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A Multi-Turbine Power Curve Approach

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Abstract: **A simple methodology is described – the multi-turbine power curve approach – a methodology to generate a qualified estimate of the time series of the aggregated power generation from planned wind turbine units distributed in an area where limited wind time series are available. This is often the situation in a planning phase where you want to simulate planned expansions in a power system with wind power. The methodology is described in a step-by-step guideline.**

Index terms: Aggregated power, power curve, power planning tool, wind power.

I. Introduction

The wind speed varies in both time and space, and the correlations between the wind speeds in two points in time or space will decrease with increasing time or distance.

The power in the wind is proportional with the cube of the wind speed

$$P_w(u) = \tfrac{1}{2}\rho A u^3 \qquad (1)$$

where ρ is the air density, A is the area of the cross-section of the 'flow tube' and u is the wind speed.

The electrical power output from a wind turbine can be expressed as

$$P_e = C_P \times P_w \qquad (2)$$

where

$$C_P(u) = \frac{P_e}{P_w} \qquad (3)$$

is the wind turbines efficiency coefficient. C_P increases with the wind speed from zero until its maximum (≈ 0.5 at 6..10 m/s, depending on the turbine design), and decreases with higher wind speed in order to limit the power output

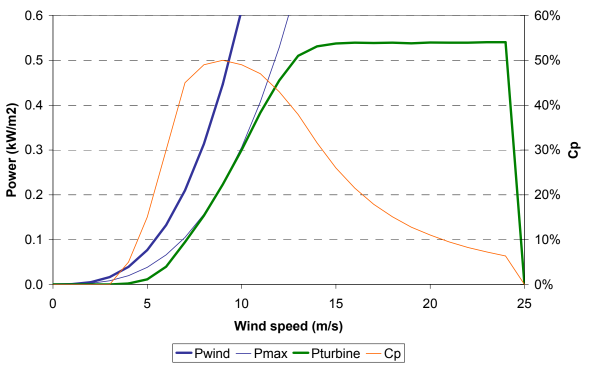

Figure 1: Illustration of the power output from a single wind turbine relative to the power in the wind (and the power in the wind multiplied with a fixed factor: 0.5). In addition, the turbines efficiency coefficient, C_p, is indicated.

to the rated level (see Figure 1).

The power output from a single wind turbine unit will therefore for the lower wind speed levels (4..8 m/s) be even more sensitive to the variation in wind speed than expressed by Eq. 1.

The rapid short-term, small-scale fluctuations in the wind speed are in some degree smoothed out at the power output from a single wind turbine unit, both by the extent of the rotor (up till 100 m rotor diameter for a modern large-scale wind turbine) and by the power control of the wind turbine (stall, pitch, variable speed etc).

The aggregated power generation from more wind turbine units in an area, P_Σ, will further smoothen out the short-term fluctuations, as the power generation from the individual units are not fully correlated. In general, the more units and the larger distance between the units, the lower level of the high frequency fluctuations in the aggregated power generation.

When modelling (e.g. on hourly time basis) potential developments of integrated power systems with wind power potentials, detailed information of the wind power potential for the areas of interest are often not available. Typically, the information of the instantaneous wind resource for an area is available in terms of one time series of the wind speed only, valid only for the specific site, but representative for the entire area.

Therefore, you need to be able to simulate a time series of the aggregated power generation from a cluster of wind turbines on the basis of the time series of the wind speed in a single point; or alternatively on the basis of a time series of the power generation from a single unit or a smaller cluster of wind turbines.

However, a qualified sample of a time series of the aggregated power output from multiple (but similar) wind turbines can be derived based on this one point wind speed time series and a standard power curve for a single wind turbine, representative for all the wind turbine units in question, by taking into account the smoothing effects in both time and space.

The methodology presented in the present paper is a simplified multi-turbine power curve approach to simulate the smoothing effects of the aggregated power output from a number of wind turbines within an area. It has been developed as part of the EU supported WILMAR project[1].

The model

Based on only one wind speed time series representative for the area, and a standard wind turbine power curve representative for the wind turbines, the methodology presented will provide a qualified sample of a time series

[1] WILMAR – Wind Power Integration in a Liberalised Electricity Market (EU Contract No: ENK5-CT-2000-00663).

of the aggregated power generation from a number of (similar) wind turbine units within an area. The extent of the area may vary from few kilometres (corresponding to a wind park) to several hundred of kilometres (representing a region). For this purpose an artificial, empiric based 'multi-turbine power curve' representative for the aggregated power generation has been developed. The methodology take into account the smoothing effects in both time and space. The methodology has been verified by real data and compared to using no smoothing and to using a standard power curve for the wind turbines.

The inputs needed are:

1. a wind speed time series representative for the area;
2. a standard wind turbine power curve representative for the wind turbines to be covered; and
3. the dimension of the area.

The methodology is described in a step-by-step guide including:

- The wind is characterised in terms of the wind speed distribution, the mean wind speed and the turbulence intensity.
- The wind speed time series is adjusted to relevant hub height and smoothed by a moving block averaging using a time slot representing the propagation time over the area.
- The 'smoothed power curve' is found based on a representative standard power curve and the standard deviation of the spatial wind speed distribution, and scaled appropriate to represent the total installed wind power capacity.
- The aggregated wind power time series is finally derived by applying the smoothed and scaled power curve to the smoothed and adjusted wind speed time series.

II. Aggregated Wind Power

The aggregated instantaneous power output, P_Σ, from a number of wind turbines within an area (e.g. a cluster, a wind farm or a region) is simply the sum of the simultaneous power output from all the individual units within the area, P_1, P_2, ...

$$P_\Sigma = \sum_i P_i \qquad (4)$$

Due to the spatial distribution of the individual wind turbine units (the distances between the units) in combination with the stochastic nature of the wind speed, the power outputs from the individual units within the area are not necessarily the same at the same time. The simultaneous power outputs from the wind turbines will be distributed around an average value. The deviation of the distribution depends on the extent of the area in question and the turbulence in the wind.

The fluctuations of the wind speeds at the individual units (and thereby of the power outputs from the units) will be more or less correlated in time – depending on the distances between the units and the time scale of interest (see Figure 2). The short-term power fluctuations from the individual units will therefore be more or less smooth out in the aggregated output – depending on the number of units, the size of the area and the time scale.

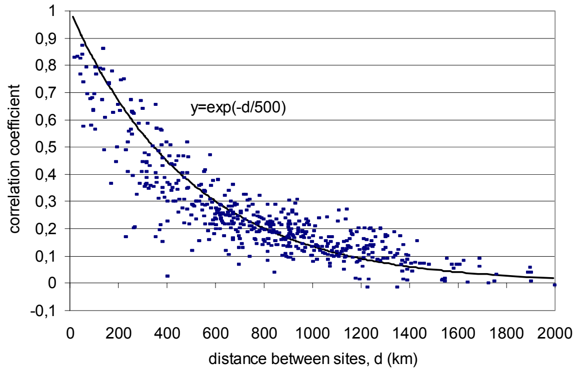

Figure 2: The power output from wind turbines correlates for sites 200-400 km apart, after which the correlation becomes weak. Hourly data from the Nordic countries, year 2001 (Holttinen, 2003)

The extent of the smoothing effect can be estimated from comparing the statistical parameters of existing single turbine, a wind farm and real production data (Holttinen, 2003). For the example of West coast Finland, year 2001 (Figure 3), the standard deviation of the hourly time series for a single turbine is 25…27 % of capacity. For the 3 clusters of altogether 8 turbines 10 km apart it is slightly less than 25 %. For 5 sites in an area stretching 200 km it is less than 21 %. As the data available does not have the same average production, it is more appropriate to compare the relative standard deviations (standard deviation / average) where we see a reduction from single turbine 1.14, wind farm 1.02 to larger area 0.93.

The reduction is more dramatic when looking at the variations from one time step to another. The hourly variations as a time series, the standard deviation is 8 % of capacity for the single turbine, 7 % of capacity for the wind farm and 4 % of capacity for a larger area, thus reduction of variations to a half of the single turbine values.

For the East Denmark (about 100 x 200 km); example in Figure 4, similar values for the standard deviations are seen for the year 2001 data. From the hourly time series, standard deviation is 24 % of capacity for a wind farm (stdev/aver 1.12) and 21 % for the whole area of East Denmark (stdev/aver 1.09). From the hourly variations time series, standard deviation is 7 % of capacity for the wind farm and 3 % of capacity for the whole East Denmark.

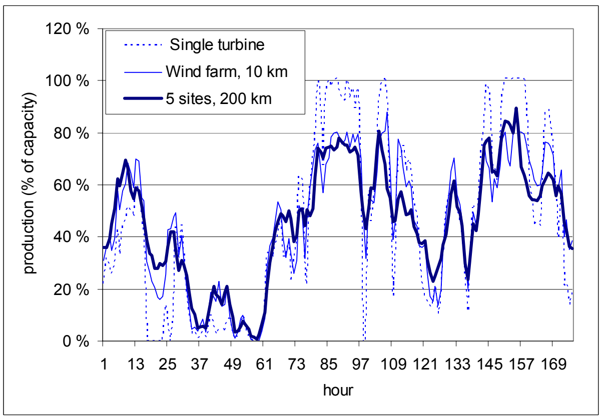

Figure 3: The power output (hourly time series) from one single unit, from a wind farm and from multiple wind farms.

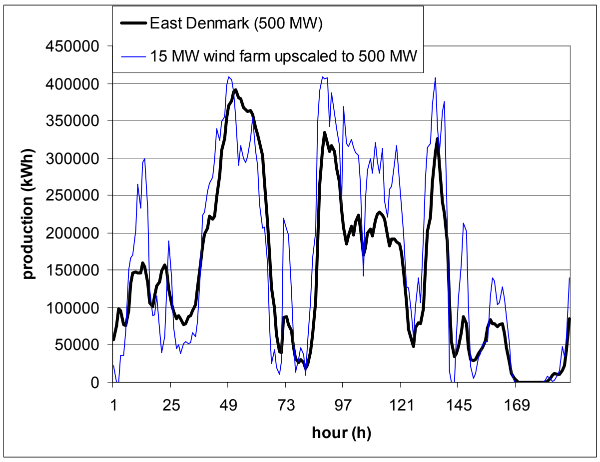

Figure 4: Year 2001 data from East Denmark.

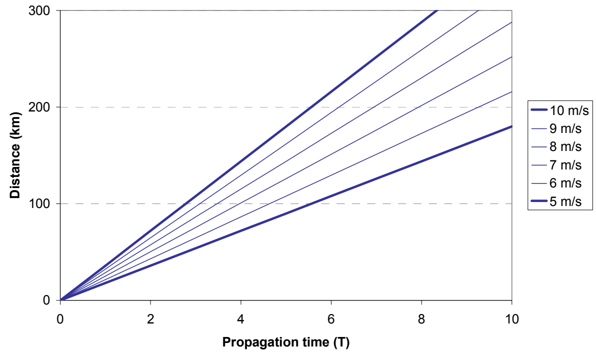

Figure 5: Illustration of the relation between the distance, D, and the propagation time, T, for the 'wind', indicated for various average wind speed levels.

III. THE METHODOLOGY

The methodology presented below (and the numbers indicated) are simplified, pragmatic and approximate, but it may be the best you can do and acceptable for an estimate. The approximations in the method on the other hands imply that the method is applicable also for a low number of wind turbines (down to 5) or wind farms (down to 3), provided that the wind turbines / farms are equally in size and equally distributed over the area. The methodology assumes that all the wind turbines within the area are similar – equal in size and control principle.

Moving block-averaged wind speed time series

As a first approximation, a change in wind speed will propagate in space in the direction of the average wind direction with a speed similar to the average wind speed. (E.g., with an average wind speed of 8 m/s a wind speed change will propagate approximately 5 km within 10 minutes, 30 km within 1 hour or 100 km within 3 hours.) A wind speed measured upfront the area relative to the wind direction will thus still be (more or less) represented within the area in a time period corresponding to the travelling time of the air to pass the area.

To represent this spatial 'memory-effect' of the wind fluctuations over the area in the aggregated power for the area, the original wind speed time series is block-averaged over a moving timeslot corresponding to a representative wind speed (the mean wind speed, w_m) and the spatial dimension of the area, D:

$$w_j = \frac{1}{N+1} \sum_{i=j-\frac{N}{2}}^{j+\frac{N}{2}} w_i \qquad (5)$$

where w_j is the j^{th} element in the generated moving averaged time series, and w_i is the i^{th} element in the original time series. The number of points to include in each averaging process is

$$N = T/\Delta t \qquad (6)$$

where T is the propagation time and Δt is the time step in the time series (N should be an even number). Figure 5 illustrates the propagation time, T, as function of the average wind speed and the dimension of the area. The new time series generated by the moving average of the original will have the same time step as the original. (E.g. for an area with a dimension, D, of 200 km and a mean wind speed, w_s, of 8 m/s, the resulting wind speed time series is derived from the original by moving block-averaging all the numbers in the original wind speed time series within a timeslot, T, of 200km / 8m/s = 7 hours around the actual time. If the time step in the time series is 10 minutes, N becomes 42.)

Spatial wind speed distribution

In addition, the various wind speeds at the individual wind turbine units will at any specific time be distributed around the average wind speed. As a first approximation, the distribution of the individual simultaneous wind speeds at a given time is normal distributed around the block-average wind speed for the corresponding timeslot as specified above (see Figure 8). The normalised standard deviation (relative to the mean wind speed) of the distribution depends on the spatial dimension, D, of the area and the wind turbulence intensity, I (see Figure 6).

The multi-turbine power curve

If the distribution in Figure 8 of the wind speed around the block-average values is applied on the power curve representative for a single unit (Figure 1), you will get a smoothed multi-turbine power curve, that is representative for the aggregated power output for the wind turbines within the area (see Figure 7).

The j^{th} element of the (discrete) multi-turbine power curve, Pm_j, is found by the sum

$$Pm_j = \sum_i Ps_{j+i} \times ps_i \qquad (7)$$

where Ps_j is the j^{th} element of the (discrete) single-turbine power curve and ps_i is the probability of the spatial distribution in Figure 8. (In practice the sum should as a minimum be done for a wind speed range from −5 m/s to +5 m/s around the j^{th} element in the power curve.)

Adjusting the energy production

The estimated annual energy productions for a given wind speed distribution based on the two power curves in Figure 7 should be equal. In the present methodology this is obtained by a minor offset adjustment of the distribution function in Figure 8. The necessary offset adjustment depends on the actual power curve, the wind

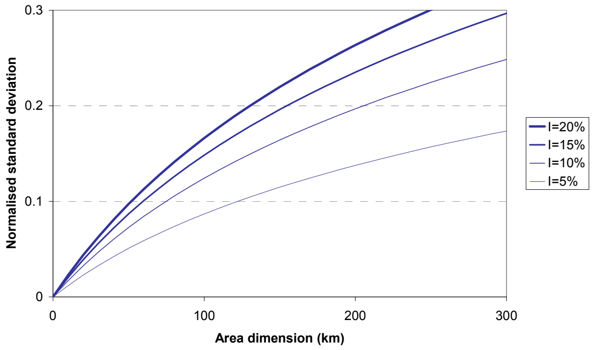

Figure 6: The normalised standard deviation of the distribution of the wind speeds at the individual wind turbine units at any given time (see the example in Figure 8) as function of the dimension of the area, D, and wind turbulence intensity, I. (Still to be further empiric validated)

speed distribution in time (the Weibull distribution) and the wind speed distribution in space (the normal distribution). The appropriate adjustment must be found by manual iteration.

Finally, the multi-turbine power curve should be appropriately up-scaled to form the aggregated power curve that matches the total wind power capacity within the area.

Wind power time series

This aggregated power curve in combination with the block-average wind speed time series can then be used for an estimation of a time series of the aggregated power generation (with the same time resolution as the original wind speed time series). The aggregated power curve will at the lower wind speed levels result in a higher average power generation per unit than for the single unit and at the higher wind speed levels result in a lower average power generation. This is also reflected in the changes of the normalised annual energy production distributions (the statistical distribution of the contribution per rotor swept area to the annual energy production) as a function of the wind speed for a given wind speed distribution (see Figure 9). The energy distribution function for the multiple-turbine power curve is wider and more flat relative to the single-turbine power curve, while the accumulated normalised annual energy production remain

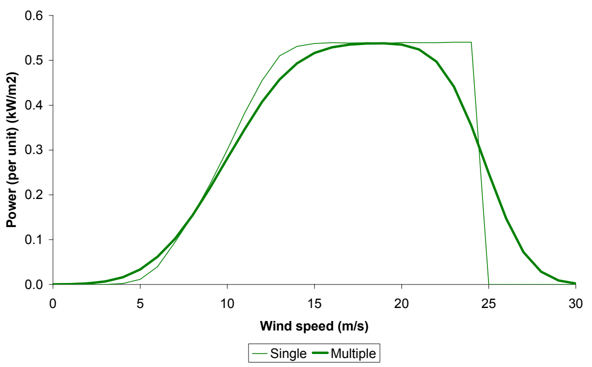

Figure 7: The distribution function of Figure 8 applied on a normalised power curve, representative for a modern large-scale wind turbine, resulting in a smoothed normalised multi-turbine power curve, representative for the aggregated power output from the multiple wind turbines within a given area.

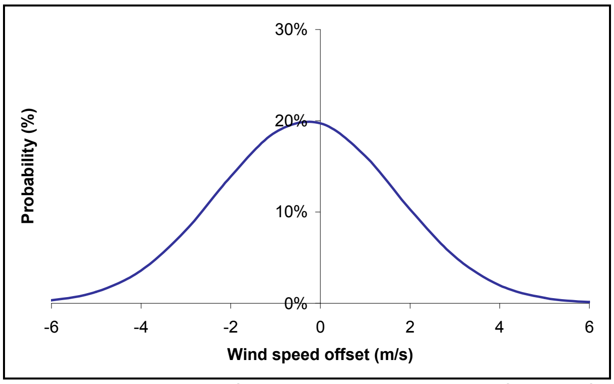

Figure 8: An example of the probability distribution function for the wind speeds for the individual wind turbines in an area at a given time (the wind speed indicated in the graph is relative to the block-average wind speed for the timeslot). The distribution indicated has a standard deviation of 1.5 m/s, corresponding to e.g. a spatial dimension of 200 km, an average wind speed of 8 m/s and a turbulence intensity of 10 % (see Figure 6). An offset adjustment of –0.15 m/s results in an unchanged accumulated production for the two power curves and the given wind speed distribution.

unchanged.

The time series for the aggregated power output from the wind farm is simply obtained by applying the time series of the block-averaged wind speed on the aggregated multi-turbine power curve for the multiple wind turbines.

The methodology – step by step

Below is a step-by-step guideline for the application of the methodology at a given set of data:

1. Specify a representative dimension of the area, D – the extent of the area.
2. Specify the wind speed distribution representative for the area (e.g. given by the two Weibull distribution parameters – the scale factor, A, and the form factor, k), the mean wind speed, w_m, and a representative wind turbulence intensity, I.
3. Generate a new wind speed time series from the original wind speed time series by applying a moving block-average of the elements in the original time series in a timeslot around the specific time corresponding to the dimension of the area, D, and the

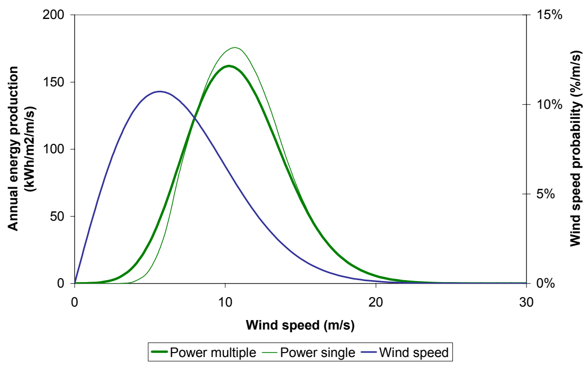

Figure 9: The normalised energy production distributions (in annual kWh/m²) as a function of the wind speed for the single-turbine power curve and for the multiple-turbine power curve of Figure 7 respectively for a given wind speed distribution (a Weibull distribution with A = 8 m/s and k = 2 has been used). For this example the accumulated normalised annual energy production is 1300 kWh/m² (the same for both power curves).

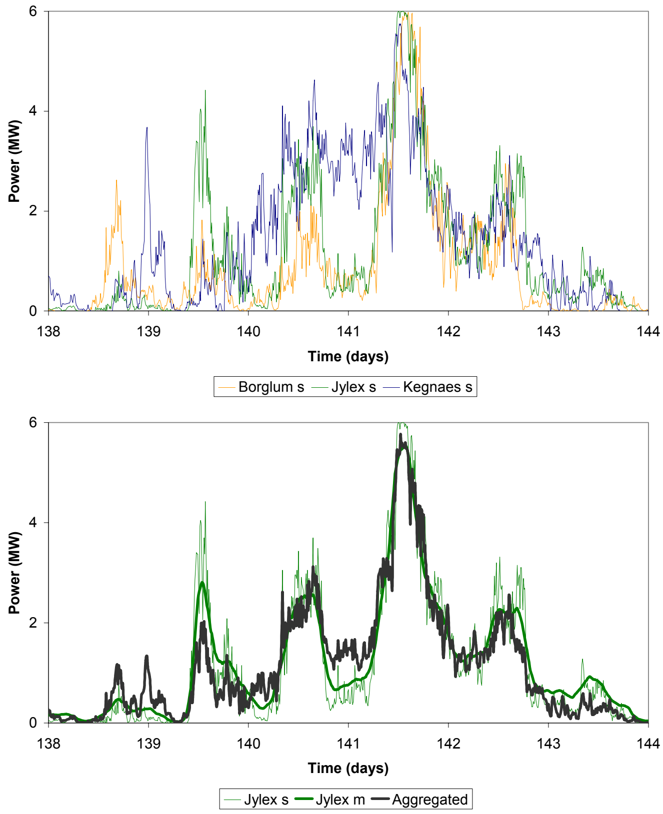

Figure 10: a) Wind power time series based on real wind data for single wind turbine units at the sites Borglum, Jylex and Kegnaes in Denmark, distributed several hundreds kilometres. b) Actual and simulated aggregated power time series of the time series in a) (simulated based on the Jylex data).

mean wind speed, w_m (see Figure 5).

4. Identify from Figure 6 the appropriate normalised standard deviation, σ_n, of the spatial wind speed distribution. Find the actual standard deviation (in m/s) to be used, σ_w, by multiply with the mean wind speed, w_m.
5. Generate the normal distribution with the given standard deviation, σ_w.
6. Identify a (normalised – kW/m^2) power curve, representative for all the wind turbines in question.
7. Generate the aggregated multi-turbine power curve by applying the normal distribution on the standard single-turbine power curve as specified in Eq. 7.
8. Apply the wind speed distribution in time (the Weibull distribution) to the two power curves, check the (normalised) annual energy production, and adjust the offset of the spatial wind speed distribution (the normal distribution) until the energy productions are equal.
9. Generate the actual aggregated power curve for the area by up-scaling the normalised multi-turbine power curve appropriately (to match the total installed wind power capacity).
10. Generate the wind power time series for the area by applying the aggregated power curve to the block-averaged wind speed time series.

Figure 11: Map indicating meteorological stations in Denmark with on-line data viewable on Risøs web-site (www.risoe.dk/vea-data).

IV. Results

The methodology is demonstrated in Figure 10. Wind data from 3 stations in Denmark (Borglum, Jylex, Kegnaes) has been used (see Figure 11). The wind power from single units at the three sites has been calculated based on measured wind data. The sum of the wind power generation from the three wind turbine units (the aggregated wind power) is compared to the simulated aggregated wind power based on only the Jylex data. The simulated time series reproduce some of the qualities in the real aggregated time series – e.g. less tendency to go to zero and max production and smoothing out the rapid, large fluctuations. Only three wind turbine units are included in the real aggregated time series, and rapid, small-scale fluctuations are therefore still present. The simulated time series don't reproduce this.

V. Conclusion

The smoothing effect of the wind power fluctuations in the aggregated power generation from distributed wind turbines has been illustrated by real data. The multi-turbine power curve approach has been described and demonstrated. The methodology is very simplified and is not able to simulate all the qualities in the aggregated wind power time series, but it's better than doing nothing, and it might be the best option.

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