.

Symmetrical single- and two-phase induction motors

In a wide range of applications, the single-phase induction machine is equipped with a cage rotor. During the steady-state operation, the stator currents vary with the stator voltage frequency ω, while the rotor currents vary with the slip frequency $\omega - \omega_r$.

The instantaneous electromagnetic torque is given by:

$$T_e = \left(\frac{P}{2}\right) L_{mas}\left(i_{QS} i'_{DR} - i_{DS} i'_{QR}\right) = \left(\frac{P}{2}\right)\left(\frac{X_m}{\omega_b}\right)\left(i_{QS} i'_{DR} - i_{DS} i'_{QR}\right) \quad (2.13)$$

where the index *DS*, *QS* refer to the stator currents and *DR'*, *QR'* to the referred rotor currents.

The steady-state electromagnetic torque equation is:

$$T_e = 2\left(\frac{P}{2}\right)\frac{X_m}{\omega_b}\mathrm{Re}\left[\mathrm{j}\underline{I}^*_{as}\underline{I}_{ar}\right] = \frac{2(P/2)(X_m^2/\omega_b)(R'_r/s)\left|\underline{I}_{as}\right|^2}{(R'_r/s)^2 + X'^2_{rr}} \quad (2.14)$$

$$T_e = \frac{2(P/2)(X_m^2/\omega_b)(R'_r\ s)\left|\underline{V}_{as}\right|^2}{\left[R_s R'_r + s\left(X_m^2 - X_{ss}X'_{rr}\right)\right]^2 + \left(R_r X_{ss} + R_s X'_{rr}\right)^2} \quad (2.15)$$

It is important to emphasise that the positive values of the torque are obtainable when the slip *s* is positive (*motor* operation) and the negative values when the slip *s* is negative (*generator* operation).

By setting the torque/slip derivative equal to zero, the relation for the critical slip (s_m) can be obtained:

$$s_m = R'_r\, G$$
$$G = \pm\sqrt{\frac{R_s^2 + X_{ss}^2}{(X_m^2 - X_{ss}X'_{rr})^2 + R_s^2 X'^2_{rr}}} \quad (2.16)\text{-}(2.17)$$

The positive value corresponds to *motor* operation, and the negative one to *generator* operation. If at start-up (s = 1) the torque is directly dependent on the variation of rotor resistance, as the magnetisation reactance value is considered to be much higher than the stator or rotor resistance value, the maximum (break-down) torque value is not dependent on the rotor resistance value:

$$T_{e,max} = \frac{2(P/2)(X_m^2/\omega)G\left|\underline{V}_{as}\right|^2}{\left[R_s + G(X_m^2 - X_{ss}X'_{rr})\right]^2 + (X_{ss} + GR_s X'_{rr})^2}, \quad (2.18)$$

where, as in the previous expressions:

$X_{ss} = X_{ls} + X_m$ represents the stator self-reactance;

$X'_{\mathrm{rr}} = X'_{\mathrm{lr}} + X_{\mathrm{m}}$ represents the rotor self-reactance referred to the stator;

Note that the stator and rotor self-reactances can not be defined for an unsymmetrical single-phase induction motor and characterise only the symmetrical motor configuration: identical stator windings and balanced supply voltage.

Unsymmetrical single-phase induction motor

A new set of equations and a new equivalent circuit for the unsymmetrical single-phase induction machine can be obtained by eliminating from the initial assumptions the one referring to identical stator windings. The instantaneous electromagnetic torque equation is:

$$T_{\mathrm{e}} = \frac{P}{2}\frac{X_{\mathrm{m}}}{\omega_{\mathrm{b}}} k(i_{\mathrm{QS}} i'_{\mathrm{DR}} - i_{\mathrm{DS}} i'_{\mathrm{QR}}) \tag{2.19}$$

Before the two-axis (*d-q*) theory was well defined and validated for the single-phase induction motor analysis, there were the original analytical models that predicted motor performance in *steady-state operation* on the basis of the double-revolving field theory proposed by Morrill (1929), the cross-field theory proposed by Puuchstein and Lloyd (1942) and symmetrical components approach that was applied by Fitzgerald and Kingsley (1961). The last approach was initially proposed by Fortescue (1918) for polyphase networks, but subsequently was successfully used for electrical motor analysis. A remarkably unified presentation of all these theories and the equivalence between all theories was made by Veinott (1959). On the basis of the symmetrical components theory it is possible to analyse any single-phase induction motor with unbalanced stator windings. A particular case is the three-phase induction motor supplied from a single-phase system. This configuration was extensively analysed by Stepina (1982), Oliveira (1990) or Tozune (1991). Fuchs et al (1990) and Huang et al. (1988) use the double-revolving field theory in their design procedure for single-phase induction motors. One should note that all these theories lack the capability of predicting the instantaneous electromagnetic torque, i.e., the dynamic motor performance.

However, the *d-q* axes theory permits a unified analysis of the single-phase induction motor; the same equivalent circuit and equations system can be employed with minor modifications [derivation operator (d/d*t*) takes place of complex operator (j ω)] for a complete estimation of motor torques. Defined as a two-reaction theory for synchronous motors by Park (1929), the *d-q* axes theory was first applied to induction motors by Clarke (1943). The next step further was taken by Kovacs and Racs (1959) when they proposed a space vector theory for AC machines. Unfortunately, this theory is applicable only to symmetrical poly-phase machines, so the usually unsymmetrical single-phase AC motors cannot be analysed using this theory.

An important contribution to the single-phase AC motors analysis in two-axis theory was made by Krause et al (1965a, 1965b, 1995). They established that the only option for using the two-axis theory in the single-phase induction motors analysis is a stationary reference frame (fixed to the stator). His suggestion of using the magnetic axes of the stator windings as references for *d-q* axes was further used in multiple works. On the basis of their approach, a series of analytical models have been further developed by Xu and Novotny (1990, 1992), Correa et al (1998, 1999), Popescu et al. (2001), Umans (1996), Liu (1995). All these models propose different methods to add non-linear effects to the well-known *d-q* equivalent circuit. An original approach that has to be highlighted is the multiple reference frame (Walls and Sudhoff, 1996) theory that also originated from *d-q* axes theory. The implementation of the multiple reference frame theory makes also possible the analysis of the single-phase induction motors with shifted stator windings. Slemon (1989) proposed *d-q* axes models for 3-phase induction motors defined as gamma and inverse gamma models, according to the relative position between the leakage and magnetization reactances. These models have been modified for single-phase induction motors by the author (Popescu, 2000).

With the emerging vector control strategy (sensorless or not) in the last decades *d-q* models for induction motors have been extensively employed for the modern electric drives implementation. DeDoncker and Novotny (1988) and DeDoncker et al (1995) developed an universal field oriented controller that uses three motor models according to the selected reference frame. On the basis of this approach the author (Popescu, 2000) proposed a similar controller for the single and two-phase induction motors. In vector control strategy, discrete models of the induction motors have been developed for three-phase case (Vainio et al., 1992) or single-phase version (Popescu, 2000).

Capacitor induction motor

Placing a capacitor in series with the auxiliary winding (considered as the starting winding) has the effect of increasing the magnitude of the current component I_{DS} which in turn increases the average electromagnetic torque during the starting period. The same technique may be used to increase the starting torque with a symmetrical two-phase machine supplied from a single-phase source. With the reference frame stationary, the operation of the unsymmetrical or symmetrical two-phase induction motor with a capacitor connected in series with the auxiliary (*bs*) winding can be easily described by replacing R_a with $R_{\mathrm{a}} + R_{\mathrm{Cap}}$, where R_{Cap} is the resistance of the capacitor, and $(p/\omega_{\mathrm{b}})\, X_{\mathrm{la}}$ with $(p/\omega_{\mathrm{b}})\, X_{\mathrm{la}} + (\omega_{\mathrm{b}}/p)\, X_{\mathrm{Cap}}$ in the voltage equations.

For the steady-state voltage equations the time derivative operator p should be replaced with the complex $j\omega$. If the motor employs two capacitor values, one for starting and one for running operation, the torque behaviour can be simply modelled by switching from one capacitor value to the other according to the requested limitation criterion of either speed, current or time.

Parameters of the single-phase induction motor

A. Computation

As previously stated, the analytical computation of the electromagnetic torque in AC motors is strongly dependent on the value of the equivalent circuit parameters. The models developed in this study employ parameters referred to the stator windings; the q-axis circuit corresponds to the main stator winding and d-axis circuit corresponds to the auxiliary stator winding. Thus the stator resistances (R_{m} and R_{a}) the leakage reactances (X_{lm} and X_{la}) are computed directly using the classical theory of electrical machinery (Veinott, 1959). The resistances values include the temperature effect, whilst the leakage reactance values may include a saturation adjustment factor. These values are considered constant during computation. A separate approach is necessary for the magnetizing reactances (X_{m} and X_{a}). The main winding magnetizing reactance has a non-linear value that considers the saturation effect on the basis of the flux-linkage vs. current curve. The auxiliary winding magnetizing reactance is assumed to exhibit a different value from the one that characterises the main winding only due to the number of turns. Details about these procedures are illustrated in P4. The rotor parameters (R_{r}, X_{lr}) are directly computed for q-axis case, while for the d-axis it is necessary to use a transformation equals to the square of the effective turns ratio between stator windings (k^2R_{r}, k^2X_{lr}).

B. Experimental determination

There are several experimental methods that allow the measurements of the equivalent circuit parameters for the single- and two-phase induction motor. The most useful methods are those proposed by Suhr, 1952 (single-phase method) and Van der Merwe, 1995 or Fuchs et al 1990. (two-phase method). The first method is based on independent tests performed for each winding, whilst the second method employs tests simultaneously performed for both stator windings.

B1. Single-phase method

The algorithm for equivalent circuit parameters determination is as follows:

1. Turns ratio estimation (k): As the stator windings are usually magnetically displaced at 90 electrical degrees, when the main winding is energized and the motor runs at rated speed, in the auxiliary winding a voltage E_a is induced. In a similar way if the auxiliary winding is energized and the motor runs at rated speed in the main winding a voltage E_m is induced. The effective turns ratio can be estimated as:

$$k = \sqrt{\frac{V_m E_a}{V_a E_m}} \tag{2.20}$$

2. No-load test is individually performed for each stator winding. Using the double revolving field theory (Veinott, 1959) and assuming equal leakage reactances for stator and rotor, the following parameters can be estimated X_{lm} , X_m , R_r , X_{lr}. Suhr (1952) and Veinott (1959) estimate that the rotor resistance that has to be employed in the forward field circuit has to be 1.2 times smaller than the computed value. Similarly, the rotor resistance that has to be employed in the backward field circuit has to be 1.8 times higher than the computed value.

3. A short-circuit test is individually performed for each stator winding. The magnetizing reactance value (X_m) will be different from the value estimated from the no-load test because of the saturation effect. This saturated value is used to validate the computed value that is further implemented in the fixed-value equivalent circuit parameters.

B2. Two-phase method

This method is based on the idea of balancing the motor operation such that the backward field is zero. Figure 2.1 illustrates the principle of the supply circuit where T1 and T2 are single-phase transformers and AT represents an auto-transformer (a variac can be used).

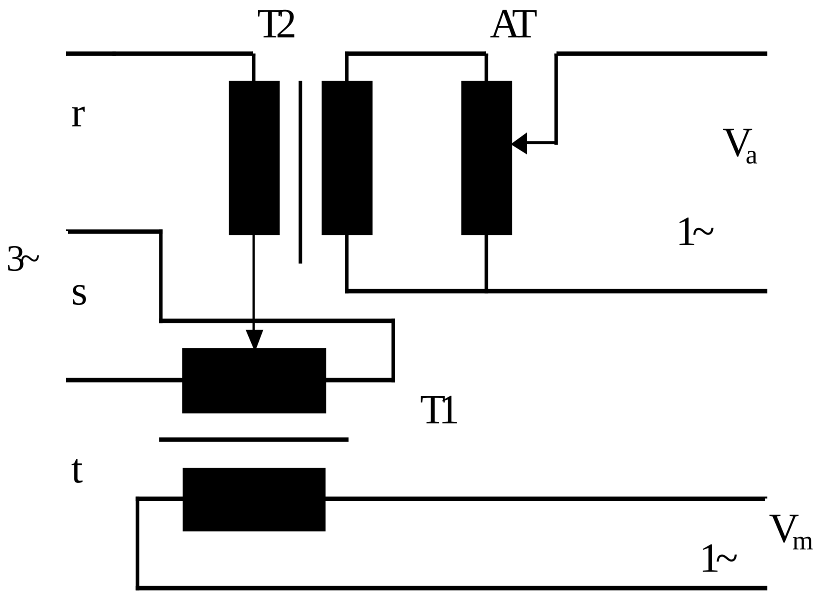

Figure 2.1 Balanced two-phase supply voltage system

The algorithm for estimating the equivalent circuit parameters using the two-phase method is as follows:

1. Stator parameters are determined with the rotor extracted from the machine. As there are no mutual fluxes between the windings, the corresponding impedances can be individually estimated. If the resistance is determined with a standard DC test, the leakage reactance can be estimated as:

$$X_{\mathrm{lm}} = \sqrt{Z_{\mathrm{m}}^2 - R_{\mathrm{m}}^2} \tag{2.21}$$

A similar method is used for the auxiliary winding.

2. Rotor parameters are determined using the short-circuit (locked rotor) test. By varying the voltage, a magnetising characteristic of the rotor leakage flux versus current can be obtained. The leakage reactance is computed accordingly as the ratio leakage flux/ current. The rotor resistance is difficult to measure. However, an accurate estimation of the rotor resistance can be made just through computation. The rotor bar and ring dimensions and resistivity are used to compute the DC value of these resistances. Then, taking inot account the skin-effect, one can obtain:

$$\begin{aligned} R_{\mathrm{r}} &= R_{\mathrm{r0}}\xi \frac{\sinh(2\xi) - \sin(2\xi)}{\cosh(2\xi) - \cos(2\xi)} \\ \xi &= k_{\mathrm{r}}\sqrt{f} = h_{\mathrm{r}}\sqrt{\frac{\pi\mu_0 f}{\rho_{\mathrm{r}}}} \end{aligned} \tag{2.22}$$

where h_{r} represents the height of the rotor bar, and ρ_{r} the bar resistivity.

3. Magnetizing characteristic. If a balanced two-phase voltage system is applied to the terminals of a single-phase induction motor with stator auxiliary winding, the phase currents waveform is sinusoidal. The resultant magnetic field will be a quasi-circular revolving field. The spatial waveform of the magnetic field is almost trapezoidal due to the saturation and the fact that it can decompose into a series of odd-order harmonics plus the fundamental. The harmonics field will induce voltages in the stator windings. Thus, even though the voltages and currents exhibit a sinusoidal variation in time the amplitude of the currents varies non-linearly with the amplitude of the voltages. Consequently, it is impossible to obtain a hysteresis loop using only one set of measured currents and voltages. However, a magnetising characteristic (λ-i) in the frequency domain can be established using the no-load tests over the balanced operated motor.
By employing sinusoidal supply voltage with different amplitudes one can determine the magnetizing characteristic (λ-i) considering that:

$$\lambda = \frac{E_{\mathrm{m}}}{\omega} \tag{2.23}$$

4. Core loss resistance. For the no-load test, the input power can be segregated by components as follows:

$$P_{\mathrm{in}} = P_{\mathrm{cu1}} + P_{\mathrm{cu2}} + P_{\mathrm{r}} + P_{\mathrm{C}} + P_{\mathrm{mec}} \tag{2.24}$$

where $P_{\mathrm{cu}1}$, $P_{\mathrm{cu}2}$ represent the copper loss in the stator windings (main and auxiliary), P_{r} represents the rotor loss caused by the backward rotating field, P_{C} is the core loss and P_{mec} represents the mechanical (windage and friction) loss. As with the three-phase machine tests, one can graphically obtain the core loss variation with square voltage. The core loss equivalent resistance will be:

$$R_{\mathrm{C}} = \frac{E_{\mathrm{m}}^2}{P_{\mathrm{C}}} \tag{2.25}$$

The stator core loss is caused by the total magnetic flux (leakage and magnetizing). If only the fundamental harmonic of the magnetic flux is considered, the magnetizing flux is much higher than the leakage flux and consequently the core loss created by the leakage flux can be ignored. The magnetic circuit of the stator magnetizing flux comprises the stator yoke and teeth, while other losses can be caused by the leakage flux (e.g., stator bridge slot leakage flux). The overall core losses will be higher if the leakage flux is considered. Also, due to the saturation the magnetizing and leakage flux present a non-linear variation to the magneto-motive force.
Due to these aspects the core loss segregation per components (hysteresis, eddy-current) is difficult to achieve, therefore the overall core loss is used in the equivalent circuits of the single- or two-phase induction motor.

The computation of the core loss created by the special and time harmonics mmf is still an unsolved problem for these motor types.

General d-q axis model for permanent magnet motor

Analysis of an *m*-phase balanced permanent magnet AC motor with salient-pole rotor is carried out using the voltage equations in the rotor reference frame under the following assumptions:

(a) Negligible spatial m.m.f. harmonics, i.e., the stator windings and rotor magnets are arranged to produce sinusoidal m.m.f. airgap distribution;

(b) The effects of stator slots may be neglected;
(c) There is no fringing of the magnet field;
(d) The magnetic field intensity is constant and radially directed across the air-gap, such that the permanent magnet may be equivalent to a super conducting winding, where the equivalent current is always constant with $i_e = E_0/X_{md}$;
(e) Eddy-current and hysteresis effects are negligible.
(f) The rotor cage is substituted with two equivalent sinusoidally distributed windings (*rd* and *rq*) with magnetic axes shifted by 90 electrical degrees.

The voltage equations are expressed through the following matrix equation:

$$\begin{bmatrix} v_d \\ v_q - \frac{\omega_r}{\omega} E_0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + \frac{p}{\omega} X_d & -\frac{\omega_r}{\omega} X_q & \frac{p}{\omega} X_{md} & -\frac{\omega_r}{\omega} X_{md} \\ \frac{\omega_r}{\omega} X_d & R_s + \frac{p}{\omega} X_q & \frac{\omega_r}{\omega} X_{mq} & \frac{p}{\omega} X_{mq} \\ \frac{p}{\omega} X_{md} & 0 & R_{rd} + \frac{p}{\omega} X_{lrd} & 0 \\ 0 & \frac{p}{\omega} X_{mq} & 0 & R_{rq} + \frac{p}{\omega} X_{lrq} \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \\ i_{rd} \\ i_{rq} \end{bmatrix} \tag{2.26}$$

The following expressions compute the instantaneous electromagnetic torque and its components:

Resultant electromagnetic:

$$\begin{aligned} T_e &= \frac{mP}{2}\left[\psi_d i_q - \psi_q i_d\right] \\ &= \frac{mP}{2\omega}\left[\left(X_{md} - X_{mq}\right) i_d\, i_q + \left(X_{md} i_{dr} i_q - X_{mq} i_{qr} i_d\right) + X_{md} i_e i_q\right] \\ &= T_{rel} + T_{cage} + T_m \end{aligned} \tag{2.27}$$

Reluctance torque that produced by torque is created by the magnetic energy variation due to the variable reluctance rotor displacement in the alternative stator flux-linkage

$$T_{rel} = \frac{m\,P}{2\omega}\left(X_{md} - X_{mq}\right) i_d\, i_q = \frac{m\,P}{2\omega}\left(X_d - X_q\right) i_d\, i_q \tag{2.28}$$

Cage torque – induction torque – that is created by the interaction between induced currents in the cage rotor and the stator flux-linkages:

$$T_{cage} = \frac{mP}{2\omega}\left(X_{md}\, i_{dr}\, i_q - X_{mq}\, i_{qr}\, i_d\right) \tag{2.29}$$

Alignment – excitation or magnet torque – that is created by the interaction between flux-linkages created by magnet rotation and stator currents:

$$T_m = \frac{mP}{2\omega} X_{md}\, i_e\, i_q = \frac{mP}{2\omega} E_0\, i_q \tag{2.30}$$

During the starting of a permanent magnet motor a specific torque component, magnet-braking torque, appears. This is produced by magnet flux generating currents in the stator windings, and is associated with the loss in the stator circuit resistance. The variation of this torque with speed follows a pattern similar to that in the induction motor, but the per-unit speed $(1 - s)$ takes the place of the slip (s). Consequently, the magnet braking torque is zero when the motor starts and negative at synchronous speed. Similarly to the break-down torque in an induction motor, the magnet braking torque exhibits a maximum value that depends on motor parameters (resistances, reactances). A detailed relation for this torque component and how it can be derived is presented in Publication 5. The magnet braking torque should not be confused with the synchronous "alignment" torque that arises at synchronous speed (2.30), even though the magnet braking torque is still present at synchronous speed and therefore diminishes the output and the efficiency.

During starting, the accelerating torque of the permanent magnet AC motor is the average cage torque minus the magnet braking torque and the load torque. The average cage torque is developed by "induction motor action", except that the saliency and the unbalanced stator voltages complicate the analysis and may compromise performance. The magnet alignment torque has a non-zero average value (i.e., averaged over one revolution or electrical cycle) only at synchronous speed. At all other speeds it contributes an oscillatory component of torque. The same is true of the reluctance torque. As the rotor approaches synchronous speed, the screening effect of the cage becomes smaller, and, as the slip is very small, the oscillatory synchronous torques (alignment and reluctance) cause large variations in speed that may impair the ability to synchronize large-inertia loads.

At synchronous speed (steady state) the time derivative terms become zero. Note that for the case of $E_0 = 0$ we can obtain the equation system for a synchronous reluctance motor. For zero excitation voltage plus the additional condition $X_d = X_q$ (no reluctance effect), the induction motor case can be obtained.

The analytical modelling of the AC permanent magnet motors is based on the original two-reaction theory developed by Park (1929) for the synchronous machine. Chang (1951) realized early approaches on the single-phase synchronous reluctance motor. His work is limited to the steady-state operation, but his choice of reference frame and symmetrical components were essentially followed in Miller's work (1985) on the single-phase permanent magnet motor. The steady-state operation of synchronous permanent magnet motors in their three-phase version was analysed using the two-axis theory by Honsinger (1980), Miller (1989), Hendershot and Miller (1994), Sebastian et al. (1986) or Slemon (1994).

Different methods of considering the core loss effect were proposed by Honsinger (1980), by connecting a fixed value resistor in parallel with the magnetizing reactances, Sebastian et al. (1986), by connecting the same resistor between the stator resistance and the stator leakage reactance, and Consoli and Racitti (1991) by employing two resistors, one of which would model the voltage effect while the other would model the stator current effect.

The asynchronous starting performance of synchronous motors (with or without permanent magnet excitation) for three-phase motors was modelled using the same two-axis theory by Adkins (1957), Honsinger (1980), Miller (1984), Rahman and Osheiba (1990), Soulard and Nee (2000) and Nee et al. (2000). The single-phase version received less attention and, until recently, only the single-phase synchronous reluctance motor was analysed, by Finch and Lawrenson (1979). Their analysis is made using the actual phase-variables, but without a complete mathematical model of the problem. The only attempts to analytically model the asynchronous performance of the line-start single-phase permanent magnet motor are made in publications 5 and 6 of this work, or by Chalmers et al (1995), or Zhou and Tseng (2002). Chalmers et al. give little in support of their test data and do not present a complete set of equations to determine the torque behaviour. Their work claims to use the actual phase-variables instead of a *d-q* axes model. Zhou and Tseng give a much more detailed analytical model formulated in *d-q* axes, but their work lacks the analysis of the torque components and a justifiable methodology showing how they derived their equations.

The synchronous operation of the single-phase permanent magnet motors was presented in only a few papers, which use the same *d*-*q* axes theory combined with symmetrical components theory approach (Miller 1985, Chaudhari and Fernandes 1999, Boldea et al. 1999). Miller's work (1985) is considered to be the first attempt to analyse and validate by test data the single-phase line-start permanent magnet motor when operating at synchronous speed. Chaudhari and Fernandes (1999) give just a theoretical model that could be applied to the motor analysis. Boldea (1999) developed a unified analytical tool dedicated to single-phase AC motors with auxiliary capacitive impedance.

As all the above analytical models for AC motors rely on the accuracy of estimating the equivalent circuit parameters, I have to mention some important contributions in this respect. Miyashita et al. (1980) proposed an original method of estimating *d*-*q* axes synchronous inductance L_d and L_q by using a rotor reluctance network corresponding to different magnetic flux paths, where saturation is prone to occur. Honsinger (1982) used simple equivalent magnetic circuits to estimate the permanent magnet motor reactances. Nee et al. (2000) developed a simple method of estimating the same parameters on the basis of no-load and load test data. Schiferl and Lipo (1989) demonstrated that interior permanent magnet motors exhibit a specific pattern for the core loss variation and proposed a harmonics analysis of the air-gap flux density to estimate the stator iron loss. Different calculation methods by analytical methods or finite-element analysis were proposed by Yamamoto et al. (1999). One should note that all the above methods refer only to the three-phase version of the synchronous permanent magnet motor. This work has used the same approaches to estimate the single-phase motor parameters. A special procedure for the single-phase permanent magnet motors with unsymmetrical stator windings would still be desirable.

2.2 Numerical modelling

Maxwell stress method

Methods based on Maxwell's stress tensor are commonly used in the computation of forces and/or torques for electromagnetic devices when numerical modelling (e.g., the finite element method) is employed. The electromagnetic torque is obtained as a surface integral

$$T_\mathrm{e} = l\iint \frac{r}{2}\left[\mathbf{H}(\mathbf{B}\cdot\mathbf{n})+\mathbf{B}(\mathbf{H}\cdot\mathbf{n})-(\mathbf{H}\cdot\mathbf{B})\mathbf{n}\right]ds \tag{2.31}$$

where l is the active length of the machine and $\mathbf{n}$ denotes the direction normal to the surface at the point of interest. The dot sign stands for scalar product.

When (2.16) is used for the calculation of the torque of a rotating electrical motor, a closed integration surface that surrounds the rotor in free space must be chosen. In a 2D model the surface integral is reduced to a line integral along the air-gap. If a circle of radius r is taken as the integration path, the relation for computing the torque becomes

$$T_\mathrm{e} = \frac{l}{\mu_0}\int_0^{2\pi} r^2 B_r B_\theta d\theta \tag{2.32}$$

where B_r and B_θ are the radial and tangential components of the flux density $\mathbf{B}$.

In the two-dimensional model of an electrical machine, the three-dimensional surface integral in (2.16) may be replaced by a volume integral over a hollow shell in free space surrounding the moving rotor. As the true torque is independent of the radius, by integrating the expression (2.18) in the radial direction over the air-gap we obtain the following expression

$$T_\mathrm{e}(r_\mathrm{s}-r_\mathrm{r}) = \int_{r_\mathrm{r}}^{r_\mathrm{s}} T_\mathrm{e}\, dr = \frac{1}{\mu_0}\int_{r_\mathrm{r}}^{r_\mathrm{s}}\left\{\int_0^{2\pi} rB_r B_\theta\, d\theta\right\} dr = \frac{1}{\mu_0}\int_{S_\mathrm{airgap}} rB_r B_\theta\, dS \tag{2.33}$$

where r_s and r_r are the outer and inner radii of the air-gap respectively and S_airgap is the cross sectional area of the air-gap. From the equations above the torque is obtained as an integral over the air-gap.

$$T_\mathrm{e} = \frac{1}{\mu_0(r_\mathrm{s}-r_\mathrm{r})}\int_{S_\mathrm{airgap}} rB_r B_\theta dS \tag{2.34}$$

As demonstrated by Arkkio (1987) expression (2.34) for torque computation in induction motors has been proven to give more reliable results than expression (2.32). However, the method has limitations when applied to predict the electromagnetic torque variation with rotor position for surface mounted permanent magnet motors. Sadowski et al (1992) showed that the results depend on the rotor step value.

Energy functional method

The method of energy functional derivative is particularly well suited for finite element analysis, since the starting point in finite element analysis is often the minimisation of an expression for stored magnetic energy. Since the stored magnetic energy is a global quantity, it is less sensitive to local errors (like Maxwell's stress method), which may be due to poor meshing or local round off. The force (or torque) that characterizes a moving object may be found as a derivative of the *magnetic energy* with respect to position at constant flux linkages or the derivative of *magnetic co-energy* with respect to position at constant current, expression (2.1).

The two approaches that are mostly employed to compute the electromagnetic torque using the energy or coenergy derivative are that developed by Coulomb 1983, which determines the local functional energy derivative for each rotor position, and another that uses two consecutive rotor positions to estimate the energy or coenergy derivative. The second method has two major drawbacks: firstly, two solutions are necessary whereas with other methods of force or torque computation (such as Maxwell's stress tensor) the force (or torque) can be computed directly from the solution; secondly, it may be necessary to adopt a trial-and-error procedure to select a suitable value of the position increment, $\Delta\theta$. If $\Delta\theta$ is too small, the energy (or co-energy) variation will be insufficient to over-come round off error. If $\Delta\theta$ is too large, the calculated torque (force) value will no longer be accurate for the specified position.

An alternative to the virtual work method based on *mean and difference potentials* variation as proposed by Hamdi et al (1993). The method is applicable if the system is linear and it was tested for cogging torque computation in a surface mounted magnet motor. This leads to important limitations of the method when analysing interior permanent magnet motors or induction motors that exhibit an important saturation level. Rather than working with two boundary value problems in A_1 and A_2, it is possible to define two new problems. The first is obtained by taking the average between two solutions and the second by taking the difference between two consecutive solutions. The electromagnetic torque expression will be

$$T_\mathrm{e} = -l \cdot \frac{\int_S B_\mathrm{d} H_\mathrm{m} dS + \int_S B_\mathrm{m} H_\mathrm{d} dS}{\Delta\theta} \tag{2.35}$$

Instead of solving two original boundary value problems, with very similar solutions and then trying to determine their difference through subtraction, the energy difference can be evaluated by solving boundary value problems for the mean and deviation potentials and flux densities respectively. The last two boundary-value problems are quite different from each other and the calculation technique may be expected to be numerically stable.

Magnetizing currents method

The magnetizing current method was proposed by Kabashima et al. (1988) and is applicable for a large range of electromagnetic devices that have a moveable part (e.g., rotor) without a solenation distribution. This means that for induction machines the implementation of this method is not recommended. The main advantages of the magnetization current method are: the field computation is required only once and there is no need of integration as in Maxwell's stress tensor or virtual work methods.

The initial assumption is that $\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M}$ holds true regardless of whether the material is iron or permanent magnet. For first-order elements, **B** and **M** are constant within the elements. Thus a magnetizing current that exists between two elements with a common border-line may be defined

$$I_\mathrm{m} = \int \mathbf{J}_\mathrm{m} dS = \frac{1}{\mu_0} \int \mathrm{rot}\,\mathbf{M}\, dS = \frac{1}{\mu_0}\left(M_{1\mathrm{t}} - M_{2\mathrm{t}}\right) l \tag{2.36}$$

According to Ampere's law, the tangential components of magnetic fields **H** on the border line are equal and the current I_m on the line ij with length l_{ij} between element *i* and *j* becomes

$$I_{\mathrm{m}} = \frac{1}{\mu_0}\left(B_{1\theta} - B_{2\theta}\right) l_{ij} \tag{2.37}$$

In a rotating motor, if the normal and tangential components of the flux density are used, we may obtain the normal and tangential components of the force. For a rotating electrical machine, the tangential force will determine the rotational torque, whilst the normal force will determine only the vibrating forces in the air-gap. The total torque can be obtained by adding all torques acting on the element lines in the region that separates rotor and air:

$$T_{\mathrm{e}} = \sum_{i=1}^{N_e}\sum_{j=1}^{N_e} I_{\mathrm{m}(ij)} B_{r(ij)} lr = \frac{r \cdot l}{\mu_0} \cdot \sum_{i=1}^{N_e}\sum_{j=1}^{N_e}\left(B_{i\theta} - B_{j\theta}\right) B_{r(ij)} l_{ij} \tag{2.38}$$

The use of numerical methods in electromagnetic torque prediction for single-phase AC motors was exercised only several times and mainly in the last decade. This is due to the uncertainty regarding the rotor bars currents magnitude and distribution when an AC motor is supplied with unbalanced supply voltage. The presence of the auxiliary impedance in series with the stator auxiliary winding complicates even further the numerical analysis of a single-phase AC motor. Successful attempts to implement a numerical method (i.e., finite element method) have been reported by Carlson et al (1994) for single-phase line-start permanent magnet motor. They have used coupled circuits in their time-stepping analysis of the motor. A similar method was proposed by Knight and Salmon (1999) or Williamson and Knight (1999). One should note that both works consider the permanent magnets effect as superimposed to the stator currents effect in a stator reference frame, which can hardly be acceptable if the rotor incremental step is much higher than the distance between two adjacent nodes in the moving air-gap regions. Knight and Salmon reported a computation time of 12 hours for 2 seconds real time. Another approach is employed by Cross and Viarouge (1998). They consider the line-start motor as equivalent to two fictitious motors: an induction motor and a permanent magnet motor with the same stator configuration. Coupling these two equivalent electromagnetic structures may lead to highly accurate results. Kang et al (2003) use an iterative procedure in their numerical method of simulating the single-phase line-start permanent magnet motor. Lee et al. (2002) focused their analysis on magnetizing the magnet in the rotor. Thus, their numerical model contains a fixed rotor position analysis. Obviously, the three-phase version of AC motors has been analysed in a large number of papers using numerical methods, especially dedicated to the induction motor analysis.

Flux-linkages and inductance

Quantities, such as flux-linkages, inductances, resistances, etc may be determined from a numerical solution by several methods.

Two definitions of the inductance can be found: one based on the flux linked with the winding (2.39) and the other based on the magnetic stored energy (2.40).

$$L = \frac{N\Phi}{i} \tag{2.39}$$

$$L = 2\frac{\int\limits_{Vol}\left(\int\limits_{B} HdB\right)dv}{i^2} \quad (2.40)$$

with *N* the number of turns, *i* the terminal current and *Vol* the volume of the analysed system.

The flux-linkage in relation to (2.39) can be determined from the magnetic vector potential inside the considered winding area:

$$\Phi = \oint\limits_{l} \mathbf{A} \cdot d\mathbf{l} = \frac{l}{Ni}\int\limits_{\Omega} A\, J d\Omega \quad (2.41)$$

where l is the active length of the analysed electromagnetic system, J the current density in the fractional winding cross section area $d\Omega$, with Ω the surface of the winding cross section (including both sides of the winding). Equation (2.41) automatically accounts for the number of turns N and the orientation of different sides of the winding, as the sign of both the magnetic vector potential **A** and the current density J are related. Note that magnetic vector potential has only a *z*-direction component.

In a linear analysis, (μ_r = constant) therefore the saturation effect can be ignored, the calculation of the inductance can be easily implemented with the use of relations (2.39) and (2.41). Also, the expression (2.40) will give the same result. If the material is non-linear, using (2.39) is not appropriate because is does not consider the effect of higher harmonics in the magnetic stored energy variation. If a sinusoidal current is applied, there will be odd harmonics in the variation of the stored energy.

2.3 Conclusions

In this chapter, the state of the art of the calculation of electromagnetic torque in single- and two-phase AC motors is discussed from the analytical and numerical point of view. An emphasis is placed on the induction motors and permanent magnet motors cases.

The general principles of the torque behaviour in AC motors have been discussed and analytical methods have been developed virtually throughout the entire history of electrical machines. However, numerical methods have only relatively recently been used for analyzing the performance of electrical motors. Both of the methods have their own advantages and disadvantages. Consequently, it is still arguable which approach is the most reliable when analyzing parameters and operation of electrical motors.

Analytical methods obtain the quickest results but due to the simplifying assumptions and depending on the equivalent circuit parameters computation method the accuracy level can exhibit a wide range of variation. Knowledge of the torque components, their dependence on different motor parameters and how they influence the overall motor performance are clear advantages of analytical methods.

The numerical results have very good accuracy but the analysis is usually time consuming when compared to the analytical methods. The choice between the speed and simplicity of the analytical methods and the flexibility and relatively straightforward applicability of the numerical method (i.e., the finite element method) relies on user-design criteria and his/her understanding of the electrical machinery phenomena. Therefore, in modern electric motor design the user should exercise judgement in choosing appropriate methods.

In this thesis, the electromagnetic torque in single and two-phase AC motors is estimated through the use of analytical models. With a focus on induction motors and permanent magnet motors, several mathematical models are presented. From the drive-system point of view, it is demonstrated that the inclusion of the asymmetries and main non-linear effects in these machines (saturation, core loss) may give sufficiently accurate information in any preliminary design. The developed analytical model also gives detailed insight into the torque dependencies. The calculation methods are presented, discussed and validated in the following chapters of this thesis.

3 NEW ANALYTICAL MODELS

This chapter presents the main achievements in developing analytical models in order to reach the goals of the study.

The mathematical model of single-phase induction motors with stator windings that are distributed in stator slots with non-orthogonal magnetic axes is presented in Publications P2 and P3 and briefly presented in Section 3.1. The first application of the mathematical model to the analytical prediction of the electromagnetic torque components in such a motor is described in Publication P2. The influence of the electrical shift angle between magnetic axes of the stator windings on motor performance when the motor is in a PWM inverter-fed configuration is analysed in detail in Publication 3.

As mentioned in the previous chapters, a single-phase induction motor can be operated as a two-phase motor if the voltage system applied to the winding terminals is always balanced. This configuration requires an electronically controlled drive system; therefore a variable-speed application can be straightforwardly implemented. The mathematical model for two-phase induction motors with unsymmetrical stator windings and with the stator core loss and saturation effects taken into account, is illustrated in Publications 1 and 4 and briefly presented in Section 3.2. The model with all its components is described for the first time in Publication 4.

An original analytical model was developed to estimate the starting performance of a single-phase line-start permanent magnet motor. This model is explained and demonstrated in Publications 5, 6 and 8 and is presented in a condensed form in Section 3.3. The first application of this approach is described in Publication 6. The developed analytical model makes possible the estimation of all the torque components, asynchronous (cage), magnet braking, pulsating and instantaneous torque.

Finally, using the symmetrical components a two-axis mathematical model was developed in order to analyse a single-phase induction motor with a Steinmetz delta connection. The analytical results are validated against the test data and finite-element analysis. This model is described in Publication 7 and briefly illustrated in Section 3.4.

3.1 Single-phase induction motor with unsymmetrical and non-orthogonal stator windings

The two-axis model of a single-phase induction motor equipped with stator windings spatially displaced such that their magnet axes are non-orthogonal (or not in quadrature) was originally developed in Publication 2. This model also takes into account the asymmetries of the windings, i.e., the different number of turns and different wire diameter. Two simplifying assumptions have been made: that the stator windings have the same distribution and the core loss and saturation effects are ignored. The model employs the stator reference frame, while the shift angle between the magnetic axes of the stator windings is measured from the orthogonal position, as proposed by Walls and Sudhoff (1996). Figure 3.1 illustrates the proposed mathematical model. One can note the cross-coupling effect between the two axes. The instantaneous electromagnetic torque can be defined by

$$T_{\mathrm{e}} = \left(\frac{P}{2}\right)\left(\frac{X_{\mathrm{m}}}{\omega_{\mathrm{b}}}\right) k \left(i_{\mathrm{QS}}\, i'_{\mathrm{DR}} - i_{\mathrm{DS}}\, i'_{\mathrm{QR}} \cos\varphi_{\mathrm{a}} - k \cdot i_{\mathrm{DS}}\, i'_{\mathrm{DR}} \sin\varphi_{\mathrm{a}} \right) \tag{3.1}$$

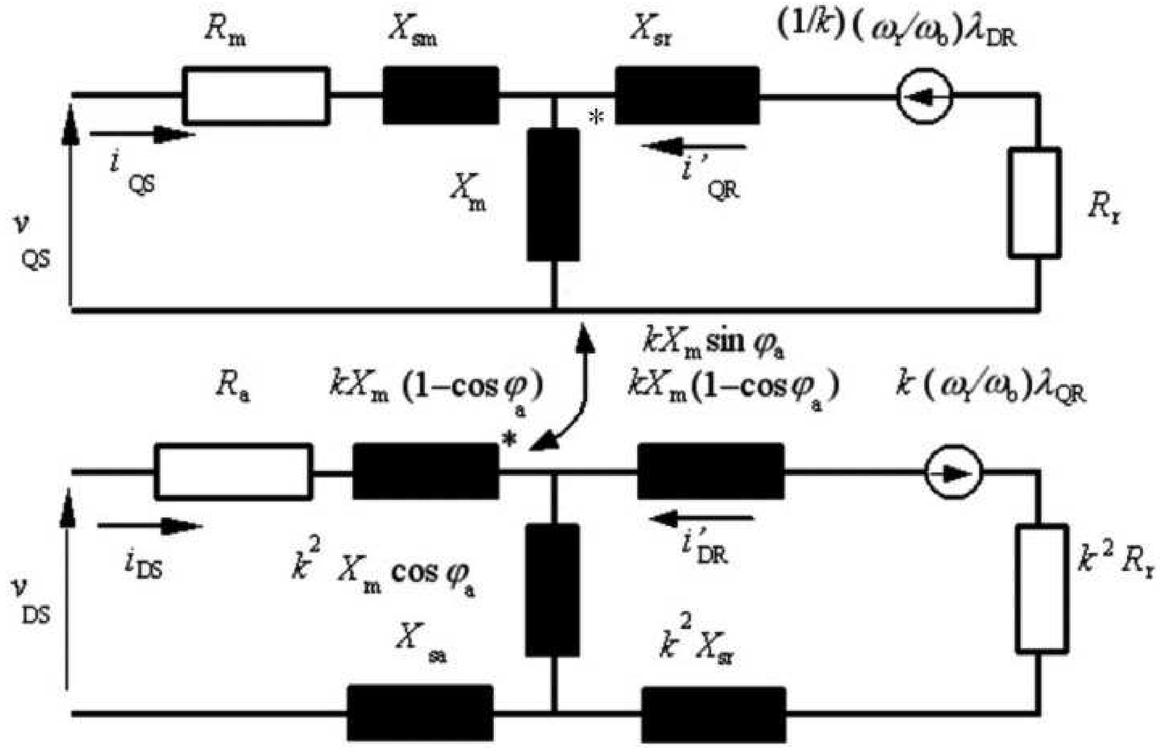

Figure 3.1 Equivalent two-axis model of a single-phase induction motor with non-orthogonal magnetic stator windings

When the stator windings magnetic axes are shifted with 90 electrical degrees ($\varphi_a = 0$) expression (3.1) is identical with (2.10).

In Figure 3.1 the stator reference frame was used to derive the two-axis equivalent circuit. The *q*-axis (quadrature) is fixed in the magnetic axis of the main stator winding. Consequently, the *q*-axis circuit is in fact the main stator winding circuit where the rotor parameters are referred to the main stator winding. The d-axis circuit represents the transformed auxiliary stator winding. The rotor parameters in the *d*-axis circuit are referred to the auxiliary stator winding. Note that the stator windings are assumed to have the same distribution such that the magnetization reactances ratio is square of the effective turns ratio (k^2) .

Because the actual magnetic axes of stator windings are displaced with an angle different from 90 electrical degrees, the main and auxiliary windings are mutually coupled. The mutual reactance is expressed as $k\,X_m \sin\varphi_a$. This coupling effect takes place between the magnetization reactance (X_m) in the *q*-axis and the leakage reactance that appears in the primary of the *d*-axis circuit due to the electrically non-orthogonal stator windings, $k\,X_m\left(1-\cos\varphi_a\right)$. The double arrow in Figure 3.1 shows where the coupling effect occurs in the equivalent two-axis model. The reactances polarisation (stars signs) shows that in flux-linkages expressions the mutual coupling effect has to be considered with negative sign.

The analysis of single-phase induction motors with windings not in quadrature was made for the first time by Puuchstein and Lloyd (1935) who used time domain equivalent circuits and Lyon and Kingley (1936) who used the symmetrical components approach for steady-state performance. Since these modelling attempts, only Walls and Sudhoff (1996) have proposed a multiple reference frame for the steady-state and dynamic analysis of the motor performance. The drawback of their model is the doubled number of equations involved in the system analysis.

The model in Figure 3.1 was used in Publications 2 and 3 to analyse the influence of different parameters (e.g., the shift angle between the two axes windings) on motor performance (average torque, starting torque, pulsating torque) when the motor is inverter fed. The results of the shift angle variation are presented in Chapter 5.

3.2 Two-phase induction motor model for vector control purposes

For low-power applications, the two-phase induction motor is a suitable replacement for the three-phase version, as the same motor performance can be achieved with a more economical solution. In Publications 1 and 4, a new mathematical model is proposed and validated. The most important non-linear phenomena, saturation and core losses are included.

The background of modelling the induction motors in two-axis and considering the cross-coupling between the axes due to saturation or the resistance limited eddy-currents effect is mainly illustrated in works authored by Boldea and Nasar (1987), Slemon (1989), Krause et al. (1995), Levi (1995) or Correa et al (1998, 1999). Boldea and Nasar were the first to propose a unified approach to the analysis of the cross-coupling in two-axis motor models. Slemon's equivalent two-axis circuits are dedicated to the modelling induction motors for electrical drives. He suggests that the most appropriate models that include saturation and core loss effects are the so called gamma and inverse gamma models. Krause et al. realized a unified approach to modelling all the AC motors in two-axis equivalent circuits. Levi's work defines the induction motor as a system with input parameters, state-space variables and outputs. Consequently, he identifies all the possible combinations in using voltages as inputs, flux-linkages and currents as state-space variables, while the outputs can be a chosen function of the application. Excepting Krause's work, all the above models refer to the three-phase induction motors. Correa et al. originally proposed a model for the two-phase induction motor that could be implemented in a low-cost vector control strategy application.

In this study, two models are dedicated to the two-phase induction motor analysis. Similarly with the model illustrated in Figure 3.1; in these two new models the *q*-axis (quadrature) is fixed in the magnetic axis of the main stator winding. Consequently, the *q*-axis circuit is, in fact, the main stator winding circuit where the rotor parameters are referred to the main stator winding. The *d*-axis circuit represents the transformed auxiliary stator winding. The rotor parameters in *d*-axis circuit are referred to the auxiliary stator winding. Note that the stator windings are assumed to have a distribution such that the magnetization reactances ratio is the square of the effective turns ratio (k^2).

The first model presented in Publication 4 considers that the stator core losses occur in the teeth and in the yoke. The loss is a function of the total flux in the teeth and in the yoke. If we consider that the yoke and most of the teeth carry a flux which is proportional to the stator flux linkage, it appears to be appropriate to connect an equivalent core loss resistance either at the terminals of the motor equivalent circuit, or more correctly after the stator resistance where the voltage across core loss resistance would be the total induced voltage in the stator winding. Thus, the equivalent circuit in Figure 3.2 may be employed. It should be emphasised that the equivalent iron-loss resistance exhibits different values with frequency variation or with current (load) and voltage variation (through the saturation effect). Also, test data make possible the extraction of the sum between iron losses and stray load losses. Even tests performed at no-load operation will only give estimations for the iron loss. Thus R_{feD} R_{feQ} are usually computed either empirically, using Steinmetz equation and coefficients, or using the finite element analysis. The instantaneous electromagnetic torque may be expressed by

$$\begin{aligned} T_{\mathrm{e}} &= \frac{P}{2}\left[kL_{\mathrm{mqs}} i_{\mathrm{DR}} \left(i_{\mathrm{QS}} + i_{\mathrm{QR}} - i_{\mathrm{feq}} \right) - \frac{1}{k} L_{\mathrm{mds}} i_{\mathrm{QR}} \left(i_{\mathrm{DS}} + i_{\mathrm{DR}} - i_{\mathrm{fed}} \right) \right] \\ &= \frac{P}{2}\left(k\lambda_{\mathrm{QR}} i_{\mathrm{DR}} - \frac{1}{k} \lambda_{\mathrm{DR}} i_{\mathrm{QR}} \right) \end{aligned} \tag{3.2}$$

A second model is developed in Publication 1. It is considered that the total iron losses may be separated into those caused by magnetization flux called core-loss, and those caused by the leakage flux called stray-load loss. The magnetization flux is proportional to the internal or air gap voltage. It follows that a resistor connected across the air gap voltage can represent the core-loss. It should be noted that the core-loss is proportional to the square of the air gap voltage and load-loss to the square of the stator current, much as experimentation shows them to be (Honsinger, 1980). In this study, the *computed* value of the equivalent iron loss resistance was used, as the value extracted from *test data* would include other losses beside core losses.

The torque behaviour can be studied with the following relation for the instantaneous resultant electromagnetic torque

$$T_{\mathrm{e}} = \left(\frac{P}{2}\right)\left(\frac{X_{\mathrm{m}}}{\omega_{\mathrm{b}}}\right) k\left[\left(i_{\mathrm{QS}} + i_{\mathrm{feq}} \right) i_{\mathrm{DR}} - \left(i_{\mathrm{DS}} + i_{\mathrm{fed}} \right) i_{\mathrm{QR}} \right] \tag{3.3}$$

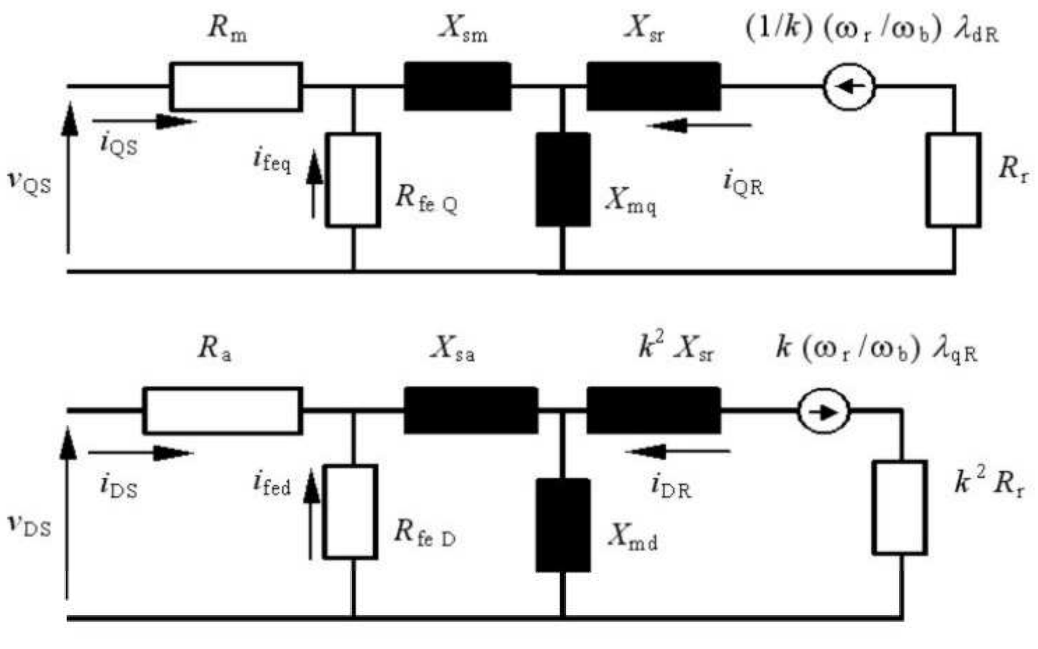

Figure 3.2 Equivalent two-axis model of a two-phase induction motor for vector control purposes

If we ignore the iron loss term we obtain:

$$T_e = \left(\frac{P}{2}\right)\left(\frac{X_m}{\omega_b}\right) k\left[i_{QS} i_{DR} - i_{DS} i_{QR}\right] \tag{3.4}$$

In *d-q* axis phasorial terms, the currents can be expressed in terms of forward (+) and backward (-) quantities. By taking into account sinusoidal supply frequency (i.e., no time-harmonics) we can express the average steady-state electromagnetic torque:

$$T_{e(avg)} = P\frac{X_m}{\omega_b} \mathrm{Re}\left[\mathrm{j}\left(\underline{I}^*_{QS+} \cdot \underline{I}_{QR+} - \underline{I}^*_{QS-} \cdot \underline{I}_{QR-} \right)\right] \tag{3.5}$$

The pulsating torque has a double frequency and superimposes over the average electromagnetic torque to determine the resultant electromagnetic torque:

$$T_{e(pls)} = P\left(\frac{X_m}{\omega_b}\right)\left\{ \mathrm{Re}\left[\mathrm{j}\left(-\underline{I}_{QS+}\underline{I}_{QR-} + \underline{I}_{QS-}\underline{I}_{QR+} \right)\right]\cos 2\omega t + \mathrm{Re}\left(\underline{I}_{QS+}\underline{I}_{QR-} - \underline{I}_{QS-}\underline{I}_{QR+} \right)\sin\ 2\omega t \right\} \tag{3.6}$$

Relation (3.6) illustrates the fact that the pulsating torque has a zero average value and will worsen the single-phase induction motor performance. For the region close to the rated-speed operation, the rotor oscillations due to the pulsating torque may be very important. Thus, the minimisation of this torque component is an important factor in the optimisation of single-phase induction motor design.
An important validation of the analytical torque prediction for a single- or two-phase induction motor is made with the use of eqs. (3.4), (3.5) and (3.6); the instantaneous torque (3.4) will have to vary close to the limits of the envelope curves obtained by superimposing the pulsating torque (3.6) zero to peak amplitude over the average electromagnetic torque (3.5).

In Publication 1, it is demonstrated that an induction motor equipped with two unsymmetrical stator windings exhibits an important pulsating torque that varies with double supply frequency (i.e., 100 Hz for 50Hz supply frequency) when it is operated as a capacitor-start, capacitor-run motor. Alternatively, the same motor can be operated as a balanced two-phase induction motor when it is fed from a PWM inverter. In the latter case the amplitude of the pulsating torque harmonics will be minimised. The results of torque behaviour for this motor type are presented in Chapter 5.

3.3 Single-phase line-start permanent magnet motor

The line-start permanent magnet motor in its single-phase version is considered to have a bright future for home appliances, such as refrigerator compressors. It may be considered either as an induction motor that has magnets buried inside the rotor, or as synchronous permanent magnet motor that is equipped with an aluminium or copper rotor cage. In fact, the motor operates as an induction motor during starting and as a synchronous permanent magnet motor during steady-state operation. Several configurations have been patented, including those that allow interior magnets to be disposed in an arc shape (Miyashita et al., 1977), rectangular shape (Steen, 1979) or even insert above the rotor slots (Kliman et al., 1996).

This motor type received little theoretical attention before the permanent magnets motors become a hot topic in the last decades. Honsinger (1979, 1980a, 1980b) has analysed the asynchronous and steady-state performance of the three-phase line-start permanent magnet motor using a two-axis model. He was first to identify the effect of the magnets during starting, when the induced currents in the stator windings due to rotor movement determine a so-called ‘magnet braking torque’. His analytical procedure was completed for the synchronous operation by Consoli and Racitti (1991) and Schiferl and Lipo (1989) who analysed the core loss effect for this motor type.

As suggested by Lefevre (2000), Nee et al. (2000), Yamamoto et al. (1999) it is possible to use finite element analysis or original analytical methods is possible to determine the motor *d*-*q* axes parameters. Miller (1985) was the first to develop an analytical model for the single-phase line-start permanent magnet motor that predicts the steady-state performance. As the *d*-*q* axes parameters depend on rotor position and load, Miller proposed an iterative procedure for extracting the motor variables (currents and voltages) and further computing the electromagnetic torque. His model uses symmetrical components in a two-axis rotor reference frame. Chaudhari and Fernandes (1999) tried to use a model where the stator parameters are expressed as actual phase-variables and rotor parameters are expressed in a rotor reference frame. A symmetrical components decomposition is then employed. However, their mathematical model has not been thoroughly tested against experimental data. Boldea et al (1999) use the same symmetrical components and two-axis theory combination to determine a unified approach on the steady-state operation of the single-phase AC motors with auxiliary capacitor.

In this study, a mathematical model that predicts the asynchronous operation of the single-phase line-start permanent magnet motor is formulated for the first time. Originally, this model was presented in Publication 6. The basic idea is to use the superposition principle in estimating the asynchronous (cage) torques caused by the rotor bars induced currents and the magnet braking torque caused by the stator winding induced currents. Thus, a series of frame transformations as suggested by Miller (1985) is employed. Then, through the application of the symmetrical components two fictitious motors in two-axes theory are defined. The variables are expressed in a rotor reference frame as complex numbers (underlined symbols). Figures 3.3 and 3.4 show the equivalent circuits for the positive and negative sequences used to compute the asynchronous torque. Assuming that the stator windings have the same copper weight, the following expression can be used

$$T_{(\mathrm{avg})+} = \left(\frac{P}{2}\right)\mathrm{Re}\left\{\left(\underline{\psi}_{\mathrm{q}+}\right)^{*}\underline{I}_{\mathrm{d}+} - \left(\underline{\psi}_{\mathrm{d}+}\right)^{*}\underline{I}_{\mathrm{q}+}\right\} \tag{3.7}$$

$$T_{(\mathrm{avg})-} = \left(\frac{P}{2}\right)\mathrm{Re}\left\{\left(\underline{\psi}_{\mathrm{q}-}\right)^{*}\underline{I}_{\mathrm{d}-} - \left(\underline{\psi}_{\mathrm{d}-}\right)^{*}\underline{I}_{\mathrm{q}-}\right\} \tag{3.8}$$

The magnet braking torque is computed independently as

$$T_{\mathrm{m}} = \frac{P}{2}\sin(\zeta)\left[\beta\cdot\psi_{\mathrm{dm}}\,I_{\mathrm{qm}} - \frac{1}{\beta}\cdot\psi_{\mathrm{qm}}I_{\mathrm{dm}}\right] \tag{3.9}$$

The overall quasi steady-state electromagnetic torque will be defined as

$$T_{\mathrm{e}} = T_{(\mathrm{avg})+} + T_{(\mathrm{avg})-} + T_{\mathrm{m}} \tag{3.10}$$

Another original contribution of the author is the computation method for the peak-to-zero amplitude of the pulsating torque fundamentals, presented in Publications 5 and 6. Considering all the currents in the rotor reference frame and their interactions, six pulsating torque components are identified. These pulsating torques vary with different frequencies: $2f$, $2sf$, $(2 - 2s)f$, $(4 - 2s)f$, sf, $(2 - s)f$.

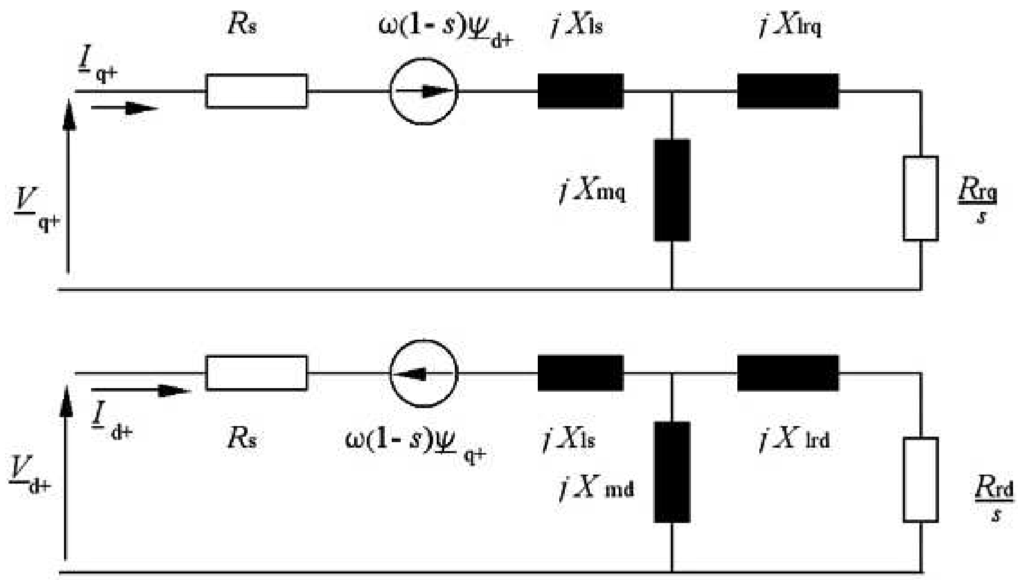

Figure 3.3 Equivalent two-axis model of the positive sequence used to compute the asynchronous torque in a single-phase line-start permanent magnet motor

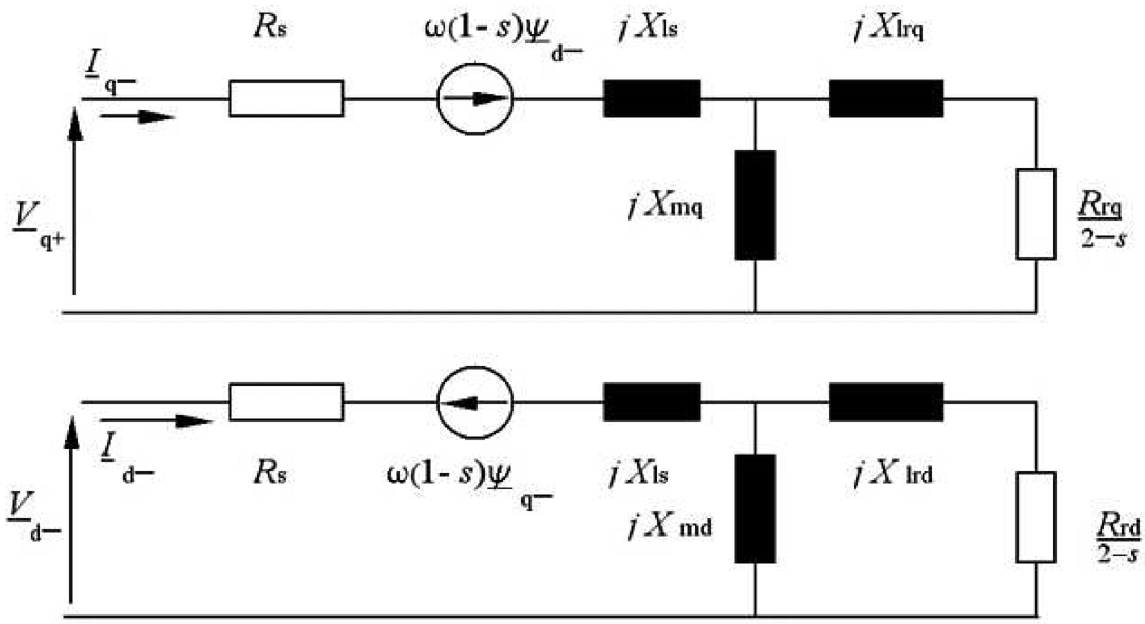

Figure 3.4 Equivalent two-axis model of the negative sequence used to compute the asynchronous torque in a single-phase line-start permanent magnet motor

On the basis of their cause we can classify the pulsating torque components as

Reluctance pulsating torques

These pulsating torque components vary with (2*sf*) and (4 – 2*s*)*f* frequency and are caused by the interaction between the stator flux-linkages and stator currents of the same symmetrical component sequence (Figures 3.3. and 3.4). If the positive sequence reluctance pulsating torque vanishes at synchronous speed and transforms into the reluctance torque (2.13), the negative sequence reluctance pulsating torque will exhibit a 4 *f* frequency.

$$T_{(\mathrm{pls})(2sf)} = \frac{P}{2}\mathrm{Abs}\left\{\left(\underline{\psi}_{\mathrm{q}+}\right)\underline{I}_{\mathrm{d}+} - \left(\underline{\psi}_{\mathrm{d}+}\right)\underline{I}_{\mathrm{q}+}\right\} \tag{3.11}$$

$$T_{(\mathrm{pls})(4-2s)f} = \frac{P}{2}\mathrm{Abs}\left\{\left(\underline{\psi}_{\mathrm{q}-}\right)\underline{I}_{\mathrm{d}-} - \left(\underline{\psi}_{\mathrm{d}-}\right)\underline{I}_{\mathrm{q}-}\right\} \tag{3.12}$$

Unbalanced stator voltage pulsating torques

These pulsating torque components vary with (2*f*) and (2 – 2*s*)*f* frequency and are caused by the interaction between the stator flux-linkages and stator currents of the different symmetrical component sequences. Both these pulsating torque components will exhibit a 2*f* frequency at synchronous speed.

$$T_{(\mathrm{pls})(2f)} = \frac{P}{2}\mathrm{Abs}\left\{\left(\underline{\psi}_{\mathrm{q}+}\right)\underline{I}_{\mathrm{d}-} - \left(\underline{\psi}_{\mathrm{d}+}\right)\underline{I}_{\mathrm{q}-}\right\} \tag{3.13}$$

$$T_{(\mathrm{pls})(2-2s)f} = \frac{P}{2}\mathrm{Abs}\left\{\left(\underline{\psi}_{\mathrm{q}-}\right)\underline{I}_{+} - \left(\underline{\psi}_{\mathrm{d}-}\right)\underline{I}_{\mathrm{q}+}\right\} \tag{3.14}$$

Permanent magnet (excitation) pulsating torques

These pulsating torque components vary with (*sf*) and (2 – *s*)*f* frequency and are caused by the interaction between the stator flux-linkages and stator currents with the flux-linkages and currents induced by the magnets rotation. If the positive sequence permanent magnet pulsating torque vanishes at synchronous speed and transforms into the reluctance torque (2.13), the negative sequence permanent magnet pulsating torque will exhibit a 2 *f* frequency.

$$T_{(\mathrm{pls})(sf)} = \frac{P}{2}\mathrm{Abs}\left[\left(\underline{\psi}_{\mathrm{q}+}\right)I_{\mathrm{dm}} + \left(\psi_{\mathrm{qm}}\right)\underline{I}_{\mathrm{d}+}\right] - \frac{P}{2}\cdot\mathrm{Abs}\left[\left(\underline{\psi}_{\mathrm{d}+}\right)I_{\mathrm{qm}} + \left(\psi_{\mathrm{dm}}\right)\underline{I}_{\mathrm{q}+}\right] \tag{3.15}$$

$$T_{(\mathrm{pls})(2\text{-}s)f} = P\mathrm{Abs}\left[\left(\underline{\psi}_{\mathrm{q}-}\right)I_{\mathrm{dm}} + \left(\psi_{\mathrm{qm}}\right)\underline{I}_{\mathrm{d}-}\right] - P\mathrm{Abs}\left[\left(\underline{\psi}_{\mathrm{d}-}\right)I_{\mathrm{qm}} + \left(\psi_{\mathrm{dm}}\right)\underline{I}_{\mathrm{q}-}\right] \tag{3.16}$$

The computation of all the torque components takes into account the asymmetries that occur on both stator and rotor. The validation of the proposed analytical method against test data is illustrated in Chapter 5.

3.4 Single-phase induction motor with Steinmetz delta connection

A three-phase winding supplied from a single-phase system is a solution commonly known as a Steinmetz connection and has been employed since the early decades of last century. However, its analytical or numerical modelling did not receive the attention a normal three-phase winding would receive. Only in recent decades have Stepina (1982), Oliveira (1990) or Tozune (1991) analysed and proposed models based on symmetrical components theory.

In Publication 7, a motor with multiple independent windings to make possible the operation in two largely different configurations of 2 and 18 poles, respectively, is analysed. The 18-pole configuration uses a 3-phase winding and a delta Steinmetz connection. The main objective of the theoretical analysis was to establish a mathematically rigorous equivalence between the Steinmetz connection and a main and auxiliary winding so that the motor can be optimally designed using single-phase motor engineering knowledge and practices.

Starting from the commonly employed theory of symmetrical components, and after further mathematical transformations, new equations and equivalent circuits for the stator winding connections have been developed in Publication 7. Also, as new contributions, the mathematical relations between the actual measurable voltages and currents in the motor real windings on the one hand, and the values in the equivalent auxiliary and main winding on the other, have been established for both the delta and the wye Steinmetz connection. A design procedure based on the new analysis method has been implemented using motor design software; the theory has been experimentally validated.

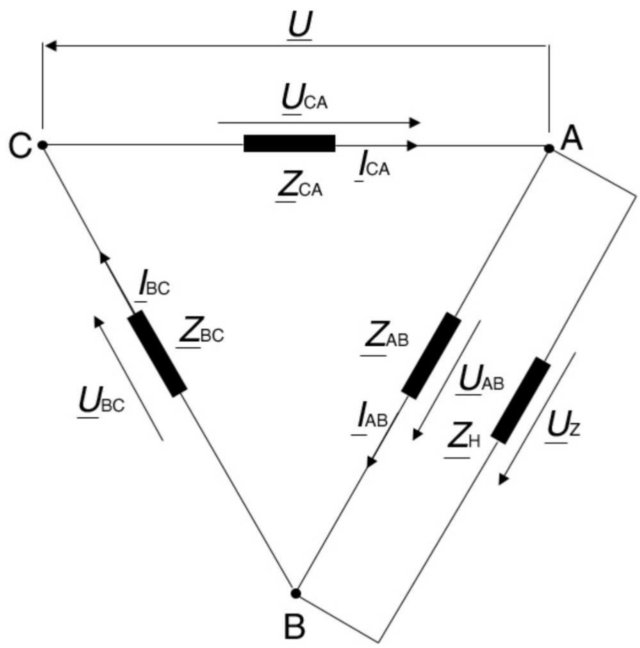

Figure 3.5. Steinmetz delta connection diagram

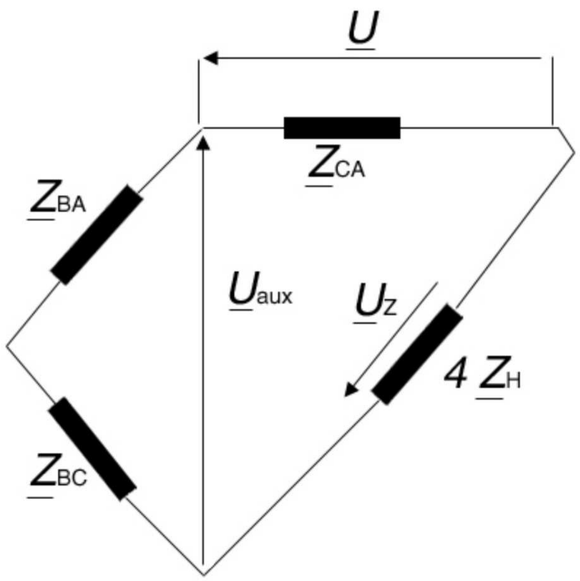

Figure 3.6.Equivalent two-axis model for a Steinmetz delta connection

The optimal design of the 2- and 18-pole motor poses interesting challenges, such as the dimensioning of a magnetic circuit capable of operation with two very different field patterns, the comparison and selection of a wye or delta Steinmetz connection, the minimisation of the torque ripple for a specific load, together with capacitor rationalisation etc. The methods employed are described and design considerations of interest to an electric motor engineer are also discussed.

Figures 3.5 and 3.6 show the schematic diagram of a delta Steinmetz connection and its two-axis equivalent circuit, respectively. The variables are expressed as a function of measurable parameters. As the equivalent main winding is represented by the impedance $\underline{Z}_{\mathrm{CA}}$, the main RMS voltage can be measured across AC, as in Figure 3.5. The equivalent auxiliary winding is represented by the impedances $\underline{Z}_{\mathrm{BA}} = -\underline{Z}_{\mathrm{AB}}$ and $\underline{Z}_{\mathrm{BC}}$. Note, that in order to obtain the equivalent auxiliary winding, the actual impedance between phases A and B has a reversed sign and is series connected with the impedance between phases B and C. An important factor in the equivalent delta connection (Figure 3.6) is that the motor employs only a quarter of the actual capacitance (4 $\underline{Z}_{\mathrm{H}}$ vs. $\underline{Z}_{\mathrm{H}}$).

The motor performance can be predicted using the classical revolving field theory (Morrill, 1929, Veinott, 1959). An example is the starting torque given by the expression

$$T_{\mathrm{st}} = \left(\frac{k U_{\mathrm{n}}^{2} R_{\mathrm{r}}}{\pi f \left| \underline{Z}_{\mathrm{m}} \right|^{2}} \right) \frac{\mathrm{Im}\left(\underline{Z}_{\mathrm{m}}\right)\mathrm{Re}\left(\underline{Z}_{\mathrm{a}}\right) + \mathrm{Re}\left(\underline{Z}_{\mathrm{m}}\right)\mathrm{Im}\left(\underline{Z}_{\mathrm{C}}\right) - k^{2}\,\mathrm{Re}\left(\underline{Z}_{\mathrm{m}}\right)\mathrm{Im}\left(\underline{Z}_{\mathrm{m}}\right)}{\left[k^{2}\,\mathrm{Im}\left(\underline{Z}_{\mathrm{m}}\right) - \mathrm{Im}\left(\underline{Z}_{\mathrm{C}}\right)\right]^{2} + \mathrm{Re}^{2}\left(\underline{Z}_{\mathrm{a}}\right)} \tag{3.17}$$

where, for the analysed motor, the following correspondences exist:
$\underline{Z}_{\mathrm{m}} = \underline{Z}_{\mathrm{CA}}$ represents the main winding impedance measured at locked-rotor conditions;
$\underline{Z}_{\mathrm{a}} = \underline{Z}_{\mathrm{BC}} + \underline{Z}_{\mathrm{BA}}$ represents the auxiliary winding impedance measured at locked-rotor conditions;
$\underline{Z}_{\mathrm{C}} = 4\underline{Z}_{\mathrm{H}}$ represents the capacitive impedance connected in series with the auxiliary winding;

The results of the analytical procedure are discussed in Chapter 5.

3.5 Conclusions

In this Chapter, new methods of analysis are presented. The mathematical models originally developed represent important tools in the preliminary estimation of a single- or two-phase AC motor performance. All the developed models give a good starting point in a preliminary design that can be further used in the initialisation of an optimisation procedure through analytical or numerical methods.

An analytical model for the single-phase induction motor with unsymmetrical stator windings that are not in quadrature is developed in this study. The influence of the shift angle between magnetic axes of the stator windings over the electromagnetic torque is discussed and validated.

Two-phase induction motors can be modelled for electric drives using several equivalent circuits that include the saturation and core loss effects. Possible vector controlled strategies are theoretically analysed and proposed for this motor type.

Asynchronous performance for a single-phase line-start permanent magnet motor is predicted using an original method based on a series of frame transformations, symmetrical components theory and two-axis theory. The proposed model permits a preliminary simulation for any single-phase AC motor whether induction, synchronous reluctance or synchronous permanent magnet.

An analytical methodology is proposed for a single-phase induction motor with special windings. The advantage of this method is the equivalence between classical theory on the single-phase induction motor with a main and an auxiliary winding, while using measurable parameters to quantify the voltages and currents in the two-axis model theory.

4 MEASUREMENTS

The developed mathematical models of analysis are partially validated by measurements. In this chapter, the measurements of the electromagnetic torque in the single-phase line-start permanent magnet motors and single-phase induction motor with two-speed and special windings are briefly described.

The original measurement set-up and the procedure for the line-start motor was developed at Zanussi Elettromeccanica Compressors, Italy and SPEED Laboratory, Glasgow University, U.K. Standard tests and a specific set-up stand were performed at AO Smith Corp. for the single-phase induction motor with two-speed and special windings. The measurements and the results of the measurements are presented in publications P5, P6, P7, P8 and discussed briefly in Chapter 5.

Figure 4.1 shows the settings for the test stand for a single-phase line-start permanent magnet motor, while the cross sectional geometry is shown in Figure 4.2. Table I presents the stator winding data for the four tested motors. A hysteresis brake (coupled with an acquisition and control system) was used to test the motor. The automatic procedure starts running the motor without load, then gradually decreases the speed measuring at each step the torque and power.

Two sets of data are presented for motors not equipped with buried magnets rotors, and with magnets inserted inside the rotor. In this way, it was possible to observe the influence of permanent magnets over motor starting performance. The experiments were intended to study the torque behaviour during starting, synchronisation and synchronous operation, for a wide range of capacitance values. The tested motors exhibited large torque oscillations at low speed. This phenomenon makes any measurement for locked rotor or low-speed conditions very difficult.

TABLE I.
STATOR WINDING DATA FOR TESTED SINGLE-PHASE LINE-START PERMANENT MAGNET MOTOR
(PUBLICATIONS 5, 6, 8)

Motor Type	Winding parameters			
	N_m [p.u.]	β	ϕ_m / ϕ_a	ζ [elec. °]
Motor A	1.46	1.42	1.22	90
Motor B	1.14	1.42	1.3	90
Motor C	1	1	1	90
Motor D	0.87	0.70	0.76	90

Where:

N_m	number of turns on the main stator winding
β	turns ratio (main/aux) between stator windings
$\phi_{m,a}$	diameter of the main/auxiliary winding
ζ	shift electrical angle between magnetic axes of stator windings

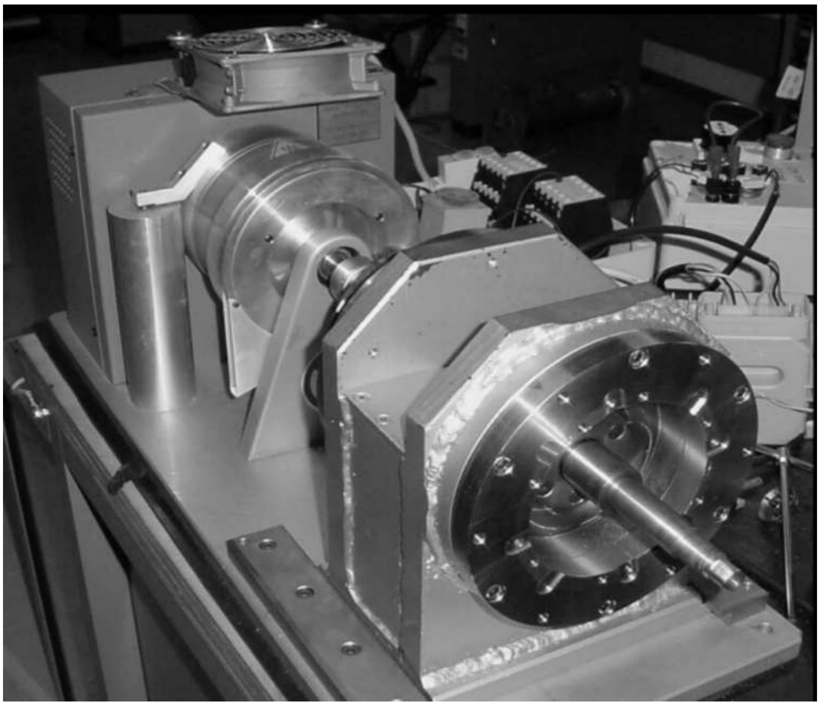

Figure 4.1. Test stand for single-phase line-start permanent magnet motor
Courtesy of Zanussi Elettromecanica Compressors, Italy

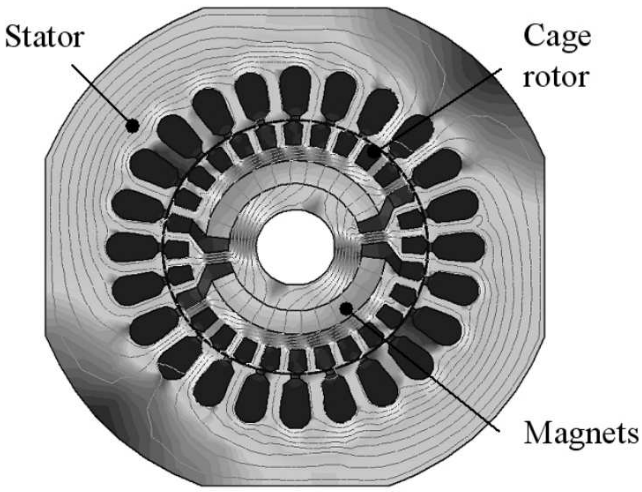

Figure 4.2. The cross sectional geometry of the tested line-start permanent magnet motor with plotted flux lines when motor operates under load

Table II presents the stator winding data for the tested single-phase induction motor with two-speed and special windings; the cross sectional geometry is shown in Figure 4.3. Standard tests (no-load, short circuit, load operation) were used to test the motor. The experiments were intended to study the torque behaviour during starting, rated load operation, pull-out torque limits, locked-rotor conditions for a variable range of capacitance values. The transient performance is simulated as the motor for a real life practical application with a very high inertial load; the main objective was to illustrate that in such conditions the relatively large torque ripple does not cause speed oscillations and that less than perfect load balancing is acceptable. The exact measurement of the time transients was very difficult in the case studied, because a very large bandwidth torque transducer was necessary, due to the high-frequency torque ripple, and a cumbersome test stand set up was required to couple the very large inertial load without shaft torsional oscillations.

TABLE II

THE MAIN PARAMETERS FOR TEST SINGLE-PHASE INDUCTION MOTOR WITH TWO SPEED AND SPECIAL WINDINGS (PUBLICATION 7)
Courtesy of AO Smith Corp., USA

Poles		18	2
R_1	Ω	6.93	1.33
R_{1a}	Ω	11.98	1.90
X_1	Ω	17.38	1.34
X_{1a}	Ω	70.03	1.14
R_2'	Ω	25.68	1.41
X_2'	Ω	60.89	3.16
Turns ratio a		1.73	0.926
X_m	Ω	29.27	95.10
Core loss equiv. resistance R_{Fe}	Ω	682.38	905.59
Rotor inertia	kgm^2	0.0195	0.0195
Load torque at rated speed	lb · ft	5.7	2.7
Load inertia	p.u.	5.2	5.2
Start capacitance C_{st}	μF	60	300
Run capacitance C_{st}	μF	60	60

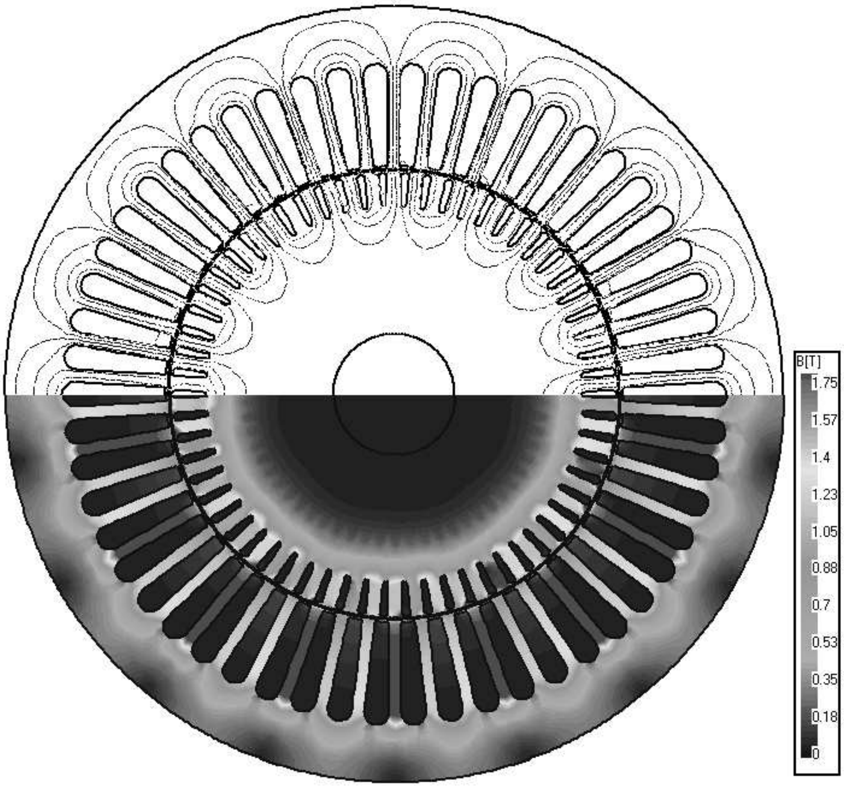

Figure 4.3. The cross sectional geometry of the tested single-phase induction motor with two speed and special windings. The plotted flux lines correspond to the 18 pole configuration

The accuracy of the software for transient simulations has been experimentally validated previously on other motor designs and applications.

Conclusions

In this chapter, the measurement of electromagnetic torque for two specific single-phase AC motor configurations is presented and discussed. The measurement is used for the validation of the analytical models and to verify the influence of different simplifying assumptions on the results. The developed mathematical models seem to predict with sufficient accuracy the value of the air-gap electromagnetic torque, provided that the equivalent circuit parameters are correctly estimated or measured.

5 DISCUSSIONS OF THE RESULTS

In this chapter, the results of the calculations and measurements are briefly introduced and discussed. The chapter is based on Publications P1, P3, P5, P6, P7 and P8. The verification results of the shift angle between the stator winding magnetic axes over the electromagnetic torque are presented at the beginning of this chapter. Then, the results of the comparison analysis of a torque harmonics of a single-phase induction motor when is used as a capacitor-run motor or an inverter-fed motor are discussed, focusing on the pulsating torque amplitude. Thirdly, the estimation of the electromagnetic torque for a single-phase line-start permanent magnet motor is presented. Different phenomena that occur due to the motor asymmetries are discussed. Last but not least, in the final section of this chapter the results of the analysis of a single-phase induction motor with two-speed and special windings are studied.

Influence of the shift angle between stator winding magnetic axes

The influence of the shift angle when the stator windings of a single-phase induction motor are not in quadrature is modelled and verified theoretically in Publication 2, with more details of the both the steady-state and transients in Publication 3. Tests and computations were performed on a two-pole single-phase induction motor with 28 stator slots and 24 rotor bars. Figure 5.1 illustrates the measured torque-slip characteristics of a two-phase induction motor for three different values of the electrical shift angle (-π/8 rad; 0 rad; +π/8 rad). Physically, for the analysed motor, the selected shift angle corresponds approximately to two stator slots, as the number of stator slots is 28. The PWM inverter supplies a range of output fundamental frequencies between 10 … 200 Hz. The analysed values are for 220 V/ 50 Hz. The positive value of the electrical shift angle (φ_a = +π/8 radians) determines increased starting and rated torque, while the negative value determines poor performances regarding torque-speed characteristics.

The explanation of this phenomenon is given by the double revolving field theory (Morrill, 1929 and Veinott, 1959), in which the reverse field component became weaker for $\varphi_a > 0$, and stronger for $\varphi_a < 0$ due to mutual inductances between the stator phases. All the equivalent circuit parameters have been measured (stator resistance, self-inductance) using standard tests or computed with classical methods (Veinott, 1959). The model set forth herein needs further experimental work for a reasonable accuracy validation of the computed results.

The influence of the sense and value of the electrical shift angle between the stator windings, on the single-phase induction machine’s torque behaviour supplied from variable voltage-variable frequency devices (type PWM inverter), is described by the following remarks:

(a) higher starting, break-down and rated torque, lower pulsating torque if a positive value of the electrical shifted angle between the stator windings (opposite to the rotation sense) is realised;

(b) higher pulsating torque, lower starting, rated and break-down torque if a negative value of the electrical shifted angle (toward the rotation sense) is used.

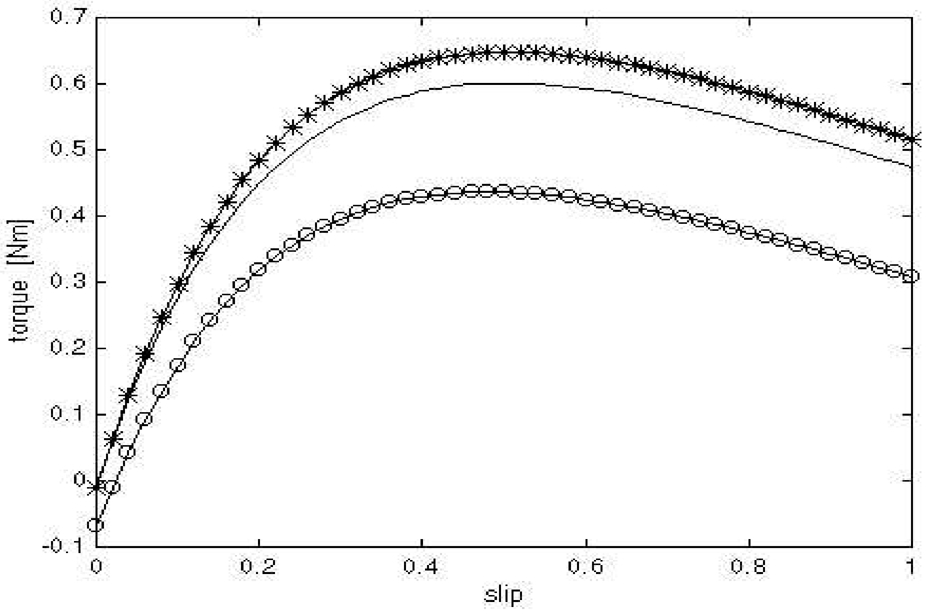

Figure 5.1 Measured torque-slip characteristics of a PWM inverter-fed TPIM with stator windings not in quadrature for constant V/f ratio (220 V/50Hz)
(-- φ_a=0 rad; - φ_a= π/8 rad; -o φ_a=$-\pi$/8 rad)*

Harmonics torque analysis in a single-phase induction motor

An induction motor with two unsymmetrical stator windings can be operated either as a single-phase motor or as a two-phase motor. In Publications 1 and 4 several solutions for a two-phase inverter-fed induction motor are proposed. One important aspect that affects the motor performance is the amplitude of pulsating torques and the harmonics content of the instantaneous electromagnetic torque. A comparative study of torque harmonics is made in Publication 1. Simulations were made using a simple mathematical model that includes the core losses effect through a fixed value resistance. The instantaneous torque values were recorded using a torque transducer only for the case when the motor was operated as a capacitor-run motor and directly connected to AC mains. For operation as a PWM inverter-fed motor with voltage open loop control, only the general magnetic noise level was recorded in order to verify that no harmonic pulsating torque exceeds the computed amplitude value.

Figure 5.2 and Figure 5.3 are obtained through a FFT decomposition of the computed or measured data. One can note in Figure 5.2 that the main pulsating torque in a capacitor-run single-phase induction motor is produced by the double frequency supply pulsating torque. The computed fundamental harmonic is 0.2Nm for the sinusoidal voltage fed motor case and 0.15Nm for the PWM inverter-fed motor. The test motor has a 0.1Nm rated torque. The other harmonic components are negligible. By comparison with the results presented in Figure 5.3, it is expected that the instantaneous torque of a two-phase induction motor connected to an inverter has a smaller pulsating component than the capacitor-run motor. However, a much wider harmonics spectrum is present in a PWM inverter fed motor solution; consequently, even though the pulsating torque amplitude level is acceptable, the interference with high-frequency equipment may lead to the conclusion that this solution is prohibitive. This phenomenon could be explained by the elimination of the negative sequence voltage and, consequently, of the unbalanced stator cage torques and pulsating torque components. The validation of this assumption represents a further research task for the author and will need accurate torque, noise and vibration measurements that can be subjected to Fourier series decomposition.

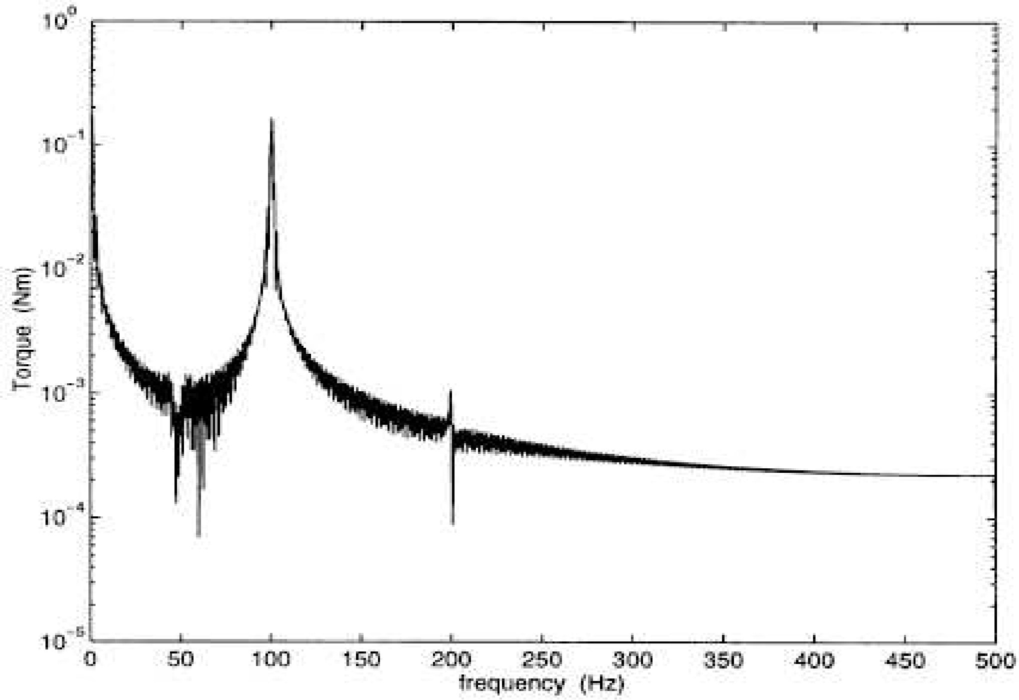

Figure 5.2 Torque harmonics content, computed from test data for capacitor-run single-phase induction motor. Fundamental frequency = 50 Hz.

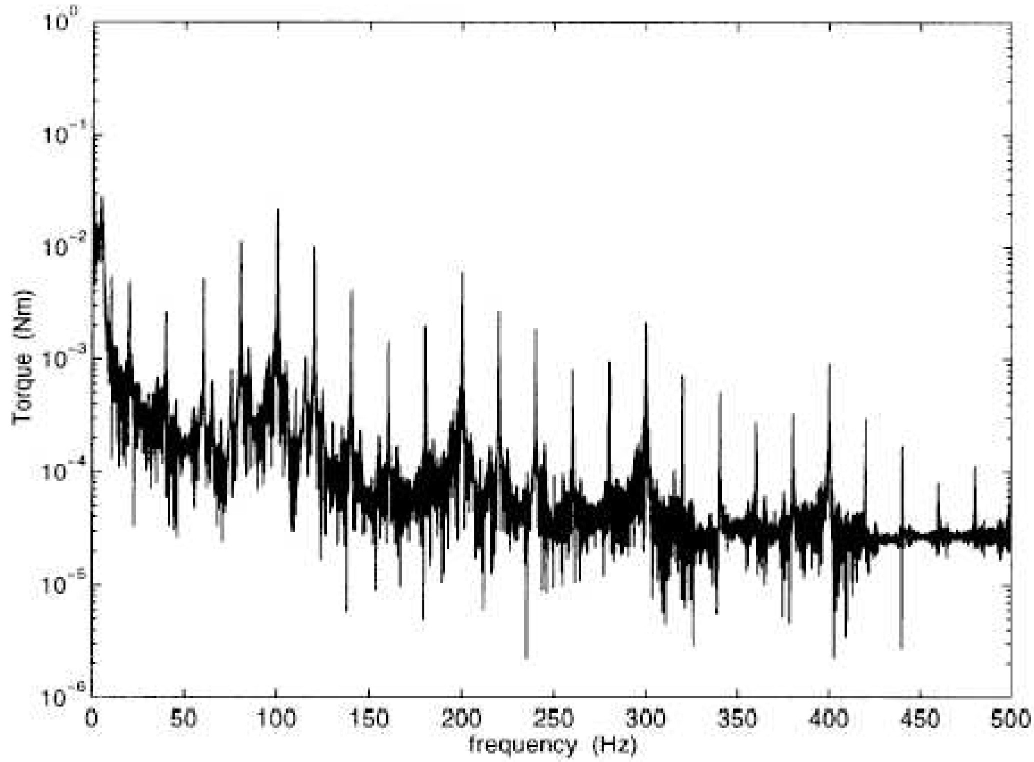

Figure 5.3 Torque harmonic content, simulated for two-phase PWM inverter-fed induction motor
Fundamental frequency = 50 Hz, Chopping frequency = 5 kHz.

In Publication 1, it is shown that from the line current point of view, the inverter-fed motor configuration exhibits a a total harmonic distortion (THD) coefficient of 62.3% as compared with 1.2% in the capacitor-run motor case. The PWM inverter was a H-bridge configuration as described by Mhango and Perryman (1997)

Starting torque performance in single-phase line-start permanent magnet motor

Publications 5, 6, 8 focus on the application of a single-phase line-start permanent magnet motor. As experiments and theory (Miller, 1985), (Williamson and Knight, 1999), (Lefevre, 2000) show, this motor configuration has a potentially higher efficiency than its equivalent induction motor. Beneficially for synchronous-speed operation, the permanent magnets determine during starting a braking torque and important oscillations in the instantaneous electromagnetic torque value. Even though the theoretical causes of torques were already identified for the three-phase motor by Honsinger (1979, 1980), the single-phase motor was completely uncovered previously. Publications 5 and 6 are the first known attempts to develop an analytical model for the asynchronous performance of the single-phase line-start motor.

In P5 and P6, an original model to compute the cage torques the magnet braking torque (determined by the stator winding induced currents due to rotor movement) and the fundamental pulsating torque components is proposed. Two main assumptions are made:

(a) effects of cage and magnets can be superimposed and treated separately;
(b) stator windings have the same copper weight and therefore the symmetrical components theory will determine two uncoupled equivalent circuits.

The validity of the model is verified by testing four motors with different stator winding configurations and distribution such that only two motors respect the assumption (b). The equivalent circuit parameters used in computations are obtained from winding data and geometrical dimensions using special algorithms developed at SPEED Laboratory (Hendershot and Miller, 1994), (Miller, 1989). The estimation of *d*-*q* axes inductance is made according to the method developed by Miller et al. (2003).

The magnet braking torque is produced by the fact that the magnet flux generates currents in the stator windings, and is associated with the loss in the stator circuit resistance. The variation of this torque with speed follows a pattern similar to that in the induction motor, but the per-unit speed takes the place of the slip. The magnet braking torque should not be confused with the synchronous “alignment” torque that arises at synchronous speed, even though the magnet braking torque is still present at synchronous speed and therefore diminishes the output and the efficiency.

During starting, the accelerating torque of the permanent magnet AC motor is the average cage torque minus the magnet braking torque and the load torque. The average cage torque is developed by “induction motor action”, except that the saliency and the unbalanced stator voltages complicate the analysis and may compromise performance. The magnet alignment torque has a non-zero average value (i.e., averaged over one revolution or electrical cycle) only at synchronous speed. At all other speeds it contributes an oscillatory component of torque. The same is true of the reluctance torque. As the rotor approaches synchronous speed, the screening effect of the cage becomes less, and, as the slip is very small, the oscillatory synchronous torques (alignment and reluctance) cause large variations in speed that may impair the ability to synchronize large-inertia loads.

Figures 5.4 and 5.5 show the validation of the simulated results versus the test data for two of the motors tested in Publications 5 and 6. In Figure 5.4, a motor with more turns in the main winding than in the auxiliary winding (β = 1.42) is analysed, while in Figure 5.5 a motor with less turns in the main winding than in the auxiliary winding (β = 0.70) is analysed. Torque measurements were performed first when the rotor was without magnets ($T_{e\ no\ magnet}$) and secondly when the rotor was equipped with magnets ($T_{e\ test}$). This way, the influence of the magnets over the starting performance could be observed. The computed resultant electromagnetic torque is shown as a dotted line ($T_{e\ sim}$). It has to be noted that the difference between $T_{e\ test}$ and $T_{e\ no\ magnet}$ is not entirely due to the magnet braking torque. The saturation level is modified when the magnets have been inserted into the rotor as compared to the case when the rotor is not equipped with magnets. Consequently, the magnetizing reactance values in *d-q* axes are changed. Another important observation can be made in relation with the equivalent circuit parameters variation during starting. As observed, the computed torque exhibits a good agreement with test data, while the parameters are kept constant during simulations; inductances are computed taking into account the saturation level while resistances are computed taking into account the temperature rise effect. The author proposes the use of a set of parameters computed or measured for saturated conditions and synchronous speed, as this approach is likely to lead to a sufficiently accurate prediction of the motor starting performance.

In Figures 5.6 – 5.7, the computed dynamic torque and quasi-steady state average resultant torque (solid line) and the envelope of the instantaneous torque are presented (dashed lines). Note the torque behaviour at the asynchronous operation when the auxiliary capacitive impedance ($\underline{Z}_C$) is switched from the capacitor-start to the capacitor-run fixed value. The minimum and maximum envelope trajectory (T_{envmax},T_{envmin}) are obtained by superimposing the effect of the pulsating torque components over the average resultant torque. This approach ignores the mechanical pulsation due to rotor/load inertia and assumes that even though pulsating torque components vary with different frequencies, their global effect may be simulated by superposition. The slight difference between the quasi steady-state torque (i.e., average torque variation with slip) and dynamic instantaneous torque is due to the rotor inertia influence and the pulsating torque variation with frequency harmonics. All simulations have been implemented ignoring saturation and core loss.

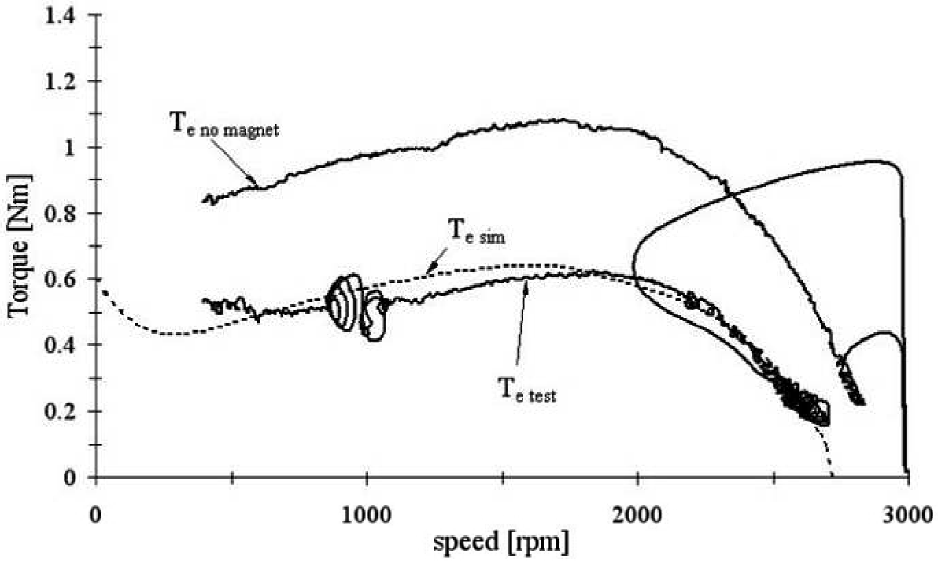

Figure 5.4 Experimental and computed torque variation vs. speed during no-load starting operation for a capacitor-start single-phase line-start permanent magnet motor with β = 1.42

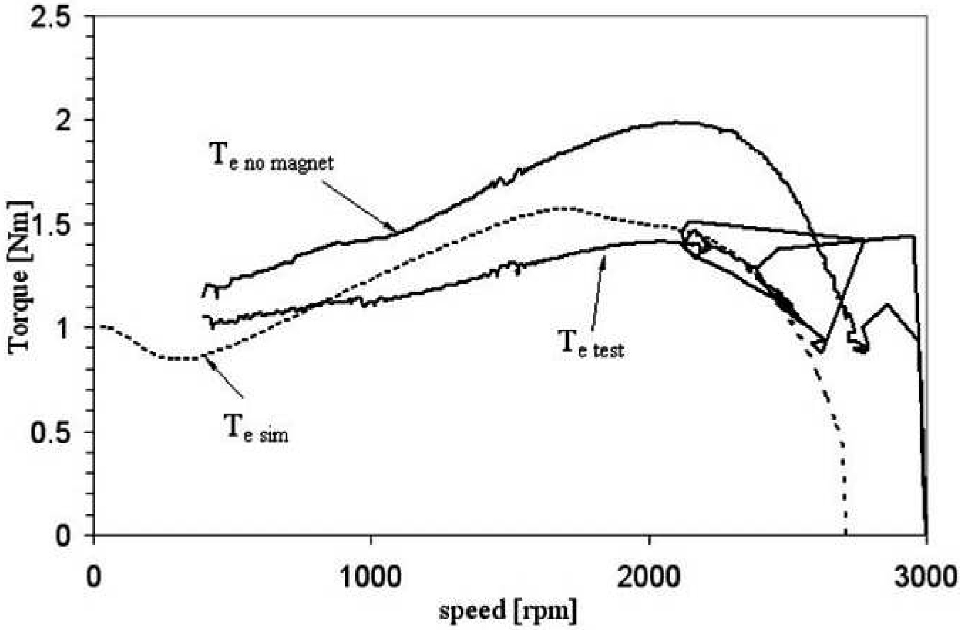

Figure 5.5 Experimental and computed torque variation vs. speed during no-load starting operation for a capacitor-start single-phase line-start permanent magnet motor with β = 0.70

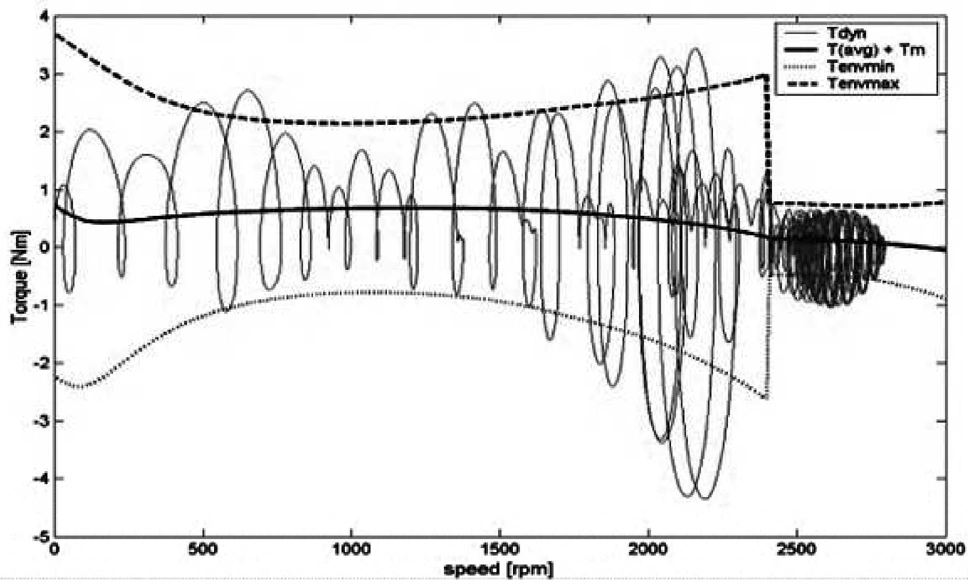

Figure 5.6 Computed dynamic torque variation vs. speed during no-load starting operation for a capacitor-start single-phase line-start permanent magnet motor with β = 1.42

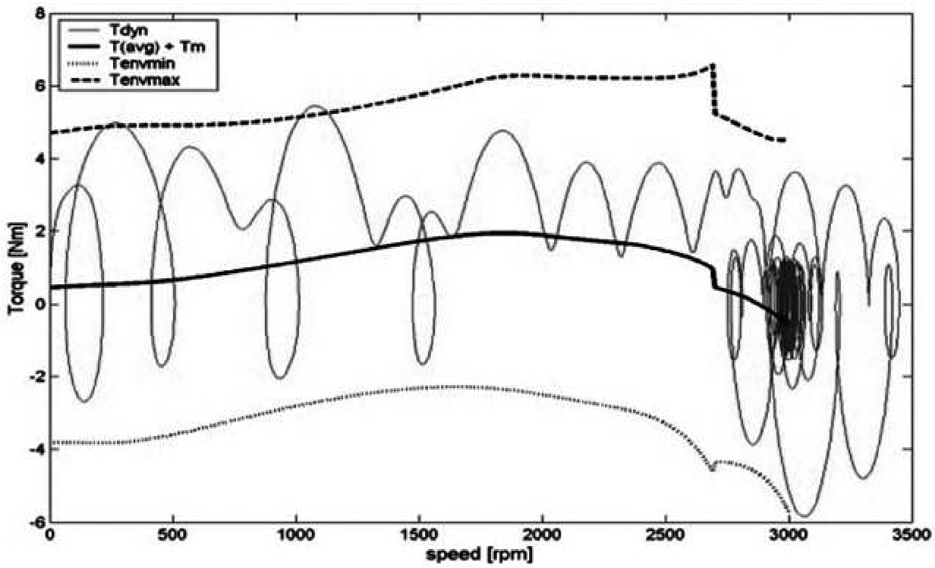

Figure 5.7 Computed dynamic torque variation vs. speed during no-load starting operation for a capacitor-start single-phase line-start permanent magnet motor with β = 0.70

All the important characteristics of the torque components are described in detail in P5 and P6. The line-start permanent magnet motors may present a rotor cage with variable shape area bars. Some examples of such an asymmetrical cage rotor are described by Myiashita et al. (1997) or Lefevre (2000). The asymmetry of the rotor cage will determine the Goerges phenomenon that may lead to an important dip around half synchronous speed. The tested motors in P5 and P6 are equipped with a symmetrical rotor cage that is realised with identical shape and area rotor bars. It is important to mention that in *d-q* axes theory the cage rotor parameters can be obtained in the following ways:

a) symmetrical rotor cage – computation of the referred rotor resistance on the basis of the rotor bars and end ring dimensions; measured by no-load and locked rotor test where the magnets are extracted from the rotor. The *d-q* axes rotor resistances (R_d, R_q) are equal to the computed or measured referred rotor resistance value;
b) asymmetrical rotor cage – computation of *d-q* axes rotor resistance values realised as in the classical theory of the synchronous machines with salient poles and damper cage; the individual measurement of the different *d-q* axes rotor resistance parameters is not possible, a combination of no-load and locked rotor tests will provide just the average of the *d-q* axes rotor resistances.

The rotor bars resistance value has to be optimised as a high value (small rotor bar area, higher number of bars) that will give good starting torque but poor synchronisation capability, as demonstrated by Miller (1984), and a low value (higher rotor bar area, lower number of bars) that will improve synchronisation but worsen the starting performance.

Magnet braking torque determines an important whirl at low-speed (see Figures 5.4, 5.5); magnet braking torque minimisation techniques (lower magnetization magnets, higher air-gap, lower number of turns) usually lead to a worse synchronous-speed performance. At low-speed, important torque oscillations occur; there are six pulsating torque components during asynchronous operation and two at synchronous speed, the most important pulsating torques vary with 2*f* and 4*f*, where *f* is the supply fundamental frequency.

The rotor asymmetry is responsible for the non-zero reluctance pulsating torque at standstill, and the stator asymmetry is responsible for the non-zero unbalanced stator pulsating torque even at synchronous-speed operation. For a single-phase permanent magnet motor, the proper selection of a capacitor to obtain a balanced stator voltage system will lead only to the minimisation of the stator asymmetry effect. The rotor asymmetry effect cannot be eliminated.

The study of the starting performance of a single-phase line-start permanent magnet motor is further developed in Publication 8. The same motor equipped with identical stator windings was operated as a capacitor-start, capacitor-run motor using different fixed values capacitors and as PWM inverter-fed. The employed inverter was a three-phase configuration and the control strategy was as described by Holmes and Kotsopoulos (1993).

In Figures 5.8 – 5.9, the measured and respectively computed torque during starting are illustrated. The starting torque is higher for the capacitor motor (up to 1.5 Nm for the tested motor) when compared to the inverter-fed motor (0.5 Nm), for the same main supply voltage amplitude. The PWM inverter-fed motor exhibits a lower starting current level. Tested data were 4 A rms for the PWM fed motor as compared to a maximum starting current 5.5 A rms for the capacitor motor. The harmonics torque content are expected to exhibit a lower zero to peak amplitude.

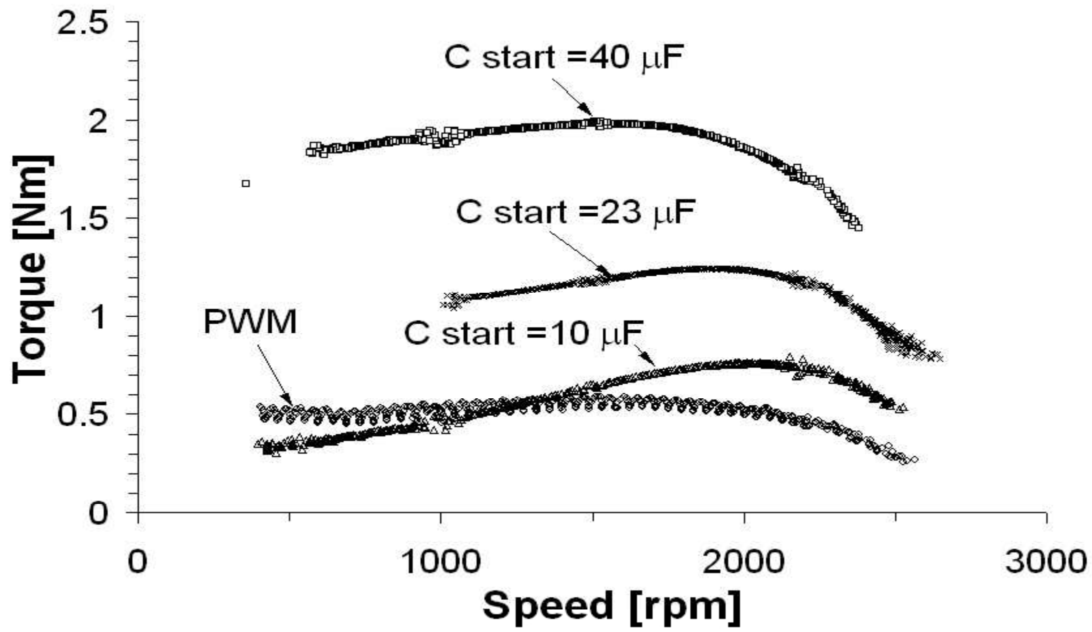

Figure 5.8 Experimental torque variation vs. speed during starting
(capacitor permanent magnet motor – C start and inverter-fed permanent magnet motor – PWM)

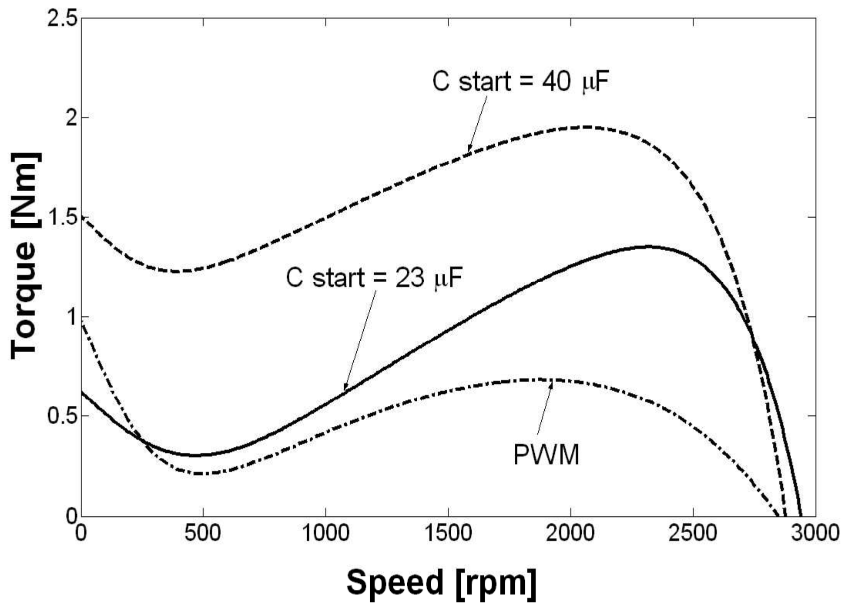

Figure 5.9 Computed torque variation vs. speed during starting
(C start – capacitor permanent magnet motor and PWM – inverter-fed permanent magnet motor)

This phenomenon could be explained by the elimination of the negative sequence voltage and consequently of the unbalanced stator cage torques and pulsating torque components. The validation of this assumption represents a further research task for the author and would need accurate torque, noise and vibration measurements that could be subjected to Fourier series decomposition.

From the starting torque capabilities point of view, the capacitor motor performance easily overcomes the open loop inverter-fed motor performance. Test data and computed data demonstrate that for high load starting conditions the capacitor motor represents the optimum solution. For low load starting conditions, the PWM inverter-fed motor represents a reasonable alternative solution.

Saturation of the magnetic circuit is particularly complex in this motor type; different sections of the machine saturate independently, causing large and sometimes time-varying changes in equivalent-circuit parameters such as inductances X_d and X_q and EMF (E_0). Accordingly, two series of finite-element calculations were carried out, one with current only in the d-axis and the other with current only in the q-axis. For each solution, X_d and X_q were obtained. The result is given in Figure 5.10, which expresses X_d as a function of I_d with $I_q = 0$, and X_q as a function of I_q with $I_d = 0$. In all cases, it is assumed that E_0 is constant.

Figure 5.10 shows a huge variation of 6:1 in X_q almost 2:1 in X_d. Calculations with current flowing simultaneously in both axes show that X_d is affected by I_q, being increased when $I_d < 0$ and decreased when $I_d > 0$, with $I_q > 0$. The discontinuity at $I_d = 0$ or $I_q = 0$ can be attributed to an error or variation of the back EMF E from the open-circuit value E_0. If E is assumed constant and equal to E_0, the value of X_d that will be inferred by using only the term E/I_d, which is indefinite when $I_d = 0$. Evidently the effect of cross-saturation is that E depends on the current components I_d and I_q, as do X_d and X_q. Note that although the finite-element method can be used to calculate E_0, X_d and X_q for use in the phasor diagram, this method applies only to motors that have sinusoidal distributed windings and sinusoidal waveforms of EMF, current, and terminal voltage. An important and accurate alternative to estimate the electromagnetic torque is the flux-MMF diagram (Miller et al., 2003).

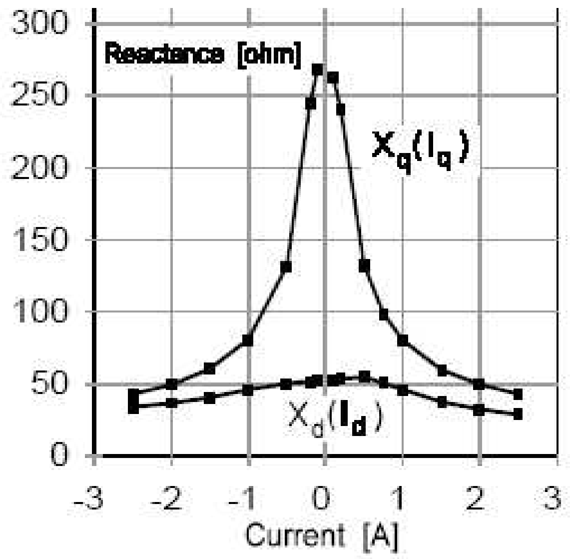

Figure 5.10 Variation of X_d vs. I_d with I_q = 0, and of X_q vs. I_q with I_d = 0, calculated using finite element data with constant E_0.

In the previous sections it was emphasised that for a correct implementation of the symmetrical components theory, all the equivalent circuit parameters have to be referred to one of the stator windings, main or auxiliary (as in the actual approach). If the stator windings have a similar distribution (coils span, total number of coils, etc.) when computing or measuring the *dq* axes synchronous reactances, there is no significant variation in the predicted results regardless of the choice of the reference stator winding. However, when the differences between stator winding are insufficiently described just by using the turns ratio, one should use only the reference winding for inductance computation and/or estimation through experimental data. The different saturation levels that might occur when the main or the auxiliary winding is used as reference justify this.

Single-phase induction motor with a three-phase delta winding

Publication 7 is dedicated to the analysis of a two-speed single-phase motor with special windings, 2 pole and 18 pole. The high ratio between the poles number of the two stator windings determines a series of challenging tasks in motor design. In this chapter, the 18-pole configuration is discussed. It is produced through a Steinmetz delta connection of a 3-phase winding to a single phase voltage supply. The Steinmetz connection is advantageous because it has a higher winding factor than a main or auxiliary winding and also eliminates the third-order harmonics of the air-gap m.m.f. (Stepina 1982). Furthermore, the end-coil dimensions are reduced and, as a result the end-leakage reactance and the end-winding resistance, are relatively low. The Steinmetz delta connection has an advantage over a Steinmetz star connection in that a smaller capacitor is required to produce the same power output. This is because for the same single-phase voltage supply the number of turns, and therefore the winding impedances are higher for a Steinmetz delta connection.

The theory presented in P7 demonstrates that a Steinmetz connection is equivalent to, and can be fictitiously replaced by an auxiliary and a main winding with an effective ratio of turns (aux/main) $k = \sqrt{3}$. Because the ratio of turns is larger than 1, the motor can be balanced only by using a capacitive impedance connected in series with the auxiliary winding (Veinott 1959, Stepina 1982). The developed mathematical model links the fictitious equivalent circuit to the measurable parameters. Similar models based on symmetrical components theory are more complicated and fail to identify a test method for their proposed methods (Oliveira 1990, Tozune 1991).

The 60 Hz, 18-pole motor configuration is operated as a permanent split capacitor type; the same capacitor is used both for starting and running operation and has to be optimised for both situations.

In Figure 5.11 the experimental and computed steady-state torque vs. speed are plotted. As the turns ratio is fixed in a Steinmetz connected motor, it was used in the double-revolving field model (Morrill 1929, Veinott 1959) to study only the influence of the capacitor selection on the example motor performance and obtained the results plotted in Figure 5.12 and 5.13. The region of most interest is in between 50 to 100μF, where the starting torque and the average torque at rated load reach a maximum, and the amplitude of the pulsating torque at rated load has a minimum. Also, within this capacitance range, the ratio of the starting torque and starting current achieves a maximum.

It is interesting to note that for the considered motor example, a phase angle of 60 degrees, which would ensure fully balanced operation, cannot be achieved at a relatively large load of 5.5lbft (7.45Nm) at 335rpm. In order to completely eliminate the torque pulsation not only the capacitance but also the turns ratio of the equivalent auxiliary and main winding should be modified, which is clearly not feasible with a Steinmetz connection. However, by optimal choice of the capacitor, the pulsating torque is minimized at a level that is totally satisfactory for typical applications, while the starting and rated torque requirements are met.

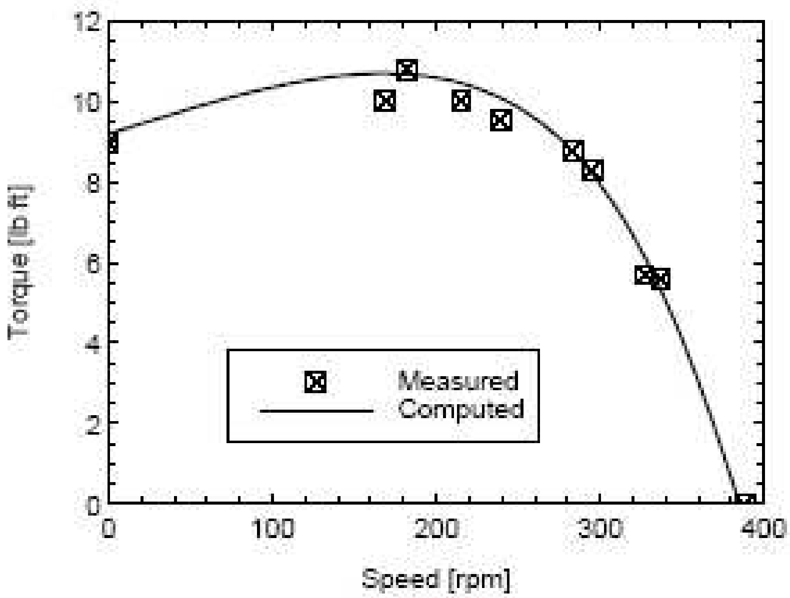

Figure 5.11 Experimental and computed torque vs. speed for an 18-pole single-phase induction motor with Steinmetz delta connected three-phase stator windings (1 lbft = 1.3557 Nm)

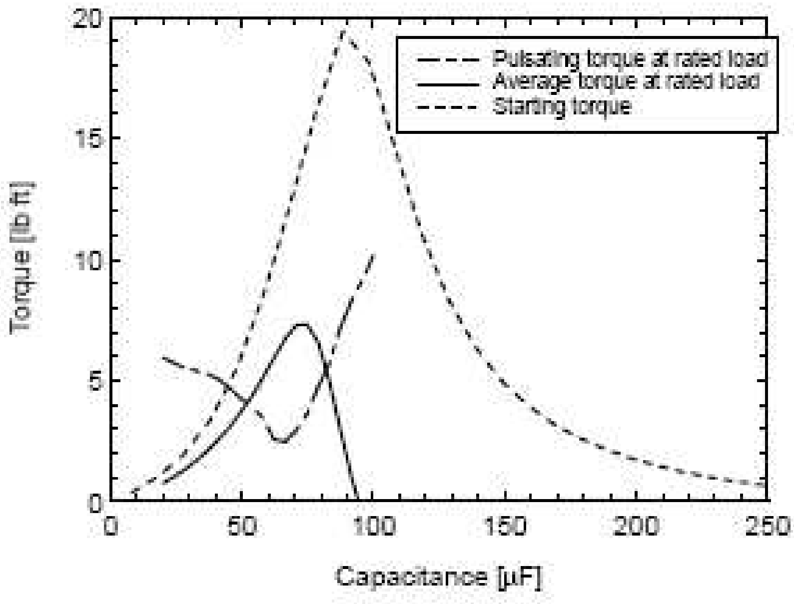

Figure 5.12 Starting and rated load torque as a function of capacitance in the 18 pole motor configuration with Steinmetz delta connected three-phase stator winding (1 lbft = 1.3557 Nm)

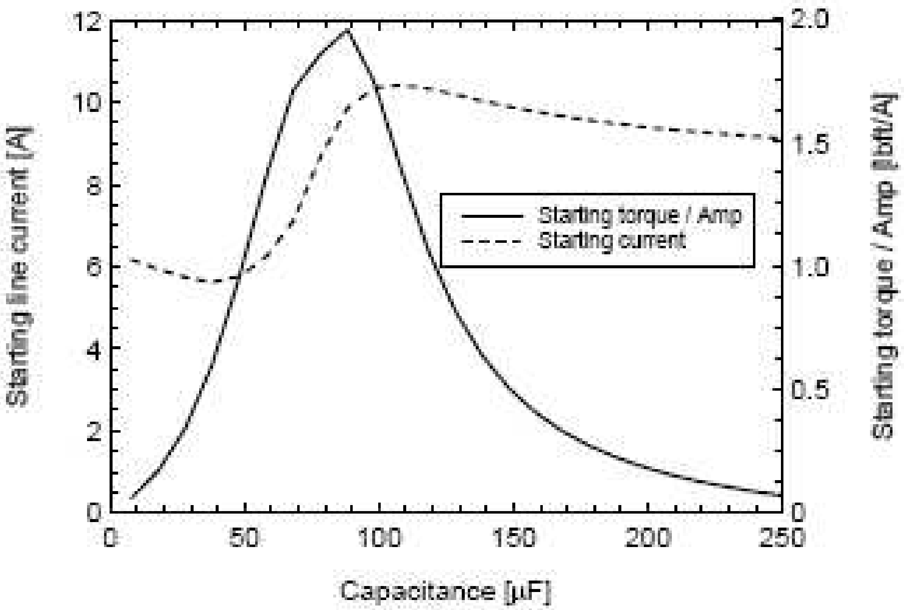

Figure 5.13 Starting torque per amp as a function of capacitance in the 18 pole motor configuration with Steinmetz delta connected three-phase stator winding

Conclusions

In this chapter, the main results of this thesis are presented and discussed briefly. A simple mathematical model verifies the influence of the shift angle between stator winding magnetic axes over the torque pattern. The results show good agreement.

Harmonic torque analysis is performed for an induction motor with two unsymmetrical windings. A comparative study is made to observe different problems that might occur when the motor is run as a capacitor-run motor or as an inverter-fed motor. All the computations are achieved using a simple mathematical model that may incorporates the core loss effect.

Also in this chapter, the permanent magnet influence over the starting performance of a single-phase line-start motor is mathematically modelled. The most important torque components (cage, magnet braking, pulsating) are computed. The test data show very good agreement with computations. Following the torque analysis, important design features are recommended. The study is completed with a comparison between motor asynchronous torques when different fixed capacitors are used, and an electronically controlled solution is proposed. The developed model may be extended for the simulation of other AC motors (induction or synchronous reluctance) or to include non-linear effects (saturation, core losses).

Finally, a motor with delta connected three-phase winding supplied from a single-phase system is analysed. Through a symmetrical components transformation, the actual winding is substituted with a fictitious two-axis winding that permits the analytical torque prediction using classical methods. Computed and experimental results are in very good correspondence.

6 SUMMARY

In this work, the electromagnetic torque in single- and two-phase AC motors has been studied. The methods of analysis have been developed in order to obtain a sufficiently accurate estimation for different torque components. Two motor types are analysed: induction motors and line-start permanent magnet motors. The mathematical models that are developed may be used for the analytical torque prediction of other single- or two-phase AC motors, i.e., synchronous reluctance or shaded pole induction motors.

For single-phase induction motors several models are described. The following torque components are analytically estimated: average, pulsating, instantaneous. Their cause and ways of improving motor performance are discussed. These models include important non-linear field effects: cross-coupling between stator windings due to the non-orthogonal magnetic axes. magnetization inductance saturation, and resistance limited eddy currents. How these non-linear phenomena influence the motor performance is presented in detail. On the basis of the new models developed, several new topologies for driving the single- or two-phase induction motors are proposed and analysed. It is believed that even though these new solutions have not been subjected to a thorough study in terms of economics, single-phase induction motors fed from PWM inverters with scalar and vector control strategies may represent reliable alternative drives in the near future.

For a line-start permanent magnet motor, an original model has been developed. The following torque components are analytically estimated: asynchronous (cage), magnet braking, pulsating, steady-state, alignment, reluctance torques. The basic assumption in the computation of torque components is that the effects of electromagnetic fields due to different causes (stator currents, permanent magnets, variable reluctance) can be superimposed. This analytical solution is the first known attempt to describe all the torque components - starting, synchronisation, and synchronous speed operation - that might occur during this motor operation. Due to its potentially higher efficiency, the single-phase version of the line-start permanent magnet motor may be an economical alternative for single-phase induction motors in some specific applications such as refrigerator compressors or air-conditioning systems. As with to the single-phase induction motor case, an electronically controlled solution is proposed for this motor type. The PWM inverter-fed motor is analysed in comparison with the capacitor-start, capacitor-run motor.

The equivalent circuit parameters in all the developed models have been estimated either analytically, using classical theory, or numerically using finite-element analysis. Where possible, test data have been used to provide a higher accuracy level for the value of the motor parameters. Specific experimental settings were employed to validate the newly proposed analytical model for the line-start permanent magnet motor.

The main results of this study can be formulated as follows: the two axis (d-q) theory may be used for a unified single-phase electrical AC-machine analysis; simple five-order mathematical models with fixed value parameters can be employed for the preliminary design of single-phase induction motors or permanent magnet motors; the accuracy level of the predicted electromagnetic torque depends on the precision of the equivalent circuit parameters estimation; the developed models can be connected to a combined circuit-finite element analysis; the developed models can be easily implemented in further electrical drive control schemes; the developed mathematical models allow the study of the influence of different factors over the electromagnetic torque pattern and have been successfully used for educational purposes.

Further work is intended to model and implement the same torque behaviour using numerical methods (2D finite element). This numerical analysis will include saturation, core loss effects and time-stepping algorithms. It is also necessary to develop analytical models of the single- and two-phase AC motors that contain the space and time harmonics effects. By completion of these further research steps, the author considers that the trade-off between accuracy and computation time can be achieved. Thus, an important tool could be available for electrical machinery design engineers and for academic purposes.

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