Slowly Decreasing Throughput Algorithms for Random Access Channel

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A multiple power level transmission system augments the maximum throughput by selecting the power levels randomly, exhibiting the similar behavior to the standard Slotted ALOHA, where the throughput decreasing abruptly at higher traffic load conditions and consequently, drops to almost zero, leading to unstable condition. Three slowly decreasing throughput algorithms for Slotted ALOHA are developed to prevent that throughput-collapse. Herein, a mobile terminal can transmit any packet to the lower power levels at a higher probability that increases the probability of only one packet at the highest power levels and thereby enhancing the capture probability. It needs no information about the network, amplifying the implementation privilege. Extensive comparisons show that the proposed slowly decreasing throughput algorithms prevent throughput-collapse under higher offered traffic load condition compared with the traditional random power level selection algorithm.

1. INTRODUCTION

Slotted ALOHA is a widely used random access protocol independently and a part of different multiple access protocols for its adequate working capability especially with bursty traffic. Unfortunately, it suffers from a lower maximum throughput and it decreases abruptly after a certain limit of offered traffic that makes the system unstable. In the popular Slotted ALOHA system, the users are separated by transmitting packets into different power levels. The receiver can capture the packet with the highest power level, even when packets with a lower power level are transmitted to the same slot. The possibility of transmitting packets at multiple power levels and receiving the packet with the highest power level was first introduced and analyzed by Metzner [1]. Shacham [2] devised the details of throughput and delay performance analysis. In this approach, the higher classes have the advantage over the lower classes, which is unjustified. To make the system fair and efficient, the user should transmit the packets into pre-selected power levels by a random selection, implying that any user belongs to any class. On the other hand, the receiver captures the packets at different power levels. The performance analysis of multiple random power levels with capture was developed in [3].

In cellular environment, the natural effects such as fading, near/far effect and slow fading regulate the power levels at the receiver. On the contrary, the multiple power level transmission system can be considered as a man made effect. The combined effect of these two phenomena for a finite number of users has been detailed in [4]. Verdone [5] modeled

the situation where the test packet is affected by only one interfering packet in the same slot. If a multiple number of copies of the same packet are transmitted at multiple power levels by random selection, better performance is achieved when they stay in multiple bands [6].

If any user transmits packets randomly without the knowledge of others, the probability of transmitting more than one packet transmission at the highest power level increases. This occurs especially with a lower number of power levels, operating with a higher offered traffic load. In this case, the throughput of a random power level transmission system decreases [6, 15]. It makes the system throughput lower and finally unstable. The main concern of the ALOHA based network is the stability consideration especially at a higher traffic load condition. If more than a limited number of packets are transmitted into the same time slot, there is good possibility of having more than one packets transmitted in the same power level. As a result, they destroy each other.

Generally, a newly generated packet is transmitted in each slot with a given probability. After an unsuccessful transmission of a packet it is buffered and retransmitted after a random delay with another given probability. Different kinds of retransmission probabilities especially the well-known and widely accepted exponential back-off retransmission probability together with different values of p (the exponent retransmission probability) was analysed in [7]. The selection of a constant value of this exponential retransmission probability p is a very difficult, particularly at a dynamic load condition [8]. Besides, the original version of Slotted ALOHA with an infinite number of users having a constant value of p is inherently unstable [9]. Therefore, a dynamic selection of the exponential retransmission probability p is the solution of this problem for pure Slotted ALOHA with an infinite number of users [10]. Several algorithms were proposed for the stabilised Slotted ALOHA system with a dynamic selection of exponential retransmission probability p [11-13]. The selection of the dynamic retransmission probability always deserves some feedback information that makes the implementation difficult.

Some times Slotted ALOHA works as a part of other multiple access where a number of retransmission cut-of is allowed. The optimum number of transmission for a given new packet generation rate that maximises the channel throughput is deduced in [14]. In this scheme, a mobile terminal has to set the optimum number of transmission depending on the average new packet generation rate. The information of the average new packet generation rate from all active transmitters must be send to all active mobile terminals continuously in order to achieve the maximum throughput, overwhelm the implementation.

A desirable scheme is such that it makes the throughput close to maximum and keeping that throughput at the same level without any feedback information. Since the scheme requires no feedback information, it is easy to implement and more stable for bursty traffic and is the main intention of this paper.

The Power level Division Multiple Access (PDMA) were introduced and analysed in [15], where a sequentially assigned power level based multiple access scheme is used. In this scheme, the maximum number of terminal that transmits a packet in a given time slot is the same as the number of random power level, and only one packet is transmitted at each power level. So the throughput is kept at its highest value, i. e. one, through successful transmission in each slot, although the traffic load is high. Every terminal is ascribed a slot and a power level in every frame. The allocated power level should be one level higher than the previous power level assigned in the previous frame.

In the original version of PDMA, the power level allocation among different terminals is not fair. In this scheme, one terminal can capture another terminal to a greater extent in a

given cycle (block). A fair PDMA scheme was introduced in [16] using *permutation* method to allocate the power levels and time slots to every mobile terminal so that each terminal can capture another terminal equally. Note that one cycle (block) consists of a number of frames and power levels. From stability point of view this scheme is attractive, but each mobile has to know which power level is assigned for it in which slot. Therefore, from the implementation point of view, the PDMA is not so attractive.

A generalised multicopy ALOHA scheme was also introduced to increase the throughput at higher traffic load conditions [17]. In this scheme, two enhancement methods of multicopy ALOHA are developed. Firstly, a relaxed multicopy transmission policy, which does not decrease the successful transmission probability especially at a higher traffic load condition and secondly, an artificial modified capture model is used. Excellent work regarding the power level selection probabilities that makes the maximum throughput was proposed and analysed in [18]. Unfortunately, this scheme needs to know the number of mobile terminals working in the system.

The main intention of the above mentioned stability improvement schemes [15-18] is to transmit *only one* packet at the highest power level especially when the traffic load is high. We contrive a very simple solution to this problem involving packet transmission at preselected power levels with a higher probability at lower power levels. In these algorithms a mobile terminal does not need any feedback information except its own success or failure, which is conducive to the implementation. The propose schemes do not reach the maximum throughput, rather close to maximum and keeping that level up to a possible higher level, without any feedback information that prevent throughput-collapse and make the channel stable.

2. THE RANDOM POWER LEVEL TRANSMISSION SCHEME

Let us assume that there are N per-selected power levels. During the transmission of any packet, a mobile terminal transmits its packet at any power level. Let us concentrate on the general jth power level (Fig. 1), j=1, 2, .., N, where N is the lowest power level and 1 is the highest power level. The receiver can receive the packet transmitted at the jth power level if only one packet is transmitted at that power level and all other interfering packets are at lower power levels than that of j. That is, all interfering packet(s) are confined between N and (j+1) power levels. We assume that the jth power level is sufficiently higher than the (j+1)th power level for the receiver to decode the packet in the jth power level successfully.



Fig.1: Random power levels.

3. SLOWLY DECREASING THROUGHPUT ALGORITHMS

The analysis for packet transmission with a random selection of power levels is performed in [6, 15]. The throughput increases with the increase in a number of power levels. The main drawback of this random power level selection is the throughput collapse at higher traffic load conditions. Herein, the disadvantage associated with the random power level selection exclusively at higher load conditions is circumvented.

Three different possibilities are considered herein: (1) Linear approach; (2) Annular approach; and (3) Circular shell approach.

3.1 Linear Approach

Any mobile terminal can transmit its any packet at any power level with the probability presented in Fig. 2.

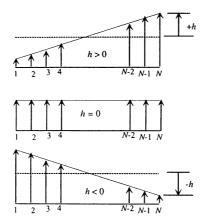


Fig. 2: Probability distribution of packet transmission into different power levels.

Three possible conditions are:

- 1. If h>0, the probability of transmission at lower power levels is *higher* than that of at higher power levels.
- 2. If h=0, the probability of transmission of packets at all N power levels is equally distributed. The power level selection is random [6, 15].
- 3. If h < 0, the probability of transmission at lower power levels is *lower* than that of at higher power levels.

Let us define P_j as the probability of transmitting a packet at the *j*th power level, which can be formulated from Fig. 2 as

$$P_{j} = \frac{2h}{N-1}(j-1) + \frac{1}{N} - h = \frac{h}{N-1}(2j-N-1) + \frac{1}{N}$$
 (1)

where P[Success | j] defines the probability of successfully receiving a packet when it falls into the *j*th power level.

The probability that the test packet is overlapped with k other interfering packets in a given slot is Poissonically distributed and is given by

$$P[\text{Overlap} \mid k] = \frac{G^k}{k!} \exp(-G)$$
 (2)

Since packets are distributed over all power levels according to Eq. (1), the probability of a packet being transmitted within (j+1) levels is

P[Success | 1] packet has lower power than j

$$= \sum_{i=j+1}^{N} \left[\frac{h}{N-1} (2i - N - 1) + \frac{1}{N} \right]$$

$$= \frac{jh(N-j)}{N-1} + (N-j) \frac{1}{N}$$
(3)

The probability that all interfering k packets are transmitted at lower power levels than the jth power level is

P[Success | all k | packets have lower power]

$$= \left\{ \frac{jh(N-j)}{N-1} + (N-j)\frac{1}{N} \right\}^k \tag{4}$$

Therefore, the probability of success of a packet transmitted at the jth power level is

 $P[Success | j] = \sum_{k=0}^{\infty} P[Overlap | k] * P[Success | all k] packets have lower power]$

$$= \sum_{k=0}^{\infty} \frac{G^k}{k!} \exp(-G) \left\{ \frac{jh(N-j)}{N-1} + (N-j)\frac{1}{N} \right\}^k$$

$$= \exp\left\{ \frac{Gjh(N-j)}{N-1} - \frac{Gj}{N} \right\}$$
(5)

Thus the probability of success of a packet, taking all N random power levels into account, can be shown as

$$P_{LA} = \sum_{j=1}^{N} P[Success \mid j] * P_{j}$$

$$= \sum_{j=1}^{N} \left[\left\{ \frac{h}{N-1} (2j - N - 1) + \frac{1}{N} \right\} exp \left\{ \frac{Gjh(N-j)}{N-1} - \frac{Gj}{N} \right\} \right]$$
(6)

Finally, the throughput of the improved stability scheme with the linear approach is

$$S_{LA} = G \sum_{j=1}^{N} \left[\left\{ \frac{h}{N-1} (2j - N - 1) + \frac{1}{N} \right\} \exp \left\{ \frac{Gjh(N-j)}{N-1} - \frac{Gj}{N} \right\} \right]$$
 (7)

The delay can be shown from Eq. (7) as

$$D_{LA} = 1 + \left(\frac{G}{S_{LA}} - 1\right) \frac{1}{r} \tag{8}$$

where 1/r is the retransmission delay.

Fig. 3 shows the numerical results for different values of **stability factor** h, and two different power levels. It can be seen that the throughput increases with increasing number of power levels.

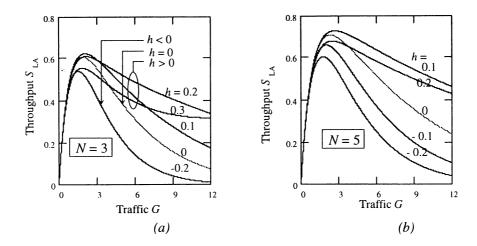


Fig. 3: Throughput S vs. traffic G for different values of stability factor, h.

From Fig. 3, we have the following observations:

- h < 0, is always ineffective.
- The maximum value of stability factor, h is equal to 1/N.
- For optimum system operation, the stability factor h increases with a higher rate at lower traffic load conditions and the value of h is approximately 1/N at a higher load.

Conclusively, depending on the value of the maximum power level N in a system, the operator can set the value of h, such that the throughput does not decline abruptly with the higher average traffic arrival rate G.

From the previous subsection, it is evident that the stable operation at a higher traffic load condition can be achieved by transmitting the packets into lower power levels with a higher probability, provided that the parameter h must be selected appropriately. The approach necessarily induces a system parameter by which the mobile terminal recognises the transmission probabilities of any power level.

3.2 Annular Approach

Consider an N annular system as shown in Fig. 4. Any mobile terminal selects the 1st power level with the probability of a fractional area of inner most annular. At the same way, any mobile terminal transmits its any packet with the 2nd power level with the probability of a fractional area of the second inner annular. In general, the packet transmission into the *i*th power level is the fractional probability of the *i*th inner annular. Referring to Fig. 4 the fractional area of the *i*th annular is

$$P_{i} = \frac{\int_{t-1}^{t} 2\pi x \, dx}{\pi R^{2}} = \frac{2i-1}{(R/r)^{2}} = \frac{2i-1}{N^{2}}$$
(9)

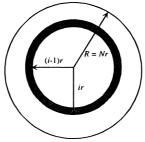


Fig. 4: Fractional area of the ith annular.

A packet transmitted into the *i*th power is successfully transmitted if and only if there is only one packet transmitted into the *i*th power level and all other k interfering packets are transmitted into the lower power levels than that of the *i*th power level. The probability that a packet is transmitted within (i+1) levels or a lower power level than the *i*th power level is

P[Success | 1 packet has lower power than i]

$$= \sum_{j=i+1}^{N} \left[\frac{2j-1}{N^2} \right] = \frac{N^2 - i^2}{N^2} \tag{10}$$

The probability that all interfering k packets are transmitted at lower power levels than the ith power level is

P[Success | all k packets have lower power than i]

$$=\left\{\frac{N^2-i^2}{N^2}\right\}^k\tag{11}$$

So the probability of success of a packet transmitted at the *i*th power level reads $P[Success | i] = \sum_{k=0}^{\infty} P[Overlap | k] * P[Success | all k packets have lower power than i]$

$$= \sum_{k=0}^{\infty} \frac{G^k}{k!} \exp(-G) \left\{ \frac{N^2 - i^2}{N^2} \right\}^k = \exp\left\{ -i^2 \frac{G}{N^2} \right\}$$
 (12)

Thus the probability of success of a packet, taking all N power levels into account, can be shown as

$$P_{AA} = \sum_{i=1}^{N} P[Success \mid i] * P_{i} = \sum_{i=1}^{N} \frac{2i-1}{N^{2}} exp\left(-i^{2} \frac{G}{N^{2}}\right)$$
 (13)

Finally, the throughput of the slowly decreasing throughput algorithm considering the annular approach yields

$$S_{AA} = G \sum_{i=1}^{N} \left\{ \frac{2i-1}{N^2} \right\} \exp\left\{ -i^2 \frac{G}{N^2} \right\}$$
 (14)

and accordingly, the packet delay with annular approach is

$$D_{AA} = 1 + \left(\frac{G}{S_{AA}} - 1\right)\frac{1}{r} \tag{15}$$

where r is the retransmission probability.

The random power level selection as shown in [6, 15] can be derived very easily by setting h = 0 in Eq. (7) providing with

$$S_N = \frac{G}{N} \sum_{j=1}^{N} \exp\left\{-j\frac{G}{N}\right\} \tag{16}$$

Leading the delay of the random power level selection scheme to

$$D_N = 1 + \left(\frac{G}{S_N} - 1\right) \frac{1}{r} \tag{17}$$

The performance comparison of the slowly decreasing throughput algorithm annular approach pertaining to Eqs. (14) and (15) and the traditional random power level selection comprising Eqs. (16) and (17), are depicted in Fig. 5.

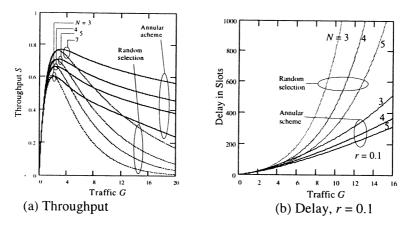


Fig. 5: Performance comparison of annular and traditional random power level selection approaches.

3.3 Circular Shell Approach

In multiple circular shell approach, the traffic generation from all mobile stations is uniformly distributed with a rigid ball of radius R. Each circular shell has an equal radius r. Total number of circular shells is R/r = N. The packet transmission into the different power levels corresponds to the fractional volume of the circular shell.

Concentrating on jth circular shell varying from 1 to N, a packet of a mobile terminal selects the jth power level with the fractional volume of the jth circular shell. Therefore, the probability of transmitting a packet, selecting the jth power is proportional to its volume.

$$P_{j} = \frac{\int_{(j-1)r}^{jr} 4\pi x^{2} dx}{\frac{4}{3}\pi R^{3}}$$

$$= \frac{(3j^{2} - 3j + 1)r^{3}}{R^{3}} = \frac{3j^{2} - 3j + 1}{N^{3}}$$
(18)

The proposed probability power level transmission system considers that during the transmission of a packet, a user can transmit its packet at any power level with some probability. Regarding the general jth power level, a packet can be captured by radio receiver successfully if and only if there is no packet at the same slot in the upper power level (i.e. from 1 to (j-1) power levels), and exactly one packet transmitted into the jth power level. Assuming that the jth power level is sufficiently higher so that if there is any packet in the (j+1)th power level, a receiver can decode the packet in the jth power level, successfully.

Consider the general jth spherical shell varying from 1 to N. Let $P[Success \mid j]$ is the probability of transmitting a packet into the jth power level received successfully. According to the multiple spherical shell concepts, it should occur if there is no packet transmitted into the higher power levels than j does (power levels from (j-1) to 1), irrespective of lower power levels. The average packet transmitted into all N power levels is G packet per time slot. Consequently, $P[Success \mid j]$ can be defined as

P[Success | j] = P[No packet transmitted at 1 to (j-1) power levels]

*P [No overlapping into the jth power level]

$$= \left\{ \exp(-GP_1)^* \dots * \exp(-GP_{j-1}) \right\} * \exp(-GP_j) = \exp\left(-G\sum_{k=1}^{j} P_k\right)$$
 (19)

The probability of success of a packet can be define as

$$P_{CS}[Success] = \sum_{j=1}^{N} P[Success | j]$$

*P[Probability of transmitting a packet into the*j*th power level]

$$= \sum_{j=1}^{N} P[Success| j] * P_{j} = \sum_{j=1}^{N} P_{j} \exp \left\{-G \sum_{k=1}^{j} P_{k}\right\}$$
 (20)

where
$$P_j = \frac{3j^2 - 3j - 1}{N^3}$$
 as shown in Eq. (18)

Throughput with the multiple circular shell approach is:

$$S_{CS} = GP_{CS}[Success] \tag{21}$$

Combining Eqs. (20) and (21) and simplifying yields

$$S_{CS} = G \sum_{j=1}^{N} \frac{3j^2 - 3j + 1}{N^3} \exp\left(-j^3 \frac{G}{N^3}\right)$$
 (22)

and the packet delay in slots is

$$D_{CS} = 1 + \left(\frac{G}{S_{CS}} - 1\right) \frac{1}{r} \tag{23}$$

The performance comparison of the slowly decreasing throughput algorithm circular shell approach and the traditional random power levels are shown in Fig. 6.

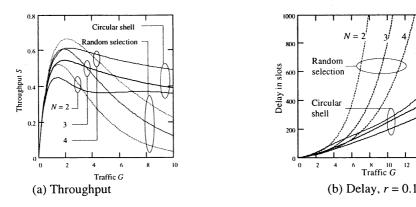


Figure 6: Performance comparison of circular shell and traditional random power level selection approaches.

4. COMPARISON OF THREE DIFFERENT SCHEMES

It is a common interest to find the optimum choice from these three different schemes. The throughput of the traditional random power level selection, annular and the circular shell approaches is depicted in Figure 7. A careful inspection shows that the circular shell approach exhibits a higher throughput at a higher average traffic load condition but a lower maximum throughput. In the circular shell approach, the multiple overlapping packets have higher probability to be in lower power levels than that of the annular approach especially at a higher traffic load condition. This is why the circular shell approach shows a higher throughput at a higher average traffic load condition compared with the annular approach. On the other hand, the annular approach achieves a higher maximum throughput but a lower throughput at a higher traffic load condition. The reason is that at a medium load condition, the probability of having only one packet at each power level is higher in the annular approach. In the case of circular shell approach, more than one packets falls into lower power

levels and destroy each other and show a lower maximum throughput. Therefore, if the system has more possibility of having higher average load for its robust burstyness of traffic, the circular shell approach is more preferable. Figure 8 deals with the probabilities of having packets into different power levels with the number of power levels N = 5 and h = 0.15. It is evident that the linear approach is quite close to the annular approach if a suitable value of h is chosen. On the other hand, the circular shell approach is excellent at a higher traffic load condition.

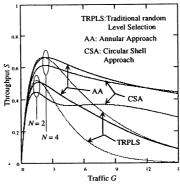


Fig. 7: Performance comparison of circular shell, annular and traditional random power level selection approaches.

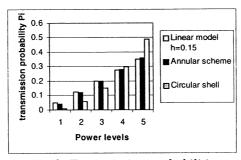


Fig. 8: Transmission probabilities

5. CONCLUSIONS

Three slowly decreasing throughput algorithms namely, linear, annular and circular shell, are proposed and analysed. Since the mobile terminals transmit their packets without the knowledge of current loading condition of the networks, the implementation is positively easier. For a given number of power levels, the probability of more than one packet into the highest power level increases if the traffic load is very high. If the transmitters transmit their packets into the higher power levels with lower probabilities, the probability of only one packet into the highest power level increases. Consequently, enhancement of the packet success probability in each slot, keeping the system stable at a higher traffic loading condition is stimulated. The rest of the packets fall automatically, into the lower power levels because of the higher selection probabilities of those power levels.

The circular shell approach shows a higher throughput at a higher average traffic load condition compared with the annular approach. On the other hand, the annular approach shows a better maximum throughput compared with the circular approach. The linear approach depends on the proper choice of parameter h involving the average packet generation rate.

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