

# Stochastic factor model for electricity spot price - the case of the Nordic market

Iivo Vehviläinen<sup>a,\*</sup>, Tuomas Pyykkönen<sup>a</sup>

<sup>a</sup>*Fortum Power and Heat Oy, 0048 FORTUM, FINLAND*

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## Abstract

This paper presents a stochastic factor based approach to mid-term modeling of spot prices in deregulated electricity markets. The fundamentals affecting the spot price are modeled independently and a market equilibrium model combines them to form spot price. Main advantage of the model is the transparency of the generated prices because each underlying factor and the dynamics between factors can be modeled and studied in detail. Paper shows realistic numerical examples on the forerunner Scandinavian electricity market. The model is used to price an exotic electricity derivative.

*Key words:* Electricity prices, stochastic factor models, deregulated markets

JEL: C51, D49

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## 1 Introduction

There are exchanges for spot electricity trading in several regions around the globe after an industry changing deregulation process. Physical delivery of electricity occurs through the exchange at the spot price or via the OTC-markets at a price linked to the spot price. In addition, a spot market with enough liquidity gives a reference index for trading with derivative instruments. Electricity markets differ from the traditional financial markets and other commodity markets due to the non-storability of electricity. Supply and demand must be in balance at each instance separately. A viable model for

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\* Corresponding author.

*Email addresses:* Iivo.Vehvilainen@iki.fi (Iivo Vehviläinen),  
Tuomas.Pyykkonen@fortum.com (Tuomas Pyykkönen).

the spot price process is of up-most importance in all the areas of deregulated power business, including derivative and sales pricing, risk analysis, portfolio management, investment analysis, and regulatory policy making.

There have been several attempts on the modeling of the electricity prices. Recent overviews are found for example in Wallace and Fleten (2002) and Skantze and Ilic (2001). Research is roughly divided to statistical models and fundamental models. Statistical models depend on a set of parameters that describe the properties of the process while fundamental electricity price models are based on competitive equilibrium models for the electricity market.

The statistical approaches model the electricity spot price process directly. Parameters of the price processes are estimated from the available historical market data. Some recent examples are found in Lucia and Schwartz (2002), Davison et al. (2002), Vehviläinen and Keppo (2003), and Deng (2000). All but the very first statistical electricity price models have elaborated over the standard finance tool, the geometric-Brownian motion. Estimation of price process parameters is possible in the traditional financial markets because there is plenty of historical market data. However, in the Nordic electricity market, the amount of historical data is very limited because of the strong seasonal and yearly variations. Koekebakker and Ollmar (2001) showed that there are no simple factors that could comprehensively explain the electricity price movements, at least not in the Nordic markets. The statistical models work best for short time intervals as the static model structure and the relatively small set of parameters are not able to capture the longer term dynamic characteristics of spot prices.

In fundamental models electricity prices are obtained from a model for the expected production costs of electricity and expected consumption of electricity. One approach is to calculate the theoretical equilibrium price of the whole market, see e.g. Fleten and Wallace (1998) and references therein. Alternatively the supply function can be modeled directly, see e.g. Skantze et al. (2000) and Eydeland and Geman (1999). The fundamental models produce spot price scenarios directly from historical or simulated data, while in the statistical process based approaches historical data is used to estimate the parameter values of the processes. The fundamental models typically require a comprehensive data set that is difficult to collect and maintain and it is often laborious to use the fundamental models to create numerous spot price scenarios.

The model of this paper combines favorable sides of both statistical and fundamental models. The fundamentals affecting the spot price are modeled as stochastic factors that follow statistical processes. The processes for fundamental factors are stabler in form and more accurately represented than the statistical models for the complicated spot price process. There is a long his-

tory available for the estimation of parameters of the fundamental factors, such as climate data. The stochastic factors are combined to form the spot price with an approximate market equilibrium model that is based on the actual Nordic market model. The parameters for the approximation are estimated from the realized market prices so that they reflect both the marginal production costs of the whole market and the behavior of market participants. The model is best suited for mid-term analysis in such applications as derivative pricing and risk management.

There are several approaches to the estimation of the model parameters and the formulation of the market equilibrium model. Detailed study of each parameter and advanced estimation of the model parameters are outside the scope of this paper. Rather, the purpose is to provide concise models that reasonably approximate the observed historical values. For details in climate related models see e.g. Skiranthan and McMahon (2001) and references therein and for modeling of electricity load see e.g. Bunn and Farmer (1985), or more recently Pardo et al. (2002).

Next section presents the framework for the model. Section 3 gives the models for the fundamental stochastic factors. Section 4 provides the details of the market equilibrium model that is used to combine the stochastic factors to the spot price. Section 5 gives numerical examples from the Nordic market and section 6 concludes.

## 2 Framework

### 2.1 *Spot market*

Physical spot prices in the Nordic market are set by an market equilibrium model where the supply and demand curves of all the market participants are matched day-ahead. The prices for the one-hour periods are calculated by matching the collaborative demand and supply curves calculated from the bid and ask prices given by the market participants, see e.g. Lucia and Schwartz (2002) for details.

Over half of the yearly production in the Nordic market is hydro-power based, around one fourth is produced with nuclear power, and the rest is a mixture of production types, including industrial and municipal CHP, condensing power, and wind power. Large amount of market participants have access to production capacity and the competitiveness of the market is high.

Winter in the Nordic area is cold with temperature commonly below freezing.

Electricity heating is widely used by households but air conditioning is rare during the mild summers. Thus the demand during the winter time is at a much higher level than during the summer time. There are a lot of energy intensive industries such as paper and metal industry that provide a rather stable baseload throughout the year.

## 2.2 Theoretical setting

The spot price process is build based on a set of stochastic factors that are defined in a discrete time probability space  $(\Omega, \mathcal{F}, P)$  for a time period  $[0, \tau]$ . Here  $\Omega$  is the set of possible outcomes,  $\mathcal{F}$  is a  $\sigma$ -algebra in  $\Omega$ , and  $P$  is a probability measure defined on  $\mathcal{F}$ . The time period  $[0, \tau]$  is divided to  $T$  intervals,  $t = 0, \dots, T$ . There are  $(n + 1)$  stochastic factors,  $\mathbf{x}^t := (x_0^t, \dots, x_n^t)$ , whose development over time is given by

$$x_i^{t+1} = \mu_i^t(\mathbf{x}^t) + \sigma_i^t(\mathbf{x}^t)\epsilon_i^t, \quad 0 \leq t \leq T, 0 \leq i \leq n, \quad (1)$$

where  $\mu_i^t(\mathbf{x}^t)$  is the local drift of  $x_i^t$  and  $\sigma_i^t(\mathbf{x}^t)$  is the local volatility from Gaussian stochastic variable  $\epsilon_i^t$ . The model has a number of parameters denoted by  $c_j$  whose value are estimated from history or given directly by physical reality.

The variables  $\epsilon_i^t$  are assumed to be correlated with the correlation matrix  $\mathbf{C}$ . A Cholesky factorization is applied to calculate correlated random variables based on independently sampled random variables.

## 2.3 Restrictions

Focus of this paper is in the so-called system price that is obtained from the total supply and demand curves in the Nordic market area. The model works on an aggregated level which causes the loss of some explanatory power. For example, physical transmission restrictions and the consequential price area differences, dynamics of individual hydro reservoirs, and variations in local demand are ignored. However, the system price is the most relevant price indicator that is used as reference for most of the derivatives trading.

The supply-demand market equilibrium is approximated by assuming that the electricity demand is inelastic, which is a feasible assumption in all but extreme cases. The parameters of the supply price function are estimated from the realized spot prices. The supply price function models the asking prices of producers rather than the genuine marginal production costs. It is assumed that the form of the supply price function does not change and purely game-theoretic aspects like investment decisions are beyond the scope of this

paper. It would be possible to incorporate, for example seasonal or price level dependent, changes in the supply conditions to the model.

The discretization time step is assumed to be relatively long, e.g. one month. Capturing the daily and hourly variations is best done with short-term price models. As such, the model is best suited for mid-term or long-term analysis, although the same principles could be applied to shorter time periods.

The stochastic nature of the estimated model parameters is ignored in the presented examples and analysis. In reality most of the parameters should be modeled as stochastic factors themselves in Bayesian spirit. For example, power plant failures, stochastic demand growth, or currency exchange rates could be modeled. Addition of such stochastic parameters is easily done, if such factors are important for the model use.

### 3 Fundamental factors

#### 3.1 Climate data

Because half of the production is hydro based, the hydrological situation affects heavily to the available supply in the Nordic area. The possibility to produce hydro-power depends on the water level in reservoirs that are in turn filled by hydro-inflow. The hydro-inflow to the reservoir system results mainly from precipitation and the melting of snow-pack. Generation of the snow-pack is directed by precipitation and temperature. Also, electricity demand is very much driven by temperature. The most important stochastic climate factors, hydro-inflow and temperature, must be handled by taking into account their correlation structure.

Temperature data exhibits serial correlation and strong seasonality. Serial correlation is assumed to be of the first order, i.e. the additions of temperature differences are independent from each other. The seasonality is modeled by the use of average temperature curve. Serial correlation coefficient, average temperature curve, and volatility of temperature are all estimated from historical data. The deviation of temperature from the normal temperature,  $x_{\Delta Temp}^t$ , is assumed to follow

$$x_{\Delta Temp}^{t+1} = c_{TempSer} x_{\Delta Temp}^t + \sigma_{\Delta Temp}^t \epsilon_{\Delta Temp}^t, \quad (2)$$

where  $c_{TempSer}$  is the serial correlation coefficient for temperature deviation and  $\sigma_{\Delta Temp}^t$  is the volatility resulting from the random variable  $\epsilon_{\Delta Temp}^t$  that follows normal Gaussian distribution. Given the process for the deviation of

the temperature from the average temperature, the realization for temperature is given by

$$x_{Temp}^t = c_{TempAve}^t + x_{\Delta Temp}^t, \quad (3)$$

where  $c_{TempAve}^t$  denotes the average temperature at time  $t$ .

Precipitation has similar characteristics to temperature and a similar model and estimation method are used. The deviation of precipitation from the normal level of precipitation,  $x_{\Delta Precip}^t$ , is assumed to follow

$$x_{\Delta Precip}^{t+1} = c_{PrecipSer} x_{\Delta Precip}^t + \sigma_{\Delta Precip}^t \epsilon_{\Delta Precip}^t, \quad (4)$$

where  $c_{PrecipSer}$  is the serial correlation coefficient for precipitation deviation,  $\sigma_{\Delta Precip}^t$  is the volatility resulting from the random variable  $\epsilon_{\Delta Precip}^t$  that is correlated with  $\epsilon_{\Delta Temp}^t$  with correlation factor  $\rho_{\Delta Temp \Delta Precip}^t$ . Again, from the process for the deviation from the average, the realization for precipitation is given by

$$x_{Precip}^t = c_{PrecipAve}^t + x_{\Delta Precip}^t, \quad (5)$$

where  $c_{PrecipAve}^t$  gives the average precipitation at time  $t$ .

### 3.2 Hydro-balance

Precipitation that occurs on those days whose temperature is below freezing increases the snow-pack. Given a temperature average for the discrete time period, e.g. for a month, the daily temperature is assumed to be independently normally distributed around the average for all the days within the period. The increase in the snow-pack occurs according to

$$x_{Freeze}^t = x_{SubZero}^t x_{Precip}^t \quad (6)$$

where

$$x_{SubZero}^t = \Phi(0^\circ\text{C}; x_{Temp}^t; \sigma_{DailyTemp}^t) \quad (7)$$

is the cumulative normal distribution function that gives the percentage of the time that the temperature is below  $0^\circ\text{C}$  within the discretization period, and  $\sigma_{DailyTemp}^t$  gives the volatility of the temperature within the discretization period.

The amount of snow that melts is proportional to the size of the snow-pack. Melting is assumed to start at temperature  $c_{MeltTemp} > 0^\circ\text{C}$ . If it is colder, even slightly above freezing temperature, it is assumed that the melting that occurs is only marginal. If it is warmer, then melting occurs proportionally to the size of the snow-pack,  $x_{Snow}^t$ , according to

$$x_{Melt}^t = c_{MeltSlope} (x_{Temp}^t - c_{MeltTemp})^+ x_{Snow}^t, \quad (8)$$

where  $c_{MeltSlope}$  gives the proportion of snow that melts and  $(\cdot)^+$  indicates that only positive part is considered.

The initial snow-pack level  $x_{Snow}^0$  is assumed to be known. The development of the snow-pack over time is given by

$$x_{Snow}^{t+1} = x_{Snow}^t + x_{Freeze}^t - x_{Melt}^t. \quad (9)$$

Note that the melting is proportional to the amount of snow so that  $x_{Snow}^t$  is always positive if the discretization error is ignored.

The hydro-inflow,  $x_{Inflow}^t$ , is given as the sum of the precipitation that does not turn into snow and the inflow that comes from the melting of the snow-pack, i.e.

$$x_{Inflow}^t = x_{Precip}^t - x_{Freeze}^t + x_{Melt}^t. \quad (10)$$

The total inflow splits to two components. Part of the inflow is arrives to run-of-river type of hydro units and is immediately and without choice produced. The rest of the inflow streams to the hydro reservoirs. Hydro-producers control their reservoir levels by deliberately letting some of the inflow pass directly in a similar manner as with the run-of-river production if the reservoir levels start to be close to the maximum levels. The idea is to avoid spillage that would be forced if the reservoirs are filled to the maximum level. The division of the inflow is done to the unregulated part

$$x_{InflowU}^t = x_{InflowU\%}^t x_{Inflow}^t \quad (11)$$

and regulated part

$$x_{InflowR}^t = (1 - x_{InflowU\%}^t) x_{Inflow}^t, \quad (12)$$

where the proportion  $x_{InflowU\%}^t$  is defined as

$$x_{InflowU\%}^t = c_{InflowU\%1} + e^{c_{InflowU\%2}(1-x_{Res\%}^t)}, \quad (13)$$

where  $c_{InflowU\%1}$  and  $c_{InflowU\%2}$  are parameters estimated from historical data and  $x_{Res\%}^t$  is the reservoir level as percentage of the full reservoir. There is always some unregulated inflow and the proportion increases rapidly as the reservoirs are filled.

The initial reservoir level,  $x_{Res}^0$ , is assumed to be known. Change in the reservoir level results from the inflow  $x_{Inflow}^t$  and the total discharge for hydro-production,  $x_{SHydro}^t$ , taking into account the physical restrictions of the reservoir. If the reservoir is too full, then a part of the inflow must be spilled,  $x_{Spill}^t$ . The change of the reservoir level is

$$x_{Res}^{t+1} = x_{Res}^t + x_{Inflow}^t - x_{SHydro}^t - x_{Spill}^t, \quad (14)$$

where the hydro-production is determined later and the spillage is assumed to be of the form

$$x_{Spill}^t = c_{SpillRate}(x_{Res}^t - c_{ResNormal}^t - c_{SpillLevel})^+, \quad (15)$$

where  $c_{SpillRate}$  gives the rate with which spilling occurs if the reservoir is over  $c_{SpillLevel}$  over the normal, i.e. long-term historical average, reservoir level given by  $c_{ResNormal}^t$ . The construction forces the model to comply with the actual physical restrictions. Reservoir level as percentage of the full reservoir is given by

$$x_{Res\%}^t = x_{Res}^t / c_{ResMax}^t. \quad (16)$$

The total hydro-balance is given by the sum of the energy in hydro-reservoirs and the energy stored in the snow-pack

$$x_{HB}^t = x_{Res}^t + x_{Snow}^t, \quad (17)$$

and given the normal historical average hydro-balance level  $c_{HBNormal}^t$ ,

$$x_{\Delta HB}^t = x_{HB}^t - c_{HBNormal}^t \quad (18)$$

gives the deviation of the hydro-balance from the normal level.

### 3.3 Demand

Electricity demand is modeled with a fixed component mainly due to industry, a temperature dependent component, and a noise term. Fixed industrial part is assumed to be known and the temperature dependent part is estimated from the history data for the time for which both temperature and demand are known. The size of the noise term is given by the estimation error.

It is assumed that the temperature dependency of demand is linear between some minimum temperature  $c_{DTemp1}$  and maximum temperature  $c_{DTemp2}$ ,  $c_{DTemp1} < c_{DTemp2}$ . The demands at those temperatures are correspondingly at maximum  $c_{D1}$  and at minimum  $c_{D2}$ ,  $c_{D1} > c_{D2}$ . Define the change of demand per change of temperature as

$$c_{DSlope} = \frac{c_{D2} - c_{D1}}{c_{DTemp2} - c_{DTemp1}} < 0. \quad (19)$$

Then the total demand  $x_D^t = x_D^t(x_{Temp}^t)$  is given by

$$x_D^t = c_{D1} + c_{DSlope}[(x_{Temp}^t - c_{DTemp1})^+ - (x_{Temp}^t - c_{DTemp2})^+] + \sigma_D^t \epsilon_D^t, \quad (20)$$

i.e. demand is at maximum level  $c_{D1}$  below temperature  $c_{DTemp1}$ . Above temperature  $c_{DTemp1}$  the demand starts to decrease with rate  $c_{DSlope}$  until temperature  $c_{DTemp2}$  and minimum demand level  $c_{D2}$  are reached. Parameter  $\sigma_D^t$



gives the noise from the independent Gaussian random variable  $\epsilon_D^t$ . Note that the baseload component is built-in to the demand model. The changes to the industrial load for example during holiday periods can be done in the demand model directly. In addition, for longer time horizons, a constant demand growth term representing the perceived growth in the market area can be included.

### 3.4 Baseload supply

Baseload production has a low or non-existing marginal cost of production and relatively high cost of adjusting production level. It is not economically feasible to change the baseload production schedule because of small variations in spot prices. Nuclear production and industrial CHP are driven almost continuously outside the revision periods if there are no failures or interruptions in the industry process.

The baseload supply is given by

$$x_{SBaseLoad}^t = c_{SBaseLoad}^t + x_{SCHP}^t + x_{SHydroU}^t, \quad (21)$$

where  $c_{SBaseLoad}^t$  gives production from nuclear production and industrial CHP,  $x_{SCHP}^t$  gives municipal CHP production and  $x_{SHydroU}^t$  unregulated hydro-production. The last two are modeled as follows.

Municipal CHP is driven by the heating demand that is in turn directly dependent on the temperature. The model of municipal CHP production is similar to the model of the temperature dependent demand. The production is assumed to be at maximum  $c_{S1}$  at some minimum temperature  $c_{STemp1}$  and at minimum  $c_{S2}$  at some maximum temperature  $c_{STemp2}$ . The change of production per change of temperature is given by

$$c_{SSlope} = \frac{c_{S2} - c_{S1}}{c_{STemp2} - c_{STemp1}} < 0 \quad (22)$$

and the municipal CHP production by

$$x_{SCHP}^t = c_{S1} + c_{SSlope}[(x_{Temp}^t - c_{STemp1})^+ - (x_{Temp}^t - c_{STemp2})^+]. \quad (23)$$

The municipal CHP has an important role in reducing the effects of temperature to the supply-demand balance and the spot price.

Hydro production is divided to two parts, unregulated hydro production that results from the unregulated hydro inflow, and regulated hydro production. The unregulated hydro production is given by

$$x_{SHydroU}^t = x_{InflowU}^t, \quad (24)$$

i.e. directly by the inflow that is directed to it.

## 4 Market equilibrium

### 4.1 Spot price discovery

The market mechanism of the Nordic spot market makes the supply and demand curves match at the spot price level. There are several theoretical equilibrium models for the supply-demand setting. The actual market is here approximated by assuming that demand is not elastic. The supply price function gives then the spot price at the level of inelastic demand. The standard economic theory states that the market equilibrium price should equal the marginal production cost of the last production unit that is activated. In longer term such running policy would not allow any capital to be raised for investments and market participants can offer their production capacity based on some strategic or game-theoretic principles.

It is assumed that demand always exceeds baseload supply and that the surplus demand is covered by regulated hydro production and condensing power production. The order in which these the two production types are used depends on the value that producers give to the energy in the hydro-reservoirs and the asking price of condensing power. The forms of the supply price functions for hydro-power and condensing power are derived in the following.

Hydrological situation determines the willingness of producers to run their hydro-power capacity. More precisely, based on prevailing hydro-balance in comparison to the normal situation, the producers are assumed to value their water as follows:

$$x_{WV}^t = c_{WVSlope} x_{ResPenalty}^t x_{\Delta HB}^t + c_{WVLevel}, \quad (25)$$

where  $x_{WV}^t$  is the water value,  $x_{\Delta HB}^t$  is the difference in hydro-balance,  $c_{WVSlope} < 0$  gives the slope and  $c_{WVLevel}$  the normal level of the value function. The penalty from being too close to the physical borders of the reservoir,  $x_{ResPenalty}^t$ , is given by

$$x_{ResPenalty}^t = e^{-c_{ResPenalty}(x_{Res\%}^t - c_{ResMin\%})} + 1 > 0. \quad (26)$$

The penalty function starts to exponentially affect the water value with coefficient  $c_{ResPenalty}$  if the minimum reservoir level  $c_{ResMin\%}$  is approached. If the reservoir is too empty then  $x_{ResPenalty}^t > 1$  and otherwise the penalty function has values close to 1. Note that the behavior near the maximum reservoir level is already included in the division of the inflow to regulated and unregulated

inflow. The model assumes that the asking price of hydro-power is dependent on the deviation of the hydro-balance from the historical long-term average, i.e. normal, value.

The asking price of condensing power at production level  $E$  is assumed to be given by

$$x_{SCondMC}^t(E) = (c_{CondMCSlope}E + c_{CondMCLevel})^+, \quad (27)$$

where  $c_{CondMCSlope}$  gives the marginal asking price per production and  $c_{CondMCLevel}$  adjusts the general ask price level. The piecewise linear form of the asking price approximates the aggregated supply curve of all the producers.

The water value and asking price of condensing power are used to derive the spot market equilibrium price. Amount of condensing production that is running with a price below or equal to the water value,  $x_{SCondWV}^t$ , is derived from the asking price of condensing power (27) by substituting the water value level (25) as the ask price and solving for energy  $E$ , yielding

$$x_{SCondWV}^t = 1/c_{CondMCSlope}(x_{WV}^t - c_{CondMCLevel})^+. \quad (28)$$

Note that no condensing power is running if the water value is below  $c_{CondMCLevel}$ . If the demand exceeding baseload supply is not completely covered by the condensing power production in (28), the next production form that becomes active is the regulated hydro production. It is assumed that regulated hydro-production  $x_{SHydroR}^t$  is run up to the level equal to the remaining uncovered demand or to the maximum hydro-production level that is remaining after unregulated production,  $c_{SHydroMax} - x_{SHydroU}^t$ , i.e.

$$x_{SHydroR}^t = \min((x_D^t - x_{SBaseLoad}^t - x_{SCondWV}^t)^+, c_{SHydroMax} - x_{SHydroU}^t), \quad (29)$$

where the function  $\min(\cdot, \cdot)$  is used to limit the hydro-production to the remaining demand or to the maximum available production level, whichever is smaller. If the hydro-production is at the maximum level and there still is uncovered demand, additional condensing power production is activated so that the total demand is covered. The final condensing power production  $x_{SCond}^t$  is then given by

$$x_{SCond}^t = x_{SCondWV}^t + (x_D^t - x_{SBaseLoad}^t - x_{SCondWV}^t - x_{SHydroR}^t)^+, \quad (30)$$

The asking price of condensing power is obtained from (27) as

$$x_{SCondMC}^t = (c_{CondMCSlope}x_{SCond}^t + c_{CondMCLevel})^+. \quad (31)$$

In the present paper the discretization period is assumed to be relatively long, from one week to one month, and it might be unrealistic to assume that the price for the whole time period is determined by the same marginal unit. The

asking price of condensing power and hydro-production are known, and the spot price is assumed to be based on them as follows,

$$x_{Spot}^t = \frac{x_{SCond}^t x_{SCondMC}^t + x_{SHydroR}^t x_{WV}^t}{x_{SCond}^t + x_{SHydroR}^t}, \quad (32)$$

i.e. the spot price is given by the production volume weighted average of the supply price of condensing power and the supply price of hydro-power. The weights are given by the amount of condensing production  $x_{SCond}^t$  and the amount of regulated hydro-production  $x_{SHydroR}^t$ .

## 4.2 Calibration

The model is calibrated using the historical data for the stochastic factors. Some of the parameters describe physical reality, such as minimum and maximum reservoir levels, and others are estimated. In the examples of this paper, parameter estimation is done by a simple least square error method, but there are of course more sophisticated approaches. The estimation has three phases that are done separately. All the phases can be independently monitored and expert opinions applied if necessary.

Firstly, the climate data is estimated possibly from a longer historical time period as the data is not dependent on the existence of a market or even electricity. For example, there is typically a long history of temperature observations that can be used for parameter estimation. Secondly, parameters for demand and supply that are directly linked to the climate data are estimated by using available historical consumption or production data together with the corresponding historical climate variables. In reality, these factors are also relatively well understood based on the physical characteristics. Thirdly, the market equilibrium model parameters are estimated from the available historical spot prices and corresponding fundamental data. The fundamental data is given by the first two steps of the estimation and the respective models. For example, the deviation of hydro-balance that is needed for the estimation of water-value function parameters is given by the temperature and precipitation. The estimated market equilibrium parameters reflect the aggregated cost structure of producers and pricing behavior of market participants.

## 5 Examples

### 5.1 General

The numerical examples are set in the Nordic market. The climate parameters have been estimated from some 10 recent years of data while the time period that has been used for the market equilibrium, demand, and supply parameter estimation is from the year 1996 to the year 2000. The data and results in the examples are in monthly granularity. Data sources are the spot exchange Nord Pool (2002) and the Nordic grid operator's organization Nordel (2002). All the prices have been converted to euros from initial Norwegian crowns with a fixed exchange rate of 0.13 *NOK/EUR*. Table 1 presents the estimated values for the scalar parameters. Some values are based directly on physical reality and others are calibrated according to the real market values. As an example, the demand parameters indicate that the sensitivity of demand to one degree change in temperature is around 1000 MWs, a value consistent with experience. All the simulation results are from a set of 10000 simulations.

Table 1  
Estimated scalar parameter values of the model.

Climate data	Value	Demand	Value	Supply	Value
$c_{TempSer}$	0.34	$c_{D1}$	34.8 TWh	$c_{S1}$	4.3 TWh
$c_{PrecipSer}$	0.39	$c_{D2}$	19.5 TWh	$c_{S2}$	0 TWh
$c_{MeltSlope}$	0.07	$c_{DTemp1}$	-4.0 °C	$c_{STemp1}$	0.0 °C
$c_{MeltTemp}$	3.7 °C	$c_{DTemp2}$	16.9 °C	$c_{STemp2}$	18.0 °C
$\sigma_{DailyTemp}^t$	5.0			$c_{SHydroMax}$	21 TWh
$c_{InflowU\%1}$	0.2			$c_{ResPenalty}$	14.37
$c_{InflowU\%2}$	-13.3			$c_{WVSlope}$	-0.49 EUR/TWh <sup>2</sup>
$c_{SpillRate}$	0.13			$c_{WVLevel}$	18.26 EUR/MWh
$c_{SpillLevel}$	19.7 TWh			$c_{CondMCSlope}$	0.29 EUR/TWh <sup>2</sup>
$c_{ResMin\%}$	19 %			$c_{CondMCLevel}$	28.61 EUR/MWh
$c_{ResMax\%}$	95 %				
$\rho_{\Delta Temp \Delta Precip}^t$	-0.3	(Summer)			
$\rho_{\Delta Temp \Delta Precip}^t$	0.3	(Winter)			

### 5.2 Spot price distribution

Figure 1 presents the simulated average and 5 % and 95 % values of the seasonally dependent temperature and precipitation, the realized values of the year 2001, normal long-term average hydro-balance and reservoir levels, and the baseload supply. The simulated averages match the averages used in the presentation. The interpretation of the probability limits is that for a given

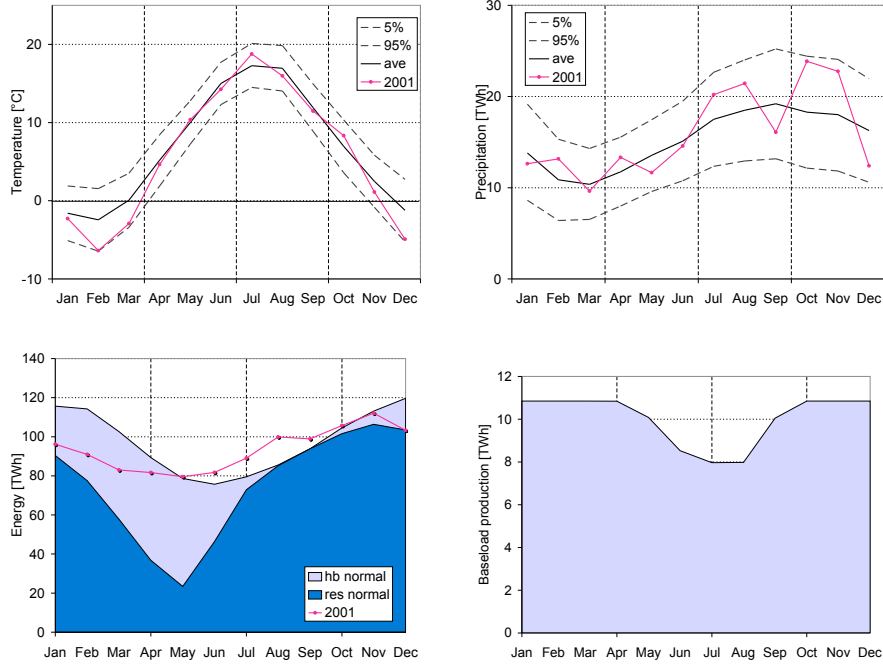


Fig. 1. Simulated average values, 5 % and 95 % percentiles, and realization for year 2001 for temperature (top left) and precipitation (top right). The long-term historical normal values and realization for 2001 for hydro-balance and reservoir levels (bottom left). The difference between hydro-balance and reservoir level is given by snowpack. The baseload supply that includes nuclear and industrial CHP production (bottom right).

period the variables are within the probability limits in 90 % of the cases over a large number of simulations.

The variable estimation period is commonly known as *ex ante* period. The model performance is demonstrated in the so called *ex post* period. Figure 2 presents the realization of the spot price and the model prices from 1996 to 2001. Model parameters are estimated from the time period from 1996 to 2000 and the year 2001 is the *ex post* period. During the *ex post* period only the temperature and precipitation values are updated according to their realized values and rest of the model parameters are equal to the *ex ante* values. The spot price for the year 1996 is higher than for the other years. The hydro-inflow during 1996 was considerably below the normal level which decreased the amount of available hydro power generation. As a result, more expensive thermal power was used to cover the yearly demand. The hydro inflow was higher than the normal level in years from 1997 to 2000 and this was one of the reasons why the prices of those years were lower than for the year 1996. The model manages to capture the drastic change from the long wet time period before 2001 to the normal hydrological conditions and cold temperatures in the beginning of 2001. To quantify the errors between the realized spot prices

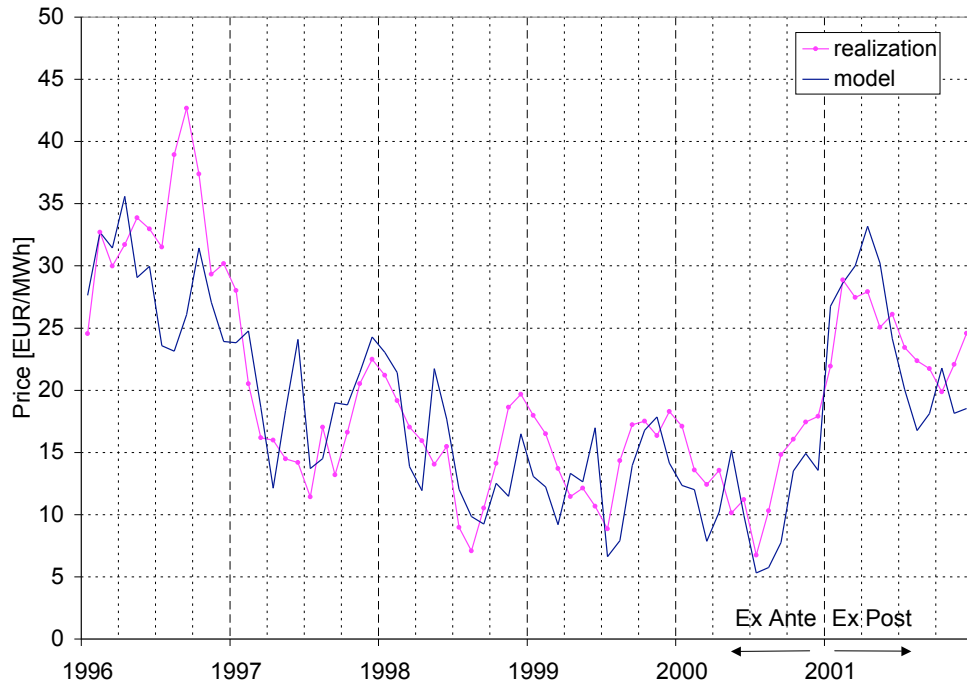


Fig. 2. Realizations of spot prices from 1996 to 2001 and the model price for 1996 to 2000 with the data used in parameter estimation (ex ante) and the model price with realization data for 2001 (ex post).

and different price forecasts, a RMS (Root Mean Square) statistic is used. A smaller RMS error value indicates smaller error. The RMS error between the *ex post* model simulation and the realized spot prices is 4.4 in EUR/MWh in 2001.

Figure 3 presents the monthly averages of the realized spot price for the year 2001, the average spot price given by model simulation, the 5 % and 95 % probability limits of the price model, and the market quotes on 29 December 2000. Initially, the model prices start on average at a lower level than the realized market prices, and there is only a slight chance for the relatively high spot price that was realized in 2001. The reason is that the fundamental hydro-balance situation changed dramatically from a long spell of wet and mild years closer to a historical average scenario. It had been a mild winter in late 2000, and the realized values of temperature and precipitation were both higher than average. Higher than average realizations affect model prices especially for the start of the year because due to the serial correlation it is more likely that the weather will continue warm and rainy. In fact, the start of the year 2001 was very cold, see Figure 1.

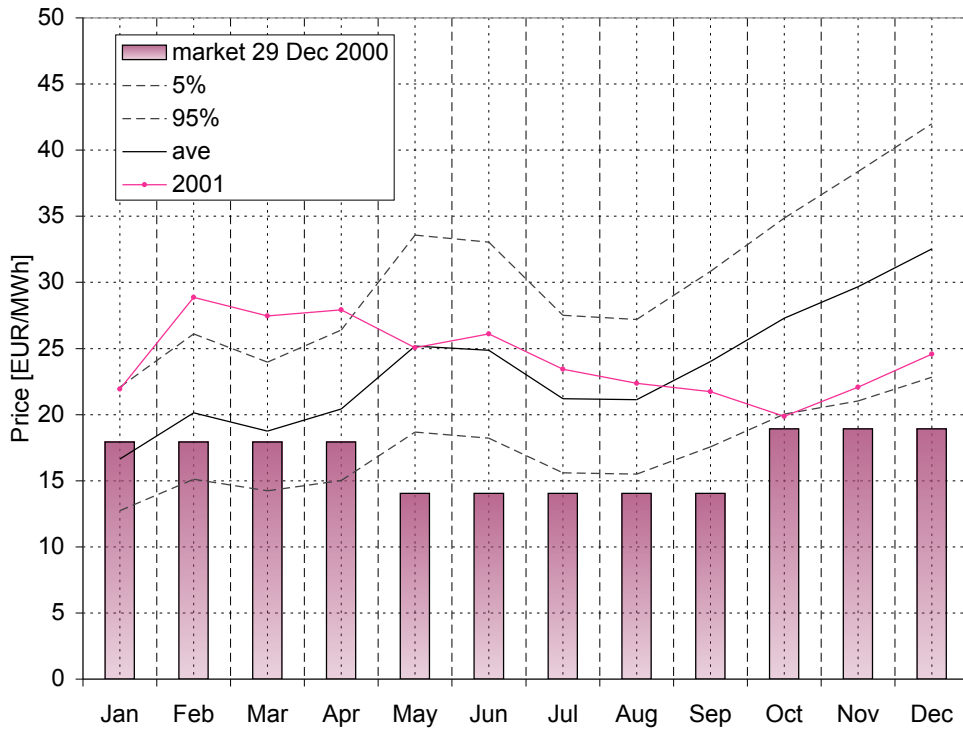


Fig. 3. The spot price forecast for 2001, the realized spot price, and the electricity forward prices on the 29 Dec 2000.

The average model price for the initial time period match quite well the market quotes from the end of year 2000. The model and the markets have a similar view on the initial fundamental conditions at the end of 2000. But the market was discounting a year similar to the previous few, in fact extremely wet, years for the whole 2001, and the market quotations on 29 December 2000 fail to predict the realized spot prices totally. The market forward curve from the end of the year 2000 can be viewed as a static market forecast for the spot price of the year 2001. The RMS error between the static market forecast and the spot prices is 7.4 EUR/MWh for the year 2001, i.e. the spot price model with an error of 4.4 EUR/MWh is more accurate than the static market forecast.

Obviously, the dynamic update of information to the spot price model improves the model accuracy. To accommodate similar update of information, the market forward curve is updated with new market quotations at the end of each month. In the granularity of the model, the market actors have reasonably accurate forecasts of the fundamentals affecting the spot price as well as updated information about their own actions. The RMS error of such dynamic market forecast is 2.6 EUR/MWh, i.e. with the same amount of information, the market was better at forecasting the spot price for the closest month than the model. The forward market prices can be considered as the best possible



predictor of the future price movements - were someone to create a better model it would be used to act in the markets and the differences would disappear. However, the market forward curve only predicts the expected spot prices, and the accuracy decreases rapidly with longer term horizons. The importance of the spot price model is that the forecast of expected spot price will always be wrong but the generated price distribution should capture the range in which the spot prices can move.

Table 2

The RMS (Root Mean Square) error in EUR/MWh between the realized spot price for the year 2001 and the *ex post* model simulation, the static market forward forecast from 29 December 2000, the dynamically updated market forward forecast, and the model dynamics test case.

Forecast	RMS error
Model <i>ex post</i>	4.4
Static market	7.4
Dynamic market	2.6
Model dynamics	1.8

Finally, to test the dynamics of the model, the market equilibrium parameters are fitted by minimizing the RMS measure for the year 2001 only. The other model parameters are as in the *ex post* forecast, i.e. with the newly estimated market equilibrium parameters the model produces the spot price for the year 2001 by using the realized temperature and precipitation. The RMS error of the model compared to the realized spot price is then 1.8 EUR/MWh which is under the error of the dynamic market forecast. Table 2 summarizes the RMS errors of different price forecasts. Given information of the realized spot prices, the model is capable of accurately reproducing these spot prices, i.e. the internal model dynamics are justifiable.

### 5.3 Pricing

The explicit models for the fundamental factors make possible the pricing of exotic derivative instruments. Simulation of fundamental factors and prices give a model dependent theoretical prices for derivatives. See e.g. Vehviläinen (2002) for other considerations on electricity derivatives pricing. As an example, the model allows the pricing of derivatives that hedge both price and volume uncertainty. As the competitive market drives down the margins for all market participants, the importance of hedging effectiveness increases.

A regular call option for electricity spot price allows purchase of electricity at

a fixed price, i.e. the strike price  $c_{Strike}$ . The option payoff is

$$F_{Regular} = \sum_t (x_{Spot}^t - c_{Strike})^+, \quad (33)$$

where  $x_{Spot}^t$  is the spot price realization. Market participants can use the option to protect against high prices in the spot market. To be protected against possible temperature dependent variations, market participants need to buy regular call options for the whole uncertain part of their volume.

In comparison, consider a tailor-made exotic derivative instrument for hedging temperature dependent volume uncertainty. The derivative provides a call option for spot electricity price only if temperature realization is below long term historical average. These low temperatures coincide with high demand for market participants in the Nordic area. The idea is to improve hedging efficiency by concentrating the option payoffs to the times where hedging is required, see Bhanot (2002) for a similar discussion. The option payoff is

$$F_{Exotic} = \sum_t (c_{TempAve}^t - x_{Temp}^t)^+ (x_{Spot}^t - c_{Strike})^+, \quad (34)$$

where  $c_{TempAve}^t$  is the average temperature and  $x_{Temp}^t$  is the temperature realization.

Spot price model simulation provides an estimate for the price of the exotic option for the year 2001 in the setting of the previous section. The strike price is fixed to 21.6 EUR/MWh which equals the simulated expected price for 2001. Simulation of spot prices and temperatures yields the expected option payoff for the exotic option as 2.2 EUR/MWh. In contrast to the exotic option, a certain amount of regular option contracts needs to be bought to cover possible temperature dependent variations. Higher volume of regular options increases the hedging efficiency but also costs more. Direct comparison to the exotic option is not possible. As an example, the amount of regular options here is equal to the deviation of temperature by one standard deviation. Simulation of spot prices yields the expected total option payoff then as 6.0 EUR/MWh. The price of the regular spot price option is higher than the exotic option. The regular spot price option covers against high spot prices even when temperatures are higher than on average, e.g. due to low inflows, and this is reflected in the price.

Taking the temperature dependency into account reduces the option price from 6.0 EUR/MWh to 2.2 EUR/MWh, or more than 60 %. This implies potential reduction of hedging costs with the exotic option. The pricing of such options is not possible using standard methods in the literature. The model of this paper can produce price estimates for the exotic volumetric option and similarly for many other more complex derivative instruments.

## 6 Discussion

Spot prices are of fundamental importance in the deregulated markets. Each market actor must take a position on the price development over time, even though judgment is sometimes based on subjective views only.

In many applications it is beneficial to be able to study correlated stochastic variables. For example in risk management, the correlation between electricity consumption and price of electricity is vital. The structure of correlations is often more complex than what simple statistical models for spot price can capture. The model of this paper provides a transparent approach to handling of the correlations, and the model is especially well suited for analysis of company risks on the deregulated markets.

The simulated results obtained with the model of this paper can be analyzed independently. The model provides simulated values for the fundamental data, demand and supply information, and pricing strategies of the consolidated market. If some of the resulting values or estimated parameters do not seem realistic, expert opinion can be applied. The accuracy of the model is comparable to the short-term market expectations but the model is better at longer time periods. Note also that the construction and calibration of the model is an educating exercise in itself.

The complex structure of many electricity deals can present a problem in risk analysis but also in pricing of the deals. The examples in the paper show that even very complex structures can be priced with the model of this paper. Some other possible applications of the model would be in more sophisticated applications in portfolio or risk management, investment and divestment decisions, and regulatory policy making.

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