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In: IEEE Transactions on Applied Superconductivity 2001.
Vol. 11, No. 1, pp. 260–262.

Analysis of the Bloch Oscillating Transistor

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Abstract— **The Bloch-oscillating transistor (BOT) is a device where coherent Cooper-pair current in mesoscopic Josephson junctions can be controlled with single-electron tunneling from the base-electrode. We show computational results on the single-junction BOT (sj BOT). Also the double-junction BOT (dj BOT) is introduced. An approximate formula of the device noise temperature is also presented.**

Keywords— **Bloch oscillations, Single electron tunneling, Mesoscopic tunnel junctions**

I. Introduction

THE Bloch-oscillations in mesoscopic Josephson junctions are coherent charge oscillations, which are a manifestation of the band structure in the energy dispersion relation[1-4]. They occur if a junction is sufficiently free of dissipation. However, also in an undamped junction Zener-tunneling tends to drive the system to higher energy bands thus suppressing the oscillations and some dissipation is needed to return the junction to the lowest energy band[5]. In this paper we analyze properties of the Bloch-oscillating transistor (BOT)[6], where the dissipation for the junction is provided in a controlled way by a normal tunnel junction. The computational results for the single junction BOT (sj BOT) show that the characteristics are qualitatively similar to those of a Bipolar Junction Transistor. Furthermore, we present an idea of the double-junction BOT (dj BOT), which is probably easier to realize.

II. Model

Two circuit equivalents of the sj BOT are shown in Fig. 1. They are equivalent up to the definition of voltages. The junction #1 connects the normal conducting base electrode to the superconducting island. The junction #2 connects the superconducting island to the superconducting emitter electrode. Its Josephson coupling energy $E_J = \Phi_0 I_{cs}/2\pi$ is assumed to be of the same order as the charging energy $E_c = e^2/2C_2$. The impedance of the LR-circuit at the collector electrode is assumed large so that the charging effects have to be taken into account when computing single-electron tunnel rates of the junction #1 [7]. The large-impedance environment also guarantees that the quasicharge is a good variable describing the state of the junction #2.

The model used in simulations is presented in detail in [6]. The junction #2 is described with the theory of Bloch-oscillations[1]. The energy versus quasicharge Q_2 is shown in Fig. 2. A band structure is formed, where the magnitude of the gap between two lowest bands is

Manuscript received September 19th 2000

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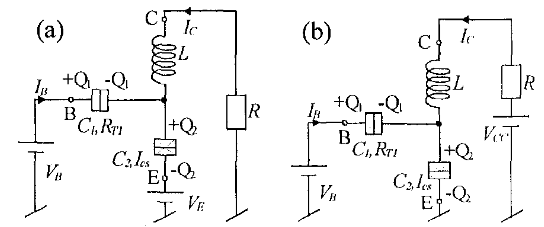

Fig. 1. The equivalent circuit of the sj BOT. The circuits in (a) and (b) are equivalent up to the definition of voltages.

the Josephson coupling energy E_J. Between higher bands the gap is assumed zero. If the junction is initially at the lowest energy band and charge is driven to or from the island via the collector, the quasicharge is increased until it reaches the value $|Q_2| = e$. At that point either a Cooper pair tunnels through the junction so that Q_2 changes from $+e$ to $-e$ or from $-e$ to $+e$, or Zener-tunneling occurs so that the charge continuously increases to $|Q_2| > e$. The probability of Zener-tunneling is $P^Z_{0\leftrightarrow 1} = \exp\left[-(\pi e^3/8\hbar I_C C)\left(E_J/E_C\right)^2\right]$[5]. We assume for the discussion that if $|Q_2| < e$ the junction is at the lowest (zeroth) energy band, if $e < |Q_2| < 2e$, it is at the first band etc.

It is assumed that no single electron tunneling [8] occurs through junction #2. This is justified if $e^2/2C_2 \lesssim 2\Delta$, where Δ is the superconducting gap, i.e. that the junction is biased below the gap at the voltage range of interest. The single electron tunnel-resistance R_{T1} is assumed constant. This is justified if the junction #1 is biased above the gap most of the time, which occurs if $e^2/2C_1 \gtrsim \Delta$. It is therefore preferable that $C_1 < C_2$. The tunnel rates are obtained using the Orthodox Theory with the local rule.

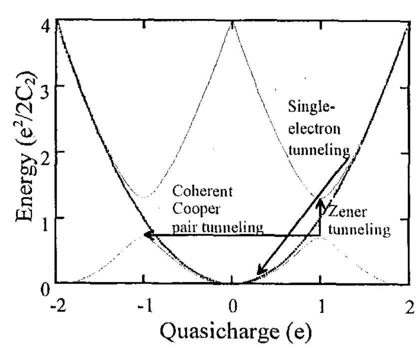

Fig. 2. The energy versus quasicharge in a mesoscopic Josephson junction. Different possible transitions are denoted with arrows.

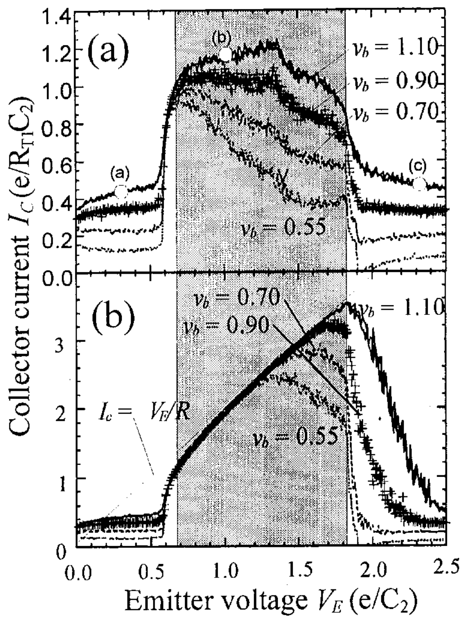

Fig. 3. The collector current as function of the emitter voltage for the connection of Fig. 1 (a). In (b) the Zener-tunneling is neglected.

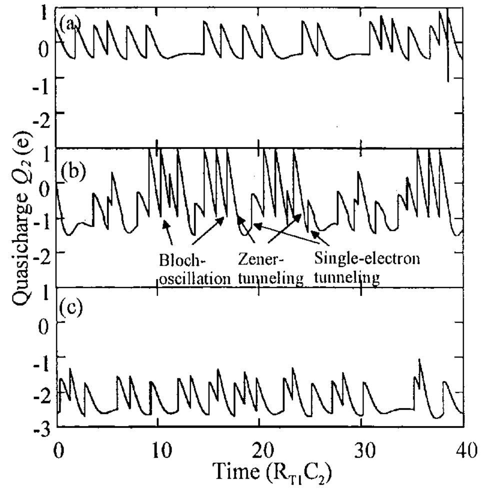

Fig. 4. The quasicharge versus time at bias points denoted in Fig. 3(a).

III. Simulations

The parameter definitions and their values used in simulations are as shown in Table 1.

A set of simulated curves using the bias arrangement of Fig. 1(a) is shown in Fig. 3. The active region, where the Bloch-oscillations occur is shown in gray. The Figs. 3(a) and 3(b) are otherwise similar, but in the latter Zener-tunneling is neglected. In Fig. 3(b) it can be seen, that in the region where Bloch-oscillations occur without dissipation, the Josephson junction acts as a short circuit and the relation $I_C = V_E/R$ is satisfied. However, when Zener-tunneling is turned on (Fig. 3(a)), I_C becomes smaller.

The curves of Fig. 3(a) can be understood by looking at Fig. 4, where the quasicharge Q_2 is shown as a function of time at different bias points. In Figs. 4(a) and (c) the system never reaches the Brillioun zone boundary $|Q_2| = e$, so Bloch oscillations do not occur. In the active region (Fig. 4(b)) one sees Bloch-oscillations interrupted occasionally by Zener-tunneling, which takes the system to the first band $|Q_2| > e$. From there it is returned to the lowest band via single-electron tunneling, the probability of which can be tuned with the base voltage enabling collector current tuning.

TABLE I
Definitions and Values of the Dimensionless Parameters Used in Simulations

Parameter	Definition	Value
c_1	C_1/C_2	0.1
l	$LR_{T1}^2C_2$	0.26
ε_J	$E_J/(e^2/2C_2)$	0.6
r	R/R_{T1}	0.5
α_t	$h/\pi^2e^2R_{T1}$	0.077

Fig. 5 shows the characteristics curves with the bias arrangement of Fig. 1(b). It can be seen that the collector current I_C has a plateau at about $V_{CC} \approx 1.8e/C_2$.At this point the BOT operates as an ideal current source. We have also shown that both current and power gain are obtained for a small signal connected at the base electrode[6].

IV. Noise

To approximately understand the noise properties of the BOT, we assume that voltage noise at the input is due to thermal noise from the tunnel resistance R_{T1}, i.e., $e_n = \sqrt{4k_BTR_{T1}}$. We assume that current noise is arising from the shot noise of the tunnel junction, i.e. $i_n = \sqrt{2eI_B} = \sqrt{2e^2f_B}$, where f_B is the frequency of single-

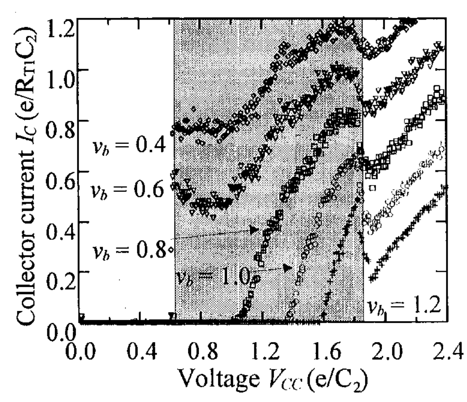

Fig. 5. The collector current as a function of the bias voltage for the circuit of Fig. 1(b).

electron tunneling. We have also assumed here that the subsequent tunnel events are not correlated. The noise temperature is then obtained as

$$T_n = \frac{e_n i_n}{2k_B} = \sqrt{\frac{2TR_{T1}e^2 f_B}{k_B}} \approx \sqrt{\frac{Thf_B}{k_B}} = \sqrt{TT_q}, \quad (1)$$

where the approximate equality assumes that the tunnel resistance R_{T1} is approximately the quantum resistance $h/2e^2$. In the last equality temperature $T_q = hf_B/k_B$ is defined. The noise temperature is smaller than the bath temperature if $T_q < T$.

The optimal input resistance to the device can be similarly estimated as

$$R_{opt} = \frac{e_n}{i_n} \approx 2R_q\sqrt{\frac{T}{T_q}}. \quad (2)$$

The value of f_B is needed for the optimization and it depends on several design parameters, which should be known before the numerical values of T_n and R_{opt} can be obtained.

V. Realization Issues

In the experimental realization of the sj BOT probably the most difficult issue is to implement the required large-impedance environment in order to keep the quasicharge a good variable and to preserve the charging effects essential for the device. For example, if the capacitance C_2 is taken to be 1 fF and the tunnel resistance 34 kΩ, the required inductance is 300 nH, which is very difficult to realize with contemporary technologies. Possible solutions might be the kinetic inductance of a nanotube, a narrow superconducting wire or the inductance of a series array of Josephson junctions. However, a possibility might also be the double junction BOT (dj BOT), shown in Fig. 6. Instead of providing the blockade by a linear circuit, an extra Josephson junction is introduced to the collector electrode. The collective Bloch-oscillations of the two junctions can now be tuned by single-electron tunneling from the base electrode, which according to our belief should result in dynamics similar to those of the sj BOT.

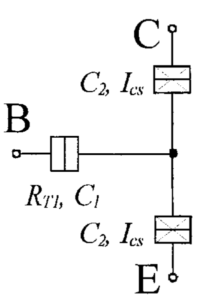

Fig. 6. The equivalent circuit of the double junction BOT.

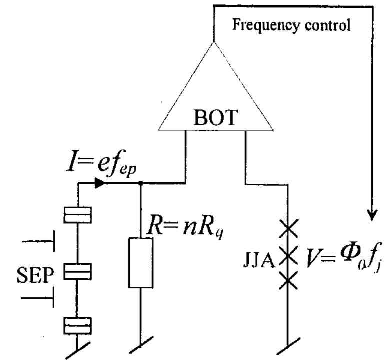

Fig. 7. A schematic of the measurement setup to provide a test bench for the metrological triangle.

VI. Applications

An active area of research today is the realization of the metrological triangle. To date there is no setup, which would serve as the test of its consistency to the level of 10^{-7} or better. One problem is to find a component serving as a preamplifier stage in a measurement similar to that of Fig. 7. There a resistor, whose value can be traced to the quantum resistance, is driven with a current from a single-electron pump (SEP). This is compared to a voltage obtained from a Josephson junction array (JJA). The measurement gives the value of the Klitzing constant.

VII. Conclusions

We have shown that the sj BOT has characteristics that resemble those of a Bipolar Junction Transistor. The realization issues were discussed, and the major difficulty in practice was found to be the large-impedance environment. Therefore an idea of the dj BOT was introduced. Intuitively, it should have properties similar to that of the sj BOT.

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