Gas Fired Power Plants: Investment Timing, Operating Flexibility and Abandonment

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Working Paper 04-03

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Abstract

We analyze investments in gas fired power plants under stochastic electricity and gas prices. A simple but realistic two-factor model is used for price processes, enabling analysis of the value of operating flexibility, the opportunity to abandon the capital equipment, as well as finding thresholds for energy prices for which it is optimal to enter into the investment. Our case study, using real data, indicates that when the decision to build is considered, the abandonment option does not have significant value, whereas the operating flexibility and time-to-build option have significant effect on the building threshold.

Key words: Real options, spark spread, gas fired power plant, forward prices

JEL classification code: G13

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1 Introduction

In the next 20 years, fossil fuels will account for 75% of all new electric power generating capacity, and 60% of this is assumed to come in the form of gas fired power plants (see, e.g., IEA, 2003). Thus, many companies in the electricity and natural gas industries are considering investments in such plants. At the same time, the restructuring of electricity and gas markets has brought price transparency in the form of easily available spot- and forward prices. This article illustrates how information on electricity and gas futures and forward markets can be used to analyze gas fired power plant investments.

A gas fired power plant may be interesting not only from the point of view of meeting increased power demand. Consider a company owning an undeveloped gas field at a distance to major gas demand hubs. Most of the world's gas reserves are in such a category of "stranded gas". Building natural gas pipelines is very costly, and the unit costs of gas transportation decreases rapidly with the capacity of the pipeline. Thus, locating a gas fired power plant at the end of a new pipeline, near electricity demand, improves the economy of scale in transmission of natural gas.

The research question addressed here is that of an energy manager having an opportunity to build a gas fired power plant. How high should electricity prices be compared to gas prices, before I start building the plant? Does it matter whether the plant is base load, running whatever the level of electricity and gas prices, or peak load, running only when electricity price is above the fuel cost? How does the opportunity to abandon the plant influence the decision to invest? How do greenhouse gas emission costs affect profitability?

Whether a new power plant will be run as a base load plant, or ramped up and down according to current energy prices, depends more on the state of the local natural gas market than the technical design of the plant itself. New gas plants will often be of combined cycle gas turbine (CCGT) type, which can be operated both as base load and peak load plants. The operating flexibility is often constrained by the flexibility of the gas inflow. If there is little local storage and/or alternative uses of the natural gas, the plant operator will seldom find it profitable to ramp down the plant.

We use a real options approach (see, e.g., Dixit and Pindyck, 1994). The gas fired power plant's operating cash flows depend on the spark spread, defined as the difference between the price of electricity and the cost of gas used for the generation of electricity. Spark spread based valuation of power plants has been studied in Deng et al. (2001). Our model makes several extensions to their model. First, by using a two-factor model, similar to that of Schwartz and Smith (2000), for the spark spread process we can incorporate the typical characteristics of non-storable commodity prices, i.e. short-term mean-reversion and long-term uncertainty. Second, our model takes into account the option to postpone investment decisions. Such postponement option analysis originates from the work of McDonald and Siegel (1986).

The long-maturity forwards on electricity and gas, e.g. three-year forwards, give the exact and certain market value of a constant electricity and gas flow. A base load plant operates with a constant electricity and gas flow, and thus a base load plant can be valued with long-term spark spread forwards. On the other hand, a peak load plant can react to short-term variations in the spark spread by ramping up and down, leading to a non-constant gas and electricity flow. Thus, the short-term dynamics of the spark spread are needed for the valuation of a peak load plant. The short-term dynamics can be estimated from short-maturity forwards.

Long-term investments, such as gas fired power plants, are never commenced due to short-term non-persistent spikes in the spark spread. Rather, investment decisions are based on long-term price levels, here called equilibrium prices. We compare the current equilibrium price estimate to a computed investment threshold, reflecting that at this threshold level of equilibrium price, the value of waiting longer is equal to the net present value received if investment is commenced. Thus, when the equilibrium price increases to the investment threshold, the implementation of the power plant project should be started. In this article we analyze the investment decision purely based on the information on electricity and gas forward and future markets. This means that we will not make any assumptions of the peak load plant's ramping policy. Omitting the ramping policy means that we can not give exact value of the plant. Instead we give upper and lower bounds for the plant value which are then used to calculate upper and lower bounds for the investment thresholds. Generally, this means that if the equilibrium price is above the upper bound of the investment threshold the project should be commenced regardless of the possible ramping policies. Correspondingly, if the equilibrium price is below the lower bound of the investment threshold whatever is the ramping policy investing will not be optimal. In case that the equilibrium price falls between the upper and lower bound the profitability is dependent on the available ramping policy and hence further analyzes of the ramping policy and its profitability is needed.

An alternative to using forward prices in the estimation of the price dynamics parameters is to focus on spot prices. Deng (2003) studies investment timing and gas plant valuation under electricity and gas price uncertainty by using separate stochastic processes for electricity and gas spot prices. His model is calibrated to historical spot data and it contains jumps and spikes in the spot price process. We do not include jumps or spikes, although these features may very well be present in the spot price history. The reason is that forward prices reflect all important and currently available information about future supply, demand and risk. Forward prices show directly the current market value of future spark spread, and are the risk-adjusted expected future spot price level. Furthermore, ignoring forward price data and only looking at spot price data easily leads to value estimates that are inconsistent with the no-arbitrage principle, i.e. the estimated real asset value can differ from the value dictated by the forward curve.

Our simplifications compared to Deng (2003), omission of price spikes and modeling the spark spread with one price process, mean that our model cannot capture operational efficiency that varies with output or over time. However, that issue is relevant only for optimization of short-term operation, and do not play a significant role when taking a strategic view as we do here, e.g., Deng and Oren (2003) find that for efficient plants, the error is small.

We illustrate the use of our model by applying it into the energy markets in Scandinavia. The electricity markets there have been restructured since the late 1980s, with North Sea gas markets still in transition. Naturally, our model can be applied to other energy markets as well. Our case study indicates that the difference of a peak and base load plant value is considerable, i.e. the value of being able to ramp up and down is significant. Our application also indicates that the addition of an abandonment option does not dramatically change the investment threshold. Thus, when investments in gas fired power plants are considered, a good overall view of the investment problem can be made by disregarding the abandonment option, whereas the operating flexibility and time-to-build option have significant effect on the investment threshold.

The model generalizes beyond the case of gas fired power plants. Any investment involving a relatively simple transformation of one commodity to another could be analyzed using this framework. The spread between output price and input costs is then an important source of uncertainty. Examples include transformation of natural gas into liquefied natural gas, a methanol factory, and a biodiesel factory.

The paper is organized as follows. We present the model of price uncertainty in Section 2, where we also argue why it is important to incorporate information in forward prices to real options analyses. In Section 3 upper and lower bounds for the plant value are calculated, whereas in Section 4 the investment problem is studied. In Section 5 we illustrate the model with an example. In Section 6 we discuss the results of the example. Finally, Section 7 concludes the study.

2 The energy price process

Seasonality in the supply and demand of electricity and natural gas, combined with limited storage opportunities, causes cycles and peaks in the electricity and gas prices. Spark spread measures the contribution margin of a gas fired power plant, thus it is defined as the difference between price of electricity S_e and the cost of gas used for the generation of electricity

$$S = S_e - K_H S_g , \tag{1}$$

where S_g is the price of gas and heat rate K_H is the amount of gas required to generate one MWh of electricity. Heat rate measures the efficiency of the plant: the lower the heat rate, the more efficient the facility. The efficiency of a gas fired power plant varies slightly over time and with the output level. Still, the use of a constant heat rate is considered plausible for long-term analyses (see, e.g., Deng et al., 2001). Note that the value of the spark spread can be negative as well as positive.

Electricity and gas are often used to same purposes, such as cooling and heating, and thus the seasonality in electricity and gas prices have similar characteristics. Due to the similar characteristics the seasonality in electricity and gas forward prices decays from the spark spread and thus the spark spread process does not have seasonality. An example of spark spread process, displayed in Figure 1 with black line, supports the hypothesis of no seasonality in the data. Descriptive statistics about spark spread data can be found, for example, in Näsäkkälä and Fleten (2004). The following assumption describes our dynamics for the spark spread process. Schwartz and Smith (2000) use similar price dynamics to evaluate oil-linked assets.

ASSUMPTION 1. *The spark spread is a sum of short-term deviations and equilibrium price*

$$S(t) = \chi(t) + \xi(t), \tag{2}$$

where the short-term deviations $\chi(t)$ *are assumed to revert toward zero following an Ornstein-Uhlenbeck process*

$$d\chi(t) = -\kappa\chi(t)dt + \sigma_\chi dB_\chi(t). \tag{3}$$

The equilibrium price $\xi(t)$ *is assumed to follow an arithmetic Brownian motion process*

$$d\xi(t) = \mu_\xi dt + \sigma_\xi dB_\xi(t), \tag{4}$$

where κ, σ_χ, μ_ξ, *and* σ_ξ *are constants.* $B_\kappa(\cdot)$ *and* $B_\xi(\cdot)$ *are standard Brownian motions, with correlation* $\rho dt = dB_\chi dB_\xi$ *and information* F_t.

Increase in the spark spread attracts high cost producers to the market putting downward pressure on prices. Conversely, when prices decrease some high cost producers will withdraw capacity temporarily, putting upward pressure on prices. As these entries and exits are not instantaneous, prices may be temporarily high or low, but will revert toward the equilibrium price ξ. The mean-reversion parameter κ describes the rate at which the short-term deviations χ are expected to decay. The uncertainty in the equilibrium price is caused by the uncertainty in fundamental changes that are expected to persist. For example, advances in gas exploration and production technology, changes in the discovery of natural gas, improved gas fired power plant technology, and political and regulatory effects can cause changes in the equilibrium price. Other studies where the two factors are interpreted as short- and long-term factors include, for example, Schwartz and Smith (2000), Ross (1997), and Pilipović (1998). Note that the decreasing forward volatility structure, typical for commodities, can be seen as a consequence of the mean-reversion in the spot prices (see, e.g., Schwartz, 1997).

Traditionally, log-normal distributions are used to model non-negative prices. Instead of modeling electricity and gas prices independently we model their difference. Our spark spread dynamics do not follow from the difference of two log-normally distributed price processes, however it has similar characteristics, namely it can have negative and positive values. The following corollary expresses the distribution of the future spark spread values.

COROLLARY 1. *When spark spread has dynamics given in* (2)-(4), *prices are normally distributed, and the expected value and variance are given by*

$$E\left[S(T) \mid \mathrm{F}_t\right] = \mathrm{e}^{-\kappa(T-t)}\,\chi(t) + \xi(t) + \mu_\xi(T-t) \tag{5}$$

$$Var\left(S(T)\right) = \frac{\sigma_\chi^2}{2\kappa}\left(1-\mathrm{e}^{-2\kappa(T-t)}\right) + \sigma_\xi^2(T-t) + 2\left(1-\mathrm{e}^{-\kappa(T-t)}\right)\frac{\rho\sigma_\chi\sigma_\xi}{\kappa}. \tag{6}$$

PROOF: See, e.g., Schwartz and Smith (2000).

Corollary 1 states that the spark spread is a sum of two normally distributed variables: equilibrium price and short-term variations. The expected value of short-term variations converges to zero as the maturity $T-t$ increases and thus the expected value of the spark spread converges to the expected value of the equilibrium price. The mean-reversion parameter κ describes the rate of this convergence. The maturity in which short-term deviations are expected to halve is given by

$$T_{1/2} = -\frac{\ln(0.5)}{\kappa}. \tag{7}$$

The spark spread variance caused by the uncertainty in the equilibrium price increases linearly as a function of maturity, whereas the spark spread variance due to the short-term variations converges towards $\sigma_\chi^2/2\kappa$.

Neither the short-term deviations χ nor the equilibrium price ξ are directly observable, but estimates for them can be obtained from forward prices. A forward price is the risk-adjusted expected future spark spread value and thus forward prices can be used to infer the risk adjusted dynamics of short-term deviations and equilibrium price. The expected short-term variations decrease to zero when the maturity increases and thus the long-maturity forwards give information about the equilibrium price. When the maturity is short, the short-term variations have not yet converged to zero. Thus, the difference of long- and short-maturity forwards gives information about the short-term dynamics. Based on this simple idea Schwartz and Smith (2000) propose a Kalman filter based estimation for the parameters of two-factor spot price process. In Näsäkkälä and Fleten (2004) the method is used to estimate spark spread data.

If there are no forward prices available the short-term deviations and equilibrium price can not be estimated from market prices. As both electricity and natural gas are difficult to store, the usual storability arguments determining the relationship between the spot and forward prices do not hold for spark spread. Thus, when there are no forward prices available, there is neither sound theory for the estimation of the spark spread parameters and for the risk adjustment selection. In this situation often ad hoc parameters together with risk-adjusted discount rate are used (see, e.g., Dixit and Pindyck, 1994).

3 Gas plant valuation

In this section we calculate upper and lower bounds for the value of the gas fired power plant. The following assumption gives the operational characteristics of the plant.

Assumption 2. The plant can be ramped up and down according to changes in the spark spread. The costs associated with starting up and shutting down the plant can be amortized into fixed costs.

In a gas fired power plant, the operation and maintenance costs do not vary much over time, thus it is realistic to assume that the fixed costs are constant. The ramping policy of a particular plant depends on local conditions associated with plant design and gas inflow arrangement. Instead of giving an exact definition for the ramping policy, we use upper and lower bounds for the gains associated with ramping. The lower bound V_L can be calculated by assuming that the plant cannot exploit unexpected changes in the spark spread, i.e. by assuming that the plant produces electricity at the rated capacity independent of the spark spread. Such a plant is often called a base load plant. The following lemma gives the value of a base load plant.

LEMMA 1. At time t, the lower bound of the plant value $V_L(\chi,\xi) \le V(\chi,\xi)$ *is given by the value of a base load plant*

$$V_L(\chi,\xi) = \bar{C}\left(\frac{\chi(t)}{\kappa+r} + \frac{\xi(t)-E}{r} + \frac{\mu_\xi}{r^2} - \mathrm{e}^{-r(\bar{T}-t)}\left(\frac{\mathrm{e}^{-\kappa(\bar{T}-t)}\chi(t)}{\kappa+r} + \frac{\xi(t)-E}{r} + \frac{\mu_\xi\left(r(\bar{T}-t)+1\right)}{r^2}\right)\right) - \frac{G}{r}\left(1-\mathrm{e}^{-r(\bar{T}-t)}\right), \quad (8)$$

where $\bar{T}$ *is the lifetime of the plant,* $\bar{C}$ *is the capacity of the plant, G are the fixed costs of running the plant, and E are the emission costs of producing given amount of electricity from natural gas.*

PROOF: The value of a base load plant is the present value of expected operating cash flows

$$\begin{aligned} V_L(\chi,\xi) &= \int_t^{\bar{T}} e^{-r(s-t)}\left(\bar{C}\left(E\left[S(s)\,|\,F_t\right]-E\right)-G\right)ds = \\ &= \int_t^{\bar{T}} e^{-r(s-t)}\left(\bar{C}\left(e^{-\kappa(s-t)}\chi(t)+\xi(t)-E+\mu_\xi(s-t)\right)-G\right)ds \end{aligned} \quad (9)$$

Integration gives (8). Q.E.D.

The lower bound is just the discounted sum of expected spark spread values less emission and fixed costs. Thus, the lower bound is not affected by the short-term and equilibrium volatilities σ_χ and σ_ξ.

An owner of a gas fired power plant can react to adverse changes in the spark spread by temporarily shutting down the plant. Such a plant is often called a peak load plant. The value of a peak load plant is the discounted sum of expected spark spread values less emission and fixed costs plus the option value of being able to ramp up and down. The value of the up and down ramping is dependent on the response times of the plant, and is maximized when ramping up and down can be done without delay. In other words, the upper bound V_U for the plant value can be calculated by assuming that the up and down ramping can be done without delay, i.e. by assuming that the plant produces electricity only when the spark spread exceeds emission costs.

LEMMA 2. At time t, the upper bound of the plant value $V(\chi,\xi) \le V_U(\chi,\xi)$ *is given by the value of an ideal peak load plant*

$$V_U(\chi,\xi) = \bar{C}\int_t^{\bar{T}} e^{-r(s-t)}\left(\frac{\sqrt{Var(S(s))}}{\sqrt{2\pi}}e^{\left(-\frac{(E-E[S(s)|F_t])^2}{2Var(S(s))}\right)} + \left(E[S(s)\mid F_t]-E\right)\left(1-\Phi\left(\frac{E-E[S(s)\mid F_t]}{\sqrt{Var(S(s))}}\right)\right)\right)ds - \frac{G}{r}\left(1-e^{-r(\bar{T}-t)}\right) \quad (10)$$

where $\Phi(\cdot)$ is the normal cumulative distribution function, and G are the fixed costs of running the plant. The expected value $E[S(s)\mid F_t]$ and variance $Var(S(s))$ for the spark spread are given in Corollary 1.

PROOF: See Appendix A.

The more the spark spread varies more valuable the option to ramp up and down is and thus the value of the peak load plant increases as a function of the variance of the spark spread.

To summarize: As we are not able to precisely characterize the response times of the plant, we do not calculate the exact valuation formula for the gas fired power plant, but we give bounds for the plant value. The lower bound is given by the base load plant (Lemma 1) and the upper bound is given by the ideal peak load plant (Lemma 2).

4 Investment analysis

In this section we calculate bounds for the investment thresholds when the gas plant value has the bounds given by Lemma 1 and Lemma 2. The following assumption characterizes the variables affecting the investment decisions.

ASSUMPTION 3. The investment decisions are made as a function of equilibrium price. In the investment decisions the lifetime of the plant is assumed to be infinite.

Assumption 3 states that when the gas plant investments, i.e. building and abandonment, are considered the decisions are made as a function of the equilibrium price ξ, i.e. the current short-term realization is omitted in investment decisions. The short-term dynamics, i.e. short-term volatility σ_χ and mean-reversion κ, still affect the value of the plant, and thus they also affect the investment decision. Thus, the short-term dynamics are important in the investment decision, even though the particular realization is assumed to be zero when investment decisions are made. The omission of the short-term realization is motivated by the fact that gas fired power plants are long-term investments, and a gas plant investment is never commenced due to a non-persistent spike in the price process. The assumption that investment decisions are made as a function of equilibrium price is a realistic approximation of the investment decision process if the expected lifetime of the short-term deviations is considerably smaller than the expected lifetime of the plant. In Section 5 we use mean-reversion $\kappa = 2.6$, which gives, with (7), that the short-term variations are expected to halve in about three months. Usually, the lifetime of a gas fired power plant is assumed to be around 30 years, and building a gas fired power plant takes usually about two years. Thus, the approximation obtained by omitting the short-term realization in the investment decision is realistic. Note that in reality there can be exceptional peak periods when the mean-reversion towards equilibrium price is lower than usual. During these peaks investment lags can make investing difficult, which makes speculative early investment options appealing, for more about speculative investment options see, e.g. Bar-Ilan and Strange (1996). The infinite lifetime assumption is motivated by the fact that the plant's lifetime is often increased by upgrading and reconstructions, and by downward shifts in the maintenance cost curve (see, e.g., Ellerman, 1998). The upper and lower bounds for the plant value as a function of lifetime will be illustrated in Section 5.

Building the plant becomes optimal when the equilibrium price rises to a building threshold ξ_H. When waiting is optimal, i.e., when $\xi < \xi_H$, the investor has an option to postpone the building decision. The value of such a time-to-build option is given by the following lemma.

LEMMA 3. The value of an option to build a gas fired power plant is

$$F_0(\xi) = A_1 e^{\beta_1 \xi} - \frac{W}{r}, \qquad \text{when} \quad \xi \le \xi_H, \tag{11}$$

where A_1 is a positive parameter and W are constant payments that the firm faces to keep the build option alive. The parameter β_1 is given by

$$\beta_1 = \frac{-\mu_\xi + \sqrt{\mu_\xi^{\,2} + 2\sigma_\xi^{\,2} r}}{\sigma_\xi^{\,2}} > 0. \tag{12}$$

PROOF: See Appendix B.

The time-to-build option value increases exponentially as a function of the equilibrium price. The parameter A_1 depends on the value of the plant and on the investment cost. As we are not able to exactly state the gas plant value, we can not state the exact building threshold, but the following proposition gives a method to calculate upper and lower bounds $\xi_{HL} \le \xi_H \le \xi_{HU}$ for the building threshold.

PROPOSITION 1. The lower bound of the building threshold $\xi_{HL} \le \xi_H$ is given by

$$F_0(\xi_{HL}) = V_U(0, \xi_{HL}) - I \tag{13}$$

$$\frac{\partial F_0(\xi_{HL})}{\partial \xi} = \frac{\partial V_U(0, \xi_{HL})}{\partial \xi}, \tag{14}$$

whereas the upper bound $\xi_H \le \xi_{HU}$ is given by

$$F_0(\xi_{HU}) = V_L(0, \xi_{HU}) - I \tag{15}$$

$$\frac{\partial F_0(\xi_{HU})}{\partial \xi} = \frac{\partial V_L(0, \xi_{HU})}{\partial \xi}. \tag{16}$$

PROOF: This is a special case of Proposition 2 and the proof will be omitted.

The equations in Proposition 1 cannot be solved analytically but a numerical solution can be attained. The more valuable the plant becomes, the more eager the firms are to invest, thus the lower bound for the building threshold is given by the upper bound of the plant's value and vice versa.

Next we will consider how the investment decision changes if there is an opportunity to abandon the gas plant and realize the plant's salvage value. In this case, when a decision to build is made the investor receives both the gas plant and an option to abandon the plant. As the lifetime of the plant is assumed to be infinite, there is a constant threshold value ξ_L for the abandonment, i.e. abandoning is not optimal when $\xi_L < \xi$. The following Lemma states the value of such an abandonment option.

LEMMA 4. The value of an abandonment option is

$$F_1(\xi) = D_2 e^{\beta_2 \xi} \qquad when \quad \xi_L \le \xi \tag{17}$$

where D_2 is a positive parameter. The parameter β_2 is given by

$$\beta_2 = \frac{-\mu_\xi - \sqrt{\mu_\xi^{\,2} + 2\sigma_\xi^{\,2} r}}{\sigma_\xi^{\,2}} < 0. \tag{18}$$

PROOF: The proof is similar to that of the build option (Appendix B), but now the option becomes less valuable as the spark spread increases. Q.E.D.

The abandonment option value decreases exponentially as a function of the equilibrium price. The parameter D_2 depends on the plant's salvage value. Again we are not able to state the exact building and abandonment thresholds, but the following Proposition gives upper and lower bounds for the thresholds, i.e. $\xi_{HL} \le \xi_H \le \xi_{HU}$ and $\xi_{LL} \le \xi_L \le \xi_{LU}$.

PROPOSITION 2. The lower bounds for the building and abandonment thresholds $\xi_{HL} \le \xi$ and $\xi_{LL} \le \xi$ are given by

$$F_0(\xi_{HL}) = V_U(0,\xi_{HL}) + F_1(\xi_{HL}) - I \tag{19}$$

$$F_1(\xi_{LL}) + V_U(0,\xi_{LL}) = D \tag{20}$$

$$\frac{\partial F_0(\xi_{HL})}{\partial \xi} = \frac{\partial V_U(0,\xi_{HL})}{\partial \xi} + \frac{\partial F_1(\xi_{HL})}{\partial \xi} \tag{21}$$

$$\frac{\partial F_1(\xi_{LL})}{\partial \xi} + \frac{\partial V_U(0,\xi_{LL})}{\partial \xi} = 0 , \tag{22}$$

whereas the upper bounds $\xi \le \xi_{HU}$ and $\xi \le \xi_{LU}$ are given by

$$F_0(\xi_{HU}) = V_L(0,\xi_{HU}) + F_1(\xi_{HU}) - I , \tag{23}$$

$$F_1(\xi_{LU}) + V_L(0,\xi_{LU}) = D , \tag{24}$$

$$\frac{\partial F_0(\xi_{HU})}{\partial \xi} = \frac{\partial V_L(0,\xi_{HU})}{\partial \xi} + \frac{\partial F_1(\xi_{HU})}{\partial \xi} \tag{25}$$

$$\frac{\partial F_1(\xi_{LU})}{\partial \xi} + \frac{\partial V_L(0,\xi_{LU})}{\partial \xi} = 0 . \tag{26}$$

PROOF: See Appendix C.

The equations in Proposition 2 cannot either be solved analytically but a numerical solution can be attained. The less valuable the plant is, the more eager the firms are to abandon the plant. Thus the upper bound of the abandonment threshold is given by the lower bound of the plant value, and vice versa.

To summarize: in this section we have derived a method to calculate lower and upper bounds for the building and abandonment thresholds. If the abandonment option is ignored the building threshold is given by Proposition 1. When both building and abandonment are studied the thresholds are given by Proposition 2.

5 Application

It is estimated that over the period 2001-2030 about 2000 *GW* of new natural gas fired power plant capacity will be built (see, e.g., IEA, 2003). Our method can be used to analyze all these investments. In this example we concentrate on the possibility to build a natural gas fired power plant in Norway. The main reason to concentrate on this particular case is the availability of good spark spread and investment cost data. Norwegian energy and environmental authorities have given four licenses to build a gas fired power plant and we take the view of an investor having one of these licenses.

The costs of building and running a natural gas fired power plant in Norway are estimated by Undrum et al. (2000). With an exchange rate of 7 *NOK/USD*, a combined cycle gas turbine (CCGT) plant costs approximately 1620 *MNOK*, and the maintenance costs G are approximately 50 *MNOK/year*. We estimate that the costs of holding the license W are 5% of the fixed costs of a running a plant. In Undrum et al. (2000) approximately 35% of the investment costs are used for capital equipment. We assume that if the plant is abandoned all the capital equipment can be realized on second hand market, i.e. the salvage value of the plant D is 567 *MNOK*. The estimated parameters are for a gas plant whose maximum capacity is 415 *MW*. We assume that the capacity factor of the plant is 90%, thus we use a production capacity of 3.27 *TWh/year*. Table 1 contains a summary of the gas plant parameters.

Table 1: The gas plant parameters

Parameter	W	$\bar{C}$	G	I	D
Unit	*MNOK/year*	*TWh/year*	*MNOK/year*	*MNOK*	*MNOK*
Value	2.5	3.27	50	1620	567

Näsäkkälä and Fleten (2004) use electricity data from Nord Pool (The Nordic Power Exchange) and gas data from International Petroleum Exchange (IPE) to estimate spark spread dynamics for a combined cycle gas turbine (CCGT) plant whose efficiency is 58.1%, i.e. they use heat rate $K_H = 1.72$. For short-maturity forwards, giving information about the short-term dynamics, they use monthly forward contracts with 1-month maturity. For long-maturity contracts, giving information about the equilibrium price dynamics, they use three year contracts with 1-year maturity.

Their estimation procedure is following: First, the mean-reversion κ, correlation ρ, and volatility σ_χ, σ_ξ parameters are estimated from the price history of short- and long-maturity forwards, more precisely from price quotes between 2nd of January 2001 and the 30th of January 2004. Second, the long-term drift μ_ξ is estimated from the long-term forwards on the 30th of January 2004. Third, current equilibrium price ξ_0 and short-term deviation χ_0 are chosen so that the expected value matches the whole forward curve at the 30th of January 2004. In Figure 1 the estimated spot price process $\chi_t + \xi_t$ over the price history, i.e. 2nd of January 2001 through the 30th of January. 2004, is indicated with a black solid line. The grey line represents the estimated equilibrium price ξ_t. The equilibrium price varies less than the spot price, whose variation is also affected by the short-term fluctuations. After the 30th of January 2004 the risk-adjusted expected future spark spread and its 68% confidence level are indicated by black solid and dashed lines. The expected value and confidence levels are calculated with Corollary 1. The expected value decreases rapidly during the first few months as the expected value of current short-term deviation converges to zero. The forward curve at the 30th of January 2004 is indicated by grey vertical lines. The spark spread parameters are summarized in Table 2.

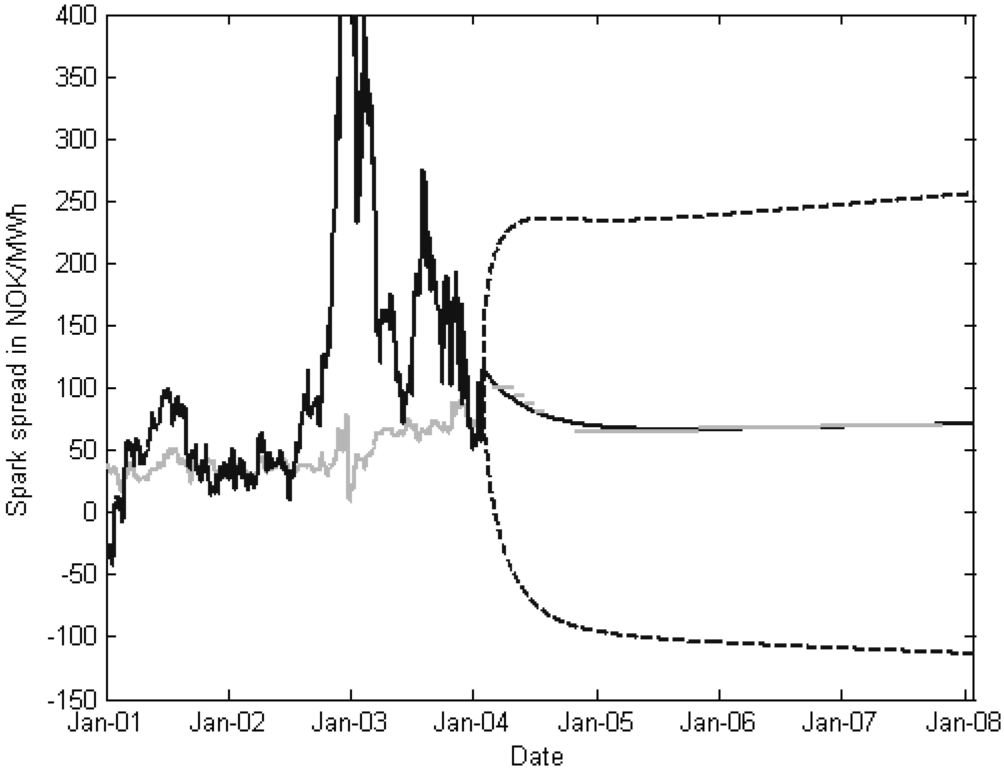

Figure 1: Equilibrium and spot price together with expected value and forward curve

Table 2: Spark spread parameter estimates

Parameter	r	κ	μ_ξ	ρ	σ_χ	σ_ξ	χ_0	ξ_0
Unit			*NOK/MWh*		*NOK/MWh*	*NOK/MWh*	*NOK/MWh*	*NOK/MWh*
Value	0.06	2.6	2.18	-0.21	382.2	47.8	52.9	62.3

When emission costs E are assumed to be zero, and the plant's lifetime $\bar{T}$ is assumed infinite, the lower bound for the plant value V_L, given by Lemma 1, is 4542 *MNOK*. Correspondingly, the upper bound for the plant value V_U, given by Lemma 2, is 7539 *MNOK*. The plant value as a function of the lifetime $\bar{T}$ is illustrated in Figure 2. Figure 2 indicates how the plant value gradually stabilizes to a given level as the lifetime increases.

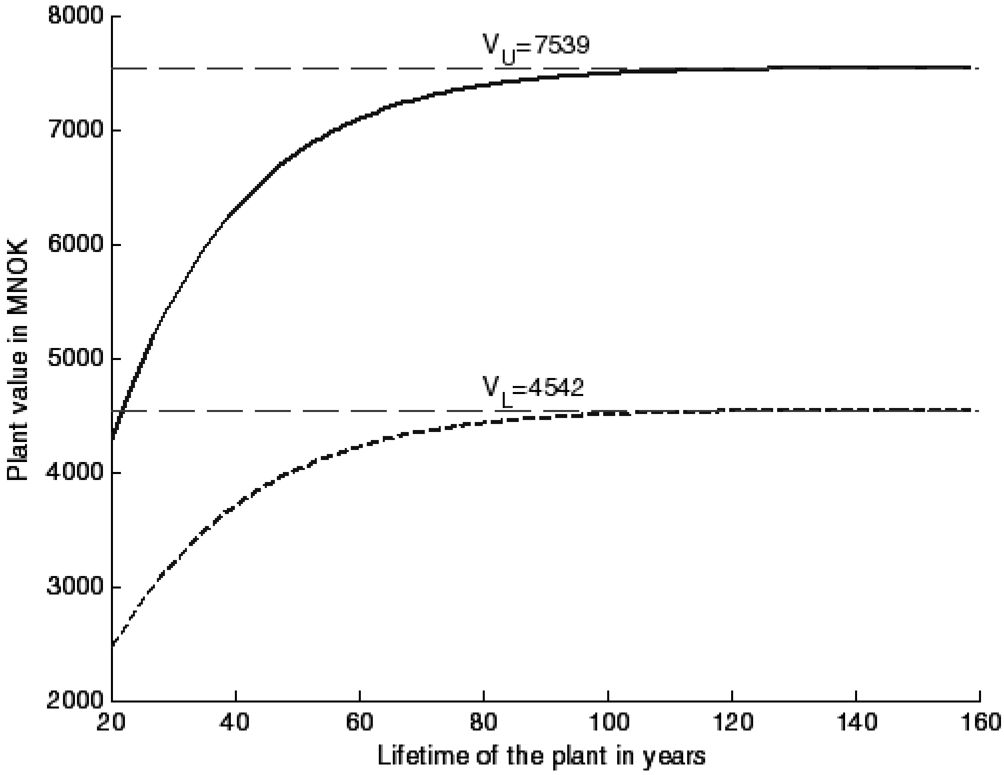

Figure 2: Plant value as a function of the plant's lifetime

Proposition 1 gives that the building threshold ξ_H when abandonment is not considered is somewhere between [46.3; 165.3] *NOK/MWh*. When also the abandonment option is taken into account the building threshold ξ_H^A is on an interval [43.8; 134.3] *NOK/MWh*, and the abandonment threshold ξ_L^A is between [-362.8; -131.6] *NOK/MWh*. In the latter case the thresholds are given by Proposition 2. If there is an option to abandon, some of the investment costs can be returned when the investment turns to be unprofitable, and thus the addition of abandonment option makes earlier investment more favorable. The abandonment option also narrows the gap between upper and lower bound of the building threshold. The abandonment makes the flexibility in the plant less valuable as the possibility to abandon partly compensates the value of being able to temporarily shut down. The bounds of the plant value and investment thresholds are summarized in Table 3. In both cases the current equilibrium price ξ_0, given in Table 2, is on the building interval, thus the building decision is dependent on the ramping policy.

Table 3: Plant value and investment thresholds

Variable	$V(0,\xi_0)$	ξ_H	ξ_H^A	ξ_L^A
Unit	*MNOK*	*NOK/MWh*	*NOK/MWh*	*NOK/MWh*
Value	[4540; 7537]	[46.3; 165.3]	[43.8; 134.3]	[-362.8; -131.6]

For comparison we calculate the thresholds with a net present value method, i.e. we assume that the plant is built when the expected value of the plant is equal to investment costs and the abandonment is done when the plant value is equal to salvage value. In this case only the options to postpone the investment decisions are ignored, and thus the uncertainty in the spark spread process still affects the investment decisions by changing the value of operating flexibility. This method gives that the investment threshold ξ_H^{NPV} is on the interval [-178.2; 8.7] *NOK/MWh* and the abandonment threshold ξ_L^{NPV} is on the interval [-271.8; -10.6] *NOK/MWh.* The options to postpone have positive value and thus the building threshold increases and the abandonment threshold decreases when the options to postpone are included. The net present value method gives that it is optimal to invest with the current equilibrium price, whatever the ramping policy is.

Figure 3 illustrates the option values F_0 and F_1 and the plant value V as a function of equilibrium price ξ. In the upper part of Figure 3 the black lines are the bounds of the plant value, and the grey lines are the bounds of the investment option value. The bounds of the investment thresholds are indicated by vertical lines. The solid lines are the lower bounds and the dashed lines are the upper bounds. The value of the build option increases exponentially as a function of the equilibrium price until it is optimal to build the plant. The owner of a gas plant has also an abandonment option whose value decreases exponentially as a function of equilibrium price. In the lower part of Figure 3 the abandonment option values are indicated together with the plant value. The gap between the bounds of the build option is small compared to the gap between bounds of the abandonment option. The peak load plant can react to decreasing prices by ramping down the plant. Therefore, the difference between the bounds of the plant value increases as the equilibrium price decreases. As the bounds for the option values are determined by the bounds of the plant value, the upper and lower bound of the abandonment option diverge when equilibrium price decreases.

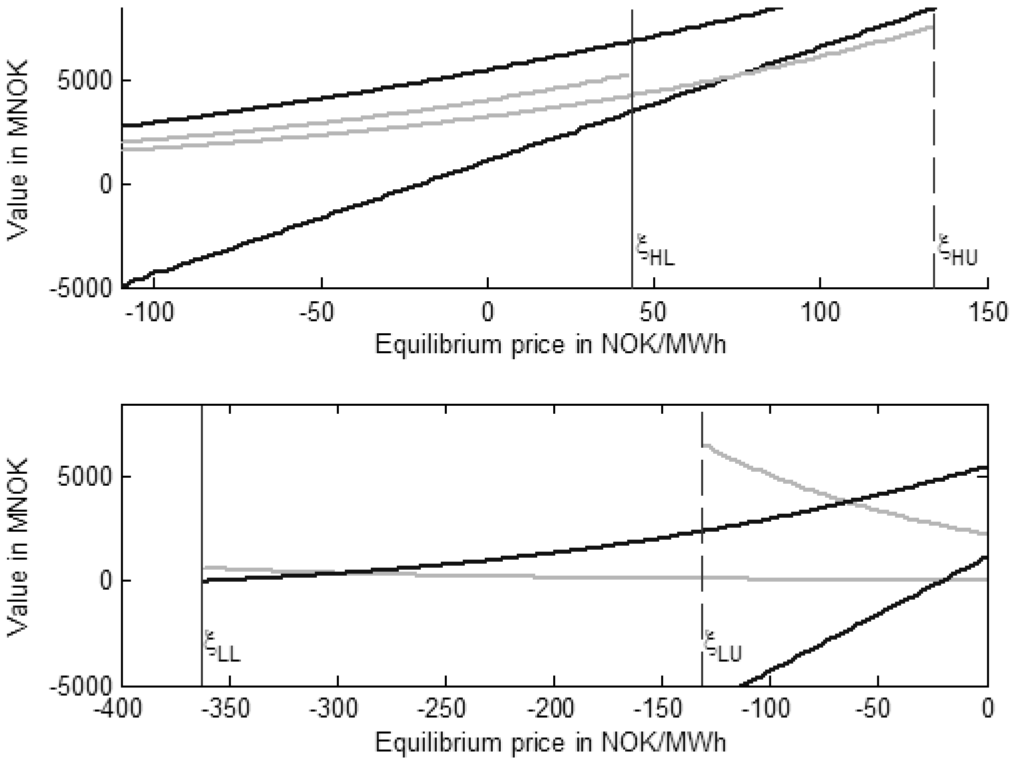

Figure 3: Plant and option values

Next we study how the thresholds change as a function of some key parameters. In Figure 4 the thresholds as a function of equilibrium volatility σ_ξ are illustrated. The grey lines are the bounds of the building threshold and the black lines are the bounds of the abandonment threshold. An increase in the equilibrium volatility increases the building threshold, but at the same time the abandonment threshold decreases, i.e. uncertainty makes waiting more favorable. In Figure 4 the gap between the bounds of the abandonment threshold increases as function of uncertainty. An increase in the equilibrium volatility does not change the value of a base load plant, but it increases the value of a peak load plant. When the equilibrium price is small and the market becomes more volatile the more valuable the peak load plant is compared to the base load plant and broader the gap between bounds of the abandonment thresholds is. On the other hand, when the equilibrium price is high the difference of base and peak load plant values is not sensitive to changes in equilibrium volatility and thus the gap between upper and lower bound of the building threshold does not increase much as a function of equilibrium volatility.

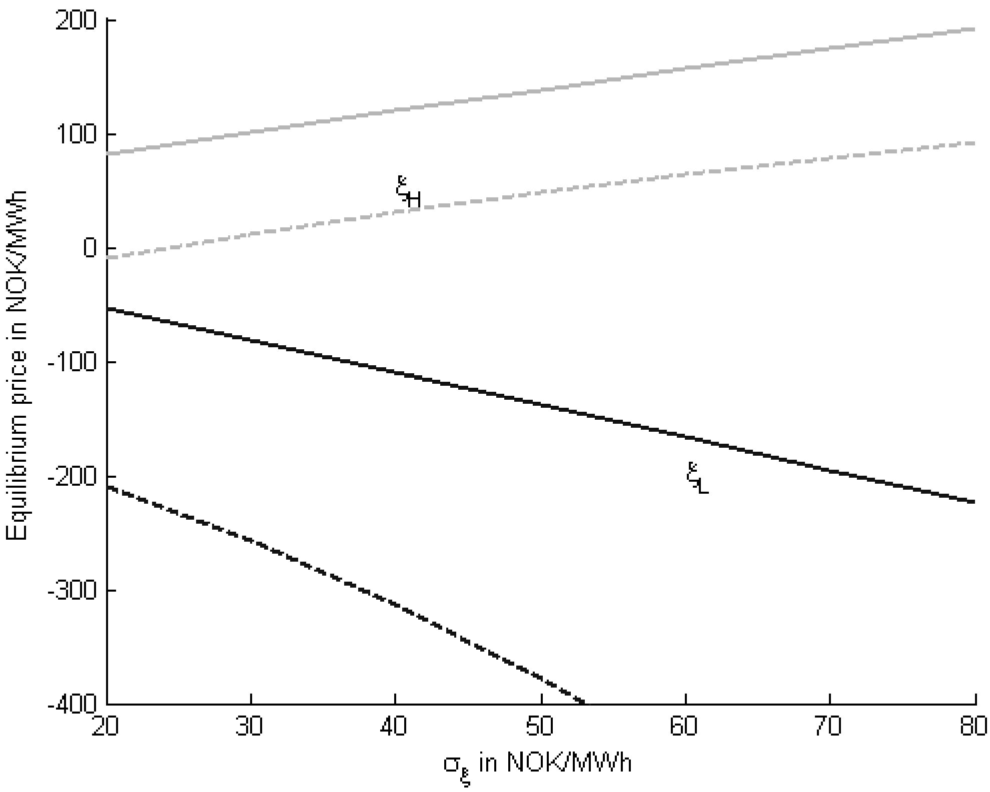

Figure 4: Investment thresholds as a function of equilibrium volatility

Figure 5 illustrates the thresholds as a function of emission costs *E*. In Figure 5 the unit of emission cost is *NOK/MWh*, whereas it usually is quoted in *USD/ton*. The CO_2 production of a gas fired power plant is 363 *kg/MWh*. With an exchange rate of 7 *NOK/USD*, an emission cost of 10 *NOK/MWh* corresponds 3.94 *USD/ton*. In Figure 5 the thresholds increase linearly, with slope one, as a function of emission costs. Thus, if the emission costs are increased by one *NOK/MWh,* both thresholds are also increased by one *NOK/MWh*. This is a consequence of a normally distributed equilibrium price. Change in emission costs can be seen as a change in initial value of the equilibrium price. Even though we have used constant emission costs, there is uncertainty in future levels of emission costs. An easy way to model the uncertainty in the emission costs is to increase the equilibrium uncertainty. Thus, not just increase in the expected value of emission costs, but also uncertainty in emission costs postpones investment decisions, i.e. increases the building threshold and decreases the abandonment threshold.

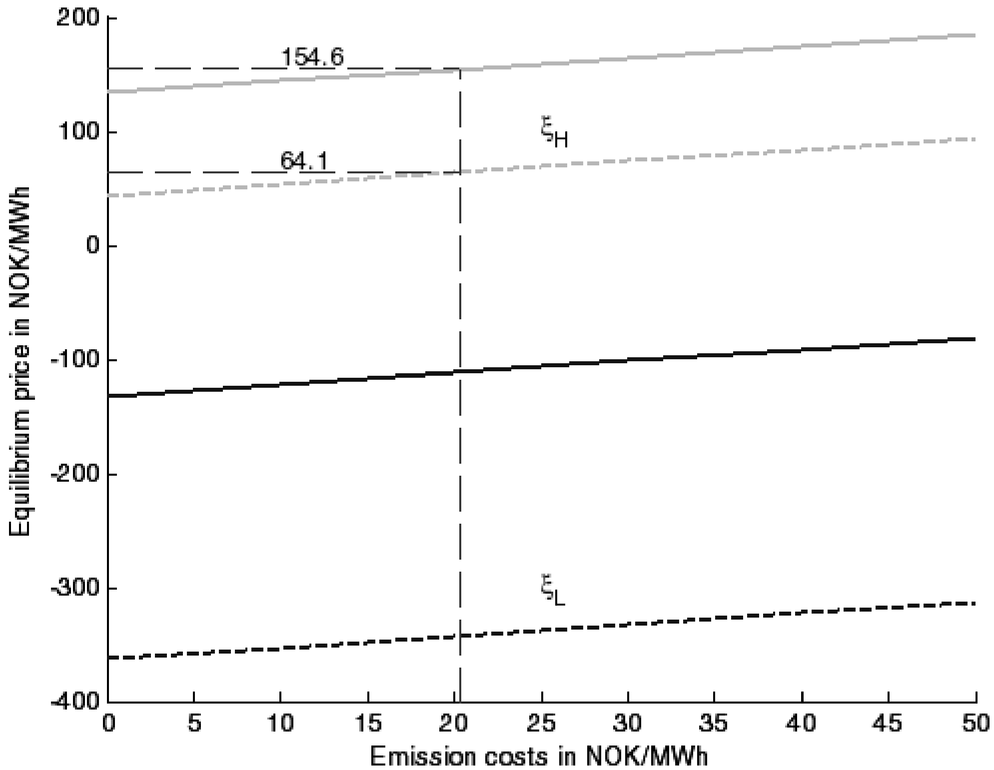

Figure 5: Investment thresholds as a function of emission costs

Undrum et al. (2000) evaluate different alternatives to capture CO_2 from gas turbine power cycles. They estimate that costs to install equipment to capture CO_2 from exhaust gas using absorption by amine solutions are 2140 *MNOK*. Thus, given the investment costs in Table 1 the costs of a gas power plant with CO_2 capture technology are 3760 *MNOK*. Figure 6 illustrates the thresholds as a function of investment costs when the salvage value is 35% of the investment costs, i.e. $D = 0.35I$. The resale value of a plant with CO_2 capture technology is 1316 *MNOK*.

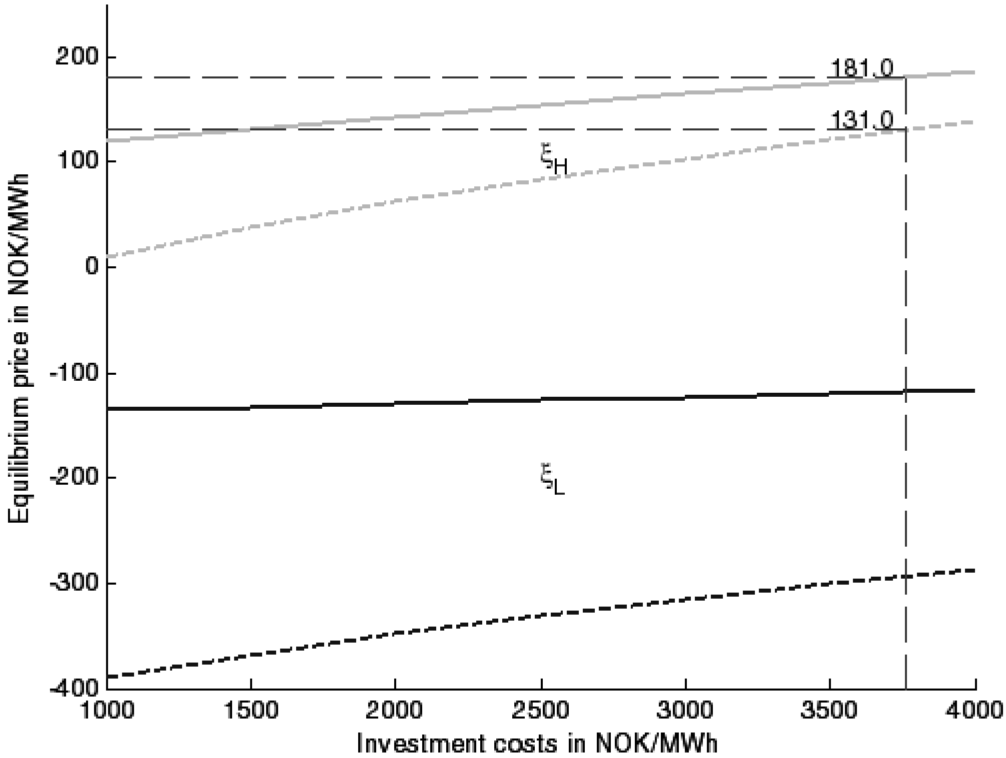

Figure 6: Investment thresholds as a function of investment costs

In Figure 6 the threshold to build a gas turbine with CO_2 capture equipment is on the interval [131.0; 181.0] *NOK/MWh*. In Table 2 the current equilibrium price is estimated at 62.3 *NOK/MWh*. Thus, with the current costs of CO_2 capture equipment it is not optimal to invest to such equipment. Let us assume that the building threshold is in the middle of its upper and lower bound, i.e. at 156 *NOK/MWh*. Note that this assumption is made only to simplify the following analyses and it is not based on any analyses of the plants operational cost structure and ramping constraints. Setting the threshold in the middle of the upper and lower bounds generally means that we assume that the plant can use half of the potential in the short-term spikes by ramping up and down. The effects of different operational cost structures and ramping constraints are analyzed, for example, in Deng and Oren (2003). Once the emission costs are around 65 *NOK/MWh,* the average of the upper and lower bound of the building threshold, for a plant without CO_2 capture equipment, is 156 *NOK/MWh*. By assuming that all emission costs are caused by CO_2, and by ignoring the reduced efficiency of the plant when the greenhouse gas capture equipment is in place and uncertainty in CO_2

emission costs, we get that it is optimal to install the CO_2 capture equipment when emission costs are greater than 25.6 *USD/ton*, i.e. 65 *NOK/MWh*.

The current estimate is that emission costs will be somewhere between 5 *USD/ton* and 20 *USD/ton*, where the lower range is most likely. Figure 5 indicates that when emission costs are 8 *USD/ton,* i.e. 20.3 *NOK/MWh,* the threshold to build a plant without CO_2 capture equipment is on the interval [64.1; 154.6] *NOK/MWh*. By assuming again that the building threshold is the average of upper and lower bound, we get that building threshold for a gas plant without CO_2 capture equipment is 109.4 *NOK/MWh*. The building threshold for the plant with CO_2 capture equipment is lowered from 155.5 *NOK/MWh* to 109.4 *NOK/MWh* if the investment costs are lowered to 2215 *MNOK*. Thus, if the costs of building a gas plant with CO_2 capture equipment are lowered with 1540 *MNOK,* it is optimal to build a gas plant with such equipment.

6 Discussion

In our case study the building threshold is on the interval [46.3; 165.3] *NOK/MWh* when the emission costs and the abandonment option are ignored. In Table 2 the current value of equilibrium price is 62.3 *NOK/MWh* which means that building of a gas fired power plant is optimal if the plants ramping constraints and operational cost structure are such that more than 87% of the potential in the short-term variations can be used. By taking the emission costs into account the situation changes slightly. In the case that the CO_2 capture equipment is not build the building threshold is on the interval [64.1; 154.6] *NOK/MWh* and when the CO_2 capture equipment is built the threshold is on the interval [131.0; 181.0] *NOK/MWh.* In both cases the current equilibrium price is below the interval which means that whatever the ramping policy is investing is not optimal. If the build option is omitted, i.e. building is commenced when the expected value of the plant is equal to investment costs, the situation changes considerably. In this case the investment threshold is on the interval [-178.2; 8.7] *NOK/MWh* which means that building is clearly optimal whatever the ramping policy is. Thus, the build option has remarkable effect on the building decision.

The rather large gap between upper and lower bounds of the investment thresholds means that the peak load plant value differs considerably from the base load plant value. Our case study also indicates that the addition of an abandonment option does not change dramatically the building threshold. Thus, as a first approximation for the investment decision it is plausible to ignore the abandonment option but the operating flexibility should not be disregarded. Note that the operational flexibility is dependent on the short-term uncertainty, i.e. if the size of the short-term variations decreases in the future, for example due to the more flexible production facilities, the potential of a peak load plant also decreases.

There are some issues that we have disregarded, in our case study, but which should be considered when the Norwegian case is analyzed more thoroughly. First, we have used the UK market as a reference for gas. There is lot of natural gas available in the Norwegian continental shelf. Due to the physical distance from the Norwegian coastline to the UK, the gas price at a Norwegian terminal will be equal to the UK price less transportation costs. It is estimated that the transportation costs for natural gas between Norway and UK is around $0.15 NOK/Sm^3$, where one Sm^3 is equal to $9.87 kWh$, this means that by using price quotes from IPE, we under estimate the spark spread by around 26 *NOK/MWh.* Second, there are also a tax issues that have not been considered. Oil and gas companies operating on the Norwegian shelf have a 78% tax rate, while onshore activities are taxed at 28%. If a company invested in a gas power plant, it could sell the gas at a loss with offshore taxation, and buy the same gas as a power plant owner with onshore taxation. Hence, the taxes can have also a positive impact on the profitability of the plant project.

The theory developed rests on an assumption that the energy company has an exclusive license, i.e. a monopoly right to invest. One may be concerned with how competition or other forms of market failure in the electricity or gas markets affect the results. However, as long as the information in efficient market prices of futures and forward contracts are incorporated in the analysis, these concerns are unfounded. Efficient forward prices will reflect any market failure. Of course, in practical cases there will be basis risk, for example due to electricity or gas being delivered or purchased at a different location or due to the quality of the gas that is underlying the forward contracts. Another problem is that long-term contracts may not be available. For a discussion of these issues, see e.g. Fama and French (1987).

7 Conclusions

We use real options theory to analyze gas fired power plant investments. Our valuation is based on electricity and gas forward prices. We have derived a method to compute upper and lower bounds for the plant value and investment thresholds when the spark spread follows a two-factor model, capturing both the short-term mean-reversion and long-term uncertainty.

In our case study we take the view of an investor having a license to build a gas fired power plant. Our results indicate that the abandonment option and the operating flexibility interact so that their joint value is less than their separate values, because an option to permanently shut down compensates the option to temporarily shut down and vice versa. However, the case study indicates that the addition of abandonment option does not dramatically change the bounds of the building threshold. On the other hand, the difference in upper and lower bound of the investment thresholds is considerable and thus the operating flexibility has significant effect on the building decision. Thus, when investments to gas fired power plants are considered a good overall view of the investment problem can be made by ignoring the abandonment option, whereas the operating flexibility and time-to-build option should not be disregarded.

Appendix A

A peak load plant operates only when the spark spread exceeds emission costs. The plant's value, at time *t*, is the expected cash flows less operational costs G

$$V_U(\chi,\xi) = \int_t^{\bar{T}} e^{-r(s-t)} \left(\bar{C} c(\chi(s),\xi(s)) - G\right) ds \qquad , \tag{A1}$$

where $\bar{T}$ is the lifetime of the plant, $\bar{C}$ is the capacity of the plant, and $c(\chi(s),\xi(s))$ is the expected value of spark spread exceeding emission costs at time *s*, i.e.

$$c(\chi(s),\xi(s)) = E\left[\max\left(S(s) - E, 0\right) \mid F_t\right] = \int_E^{\infty} (y - E) h(y) dy . \tag{A2}$$

The second equality follows from the normally distributed spark spread process. In (A2) $h(y)$ is the density function of a normally distributed variable *y*, whose mean and variance are the mean and variance of spark spread at time *s*, given in Corollary 1. For clarity we rewrite the mean and variance here

$$E\left[S(T) \mid \mathrm{F}_s\right] = \mathrm{e}^{-\kappa(T-s)} \chi(s) + \xi(s) + \mu_\xi (T - s) \tag{A3}$$

$$Var\left(S(T)\right) = \frac{\sigma_\chi^2}{2\kappa}\left(1 - \mathrm{e}^{-2\kappa(T-s)}\right) + \sigma_\xi^2 (T - s) + 2\left(1 - \mathrm{e}^{-\kappa(T-s)}\right)\frac{\rho\sigma_\chi\sigma_\xi}{\kappa} . \tag{A4}$$

Integration gives

$$c(\chi(s),\xi(s)) = \frac{\sqrt{Var\left(S(s)\right)}}{\sqrt{2\pi}} e^{\left(-\frac{(E - E[S(s)|F_t])^2}{2Var(S(s))}\right)} + \left(E\left[S(s) \mid F_t\right] - E\right)\left(1 - \Phi\left(\frac{E - E\left[S(s) \mid F_t\right]}{\sqrt{Var\left(S(s)\right)}}\right)\right), \tag{A5}$$

where $\Phi(\cdot)$ is the normal cumulative distribution function. Equations (A1) and (A5) give the value of the peak load plant

$$V_U(\chi,\xi)$$

$$= \bar{C}\int_t^{\bar{T}} e^{-r(s-t)} \left(\frac{\sqrt{Var\left(S(s)\right)}}{\sqrt{2\pi}} e^{\left(-\frac{(E - E[S(s)|F_t])^2}{2Var(S(s))}\right)} + \left(E\left[S(s) \mid F_t\right] - E\right)\left(1 - \Phi\left(\frac{E - E\left[S(s) \mid F_t\right]}{\sqrt{Var\left(S(s)\right)}}\right)\right)\right) ds - \frac{G}{r}\left(1 - \mathrm{e}^{-r(\bar{T}-t)}\right) \tag{A6}$$

Appendix B

When it is not optimal to exercise the build option, i.e. when $\xi < \xi_H$, the option to build F_0 must satisfy following Bellman equation

$$rF_0(\xi)dt = E\left[dF_0(\xi)\right] - Wdt, \qquad when\, \xi < \xi_H . \tag{B1}$$

Using Itô's lemma and taking the expectation we get following differential equation for the option value

$$\frac{1}{2}\sigma_\xi^2 \frac{\partial^2 F_0(\xi)}{\partial^2 \xi} + \mu_\zeta \frac{\partial F_0(\xi)}{\partial \xi} - rF_0(\xi) - W = 0, \qquad when\, \xi < \xi_H . \tag{B2}$$

A solution to the differential equation is a linear combination of two independent solutions plus any particular solution (see, e.g., Dixit and Pindyck, 1994). Thus, the value of the build option is

$$F_0(\xi) = A_1\, e^{\beta_1 \xi} + A_2\, e^{\beta_2 \xi} - \frac{W}{r}, \qquad when\, \xi < \xi_H , \tag{B3}$$

where A_1, A_2 are unknown non-negative parameters and β_1 and β_2 are the roots of the fundamental quadratic equation, and are given by

$$\beta_1 = \frac{-\mu_\xi + \sqrt{{\mu_\xi}^2 + 2{\sigma_\xi}^2 r}}{{\sigma_\xi}^2} > 0 \tag{B4}$$

$$\beta_2 = \frac{-\mu_\xi - \sqrt{{\mu_\xi}^2 + 2{\sigma_\xi}^2 r}}{{\sigma_\xi}^2} < 0 . \tag{B5}$$

The build option value approaches zero as the spark spread decreases, i.e. A_2 must be equal to zero, and thus

$$F_0(\xi) = A_1\, e^{\beta_1 \xi} - \frac{W}{r}, \qquad when\, \xi < \xi_{H0} . \tag{B6}$$

Appendix C

It is optimal to exercise the build option when the option value becomes equal to the values gained by exercising the option

$$F_0(\xi_H) = V(0, \xi_H) - I + F_1(\xi_H) . \tag{C1}$$

Correspondingly, it is optimal to abandon when values gained by abandoning are equal to values lost

$$F_1(\xi_L) + V(0, \xi_L) = D . \tag{C2}$$

The smooth-pasting conditions must also hold when the options are exercised (for an intuitive proof see, e.g., Dixit and Pindyck, 1994 and for a rigorous derivation see Samuelson, 1965)

$$\frac{\partial F_0(\xi_H)}{\partial \xi} = \frac{\partial V(0,\xi_H)}{\partial \xi} + \frac{\partial F_1(\xi_H)}{\partial \xi} \tag{C3}$$

$$\frac{\partial F_1(\xi_L)}{\partial \xi} + \frac{\partial V(0,\xi_L)}{\partial \xi} = 0 \,. \tag{C4}$$

The building and abandonment thresholds ξ_H and ξ_L as well as the option parameters A_1 and D_2 for all plant values V must satisfy (C1)- (C4). It remains to show that increase in the plant value decreases the investment and abandonment thresholds. Let us denote

$$G^U\left(\xi_H, A_1, D_2\right) = F_0(\xi_H) - V(0,\xi_H) - F_1(\xi_H) + I \tag{C5}$$

$$G^L\left(\xi_L, A_1\right) = F_1(\xi_L) + V(0,\xi_H) - D \,, \tag{C6}$$

where A_1 and D_2 are the parameters of investment and abandonment options and ξ_H and ξ_L are the investment thresholds when the plant value is V. By denoting the partial derivatives with subscripts, the value-matching and smooth-pasting conditions for plant value V are

$$G^U\left(\xi_H, A_1, D_2\right) = 0 \tag{C7}$$

$$G^L\left(\xi_L, A_1\right) = 0 \tag{C8}$$

$$G^U_{\xi_H}\left(\xi_H, A_1, D_2\right) = 0 \tag{C9}$$

$$G^L_{\xi_L}\left(\xi_L, D_2\right) = 0 \,. \tag{C10}$$

When the plant value V is changed with df differentiation gives

$$G^U_{A_1}\left(\xi_H, A_1, D_2\right) dA_1 + G^U_{D_2}\left(\xi_H, A_1, D_2\right) dD_2 + G^U_{\xi_H}\left(\xi_H, A_1, D_2\right) d\xi_H = df \tag{C11}$$

$$G^L_{D_2}\left(\xi_L, D_2\right) dD_2 + G^L_{\xi_L}\left(\xi_L, D_2\right) d\xi_L = -df \,. \tag{C12}$$

Differentiation of the smooth-pasting condition gives

$$G^U_{\xi_H\xi_H}\left(\xi_H, A_1, D_2\right) d\xi_H + G^U_{\xi_H A_1}\left(\xi_H, A_1, D_2\right) dA_1 + G^U_{\xi_H D_2}\left(\xi_H, A_1, D_2\right) dD_2 = 0 \tag{C13}$$

$$G^L_{\xi_L\xi_L}\left(\xi_L, D_2\right) d\xi_L + G^L_{\xi_L D_2}\left(\xi_L, D_2\right) dD_2 = 0 \,. \tag{C14}$$

Equations (C10), (C12), and (C14) give for the change of the abandonment threshold

$$d\xi_L = \frac{G^L_{\xi_L D_2}\left(\xi_L, D_2\right) df}{G^L_{\xi_L\xi_L}\left(\xi_L, D_2\right) G^L_{D_2}\left(\xi_L, D_2\right)} = \frac{\beta_2 df}{G^L_{\xi_L\xi_L}\left(\xi_L, D_2\right)} \,. \tag{C15}$$

The second equality is obtained by calculating the derivatives of the abandonment option given in (17). Before abandonment, in the value-matching condition, $G^L\left(\xi_H, A_1\right)$ approaches zero from above, thus $G^L(\xi, A_1)$ must be convex in ξ. When the plant value is increased with positive amount, i.e. $df > 0$, we get

$$d\xi_L < 0. \tag{C16}$$

Hence when the plant value increases the abandonment threshold decreases. Equations (C9), (C11), (C13) and (C15) give the change of the building threshold

$$\begin{aligned} d\xi_H &= \frac{G^U_{\xi_H A_1}(\xi_H, A_1, D_2)\left(\dfrac{df + G^U_{D_2}(\xi_H, A_1, D_2)\dfrac{df}{G^L_{D_2}(\xi_L, D_2)}}{G^U_{A_1}(\xi_H, A_1, D_2)}\right) + df\,\dfrac{G^U_{\xi_H D_2}(\xi_H, A_1, D_2)}{G^L_{D_2}(\xi_L, D_2)}}{G^U_{\xi_H \xi_H}(\xi_H, D_2, A_1)}, \\ &= \frac{-\beta_1\left(1 + e^{\beta_2(\xi_H - \xi_L)}\right) + \beta_2 e^{\beta_2(\xi_H - \xi_L)}}{G^U_{\xi_H \xi_H}(\xi_H, D_2, A_1)} df \end{aligned} \tag{C17}$$

where the second equality is obtained by calculating the derivatives of the build and abandonment options given in (11) and (17). Before building, in the value-matching condition, $G^U(\xi_H, A_1, D_2)$ approaches zero from above, thus $G^U(\xi, A_1, D_2)$ must be convex in ξ. When the plant value is increased with positive amount, i.e. $df > 0$, we get

$$d\xi_H < 0. \tag{C18}$$

Q.E.D.

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