Improved MIMO Performance with Non-Orthogonal Space-Time Block Codes

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Abstract— **Transmission over Multiple Input Multiple Output (MIMO) radio channels is considered. When multiple Rx antennas are present, one may simultaneously increase the rate and improve performance by optimizing transmit diversity, using non-orthogonal space-time block codes. This improves the performance of MIMO systems considerably, especially when the number of Tx and Rx antennas is small. As an example, we consider a 2 Tx, 2 Rx system, and construct a 2×2 non-orthogonal space-time block code with symbol rate 2, which performs 2.5 dB better than the corresponding BLAST system at BER 10^{-3}.**

I. INTRODUCTION

It is a remarkable fact that employing multiple antennas both at the transmitter and receiver promises huge capacity increases [1], [2]. Indeed, the capacity increases linearly with the minimum of $N_{\rm Tx}$ and $N_{\rm Rx}$, the number of Tx and Rx antennas.

Methods to realize this capacity increase were suggested in [1] (D-BLAST) and in [3] (V-BLAST). The basic idea of the BLAST systems is to transmit a different data symbol from each antenna during each symbol interval, and to use interference cancelling methods to decorrelate the symbols, based on the received signals from a multitude of uncorrelated Rx antennas.

A different way to use a multiple Tx-antenna resource is to create transmit diversity.The most effective methods for this are various methods of space-time coding [4], [5], [6], [7].

The dominating view in the community has been that these uses of the multiple Tx antenna resource are mutually exclusive; for pure MIMO systems with $N_{\rm Tx} \leq N_{\rm Rx}$, BLAST modulation is used, and for systems with $N_{\rm Tx} > N_{\rm Rx}$ space-time coding is used at least partly. Combining space-time coding and BLAST for $N_{\rm Tx} > N_{\rm Rx}$ has been investigated in e.g. [8], [9], [10]. The space-time coding part of BLAST-systems is usually effected by a horizontally, vertically or diagonally concatenated channel code. These are effective methods, when $N_{\rm Rx}$ is large. With small $N_{\rm Rx} \sim N_{\rm Tx} > 1$, the transmit diversity capabilities omitted by the BLAST modulation (and only partially tapped by the concatenated channel codes) become important [11]. This leads to the idea that diversity and rate increase should be incorporated simultaneously, in a well designed space-time modulation. One way to do this is to use codes based upon unitary constellations. These are capacity achieving [12], and thus rate efficient, but simultaneously improve performance due to transmit diversity. Generic unitary space-time modulations are non-linear, however.

From decoding point of view, linearity is a desirable trait as it enables the use of linear algebraic methods keeping decoding complexity polynomic in the number of antennas and data rate. Linear unitary space time modulations are nothing but space-time block codes based on orthogonal designs [6], [7]. These have the unfortunate property that the achievable rate falls off exponentially with the number of antennas, which is a consequence of the incommensurability of linearity and unitarity [13]. With complex modulation alphabets and $N_{\rm Tx} > 2$, it is impossible to reach symbol rate 1 [7], let alone the multiple rates needed to fully exploit the MIMO channel capacity. This impairs the achievable capacity if multiple receive antennas are deployed [14], [11].

To increase the rate, keeping linearity, one has to sacrifice orthogonality. This leads to the concept of non-orthogonal space-time codes [15], [16], [11], [17][1] The idea of non-orthogonal space-time block codes is to increase the rate of a space-time block code by partly relaxing the orthogonality requirement in a controlled fashion. Explicit constructions for MIMO channels were pioneered in [11].

In this paper, we investigate non-orthogonal space-time block codes designed for a MIMO channel, MIMO-NOSTBC's. We shall simultaneously increase rate and aim for optimal performance through increased Tx-diversity. We shall see that the the non-orthogonal space-time block codes with multiple rates presented in the literature [11] are deficient when it comes to transmit diversity. This is due to singular points in the codeword difference matrices. Here we make use of the full algebraic machinery developed for non-orthogonal space-time block codes in [17]. Performance is optimized with possibly matrix valued constellation rotations, which rotate away from the singular points. The design principles developed in these references, combined with the well known design principles of space-time codes in general [4], [5] make the problem of designing optimal MIMO-NOSTBC schemes transparent.

[1] In [11], these codes were called linear dispersion codes. In [16] they were called quasi-orthogonal codes.

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The performance benefits of MIMO-NOSTBC's are is likely to decrease with an increasing number of antennas. With increasing numbers of Tx antennas, the scheme becomes further removed from orthogonality, with more and more self-interference. Also, the Rx diversity from the increased number of Rx antennas is already high, and it need not be improved with additional Tx diversity. Indeed, for the number of antennas going to infinity, a random space-time code is optimal, and the easiest way to construct such a code is to let the random channels do the job. This is what happens in the BLAST schemes.

For low number of antennas, adding simultaneous Tx diversity to MIMO schemes may improve performance significantly. Here we consider a system with 2 Tx and 2 Rx antennas, and a doubled data rate, i.e. four data symbols are transmitted in the two symbol period duration of the code. Compared to VBLAST at bit error rate 10^{-3}, the gain is 2.5 dB. Comparing to the scheme in [11], the benefit of optimizing the Tx diversity is 1.5 dB.

II. System Model

We investigate a communication system over Multiple Input, Multiple Output (MIMO) Rayleigh fading channels. For concreteness, two Tx and two Rx antennas are employed. No multipath is assumed. The channel realizations between the antennas are denoted by the matrix

$$\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} , \tag{1}$$

and the α_{ij} are assumed uncorrelated. Furthermore, we assume block fading; the α_{ij} are constant for the duration of a block of two symbol periods, and change arbitrarily from block to block, subject to Rayleigh statistics.

A baseband description of the signaling goes as follows. During each block of two symbol periods, four symbols $z_i,\ i = 1, \ldots 4$ are transmitted. The symbols are arranged in a 2×2 matrix C. The rows of C are transmitted during one symbol period from the two Tx antennas, the columns are transmitted from one antenna during two symbol periods. Thus the received signal at the two Rx antennas during the two symbol periods of the block turns out to be the matrix

$$r = C\vec{\alpha} \ + \ \text{noise} . \tag{2}$$

We assume full channel state information at the receiver and no channel state information at the transmitter. As the number of states is small, we use maximal likelihood (ML) decoding.

The performance of the optimal 2×2 non-orthogonal space-time code with rate 2 is compared to the performance of rate 2 BLAST, which transmits two symbols during one symbol period, i.e.

$$C_{\text{BLAST}} = \begin{bmatrix} z_1 & z_2 \end{bmatrix} , \tag{3}$$

and ML decoding. We also compare to the performance of the rate 1 orthogonal space-time block code of Alamouti [6],

$$C_{\text{Ala}} = \begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1 \end{bmatrix} , \tag{4}$$

received with two Rx antennas, and with the symbols in a modulation constellation with double the number of bits.

III. Non-Orthogonal Space-Time Block Codes

The two essential features of space-time block codes based on orthogonal designs [6], [7] are linearity and orthogonality (unitarity). In [13] it was shown that these two properties do not fit well together, which leads to stringent conditions on the number of dimensions in an unitary design, i.e. on the achievable symbol rate of the space-time block code. The maximal rate of a square matrix embeddable[2] unitary design is[3]

$$R_{\max} = \frac{\lceil \log_2 N_{\text{Tx}} \rceil + 1}{2^{\lceil \log_2 N_{\text{Tx}} \rceil}} , \tag{5}$$

To increase the rate over this, orthogonality has to be sacrificed.

First we consider the consequences of requiring linearity of the code. The code matrix can be expanded as

$$C = \sum_{k=1}^{K} C_k(z_k) , \tag{6}$$

where C_k are K matrices of dimensions $T \times N_{\text{Tx}}$ that are linear functions of the individual symbols z_k and their complex conjugates. The Hermitian square of C is

$$C^\dagger C = \sum_{k=1}^{K} C_k^\dagger C_k + \sum_{i<k} \left(C_i^\dagger C_k + C_k^\dagger C_i \right) \tag{7}$$

If one requires unitarity in the sense of orthogonal designs, the design rules are

$$C_k^\dagger C_k = |z_k|^2 \, I_N \quad \forall \, k , \tag{8}$$

and

$$C_i^\dagger C_k + C_k^\dagger C_i = 0 \quad \forall \, (i \neq k) , \tag{9}$$

with I_N the N_{Tx} dimensional identity matrix. The former condition requires each individual symbol to be encoded by an unitary matrix.[4] The latter, due to the linearity of C_k, leads to the complexified Radon-Hurwitz equation, see [7], eventually giving the constraint (5) on rates.

[2] One that can be constructed from a square matrix unitary design by canceling columns.

[3] The smallest integer larger or equal than $\circ$ is denoted by $\lceil \circ \rceil$.

[4] Here we use the term "unitary" loosely, to indicate a matrix for which the inverse is *proportional to* its hermitian conjugate.

IV. SPACE-TIME CODE DESIGN CRITERIA

A. Rank, Determinant and Trace

Design criteria for space-time codes presented in the literature are formulated in terms of the codeword difference matrix D^{ce}. Minimizing the pairwise error probability of deciding in favour of C_e when transmitting C_c leads to the well known rank [4] and determinant [5] criteria. Less known, but paramount for designing non-orthogonal space-time block codes, is the trace criterion [18]. This criterion is the following:

To optimize performance in Rayleigh fading C should be designed so that the eigenvalues of $D^\dagger\ D$ are as close as possible to each other *and* to $\mathrm{Tr}\left[D^\dagger D\right]/N$, and for which the row-wise sum of the absolute values of the elements off the main diagonal is as small as possible. Moreover, $\mathrm{Tr}\left[D^\dagger D\right]$ plays the role of Euclidean distance between codeword pairs.

From the linearity of the codes investigated here, it follows that the codeword difference matrix is linear in the symbol differences $\Delta_k = z_k^{(c)} - z_k^{(e)}$,

$$D^{(ce)} = \sum_{k=1}^{K} C_k(\Delta_k) \equiv \sum_{k=1}^{K} D_k\ . \tag{10}$$

The matrices D_k give the linear dependence of $D^{(ce)}$ on Δ_k. The distance matrix (the hermitian square of the codeword difference matrix), reads

$$D^\dagger D = \sum_{k=1}^{K} |\Delta_k|^2\ I + \mathcal{N} \tag{11}$$

with the non-orthogonality matrix

$$\mathcal{N} = \sum_{i<k} D_i^\dagger D_k + D_k^\dagger D_i\ . \tag{12}$$

B. Maximal Mutual Information (Minimal Non-Orthogonality)

The principle of maximizing the mutual information provided by a non-orthogonal space-time block code was suggested in [11]. It is equivalent to the principle of minimal non-orthogonality suggested in [15]. This means that the average ratio of $\mathcal{N}$ and $\sum|\Delta_k|^2$ in (11) should be minimized, which minimizes the inter-symbol-interference directly caused by the non-orthogonality of the code.

C. Maximal Symbolwise Diversity

Consider the Euclidean distance squared $\mathrm{Tr}\left[D^\dagger D\right]$. This is a real valued positive semidefinite quadratic function of the symbol differences Δ_k (and their complex conjugates). In a well designed code, the Euclidean distance is a monotonically increasing function of the number of bit-errors in a codeword pair. Preferably, the Euclidean distance squared is proportional to a sum of the symbolwise Euclidean distances squared, $|\Delta_k|^2$.

Next, from the linearity of the code it is clear that if the code does not provide full diversity protection against one-symbol errors, it cannot provide full diversity protection against multiple-symbol errors.

This leads us to the requirement of *Maximal Symbolwise Diversity* (MSD) [15] i.e. that also in a non-orthogonal case, the individual code matrices C_k should be unitary matrices with $C_k^\dagger C_k = |z_k|^2\ I$. Thus (8) is fulfilled. For a maximal symbolwise diversity code, the distance matrix is $D^\dagger D = \sum_{k=1}^{K} |\Delta_k|^2\ I + \mathcal{N}$. Comparing to the design rules in [11], MSD is a complex symbol version of the strictest (iii) of the real symbol constraints in [11]. This rule requires unitarity separately of the matrices A_k and B_k encoding respectively the real and imaginary parts of a given symbol z_k. MSD requires unitarity from the full matrix C_k, which poses an additional constraint on the Radon-Hurwitz commutator $A_k^\dagger B_k - B_k^\dagger A_k$ of the coefficient matrices.

The trace criterion now lends itself to optimizing the code by minimizing the effect of $\mathcal{N}$.

As an example, we consider $N_{\mathrm{Tx}} = 4, N_{\mathrm{Rx}} = 1$. The maximal rate allowed by orthogonality is 3/4. To optimally use the channel resource in this case, the symbol rate has to equal the number of degrees of freedom in the channel, which is $\min\left(N_{\mathrm{Tx}}, N_{\mathrm{Rx}}\right) = 1$ for uncorrelated Rayleigh fading. Schemes achieving this rate, with minimal non-orthogonality, and correspondingly maximal mutual information, have been designed in [15], [16]. A $N_{\mathrm{Tx}} = 3$ code belonging to the same equivalence class has been proposed in [11]. For a rate 1 code, it is possible to design the 4×4 matrices $C_k,\ k = 1, \ldots, 4$ so that they have no overlapping matrix elements. From this it follows that $\mathcal{N}$ is traceless, and the two branches of the trace criterion coalesce, namely that the eigenvalues of $D^\dagger D$ should be as close as possible to each other *and* to $\mathrm{Tr}[D^\dagger D]$. Thus the trace criterion is fulfilled if the ratio of the Euclidean distance square $\mathrm{Tr}[D^\dagger D]$ to the sum of the offdiagonal power in $\mathcal{N}$ is minimized. This is achieved by codes of the form [15], [16],

$$C_{4\times 4} = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 \\ -z_2^* & z_1^* & -z_4^* & z_3^* \\ z_3 & z_4 & z_1 & z_2 \\ -z_4^* & z_3^* & -z_2^* & z_1^* \end{bmatrix}\ . \tag{13}$$

These codes are deficient with respect to diversity. E.g. if the same error is made in z_1 and z_3, and no error is made in z_2, z_4, the distance matrix has rank 2, not 4. As shown in [17], these singularities may be avoided by (possibly matrix valued) constellation rotations, which prevent the same error to be made in e.g. z_1 and z_3.

D. Traceless Non-Orthogonality

Now investigate the distance matrix when the matrices C_k partly overlap. Maximal symbolwise diversity is required, as explained in the previous Section. In this case, the C_k should be designed so that the non-orthogonality matrix $\mathcal{N}$ is traceless [17]:

$$\mathrm{Tr}\mathcal{N} = 0 \,. \tag{14}$$

The reason for this is the following. Due to linearity, $D^\dagger D$ is a quadratic function of the symbol differences Δ_k. This is true for the non-orthogonal part of $D^\dagger\, D$ as well, and for the part of the Euclidean distance arising from non-orthogonality, namely $\mathrm{Tr}\mathcal{N}$.

As $D^\dagger D$ is a Hermitian matrix, its diagonal elements are real. The dominant diagonal terms in (11) are real. Thus the diagonal elements of $\mathcal{N}$ are real as well. The only possibility is that they are some real combinations of two symbol differences, i.e. $\mathrm{Re}\,[\Delta_k \Delta_l]$ and/or $\mathrm{Im}\,[\Delta_k \Delta_l]$.

In a generic complex modulation scheme, a number of Δ_k:s may be related by rotations in the complex plane. Now, if $\mathcal{N}$ has a trace, the value of the trace, and thus the Euclidean distance changes by rotating Δ_k, even if the other Δ_l are averaged over. This means that the Euclidean distance between a symbol and its nearest neighbors differs from the distance between an equivalent rotated symbol and its nearest neighbors. Thus the modulation points are not homogeneously situated in the multidimensional space-time code space, and the space-time block code cannot be optimal.

E. Symbol-to-Symbol Homogeneity

Another principle which follows directly from requiring homogeneity in constellation space is symbol-to-symbol homogeneity [17], which means that all symbols should be coded with similarly in relation to the others.

V. MIMO-NOSTBC with 2 Tx, 2 Rx, Rate 2

Now we want to design a code with symbol-rate 2, employing 2 Tx antennas, designed for reception with more than 2 Rx antennas. The temporal length of the code is taken to be 2. Thus the ensuing 2×2 code comprises 4 symbols. Applying maximal symbolwise diversity, minimal non-orthogonality and symbol-to-symbol homogeneity, it is clear that the symbols should be encoded in two orthogonally encoded pairs:

$$C(s_1,s_2,s_3,s_4) = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} + U \begin{bmatrix} s_3 & s_4 \\ -s_4^* & s_3^* \end{bmatrix} V \,. \tag{15}$$

Here, U and V are two 2×2 unitary matrices. Requiring traceless non-orthogonality leads to

$$\begin{aligned} \det U &= -1 \\ V &= I \end{aligned} \tag{16}$$

or vice-versa. Here I is the two-dimensional identity matrix. Isolating the negative sign of the determinant to a multiplying matrix $W = \mathrm{diag}[1,-1]$, UW is an element in SU(2), the group of special unitary 2×2 matrices. This group is a three-dimensional manifold. The rank and determinant criteria may now be used to find the optimal rotation matrices UW within this manifold.

In [11], the MIMO-NOSTBC code (15) with diagonal

$$UW = \begin{bmatrix} \mathrm{e}^{\mathrm{j}\,\pi/4} & 0 \\ 0 & \mathrm{e}^{-\mathrm{j}\,\pi/4} \end{bmatrix} \tag{17}$$

was proposed. It was shown that this code (and correspondingly, all codes in the family satisfying (16) provide maximal mutual information. The code with (17), however, is not transmit diversity nor performance optimal. Consider an error event, where the same error Δ is made in z_1 and z_4, and no error is made in z_2, z_3. The product distance is

$$\det\left(D_{ce}^\dagger D_{ce}\right) = \det \begin{bmatrix} \Delta & \mathrm{e}^{\mathrm{j}\,\pi/4}\Delta \\ \mathrm{e}^{-\mathrm{j}\,\pi/4}\Delta^* & \Delta^* \end{bmatrix} = 0 \,. \tag{18}$$

For this kind of error events, the transmit diversity protection is only one. To improve on this, one may take an off-diagonal matrix UW. The union bound of pairwise error probabilities is minimized at $E_b/N_0 = 10$ dB with the choice

$$UW = \begin{bmatrix} \mathrm{e}^{\mathrm{j}\,7\pi/20}\cos\left(\frac{9\pi}{50}\right) & \mathrm{e}^{\mathrm{j}\,\pi/4}\sin\left(\frac{9\pi}{50}\right) \\ -\,\mathrm{e}^{-\mathrm{j}\,\pi/4}\sin\left(\frac{9\pi}{50}\right) & \mathrm{e}^{-\mathrm{j}\,7\pi/20}\cos\left(\frac{9\pi}{50}\right) \end{bmatrix} \tag{19}$$

Performance of these two rate 2 MIMO-NOSTBC schemes for 2 Tx, 2 Rx antennas can be found in Figure 1. As the space-time block codes are considered as modulation schemes, performance is measured in BER, without concatenated coding. The symbols z_k were taken in QPSK, the channel is block Rayleigh fading with block length 2, the receiver has perfect channel state information, the transmitter none. Due to the low number of bits, maximal likelihood detection was used for all schemes. Alternatively, MMSE-OSIC detectors, or sphere detectors, as suggested in [11], may be used.

The first plot, with legend "NOSTBC(HH)", is the code of [11] with diagonal UW (17). The second, with legend "NOSTBC(OPT)", has optimal, non-diagonal UW (19). The different degree of transmit diversity of these schemes is clearly visible. NOSTBC(OPT) has diversity degree 4, corresponding to transmit diversity degree 2. On the contrary, NOSTB(HH) has diversity degree 2, corresponding to no Tx diversity. Error events of the kind (18) with no transmit diversity protection are

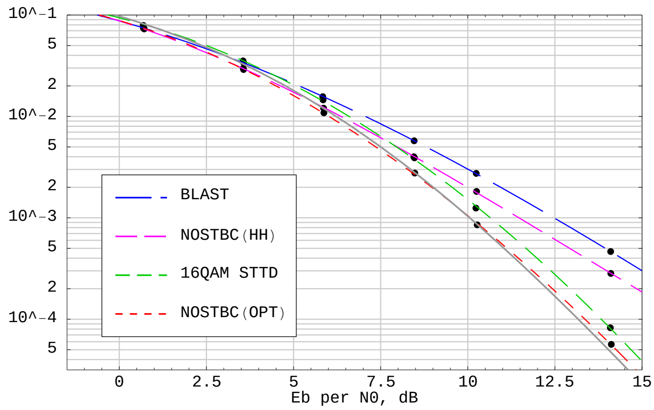

Fig. 1. Uncoded BER performance of various MIMO schemes, received with two Rx antennas. The continuous plot is the theoretical performance of MRC-combining of 4 channels.

frequent enough to significantly effect the performance already at these comparatively low E_b/N_0's.

From the performance of NOSTBC(OPT) it should be noted that not only can the rate be doubled by adding a second transmit antenna, in addition one may enjoy improved performance. The 2×2 NOSTBC(OPT) scheme with rate 2 performs closely to a completely orthogonal scheme with 4 Tx, 1 Rx, and rate 1 (the continuous line).

The MIMO-NOSTBC's are compared to transmitting the same data rate with BLAST, as well as using two 16QAM symbols in the rate 1 Alamouti code, legend "16QAM STTD". The gain compared to BLAST with the same data rate grows with SNR, at raw BER 0.1 % it is 2.5 dB. The gain compared to 16-QAM Alamouti code is a constant shift of 0.7 dB. This is the penalty from using the capacity deficient orthogonal scheme when receiving with two antennas.

VI. Conclusion

We have considered a MIMO-NOSTBC scheme, where the same antenna resource is simultaneously used for rate increase and optimal Tx-diversity. Design methods for such codes are considered, and it is shown how considerable improvement in performance can be achieved by this scheme. These codes are based on the rate increase promised by the increasing number of independently codable degrees of freedom (the rank) in uncorrelated MIMO channels. With channel correlations, or with considerable differences in SNR's between the Tx (or Rx) antennas, the rank of the MIMO channel deteriorates rapidly. In such situations, using an orthogonal scheme with higher constellations would provide more robust performance.

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