To appear in Review of Financial Economics

Flexibility and Technology Choice in Gas Fired Power Plant Investments

Erkka Näsäkkälä[1], Stein-Erik Fleten[2]

Abstract

The value of a gas fired power depends on the spark spread, defined as the difference between the price of electricity and the cost of gas used for the generation of electricity. We model the spark spread using a two-factor model, allowing mean-reversion in short-term variations and uncertainty in the equilibrium price to which prices revert. We analyze two types of gas plants: peak and base load plants. A peak load plant generates electricity when spark spread exceeds emission costs, whereas a base load plant generates electricity at all levels of spark spread. A base load plant can be upgraded to a peak load plant. First, we find the upgrading threshold for a base load plant. The upgrading threshold gives the optimal type of gas plant as a function of spark spread. Second, we calculate building threshold for the investment costs. When the investment costs are below the threshold it is optimal to build the plant with the previously solved optimal technology. In the numerical example, we illustrate how our model can be used when investments in gas fired power plants are considered.

Key words: Real options, spark spread, gas fired power plants, investment flexibility

[1] Corresponding author. Systems Analysis Laboratory, Helsinki University of Technology, P.O. Box 1100, FIN-02015 HUT, Finland. E-mail: erkka.nasakkala@hut.fi

[2] Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway. E-mail: Stein-Erik.Fleten@iot.ntnu.no

1 Introduction

We study flexibility in the choice of technology regarding investments in gas fired power plants. The model is motivated by the advances in combined cycle gas turbine (CCGT) technology, and the emergence of energy commodity markets worldwide. It has been customary for investors holding licenses to build gas fired power plants to plan as if the plants are to be run around the clock, year round. The reason for this is that gas is often purchased on long-term physical take-or-pay contracts. However, the gas market is being liberalized, and in the future we expect a more market-oriented and flexible gas supply system.

The value of a gas fired power plant depends on the spark spread, defined as the difference between the price of electricity and the cost of gas used for the generation of electricity. The cash flows of an operating gas fired power plant are given by the spark spread less operational and emission costs. If the emission costs exceed the spark spread some losses can be avoided by ramping down the plant. When the plant is ramped down only operational costs remain. We assume that there are two technologies for a gas fired power plant. A base load plant produces electricity independent of the spark spread and emission costs. The inflexibility of a base load plant can be caused by constraints in the gas inflow. If a plant is ramped down, but the gas inflow cannot be changed, the gas will be lost. Thus, it is never optimal to ramp down a plant with a rigid gas supply system. A peak load plant is ramped up and down according to price changes. Due to the ramping the peak load plant's operational costs are higher. Base load plants can often be upgraded to peak load plants.

We take a viewpoint of an investor holding a license to build a gas fired power plant and study following questions. Should the plant be operated as a peak or as a base load plant? When should the base load plant be upgraded to a peak load plant? How low should the investment costs be in order to entice an investment at the current level of spark spread? Valuation of gas fired power plants has been studied in Deng et al. (2001). Our model makes several extensions to their model. First, we take into account the option to postpone the investment decision. Such postponement option analysis originates from the work of McDonald and Siegel (1986). Second, our study considers the flexibility in the choice of production technology. He and Pindyck (1992) study the output flexibility for a firm having two possible products to produce. Brekke and Schieldrop (2000) study the input flexibility in a power plant when the plant can be either gas or oil fired. In our case, the flexibility is in the choice of production strategy, not in the choice of input or output.

A number of authors have argued that mean-reverting price processes, instead of geometric Brownian motion based models, are more appropriate for commodities (see, e.g., Laughton and Jacoby 1993 and 1995, Cortazar and Schwartz, 1994, and Smith and McCardle, 1999). Schwartz and Smith (2000) develop a two-factor model where the short-term deviations are modeled with a mean-reverting process and the equilibrium price evolves according to a Brownian motion. We use a similar two-factor model for the spark spread. The short-term deviations reflect non-persistent changes, for example, changes in the demand resulting from variations in the weather. The equilibrium price reflects fundamental changes that are expected to persist, for example, expectations regarding the discovery of natural gas. Other two-factor models with long- and short-term factors include, among others, Ross (1997) and Pilipović (1998).

Deng et al. (2001) use separate price processes for electricity and gas. Instead of modeling the electricity and gas prices separately, we model the spark spread process directly. By using the spark spread process we have a single reference price process for gas fired power plant investments. When electricity and gas processes are used there is no such reference price. When separate price processes are used the spark spread variance can be a function of electricity and gas prices, whereas when spark spread model is used this is not possible, as electricity and gas prices are not observed. This means that we may loose some information about the spark spread's uncertainty structure compared to models with separate electricity and gas price processes. In Section 6 we compare the numerical gas plant values obtained by the two approaches. The results indicate that, with our data, the difference is small.

The decision to upgrade a base load plant to a peak load plant is not affected by the previously made decision to build a base load plant. Thus, we first calculate the upgrading threshold, which gives the optimal type of gas plant as a function of spark spread. Second, we use the upgrading threshold to find the threshold for the investment costs. If the investment costs are above the investment cost threshold it is optimal to postpone the building decision.

An increase in the variability of spark spread increases the value of up and down ramping making a peak load plant the more valuable choice. On the other hand, uncertainty also delays investments (see, e.g. Dixit and Pindyck, 1994). In the numerical example we illustrate how our model can be used to measure the relative strengths of these opposite effects. In our example, an increase in short-term variations makes a peak load plant more attractive, whereas an increase in long-term uncertainty makes base load plant more favorable by postponing the upgrading decision. We also study how the change in the upgrading decision affects the investment cost threshold. Our sensitivity analysis illustrates that both upgrading and building decisions are highly dependent on the peak load plant's operational costs and spark spread's uncertainty structure.

The paper is organized as follows. Our spark spread dynamics are introduced in Section 2 and in Section 3 the price data together with the method to estimate the process parameters are presented. The basic valuation formulas for the plant values and investment options are given in Section 4. The optimal technology and thresholds for investment costs and upgrading are computed in Section 5. The model is illustrated with an example in Section 6. Section 7 concludes the study.

2 Spark spread process

The contribution margin of a gas fired power plant is measured by spark spread, which is defined as the difference between the price of electricity S_e and the cost of gas S_g used for the generation of electricity

$$S = S_e - K_H S_g . \tag{1}$$

The heat rate K_H is the amount of gas required to generate 1 MWh of electricity. Heat rate measures the efficiency of the gas plant: the lower the heat rate, the more efficient the facility. The efficiency of a gas fired power plant varies slightly over time and with the output level. Still, the use of a constant heat rate is considered plausible for long-term analyses (see, e.g., Deng et al., 2001).

The current electricity and gas prices do not independently affect the value of a gas fired power plant. Thus, instead of modeling electricity and gas prices we model their difference. Traditionally, log-normal distributions are used to model non-negative prices. We use following dynamics to model the difference of two positive prices. Our spark spread dynamics do not follow from the difference of two log-normally distributed price processes, however it has similar characteristics, namely it can have negative and positive values.

ASSUMPTION 1. *The spark spread is a sum of short-term deviations and equilibrium price*

$$S(t) = \chi(t) + \xi(t), \tag{2}$$

where the short-term deviations χ *are assumed to revert toward zero following an Ornstein-Uhlenbeck process*

$$d\chi(t) = -\kappa\chi(t)dt + \sigma_\chi dB_\chi(t). \tag{3}$$

The equilibrium price ξ *is assumed to follow an arithmetic Brownian motion process*

$$d\xi(t) = \mu_\xi dt + \sigma_\xi dB_\xi(t). \tag{4}$$

Parameters κ, σ_χ, μ_ξ, *and* σ_ξ *are constants and* $B_\kappa(\cdot)$ *and* $B_\xi(\cdot)$ *are standard Brownian motions, with correlation* $\rho dt = dB_\chi dB_\xi$ *and information* F_t.

An increase in the spark spread attracts high cost producers to the market, putting downward pressure on prices. Conversely, when prices decrease some high cost producers will withdraw capacity temporarily, putting upward pressure on prices. As these entries and exits are not instantaneous, prices may be temporarily high or low, but will tend to revert toward equilibrium price ξ. The mean-reversion parameter κ describes the rate at which the short-term deviations χ are expected to decay. The uncertainty in the equilibrium price is caused by the uncertainty in the fundamental changes that are expected to persist. For example, advances in gas exploration and production technology, changes in the discovery of natural gas, improved gas fired power plant technology, and political and regulatory effects can cause changes to the equilibrium price. Other models where the two-factors are interpreted as short- and long-term factors are, for example, Schwartz and Smith (2000), Ross (1997), and Pilipović (1998).

Seasonality in electricity and gas supply and demand combined with limited storage opportunities cause seasonality into electricity and gas prices. As electricity and gas are often used for the same purposes, such as heating or cooling, their seasonal variations have similar phases and thus the seasonality decays from the spark spread. The seasonality in the spark spread process can be modeled by including a deterministic seasonality function to (2). In Section 3 our analyses of the spark spread data suggest that there are no seasonal variations in the spark spread, so we do not include a seasonality function. The following corollary states the distribution of the future spark spread values.

COROLLARY 1. *When spark spread has dynamics given in* (2)-(4), *prices are normally distributed, and the expected value and variance, at time t, are given by*

$$E\left[S(T) \mid \mathrm{F}_t\right] = \mathrm{e}^{-\kappa(T-t)}\, \chi(t) + \xi(t) + \mu_\xi (T-t) \tag{5}$$

$$Var\left(S(T)\right) = \frac{\sigma_\chi^2}{2\kappa}\left(1-\mathrm{e}^{-2\kappa(T-t)}\right) + \sigma_\xi^2 (T-t) + 2\left(1-\mathrm{e}^{-\kappa(T-t)}\right)\frac{\rho\sigma_\chi\sigma_\xi}{\kappa}. \tag{6}$$

PROOF: See, e.g. Schwartz and Smith (2000).

Corollary 1 states that the spark spread is a sum of two normally distributed variables: equilibrium price and short-term variations. The expected value of short-term variations converges to zero as the maturity $T-t$ increases and thus the expected value of the spark spread converges to the expected value of the equilibrium price. The mean-reversion parameter κ describes the rate of this convergence. The maturity in which short-term deviations are expected to halve is given by

$$T_{1/2} = -\frac{\ln(0.5)}{\kappa}. \tag{7}$$

The variance of the spark spread process is independent of the electricity and gas prices. When prices are modeled with log-normal distributions the variance is an increasing function of price. Thus, when the spark spread is modeled as a difference of two log-normal prices its variance varies depending on the electricity and gas prices. For instance, consider a situation that the electricity and gas prices are high and the spark spread is zero. In this case the value is zero and the uncertainty is large. On the other hand, the spark spread can be also zero when both of the prices are small. This time the value of the spark spread is the same but the uncertainty is small. In our example data, introduced in Section 3, there is some indication the spark spread variance increases as a function of electricity and gas prices. Our price process in Assumption 1 does not capture this variation in spark spread variance as a function of electricity and gas prices. Nevertheless, our numerical example, in Section 6, illustrates that the difference in plant values calculated with our spark spread process and two log-normally distributed price processes is small.

Note that even though we loose some information when using the spark spread process its use is beneficial. It enables a more thorough and intuitive description of the investment decision process. When the plant value is a function of both electricity and gas prices, there is no single reference price for building and upgrading decision and thus the analysis of these decisions becomes a much more complicated two-dimensional problem.

3 Data and process estimation

Neither the short-term deviations χ nor the equilibrium price ξ are directly observable, however, they can be estimated from forward prices. A forward price is the risk-adjusted expected future spark spread value and thus forward prices can be used to infer the risk adjusted dynamics of short-term deviations and equilibrium price. The expected short-term variations decrease to zero when the maturity increases and thus the long-maturity forwards give information about the equilibrium price. When the maturity is short the short-term variations have not yet converged to zero. Thus, the difference of long- and short-maturity forwards gives information about the short-term dynamics. Our Kalman filter based estimation method for spark spread parameters is based on this simple idea. Before going to the details of the estimation method, let us introduce our electricity and gas data.

We use a price history of forward contracts over a four year time period starting at the 2nd of January 2001 and ending at the 30th of January 2004. For short-maturity forwards we use monthly forward contracts with 1-month maturity, whereas the long-maturity contracts are three year contracts with 1-year maturity. The electricity data, obtained from Nord Pool (The Nordic Power Exchange), is graphed in the upper picture of Figure 1. The currency used in Nord Pool is Norwegian krone (NOK), whose exchange rate is around 7 *NOK/USD*. The natural gas data, obtained from International Petroleum Exchange (IPE), is given in the lower picture of Figure 1. The short-maturity forwards are the black lines and the grey lines are the long-maturity forwards. Figure 1 indicates how the values of short-maturity forwards vary more than the values of long-maturity forwards. For example, during the so-called Scandinavian power crisis in December 2002 and January 2003 the highest quote of 1-month electricity forward was as high as 770 *NOK/MWh* but at the same time the highest quote of 1-year forward was only 256 *NOK/MWh*. Thus, the market did not expect high prices to last long. Similar patterns can be seen in the gas prices, for example in January 2002. The smaller volatility in the long-term forwards is typical for commodity prices (see, e.g., Schwartz, 1997), and it means that the price process contains short-term fluctuations which are expected to disappear. Thus, the decreasing volatility as a function of maturity can be seen as a consequence of the mean-reversion in the spot price process. Figure 1 also indicates that both the electricity and gas prices have seasonality. The 1-month prices tend to be high during the winter months and low during the summer months.

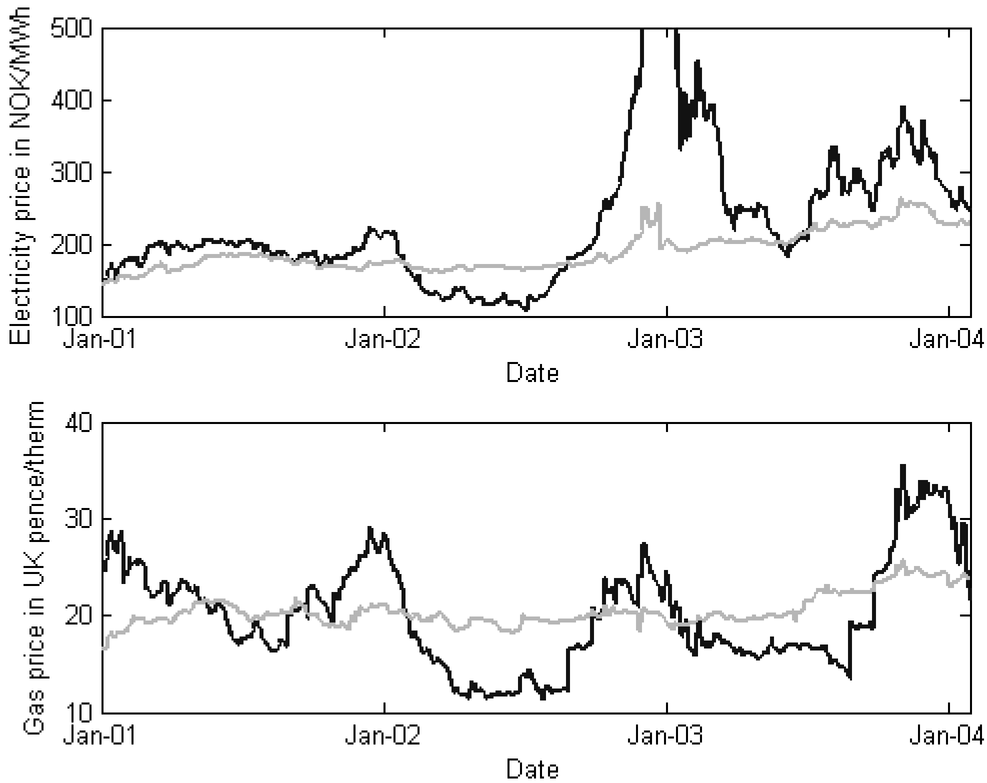

Figure 1: Electricity and gas forwards

Figure 2 graphs the time series of 1-month and 1-year spark spread forwards calculated from the data in Figure 1. We calculate the spark spread process for a new combined cycle gas turbine (CCGT) whose efficiency is 58.1%, i.e. we use a heat rate $K_H = 1.72$. The natural gas forwards are quoted in *GBP/therm*, where one *therm* is equal to 29.3071 *kWh*. From 2001 to 2004 the exchange rate has varied between 11-13 *NOK/GBP*. The spark spread process is calculated from gas prices in the UK and electricity prices in Norway and in the calculation the transportation costs between the two locations are omitted. It is estimated that the transportation costs for natural gas between Norway and UK is around $0.15 NOK/Sm^3$, where one Sm^3 is equal to $9.87 kWh$ As there is natural gas in Norwegian continental shelf, this means that we under estimate the spark spread by around 26 NOK/MWh for a plant at a Norwegian terminal.

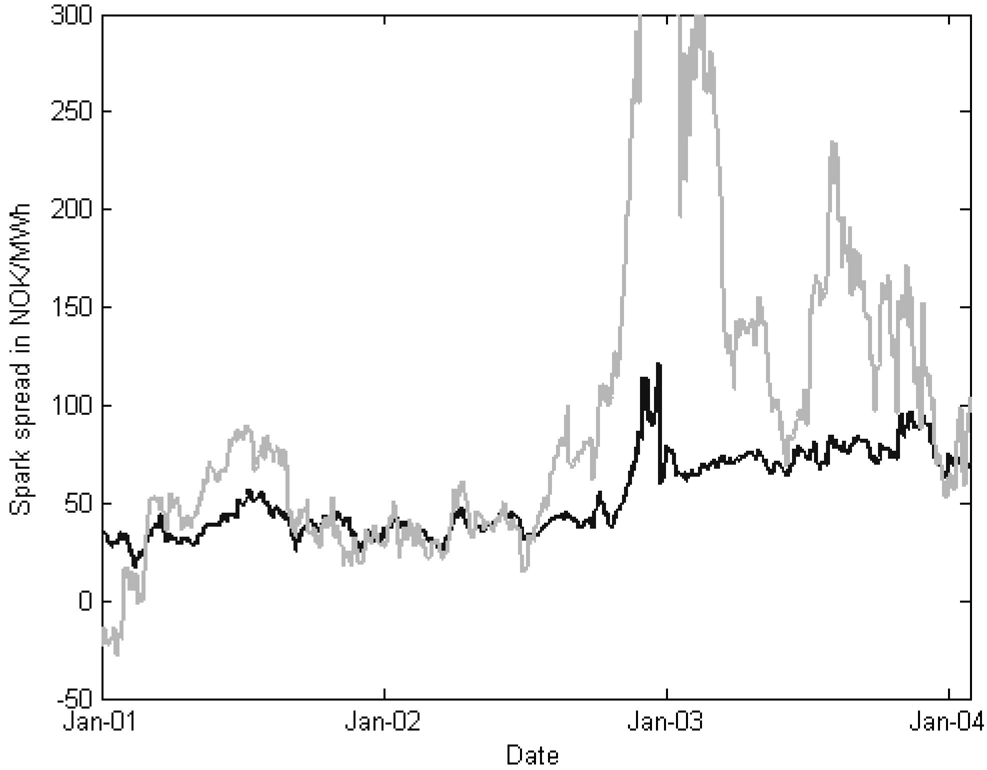

Figure 2: Spark spread forwards

The mean-reversion in electricity and gas prices leads to mean-reversion in spark spread. The seasonality in the electricity and natural gas prices has decayed from the spark spread process. For example, during year 2001 the spark spread was at the lowest in winter but increased towards the summer, while in 2003 it was the reverse. In Figure 3 the seasonality in the spark spread is analyzed by computing the autocorrelations of the changes in the monthly average prices of 1-month forwards. In Figures 3 and 4 we use seven year time series, starting at 31st of January 1997, for 1-month forwards. In the parameter estimation the longer time series can not be used as IPE did not quote long-term natural gas forwards before 2001. In Figure 3 the gas process, indicated by the black dashed line, and the electricity process, indicated by the black line, have a clear correlation between observations that are 12 months apart. In the spark spread process, indicated by the grey line, this kind of correlation does not exist. This justifies our decision to use a non-seasonal spark spread process.

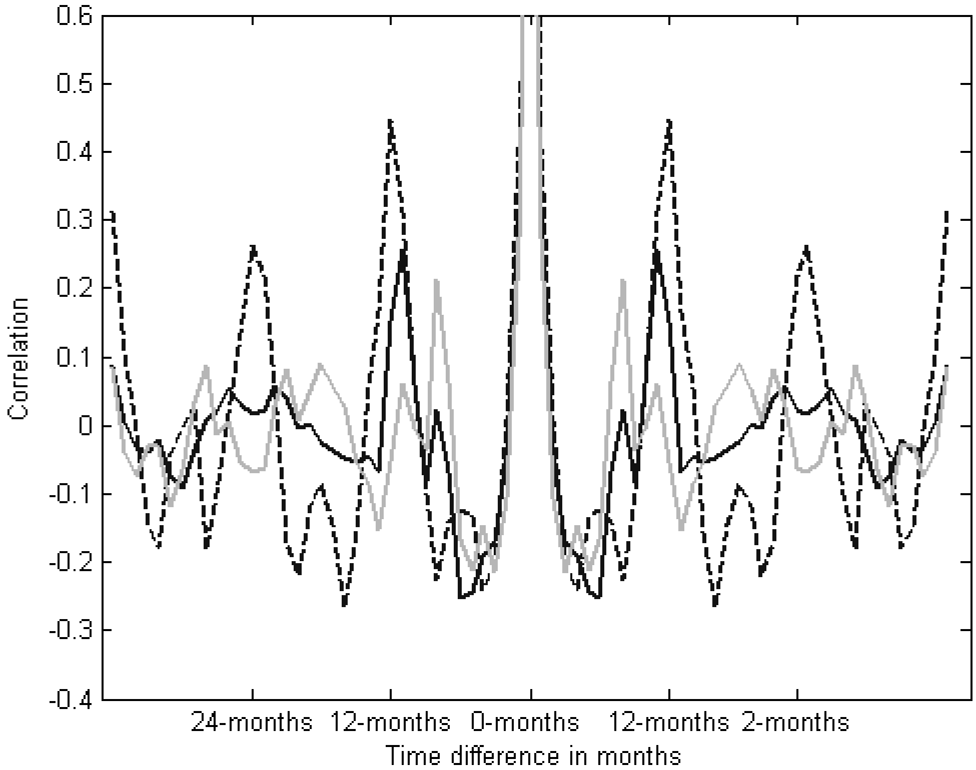

Figure 3: Autocorrelation in 1-month forwards

In Figure 4 we illustrate the absolute values of spark spread changes as a function of electricity and gas prices. Figure 4 illustrates that the electricity and gas prices are positively correlated. This can also be seen in Figure 1 where electricity prices tend to be high when gas prices are high. The 95% confidence interval for correlation between electricity and gas prices is [0.54; 0.57]. Figure 4 also indicates that the variations in spark spread are greater when the electricity and/or gas prices are high. The confidence intervals for the correlation between electricity price and absolute value of spark spread change is [0.52; 0.58], whereas for gas price it is [0.18; 0.27]. As discussed in previous section, our model does not capture the changes in the spark spread variance as a function of electricity and gas prices. In Section 6 we illustrate how this simplification affects the peak and base load plant values.

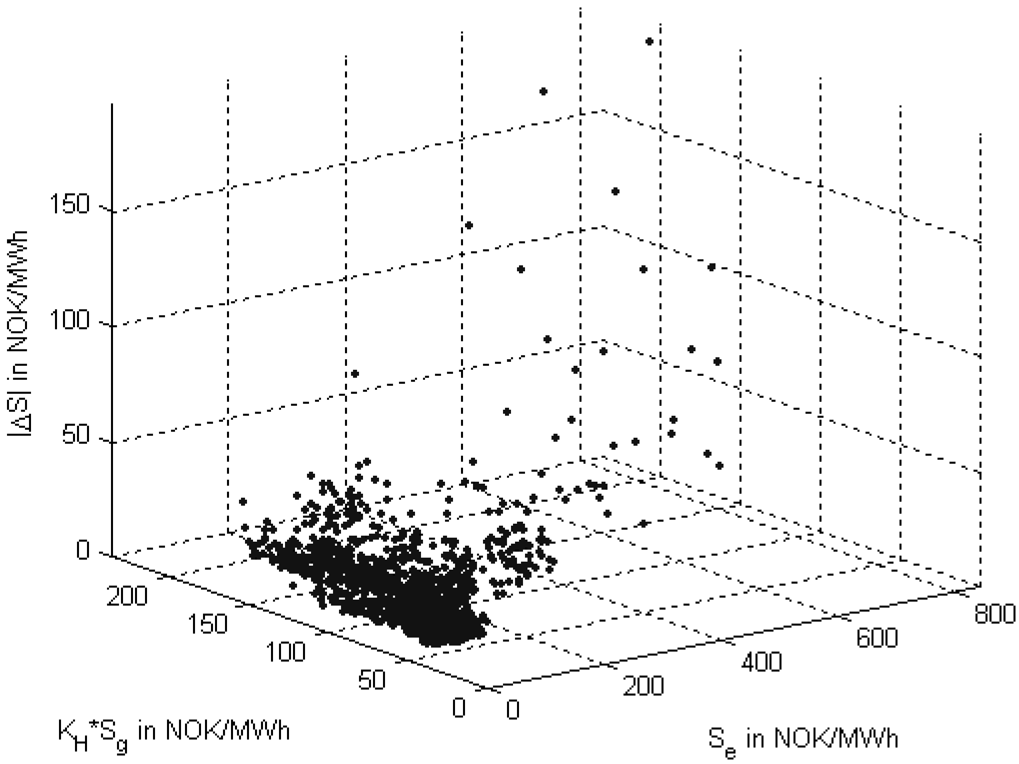

Figure 4: Changes in spark spread as a function of electricity and gas price

Next we outline a method to estimate the spark spread parameters. We operate in discrete periods $i = 1,..,N$, where N is the number of price observations. The price observations are denoted by $y_i = \left[S_i^{T_1}, S_i^{T_2}\right]$, where $S_i^{T_1}$ denotes the price of a forward contract with maturity $T_j \in \{1,2\}$ at period $i \in \{1,..,N\}$. For example, in Figures 1 and 2 the maturities are one month and one year and the periods are all trading days between the 2nd of January 2001 and the 30th of January 2004. The time difference between observations y_i and y_{i-1} is Δt_i. The forward prices $S_i^{T_1}$ and $S_i^{T_2}$ can be seen as observations depending on unobservable state variables, which are the short-term fluctuation χ_i and equilibrium price ξ_i, denoted by $x_i = \left[\chi_i, \xi_i\right]$. A Kalman filter is a recursive procedure for computing estimates for unobserved state variables based on observations that depend on these state variables. A thorough survey of Kalman filtering techniques is given, for example, in Harvey (1989). Schwartz and Smith (2000) use a Kalman filter to estimate parameters for two-factor spot price process from oil-linked forward prices. Our method follows their guidelines with a different price process and with non-constant time periods. From Assumption 1 we obtain transition equation for the state variables

$$x_i = c_i + G_i x_{i-1} + \omega_i \quad for\,all \quad i = 1,..,N\,, \tag{8}$$

where

$$c_i = \left[0, \mu_\xi \Delta t_i\right] \tag{9}$$

$$G_i = \begin{bmatrix} e^{-\kappa \Delta t_i} & 0 \\ 0 & 1 \end{bmatrix}. \tag{10}$$

The serially uncorrelated normally distributed disturbance vectors ω_i have zero expected value and the following covariance matrix

$$Cov\left(\chi_{\Delta t_i}, \xi_{\Delta t_i,}\right) = W = \begin{bmatrix} \left(1 - e^{-2\kappa \Delta t_i}\right)\dfrac{\sigma_\chi^2}{2\kappa} & \left(1 - e^{-\kappa \Delta t_i}\right)\dfrac{\rho \sigma_\chi \sigma_\xi}{2\kappa} \\ \left(1 - e^{-\kappa \Delta t_i}\right)\dfrac{\rho \sigma_\chi \sigma_\xi}{2\kappa} & \sigma_\xi^2 \Delta t_i \end{bmatrix}. \tag{11}$$

The measurement equation, describing the relationship between the state variables $x_i = \left[\chi_i,\, \xi_i\right]$ and price observations $y_i = \left[S_i^{T_1}, S_i^{T_2}\right]$, follows from Corollary 1

$$y_i = d + F x_i \quad for\ all \quad i = 1,..,N\,, \tag{12}$$

where

$$d = \left[\mu_\xi T_1, \mu_\xi T_2\right] \tag{13}$$

$$F = \begin{bmatrix} e^{-\kappa T_1} & 1 \\ e^{-\kappa T_2} & 1 \end{bmatrix}. \tag{14}$$

With the transition equation (8) and measurement equation (12) the likelihood function of a given parameter set $\mu, \kappa, \rho, \sigma_\chi, \sigma_\xi$ with a prior mean and covariance matrix can be derived as in Harvey (1989, Chapter 3). For the maximum likelihood estimation initial values for the parameters together with prior mean and covariance are needed. We reran the optimization routine from a variety of initial values and the parameters did not appear to be very sensitive to changes in the initial values. In Figure 5 the estimated spot price process $\chi_t + \xi_t$ over the estimation interval, i.e. 2nd of January 2001 through the 30th of January. 2004, is indicated with a black solid line. The grey line represents the estimated equilibrium price ξ_t. The equilibrium price varies less than the spot price, whose variation is also affected by the short-term fluctuations.

The maximum likelihood optimization gives the parameters describing the past realizations of spark spread forwards. In the valuation we want to use price dynamics that describe the future expectations of spark spread. In Figure 5 the grey vertical lines after the 30th of January 2004 represent the forward prices on that date. The forward prices give the risk adjusted future spark spread estimates, which differs from the true expected spark spread by the market price of risk. Thus by selecting the spark spread process so that it describes the forward prices as well as possible we get a price process which describes the risk adjusted future spark spread expectations. With linear regression we get that the drift in the long term forwards is 2.18 *NOK/MWh*. By using this long-term drift and mean-reversion $\kappa = 2.6$, estimated from the price history, we get the current equilibrium price $\xi_0 = 62.3$ *NOK/MWh* and short-term deviation $\chi_0 = 52.9$ *NOK/MWh*. In Figure 5 the risk-adjusted expected future spark spread and its 68% confidence level are indicated by black solid and dashed lines. The expected value and confidence levels are calculated with Corollary 1. The expected value decreases rapidly during the first few months as the expected value of current short-term deviation converges to zero.

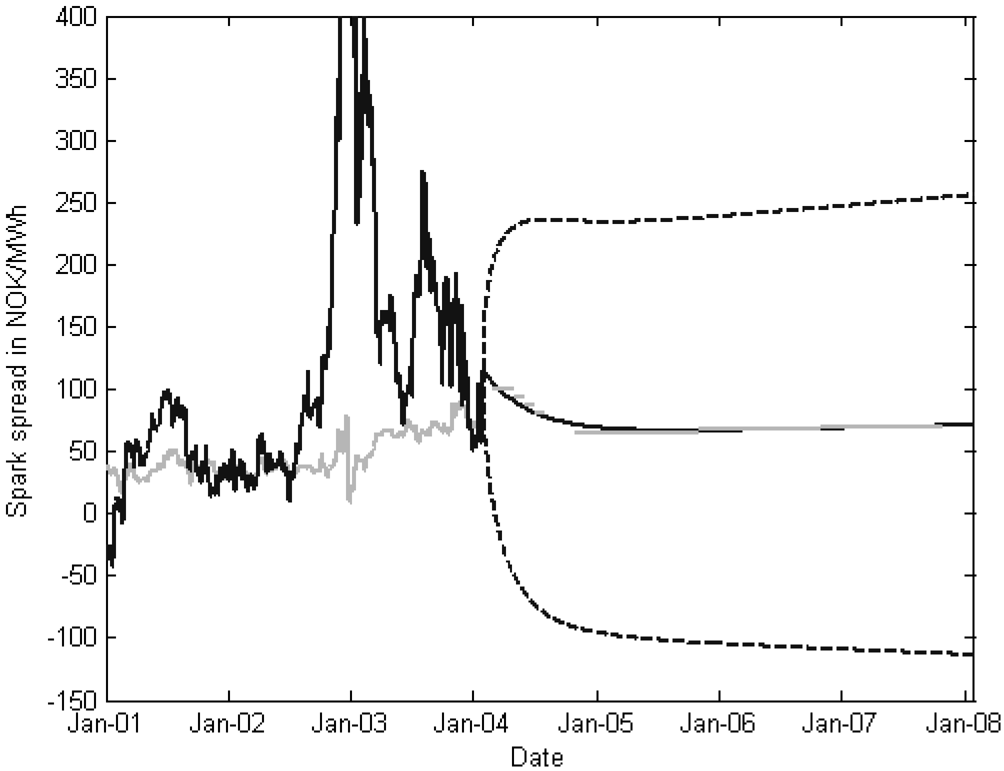

Figure 5: Estimated spot- and equilibrium spark spread together with forward curve

Let us summarize our parameter estimation. First, we estimate mean-reversion κ, correlation ρ, and volatility σ_χ, σ_ξ parameters from the forward price history with the Kalman filtering procedure. Second, the long-term drift μ_ξ is estimated from the long-term forwards. Third, current equilibrium price ξ_0 and short-term deviation χ_0 are chosen so that the expected value matches the whole forward curve. The parameter values are summarized in Table 1.

Table 1: Spark spread parameters

Parameter	κ	μ_ξ	ρ	σ_χ	σ_ξ	ξ_0	χ_0
Unit		*NOK/MWh*		*NOK/MWh*	*NOK/MWh*	*NOK/MWh*	*NOK/MWh*
Value	2.6	2.18	-0.21	382.2	47.8	62.3	52.9

4 Valuation

In this section we give the basic valuation formulas. In Section 4.1 the peak and base load plant values are given. In Section 4.2 the building and upgrading options are valued.

4.1 The plant values

The following assumption characterizes the two technologies for a gas fired power plant.

ASSUMPTION 2. A base load plant produces electricity with constant capacity. A peak load plant can be ramped up and down without delay. The costs associated with ramping up and down can be amortized into operational costs.

A base load plant produces electricity even if it is unprofitable, whereas a peak load plant can be ramped down whenever it is unprofitable to use the plant. In a gas fired power plant, the operational and maintenance costs do not vary much over time and the response times are in the order of several hours. Thus, we are assuming that the peak load plant is more flexible than it really is, but for efficient plants the error in the long-term valuation analysis will be small (see, e.g., Deng and Oren, 2003).

For simplicity, we assume that the (nonfuel) operational costs are deterministic. We use a constant risk free interest rate r. For every unit of electrical energy generated, the plant must pay environmental taxes due to emission of greenhouse gases and other pollutants. This is represented by the emission cost E. The following lemma gives the value of a base load plant.

LEMMA 1. The base load plant value, at time t, is

$$V_B\left(\chi(t),\xi(t),\kappa,\mu_\xi\right)=$$
$$\bar{C}\left(\frac{\chi(t)}{\kappa+r}+\frac{\xi(t)-E}{r}+\frac{\mu_\xi}{r^2}-\mathrm{e}^{-r(\bar{T}-t)}\left(\frac{\mathrm{e}^{-\kappa(\bar{T}-t)}\chi(t)}{\kappa+r}+\frac{\xi(t)-E}{r}+\frac{\mu_\xi\left(r(\bar{T}-t)+1\right)}{r^2}\right)\right)-\frac{G_B}{r}\left(1-\mathrm{e}^{-r(\bar{T}-t)}\right), \tag{15}$$

where $\bar{T}$ is the lifetime of the plant, $\bar{C}$ is the capacity of the plant, and G_B are the operational costs of a base load plant.

PROOF: The value of a base load plant is the present value of expected cash flows less operational costs

$$\begin{aligned} V_B\left(\chi(t),\xi(t),\kappa,\mu_\xi\right) &= \int_t^{\bar{T}} e^{-r(s-t)}\left(\bar{C}\left(E\left[S(s)\mid \mathrm{F}_t\right]-E\right)-G_B\right)ds = \\ &= \int_t^{\bar{T}} e^{-r(s-t)}\left(\bar{C}\left(e^{-\kappa(s-t)}\chi(t)+\xi(t)-E+\mu_\xi(s-t)\right)-G_B\right)ds \end{aligned} \tag{16}$$

Integration gives (15). Q.E.D.

The value of base load plant is just the discounted sum of expected spark spread values less emission and operational costs. Thus, the base load plant is not affected by the short-term and equilibrium volatilities σ_χ, σ_ξ. A peak load plant can react to unexpected changes in the spark spread by ramping down when spark spread decreases below emission costs. The following lemma gives the value of a peak load plant.

LEMMA 2. The peak load plant value, at time t, is

$$V_P\left(\chi(t),\xi(t),\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho\right)=$$
$$\bar{C}\int_t^{\bar{T}} e^{-r(s-t)}\left(\frac{\sqrt{Var\left(S(s)\right)}}{\sqrt{2\pi}}e^{\left(-\frac{(E-E[S(s)|\mathrm{F}_t])^2}{2Var(S(s))}\right)}+\left(E\left[S(s)\mid \mathrm{F}_t\right]-E\right)\left(1-\Phi\left(\frac{E-E\left[S(s)\mid \mathrm{F}_t\right]}{\sqrt{Var\left(S(s)\right)}}\right)\right)\right)ds-\frac{G_P}{r}\left(1-\mathrm{e}^{-r(\bar{T}-t)}\right) \tag{17}$$

where $\Phi(\cdot)$ is the normal cumulative distribution function, and G_P are the operational costs of a peak load plant. The expected value $E\left[S(s)\,|\,\mathrm{F}_t\right]$ and variance $Var\left(S(s)\right)$ for the spark spread are given in Corollary 1.

PROOF: See Appendix A.

The more the spark spread varies the more valuable is the ability to ramp up and down. Thus, the value of a peak load plant increases as a function of spark spread variance. The peak load plant value is the discounted sum of expected cash flows minus operational costs plus the option value of being able to ramp up and down. Note that the difference of peak and base load plant values, i.e. $V_P - V_B$, is negative if the value of up and down ramping is smaller than the difference of operational costs. The plant values will be illustrated with an example in Section 6.

4.2 The investment options

The following assumption characterizes the variables affecting the building and upgrading decision.

ASSUMPTION 3. The investment decisions are made as a function of equilibrium price. In the investment decisions the lifetime of the plant is assumed to be infinite.

Assumption 3 states that when building and upgrading are considered the current short-term realization is omitted. Thus, investments are not done due to the temporarily spiking spark spread. Note that even though the current short-term realization is omitted the short-term dynamics, i.e. short-term volatility σ_χ and mean-reversion κ, affect the value of a peak load plant, which affects building and upgrading decision. Thus, the short-term dynamics affect the investment decisions. The omission of the short-term realization is motivated by the fact that gas fired power plants are long-term investments. A gas plant investment is never commenced due to a non-persistent spike in the price process. This approximation is realistic as long as the expected lifetime of the short-term deviations is considerably smaller than the expected lifetime of the plant. The mean-reversion parameter κ=2.6, from Table 1, gives with (7) that the short-term variations are expected to halve in about three months. Usually, the lifetime of a gas fired power plant is assumed to be around 30 years. Thus, the omission of the short-term realization in the investment decision gives an realistic approximation of the true building and upgrading decisions. The infinite lifetime assumption is motivated by the fact that the plant's lifetime is often increased by upgrading and reconstructions, and by downward shifts in the maintenance costs (see, e.g. Ellerman, 1998).

As the building decision can always be postponed, there is a threshold ξ_{H0} for the equilibrium price below which building is not optimal. The following lemma gives the value of the build option when it is not optimal to build the plant, i.e. when $\xi < \xi_{H0}$.

LEMMA 3. The value of an option to build a gas fired power plant is

$$F_0(A_1, \xi, \mu_\xi, \sigma_\xi) = A_1 e^{\beta_1 \xi} \qquad when \quad \xi < \xi_{H0}, \tag{18}$$

where A_1 is an unknown positive parameter. The parameter β_1 is given by

$$\beta_1 = \frac{-\mu_\xi + \sqrt{\mu_\xi^{\ 2} + 2\sigma_\xi^{\ 2} r}}{\sigma_\xi^{\ 2}} > 0. \tag{19}$$

PROOF: See Appendix B.

The build option value increases exponentially as a function of the equilibrium price. The parameter A_1 depends on what type of plant is optimal. A method for finding the parameter A_1 is derived in the next section. When a decision to build a base load plant is made, the investor receives both the base load plant and an option to upgrade the plant. As the plant's lifetime is assumed to be infinite there is an upgrading threshold ξ_{H1} above which it is not optimal to upgrade. When it is not optimal to exercise the upgrade option, i.e. when $\xi > \xi_{H1}$, the upgrade option value is given by the following lemma.

LEMMA 4. The value of an option to upgrade a base load plant to a peak load plant is

$$F_1(D_2, \xi, \mu_\xi, \sigma_\xi) = D_2 e^{\beta_2 \xi} \qquad when \quad \xi > \xi_{H1} \tag{20}$$

where D_2 is an unknown positive parameter. The parameter β_2 is given by

$$\beta_2 = \frac{-\mu_\xi - \sqrt{\mu_\xi^{\ 2} + 2\sigma_\xi^{\ 2} r}}{\sigma_\xi^{\ 2}} < 0. \tag{21}$$

PROOF: The proof is similar to that of the build option (Appendix B), but now the option becomes less valuable as the spark spread increases. On the other hand, it becomes more valuable as spark spread decreases. Q.E.D.

The upgrade option value decreases exponentially as a function of the equilibrium price. The parameter D_2 depends on the difference of plant values and their operational costs. A method for finding the parameter D_2 is derived in the next section. The values gained, if one invests in a base load plant, is the plant value plus an option to upgrade, i.e. $V_B + F_1$, whereas when a peak load plant is built, the values gained are only the plant value, i.e. V_P.

5 Investment strategy

In this section we first derive a method to select the optimal technology for a gas plant. After the optimal technology is known we calculate building threshold for the investment costs. When the investment costs are below the threshold it is optimal to build a plant with the previously selected technology. We also calculate an upgrading threshold for the equilibrium price. If the equilibrium price decreases to the upgrading threshold, it is optimal to upgrade a base load plant to a peak load plant.

Let us start by considering the choice of technology. A company owning a base load plant has an option to upgrade the plant to a peak load plant. Lemma 4 states that the upgrade option has an upgrading threshold above which it is optimal to wait. Once the equilibrium price decreases to the upgrade threshold, upgrading becomes optimal. The following proposition gives the upgrading threshold and the value of the upgrade option.

PROPOSITION 1. The upgrading threshold ξ_{H1} is given by the following equation

$$\left(V_P(0,\xi_{H1},\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho)-V_B(0,\xi_{H1},\kappa,\mu_\xi)\right)\beta_2=\frac{\partial V_P(0,\xi_{H1},\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho)}{\partial\xi}-\frac{\partial V_B(0,\xi_{H1},\kappa,\mu_\xi)}{\partial\xi}. \tag{22}$$

Once the upgrading threshold is known the value of the upgrade option can be written as

$$F_1(\xi,\xi_{H1},\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho)=\left(V_P(0,\xi_{H1},\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho)-V_B(0,\xi_{H1},\kappa,\mu_\xi)\right)e^{\beta_2(\xi-\xi_{H1})}\quad \textit{when}\ \ \xi>\xi_{H1}. \tag{23}$$

PROOF: When the upgrading decision is made, the values lost must be equal to values gained. By assumption 3 the threshold is chosen using a current short-term variation of zero

$$F_1(D_2,\xi_{H1},\mu_\xi,\sigma_\xi)=V_P(0,\xi_{H1},\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho)-V_B(0,\xi_{H1},\kappa,\mu_\xi). \tag{24}$$

Also the smooth-pasting conditions must hold when the upgrade option is exercised (for an intuitive proof see, e.g., Dixit and Pindyck, 1994 and for a rigorous derivation see Samuelson, 1965)

$$\frac{\partial F_1(D_2,\xi_{H1},\mu_\xi,\sigma_\xi)}{\partial \xi}=\frac{\partial V_P(0,\xi_{H1},\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho)}{\partial \xi}-\frac{\partial V_B(0,\xi_{H1},\kappa,\mu_\xi)}{\partial \xi}. \tag{25}$$

Equations (24) and (25) together with the value of an upgrade option in Lemma 4 give (22) and (23).

Q.E.D.

In Proposition 1 we do not include costs for upgrading from base to a peak load plant. This is not a simplifying assumption, as the upgrading costs can be represented by peak load plant's extra operational costs whose present value is equal to the investment costs. This is possible as the lifetime of the peak load plant is known with certainty.

The value of the upgrade option is not affected by the building costs. When a base load plant is built, the investment costs are lost, and thus the upgrading decision is based solely on the value of the upgrade option and values gained and lost by upgrading. Equation (23) implies that the value of the upgrade option increases as the difference between equilibrium price and upgrade threshold decreases. When upgrading is optimal, the value of the upgrade option is equal to the values obtained by exercising it, i.e. the difference of peak and base load plant values. The peak load plant value increases as a function of short-term volatility, which means that the value of the upgrade option increases as a function of short-term uncertainty. Thus, an increase in short-term variations increases the option value. By equation (21) the parameter β_2 increases towards zero as the equilibrium uncertainty increases, thus also the equilibrium uncertainty increases the upgrade option value.

The upgrade option value indicates the additional value in a base load plant due to the possibility to upgrade to a peak load plant. The upgrade option value minus the values gained by exercising the option, i.e. $F_1 - V_P + V_B$, tells how much more valuable a base load plant with a possibility to upgrade is compared to a peak load plant. The choice of optimal technology is summarized in the following corollary.

COROLLARY 2. A base load plant is the optimal technology if the current equilibrium price ξ_0 and the upgrading threshold ξ_{H1} satisfy

$$\xi_{H1} < \xi_0 . \tag{26}$$

Correspondingly, when

$$\xi_{H1} \geq \xi_0 , \tag{27}$$

the peak load plant is the optimal technology.

PROOF: If the current equilibrium price ξ_0 is greater than the upgrading threshold ξ_{H1} it is optimal to build a base load plant and wait with the upgrading. If the equilibrium price ξ_0 is smaller than the upgrading threshold ξ_{H1}, it is optimal to exercise the upgrade option immediately, i.e. to build a peak load plant. Q.E.D.

So far, we have derived the optimal technology for a gas fired power plant and the value of the upgrade option. The following proposition gives a threshold for the investment costs as a function of equilibrium price. If investment costs are below the threshold, it is optimal to build a gas fired power plant with the technology given by Proposition 1 and Corollary 2.

PROPOSITION 2. When a base load plant is the optimal technology, i.e. when $\xi_{H1} < \xi_0$, building is optimal if the investment costs satisfy

$$I \leq V_B(0,\xi_0,\kappa,\mu_\xi) + F_1(\xi_0,\xi_{H1},\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho) - \left(\frac{\partial V_B(0,\xi_0,\kappa,\mu_\xi)}{\partial \xi} + \frac{\partial F_1(\xi_0,\xi_{H1},\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho)}{\partial \xi} \right) \frac{1}{\beta_1} . \tag{28}$$

When the peak load plant is optimal, i.e. when $\xi_{H1} \geq \xi_0$, building is optimal if the investment costs satisfy

$$I \leq V_P(0,\xi_0,\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho) - \frac{\partial V_P(0,\xi_0,\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho)}{\partial \xi} \frac{1}{\beta_1} \tag{29}$$

PROOF: Similar to that of Proposition 1. When the option is exercised, both the value matching and smooth-pasting conditions must hold. Q.E.D.

Investing is optimal when the values gained are equal or greater than the values lost. Proposition 2 states that building is optimal when investment costs are equal to or smaller than the difference of values gained and build option value. The build option value at the building threshold is the derivative of values gained, i.e. $\frac{\partial V_B}{\partial \xi} + \frac{\partial F_1}{\partial \xi}$ for a base load plant and $\frac{\partial V_P}{\partial \xi}$ for a peak load plant, divided by the parameter β_1. Equation (19) implies that the parameter β_1 decreases towards zero as equilibrium volatility increases, and equation (23) gives that the derivative of the upgrade option increases as a function of short-term and equilibrium volatility. Thus, the value of the build option increases as a function of both equilibrium and short-term volatility. On the other hand, peak load plant and upgrade option values increase as a function of equilibrium and short-term volatility. Thus, it is not unambiguous how changes in the volatility structure change the investment cost threshold. In our numerical example, in Section 6, we illustrate how our model can be used to measure the relative strengths of these opposite effects.

The value-matching and smooth-pasting conditions for the upgrade option imply the following inequality for the build option

$$\frac{\partial V_B(0,\xi,\kappa,\mu_\xi)}{\partial \xi}+\frac{\partial F_1(\xi,\mu_\xi,\sigma_\xi)}{\partial \xi}\geq\frac{\partial V_P(0,\xi,\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho)}{\partial \xi} \quad when \;\; \xi\geq\xi_{H1}\,. \tag{30}$$

The value of a peak load plant in Lemma 2 implies the following inequality

$$\frac{\partial V_P(0,\xi,\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho)}{\partial \xi}>0\ . \tag{31}$$

Conditions (30) and (31) together with Proposition 2 mean that the value of the investment option is always positive, thus investing is never optimal when the investment costs are equal to the present values gained by exercising the option. The investment option can be seen as a positive premium needed to cover the risks in the investment's present value. Condition (30) and Proposition 2 imply that this premium is always greater for an investment in a base load plant. In other words, when a peak load plant is the optimal technology, the investment cost threshold is closer to the present value of the plant. The decrease in the risk premium follows from the peak load plant's ability to react to adverse price changes.

6 Example

We study a combined cycle gas fired power plant whose maximum capacity is 415 *MW*. The utilization rate of the plant is approximately 90%. Thus, the output capacity is $\bar{C}$ = 3.27 *TWh/year*. We assume that the plant's CO_2 emission costs are 8 *\$/ton* and ignore its uncertainty. The CO_2 production of a combined cycle gas fired power plant is 363 *kg/MWh*, thus a CO_2 emission cost of 8 *\$/ton* corresponds 20.3 *NOK/MWh* when the exchange rate of 7 *NOK/USD* is used. Instead of emitting CO_2 one may also consider building equipment capturing it, however the current cost of capture technology renders this option unprofitable. Undrum et al. (2000) estimate that building a combined cycle gas fired power plant without CO_2 capture equipment in Norway costs approximately 1620 *MNOK.* We assume that the operational costs of a base load plant are 20 *MNOK/year* and the operational costs of a peak load plant are 80 *MNOK/year*. Remember that the operational costs of a peak load plant contain the augmented upgrading costs. For instance, if it costs 300 *MNOK* to upgrade a base load plant to a peak load plant, the peak load plants operational costs must be increased by 18 *MNOK/year.* In this example, we omit the possibility to sell steam. This could be due to location in a rural area. The parameters characterizing the gas plant are summarized in Table 2, where also the used risk-free interest rate is given. For the spark spread process we use the parameters in Table 1.

Table 2: The gas plant parameters

Parameter	$\bar{C}$	G_B	G_P	I	E	r
Unit	*TWh/year*	*MNOK/year*	*MNOK/year*	*MNOK*	*NOK/MWh*	
Value	3.27	20	80	1620	20.3	0.06

In Figure 6 the plant values are illustrated. In the upper part of Figure 6 the solid lines illustrate the base and peak load plant values as a function of lifetime. The base load plant is the grey line, whereas the black line is the peak load plant. The dashed lines illustrate the peak and base load plant values calculated with separate electricity and gas processes. When the lifetime is short our model underestimates the difference of peak and base load plant values compared to models using separate electricity and gas processes. Correspondingly, with longer lifetimes we overestimate the difference. The difference in the plant values is not caused just by the different volatility structure. In our model the equilibrium price increases linearly, whereas when log-normal price distributions are used the equilibrium price increases as a difference of two exponentially increasing processes. Note that the calculation of electricity and gas price process parameters is not as straightforward as calculation of spark spread parameters; before the parameters are estimated seasonality must be filtered away from electricity and gas data.

Both technologies are highly NPV positive with the investment costs given in Table 2. In the lower part of Figure 6 the plant values as a function of equilibrium volatility and short-term uncertainty are illustrated. As the uncertainty in the spark spread increases the flexibility in the peak load plant becomes more valuable and the difference of peak and base load plant values increases. When the uncertainty becomes small enough the peak load plant value is smaller than the value of a base load plant, which means that the operational costs of a peak load plant are greater than the costs saved by being able to ramp up and down.

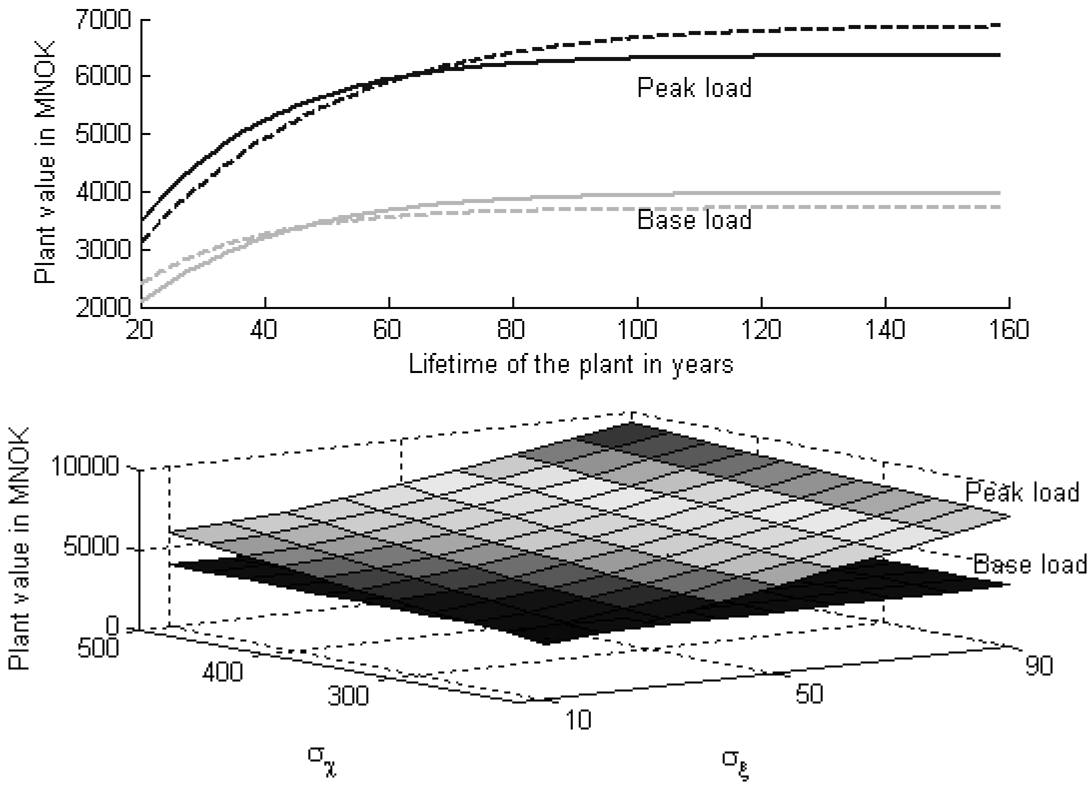

Figure 6: Peak and base load plant values

The plant values, illustrated in Figure 6, do not indicate what the optimal technology for the plant is. Proposition 1 together with Lemmas 1, 2, and 4, give an equation for the upgrading threshold ξ_{H1}. Numerically solving this equation, with parameters in Tables 1 and 2, gives $\xi_{H1}=$ 52.8 *NOK/MWh*. In Table 1 the current equilibrium price estimate is $\xi_0 = 62.3$ *NOK/MWh*. Thus, by Corollary 2, the base load plant is the optimal technology. In the upper part of Figure 7, the optimal technology as a function of the peak load plant's operational costs is illustrated. The base load plant is optimal when peak load plant's operational costs are larger than the vertical dashed line. In the upper part of Figure 7 the equilibrium price varies along the y-axis as time passes. It is optimal to upgrade a base load plant to a peak load plant when equilibrium price decreases below upgrading threshold, denoted by black line. It is less attractive to upgrade to a peak load plant with higher operational costs, hence the upgrading threshold decreases as a function of the peak load plant's operational costs.

So far we have studied the choice of technology. Next, we will find the level of investment costs at which it is optimal to invest now in a plant with the previously solved optimal technology, i.e. in a base load plant. Using proposition 2, we find that if the investment costs I are below 837 *MNOK,* it is optimal to build a base load plant now. The inequality in Proposition 2 is solved numerically by using Lemmas 1, 2, 3, and 4. Even though both of the available technologies are highly NPV positive, it is not optimal to invest now, given the cost estimates and the current estimate for the equilibrium price. In the lower part of Figure 7 we illustrate the investment cost threshold's sensitivity to peak load plant's operational costs. The investment cost threshold for a base load plant is denoted with a black line, whereas the investment cost threshold for a peak load plant is illustrated with a grey line. An investor building a base load plant receives both a base load plant and an option to upgrade to a peak load plant. The upgrade option can be seen as an American call option to exchange base load plant into a peak load plant. An increase in the peak load plant's operational costs decreases the value of a peak load plant, which also decreases the value of the upgrade option. Thus, an increase in the peak load plant's operational costs decreases investment cost threshold for both technologies. The value of the upgrade option decreases towards zero as peak load plant's operational costs increase. In other words, the investment cost threshold asymptotically decreases towards the investment cost threshold for a base load plant without an upgrade option. In the lower part of the Figure 7 the investment cost threshold decreases below zero when the peak load plant's operational costs are above 108 *MNOK/year*. Thus, with our parameter estimates it is never optimal to build a non-upgradeable base load plant.

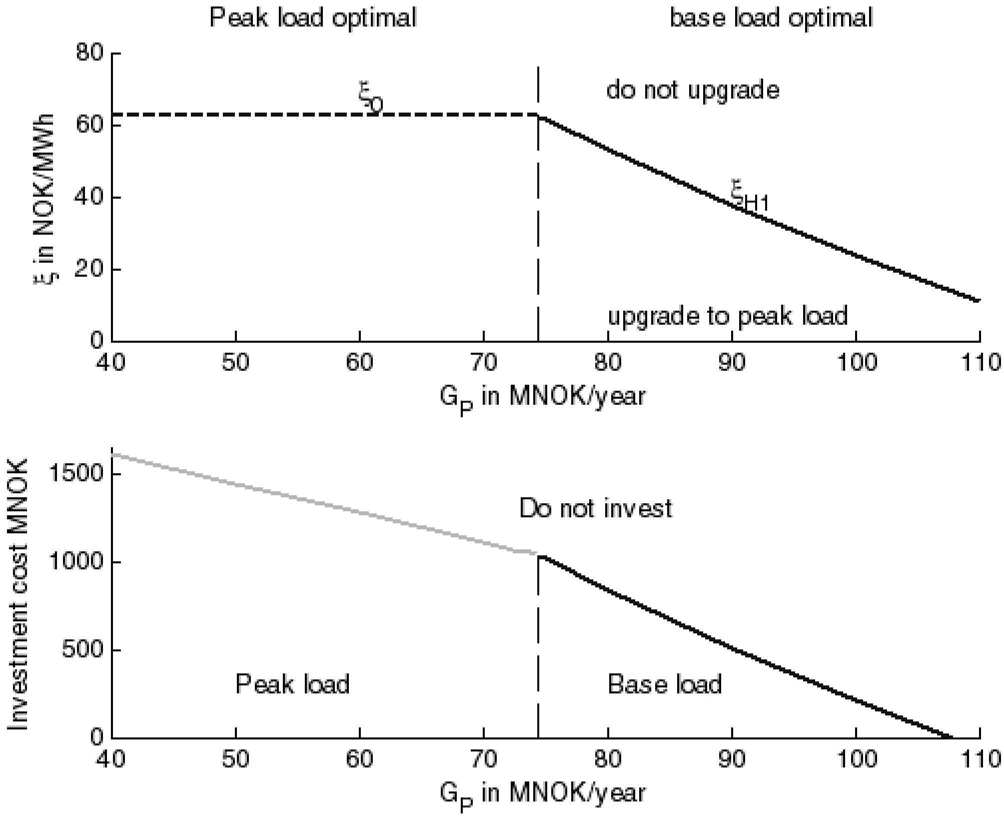

Figure 7: Upgrading and investment cost thresholds as a function of peak load plant's operational costs

Next, we study how the optimal technology changes as a function of the spark spread variance. An increase in the spark spread variance increases the value of a peak load plant, making a peak load plant a more attractive technology. On the other hand, an increase in spark spread variance also makes upgrade option more valuable, which makes base load plant more attractive. Thus, changes in the spark spread variance have an ambiguous effect on the optimal technology. In Figure 8 we illustrate the optimal technology as a function of equilibrium volatility and short-term uncertainty. When short-term uncertainty increases, the peak load plant becomes a more attractive choice. However, when equilibrium volatility increases, the base load plant becomes a more attractive choice. An increase in equilibrium volatility increases the upgrade option value more than the peak load plant's value, since the upgrade option is like an American call option to exchange a base load plant for a peak load plant. Thus, an increase in equilibrium volatility makes a base load plant more attractive. Regarding an increase in short-term variations, the situation is reverse. The only way short-term variations affect the upgrade option is through the value of the peak load plant. Thus, an increase in short-term variations is akin to an increase in the value of the underlying of an

American call. Generally, it is optimal to exercise an American option when the option value is equal to values gained by exercising the option. When exercising is not optimal, the option is more valuable than the values gained by exercising it, meaning that the option value must increase less than the underlying. The upgrade option that comes with the base load plant thus becomes relatively less worth compared to a peak load plant, and the peak load plant becomes a more attractive choice when short-term uncertainty increases. In the right part of Figure 9 the upgrading threshold's dependence on short-term and equilibrium volatility is illustrated. Naturally, when the base load plant becomes more optimal, i.e. in Figure 8 we move towards the lower right corner, upgrading is postponed. This means that the upgrading threshold increases as a function of short-term variations but decreases as a function of equilibrium volatility.

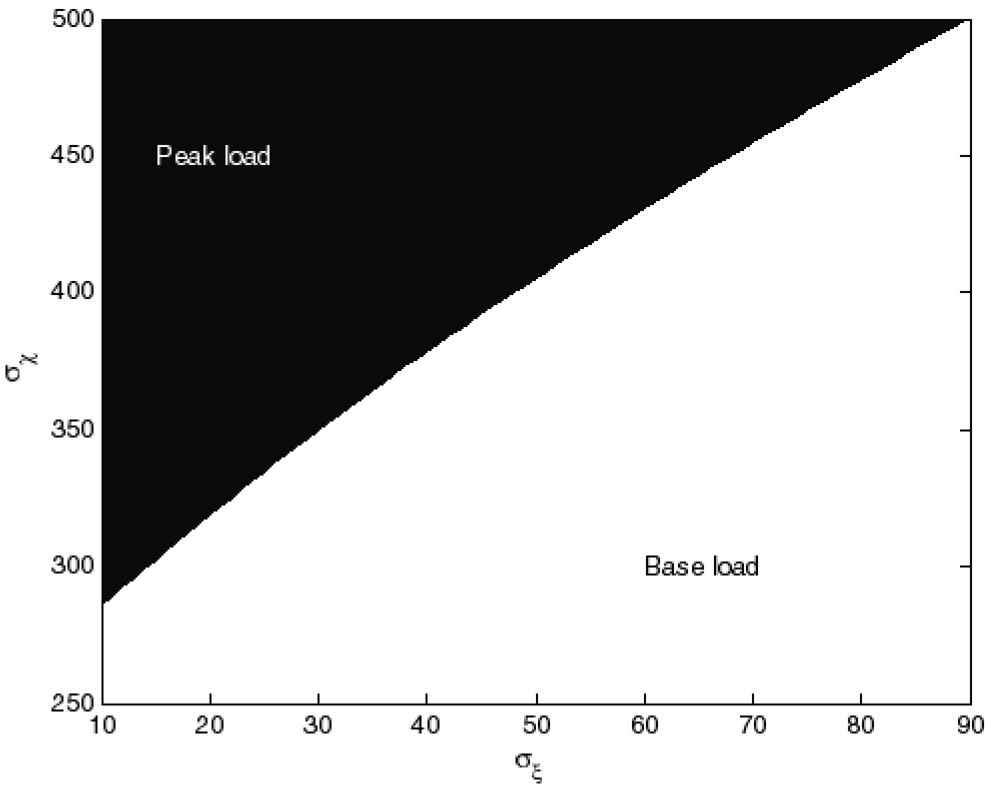

Figure 8: Optimal technology as a function of short-term and equilibrium volatility

In the left part of Figure 9 the sensitivity of the investment cost threshold as a function of equilibrium and short-term volatility is illustrated. The investment cost threshold increases as a function of short-term variations because an increase in short-term variations increases the value of the peak load plant. However, when equilibrium volatility increases the increase in peak load plant value is dominated by increase in the investment option value, and thus the investment cost threshold decreases as a function of equilibrium volatility. This is rather intuitive, uncertainty in the future environment makes long-term investments more risky, whereas due to the peak load plant's ability to react to adverse price changes, an increase in short-term variations increases the expected future cash flows from a peak load plant. At the same time, due to the mean-reversion in prices, an increase in short-term variations does not change the long-term risks in the prices.

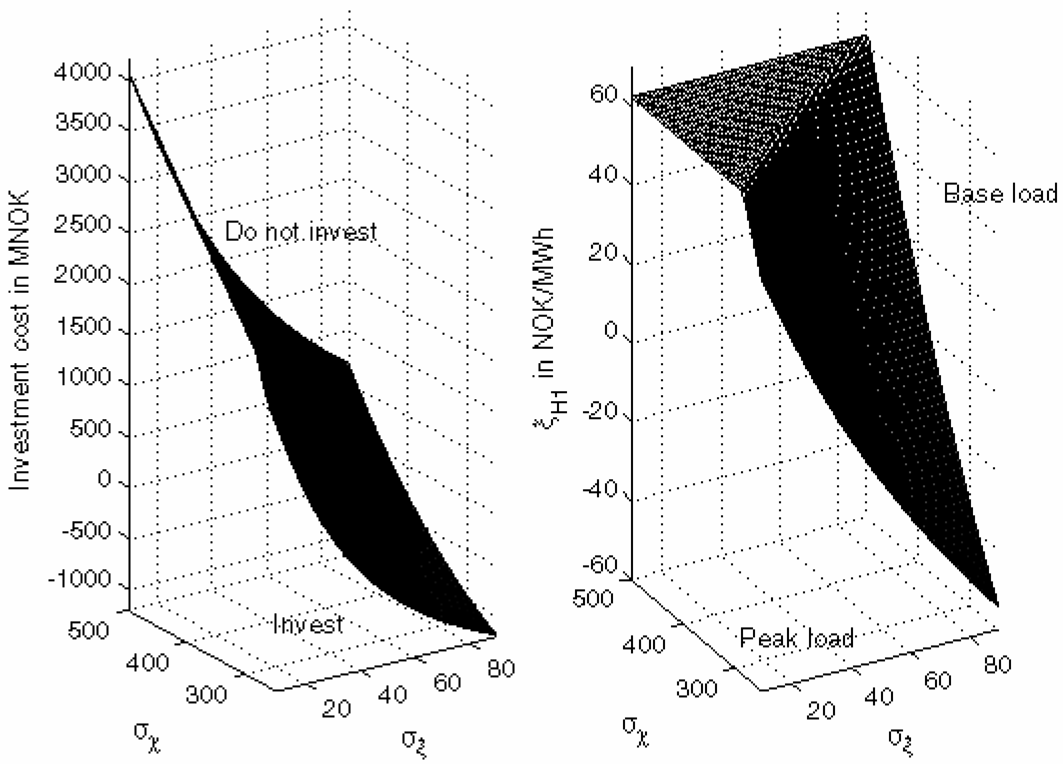

Figure 9: Investment cost and upgrading thresholds as a function of short-term and equilibrium volatility

7 Conclusions

We study gas fired power plant investments taking into account flexibility in the choice of production technology and in investment decision. First, we find the optimal technology for a gas fired power plant and then we use this optimal technology to calculate investment cost threshold. When the investment costs are below the threshold investing is optimal. The analysis is based on data from electricity and gas forward markets.

We illustrate that an increase in the variability of spark spread has an ambiguous effect on the investment decisions. An increase in the variability of spark spread increases the value of a peak load plant making investments to such plants more attractive. On the other hand, uncertainty also delays investments. In the numerical example we illustrate how our model can be used to measure the relative strengths of these opposite effects. We illustrate with a numerical example how an increase in short-term variations hastens investment decisions, whereas an increase in long-term uncertainty postpones investment decisions. This result is rather intuitive due to the mean-reversion in the short-term variations, increase in short-term variations does not increase the uncertainty in the future price levels and thus it does not postpone investment and upgrading decisions.

Acknowledgements

We would like to thank Pauli Murto and the anonymous referee for helpful comments on earlier versions of the paper. Partial financing by the Nordic Energy Research is gratefully acknowledged.

References

Brekke, K.A. and B. Schieldrop, (1999). Investment in flexible technologies under uncertainty, in *Project Flexibility, Agency, and Competition: New Developments in the Theory of Real Options,* edited by M.J. Brennan and L. Trigeorgis, New York: Oxford University Press, 34-49.

Cortazar, G. and E. S. Schwartz. (1994) The evaluation of commodity contingent claims. *Journal of Derivatives*, 1, 27-39.

Deng, S.J., Johnson B., and A. Sogomonian. (2001) Exotic electricity options and the valuation of electricity generation and transmission assets. *Decision Support Systems*, (30) 3, 383-392.

Deng, S.J. and S.S. Oren. (2003) Valuation of Electricity Generation Assets with Operational Characteristics. *Probability in the Engineering and Informational Sciences*, forthcoming.

Dixit, A.K. and R.S. Pindyck, (1994). *Investment under Uncertainty*, Princeton University Press.

Ellerman, D. (1998) Note on the Seemingly Indefinite Extension of Power Plant Lives, A Panel Contribution. *The Energy Journal*, 19 (2), 129-132.

Harvey, A. C. (1989). *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge, U.K: Cambridge University Press.

He, H. and R.S. Pindyck. (1992) Investment In Flexible Production Capacity. *Journal of Economic Dynamics and Control*, 16, 575-599.

Laughton, D.G. and H.D. Jacoby. (1993) Reversion, timing options, and long-term decision making. *Financial Management*, 33, 225-240.

Laughton, D.G. and H.D. Jacoby. (1995) The effects of reversion on commodity projects of different length. In *Real Options in Capital Investment: Models, Strategies and Applications*, edited by L. Trigeorgis, Westport: Praeger, 185-205.

McDonald, R. and D. Siegel. (1986) The value of waiting to invest. *Quarterly Journal of Economics*, 101 (4), 707-727.

Pilipović, D. (1998). *Energy Risk: Valuing and Managing Energy Derivatives*, McGraw-Hill

Ross, S. (1997) Hedging long run commitments: Exercises in incomplete market pricing. *Banca Monte Economics*, 25, 141-159.

Samuelson, P.A. (1965) Rational theory of warrant pricing. *Industrial Management*, 6, 13-31.

Schwartz, E.S. (1997) The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. *The Journal of Finance*, 52 (3), 923-973.

Schwartz, E.S. and J. E. Smith. (2000) Short-term Variations and Long-Term dynamics in Commodity Prices. *Management Science*, 46 (7), 893-911.

Smith, J.E. and K.F. McCardle. (1998) Valuing oil properties: Integrating option pricing and decision analysis approaches. *Operations Research*, 46, 1-15.

Undrum, H., Bolland, O., and E. Aarebrot. (2000) Economical assessment of natural gas fired combined cycle power plant with CO2 capture and sequestration. In *the Fifth International Conference on Greenhouse Gas Control Technologies*, Cairns, Australia.

Appendix A

A peak load plant operates only when the spark spread exceeds emission costs. The plant's value, at time *t,* is the expected cash flows less operational costs G_P

$$V_P\left(\chi(t),\xi(t),\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho\right)=\int_t^{\bar{T}} e^{-r(s-t)}\left(\bar{C}c\left(\chi(s),\xi(s),\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho\right)-G_P\right)ds, \quad \text{(A1)}$$

where $\overline{T}$ is the lifetime of the plant, $\overline{C}$ is the capacity of the plant, and $c(s)$ is the expected value of spark spread exceeding emission costs at time s, i.e.

$$c\left(\chi(s),\xi(s),\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho\right)=E\left[\max\left(S(s)-E,0\right)\mid \mathrm{F}_t\right]=\int_E^{\infty}(y-E)\,h(y)dy\,, \quad \text{(A2)}$$

where the second equality follows from the normally distributed spark spread process. In (A2) $h(y)$ is the density function of a normally distributed variable y, whose mean and variance are the mean and variance of spark spread at time s, given in Corollary 1. For clarity we rewrite the mean and variance here

$$E\left[S(T)\mid \mathrm{F}_s\right]=\mathrm{e}^{-\kappa(T-s)}\,\chi(s)+\xi(s)+\mu_\xi(T-s) \quad \text{(A3)}$$

$$Var\left(S(T)\right)=\frac{\sigma_\chi^2}{2\kappa}\left(1-\mathrm{e}^{-2\kappa(T-s)}\right)+\sigma_\xi^2(T-s)+2\left(1-\mathrm{e}^{-\kappa(T-s)}\right)\frac{\rho\sigma_\chi\sigma_\xi}{\kappa}\,. \quad \text{(A4)}$$

Integration gives

$$\begin{aligned} &c\left(\chi(s),\xi(s),\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho\right)\\ &=\frac{\sqrt{Var\left(S(s)\right)}}{\sqrt{2\pi}}e^{\left(-\frac{(E-E[S(s)\mid \mathrm{F}_t])^2}{2Var(S(s))}\right)}+\left(E\left[S(s)\mid \mathrm{F}_t\right]-E\right)\left(1-\Phi\left(\frac{E-E\left[S(s)\mid \mathrm{F}_t\right]}{\sqrt{Var\left(S(s)\right)}}\right)\right), \end{aligned} \quad \text{(A5)}$$

where $\Phi(\cdot)$ is the normal cumulative distribution function. Equations (A1) and (A5) give the value of the peak load plant

$$\begin{aligned} &V_P\left(\chi(t),\xi(t),\kappa,\mu_\xi,\sigma_\xi,\sigma_\chi,\rho\right)\\ &=\bar{C}\int_t^{\bar{T}} e^{-r(s-t)}\left(\frac{\sqrt{Var\left(S(s)\right)}}{\sqrt{2\pi}}e^{\left(-\frac{(E-E[S(s)\mid \mathrm{F}_t])^2}{2Var(S(s))}\right)}+\left(E\left[S(s)\mid \mathrm{F}_t\right]-E\right)\left(1-\Phi\left(\frac{E-E\left[S(s)\mid \mathrm{F}_t\right]}{\sqrt{Var\left(S(s)\right)}}\right)\right)\right)ds-\frac{G_P}{r}\left(1-\mathrm{e}^{-r(\bar{T}-t)}\right) \end{aligned}$$

(A6)

Appendix B

When it is not optimal to exercise the build option (i.e., when $\xi<\xi_{H0}$), the option to build F_0 must satisfy following Bellman equation

$$rF_0(\xi)dt=E\left[dF_0(\xi)\right],\ \ when\,\xi<\xi_{H0}\,. \quad \text{(B1)}$$

Itô’s lemma gives following differential equation for the option value

$$\frac{1}{2}\sigma^2\frac{\partial^2 F_0(\xi)}{\partial^2 S}+\alpha\frac{\partial F_0(\xi)}{\partial S}-rF_0(\xi)=0, \quad when\,\xi<\xi_{H0}. \tag{B2}$$

A solution to (B2) is of the form

$$F_0(\xi)=A_1\,\mathrm{e}^{\beta_1\xi}+A_2\,\mathrm{e}^{\beta_2\xi}, \quad when\,\xi<\xi_{H0}, \tag{B3}$$

where A_1, A_2 are unknown parameters and β_1 and β_2 are the roots of the fundamental quadratic equations given by

$$\beta_1=\frac{-\mu_\xi+\sqrt{\mu_\xi^{\;2}+2\sigma_\xi^{\;2}r}}{\sigma_\xi^{\;2}}>0 \tag{B4}$$

$$\beta_2=\frac{-\mu_\xi-\sqrt{\mu_\xi^{\;2}+2\sigma_\xi^{\;2}r}}{\sigma_\xi^{\;2}}<0. \tag{B5}$$

The value of the option to build approaches zero as the spark spread decreases (i.e. $A_2=0$), and thus

$$F_0(A_1,\xi,\mu_\xi,\sigma_\xi)=A_1\,\mathrm{e}^{\beta_1\xi}, \quad when\,\xi<\xi_{H0}. \tag{B6}$$