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Invisible hand effect in an evolutionary minority game model

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Abstract

In this paper, we study the properties of a minority game with evolution realized by using genetic crossover to modify fixed-length decision-making strategies of agents. Although the agents in this evolutionary game act selfishly by trying to maximize their own performances only, it turns out that the whole society will eventually be rewarded optimally. This "invisible hand" effect is what Adam Smith over two centuries ago expected to take place in the context of free market mechanism. However, this behaviour of the society of agents is realized only under idealized conditions, where all agents are utilizing the same efficient evolutionary mechanism. If on the other hand part of the agents are adaptive, but not evolutionary, the system does not reach optimum performance, which is also the case if part of the evolutionary agents form a uniformly acting "cartel".

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1. Introduction

In his book of 1776 Adam Smith outlined a mechanism which he supposed to describe the behaviour of economic societies [1]. He postulated that individuals who try to maximize their own gain without active regard to the society's welfare will eventually reward the society most effectively. As the mechanism how this should actually happen Smith described it as an ''invisible hand'' of a benevolent deity administering human happiness by leading individuals to act in a certain way. In the modern context invisible hand processes have been studied as part of game theory, a branch of mathematics dealing with payoffs and strategies, where the interrelationships between the best productivity of individual actors and the society has been refined by John Nash through equilibrium concept [2–4]. He indicates that individuals could only maximize their own benefit by taking other individuals into account. However, Smith's assumption about the optimal performance of the society through selfish individuals turns out to be valid in certain circumstances. For example, this is the situation for the minority game introduced by Challet and Zhang [5], see also Refs. [6–12].

Minority games are repeated coordination games [2,3] where agents use a number of different strategies in order to join one of the two available groups, A or B, and those who belong to the minority group are rewarded. In the original MG [5] the agents are exposed to P different histories and the strategy of an agent determines the choice of the group for each history. Thus, the length or dimension of a strategy equals P, and the set of all possible 2^P strategies composes a strategy space from which the agents' strategies are randomly drawn in the beginning of the game. Strategies are cumulatively scored based on correct minority group choices, and at each step of the game the choices of the agents are determined by their highest-scoring strategies. In the following, we shall refer to this basic minority game with the above-described adaptation mechanism as BMG, and use the abbreviation MG to refer to the minority game concept in a more general fashion.

Minority games can be viewed as simulating the performances of competing individuals and the welfare of the society they compose. This kind of mechanism could coarsely speaking be involved in a stock market where investors share information and make buy-or-sell decisions in order to gain profit. If the number of sellers of a particular stock is larger than the number of buyers, supply exceeds demand and one expects a decrease in the stock price [13]. Then the buyers, being in minority, would win due to the low price levels. In the opposite case sellers would win, because excess demand would increase the price of the stock. In the long run, the price of the stock eventually settles down to its equilibrium value, i.e., supply and demand are, on average, close to each other and the public information has been efficiently utilized. In relation to this the utility or performance of the society can be viewed as the number of content individuals. In other words if everybody agrees on the price, both the sellers and buyers are content. In the framework of MG, this means that the numbers of buyers and sellers are as close to each other as possible and the game is in one of its pure-strategy Nash equilibria [2,3,14–16]. On the other hand if the numbers deviate from equilibrium, either one of the groups is dissatisfied and thus the overall "happiness" of the society decreases.

In these games the long-term system performance is of major interest. It is measured as the variation of the minority group size around its maximum, such that the larger the minority group size at every time step is, the better the aggregate system performance is. Usually, the behaviour of the system performance depends on a control parameter z [8], which combines the dimension of the strategy space and the number of players in the game. In the BMG the best system performance occurs at $z = z_c$, which depends on the number of the strategies each agent has [17]. On the other hand, applying an evolutionary mechanism to the agents' strategies usually changes the behaviour of the game remarkably. Roughly speaking, the evolutionary mechanisms studied in the context of minority games can be divided into two groups, i.e., to those mechanisms that are applied to pure strategies and do not explicitly include probabilities in strategy selection or decision-making of the agents (e.g. Ref. [18]), and those that do so (e.g. Refs. [15,16,19,20]). A further division can be made between fixed (e.g. Ref. [18]) and variable-length strategies (e.g. Ref. [21]).

Our evolutionary MG belongs to the pure-strategy class with fixed-length strategies. However, the main difference between our game and the game discussed in Ref. [18], belonging also to the same class, is the genetic-algorithm-based mechanism by which the strategies of an agent are modified. We find that enhancing the BMG with one-point genetic crossover mechanism results in the birth of new strategies based on well-performing parent strategies and leads to behaviour resembling Smith's "invisible hand". Previously, we have studied the effect of genetic crossover of strategies on the MG performance [22–24], and shown that our simple pure-strategy evolutionary mechanism leads to highly enhanced performance, both at the system as well as at the individual agent level. Recently, Yang et al. [25] have reported results of a study using a genetic-algorithm-based evolutionary mechanism, which turned out to be quite similar to those of ours [22–24]. Below we will show that with our evolutionary mechanism the optimal system performance can be reached for a wide range of control parameter values. In contrast to Ref. [18], increasing the number of strategies increases the system performance. Furthermore, the optimal performance is typically reached for all possible histories independent of their order of appearance, as the histories are randomly drawn from a uniform distribution in order to avoid any repetitive history cycles.

This paper is organized such a way that we first introduce our evolutionary minority game (EMG) model. Then we show simulation results on the system performance and compare them with the optimal limit as well as with results of simulations using the BMG. Furthermore, we investigate using the minimum and maximum spanning tree methods whether similarly performing evolutionary agents form clusters in the sense that they would play similar strategies, and whether well-performing agents' strategies tend to be different from those of the badly performing ones. In addition, we briefly discuss the effect of a fraction of the agents forming a uniformly acting group, "cartel", on society utility. Finally we draw conclusions.

2. Model

Let us first briefly describe the BMG, and then discuss the strategy evolution method we have applied. The BMG [5] consists of (odd) N agents who simultaneously choose between two options, denoted 1 and -1. After the decisions of the agents, votes are counted, and those who belong to the minority group gain profit. The winning minority is publicly announced after every round. The game is repeated, and at each round the choice of an agent is determined by a component of a P-dimensional binary vector called the strategy of the agent. Each of the P components indicates a response corresponding to a particular history vector of length M, which comprises of the minority choices during the last M rounds. As there are 2^M possible histories, $P = 2^M$ [5].

In the BMG histories are explicitly determined by the choices of the agents, but instead we have decided to draw the histories randomly from a uniform distribution. The motivation for this was to avoid occurrences of any cyclic patterns of repeating histories, thus pre-empting the agents' possibilities for history-pattern-based coordination. Furthermore, previous results [22] with deterministic histories show that with our evolutionary mechanism, the game would finally repeat a single history only. The method of randomly selecting histories provides a stronger basis to justify the success of the chosen evolutionary mechanism in explaining the observed highly efficient system performance. (For discussion on the effect of using random versus non-random histories, see e.g. Refs. [26–28]). In the BMG each agent has S randomly chosen strategy vectors s_i that are scored according to their cumulative success in predicting the minority group, with unit score added for the right choice and deducted in the opposite case. At each round an agent uses the strategy vector s_i with the highest score.

We define the performance of an agent at each round to be the number of times it has belonged to the minority minus the number of times it has belonged to the majority, and then scaled into the interval $[0, 1]$. In order to measure characteristics of the whole system of agents we define the society utility $u(t) \in [0, 1]$ at each round t to be the number of agents who belong to the minority group divided by $(N-1)/2$ (the maximum size of the minority group). In the minority game studies a common measure to characterize the model is the attendance [17]

$$a(t) = \sum_{i=1}^{N} \sigma_i(t) , \tag{1}$$

where $\sigma_i(t) \in \{-1, 1\}$ denotes the action which the agent i takes at round t. Thus the attendance gets values $a \in \{-N, -N+2, ..., 1, -1, ..., N-2, N\}$ and it is related to the society utility as

$$u(t) = \frac{N - |a(t)|}{N-1} . \tag{2}$$

If $a = 1$ or -1, the society utility is at its maximum $u = 1$. When a increases, the society utility u decreases. In the minority game studies it is a common practice to observe the normalized fluctuations of attendance

$$\langle a^2 \rangle / N = \frac{1}{NT} \sum_{t=k+1}^{k+T} a^2(t) \tag{3}$$

as function of the control parameter $z = 2^M/N$, see Ref. [8]. The square of attendance Eq. (1) is averaged over T time-steps and then normalized by the number of agents. In the BMG with a fixed number of strategies S per agent one can separate three regions in the normalized fluctuations as z changes: for small values of z fluctuations are large, for intermediate values of z they reach a minimum, and for large values of z they start to converge towards the limit of random decisions, being unity (decisions taken by flipping a coin) [17]. According to Eq. (2), small normalized fluctuation values indicate large values of the society utility. If we increase S in the BMG, normalized fluctuations also increase and thus the society utility decreases. As we will see later, the behaviour is very different in our EMG: increasing S leads to larger society utility values, and separate regions of fluctuation levels do not exist.

In contrast to the BMG [5], where the strategies remain the same throughout the game, we utilize an evolutionary mechanism that allows agents to change their strategies for better personal gain. This mechanism is as follows: after every r rounds the agents observe the performances of their neighbours, and if they are doing worse than a neighbour, they cross two of their best S strategies and replace the two worst strategies with the resulting ones. The crossover is done in a typical genetic algorithm fashion [29,30]: a crossover point $p_c \in [0, P]$, is randomly selected, and the children inherit p_c strategy components from one parent and $P - p_c$ from the other. For example, if the parent vectors were $(1\ 1\ 1\ 1)$ and $(-1 -1 -1 -1)$ and the crossover point $p_c = 2$, the resulting vectors would be $(1\ 1 -1 -1)$ and $(-1 -1\ 1\ 1)$.

The rule of when and which agents will attempt to improve their strategies can be implemented in many ways. The only aim of determining the rule is to obtain large enough rate of convergence in the fluctuation of attendance Eq. (1). In these studies we have determined the neighbours by spanning a scale-free tree whose nodes denote the agents, and whose links determine the neighbours of an agent. With this approach we get fast convergence in fluctuations of attendance, because the typical node-to-node distance within a scale-free network is short, and thus the information on the performance of "good" agents spreads rapidly. Other possibilities include, for example, taking the worst fraction of agents and making them cross, or letting an agent observe its own performance only, and if it continuously decreases, allowing the agent cross its strategies.

3. Results

In our numerical simulations we have observed that in our EMG the society utility tends to maximize within a wide range of control parameter $z = 2^M/N$ values, provided that agents are given enough strategies at the beginning. In addition we observed that agents whose performance is close to each other do not form groups in the sense that they would use similar strategies. Also we investigated the effects of group decision-making as well as endowing only part of the agents with strategy improvement capability. These results are explained in detail below.

In what follows, we define the time scale of the simulation in terms of P rounds, such that $C = const. \times P$. The strategy length P is a natural measure of time, since on average, it takes P rounds to go through all the components of a particular strategy, and thus an agent can for each history get response to the success or failure of its choice. In Fig. 1 we show the development of attendance Eq. (1) during one simulation run. We see that the fluctuations start at a high level, but are then rapidly damped towards the minimum, indicating that the society utility Eq. (2) maximizes. In terms of the trading analogy this means that the numbers of sellers and buyers become as close to each other as possible, and thus the price of the commodity settles down to its equilibrium value. In this simulation run we have used $M = 6$, $S = 21$, $r = 2P$ and $C = 1000P$ (64,000 rounds). The behaviour of our EMG differs considerably from that of the BMG, in which the fluctuation level would remain high because the control parameter value ($z \approx 0.06$) lies within the low-z-high-fluctuation region, and because the number of strategies S is high [17].

In our EMG model the evolutionary strategy changes mean that agents can develop and strive to optimize their strategies with the proven crossover method [22], whereas in the BMG model the agents are restricted to their original strategies. According to Smith, individuals who are striving towards maximizing personal gain eventually promote the whole society most effectively. This is exactly what happens in our EMG, as the agents do not have any explicit rules to lead the society utility to the maximum. Note, however, that unlike in the real world, all

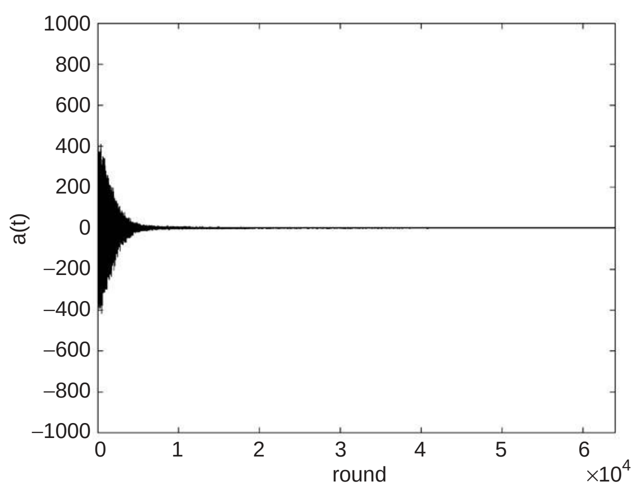

Fig. 1. The evolution of fluctuations in the EMG during one simulation run, with the number of agents $N = 1069$, memory length $M = 6$, number of strategies per agent $S = 21$, crossover period $r = 2P$, and simulation length $C = 1000P$, where $P = 2^M$. The numbers of minority and majority group members (“sellers” and “buyers”) eventually become as close to each other as possible.

agents are here equal in their "skills". The effect of differing agent abilities will be discussed below.

In Fig. 1 we gave an example of the evolution of attendance Eq. (1) and maximization of the society utility Eq. (2) (minimization of $|a(t)|$) in one particular realization of the game with fixed parameter set. Fig. 2 shows the normalized fluctuations of attendance Eq. (3) versus the control parameter $z = 2^M/N$, illustrating how the optimum is reached, if enough strategies are given to the agents at the beginning of the game. For each point on the curve we have used $M = 6$, $r = 2P$ and $C = 1000P$, and averaged over 200 estimates. An estimate for the normalized fluctuations Eq. (3) is calculated using the last 1000 simulation rounds. The number of rounds r after which the agents check their neighbours and decide about crossing their strategies is two whole periods. In this time an agent gets, on the average, two responses for its actions for every history, and thus has time to learn which of its strategies perform better than others.

There are two reference lines in Fig. 2: the horizontal dotted line $\langle a^2 \rangle / N = 1$ and the solid line $\langle a^2 \rangle / N = 1/N$. The former indicates the level of normalized fluctuations, if the random decision (coin flipping) strategy is used, and the latter the minimum value of the normalized fluctuations (maximum society utility). The four series below the random decision strategy line display normalized fluctuations for $S = 5$ (triangles), $S = 8$ (asterisks), $S = 13$ (plus-signs) and $S = 21$ (circles). All

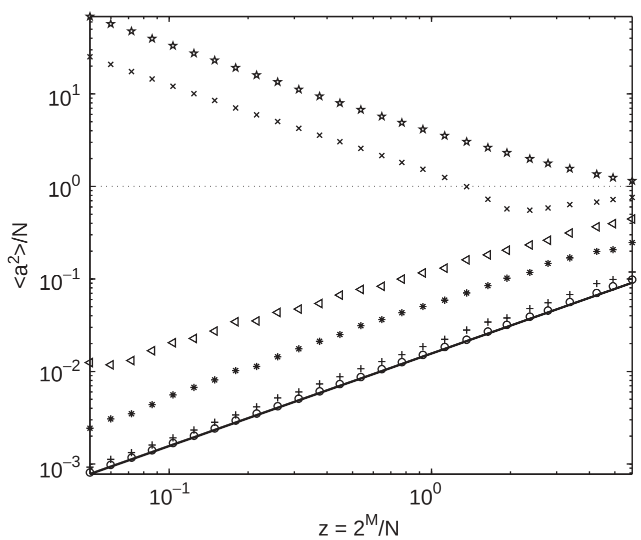

Fig. 2. Normalized fluctuations versus the control parameter $z = 2^M/N$ for $S = 5$ (triangles), $S = 8$ (asterisks), $S = 13$ (plus-signs), and $S = 21$ (circles) in our EMG. For comparison, we plotted also the normalized fluctuations in the BMG for $S = 5$ (crosses) and $S = 21$ (stars). We used $M = 6$, $r = 2P$, and $C = 1000P$, and averaged over 200 estimates. Increasing S leads the normalized fluctuations into the minimum line.

series fall around lines whose slopes ≈ 1 which is the same as that of the minimum normalized fluctuations line. As the number of strategies increases, the lines start to converge towards the minimum normalized fluctuation line. This indicates that the society utility is maximized. With $S = 21$ the level of normalized fluctuations Eq. (3) is very close to its minimum value for all values of the control parameter z. Thus, in our EMG the society utility Eq. (2) increases, if we increase the number of strategies per agent S. The reason for this is that larger initial strategy sets allow more crossover combinations, among which the agents can find good ones with higher probability. Thus, the strategy set size can be seen as representing the initial capabilities of agents, and also sets a limit for improvement. If S is too small, it is possible that combinations do not include those strategies which finally lead to the society utility maximum.

A remarkable property in the case of $S \geqslant 21$ is that the normalized fluctuation values Eq. (3) are minimized for all simulated z. The result is robust and shows how efficient the utilized evolutionary method is. For comparison, we have also plotted normalized fluctuations for the BMG for $S = 5$ (crosses) and $S = 21$ (stars). Contrary to our EMG, the normalized fluctuations increase, if more strategies are added to agents' initial strategy sets [17]. Here we can also separate the behaviour of normalized fluctuations in the low, middle and high-value regions of the control parameter z. In our EMG, $S = 5$ case is the most inefficient compared to the games with higher S in the sense of society utility, but still considerably more efficient than the BMG for $S = 5$. In fact the difference is huge—of the order of $\sim$ 100–1000—in the low z region. This difference is even bigger for higher values of S.

The assumption that agents are potentially equal in their skills is important for reaching the minimum of normalized fluctuations. If a fraction of agents is not able to adapt by crossing their strategies, the system utility will not reach its maximum value. This can be seen in Fig. 3 where we studied the development of normalized fluctuations Eq. (3) as the percentage of evolutionary agents increases for $N = 99, 205, 429$, and 891. We used $M = 6$, $S = 21$, $r = 2P$, $C = 1000P$ for one sample run, and averaged over 50 samples. If none of the agents is evolutionary, the level of normalized fluctuations is that of the BMG using the same parameters. If all agents are evolutionary, the normalized fluctuations Eq. (3) are minimized as in Fig. 2. Between these two extremes the normalized fluctuations decrease monotonically as the percentage of evolutionary agents increases. We found that the results are best described by parabolic decrease ($\langle a^2 \rangle / N = q_2 x^2 - q_1 x + q_0$) of fluctuations as function of the fraction x of evolutionary agents (see dashed lines in Fig. 3). The values of the coefficients of parabolas seem to obey a power-law $q_i \propto z^{-1}$, as indicated by the inset in Fig. 3.

Returning to the setting where all agents are evolutionary, we studied whether agents whose performance is close to each other form groups within which agents use similar strategies. If such groups exist, there might be particular strategies in the whole strategy space which are preferred compared to others, and agents who use the same or similar strategies would perform about equally

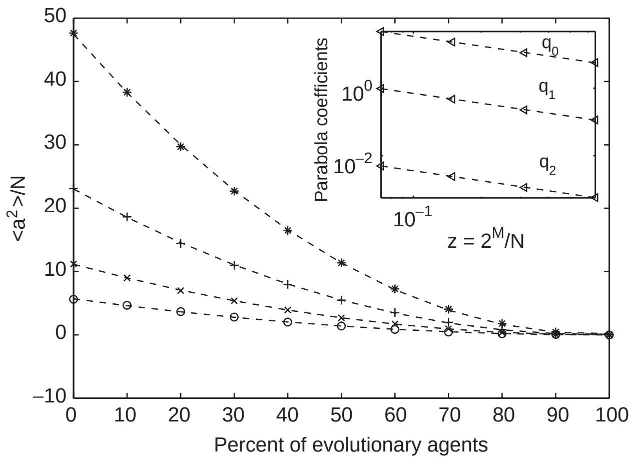

Fig. 3. Normalized fluctuations versus the percentage of evolutionary agents for $z = 0.65$ ($N = 99$, $M = 6$) (circles), $z = 0.31$ ($N = 205$, $M = 6$) (crosses), $z = 0.15$ ($N = 429$, $M = 6$) (plus-signs), and $z = 0.07$ ($N = 891$, $M = 6$) (asterisks). We used $S = 21$, $r = 2P$, $C = 1000P$, and averaged over 50 samples. The dashed lines are parabolas fitted to the observations using the minimum square error criterion. The inset shows parabolic coefficients q_i as function of the control parameter, displaying power-law decrease $q_i \propto z^{-1}$.

successfully. Here we measure the similarity of two strategies k_1, and k_2 with the Hamming distance

$$d_{k_1k_2} = \frac{\sum_{i=1}^{P} |k_1(i) - k_2(i)|}{P} \; . \tag{4}$$

In order to study the formation of groups we have simulated our EMG with $N = 65$, $M = 6$, $S = 21$, $r = 2P$, $C = 3000P$, and observed the performances of the agents from the last $100P$ rounds of the simulation. The large number of simulation rounds guarantees that agents make a sufficient number of crossovers and that the evolution of their strategy pools has more or less stopped. In our simulations such a stabilization happens often in $C = 1000P$ rounds. At the end of the simulation run we take notice of the used strategy of each agent and calculate the Hamming distance for all possible strategy pairs between agents.

In order to visualize the clustering of either winning or losing strategies we have used the minimum/maximum spanning tree methods formed by using pairwise Hamming distances. The minimum/maximum spanning tree is the shortest/longest tree graph which can be spanned between the nodes [31]. If some strategy pairs resemble each other, their Hamming distances are small, and thus these distance pairs of the whole Hamming distance matrix with $N(N-1)/2$ elements will be extracted for the minimum spanning tree, whereas in the maximum spanning tree, interconnected strategies are far from each other in the strategy space. In Fig. 4 we show the resulting spanning trees—minimum on the left, maximum on the right—which are coloured according to the performance of an agent, scaled into the range

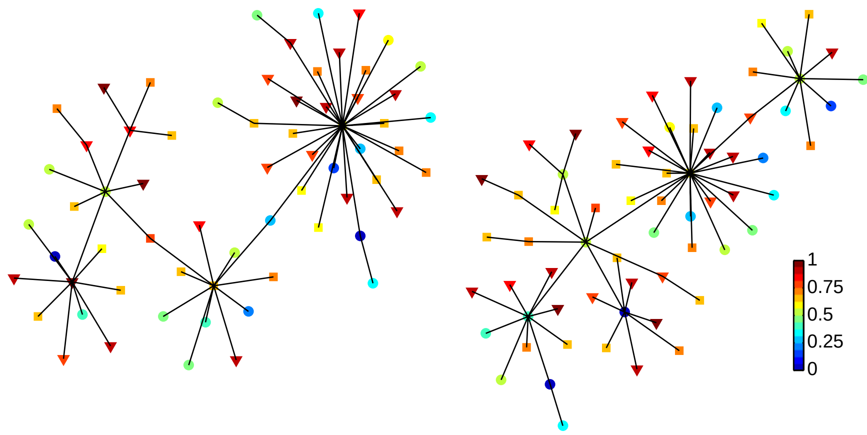

Fig. 4. The minimum spanning tree (left) and maximum spanning tree (right) of pairwise Hamming distances between strategies used at the end of simulation. $N = 65$, $M = 6$, $S = 21$, $r = 2P$, and $C = 3000P$. Performances are observed from the last $100P$ rounds.

of [0, 1] such that red stands for the best-performing agents while blue for the worst-performing agents. A more coarse-grained division of the performances of agents is indicated by three symbols: triangles for the best-performing third of the agents, circles for the worst-performing third and squares for the agents whose performance is in the middle of these two. On one hand, the minimum spanning tree shows that there is no clear clustering of strategies for similarly performing agents, because if such clusters existed, these would be seen as similarly coloured clusters of agents. On the other hand, the best-performing agents are typically connected to less well-performing agents in the maximum spanning tree. This indicates that the strategies of well-performing agents tend to be far from the strategies of less well-performing agents. Furthermore, because well-performing agents are never connected to other well-performing agents in the maximum spanning tree, their strategies cannot be very far from each other in the strategy space. In addition it is worth mentioning that distribution of the performances of agents turned out to be approximatively Gaussian.

So far, we have considered the agents in the game as individuals making independent decisions. To investigate effects of the presence of a uniformly acting group, ‘‘cartel’’, on the outcome of the game, we have developed a variation of our EMG where a certain fraction of agents make a group decision and its members always obey this decision in their actions. The group decision is done in such a way that every round the agents of the ‘‘cartel’’ make tentative minority group votes according to their strategies, and then the ‘‘cartel’’ decides the final minority group choice for all its members based on these votes. This is done taking into account the ‘‘minority wins’’—aspect of the game, such that the final group decision is the one for which the *minority* of the agents voted. All agents in the group then act according to this decision.

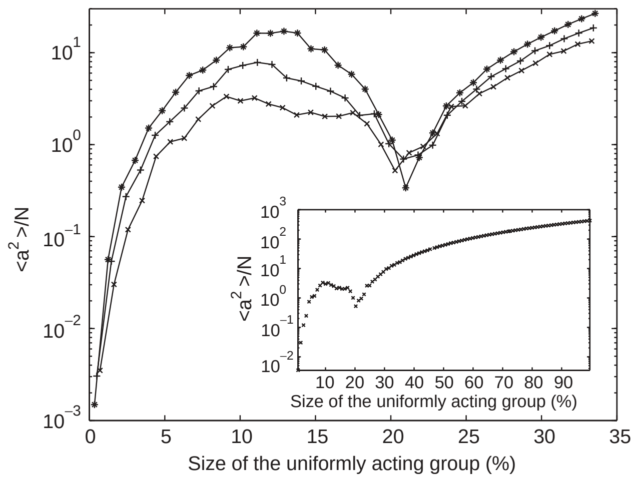

Fig. 5. Normalized fluctuations versus the group size of uniformly acting agents for $N = 891$ (asterisks), $N = 619$ (plus-signs), and $N = 429$ (crosses). We used $M = 6$, $S = 21$, $r = 2P$, and $C = 1000P$, and averaged over 50 sample runs. For group sizes over ~ 21 percent of the whole population the fluctuations grow monotonically, as the group size is enlarged, and finally reach N for 100 percent. As an example, the full range for $N = 429$ is shown in the inset. However, group sizes below ~ 21 show counter-intuitive behaviour as there are local maxima and minima on the curves.

In Fig. 5 we show the normalized fluctuations as the size of the uniformly acting group increases for $N = 891$ (asterisks), $N = 619$ (plus-signs), and $N = 429$ (crosses). We used $M = 6$, $S = 21$, $r = 2P$, $C = 1000P$, and averaged over 50 samples. For very small group sizes, i.e., 0–2 percent of the whole population, the system does not suffer from a big loss in the society utility, but as the group size grows, the normalized fluctuations increase until they reach a local maximum at group size of about ~ 10–13 percent of the agent population. After this, there is as yet unexplained decrease in the fluctuation values. As the group size further increases over ≈ 21 percent, the fluctuations begin increasing monotonically with the group size, and finally reach N in accordance with the Eq. (3). The inset in Fig. 5 shows an example of this growth in the case $N = 429$. The local peaks and the minima of the curves are rather counter-intuitive, as group decisions mean that part of the agents are forced to vote similarly. Therefore, one could expect that the normalized fluctuations always increase with the group size. One possible explanation for the minimum might be that as the group size increases, while still remaining below some certain limit, the group is better able to predict the minority choice using statistics provided by its own members.

4. Conclusions

To summarize, we have presented an evolutionary modification to the original minority game model [5], where individual agents are capable of learning from the outcomes of their past decisions and changing their strategies accordingly. This modified game leads to a stable situation where the majority and minority group sizes become almost equal for a wide range of simulation parameters. Typically, this happens for all possible histories, and each agent's choice for each history corresponds to one of the game's pure-strategy Nash equilibria. This phenomenon can be seen as an example of self-organization in a complex evolutionary system [32,33], where the evolution is driven by competition among agents. The optimized state emerges as a result of the selfish pursuits of individual agents. If the game is viewed as a toy model of market economy, the equal group sizes mean that the amounts of buyers and sellers of commodity are identical, which drives the commodity price to its equilibrium value, and society utility to its maximum. This is analogous to the "invisible hand"-effect predicted by Adam Smith, stating that selfishly acting individuals who are not actively concerned with the welfare of the whole society still eventually reward the whole society in an optimal way.

In principle, a similar equilibrium state of minimal fluctuations could be reached by dividing the agents into two almost equal-sized groups, A and B. Then, group A's minority choice would always be 1 (or -1), and group B's choice the opposite. Our studies show that this does not happen in the EMG and thus clear clusters of agents utilizing the same or similar strategies do not form. This is intuitively quite evident, as using strategies different from those of other agents increases the probability of being in the minority group. Hence, the agents' strategies tend to move away from each other in the strategy space, rather than converge. In addition, the strategies of well-performing agents tend to be far from the strategies of agents with worse performances.

We have also observed that the optimal outcome of the society is reached only under idealized conditions, where all the agents are equally capable of modifying their actions—a condition which is rarely met in real-world economic systems. If only part of the agents are allowed to evolve their strategies, the fluctuations do not reach a minimum. Furthermore, we have found that the fluctuations decrease quadratically as a function of the fraction of evolving agents. As for the homogeneity of the agent population is concerned, we have also investigated the effect of simulated uniformly acting "cartels". The expected result emerges such that if there is a cartel in the society its utility is not maximized, i.e., true price equilibrium is not reached. Surprisingly we also find that introducing a uniformly acting cartel does not lead to steadily increasing amplitude of the fluctuations as would be expected if there is a cartel in a game where the agents chose their side randomly. Instead of a steady increase in fluctuations we observe a local minimum when the uniformly acting cartel includes ~ 20 percent of the agents, for several system sizes studied. One possible reason might be that the minima arise from a combination of two factors. On one hand, the agents involved in the uniformly acting cartel are able to better estimate the winning side when the cartel size becomes large enough. On the other hand, as the cartel size grows, the fraction of agents, which are free to choose their actions and thus counter the effect of the cartel decreases, leading to increased fluctuations.

Previously, the existence of minimal fluctuations in minority games has been discussed and observed by several authors for systems of agents using probabilistic rules for strategy selection [14,16,19,34]. In contrast, in our model, the mechanisms for strategy selection are deterministic and do not include probabilities. In the case of pure strategies, in Ref. [16] the authors show that minimum fluctuations can be achieved if the agents are allowed to remove their contribution from the outcome of the game. In other words the agents are allowed to subtract their choice from the attendance, Eq. (1), used in determining the change in their personal utility function, $\Delta u_i = -\mathrm{sgn}(a - \eta\sigma_i)$. However, in our model the strategy score updating rules are the same as those in the original BMG [5], without any extensions or alterations. Nevertheless, the state of minimum fluctuations is typically reached in our game, due to the effectiveness of the genetic crossover mechanism.

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