ELSEVIER

European Journal of Operational Research 145 (2003) 136–147

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

www.elsevier.com/locate/dsw

O.R. Applications

Managing electricity market price risk

Iivo Vehviläinen [a,*,1], Jussi Keppo [b,2]

[a] Fortum Power and Heat Oy, P.O. Box 40, 00048 Fortum, Finland

[b] Department of Industrial and Operations Engineering, University of Michigan, 1205 Beal Avenue, Ann Arbor, MI 48109-2117, USA

Received 7 March 2000; accepted 26 September 2001

Abstract

This paper introduces an application of financial risk management methods to the deregulated electricity markets. A framework for the Monte Carlo performance simulation of a power portfolio is presented. The optimal portfolio selection problem is addressed and a numerical method is implemented. Numerical results of simulation and optimization are presented in the Nordic electricity market. The results suggest that the risk management methods of the paper can be applied to the everyday electricity market practice.

Keywords: Energy; Risk analysis; Stochastic processes; Simulation; Portfolio optimization

1. Introduction

The deregulation of the electricity market brings competition to the previously monopolistic market, where the risk of loss has been small. Electricity cannot be stored and it is bought primarily for consumption. The new electricity market is highly volatile in comparison to any security or commodity market. Actors in the market are exposed to substantial risks caused by the volatile market conditions. In the tightening competitive environment, the optimal management of these new risks is the focus of energy utilities worldwide. This study provides a solution for the problem of optimal risk aware power portfolio management in the deregulated electricity markets.

The deregulation of the electricity markets has introduced electricity exchanges that trade spot electricity and electricity derivatives in a similar manner as stocks and other securities are traded in the financial market. Common electricity derivatives are forward and futures contracts, European options on forward contracts, and Asian options on spot electricity. The customary bilateral electricity contracts are often more complex. The nonstorability of physical electricity and the seasonal effects make the electricity market different from the financial markets. Most importantly, there are no analytical formulas for the majority of electricity derivatives prices and all analysis must rely on numerical methods. This paper shows that with proper modifications some of the methods of the

* Corresponding author. Tel.: +358-10-45-36486; fax: +358-10-45-36700.

E-mail addresses: iivo.vehvilainen@iki.fi (I. Vehviläinen), keppo@umich.edu (J. Keppo).

1 The views of the author are his own and do not necessarily reflect those of Fortum Power and Heat Oy.

2 Tel.: +1-734-615-4045; fax: +1-734-576-9015.

0377-2217/03/$ - see front matter © 2002 Published by Elsevier Science B.V.
PII: S0377-2217(01)00399-X

financial markets are applicable in the electricity markets.

A classical problem in the risk management field is that of the optimal portfolio selection. A general numerical framework for the optimal portfolio hedging in the financial market is presented by Keppo and Peura [12]. This paper uses the same approach, but the financial setting is converted to the electricity markets and the optimal portfolio selection problem is formulated as a static optimization problem in order to get fast and practical results for energy companies. The standard methods in the energy sector rely on the well-known Markowitzian portfolio optimization. These methods are restricted to only few basic instruments and are not easily adaptable to cover more complex energy derivatives. The portfolio optimization framework of this paper is capable of covering wide range of instruments, for example, end consumer tariff sales as formulated by Keppo and Räsänen [13] and generation assets like in [6].

The optimization problem considered in this paper is the maximization of the expected utility from the electricity portfolio. Utility functions model the agent's preferences to expected profit making in comparison to the risks taken. The general utility functions are approximated in the numerical procedure where the stochastic utility maximization problem is converted to a deterministic non-linear programming problem with the use of the Monte Carlo simulation. For a discussion about the performance and implementation of Monte Carlo methods, see e.g. [3].

Fleten and Wallace [8] and Fleten et al. [9] study a scenario-based approach for solving the optimal portfolio management problem in electricity markets. The main advantage of the Monte Carlo approach over the scenario-based approaches is the possibility to easily include several stochastic variables that are evaluated simultaneously without seriously affecting the computational performance.

A central problem in the financial risk analysis is the actual quantification of the risks. A popular approach is the so-called value at risk measure that gives the worst potential loss at a given risk level, see e.g. [11]. Introductory work for the implementation of value at risk techniques to the electricity market is presented by Blanco [2] and Pilipovic [16]. The Monte Carlo method of this paper is capable of producing the value at risk measure even for complex portfolios in the electricity markets. The advantage of the method in this paper is the possibility to optimize power portfolios against the given risk measure.

The rest of the paper is organized as follows: Section 2 introduces the stochastic processes used in the paper. Section 3 derives the optimal portfolio by using the stochastic processes and Section 4 describes how Monte Carlo simulation is used in solving the optimization model parameters. Section 5 illustrates the model with numerical examples. Finally Section 6 concludes.

2. The model

2.1. Overview

This paper presents a basic stochastic model where a static portfolio that contains the instruments of interest is simulated in the electricity market over time. The stochastic processes that model the uncertainties in the market are for example electricity spot price, marginal production cost, and consumption processes. A model for the electricity spot price is presented. Inclusion of new stochastic components to represent demand and other load uncertainty is possible in the general stochastic framework of the paper. Demand models have been extensively studied previously, see e.g. [4] or [17].

It is assumed that the incomplete electricity spot market is completed with the derivatives market, i.e., it is assumed that there are electricity futures for each future spot quote. Market frictions, such as the transaction costs and taxation, are ignored. This idealized setting guarantees preliminary results and gives a basis for further development. Despite its shortcomings the model is of immediate practical value to the market participants.

2.2. Electricity price processes

In the every day financial practice it is often assumed that the price processes of the underlying assets follow geometric Brownian motion with constant parameters. The modeling of the electricity spot price process needs to be done differently because of the seasonal effects and other commodity like characteristics of electricity.

Exact formulation of the stochastic spot price process is beyond the scope of this paper. Electricity price process modeling has been studied by Deng [5] and Pilipovic [16]. General discussion about commodity price processes is found for example in [7] or [18]. Another approach is to use a market model to calculate the theoretical equilibrium price of the market, see e.g. [8]. The equilibrium model produces spot price scenarios from historical or simulated data, while in the statistical process-based approaches these data are used to estimate the parameter values of the processes.

The possible weakness with some of the presented models is the inconsistency between the spot price forecast that the models explicitly or implicitly give and the real futures prices in the market, i.e., the model is likely to show fictional arbitrage opportunities between the spot and futures market. The modeler must take care in order to avoid unintentional speculation against the market with her or his model. Direct quantitative methods like the one in this paper could be hazardous unless real market prices are used.

Non-storable physical electricity is not a tradable asset and there is no arbitrage connection between expected future spot prices and corresponding futures prices. However, it may be argued that if a futures price is higher (lower) than the corresponding expected spot price, then the participants in the market are selling (buying) futures contracts and this excess selling (buying) will remove the difference between the expected spot price and future price. In this paper, it is assumed that the expected spot price for time T, $\mathrm{E}(x(T))$, is equal to the current time t futures price, $f(t,T)$, for the same time period. In other words, it is assumed that the forward curve of electricity prices is used as the forecast for the spot electricity prices. Further, it is assumed that the price distribution around the expected value is lognormal. It is well known that this is not a perfect approximation for spot electricity because the price distributions for the prices for physical spot electricity are in reality more fat-tailed. In the numerical calculations it is necessary to use average values to describe the spot prices of discrete time periods and then the lognormality assumption holds better.

The volatility of the price process needs to be either estimated from the historical data or calculated implicitly from the available option quotes. In most markets there are not enough historical data to provide reliable estimates of the volatilities. The scarcity of option prices and the lack of analytical pricing formulas make the calculation of the implied volatilities difficult. In the numerical examples of this paper, historical volatility is used.

2.3. Financial model

The model is set in a continuous-time probability space $(\Omega, \mathscr{F}, P)$ for a time period $[0,T]$. Here Ω is the set of possible realizations, $\mathscr{F}$ is a σ-algebra in Ω, and P is a probability measure defined on $\mathscr{F}$. The N stochastic factors in the market are given by $\boldsymbol{x}(t) = (x_0(t), x_1(t), \ldots, x_N(t))$ that follows a continuous-time Itô process

$$\mathrm{d}\boldsymbol{x}(t) = \boldsymbol{\mu}_{\boldsymbol{x}}(t,\omega)\,\mathrm{d}t + \boldsymbol{\sigma}_{\boldsymbol{x}}(t,\omega)\,\mathrm{d}\boldsymbol{z}(t), \tag{1}$$

where $\boldsymbol{\mu}_{\boldsymbol{x}}(t,\omega) : [0,T] \times \Omega \to \mathbb{R}^N$ is the local growth of $\boldsymbol{x}(t)$ and $\boldsymbol{\sigma}_{\boldsymbol{x}}(t,\omega) : [0,T] \times \Omega \to \mathbb{R}^N \times \mathbb{R}^N$ is the local volatility of $\boldsymbol{x}(t)$. These functions are assumed to satisfy the technical growth and Lipschitz conditions, which means that the functions that give the growth and volatility of the stochastic factors are finite and smooth enough, see e.g. [15]. The vector $\boldsymbol{z}(t)$ consists of N uncorrelated Wiener processes that determine the uncertainties in the market. The change of the value of a variable $z(t)$ in an infinitesimal time interval $\mathrm{d}t$ is $\epsilon\sqrt{\mathrm{d}t}$, where ϵ is a Gaussian stochastic variable. The components of $\boldsymbol{x}(t)$ include e.g. spot price, marginal cost, and consumption processes. It is possible to include other stochastic factors like currency exchange rates for the use of currency derivatives, or weather indexes for weather derivatives.

One possible Itô process is the market price model for the electricity spot price described above. Assume that there are market quotes that give the expected value, $\mathrm{E}(x(t))$, of spot price and the estimated variance, $\mathrm{var}(x(t))$, is known for all times $t \in [0,T]$. The market price model assumes that the returns of the prices are lognormally distributed. The expected value and variance given by the forecast are

$$\mathrm{E}(x(t)) = x(0)\mathrm{e}^{t\mu(0,t)}, \tag{2}$$

$$\mathrm{var}(x(t)) = x(0)^2\mathrm{e}^{2t\mu(0,t)}[\mathrm{e}^{\sigma(0,t)^2} - 1], \tag{3}$$

where $x(0)$ is the current spot price, $\mu(0,t)$ is the growth rate and $\sigma(0,t)$ is the volatility of the underlying asset from current time to time t. The growth rate $\mu(0,t)$ is readily solved from Eq. (2). Substituting (2) into (3) and doing some algebraic manipulation give

$$\sigma(0,t)^2 = \ln[\mathrm{var}(x(t))/\mathrm{E}(x(t))^2 + 1]. \tag{4}$$

It is also possible to calculate the local time dependent growth rate $\mu(t)$ from the given forward curve because the growth from current time to time t must result from the local growth in $[0,t]$, and the same applies to the local volatility. Alternative method is to estimate the local functions directly from the available data. The local growth, $\mu(t)$, and the local volatility, $\sigma(t)$, can then be used in combination with the formulation of the price processes, $\boldsymbol{x}(t)$, to take into account the forward curves.

2.4. Instruments and portfolio

There are M derivative instruments in the market whose prices are given by a state price vector $\boldsymbol{s}(\boldsymbol{x}(t),t) \in \mathbb{R}^M$. From Itô's lemma the process followed by $\boldsymbol{s}(\boldsymbol{x}(t),t)$ is

$$\mathrm{d}\boldsymbol{s}(\boldsymbol{x}(t),t) = \boldsymbol{\mu}(\boldsymbol{x}(t),t)\,\mathrm{d}t + \boldsymbol{\sigma}(\boldsymbol{x}(t),t)\,\mathrm{d}\boldsymbol{z}(t), \tag{5}$$

where $\boldsymbol{\mu}(\boldsymbol{x}(t),t) : \mathbb{R}^N \times [0,T] \to \mathbb{R}^M$ and $\boldsymbol{\sigma}(\boldsymbol{x}(t),t) : \mathbb{R}^N \times [0,T] \to \mathbb{R}^M \times \mathbb{R}^N$ are continuous functions that satisfy the growth and Lipschitz conditions and $\boldsymbol{z}(t)$ contains the same Wiener processes as in the factor process (1). Instruments are physical electricity contracts, electricity derivatives, or other financial instruments, for example, currency derivatives. The exact formulation of the price of an instrument depends on the characteristics of the instrument.

The physical and financial electricity contracts and other financial instruments are combined in a portfolio. The vector $\boldsymbol{\pi} \in \mathbb{R}^M$ gives the contents of the portfolio that is held static over the whole time period. The prices of the instruments are given by $\boldsymbol{s}(\boldsymbol{x}(t),t)$, and the underlying market variables are in $\boldsymbol{x}(t)$. The underlying assets of the instruments can be assigned freely. For example, a production unit can be modeled as an exchange option from marginal cost process to the spot price process.

The wealth from the portfolio at the end of the simulation period is

$$W = \boldsymbol{\pi}^{\mathrm{T}}\boldsymbol{s}(\boldsymbol{x}(T),T), \tag{6}$$

where $\boldsymbol{s}(\boldsymbol{x}(T),T) \in \mathbb{R}^M$ contains the prices of the instruments at the end of the inspection period and $\boldsymbol{\pi}^{\mathrm{T}}$ is the transpose of $\boldsymbol{\pi}$.

3. Optimal portfolio

The optimality of the portfolio depends on the risk preferences of the optimizing agent. The use of a utility function models these risk preferences. The utility function, U, is assumed to be strictly increasing, concave and twice continuously differentiable. The estimation of the decision makers' risk preferences and utility function to a quantitative level is a formidable task. A rough estimate may be found by examining the decision makers' previous actions and reactions to presented alternatives that can be generated e.g. with the model of this paper.

The optimal portfolio is searched starting from a given initial portfolio, $\boldsymbol{\pi}_0$. Optimization is done with respect to the change in the position of the portfolio, $\boldsymbol{\theta} \in \mathbb{R}^M$, given by

$$\boldsymbol{\theta} = \boldsymbol{\pi} - \boldsymbol{\pi}_0. \tag{7}$$

The objective function of the optimization is the expected utility given from the portfolio wealth. The wealth of the portfolio depends on the contents of the portfolio and the instrument prices and payoffs. The change of any instrument in the portfolio may be constrained from below and above. If the item is non-tradable then both limits are set to zero, and for tradable products these limits can reflect the liquidity of the market and trading limits of the agent. The optimization problem is

$$\max_{\boldsymbol{\theta}} \quad \mathrm{E}\{U[W]\}, \tag{8}$$

$$\text{s.t.} \quad \boldsymbol{A\theta} \leqslant \boldsymbol{b}, \tag{9}$$

where matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$ give the constraints to the change. The feasible area given by the constraints is a convex set. In general, an analytical solution to the optimization problem (8) and (9) is not available due to the complexity of the price processes and derivative instruments, and a numerical approximation of the problem is required. The approximation is done in two stages, first the optimization problem is converted to a non-linear programming problem and then the estimates that are needed to solve the optimization problem are estimated with Monte Carlo simulation.

Taylor's approximation of $U[W]$ around the initial wealth $W_0 = \boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s}$ is

$$U(\boldsymbol{\pi}^{\mathrm{T}}\boldsymbol{s}) \approx U(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s}) + \boldsymbol{\theta}^{\mathrm{T}}\frac{\partial U}{\partial \boldsymbol{\pi}}(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s}) + \frac{1}{2}\boldsymbol{\theta}^{\mathrm{T}}\frac{\partial U^2}{\partial^2 \boldsymbol{\pi}}(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s})\boldsymbol{\theta} + \epsilon(\boldsymbol{\theta}^3), \tag{10}$$

where the residual term $\epsilon(\boldsymbol{\theta}^3)$ is such that

$$\lim_{\boldsymbol{\theta}\to 0}\frac{\epsilon(\boldsymbol{\theta}^3)}{\|\boldsymbol{\theta}\|^3} = 0. \tag{11}$$

The residual term is assumed to be small with small changes around the initial position and it is ignored.

Substituting Taylor's expansion (10) to the optimization problem (8) gives

$$\max_{\boldsymbol{\theta}} \mathrm{E}\left[U(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s}) + \boldsymbol{\theta}^{\mathrm{T}}\frac{\partial U}{\partial \boldsymbol{\pi}}(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s}) + \frac{1}{2}\boldsymbol{\theta}^{\mathrm{T}}\frac{\partial U^2}{\partial^2 \boldsymbol{\pi}}(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s})\boldsymbol{\theta}\right]. \tag{12}$$

The constant term $U(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s})$ does not have an effect on the optimization and it is ignored. Define vector $\boldsymbol{a} \in \mathbf{R}^N$ as

$$\boldsymbol{a} = \mathrm{E}\left[\frac{\partial U}{\partial \boldsymbol{\pi}}(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s})\right], \tag{13}$$

and matrix $\boldsymbol{V} \in \mathbf{R}^{N\times N}$ as

$$\boldsymbol{V} = \mathrm{E}\left[\frac{\partial U^2}{\partial^2 \boldsymbol{\pi}}(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s})\right]. \tag{14}$$

It is assumed that the matrix $\boldsymbol{V}$ is invertible. The assumption is valid almost surely if the tradable assets are linearly independent, i.e., if the payoffs of the instruments are not identical or close to identical. With the definitions (13) and (14), the problem is to maximize

$$J(\boldsymbol{\theta}) = \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{a} + \frac{1}{2}\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{V}\boldsymbol{\theta} \tag{15}$$

over θ so that the constraints on the change of the portfolio given by (9) hold. The objective function is concave because of the concavity of the utility function. If the values of $\boldsymbol{a}$ and $\boldsymbol{V}$ are known, then the optimization problem can be solved using traditional methods. The estimation of $\boldsymbol{a}$ and $\boldsymbol{V}$ with Monte Carlo simulation is presented in the following section.

If there are no constraints, the necessary condition for the optimal solution gives the solution to the problem as

$$\boldsymbol{\theta}^* = -\boldsymbol{V}^{-1}\boldsymbol{a}. \tag{16}$$

The concavity of the utility function guarantees that the second-order necessary condition for the optimality of the solution is fulfilled. The solution for the non-constrained case (16) resembles very closely the solution of the static Markowitzian portfolio optimization problem and gives practical and very fast solutions.

A method of constrained optimization is called for in the more general case with constraints on the changes in the portfolio. The solution is guaranteed to exist because the objective function is concave and the feasible area is convex, see e.g. [14]. The well-posedness of the problem makes it readily solvable with a wide variety of non-linear programming methods that are available today.

4. Portfolio simulation

4.1. The Monte Carlo method

There are several methods for derivative valuation in the financial market, see e.g. [10]. The Monte Carlo simulation method suits for large portfolios, complex and possibly path dependent instruments, and several market variables. Simulation is a feasible method in the electricity market because of the lack of analytical formulas for the prices of the derivatives and the exotic instruments with links to the physical spot market.

The Monte Carlo method simulates the continuous-time stochastic factors that are given by $\boldsymbol{x}(t)$. When the realization of $\boldsymbol{x}(t)$ is known in one simulation round, the prices of the instruments, $\boldsymbol{s}(\boldsymbol{x}(t), t)$, are calculated from the given analytical formulas or approximations. Several rounds of simulation give the estimates needed to the portfolio optimization, and an approximation of the distribution of the future payoffs of the portfolio.

4.2. Evaluating portfolio

Consider a continuous-time Itô process

$$\mathrm{d}x(t) = \mu(t, \omega)x(t)\,\mathrm{d}t + \sigma(t, \omega)x(t)\,\mathrm{d}z(t), \tag{17}$$

where $\mu(t, \omega)$ is the growth of $x(t)$, $\sigma(t, \omega)$ is the volatility of $x(t)$ and $z(t)$ follows a basic Wiener process. The process is discretized by dividing the time interval $[0, T]$ into N subintervals of equal length Δt. Assuming that μ and σ are non-stochastic and constants, the discrete version of the Itô process in (17) is

$$\Delta x = \mu x \Delta t + \sigma x \Delta z. \tag{18}$$

Change in the variable z following a basic Wiener process is

$$\Delta z = \epsilon\sqrt{\Delta t}, \tag{19}$$

where ϵ is a random sample from a standardized normal distribution. Substituting this to the process (18) gives

$$\Delta x = \mu x \Delta t + \sigma x \epsilon \sqrt{\Delta t}. \tag{20}$$

In the case of the price processes presented in this paper the values of the growth μ and volatility σ are estimated from market data for each time step, and it is assumed that they are constant over the length of the discrete time step. The parameter values are assumed to be known also with other models. The values of the normal distributed random variable ϵ are drawn from a random number generator. The initial state of the simulation is fixed and Eq. (20) is used to recursively calculate the following states. The paths of the assets, $\boldsymbol{x}(t)$, can be stored and the value of any instrument whose price, $\boldsymbol{s}(\boldsymbol{x}(t), t)$, depends only on time and the paths of the assets up to the current stage in the simulation can be determined.

A large number of simulation runs produce an approximation of the distribution of the outcomes of all the instruments in the portfolio. The total portfolio or any subset of it is available for risk analysis. The approximation of the distribution makes it possible to estimate the expected value or a risk measure of the portfolio. The number of simulations that is required to give reliable results is relatively small. For simple portfolios, a few thousand simulation rounds usually provide an indication of true values of the observed variables. Performing a quick analysis for small portfolios is therefore very fast even with moderate computing capacity. The exact number of simulation rounds and convergence properties depends on the complexity of the stochastic processes and derivative instruments that are simulated. A more detailed analysis of the convergence of the simulation is found in [12].

4.3. Solving the optimization problem

Simulation gives the estimates that are needed when the optimal portfolio selection problem is solved. An estimate for the objective function is given by

$$J(\boldsymbol{\theta}) = \boldsymbol{\theta}^{\mathrm{T}}\tilde{\boldsymbol{a}} + \frac{1}{2}\boldsymbol{\theta}^{\mathrm{T}}\tilde{\boldsymbol{V}}\boldsymbol{\theta}, \tag{21}$$

where the vector $\tilde{\boldsymbol{a}}$ and matrix $\tilde{\boldsymbol{V}}$ are given by

$$\tilde{\boldsymbol{a}} = \frac{1}{I}\sum_{i=1}^{I}\left[\frac{\partial U}{\partial \boldsymbol{\pi}}(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s}_i)\right] \approx \mathrm{E}\left[\frac{\partial U}{\partial \boldsymbol{\pi}}(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s})\right], \tag{22}$$

and

$$\tilde{\boldsymbol{V}} = \frac{1}{I}\sum_{i=1}^{I}\left[\frac{\partial U^2}{\partial^2 \boldsymbol{\pi}}(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s}_i)\right] \approx \mathrm{E}\left[\frac{\partial U^2}{\partial^2 \boldsymbol{\pi}}(\boldsymbol{\pi}_0^{\mathrm{T}}\boldsymbol{s})\right]. \tag{23}$$

A single simulation run i gives a single realization of the prices of instruments in $\boldsymbol{s}_i$. The derivatives of the utility function give single realizations $\boldsymbol{a}_i$ and $\boldsymbol{V}_i$. The recursive updating rules for $\tilde{\boldsymbol{a}}$ and $\tilde{\boldsymbol{V}}$ in the course of simulation are

$$\tilde{\boldsymbol{a}}_i = \tilde{\boldsymbol{a}}_{i-1} + \frac{1}{i}(\boldsymbol{a}_i - \tilde{\boldsymbol{a}}_{i-1}), \tag{24}$$

$$\tilde{\boldsymbol{V}}_i = \tilde{\boldsymbol{V}}_{i-1} + \frac{1}{i}(\boldsymbol{V}_i - \tilde{\boldsymbol{V}}_{i-1}). \tag{25}$$

At the beginning of simulation, $\tilde{\boldsymbol{a}}_0$ and $\tilde{\boldsymbol{V}}_0$ are set equal to zero vector and matrix. The outputs, $\boldsymbol{a}_i$'s and $\boldsymbol{V}_i$'s, of simulation runs are independent and identically distributed random variables, and the first two moments of $\boldsymbol{a}_i$ and $\boldsymbol{V}_i$ are finite. The simulated averages, $\tilde{\boldsymbol{a}}_i$ in (24) and $\tilde{\boldsymbol{V}}_i$ in (25), converge to the expected values given by (22) and (23) by the law of large numbers when the number of simulation runs goes to infinity.

5. Numerical results

5.1. Framework

The electricity market in the examples is the Scandinavian market with spot and futures trading. Physical electricity is traded in the spot market and financial trading concentrates on futures contracts that are settled against the average spot price of the delivery period of the contract. There is also a market for options and more exotic products. The focus of the examples is on the futures and forward contracts because of their profound importance and best liquidity at the moment. All the monetary values are given in euros (EUR).

Exchange quoted futures prices give the expected value of the electricity spot price in the model. The Scandinavian market quotes futures prices for several years ahead. The first few weeks are quoted directly, then the weeks are combined to blocks of four to five weeks, and later the blocks are combined to season products, which cover the year in three periods roughly corresponding to the winter and summer seasons. Spot prices are quoted for each hour of the day in the spot market. The intraday and day-to-day fluctuations of the electricity prices do affect the risk management problem especially if instruments in the portfolio have large variation in their load profiles. However, forecasting spot prices in an hourly level for long time periods is not feasible. Weekly and daily profiles for the prices can be used if there are data for the profiles of the instruments in the portfolio. The examples of this paper do not consider the weekly profiles but only weekly average values. The aggregation of the hourly data for longer time periods removes the extreme values from the price distribution and it is assumed that the weekly averages are lognormally distributed.

Fig. 1 presents the weekly averages of the spot price realizations for each year from 1996 to 2000, the expected value of the spot price model, and the 5% and 95% probability limits of the price model. The futures quotes that give the expected value of the spot price are from the closing prices of the Nord Pool power exchange on the 2 January 2001, and the historical data are obtained from the Nord Pool power exchange. The weekly expected prices of the spot price forecast are obtained from the quotations by fitting a smooth curve continuously to the whole time period and taking care that the averages for the time periods remain the same as the market quotes. The probability limits are obtained using the spot price model and historical volatility that is estimated from the spot price realizations of the years from 1996 to 2000. The interpretation of the probability limits is that the spot price is within the probability limits 90% of the time over a large number of simulations. It is clear that using more than five sample values to the estimation of the limits would improve their reliability but there are no more reliable market data available at the moment.

The spot price for the year 1996 is higher than for the other years. The hydro inflow during 1996 was below the normal level and this caused a shortage of hydropower generation that is usually around 50% of the yearly production. As a result, more expensive thermal power was used to cover the yearly demand. The years from 1997 to 2000 had all more than the normal level of hydro inflow and this was one of the reasons for which the prices of those years were lower than for the year 1996. The market expectations on the 2 January 2001 are discounting a year similar to the years from 1997 to 2000.

The expected values of spot price are used to calculate the local drift parameters of the spot price process according to formula (2). The local volatilities are estimated directly from the historical years by calculating the variance of five sample values for each week. The calculated values enable the simulation of the spot price processes using the Monte Carlo methodology presented in Section 4.

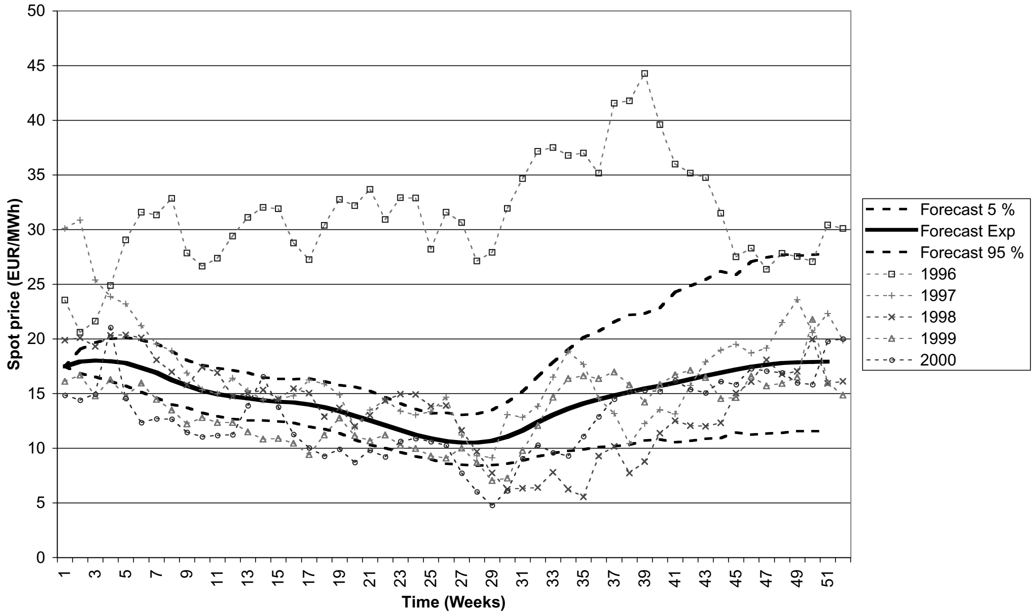

Fig. 1. Realizations of Nord Pool spot prices in years 1996–2000, the expected value of the spot price and the 5% and 95% probability limits for the spot price assuming a lognormal distribution used in the examples.

The particular results of optimization are optimal with respect to the utility function and selection of risk parameters. The utility function in the examples has the form

$$U(W) = -\mathrm{e}^{-\lambda(W-W_0)/W_0}, \tag{26}$$

where λ is the value of the risk parameter, W is the wealth of the portfolio, and W_0 is the wealth of the initial portfolio. The utility function is concave when λ is non-negative. A larger risk parameter represents more risk averse preferences while a smaller risk parameter gives a more neutral attitude towards risks. The risk parameter is 2.0 in the examples, which represents a fairly conservative risk attitude.

For each example, Monte Carlo simulation is performed and the expected values of the portfolio and instruments are calculated. The value at risk measure is presented with the risk level of 5% in order to follow the change of the risks of the portfolios. The non-linear programming problem of expected utility maximization is solved with the gradient projection method of Rosen, see [1], in all the examples.

5.2. Electricity user

The first example considers an industrial electricity end user that has a fixed electricity consumption of 50 MW for the time period from 8 January to 30 December 2001, i.e., from the beginning of week 2 to the end of week 52. The market is analyzed on the 2 January 2001. The end user purchases physical electricity from the spot market at the spot price. The optimization problem of the end user is to decide how much of the electricity is bought from the spot market at spot price and how much is hedged beforehand with the use of financial futures contracts. The end user benefits from leaving the spot purchases unhedged if the realization of the spot price is lower than the current futures prices, but the user is exposed to spot price realizations that are higher than the futures prices.

First the totally unhedged case is considered. The portfolio of the end user consist only of the 50 MW of consumption between weeks 2 and 52 of 2001. The expected value of the spot price for the time period in question is around 14.8 EUR/MWh and the total consumption is 428.4 GWh. Thus, the analytical expected value of consumption is EUR −6.33 million but there is no way of directly calculating the analytical value at risk measure. The expected result and risk for the end user are estimated with 10,000 rounds of Monte Carlo simulation. The expected value of the portfolio is EUR −6.3 million which matches the analytical value, and the value at risk is found to be EUR 1.4 million at the 5% risk level. This means that in 95% of the cases the portfolio is not expected to devalue more than EUR 1.4 million down from the expected value.

Because of the electricity price risk, the user takes an initial hedge position of 30 MW of futures contracts for the whole time period in order to reduce the uncertainty of her position. The expected result and risk for the portfolio containing both the 50 MW of consumption load and 30 MW of futures contracts are estimated with 10,000 rounds of simulation. The expected value of the portfolio is EUR −6.3 million after the hedge and the value at risk drops to EUR 0.6 million. The expected value of the portfolio does not change because futures price is equal to the expected value of the forecasted spot price.

The optimization model presented in this paper is used to find an optimal hedge level for the end user, given the chosen initial position, the utility function (26), and two risk parameters of $\lambda = 1.0$ and $\lambda = 2.0$ representing two different attitudes towards risk. Table 1 presents the results of optimization for the portfolio. The change in consumption of electricity is constrained to zero, but the results of the optimization show that a further 6 MW should be hedged with futures contracts if the attitude towards risk is more neutral, i.e., the risk parameter is equal to 1.0. If the risk preferences are more risk averse, i.e., $\lambda = 2.0$, then an additional of 17 MW is hedged. The hedge position changes because the arbitrarily chosen initial hedge position was not in accordance with the risk attitude implicitly given by utility function and risk parameter. The changes in the expected value and value at risk of the portfolio are again simulated with 10,000 simulation rounds. The uncertainty of the portfolio is reduced as the unhedged position reduces with both chosen risk preferences. The value at risk at the 5% risk level drops accordingly to EUR 0.4 million with larger risk taking capability and to EUR 0.1 million with the more conservative attitude.

The electricity consumption is constant over time in the example, but it is possible to use a more complicated model for the demand. For example, a stochastic consumption process that is correlated with the spot price process can be simulated with the Monte Carlo method. The end user consumption can be linked to the consumption process and included to the portfolio, after which the simulation proceeds in a similar manner as above.

5.3. Electricity generator

The second example presents a baseload electricity generator with a production capacity of 50 MW for the time period from the week 2 to week 52 in 2001. The production capacity is held fixed, but the marginal cost of production varies due to the changes in fuel costs. The electricity generation unit is modeled with two contracts. First, 50 MW is “bought” from the production unit with the marginal production cost and then the same 50 MW are sold to the spot market with the spot price. The optimization problem of the generator is to decide how much of the production capacity

Table 1
The contents and the expected result and value at risk measure of the initial portfolio and optimized portfolios of the electricity end user with risk parameters $\lambda = 1.0$ and $\lambda = 2.0$

Item	Before optimization		Optimization with $\lambda = 1.0$	Optimization with $\lambda = 2.0$
Consumption (MW)	−50	→	−50	−50
Hedge position (MW)	30	→	36	47
Expected value (millions of EUR)	−6.3	→	−6.3	−6.3
Value at risk (millions of EUR)	0.6	→	0.4	0.1

is hedged now and how much is sold to the spot market at the spot price.

Fig. 2 presents the expected value and the 5% and 95% probability limits of marginal production cost of the generating company, and the expected value of the electricity spot price. The marginal production cost is modeled as a stochastic variable with a given forecast for the expected value and volatility of the process. The marginal production cost corresponds roughly to the production costs of a coal-fired combined heat and power production plant. The use of forecasts implicitly takes into account the possible long-term correlations between the marginal cost and spot price processes, and in the short term the processes are assumed to be uncorrelated. It is assumed that the observed company is relatively small compared to the market and thus the production of the company does not have an effect on the spot price.

First the unhedged situation is analyzed. The average expected marginal production cost is 12.5 EUR/MWh and the production volume is 428.4 GWh. The analytical expected total cost of production is EUR 5.37 million and the analytical expected value of spot sales is EUR 6.33 million, just as in the case of electricity end user. The expected value of production is EUR 0.96 million. A set of 10,000 simulations is performed. The expected value of the total cost of production is EUR 5.3 million and the expected value of the spot electricity sales is EUR 6.3 million. The total expected value of this portfolio is EUR 1.0 million and the value at risk is EUR 1.4 million. The value at risk is very large compared to the expected value of the portfolio and the generator decides to hedge 30 MW of the electricity sales with a futures contract. A new set of 10,000 simulations shows that the value at risk reduces to EUR 0.8 million as a result of the hedge.

The optimal hedging level is determined by the optimization method using conservative risk parameter of $\lambda = 2.0$. Table 2 gives the results of the optimization starting from the initial portfolio with the generation unit and a 30 MW hedge position in a futures contract. Changes in the physical generation and sales are restricted to zero. The hedge position with the futures contract increases

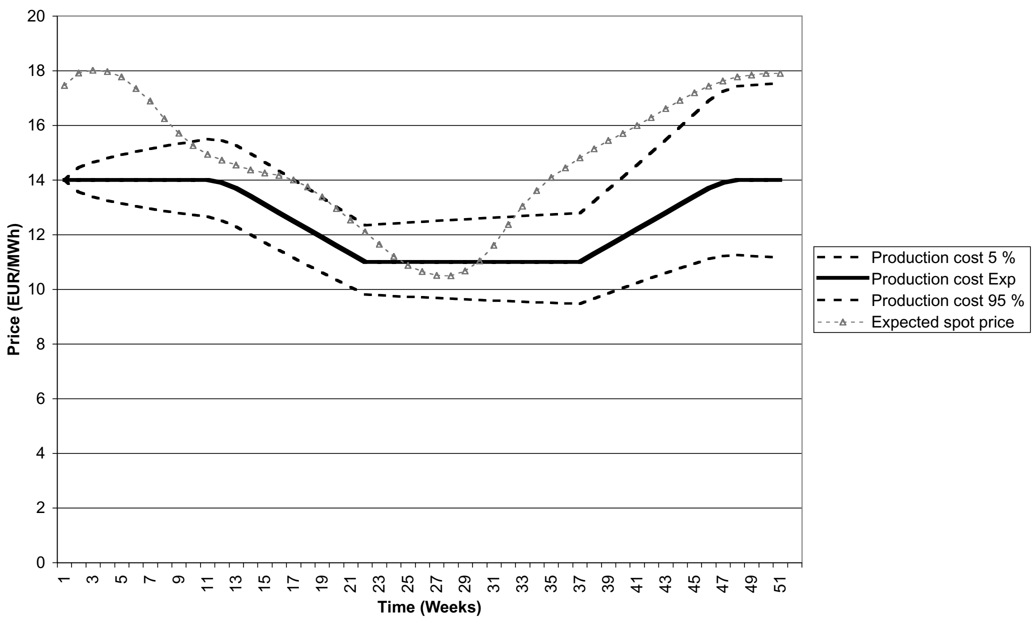

Fig. 2. The expected value of the marginal production cost price and the 5% and 95% confidence intervals for the production cost assuming a lognormal distribution.

Table 2
The contents and the expected result and value at risk measure of the initial portfolio and optimized portfolio of the electricity generator with risk parameter $\lambda = 2.0$

Item	Before optimization		After optimization
Generation (MW)	50	→	50
Spot sales (MW)	−50	→	−50
Hedge position (MW)	−30	→	−45
Expected value (millions of EUR)	1.0	→	1.0
Value at risk (millions of EUR)	1.4	→	0.7

to 45 MW and as a result the value at risk reduces to EUR 0.7 million. Note that although it is assumed that the electricity generator and end user have the same attitude towards risks in the case where $\lambda = 2.0$, the generator hedges 2 MW less of the capacity. This is due to the uncertainty in the marginal production costs. Hedging too much of the electricity sales would expose the producer to the price risk from the marginal production costs as there would be no possibility of gaining extra profits from electricity sales even if the production costs are high.

As in the situation of electricity end user, the framework is capable of covering more complicated production assets. For example, the production capacity of a combined heat and power plant may be tied to a stochastic temperature index that is simulated with the Monte Carlo method, or if there are derivatives for hedging against fuel price movements, the optimal hedging levels can be obtained in a similar manner as for the electricity derivatives.

6. Conclusions

This paper presents an integrated framework for the optimal management of a combined physical and financial power portfolio. As illustrated in the examples the model can be applied to the Scandinavian electricity market. The framework is able to cover other energy markets, telecommunications, and other non-financial markets if the processes determining the uncertainties are known and can be modeled, and that the instruments in the market can be described using these processes.

Direct application of the financial theory to the electricity market is not possible. Electricity is a non-tradable asset that cannot be stored. If there are futures contracts for each time interval, they can be used as tradable assets and the electricity market becomes more like the financial market. However, the analytical formulas from the financial market must still be applied carefully. In this paper, the electricity instruments are evaluated using numerical methods that adapt more easily to the electricity markets.

The accuracy of analysis depends on the models for the price processes that determine the values of the electricity contracts. In this study, the expected value of the spot price is taken directly from the futures quotations in the market and a lognormal weekly price distribution is assumed using historical data to estimate the volatility of the price. There are not enough historical data to provide reliable estimates on model parameters at the moment, and further research of the price process models is needed. However, the model used in this paper gives one possible starting point for further analysis.

The numerical results given by the approach in this paper are reasonable when compared with the corresponding results from the financial market. The results of the optimization are consistent with intuition and e.g. the value at risk measure. The distribution of the outcomes is sufficient to analyze and manage most of the risks. Handmade scenario analysis would bring additional information about the risks of the portfolio.

The deregulation of the electricity markets inevitably gives birth to competitive market places and with them a new set of rules for risk management. This paper suggests that the financial methods can be helpful in the risk analysis if the unique properties of the electricity market are taken into account.

Acknowledgements

The authors are grateful for the helpful comments and suggestions of Tuomas Pyykkönen, Jukka Ruusunen, and the anonymous referees.

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