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Basics of electricity derivative pricing in competitive markets

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This paper studies the application of the available financial theory to the deregulated electricity market. The special characteristics of electricity make the market different from all other commodity markets. The paper introduces a coherent framework for the assets and instruments in the electricity markets in the financial tradition. Properties of the instruments that are available in the Scandinavian electricity market are studied in more detail.

Keywords: electricity derivatives, electricity forwards, exotic options, pricing

1. Introduction

Electricity market deregulation is reality in several regions, including the US, UK, German, Spanish, and Scandinavian markets. Physical electricity is traded in spot markets on several exchanges as a result of the deregulation. The introduction of a liquid spot electricity market creates a reference index for trading with derivative instruments. This paper studies the available derivative products in the Scandinavian market, which is leading the global deregulation process.

The general theory of pricing derivative instruments is well studied and understood in the financial market, building on the seminal work by Samuelson (1965), Black and Scholes (1973), Merton (1973), and others. The advances in financial theory have also been applied to the pricing of commodity derivatives. Harrison and Kreps (1979) presented a unified framework for dynamic asset pricing, building on Cox and Ross (1976) and Ross (1978). The specifics of commodity derivative pricing were clarified by Brennan and Schwartz (1985). For the current status of dynamic asset and derivatives pricing, see Duffie (1996) and Hull (1997).

The non-storability of electricity makes the electricity market different from the financial markets and other commodity markets. Shortages in electricity generation or peaks in electricity demand results in unparalleled jumps, spikes, and volatility in spot electricity prices. For physical spot electricity, there are no replicating portfolios that are the basis of non-arbitrage pricing in the financial markets. No analytical connection has been established between the spot price and forward prices. Despite these limitations, instruments in the market have complexity of remarkable scale.

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Electricity derivative pricing has been pursued by several authors. Available research has been approaching the pricing problem from either a fundamental or a statistical point of view. Focus has been on the modelling of electricity spot price processes as the standard Black and Scholes assumptions of geometric Brownian motion are clearly not valid in electricity markets.

The fundamental approaches of electricity derivative pricing are based on competitive equilibrium models for the electricity market. Electricity prices are obtained from a model for the marginal generation cost of electricity and expected consumption of electricity. The first studies on the pricing of forward contracts in competitive markets were by Kaye *et al.* (1990). Eydeland and Geman (1999) provide a production-based model for forward contract pricing. Pricing issues of financial instruments have been covered in connection to the design of deregulated market models by Gedra (1994), Ghosh and Ramesh (1997), Lowrey (1997), and Oren (1999).

The statistical approaches work with explicit formulae for the electricity price processes. Parameters of the price processes are estimated from the available market data. Several possibilities for the spot price process are studied by Pilipovic (1997) and Deng (2000). A general discussion about commodity price processes is found, for example, in Duffie and Gray (1995) or Schwartz (1997). The assumption that there are tradable forward contracts for all future time points creates a complete forward commodity market over the spot electricity market. On the forward market, it is possible to introduce pricing models for complicated instruments such as generation assets in Deng *et al.* (1999), or end consumer sales in Keppo and Räsänen (1999).

With all models, assumptions on the spot price process set the prices for the electricity derivatives. The calibration of the models is often not possible in a consistent manner. Assuming a complete forward market is not realistic even for the most developed and liquid Scandinavian market. A general framework for the electricity market and an introduction to the real pricing problems are the main contributions of this study.

2. General theory

2.1 *Market setting*

This paper presents a mathematical model of the electricity market in the tradition of financial theory. It is assumed that trading in the market is conducted by a large number of participants who provide enough liquidity to the market and take advantage of any potential arbitrage opportunities. The effects of transaction costs and taxation are ignored throughout the paper. The interest rates are assumed to be non-stochastic, constant, and available for investing and borrowing money. The concentration is on one electricity spot market area and single currency.

The model is set in a continuous time probability space $(\Omega, \mathcal{F}, P)$ for a time period $[0, \tau]$. Here Ω is the set of possible outcomes, $\mathcal{F}$ is a σ-algebra in Ω, and P is a probability measure defined on $\mathcal{F}$. The asset prices in the market follow a $(n+1)$-dimensional Itô process $\boldsymbol{x}(t,\omega) := (x_0(t,\omega), x_1(t,\omega), \ldots, x_n(t,\omega)) : [0,\tau] \times \Omega \to \mathbb{R}^{n+1}$. The uncertainty in the market is generated by an m-dimensional Brownian motion, $\boldsymbol{B}(t,\omega) : [0,\tau] \times \Omega \to \mathbb{R}^m$. $\mathcal{F}_t$ is the standard filtration on Ω that is generated by the continuous Brownian motion, and that is completed with respect to the measure P. The market prices are given by

$$\mathrm{d}x_i(t,\omega) = \mu_i(t,\omega)\mathrm{d}t + \sum_{j=1}^{m} \sigma_{ij}(t,\omega)\mathrm{d}B_j(t) \qquad 0 \le i < n \tag{1}$$

$$\mathrm{d}x_n(t) = rx_n(t)\mathrm{d}t \tag{2}$$

where $\mu_i(t,\omega) : [0,\tau] \times \Omega \to \mathbb{R}$ is the local drift of $x_i(t,\omega)$ and $\sigma_{ij}(t,\omega) : [0,\tau] \times \Omega \to \mathbb{R}$ is the local volatility from Brownian stochastic factor j to asset price i. These functions are assumed to satisfy the normal growth and Lipschitz conditions. The prices of assets $x_i(t,\omega), 0 \leq i < n$, depend on the Brownian motion and are stochastic, emphasized by the use of ω. The price of the asset n, $x_n(t)$, is non-stochastic, dependent on the risk-free interest rate r, and represents a risk-free investment.

In an arbitrage-free financial market the prices of all derivative instruments are set in a consistent manner. There is a unique equivalent martingale measure Q, i.e. such measure that $\boldsymbol{x}(t,\omega)$ normalized with the risk-free investment process is a martingale with respect to measure Q (Øksendal, 1998). Measure Q is also called the risk neutral measure. The existence of such measure follows from the assumption that there is no arbitrage. It is assumed that the markets price all the instruments in a consistent manner also in the electricity markets, i.e. that the market expectations of the instrument prices are calculated against a risk neutral measure Q.

The derivative instruments in this paper take the form of contingent claims that have

- a payoff rate $g(t,\omega) \in \mathcal{F}_t, \ \forall\ 0 \leq t \leq T$, and
- a payoff $F(\omega;T) \in \mathcal{F}_T$ at maturity, $t = T$.

The basic idea of pricing derivative instruments in the financial market is to construct a hedging portfolio from available instruments and to calculate the cost of the hedging portfolio. If there is a unique price for the instrument then it is equal to the minimum hedging cost. The market is complete if there is a hedging portfolio for all bounded contingent claims.

2.2 *Assets and instruments*

A primary underlying asset in an electricity market is spot electricity. Physical spot prices in the Scandinavian market are set by an equilibrium model where the supply and demand curves of all the market participants are matched day-ahead; see electricity exchange Nord Pool (2001) for details.

The spot price is represented by a stochastic Itô process that is symbolized by $x_0(t,\omega)$. The disruptions in the electricity system create spikes and jumps in prices that are not included in the possible paths of an Itô process. A detailed formulation would probably require the use of Poisson jumps and more general class of Lévy processes. The exact formulation of the spot price process is beyond the scope of this study.

An additional restriction of non-tradability is set to physical spot electricity. There is no economical way of storing electricity even for short time intervals, which forces electricity to be generated and consumed instantaneously. Cash flows from generation or consumption may be a directly linked to the spot price $x_0(t,\omega)$ but at no point in time is it allowed to own spot electricity as an asset. It is not possible to hedge derivatives on spot price with just spot electricity and a bank account, i.e. the electricity market with spot electricity is incomplete.

The Scandinavian electricity market trades financial instruments similar to interest rate swaps to fix the price level of electricity that is delivered at some future time. A fixed reference price is agreed before the delivery period and the difference from a floating realization of the spot price is settled financially. In the Nordic electricity market, these financial contracts are often named forward and futures contracts instead of, more accurately, swaps. In this paper the distinction is made by using the terms electricity forwards and electricity futures contracts. No differentiation is made between the electricity forward and futures contracts in this paper as their prices are the same under the assumptions of deterministic and constant interest rates.

The underlying asset for an electricity forward contract is the average spot price of some time period. The forward price curve of electricity is defined by $n-1$ electricity forward contracts that span the time frame $[0,\tau]$. It is assumed that the time periods of the electricity forward contracts do not overlap each other. An electricity forward contract is a tradable instrument in the market, but trading ends before the delivery period of the contract.

Taking a long position in an electricity forward contract at time t with forward price $x_1(t)$ and delivery period $T=[t_1,t_2],\ t<t_1\leq t_2$ gives a payoff rate

$$g(s,\omega) := x_0(s,\omega) - x_1(t) \qquad \forall s\in T \tag{3}$$

or equivalently, disregarding interest rates, the payoff at the end of the delivery period

$$F(t,\omega;T) := (t_2 - t_1)[x_0^{ave}(\omega;T) - x_1(t)] \tag{4}$$

where

$$x_0^{ave}(\omega;T) := \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} x_0(t,\omega)\, \mathrm{d}t \tag{5}$$

i.e. $x_0^{ave}(\omega;T)$ is the average of the spot price of the time period $T=[t_1,t_2]$.

Upon the basic assets of spot electricity and electricity forward contracts markets have developed derivative instruments. In the Nord Pool exchange, trading is conducted with European options on the electricity forward contracts and Asian options on the average spot electricity price in addition to trading with electricity futures and forward contracts. The active Scandinavian OTC-market is in addition commonly trading with spot price area swaps, swing options, and American options. There are more complex contract structures in the wholesale and end user markets, and even generation assets may be viewed as derivative instruments in the light of the modern real options analysis.

Table 1 gives a summary of the instruments that this paper discusses. Forward contracts are the most simple instruments as their payoff is linear and delivery volume is fixed beforehand. Financial forward contracts have clear pricing models based on the nonarbitrage relations and with some commodity forwards convenience yield is used. Electricity forward contracts have no pricing model because the

Table 1. Contract specifications for the electricity derivative instruments that are studied in this paper: underlying assets for the derivatives, the type of the payoff functions, the volume involved in the payoff, the availabe pricing models, and sections that offer more detailed discussion

Instrument	*Underlying asset*	*Payoff*	*Volume*	*Available model*	*Section*
Forward contract	Financial	Linear	Fixed	No-arbitrage rules	3.1
	Commodity	Linear	Fixed	Convenience yield	3.1
Electricity forward	Spot	Linear	Fixed	Ð	2.2 & 3.2
European option	Elec. forwards	Non-linear	Fixed	Black–Scholes	4
Asian option	Spot	Non-linear	Fixed	Ð	5.2
Swing option	Spot	Non-linear	Volatile	Ð	5.3

underlying asset is a non-storable spot price. European and Asian options have nonlinear payoffs but the volume of the contracts is fixed. The underlying assets for European options are electricity forwards that are tradable in the market and the Black–Scholes model is applicable. Asian options on non-tradable spot electricity have no ready pricing methodology. Swing options on spot electricity are even more complex with nonlinear payoffs and uncertain volume.

3. Forward prices

3.1 *Other markets*

The importance of forward and futures contracts is far greater in the commodity markets than in the equity, currency, or interest rate markets. In the financial markets, the forward prices are usually direct consequences of the spot prices of the underlying assets. Contractual obligations, seasonal variations, consumption needs, forced production, or the cost of storage are a few of the reasons why the forward prices in the commodity markets behave differently and include additional information about the future spot prices.

Due to the non-arbitrage conditions, there is an analytical formula for the forward prices in the financial markets:

$$x_1(t;T) = \mathrm{e}^{r(T-t)} x_0(t) \tag{6}$$

where $x_1(t;T)$ is the time T forward price of the underlying asset at time t, $x_0(t)$ is the spot price of the asset, and r is the risk-free interest rate. Formula 6 gives the forward prices directly under the assumptions of constant and non-stochastic interest rates. Forward and futures contracts in financial markets do not normally lead to physical delivery but are settled in cash.

Brennan and Schwartz (1985) established an expansion to the pricing of forward contracts in those commodity markets where the underlying asset can be stored, i.e.

$$x_1(t;T) = \mathrm{e}^{(r-y)(T-t)} x_0(t) \tag{7}$$

where y is called the convenience yield. The convenience yield is defined as the rate of return equal to the additional costs and market preferences of owning the commodity. In general, convenience yield is time dependent and stochastic and it may be used to explain almost any forward price structure. Convenience yield provides a simple model that explains the deviations of the commodity forward prices from purely economical reasoning according to Formula 6. The forward contracts in commodity markets often include physical delivery whilst commodity futures contracts are usually settled in cash.

3.2 *Electricity markets*

The link between the current spot price and forward prices is weaker in the electricity markets because of the non-storability of electricity. Electricity that is delivered at any given future time is a separate asset from the electricity that is delivered now.

In financial markets and other commodity markets, the forward prices of the commodity converge to the spot price of the underlying asset at the maturity of the forward contracts. There is no convergence of forward prices in the electricity market. The underlying asset of electricity forward contracts is the average spot price of the delivery period. Trading with electricity forward contracts ceases before the delivery period, but the average spot price of the delivery period is set based on the events of that period. The following two propositions provide convergence results that are valid in the electricity markets.

Proposition 3.2.1. Electricity forward prices converge to the risk-adjusted market expectations of the average spot price of the delivery period.

Proof. The payoff of an electricity forward contract with nominal forward price $x_1(t;T) \in \mathcal{F}_t$ for the time period $T := [t_1, t_2]$ has the discounted time t value of

$$\tilde{F}(t,\omega;T) := \int_{t_1}^{t_2} \mathrm{e}^{-r(s-t)}(x_0(s,\omega) - x_1(t;T))\,\mathrm{d}s \tag{8}$$

$$= (t_2 - t_1)[\tilde{x}_0^{ave}(\omega;T) - \tilde{x}_1(t;T)], \tag{9}$$

where

$$\tilde{x}_1(t;T) := \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \mathrm{e}^{-r(s-t)} x_1(t;T)\,\mathrm{d}s \tag{10}$$

is the real forward price, and $\tilde{x}_0^{ave}(\omega;T) \in \mathcal{F}_{t_2}$ is the discounted average spot price of the delivery period.

It costs nothing to enter the contract in the market at time $t < t_1$. The present value of the electricity forward contract, $f(t;T)$, under the market expectations is

$$0 = f(t;T) := \mathrm{E}_Q[\tilde{F}(t,\omega;T) \mid \mathcal{F}_t] \tag{11}$$

where the expected value is evaluated against the measure Q. It follows that

$$0 = \mathrm{E}_Q[\tilde{x}_0^{ave}(\omega;T) - \tilde{x}_1(t,\omega;T) \mid \mathcal{F}_t] \tag{12}$$

$$\mathrm{E}_Q[\tilde{x}_1(t,\omega;T) \mid \mathcal{F}_t] = \mathrm{E}_Q[\tilde{x}_0^{ave}(\omega;T) \mid \mathcal{F}_t] \tag{13}$$

$$\tilde{x}_1(t;T) = \mathrm{E}_Q[\tilde{x}_0^{ave}(\omega;T) \mid \mathcal{F}_t] \tag{14}$$

The possible market expectations of the future forward prices disappear when the trading with the electricity forward contract ceases at time t_1. The final forward price gives the market's expectation of the average spot price during the delivery period.

Proposition 3.2.2. The average spot price of the delivery period differs from the last quotation of the forward price $x_1(t_1,\omega) \in \mathcal{F}_{t_1}$, almost surely.

Proof. Divide the last quotation of the forward price $\tilde{x}_1(t_1;T)$ to $\tilde{y}_1(t_1;[t_1,t])$ and $\tilde{z}_1(t;[t,t_2])$ so that

$$\tilde{x}_1(t_1;T) = \tilde{y}_1(t_1;[t_1,t]) + \tilde{z}_1(t;[t,t_2]) \tag{15}$$

$\tilde{y}_1(t_1;[t_1,t])$ and $\tilde{z}_1(t;[t,t_2])$ represent the market expectations for the time periods $[t_1,t]$ and $[t,t_2]$ at time t_1. The realized value of the electricity forward contract up to time $t, t_1 \le t \le t_2$, against market expectation for that time period is

$$F(t) = (t_2 - t_1)\{\tilde{y}_1(t_1;[t_1,t]) - \frac{1}{t - t_1}\int_{t_1}^{t} \tilde{x}_0(s,\omega)\,\mathrm{d}s\} \tag{16}$$

If $x_0(t,\omega)$ is stochastic then the probability that the average spot price until time t is equal to the market expectations is

$$P\left[\frac{1}{t - t_1}\int_{t_1}^{t} \tilde{x}_0(s,\omega)\,\mathrm{d}s = \tilde{y}_1(t_1;[t_1,t])\right] = 0 \tag{17}$$

because the measure of the singleton $\tilde{y}_1(t_1;[t_1,t])$, or any set with null measure, is zero. Taking $t \to t_2$ gives $\tilde{y}_1(t_1;[t_1,t_2]) = \tilde{x}_1(t_1;T)$. It follows that

$$f(t_2;T) = \tilde{x}_1(t_1) - \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \tilde{x}_0(s)\,\mathrm{d}s \neq 0 \qquad \text{a.s.} \tag{18}$$

i.e. it is almost certain that the market expectation of the average spot price was not correct.

The forward price is not necessarily the same as the average spot price of the delivery period and there is no explicit connection between the forward price and average spot price. Speculative trading with no interest in spot delivery may alter the forward prices. Therefore, before the last day of trading, it may be difficult to analyse the market expectations of the spot prices from the prevailing forward prices.

Corollary 3.2.3. Measure Q includes also the market expectations of the future forward prices.

The incompleteness of the electricity market is due to the non-tradability of spot electricity. Assuming that there is a normal forward contract for each future time point completes the market. Forward contracts are tradable assets and the forward price for a single time period converges to the spot price for the same time period under a proper market structure. The assumption is not viable in practice. The spot price risk resulting from disruptions in the physical electricity system is always present in derivative instruments on spot price and should not be ignored. The number of forward contracts required to complete the market is too large to sustain an acceptable level of liquidity for longer time periods.

4. European options on forwards

Underlying assets of the European options in the Scandinavian market are the electricity forward contracts. The maturity of the options is before the start of the delivery period of the electricity forwards. The payoff of an option is dependent only on the price of the electricity forward contract. Explicitly, the payoff of the option is not dependent on the current spot price, spot price at maturity of the option, or any other spot price.

Electricity forward contracts are tradable instruments in the market, and it is possible to create a replicating hedging portfolio for the European options on electricity forwards. The problem of pricing European options in this setting has been well studied in the financial market. The price of a European call option with strike price K and maturity at t_0 is the expected value of the option payoff at the time of its maturity discounted to present time, i.e.

$$c(t, x_1(t;T); t_0, K) = E_Q[\mathrm{e}^{-r(t_0-t)} \max(x_1(t_0;T) - K, 0)] \tag{19}$$

where the expected value is evaluated against the risk-neutral probability measure of the market. Note, it is assumed that the markets use the same risk-neutral measure for the pricing of derivatives on all assets.

As with the spot price, the explicit price processes of electricity forwards are not assumed to be known. As an example, consider the Black–Scholes framework with one electricity forward contract, $x_1(t, \omega; T)$, and a bank account, $x_2(t)$, whose prices follow

$$dx_1(t, \omega; T) = \mu_1 x_1(t, \omega; T)\,dt + \sigma_1 x_1(t, \omega; T)\,dB_1(t) \tag{20}$$

$$dx_2(t) = r x_2(t)\,dt \tag{21}$$

i.e. the stochastic process followed by the price of the electricity forward is a geometric Brownian motion and the interest rate is deterministic and constant. By the Black-76 model, the price of a European call option on the forward contract is

$$c(t, x_1(t; T); t_0, K) = \tilde{x}_1(t; T)\Phi(d_1) - e^{-r(t_0 - t)} K\Phi(d_2), \tag{22}$$

where $\Phi(d)$ is the standard normal distribution function and

$$d_1 = \frac{\ln x_1(t; T)/K + (r + \sigma_1^2/2)(t_0 - t)}{\sigma_1\sqrt{t_0 - t},} \tag{23}$$

$$d_2 = d_1 - \sigma_1\sqrt{t_0 - t} \tag{24}$$

The pricing of European options is relatively straightforward with more general assumptions about the market and the price processes. There are no additional difficulties with the pricing of European or American electricity forward options in comparison to other commodity markets due to the tradability of the electricity forwards.

5. Options on spot electricity

5.1 *General*

Options on spot electricity price are necessarily more exotic than options on tradable products. Normal European options on spot electricity for single future time points would not have enough liquidity. The market has adopted some of the structures from the financial markets, such as Asian options, and some of the structures from the regulated old markets, such as swing options.

Physical generation assets may be considered as options on spot electricity in the spirit of the modern real options analysis. For example, a condensing power plant is roughly equivalent to a call option on spot electricity, and a hydro power plant has similar characteristics to a swing option. The complexity of these options is hardly matched in the financial markets because of the physical constraints on the plant operation.

Market prices exhibit the risk adjusted market expectations of the future cash flows. The market prices of commodity derivatives may differ from the theoretical model prices because the market participants have subjective preferences to owning those derivatives. Construction of a hedging portfolio for the derivative instrument would be required to make the actual market prices equal to the theoretical prices. In the financial markets the availability of replicating portfolios removes the dependency of prices to individual risk preferences. There are equivalent non-arbitrage relations in most commodity markets. It is yet to be shown that there are such relations in the electricity markets.

A theoretical price for any derivative instrument on spot electricity requires a formulation for the spot price process. In the financial markets, geometric Brownian motion explains many of the properties of real market prices and it is used as the basis of instrument pricing. The observed electricity spot prices make it clear that the structure of the spot price process is more complex. In addition to finding a valid formulation for the spot price process, the estimation of realistic parameters of the process is a formidable task due to the possibility of jumps in electricity spot prices and the lack of historical data.

5.2 *Asian electricity options*

The settlement rules of Asian options in the electricity market are the same as in the financial markets. The reference price of an Asian option is the arithmetic average spot price, $x_0^{ave}(T,\omega)$, of some time period $T := [t_1, t_2]$. The payoff of an Asian call option with strike price K at maturity, at time t_2, is

$$A_c(t,\omega;T,K) := (t_2 - t_1)\max\{x_0^{ave}(T,\omega) - K, 0\} \tag{25}$$

and the payoff of the put option is

$$A_p(t,\omega;T,K) := (t_2 - t_1)\max\{K - x_0^{ave}(T,\omega), 0\} \tag{26}$$

Define $a_c(t;T,K)$ as the price of an Asian call option for time period T and strike price K, and equally $a_p(t;T,K)$ as the price of an Asian put option. The prices of the options under the market expectations are for the call option

$$a_c(t;T,K) := (t_2 - t_1)\mathrm{E}_Q[\mathrm{e}^{-r(t_2-t)}\max\{x_0^{ave}(T) - K, 0\}] \tag{27}$$

and for the put option

$$a_p(t;T,K) := (t_2 - t_1)\mathrm{E}_Q[\mathrm{e}^{-r(t_2-t)}\max\{K - x_0^{ave}(T), 0\}] \tag{28}$$

Pricing of Asian options in the financial market is based on the fact that the underlying asset is tradable and the assumptions of the price processes. As argued above, trading with spot electricity is not possible and the exact formulation of the electricity spot price process $x_0(t,\omega)$ is not easily defined. Theoretical price setting for Asian options is an open issue.

The put–call parity gives the relationship between the prices of European put and call options in the financial market. It is possible to consider the Asian option as a European option on the average spot price and derive a relationship similar to the put–call parity for the prices of the Asian options. It is assumed that there is an electricity forward contract with current forward price x_1 for the time period of the Asian option. Consider two portfolios:

- Portfolio A: a long position in an Asian call option with strike price K and a cash amount of $(t_2 - t_1)\mathrm{e}^{-r(t_2-t)}(K - x_1)$.
- Portfolio B: a long position in an Asian put option with strike price K and an electricity forward contract with forward price x_1.

At the end of the delivery period of the options, the call option in Portfolio A has a payoff of $(t_2 - t_1)\max\{x_0^{ave}(T) - K, 0\}$ and the money is worth $(t_2 - t_1)(K - x_1)$. In portfolio B, the put option has a payoff of $(t_2 - t_1)\max\{K - x_0^{ave}(T), 0\}$ and the electricity forward a payoff of $(t_2 - t_1)$ $[x_0^{ave}(T) - x_1]$. Both portfolios have the same total payoff of

$$(t_2 - t_1)\max\{x_0^{ave}(T) - x_1, K - x_1\} \tag{29}$$

at the end of the delivery period. The no-arbitrage assumption states that the present values of the portfolios must be equal. This yields

$$(t_2 - t_1)\mathrm{e}^{-r(t_2-t)}(x_1 - K) = -a_c(t;T,K) + a_p(t;T,K) \tag{30}$$

as it costs nothing to enter the electricity forward contract.

It is not possible to create hedging portfolios for Asian options with electricity forwards, despite the connection of the payoffs. Trading with an electricity forward contract ceases before the delivery period of the corresponding Asian option but the payoff of the Asian option depends on the events of the delivery period. Dynamic adjustment of the hedging portfolio is not possible after the delivery period has started.

5.3 *Swing options*

A swing option was widely used in the regulated Scandinavian market to address the problems created by the non-storability of electricity. Both demand and generation of electricity exhibit variability but the variations tend to smooth out over time. For example, an electricity distribution company with electricity heating or cooling customers has a varying load according to the temperature, and a hydro-power owner has the possibility to adjust his generation according to the hydrological situation.

A swing option in the electricity market gives its owner the right to use energy up to a certain limit at a fixed price during a fixed time interval. It may also include the obligation to use at least a certain amount of energy during the same interval. There are limits to the power, $u(t)$, that can be used at any given time.

The set $\mathcal{U}$ of feasible strategies for the owner of a swing option includes those $u(t) \in \mathcal{F}_t$ that satisfy the conditions

$$\begin{aligned} E_{min} \le \int_{t_1}^{t_2} u(t)\,\mathrm{d}t \le E_{max} \\ u_{min}(t) \le u(t) \le u_{max}(t) \qquad \forall t \in [t_1, t_2] \end{aligned} \tag{31}$$

The payoff of a swing option with strike price K is

$$W(\omega; T, K, u') := \int_{t_1}^{t_2} u'(t)[x_0(t,\omega) - K]\,\mathrm{d}t \tag{32}$$

when strategy $u' \in \mathcal{U}$ is employed.

The optimal use of a swing option against the spot market price is given by the solution of the following stochastic dynamic optimization problem

$$\sup_{u\in\mathcal{U}} \mathrm{E}_Q\left[\int_{t_1}^{t_2} \mathrm{e}^{-r(s-t)} u(s)(x_0(s,\omega) - K)\,\mathrm{d}s\right] \tag{33}$$

The optimality is considered against the market expectations under the measure Q. If the optimal policy is $u^*(t)$, then the option price is

$$w(t;T,K,\mathcal{U}) := \mathrm{E}_Q\left[\int_{t_1}^{t_2} \mathrm{e}^{-r(s-t)} u^*(s)(x_0(s,\omega) - K)\,\mathrm{d}s\right] \tag{34}$$

By optimal policy, it is referred to such $u^*(t)$ that

$$\begin{aligned} &\mathrm{E}_Q\left[\int_{t_1}^{t_2} \mathrm{e}^{-r(s-t)} u^*(s)(x_0(s,\omega) - K)\,\mathrm{d}s\right] \\ &\geq \mathrm{E}_Q\left[\int_{t_1}^{t_2} \mathrm{e}^{-r(s-t)} u(s)(x_0(s,\omega) - K)\,\mathrm{d}s\right] \qquad \forall u \in \mathcal{U} \end{aligned} \tag{35}$$

The solution to the stochastic optimization problem (33) is not Markovian because of the integral constraint on the total energy use. In addition, the spot price process $x_0(t,\omega)$ is not known, it is not possible to construct a hedging portfolio, and the risk preferences of the market are not known. Theoretical price setting for the swing option is an open issue.

Given the forward curve of electricity prices $x_f(t)$ for all $t \in [t_1, t_2]$ it is possible to decide the policy for the use of the swing option beforehand and hedge it completely with electricity forwards. The optimal policy against the forward curve is then given by the solution to the following deterministic optimization problem.

$$\sup_{u_f \in \mathcal{U}} \int_{t_1}^{t_2} \mathrm{e}^{-r(s-t)} u_f(s)(x_f(s) - K)\,\mathrm{d}s \tag{36}$$

Under sufficient regularity constraints, there is a unique solution, $u_f^*(t)$, to the deterministic optimization problem (36). The lower boundary for the price of the swing option is given by

$$w(t;T,K,\mathcal{U}) \geq \int_{t_1}^{t_2} \mathrm{e}^{-r(s-t)} u_f^*(s)(x_f(s) - K)\,\mathrm{d}s \tag{37}$$

The optionality of a swing contract refers to the choice in the use of energy. There is no strategy that would guarantee a better profit from a swing option than from a corresponding Asian option. More precisely, the following proposition holds.

Proposition 5.3.1. For any given strategy $u' \in \mathcal{U}$ there is a price realization $x_0'(t)$ that gives

$$W(x_0'(t);T,K,u'(t)) \leq u^{ave} A_c(t, x_0'(t);T,K) \tag{38}$$

where

$$u^{\mathrm{ave}} := \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} u'(s)\,\mathrm{d}s \tag{39}$$

Proof. Without loss of generality choose $K = 0$. Take any strategy $u' \in \mathcal{U}$. Choose a set $S \subset T$ so that

1. $u^{sup} := \sup_{s\in S} u(s) \leq u(t)$ for all $t \in T \cap S^{\mathrm{C}}$,
2. $m(S) = 1/2m(T)$, where m is the Lebesgue measure.

Such S exists because $u(t)$ is measurable. Then $u^{sup} \leq u^{ave}$. Set $x_0'(t) = 1\ \forall t \in S$ and $x_0'(t) = 0\ \forall t \in T \cap S$. Define x^{ave} as the average spot price of the time period T, then $x^{ave} = m(S)/m(T)$. It follows that

$$W'(t;T,0,u') = \int_T u'(s)x(s,\omega)\,\mathrm{d}s = \int_S u'(s)\,\mathrm{d}s \tag{40}$$

$$\leq \int_S u^{sup}\,\mathrm{d}s = u^{sup}m(S) \leq u^{ave}m(S) \tag{41}$$

$$= u^{ave}x^{ave}m(T) = u^{ave}\int_T x_0'(s)\,\mathrm{d}s \tag{42}$$

$$= u^{ave}A_c(t,x_0'(t);T,0) \tag{43}$$

i.e. for any use policy there is a price realization under which the payoff of the swing option is less or equal to the payoff of the corresponding position in an Asian option.

There is no risk-free premium from a swing option in comparison to the Asian option because any chosen exercise strategy may result in unfavourable cash flows. The payoff from a swing option may be negative with a fixed strike price $K > 0$.

6. Example

The market data is from the Scandinavian electricity market. All prices are in Norwegian crowns (NOK), and the interest rate is assumed to be constant and fixed to $r = 7.0\%$, corresponding roughly to the risk free interest rate in Norway.

Table 2 summarizes the details of the electricity forward contract Year 2001 whose delivery period is the calender year 2001. Trading with the electricity forward ceases on 29 December 2000 and the settlement date for the European options on the electricity forward is 21 December 2000. The market price for the year 2001 was around 134 Norwegian crowns on 1 November 2000.

Table 3 gives the market prices for the year 2001 options, the Black–Scholes prices of those options with historical volatilities when available, the estimated historical volatilities, and the implied Black–Scholes volatilities. The first six options are European options on the forward contract, followed by an Asian option with delivery period equal to the year 2001. Finally, there are two swing options with the energy to be used restricted to roughly 40% of full capacity and roughly 57% of full capacity and strike price equal to the current market price of 134 NOKs. With these swing options it is required that the minimum used energy is equal to the maximum used energy and the exercise of the options is made on an hourly basis. The market prices are indicative prices of the options traded on 1 November 2000.

Table 2. Details of the forward contract year 2001

Contract	*Maturity Eur. option*	*Maturity forward*	*Delivery period*	*Market closing price 1/11/00*
Year 2001	21/12/00	30/12/00	1/1/01–31/12/01	134.00

Table 3. Available market data for several options on the year 2001 electricity forward contract and spot price of the year 2001, market prices for the options, theoretical Black–Scholes prices for European options that are calculated with historical volatilities, implied Black–Scholes volatilities for European options, and historical volatilities

Option type	*Strike price*	*Price market*	*B & S*	*Volatility implied*	*Historical*
European call	140.00	0.65	0.38	13.1%	11%
European call	135.00	2.05	1.68	12.9%	11%
European call	130.00	4.95	4.65	12.9%	11%
European put	135.00	3.05	2.67	12.9%	11%
European put	130.00	1.00	0.69	13.0%	11%
European put	125.00	0.25	0.09	13.6%	11%
Asian call	140.00	13.0	Ð	Ð	Ð
Swing 40%	134.00	26.0	Ð	Ð	Ð
Swing 57%	134.00	17.0	Ð	Ð	Ð

The historical volatilities are estimated from the log-normalized asset returns assuming that the prices of the forward contracts follow geometric Brownian motion. The data for analysis is taken for the 30 trading days from the period 20 September–31 October 2000. The implied Black–Scholes volatilities are calculated using the pricing formula directly.

The market prices of the European options follow closely to the theoretical Black–Scholes prices of those options with higher implied volatility than the historical volatility. The market actually expects the volatility to grow because the trading period with the forward contract is ending and the volatility of the forward prices usually increases. The prices probably also include additional premium for the actual market imperfections compared to the Black–Scholes model, such as the lack of continuous trading, transaction costs, and stochastic volatilities and interest rates. Options that have a strike price close to the market price of the underlying are cheaper in the market than options whose strike price is far from the price of the underlying. Similar phenomenon is found in the prices of options on other markets, and it is due to the shortfalls of the Black–Scholes model.

The price of the Asian call option is notably high in comparison to the prices of the European call options. This is because the settlement of the Asian option is done against the average spot price of the year 2001 which includes greater uncertainty than the forward price of the year 2001 that is the underlying asset of the European option. In the light that the Asian option cannot be hedged with other market instruments, the price of the Asian call option may even be considered low.

The prices of the swing options are higher than the price for the Asian option. It has been shown in this paper that there are price realizations that may result in poorer performance for the swing options than Asian options. In practice, the structure of the swing option allows the utilization of the difference between the peak and off-peak hours and seasonal variations that very probably result in favourable cash flows from the swing options in comparison to the Asian option. Note that the swing option with the 40% energy limit has a greater market value than the swing option with the 57% energy limit because of the greater flexibility in the selection of optimal exercise times with the smaller energy constraint.

Table 4. Alternative hedging strategies for an industrial user with 5000 hours of varying consumption need of 10 MW: the instrument used for hedging, the strike prices of the options, and the estimated and maximum unit cost for the electricity consumption based on market prices on 1 November 2000

Instrument for hedging	*Strike*	*Estimated unit cost*	*Estimated max. unit cost*
Electricity forward	Ð	134.00 NOK	134.00 NOK
European option	140.00 NOK	134.65 NOK	140.65 NOK
Asian option	140.00 NOK	147.00 NOK	153.00 NOK
Swing option	134.00 NOK	151.00 NOK	151.00 NOK

Consider an industrial electricity consumer with varying annual consumption need of 10 MW for 5000 hours during the year 2001. Physical electricity is obtained from the electricity exchange at the spot electricity price. The consumer considers four alternatives for hedging against the volatility in the cost of electricity procurement on 1 November 2000. Table 4 summarizes the alternatives of using electricity forward at market price 134 NOKs, European option with strike price 140 NOKs, Asian option with strike price 140 NOKs, or swing option with strike price 134 NOKs. The volume of all the hedging strategies is assumed to match the total consumption. The estimated unit costs include the option premiums. The estimated unit cost is based on Proposition 3.2.1, i.e. that the forward prices give a forecast for the spot prices. The estimated maximum unit cost is based on the most unfavourable price movements and shows how the hedging strategy protects against those.

Hedging with an electricity forward locks the electricity procurement price to the level on 1 November 2000. The hedge is only approximative because of the differences between the flat profile of the electricity forward and the true varying consumption. Hedging with a European option enables the consumer to take advantage of favourable price movements of the forward prices before the maturity of the option while maintaining protection against the unfavourable price developments. It is assumed that an electricity forward is entered on the maturity date of the option. Hedging with an Asian option offers protection against high average spot prices, but enables the consumer to profit from low average spot prices. The Asian option strategy still has the same profile risk as with the electricity forwards. A swing option offers the best protection for the consumer because he/she may use it to match his/her consumption profile exactly. The choice between the alternatives depends on the consumer's views of the future, risk preferences, type of business, etc. This simplified portfolio selection problem added with more complexity is encountered by real market participants all the time.

7. Discussion

This paper suggests that the straightforward application of the methodologies of the financial markets is not feasible in the electricity markets. The most important distinction is the fact that spot electricity is not a tradable asset. There are derivatives for both the spot electricity and electricity forward contracts in the markets. Electricity forwards are tradable instruments and it is shown that there are no additional difficulties in pricing derivatives on electricity forwards.

There are very few pricing results available for derivatives whose payoff is dependent on spot electricity. Even the electricity forward prices lack an arbitrage-free price relation to the prevailing spot price. The underlying source of uncertainty is not tradable and the price process followed by the spot price is not easily formulated. The combination of these factors makes the pricing problems in the electricity markets uniquely difficult.

The paper provides a few price relations between the prices of spot electricity derivatives. Numerical market data supports these results. Several open problems are presented for further study.

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