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## Design System of Composite Laminates

### Report A-2

Olli Saarela

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## ABSTRACT

A design system of composite laminates has been developed. The system is capable of finding solutions to commonly faced design problems of continuous laminates.

The target laminate is defined with a design specification consisting of constraints and objectives that can be set for important design attributes of laminates. The objectives are accompanied by weighting factors that specify their importance with respect to each other.

Two tools are provided for problem solving: one for laminate evaluation, the other for laminate creation. The tools are called, respectively, the laminate evaluation tool and the laminate creation tool.

A design specification, a set of candidate laminates, and analysis option settings define a laminate evaluation problem. The problem is solved in two steps. Feasible laminates satisfying all constraints are first sought. The multiobjective design technique is then applied to determine how well these laminates meet the objectives.

A design specification, one or several candidate plies, and analysis option settings define a laminate creation problem. The creation process is divided into two phases. Feasible plies are first sought and ranked by creating and evaluating a set of laminates representing ply performance. The design space is constrained and approximate failure analysis techniques are used to obtain a solution in decent time. As a result of the phase, the laminate with the best performance is initially identified for each feasible ply. An attempt to improve the laminate can further be made in an extended design space and with generally accepted failure analysis techniques.

The thesis describes the structure of the design system and the theories used in problem solving. Two sets of laminates are evaluated against typical design specifications to demonstrate the performance of the laminate evaluation tool. The performance of the laminate creation tool is demonstrated by solving typical design problems. The results indicate that the developed tools are capable of finding optimal solutions in the specified design space.

**Keywords:** composite laminates, multiobjective design, optimisation



## PREFACE

The research presented has been carried out in the Laboratory of Lightweight Structures at Helsinki University of Technology as part of a composites analysis and design system development project during 1992-2004. My role as a researcher was to specify the design system with its design approaches, to instruct in the implementation of the system, to verify the system, and to evaluate its performance.

I owe my sincere thanks to my supervisors, Professor (emeritus) Seppo Laine and Professor Mauri Määttänen. I am also very grateful to Markku Palanterä, Petri Kere, Francesco Pento, Timo Brander, Pauli Leppänen, Pekka Kleimola and Jani Kosonen, as well as to all the others who participated in the development project and contributed to this thesis work.

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Finally, I wish to express my warmest thanks to Kaija Leena.

Espoo, May 2005

Olli Saarela



## ORIGINAL FEATURES

The following features are believed to be original in this thesis:

- 1 Formulation of a multiobjective design problem with a design specification that allows defining constraints and objectives for important design attributes of continuous solid and sandwich laminates.
- 2 Solution to a multiobjective laminate evaluation problem utilising a laminate analysis system. Isotropic and orthotropic solid laminates, as well as sandwich laminates with isotropic and orthotropic face sheets can be evaluated simultaneously.
- 3 Solution to a multiobjective laminate creation problem utilising the concept of homogenised laminates and tailored search techniques.

The author was assisted in the work by Petri Kere, who detailed mathematical formulation of the preference function applied in laminate ranking and built a prototype code with which the specified laminate evaluation approach was tested. The laminate level failure analysis technique used in laminate creation was developed by Markku Palanterä, Jukka-Pekka Karjalainen and the author. The specified design system has been coded and integrated in the ESAComp software by the software development team.



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## LIST OF SYMBOLS

$c$	Core thickness of a sandwich laminate
$E$	Young's modulus
$\{F\}$	Load vector (nominal load)
$G$	Shear modulus
$h$	Laminate thickness
$i,j,k$	Indexes
$M$	Laminate resultant moment
$MoS$	Margin of safety
$N$	Laminate resultant in-plane force
$m$	Number of objectives; number of loads
$n$	Number of layers in a laminate; number of loads
$p$	Proportion of layer orientation; price
$Q$	Shear force; ply / laminate shear strength in the 23- / $yz$ -plane
$R$	Ply / laminate shear strength in the 31- / $zx$ -plane
$RF$	Reserve factor
$S$	Ply / laminate shear strength in the 12- / $xy$ -plane
$s$	Preference function
$s_k$	Component objective function
$t$	Ply thickness
$w$	Weighting factor
$X$	Ply / laminate strength in the direction of the axis 1 / $x$
$x$	Design attribute value
$x,y,z$	Laminate coordinate system
$Y$	Ply / laminate strength in the direction of the axis 2 / $y$
$Z$	Ply strength in the direction of the axis 3
$\alpha$	Thermal expansion coefficient
$\beta$	Moisture expansion coefficient
$\Delta c$	Thickness range or step of the core layer in a sandwich laminate
$\Delta h$	Laminate thickness range or step

$\Delta m$	Moisture content difference between the reference environment and the operating environment
$\Delta p$	Step of the proportion of layer orientation
$\Delta T$	Temperature difference between the reference environment and the operating environment
$\Delta x$	Range of a design attribute value
$\Delta \theta$	Step of the layer orientation angle
$\gamma$	Shear strain; allowed shear strain in the $xy$ -plane
$\varepsilon$	Normal strain
$\varepsilon_c$	Allowed compressive strain in the $x$ - and $y$ -directions
$\varepsilon_t$	Allowed tensile strain in the $x$ - and $y$ -directions
$\theta$	Layer orientation angle: rotation angle between the $xyz$ - and 123-coordinate systems around the $z$ -axis
$\kappa$	Curvature
$\lambda$	Relative weight
$\rho$	Density
$\sigma$	Normal stress
$\tau$	Shear stress
$\nu_{ij}$	Poisson's ratio ( $-\varepsilon_j / \varepsilon_i$ , $\sigma_i$ applied)
1,2,3	Principal coordinate system

### Subscripts

$A$	Per unit area
$a$	Applicable (range)
$all$	Specified by all constraints
$base$	Base solution
$c$	Compressive; core ply/layer
$effective$	Effective load
$f$	Face sheet of a sandwich laminate; failure
$FPF$	First ply failure
$h$	Specified by the thickness constraint
$hom$	Homogenised

<i>ini</i>	Initial value
<i>k</i>	Index
<i>L</i>	Specified by loads
<i>lb</i>	Lower bound
<i>m</i>	In maximisation and in minimisation; specified by the mass constraint
max	Maximum
min	Minimum
<i>o</i>	Operating temperature / pressure
<i>p</i>	Specified by the material cost constraint
<i>r</i>	In the search of an attribute value in a closed range; representative
<i>ref</i>	Reference value
SE	Symmetric even
<i>sf</i>	Stress-free
SO	Symmetric odd
<i>t</i>	Tensile
<i>ub</i>	Upper bound
<i>V</i>	Per unit volume
<i>x,y,z</i>	In the direction of the axis <i>x,y,z</i>
<i>xy</i>	In the <i>xy</i> -plane
$\gamma$	Specified by the allowed shear strain
$\epsilon_c$	Specified by the allowed compressive strain
$\epsilon_t$	Specified by the allowed tensile strain
$\theta$	With the orientation $\theta$
1,2,3	In the direction of the axis 1,2,3
12,23,31	In the plane 12,23,31

### Superscripts

<i>b</i>	Bottom surface of the laminate
<i>c</i>	Constant load vector; assigned/owing to the constant load vector
<i>f</i>	Flexural engineering constant / strain / normalised stress
<i>NM</i>	Due to a load specified by in-plane forces and moments
<i>r</i>	Resultant load vector; assigned/owing to the resultant load vector
<i>t</i>	Top surface of the laminate

$v$	Variable load vector; assigned/owing to the variable load vector
$\epsilon\kappa$	Owing to a load specified by midplane strains and curvatures
$\sigma$	Laminate midplane strain / normalised in-plane stress

# 1 INTRODUCTION

## 1.1 ANALYSIS AND DESIGN OF LAMINATE STRUCTURES

Fibre-reinforced laminates have several advantages compared to conventional structural materials. High stiffness-to-density ( $E/\rho$ ) and strength-to-density ( $\sigma/\rho$ ) ratios, thermal stability, good environmental resistance and excellent formability make them desirable especially in lightweight structures. An additional advantage is the possibility to tailor the mechanical and hygrothermal properties of laminates to a large extent with material constituents and their fractions, as well as with orientations and stacking of layers.

Fibre-reinforced laminates are anisotropic. Their mechanical behaviour is rather complicated but can be predicted fairly well with existing analysis techniques. The Classical Lamination Theory (CLT) provides a means to evaluate how laminates respond to mechanical and hygrothermal loads. Enhanced theories are available for more accurate analyses, for instance for studying out-of-plane stresses near the free edges of laminates. Numerous criteria have been developed for failure analyses of laminates.

For the designer, a laminated structure is a challenge owing to the wide range of tailoring options. The design process is typically one of trial-and-error, where applicable materials and plies are identified, candidate laminates are created, their performance is studied, and the laminates are modified as needed.

To speed up the design process, the designer normally uses heuristic knowledge. A classical example is the use of symmetric laminates to avoid distortion in free hygrothermal expansion. In addition, layer orientations are normally restricted to  $0^\circ$ ,  $90^\circ$  and  $\pm 45^\circ$ . These orientations already provide satisfactory performance and result in laminates that are relatively easy to manufacture.

Design charts are another technique to rationalise the design process. Examples of such charts are so-called ‘carpet plots’, which typically give values of an engineering constant, hygrothermal expansion coefficient or strength for  $0^\circ/90^\circ/\pm 45^\circ$  laminates with different proportions of layer orientations.<sup>1</sup> Buckling charts<sup>2</sup> and failure envelopes<sup>3</sup> are also commonly used in the initial design. Examples of the few closed-form solutions of design problems are the expressions for zeros and extrema of thermal expansion coefficients of symmetric balanced laminates.<sup>4</sup> Methods to create specially orthotropic and isotropic laminates have also been developed.<sup>5-7</sup>

The approaches described above usually result in a solution that is satisfactory but not optimal. Therefore, the optimisation of composite laminates and laminated structures has been extensively studied and is widely covered in the literature. Reviews on the subject are given, for instance, by Vanderplaats & Weisshaar<sup>8</sup> and by Haftka *et al.*<sup>9</sup>

The optimisation topics discussed in published papers include, for instance, the design of laminates with required stiffness properties,<sup>10,11</sup> the design of plates for maximum stiffness<sup>12-15</sup> and maximisation of the buckling load or free vibration frequencies of laminated plates.<sup>16-26</sup>

Other common objectives are minimisation of weight or cost and maximisation of strength.<sup>25-40</sup> Typically, symmetric and balanced laminate structures are only considered in these studies. The lay-up parameters, i.e. layer angles and/or layer thicknesses, are the common design variables. Thicknesses are most often treated as continuous design variables to simplify problem solving. Many researchers have also used the so-called lamination parameters as intermediate design variables.<sup>10,18,19,21,26,31,38</sup> In addition to the lay-up or lamination parameters, processing parameters have been included in the design variables.<sup>40</sup>

An optimisation problem is most often expressed in the form of a constrained minimisation or maximisation problem that is solved with an applicable mathematical algorithm. Analytical solutions have been derived for some relatively simple cases.<sup>20-22,32,38</sup> Genetic algorithms have also been applied in problem solving,<sup>24</sup> as well as multilevel search methods utilising heuristics.<sup>33,34</sup> Depending on the complexity of the structure, its load response is computed analytically or numerically.

In *multiobjective design*, objectives are set for several design attributes. *Weighting factors* can further be assigned to the objectives to specify their importance with respect to each other. Such a problem is typically solved by constructing an objective function from the objectives and their weighting factors, and from the attribute values of the candidate objects. The function is then maximised (or minimised) by requiring that the solution must lie in the feasible region specified by the *constraints* set for the design attributes. The multiobjective design technique allows evaluation of the mutual quality of the objects and, when the necessary search procedures are included, creating an object that satisfies constraints and meets objectives as well as possible.

The multiobjective design technique has been the subject of many studies. Zhang & Evans<sup>41</sup> developed a method for the design of laminates with specified properties. They defined the objective function as the sum of square differences between the calculated and required properties, using weighting factors to specify the importance of different objectives, and scale multipliers to make uniform the order of all terms. The technique was applied in searching for laminated plates with specified elastic properties. The design variables considered were the fibre orientation angles, the layer thickness and the layer engineering constants. The constrained optimisation problem was solved using a mathematical algorithm.

Saravanos & Chamis<sup>42</sup> solved a multiobjective design problem in two phases. Firstly, constrained optimisation techniques were applied to find an optimal solution separately for each objective. Secondly, a feasible point closest to the target point defined by these optimum solutions was searched for in the objective function subspace. The design objectives in the study were the minimisation of damped resonance amplitudes, the weight, and the material cost of a composite beam and plate. Constraints were imposed on static displacements, static and dynamic ply stresses, dynamic amplitudes and natural frequencies. Another example for solving a multiobjective design problem in two phases is given by Adali *et al.*<sup>43</sup> The objective of this study was to optimise a plate formed from a symmetric and balanced  $0^\circ/90^\circ/\theta/-\theta$ -type laminate for the maximum prebuckling stiffness, postbuckling stiffness and buckling load. The objective function applied was a weighted sum of these attributes. The thickness of the laminate was fixed. The layer angle  $\theta$  maximising the objective function was searched for in the first phase for different stacking sequences. In the second phase, the optimal stacking

sequence maximising the objective function was identified.

Wu *et al.*<sup>44</sup> developed an expert system for the design of composite bars under a compression load. The system searches for a solution in two phases. In the first phase, weight optimal solutions are searched for each specified cross-sectional shape and material combination. In the second phase, the solutions are evaluated with design rules to find the one that best meets the specified design rules. The system also takes into account uncertainties of information, definition and decision when assessing competing designs. Later, Wu<sup>45</sup> extended the system for the design of bolted joints of the bars.

LeRiche & Gaudin<sup>46</sup> applied enumerative and evolutionary algorithms for solving multiobjective design problems of plates under mechanical and hygrothermal loads. They solved a design problem in three phases. An enumerative algorithm was first applied to find the fibre volume fraction and the number of plies that best meet the purpose. In the second phase, evolutionary optimisation was carried out to find the best layer orientations. An evolutionary algorithm was also applied in the third phase to find the best possible stacking sequence.

Autio<sup>47</sup> performed several studies on tailoring the thermomechanical properties of composite plates. She solved single and multiobjective problems by applying mathematical algorithms in the minimisation of the objective function. Both the lay-up parameters (layer thicknesses and orientations) and the lamination parameters were used as design variables. The use of genetic algorithms was also studied for the determination of lay-up parameters from the optimised values of lamination parameters. Genetic algorithms were also applied by Walker & Smith<sup>48</sup> to find the discrete layer angles and layer thicknesses that minimise a weighted sum of the mass and deflection of a composite plate.

Another example of the multiobjective design of composites is given by Wang *et al.*<sup>49</sup> who describe a procedure for optimising both the cost and weight of a composite structure. They used the so-called cost parameter as a primary design driver. The method was applied to simplified aileron and flap structures of an aircraft. The design variables in this study were the number of ribs and spars and the laminate thicknesses. Kere & Koski<sup>50</sup> have applied the multiobjective design technique to find, from a specified subfamily of laminates, the optimal structure for multiple loading conditions. They treat laminate failure margins with respect to the various loading conditions as criteria. This approach is also applied by Kere *et al.*<sup>51</sup> to find the weight optimal laminate with the maximum value of the reserve factor.

The studies reviewed indicate that a wide variety of methods is available for the design of laminated structures. Typically, however, the methods are capable of solving specific types of problems only. The capabilities of the multiobjective approaches are also limited in the way that the constraints and objectives cannot be set for all the important design attributes. Another serious limitation is that the tools applied in problem solving have typically been developed for the purpose of the respective research studies. Thus, they are not available for the designer of composite structures who needs to solve design problems with different initial data and with varying combinations of constraints and objectives. Since the designer faces such problems continuously, it can be concluded that there is a need for versatile design methods and tools, with which the designer is able to find efficient solutions for structural design problems in his/her everyday work.

## **1.2 SCOPE AND OBJECTIVES OF THE WORK**

This thesis is related to the development of composite analysis and design software. The software, called ESAComp, is aimed to be used in a structural design process with an applicable finite element (FE) program. During the early design phases, it helps in the search for efficient materials, plies and laminate lay-ups. During the later phases, it allows the results given by FE analyses to be post-processed, that is, to evaluate how efficient the laminates are in their planned use.

To fulfil its purpose, the ESAComp software was originally specified to contain versatile analysis tools for laminates and laminated structural elements.<sup>52</sup> It was further defined that the software should contain tools that assist the user in the design of laminated structures. Consequently, the software was structured to consist of two parts, which are called the analysis system and the design system. The former allows the laminates and laminated structural elements to be specified and analysed. The latter provides a possibility to specify and solve inverse problems, i.e. to find efficient laminates for an application.

The thesis focuses on the development of the design system, the overall objective being to specify the system with its design approaches. The first aim is to specify the design capabilities, the operational features and the development tool for the design system. The second aim is to develop design approaches to the level that allows realising the first design tools. The third aim is to confirm that the developed system fulfils its purpose. Finally, the fourth aim is to provide guidelines for further development of the system.

## **1.3 METHODS**

The requirements for the design system were originally given on a very general level.<sup>52</sup> Therefore, the needs of a structural designer were evaluated first, together with the capabilities of the analysis system by which the design system was to be implemented. The evaluation led to the following conclusions:

- Finding the best possible lay-up for laminates and laminated structural elements is the key problem of a designer.
- Tools provided by the analysis program are adequate for solving such problems as far as they are limited and well defined, for instance when a laminate with maximum stiffness or zero thermal expansion in one direction is being sought.
- Compromise solutions that satisfy many constraints and meet, as well as possible, many objectives are difficult to find with the analysis tools.

Consequently, it was specified that the design system should be capable of finding lay-ups that simultaneously satisfy many constraints and meet, as well as possible, many objectives. It was further decided that the design tools would first be developed for solid and sandwich laminates, following the order applied in the development of the analysis system. Later on, the corresponding tools would be developed for laminated structural elements such as bars, beams and plates.

A possibility to define a design target was introduced in the form of a *design specification* that allows setting the constraints and objectives for important design attributes of a laminate. Two tools were further specified for laminate design: a *laminate evaluation tool* for evaluating the feasibility and mutual quality of candidate laminate lay-ups, and a *laminate creation tool* for creating feasible and efficient laminate lay-ups from candidate plies.

The laminate evaluation tool was introduced since the designer may face a problem where he or she has a specified set of laminates to choose from. The tool is also a valuable aid for studying the feasibility and quality of candidate lay-ups in the laminate creation process. The need for the laminate creation tool is obvious since the search for a feasible and efficient laminate is one of the key tasks of a composite designer.

It was originally agreed that the design system would be realised with an Expert System (ES) development tool.<sup>52</sup> The following advantages can normally be achieved with such a tool:

- Heuristics can be used to narrow down the solution space.
- A system developed with an ES tool is easily extendable.
- The end-user, having no access to the source code, is able to tailor design procedures with a natural or close-to-natural language.
- It is relatively easy to develop an explanation facility that provides a clear report on the reasoning process.

A study on ES development tools was carried out during the system specification phase to confirm the advantages and to identify possible limitations of the tools.<sup>53</sup> It was concluded that an ES development tool is especially needed to enable the user to tailor design procedures. The possibility to develop design tools step by step was seen to be another advantage of an ES development tool. Evaluation of the tools resulted in the selection of CLIPS (C Language Integrated Production System) developed by the Software Technology Branch, NASA/Lyndon B. Johnson Space Center. Other tools were ruled out mainly by the portability and platform requirements set for the analysis and design software.<sup>52</sup>

The development of the laminate evaluation tool was relatively easy since the analysis program provided all the necessary tools for computing the design attribute values of candidate laminates. The multiobjective design technique selected for the ranking of candidate solutions was specified on the basis of experiments made with a prototype code.<sup>53</sup>

The literature reviewed in the beginning of the chapter was only partly available when the laminate creation tool was developed. The literature provided useful background information for the development work but did not reveal any design approach that could be directly applied. Therefore, a tailored design approach was developed. For convenience, laminate creation was further divided into two phases called *ply evaluation* and *laminate search*. The former is aimed at finding those plies from which feasible laminates can be formed. It also provides an initial solution to the latter phase, the aim of which is to improve the solution in an extended design space by utilising simple search techniques.

The design system was realised in steps. First, a possibility to prepare design specifications was introduced. This was followed by the development and implementation of the laminate evaluation tool.<sup>54,55</sup> Next, the ply evaluation module of the laminate creation tool was

specified, developed and implemented.<sup>56</sup> The developed system was subjected to testing to ensure that it operated as specified.<sup>57</sup> The system performance was further examined with test problems to confirm that the developed tools were capable of finding optimal solutions within their operational limits.

Owing to a lack of resources and problems encountered in coding, the laminate search module of the laminate creation tool could not be finalised within the time frame of this thesis work. Instead, the functionality of the specified search procedure was verified with a prototype code that utilises the developed laminate evaluation tool in batch mode.<sup>58,59</sup>

## **1.4 OUTLINE**

The following chapters of the thesis describe the developed design system. Chapter 2 introduces the analysis program and describes briefly how the design system is integrated into the program. Chapter 3 details the design specification that defines a design target for a laminate. Chapter 4 specifies the methods used in laminate evaluation. Chapters 5 and 6 describe, respectively, how plies are evaluated in the creation of solid and sandwich laminates. Chapter 7 defines the laminate search procedure. Chapter 8 demonstrates the system performance. The results of the work and future development plans are discussed in Chapter 9. Chapter 10 summarises the work.

## 2 ESAComp SOFTWARE

This chapter gives an overview of the ESAComp analysis system and describes how the design system is integrated into the program. The notation and conventions used in ESAComp and throughout in this thesis are summarised in Appendix A.

### 2.1 ANALYSIS SYSTEM

Figure 1 illustrates the capabilities of the analysis system in the program version 3.0:

- *Micromechanical analysis tools* are provided for the evaluation of ply properties that can be achieved with a specified fibre, matrix material and fibre volume content.
- *Macromechanical analysis tools* evaluate mechanical and hygrothermal behaviour and load-carrying capability of plies, laminates, bars/beams, plates and joints.
- *Interfaces* export material data to finite element (FE) programs, import load data from FE programs, and import material/design data from external databases.
- *User extensions* can be integrated into the system.

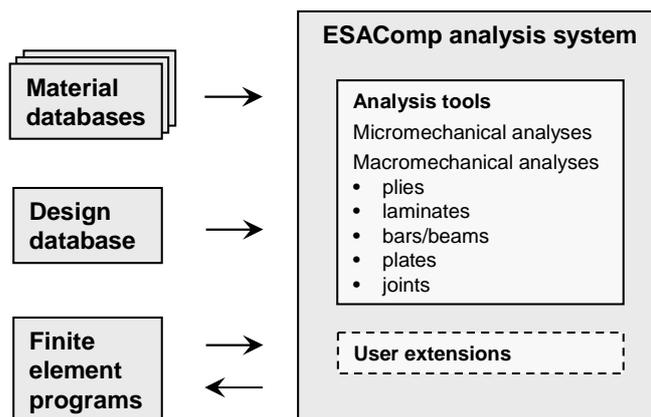


Figure 1 ESAComp analysis system.

A detailed description of the system is given in the on-line documentation of the software.<sup>60</sup> The document *Theoretical Background of ESAComp Analyses*<sup>61</sup> gives detailed information on the analysis approaches.

#### 2.1.1 Architecture

The important concepts introduced in ESAComp are *objects* and *cases*. An ESAComp object is a laminated structural element, a constituent of an element, or a load applied to an element. It may be an independent or sub-object of another object. An ESAComp case is a design study formed by a set of objects.

Figure 2 illustrates the structure of the analysis system:

- The design study in the working area is called the *active case*.
- *Specification tools* are provided for creating objects to the active case.
- *Analysis tools* perform analysis tasks related to the objects of the active case.
- Cases are stored in the *database*.
- *Database support system* allows saving the active case to the database, to activate any case in the database, to transfer objects from the database to the active case, and to export/import data from/to the program.
- *Option settings* define which of the alternative approaches the program uses.
- *Online help* guides to use the program and to design with the program.

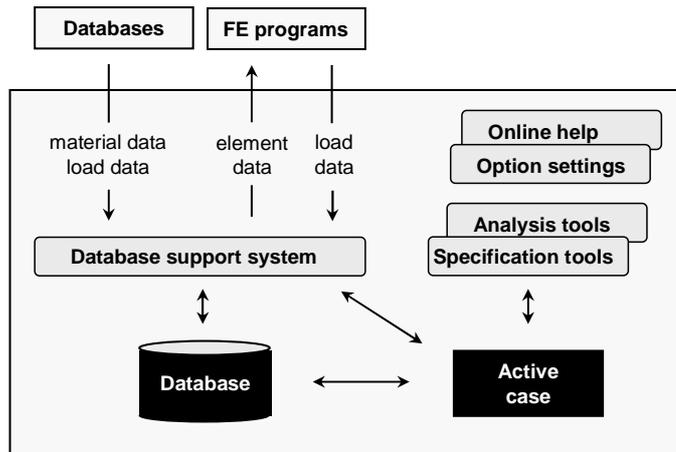


Figure 2 ESAComp analysis system - architecture.

### 2.1.2 Objects

The objects available in the software version 3.0 are fibres, matrix materials, plies, laminates, bars/beams, plates, mechanical joints, adhesively bonded joints, and loads applied to laminates and laminated structural elements. According to Figure 3, the laminates in a case are formed from plies of the case. Analogously, laminated structural elements are formed from the laminates of the case. A load is a sub-object of a laminate or of a laminated structural element.

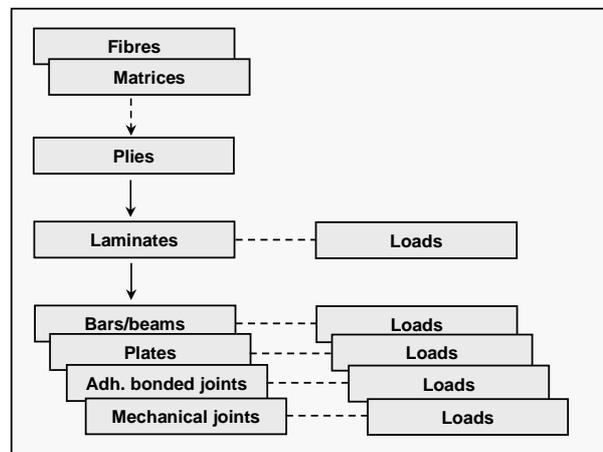


Figure 3 ESAComp case.

By default, fibres are assumed to be transversely isotropic, the plane 23 perpendicular to the longitudinal axis being the plane of isotropy. Matrix materials are assumed to be isotropic. Plies are classified on the basis of their physical nature and constitutive behaviour. The ply classes available are reinforced, homogeneous, adhesive, homogeneous core and honeycomb core for the physical nature, and orthotropic, 12 transversely isotropic, 23 transversely isotropic and isotropic for the constitutive behaviour.

Laminates may contain any types of plies. Based on their general arrangement, the laminates are classified into three groups that are (1) solid laminates, (2) sandwich laminates and (3) mixed laminates. The last group is for laminates that contain core plies but are not “classical” sandwich laminates formed by a core layer in between two reinforced or homogeneous face sheets.

Loads applied to a laminate are defined with *load vectors*. One laminate load may contain two load vectors that are called the *variable load vector* and the *constant load vector*. The concept of two vectors is used to enable a realistic description of loads that are independent of each other (i.e. of different origin) and applied simultaneously. An example of structures experiencing such loads is a spacecraft that can be subjected to a constant thrust load and to variable loads resulting from wind gusts. Also, hygrothermal and mechanical loads are often independent of each other.

A load vector consists of *load components*. These are divided into two groups, to *external loads* and to *internal loads*. Mechanical forces and moments and forced deformations are examples of the external loads that can be specified. Thermal loads and moisture loads are internal by their nature and thus referred to as internal loads. All loads are specified to be *nominal loads* which, when multiplied by a factor of safety, result in so-called *effective loads* that are used in the failure analyses of laminates.

Structural elements included in the current program version are bars and beams with different cross-sections, rectangular plates with different edge supports, mechanical joints of laminates, and adhesively bonded joints of laminates. Loads that can be applied to structural elements may contain one load vector only.

### 2.1.3 Analysis tools

The current program version contains both micro- and macromechanical analysis tools. Micromechanical tools evaluate the mechanical and hygrothermal properties of fibre-reinforced plies. They are based on the so-called rules-of-mixtures relations.

Macromechanical analysis tools are available for plies, laminates and structural elements. Three of them are used by the current design system to compute the laminate properties:

- *Laminate 2.5D behaviour* for computing the mechanical and hygrothermal properties of a laminate.
- *Laminate strength* for computing the laminate strengths in principal loading conditions.
- *Laminate load response/failure* for computing how a laminate responds to an applied load and how it is able to withstand that load.

The analyses are based on the Classical Lamination Theory (CLT). Two linear models are provided for failure analyses: The First Ply Failure (FPF) analysis assumes that a laminate fails when first failure occurs in some layer of the laminate. In the Degraded Laminate Failure (DLF) analysis, each layer of a laminate is assumed to be degraded and, thus, to carry loads with a reduced performance. The latter model resembles the so-called Last Ply Failure (LPF) analysis introduced by Tsai.<sup>3</sup>

Analysis option settings specify how the program performs the analyses. The options of interest for this study are:

- Failure criterion that can be selected separately for fibre-reinforced plies, homogeneous plies and core plies.
- Factors of safety applied to the constant and variable load vector.
- Stability factor which allows using higher effective loads in wrinkling analyses of sandwich laminates owing to the non-conservatism of the analysis.
- The option that allows defining the plane where the failure margin is computed for a layer.
- The option that allows defining whether the local instability of face sheets, i.e. wrinkling failure, is predicted or not in failure analyses of sandwich laminates.

The most commonly used failure criteria are provided for failure analyses. The end-user is further able to extend the system with new criteria.

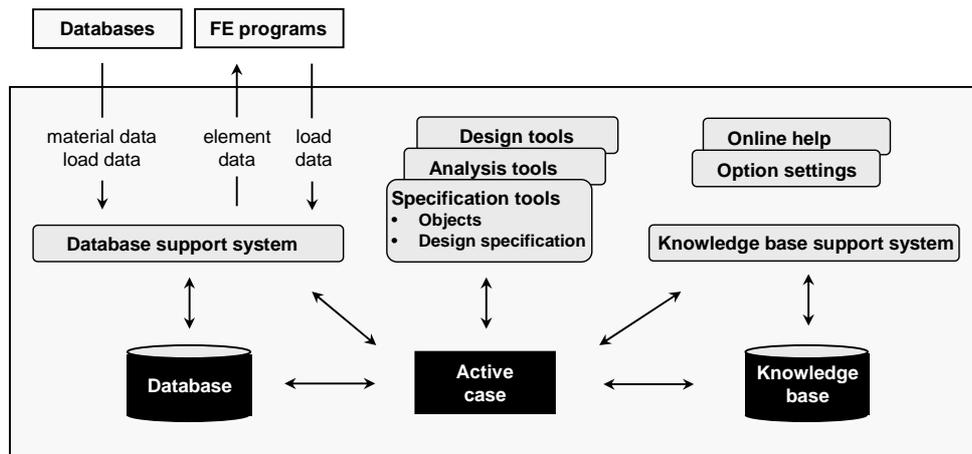
## 2.2 INTEGRATION OF THE DESIGN SYSTEM

According to the introductory chapter, the design system contains three elements. These are the design specification, the laminate evaluation tool and the laminate creation tool.

The design specification is described in detail in the following chapter. The specification is introduced in the software as an object. The design specification is case-specific and common for both design tools. This means that a case may contain only one design specification which is used both in laminate evaluation and in laminate creation. A tool for the creation and modification of a design specification has been realised analogously with the tools provided for the other ESAComp objects.

The design tools were realised by utilising the expert system development tool CLIPS. It is used for the knowledge definition of the design tools, i.e. for the definition of the design procedures described in the later chapters of the thesis. The analysis system is available for design through the CLIPS shell. This provides access to the object specification data. ESAComp tools can also be used for creating candidate solutions and for performing the necessary analyses for the solutions.

Figure 4 illustrates the ESAComp architecture after the integration of the design system. When compared with the architecture of the analysis system (Fig. 3), the new elements are the tool for creating design specifications, the knowledge base that contains knowledge definition of the design tools, and the support system provided by CLIPS for editing the knowledge. The integration of CLIPS and the realisation of the design tools are described in detail by Kere.<sup>55</sup>



**Figure 4** ESAComp analysis and design system – architecture.

### 3 DESIGN SPECIFICATION

A design specification defines the laminate being sought. This chapter describes the structure of the specification, as well as design attributes included in the specification.

#### 3.1 STRUCTURE

The design specification contains design attributes for which constraints and, as applicable, objectives can be set. Some of the attributes are qualitative since all the essential properties of laminates and laminated structural elements cannot be expressed quantitatively.

A constraint can be set for any design attribute. A constraint set for a qualitative attribute is an acceptable value or a set of acceptable values. The types of constraints that can be set for a quantitative attribute are (1) an acceptable value, (2) a set of discrete acceptable values, and (3) a range of acceptable values. A range may further be defined (3a) by the lowest acceptable value, (3b) by the highest acceptable value, or (3c) by the lowest and highest acceptable values. The first or second alternative of a range is only provided, when there is practically no need to specify other types of constraints.

Objectives can be set for quantitative attributes only. Possible types of objectives are (1) maximisation, (2) minimisation, (3) a preferred value and (4) a range of preferred values. It is further defined that an objective shall not violate a constraint set for an attribute. In other words, if a constraint has been set for an attribute, its preferred values must always lie within the feasible region specified by that constraint. This rule is also applied in maximisation and minimisation, i.e. maximisation (minimisation) is possible only when the highest (lowest) acceptable value has not been defined. Worth of noting is that the system allows defining a range of preferred values as a subset of the acceptable values. Such an option may be beneficial e.g. when the designer prefers laminates that operate at lower strain levels than the limit level specified by the constraint.

One objective may be more important than another. Therefore, an objective is always accompanied by a weighting factor that indicates the importance of the objective. Weighting factors  $w_k$  are defined within the range  $0 < w_k \leq 1$ , the value 1 indicating the highest level of importance. The default value of a weighting factor is 1. Thus, by default, all the objectives are assumed to be of equal importance. This simple approach is commonly used in multiobjective design, allowing the designer to weigh different objectives according to his/her needs.

#### 3.2 DESIGN ATTRIBUTES

The design attributes of a laminate are listed in Table 1. The type of each attribute is identified in the table, as well as the types of constraints that can be set for the attributes:

- *One* indicates that a constraint is specified with one discrete value.
- *Set* indicates that a constraint is specified with a set of discrete values.
- *Range* indicates that a constraint for a quantitative attribute is specified with a lower bound, with an upper bound, or with lower and upper bounds of acceptable values.

The reference values  $(x_k)_{ref}$  given in the last column are used to define the worst levels of attribute values in the laminate ranking. The existence of a reference value thus indicates that an objective can be set for the attribute. Most of the reference values are natural as such, since they represent the physical limits of the attribute values.

The attributes, constraints and objectives that can be set for the attributes, as well as reference values are further described in the following subsections.

**Table 1** Attributes included in the design specification of a laminate.

Category / Attribute	Type	Constraint	$(x_k)_{ref}$
<b>Laminate lay-up</b> - Ply types - Layer angles - Stacking	Qualitative Quantitative Qualitative	Set Set, Range One	
<b>Mechanical and hygrothermal behaviour</b> - In-plane modulus $E_x$ - In-plane modulus $E_y$ - In-plane modulus $G_{xy}$ - Thermal expansion coefficient $\alpha_x$ - Thermal expansion coefficient $\alpha_y$ - Moisture expansion coefficient $\beta_x$ - Moisture expansion coefficient $\beta_y$	Quantitative Quantitative Quantitative Quantitative Quantitative Quantitative Quantitative	Range Range Range Range Range Range Range	0 0 0 0 0 0 0
<b>Strength</b> - Tensile strength $X_t$ - Compressive strength $X_c$ - Tensile strength $Y_t$ - Compressive strength $Y_c$ - Shear strength $S$	Quantitative Quantitative Quantitative Quantitative Quantitative	Range Range Range Range Range	0 0 0 0 0
<b>Load-carrying capability</b>	Quantitative	Set	
<b>Deformations</b> - Maximum tensile strain $\epsilon_t$ - Maximum compressive strain $\epsilon_c$ - Maximum shear strain $\gamma$	Quantitative Quantitative Quantitative	Range Range Range	0 0 0
<b>Other design attributes</b> - Thickness $h$ - Mass per unit area $m_A$ - Material cost per unit area $p_A$ - Maximum operating temperature $T_{omax}$ - Minimum operating temperature $T_{omin}$ - Maximum operating pressure $p_{omax}$ - Minimum operating pressure $p_{omin}$ - Manufacturing technique	Quantitative Quantitative Quantitative Quantitative Quantitative Quantitative Quantitative Quantitative Qualitative	Range Range Range Range Range Range Range Range Set	0 0 0 0 °C 0 °C 1 bar 1 bar 1 bar

### 3.2.1 Laminate lay-up

#### Ply types

According to Chapter 2, plies with five different types of physical nature can be specified in the ESAComp system. The attribute *Ply types*, with a possibility to set a constraint for it, is included in the specification since the designer often wants to rule out certain ply types. For instance, he or she may rule out core plies, thus indicating that sandwich laminates are not acceptable.

A constraint specified for the attribute defines the acceptable ply types in a laminate. By default, all ply types are acceptable.

#### Layer angles

The attribute *Layer angles*, with a possibility to set a constraint for it, is included in the specification since it is often necessary to restrict the layer orientations in a laminate. Typically, for instance, layer orientations in hand lay-up are restricted to  $0^\circ$ ,  $90^\circ$  and  $\pm 45^\circ$  to simplify the manufacturing process. Some processes, e.g. filament winding, may also set physical limitations on layer orientations.

A constraint set for the attribute *Layer angles* may be a range of acceptable angles or a set of discrete acceptable angles. Positive and negative angles always appear in pairs. By default, layer angles are not constrained, i.e. all angles in the range  $0^\circ \dots \pm 90^\circ$  are acceptable.

#### Stacking

The symmetry of the lay-up with respect to the laminate midplane is probably the most common constraint set for a laminate. Typically, the lay-up must also be balanced, which means that for each off-axis layer with a positive angle, an identical layer oriented to a negative angle of the same magnitude must exist.

The attribute *Stacking*, with a possibility to set a constraint for it, allows specifying whether the laminate being sought must be symmetric and/or balanced. Thus, constraints of the type *symmetric and balanced*, *symmetric*, and *balanced* can be set. By default, any type of stacking is acceptable in a laminate evaluation. In laminate creation, only symmetrical and balanced laminates are considered, i.e. the system default constraint is *symmetric and balanced*.

### 3.2.2 Mechanical and hygrothermal behaviour

#### Engineering constants

A common problem in the design of laminated structures is to find a laminate that fulfils the constraints and meets, as well as possible, objectives set for the engineering constants. An example of such problems is the search for a laminate that maximises the modulus  $E_x$  and satisfies constraints set for the moduli  $E_y$  and  $G_{xy}$ . These in-plane moduli have been included in the design specification. They are defined as follows:

- For unsymmetrical laminates, the moduli correspond to the case where the laminate curvature is suppressed.
- For sandwich laminates, the moduli are computed by ignoring the core layer. In other words, the moduli of a laminate formed from the face sheets represent the performance of a sandwich laminate. These moduli are seen to be more informative than the actual moduli. Their use also makes in-plane stiffness comparisons of solid and sandwich laminates sensible.

For each modulus, a constraint defining the lowest acceptable value, the highest acceptable value, or a range of acceptable values can be set. Possible types of objectives are specified in Section 3.1. The physical limit value zero is used as a reference value in the laminate ranking.

### Expansion coefficients

Constraints and objectives are often set for hygrothermal expansion of a laminate. Therefore, the design specification contains thermal expansion coefficients ( $\alpha_x$ ,  $\alpha_y$ ) and moisture expansion coefficients ( $\beta_x$ ,  $\beta_y$ ) in the  $x$ - and  $y$ -directions of the laminate. For unsymmetrical laminates, they are defined to correspond to the case where the laminate curvature is suppressed.

For each expansion coefficient, a constraint defining the lowest acceptable value, the highest acceptable value, or the lowest and highest acceptable values can be set. Possible types of objectives are specified in Section 3.1. The value zero is used as a reference value when laminates are ranked on the basis of their expansion coefficient values. It is not the physical limit value of the attributes but still a logical choice for the reference value.

### 3.2.3 Strength

Laminates with specified strength properties are often sought. The strength attributes included in the design specification are the tensile and compressive strengths in the  $x$ - and  $y$ -directions ( $X_t$ ,  $X_c$ ,  $Y_t$ ,  $Y_c$ ) and the shear strength in the  $xy$ -plane ( $S$ ) of the laminate.

The strength attributes are defined as follows:

- A strength value defines the maximum load level that a laminate is able to withstand without any failure in the stress-free environment of the laminate.
- Strength values of unsymmetrical laminates correspond to the case where the laminate curvature is suppressed.
- The shear strength value of an unbalanced laminate is the lower of the two strengths computed by applying a positive and negative shear load to the laminate.
- Strength values of sandwich laminates are computed analogously with the moduli, i.e. by ignoring the core layer.
- Constraints and objectives for the attributes are specified with absolute values.

Since the designer is normally interested in achieving a specific strength level, a constraint specifying the lowest acceptable value can only be set for a strength attribute. Possible types of objectives are specified in Section 3.1. The physical limit value zero is used as a reference value in the laminate ranking.

### 3.2.4 Load-carrying capability

Only a constraint can be set for the attribute *Load-carrying capability*. It is specified with a load, or with a set of loads, that the laminate being sought must withstand. The loads are considered to be nominal loads which, when multiplied by specified factors of safety, result in effective loads that the laminate has to withstand without any failure.

### 3.2.5 Deformations

Laminate strains are often constrained to achieve the required damage tolerance and service life. Design attributes in the category *Deformations* provide a possibility to define allowed and/or desired strain levels.

The three deformation attributes available are the maximum tensile strain  $\varepsilon_t$ , the maximum compressive strain  $\varepsilon_c$  and the maximum shear strain  $\gamma$ . They are defined to be the highest absolute in-plane strain values in the laminate coordinate system  $xyz$  due to the effective forces and moments applied to the laminate. According to this definition, free hygrothermal strains, if such exist, are subtracted from actual strains to obtain the attribute values.

Constraints specifying the highest acceptable values can only be set for the deformation attributes. Possible types of objectives are specified in Section 3.1. As for the strength attributes, the physical limit value zero is used as a reference value when laminates are ranked on the basis of their deformation values.

### 3.2.6 Other design attributes

#### Thickness

A constraint or objective is often set for the thickness of the laminate being sought. Therefore, the attribute *Thickness* is included in the design specification.

A constraint defining the lowest acceptable value, the highest acceptable value, or a range of acceptable values can be set for the attribute. Except for the compatibility with the constraint, no restrictions are set for an objective. Maximisation of the attribute is thus possible though it is normally not sensible when applied alone. Together with the constraints and/or objectives set for other attributes, such an objective may, however, lead to interesting results. The physical limit value zero is used as a reference value in the laminate ranking.

#### Mass

Minimisation of mass is probably the most common structural optimisation problem. Thus, it is natural to include the attribute *Mass* in the design specification. Its value defines the mass per unit area of a laminate. A constraint specifying the highest acceptable value can only be set for the attribute. All types of objectives specified in Section 3.1 are possible. The physical limit value zero is used as a reference value in the laminate ranking.

## Material cost

Costs need to be optimised or constrained in practically all structures. The attribute *Material cost* is therefore included in the design specification. The value of the attribute defines the material cost per unit area of a laminate.

A constraint specifying the highest acceptable value can only be set for the attribute *Material cost*. All types of objectives specified in Section 3.1 are possible. The physical limit value zero is used as a reference value in the laminate ranking.

## Operating temperatures

Many commonly used material constituents of laminates have poor temperature resistance. Examples of such constituents are the polymer matrices and foamed cores of sandwich laminates. Consequently, the designer must always be aware of the operating temperatures of a structure and use materials that are capable of withstanding the temperatures.

Introducing one attribute called, for example, *Operating temperature* would enable the designer to specify the required operating temperature range of a laminate. However, setting a constraint and a sensible objective would not be possible since, according to Section 3.1, the preferred values must always lie within the feasible region specified by the constraint. Therefore, two attributes called *Maximum operating temperature* and *Minimum operating temperature* are introduced. With these two attributes, constraints and objectives can be set separately for the high- and low-temperature performance of a laminate.

A constraint defining the minimum required performance can only be set for the temperature attributes. A constraint set for *Maximum operating temperature* thus defines the lowest acceptable value of the attribute. Analogously, a constraint set for *Minimum operating temperature* defines the highest acceptable value of the attribute.

All types of objectives specified in Section 3.1 can be set for the temperature attributes. A logical choice for the reference value of both attributes is a transition temperature in between the hot and cold environment. The temperature 0 °C well represents such a temperature and is used as a reference value in the laminate ranking.

## Operating pressures

Constraints and objectives set for pressure resistance are typical in laminate design since both high- and low-operating pressures may damage a laminate. Especially sandwich laminates with lightweight core layers cannot withstand high pressures. On the other hand, low surface pressures around a laminate may cause evaporation of material constituents, which results in the degradation of the laminate.

Pressure resistance is analogous with temperature resistance such that low pressures and high pressures normally need to be considered separately. Thus, two attributes called *Maximum operating pressure* and *Minimum operating pressure* are introduced.

Constraints can be set for the pressure attributes just as for the temperature attributes. All types of objectives specified in Section 3.1 are possible. A logical choice for the reference value of both attributes is the normal ambient pressure. This is represented by the numeric value 1 bar in accordance with the default pressure unit of ESAComp.

### **Manufacturing technique**

The ESAComp system allows specifying the applicable manufacturing techniques for each ply, i.e. the techniques that can be applied when the laminates are manufactured from the ply. The possible techniques are:

- Wet lay-up
- Prepreg lay-up
- Spray lay-up
- Filament winding
- Resin transfer moulding
- Press moulding
- Pultrusion.

Only some of the techniques are normally applicable in the manufacture of a laminate structure. Therefore, the design attribute *Manufacturing technique*, with a possibility to set a constraint for it, is included in the design specification. The constraint specifies the set of acceptable manufacturing techniques.

## 4 LAMINATE EVALUATION

The laminate evaluation tool searches for feasible and efficient laminates amongst candidate laminates. This chapter describes how a laminate evaluation problem is specified and solved.

### 4.1 PROBLEM SPECIFICATION

The following data specifies a laminate evaluation problem:

- Design specification defining constraints, objectives and weighting factors of the objectives for design attributes
- Specifications of laminates being evaluated
- Specifications of plies forming the laminates
- Analysis option settings, as applicable.

The analysis option settings define how the laminate failure analyses are performed. Thus, they are relevant when constraints and/or objectives have been set for the strength attributes or when a constraint has been set for the attribute *Load-carrying capability*.

### 4.2 EVALUATION TECHNIQUES

The evaluation of candidate laminates against a design specification is a relatively straightforward task. Firstly, design attribute values are determined for the laminates. Secondly, each laminate is evaluated against the constraints to find whether the laminate is feasible or not. Finally, the feasible laminates are evaluated against the objectives to determine their mutual quality. A detailed description of the phases is given in the following subsections.

### 4.3 ATTRIBUTE VALUES OF A LAMINATE

By utilising the analysis system, the value of any design attribute is easy to determine for any candidate laminate: it is either available in the ply or laminate specification data, it can be derived from the specification data, or it can be computed with a tool of the analysis system.

#### 4.3.1 Lay-up

A laminate specification in ESAComp contains the information needed to determine design attribute values in the category *Laminate lay-up*:

- Value of the design attribute *Ply types* is readily available in the specification.
- Value of the attribute *Layer angles* is derived from the lay-up by forming a set of angles that contains all different layer orientations of the laminate.
- General arrangement of layer orientations is identified in the specification as cross-ply, balanced or unbalanced. Additionally, midplane symmetry is identified as symmetrical, antisymmetrical or unsymmetrical. The value of the attribute *Stacking* (symmetric and balanced, symmetric, balanced or other) is derived from this information by taking into account that cross-ply laminates are a subgroup of balanced laminates.

### 4.3.2 Mechanical and hygrothermal behaviour

The tool *Laminate 2.5D behaviour* of the analysis system is used to compute the engineering constants and expansion coefficients of a candidate laminate. For an unsymmetrical laminate, the tool computes the constants and coefficients both in the zero-curvature state and by allowing the laminate to curve. In accordance with the definitions in Chapter 3, the former values are selected to represent the laminate performance.

Before computing the engineering constants of a sandwich laminate, the design system modifies its lay-up by ignoring the core layer. Thus, as defined in Chapter 3, the moduli are computed for a solid laminate formed from the face sheets. The expansion coefficients are, however, computed for the actual laminate.

### 4.3.3 Strength

The tool *Laminate strength* of the analysis system is used to compute the values of strength attributes for candidate laminates. In accordance with the definitions of Chapter 3, the constraints/objectives set for laminate strengths define the load levels that the laminates are required/desired to withstand without any failure. Thus, the FPF-type analysis providing strength values for intact laminates is used. As in the computation of engineering constants, the design system modifies the lay-up of a sandwich laminate by ignoring the core layer before computing the laminate strengths.

### 4.3.4 Load-carrying capability

The tool *Laminate failure* of the analysis system is used to determine how a candidate laminate withstands a load included in the design specification. Since no ply failure is allowed, the tool is used in the FPF analysis mode. The tool provides the result in the form of a reserve factor  $RF$  that defines the ratio of the failure load to the effective load. A reserve factor  $RF \geq 1$  indicates that the laminate is able to withstand the load.

When a load consists of a constant and variable load vector, the failure analysis tool computes reserve factors in the five possible load cases specified by the load, i.e. when

- Constant load is applied alone
- Variable load is applied alone
- Constant and variable loads are applied simultaneously and the load is assumed to increase in the direction of the variable load
- Constant and variable loads are applied simultaneously and the load is assumed to increase in the direction of the constant load
- Constant and variable loads are applied simultaneously and the load is assumed to increase in the direction of the resultant load.

The evaluation tool selects the lowest of these to represent laminate performance. Worth of noting is that some reserve factors have indefinite values when the constant or variable load applied alone results in failure. These factors are ignored in the evaluation.

When the design specification contains several loads, the evaluation tool performs a failure analysis by applying the loads to the laminate one by one. The tool further selects the lowest of the reserve factors to represent load-carrying capability of the laminate.

### 4.3.5 Deformations

The tool *Laminate failure* of the analysis system also provides laminate strains owing to the effective load. Thus, the laminate deformations are computed simultaneously with the reserve factors. The attribute values are derived from surface strains since the highest and lowest strains always appear on the laminate surfaces. In accordance with the specifications of Chapter 3, the attribute values are determined as follows:

- Maximum tensile strain  $\varepsilon_t$  is the highest of the tensile surface strains in the  $x$ - and  $y$ -directions owing to the applied in-plane forces and moments specified by the load.
- Maximum compressive strain  $\varepsilon_c$  is the highest of the absolute compressive surface strains in the  $x$ - and  $y$ -directions owing to the applied in-plane forces and moments specified by the load.
- Maximum shear strain  $\gamma$  is the higher of the absolute surface shear strains in the  $xy$ -plane owing to the applied in-plane forces and moments specified by the load.

When the load consists of a constant and variable load vector, the values of an attribute are first determined in the three load cases specified by the constant, variable and resultant load vectors. The highest value is further given for the attribute. Analogously, when the design specification contains several loads, the value of an attribute is first determined for each load and the highest of these values is given for the attribute.

### 4.3.6 Other design attributes

The values of other design attributes are derived for candidate laminates from the laminate specification and from the specifications of plies forming the laminate:

- Values of the design attributes *Thickness*, *Mass* and *Material cost* are read from the laminate specification.
- Maximum and minimum operating temperatures and pressures are derived from ply specifications by selecting the ply with the poorest performance to represent laminate performance (e.g. when maximum operating temperatures of two plies forming a laminate are 80 °C and 120 °C, the tool defines the maximum operating temperature of the laminate to be 80 °C).
- A value for the attribute *Manufacturing technique* is derived from the ply specifications, the value being the set of techniques applicable for each ply of the laminate.

## 4.4 FEASIBILITY OF A LAMINATE

The feasibility of a candidate laminate is studied by comparing its design attribute values with constraints. All constraints are considered with an exception that feasibility against deformation constraints is determined only when the load-carrying capability of the laminate is feasible. This exception is natural since deformations of a failed laminate are meaningless. When the feasibility of each design attribute value is known for each candidate laminate, feasible laminates satisfying all constraints are identified.

Feasibility of attribute values is determined as follows:

- Values of the attributes *Ply types* and *Layer angles* are feasible if they are subsets of acceptable values specified by the constraints.
- Value of the attribute *Stacking* is feasible, as defined in Table 2.
- Value of the attribute *Load-carrying capability* is feasible if the lowest reserve factor  $RF_{\min}$ , as specified in Subsection 4.3.4, satisfies the condition  $RF_{\min} \geq 1$ .
- Values of other quantitative design attributes are feasible if they are within the limits specified by the constraints.
- Value of the attribute *Manufacturing technique* is feasible if at least one common technique exists in the sets that specify the attribute value and the constraint.

**Table 2** Feasible values of the attribute *Stacking*.

Constraint	Feasible values
Symmetric and balanced	Symmetric and balanced
Symmetric	Symmetric and balanced, symmetric
Balanced	Symmetric and balanced, balanced

## 4.5 MUTUAL QUALITY OF LAMINATES

Multiobjective design is used to rank feasible laminates with respect to each other. The aim is to determine, quantitatively, how closely the laminates meet the goal defined by the objectives and their weighting factors. The following procedure is used in ranking:

1. Design attribute values of the laminates are computed corresponding to each objective.
2. A component objective function is constructed corresponding to each objective.
3. The preference function is formed from the component objective functions.
4. Values of the component objective functions and the preference function are computed for the laminates.
5. The laminates are arranged according to their values of the preference function.

The next two subsections summarise how the component objective functions and the preference function are formed.

#### 4.5.1 Component objective functions

Component objective functions are constructed so that values of the functions change in the objective space linearly and monotonically in the interval  $[0,1]$ . At the best level the functions get the value 1 and at the worst level the value 0. The type of the objective defines the form of the function.

When the objective is to find a laminate with the highest design attribute value, the component objective function is of the form (Fig. 5a):

$$s_k(x_k) = 1 - \frac{(x_k)_{\max} - x_k}{(\Delta x_k)_m}, \quad (x_k)_{\min} \leq x_k \leq (x_k)_{\max} \quad (1)$$

where  $(x_k)_{\min}$  and  $(x_k)_{\max}$  are, respectively, the lowest and highest values found for the attribute. The term  $(\Delta x_k)_m$ , defining the worst level of  $x_k$  with respect to  $(x_k)_{\max}$ , is specified by  $(x_k)_{\min}$  and  $(x_k)_{\max}$ , and by the attribute specific reference value  $(x_k)_{ref}$ :

$$\begin{aligned} (\Delta x_k)_m &= (x_k)_{\max} - (x_k)_{ref}, & (x_k)_{ref} &< (x_k)_{\min} \\ (\Delta x_k)_m &= (x_k)_{\max} - (x_k)_{\min}, & (x_k)_{\min} &\leq (x_k)_{ref} \leq (x_k)_{\max} \\ (\Delta x_k)_m &= (x_k)_{ref} - (x_k)_{\min}, & (x_k)_{ref} &> (x_k)_{\max} \end{aligned} \quad (2)$$

Analogously, with an objective to find a laminate with the lowest attribute value, the function is of the form (Fig. 5b):

$$s_k(x_k) = 1 - \frac{x_k - (x_k)_{\min}}{(\Delta x_k)_m}, \quad (x_k)_{\min} \leq x_k \leq (x_k)_{\max} \quad (3)$$

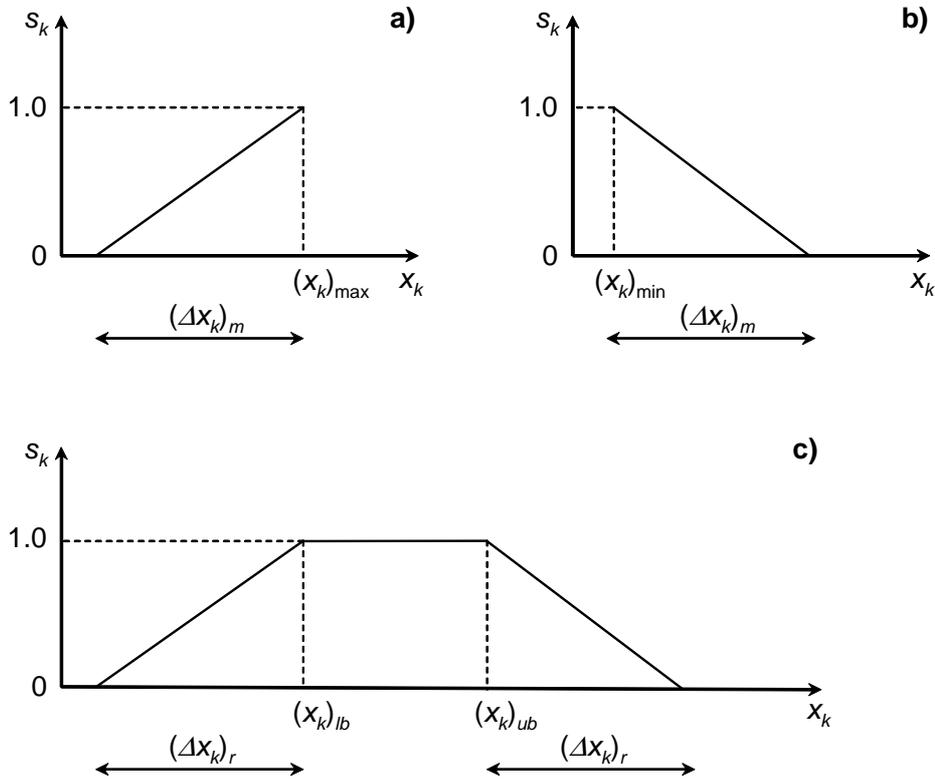
With an objective specifying a closed range of desired values, the component objective function is of the form (Fig. 5c):

$$\begin{aligned} s_k(x_k) &= 1 - \frac{(x_k)_{lb} - x_k}{(\Delta x_k)_r}, & (x_k)_{\min} &\leq x_k < (x_k)_{lb} \\ s_k(x_k) &= 1, & (x_k)_{lb} &\leq x_k \leq (x_k)_{ub} \\ s_k(x_k) &= 1 - \frac{x_k - (x_k)_{ub}}{(\Delta x_k)_r}, & (x_k)_{ub} &< x_k \leq (x_k)_{\max} \end{aligned} \quad (4)$$

where  $(x_k)_{lb}$  and  $(x_k)_{ub}$  are, respectively, the lower and upper bounds of the specified range. The term  $(\Delta x_k)_r$  is:

$$\begin{aligned}
 (\Delta x_k)_r &= \max \left[ (x_k)_{\max} - (x_k)_{ub} ; (x_k)_{lb} - (x_k)_{ref} \right] , \quad (x_k)_{ref} < (x_k)_{\min} \\
 (\Delta x_k)_r &= \max \left[ (x_k)_{\max} - (x_k)_{ub} ; (x_k)_{lb} - (x_k)_{\min} \right] , \quad (x_k)_{\min} \leq (x_k)_{ref} \leq (x_k)_{\max} \quad (5) \\
 (\Delta x_k)_r &= \max \left[ (x_k)_{ref} - (x_k)_{ub} ; (x_k)_{lb} - (x_k)_{\min} \right] , \quad (x_k)_{ref} > (x_k)_{\max}
 \end{aligned}$$

Component objective functions for other types of objectives, i.e. for an objective to find a value in a range specified by an upper or lower bound and for an objective to find a specified (exact) attribute value, are derivatives of Eqs. (4) and (5).



**Figure 5** Forms of component objective functions: **a)** maximisation, **b)** minimisation, and **c)** a closed range of desired values.

#### 4.5.2 Preference function

Each of the  $k$  objectives set for quantitative attributes is accompanied by a weighting factor  $w_k$  that indicates the importance of the objective. As was noted in Section 3.1, the factors are specified within the range  $0 < w_k \leq 1$ , the value 1 indicating the highest level of importance.

To form the preference function, relative weights  $\lambda_k$  of the objectives are derived from the weighting factors by rescaling them so that their sum equals 1:

$$\lambda_k = \frac{w_k}{\sum_{k=1}^m w_k} \quad (6)$$

Summing the products of the component objective functions and their relative weights forms the preference function:

$$s = \sum_{k=1}^m \lambda_k s_k(x_k) \quad (7)$$

With the adopted technique, the values of the preference function are always in the range  $[0,1]$ , the best possible solution having the value 1.

## 5 PLY EVALUATION - SOLID LAMINATES

According to Chapter 1, laminate creation is performed in two phases. The first is called the ply evaluation phase, whose aim is to find feasible plies amongst the set of candidate plies. The process also provides for each feasible ply an initial solution of the laminate creation problem. This chapter describes the evaluation process when solid laminates are being created.

### 5.1 PROBLEM SPECIFICATION

A laminate creation problem is restricted to solid laminates when the set of candidate plies does not contain core plies. The following data specify the ply evaluation phase:

- Design specification defining constraints, objectives and weighting factors of the objectives for design attributes
- Specifications of candidate plies from which laminates are to be formed
- Reference environments of candidate plies, as applicable
- Analysis option settings, as applicable.

Reference environment defines the stress-free temperature and moisture content of a laminate. It is ply-specific, since laminates formed from two or more different plies are not considered in the evaluation (see Section 5.2). Stress-free temperature must be defined for each ply if the set of loads in the design specification contains thermal loads. Analogously, the stress-free moisture content must be defined if the set of loads contains moisture loads.

The analysis option settings, listed in Chapter 2, define how the laminate failure analyses are performed. Thus, they are relevant when constraints and/or objectives have been set for the strength attributes or when a constraint has been set for the attribute *Load-carrying capability*.

### 5.2 EVALUATION TECHNIQUES

Ply evaluation is performed by evaluating feasibility and quality of laminates formed from the candidate plies. The laminates formed from two or more different plies are ignored in the evaluation process though the so-called hybrid laminates are beneficial in some applications. The reason for this is simple: laminate creation would become far too complicated if all combinations of candidate plies with different fractions of the plies were considered.

#### 5.2.1 Classification of plies and design attributes

The set of laminates needed to represent ply performance depends on the constitutive behaviour of the ply. By considering only the design attributes for which constraints and objectives can be set, the plies fall into two groups. One group is formed by isotropic and 12 transversely isotropic plies, the other by orthotropic and 23 transversely isotropic plies. For simplicity, the groups are later referred to as *in-plane isotropic* and *in-plane orthotropic plies*.

The design attributes are divided into groups on the basis of parameters affecting their values. The first group is formed by the attributes, the values of which are dependent on the ply

properties only. Such attributes are *Ply types*, *Maximum operating temperature*, *Minimum operating temperature*, *Maximum operating pressure*, *Minimum operating pressure*, and *Manufacturing technique*. These attributes are later referred to as *ply-specific attributes*. The second group is formed by the attributes, the values of which are dependent on the ply properties and on the laminate structure but not on the laminate thickness. Such attributes are the engineering constants, hygrothermal expansion coefficients and strengths. The attributes are referred to as *lay-up-dependent attributes*. The rest of the attributes fall into the third group, in which attribute values are dependent also on the laminate thickness. These attributes are referred to as *thickness-dependent attributes*.

### 5.2.2 Laminates representing ply performance

A huge number of laminates can be created even from one orthotropic ply. The set of laminates representing the performance of a candidate ply must therefore be limited to find a solution for the ply evaluation problem in decent time.

As a first simplification the ply thickness, if such exists in the ply specification, is ignored. This makes laminate thickness a continuous design variable. As a consequence, only one single-layer laminate is needed to represent the performance of an in-plane isotropic ply. The thickness of the laminate is defined by constraints and objectives set for the thickness-dependent attributes.

Laminate thickness is further constrained for computational reasons. The lower limit of the range is set to 0.1 mm. Laminates thinner than that seldom have any practical use. The upper limit is set to a high value to avoid additional limitations of the tool. The value 250 mm is used though it is known that the laminates with such thickness are not normally needed and cannot normally be manufactured at least in one cure cycle.

The laminate representing the performance of an in-plane orthotropic ply must be sought amongst a set of laminates. An important simplification applied in the search is the use of *homogenised laminates* instead of actual laminates. A homogenised laminate in this context refers to a laminate that is formed by merging layers with different orientations in the specified proportions of the orientations. The laminate thus possesses constant mechanical and hygrothermal properties through its thickness. Its in-plane behaviour corresponds to the behaviour of an actual symmetrical laminate formed from the ply with same layer orientations and proportions of layer orientations. The flexural engineering constants equal with the corresponding in-plane engineering constants.

The concept of homogenised laminates rules out unsymmetrical laminates. This is seen to be acceptable, since such laminates are normally not used owing to their distortion in free hygrothermal expansion. A more serious drawback of the concept is that one design variable of actual laminates, the stacking sequence, is not available for obtaining the best performance e.g. under bending loads. The drawback is compensated by the common practice to disperse layers with different orientations evenly through the thickness of the laminate to avoid thick stacks of unidirectional layers that are known to be susceptible to matrix cracking.

Layer orientations of the laminates considered in the ply evaluation phase are further constrained as follows:

- Positive and negative layer orientation angles are assumed to appear in pairs. In other words, the laminates are assumed to have a balanced structure.
- The number of positive and negative layer orientation angles is restricted to three. The laminates created are thus of the type  $[\pm\theta_{\min}/\pm\theta/\pm\theta_{\max}]$ .
- The values  $0^\circ$  and  $90^\circ$  are given, respectively, for the angles  $\theta_{\min}$  and  $\theta_{\max}$  if these values are feasible. If not, the closest feasible value is used.
- The values given for the angle  $\theta$  are:
  - $\theta = 45^\circ$  or, if this value is infeasible, the closest feasible value
  - the angle that minimises longitudinal expansion coefficients of balanced  $\pm\theta$  laminates, if such an angle exists and constraints or objectives have been set for hygrothermal expansion coefficients or, if this value is infeasible, the closest feasible value
  - the angle that minimises transverse expansion coefficients of balanced  $\pm\theta$  laminates, if such an angle exists and constraints or objectives have been set for hygrothermal expansion coefficients or, if this value is infeasible, the closest feasible value.

The existence of the minima is checked and the layer orientation angles minimising the expansion coefficients are determined with the closed form solutions available.<sup>4</sup>

- Proportions of layers with orientations  $\pm\theta_{\min}$ ,  $\pm\theta$  and  $\pm\theta_{\max}$  are constrained to values:

$$p_{\theta_{\min},\theta,\theta_{\max}} = k \cdot 0.25 \quad , \quad k = 0,1,2,3,4 \quad (8)$$

$$p_{\theta_{\min}} + p_{\theta} + p_{\theta_{\max}} = 1$$

The number of homogenised laminates being created from each in-plane orthotropic ply thus sums up to 15 when the constraints or objectives have not been set for hygrothermal expansion coefficients, and to  $3 \times 15 = 45$  when the constraints or objectives have been set for hygrothermal expansion coefficients and the angles minimising longitudinal and transverse expansion coefficients of balanced  $\pm\theta$ -laminates exist.

The following arguments are given for the specified set of laminates:

- Laminates in practical applications normally meet the in-built thickness constraint.
- Unbalanced laminates are not considered since they are seldom used in practical applications.

- Layer orientation angles are restricted to three since the use of more orientations seldom improves the design. The restriction is also a recommended design practice since manufacturing costs increase with an increasing number of layer orientations.
- Laminate analyses indicate that the range of laminate properties obtainable with in-plane orthotropic plies can be represented reasonably well with the specified layer orientations and proportions of the layer orientations.

### 5.2.3 Laminate analyses

Different types of analyses must be performed for laminates during the ply evaluation process. The techniques applied in the analyses are summarised below.

#### **Stiffness, hygrothermal expansion and strength**

The engineering constants, expansion coefficients and strengths of an in-plane isotropic laminate are equal to the corresponding values of the ply forming the laminate. The values for the attributes are thus read from the ply specification data. The values of first failure stresses are given for the strength attributes since the constraints define the required performance of an intact laminate.

The engineering constants, hygrothermal expansion coefficients and strengths of a homogenised laminate are determined by creating and analysing an actual laminate with identical layer orientations and proportions of layer orientations. The attribute values are computed with the tools *Laminate 2.5D behaviour* and *Laminate strength* of the analysis system. To minimise the computation time, an unsymmetrical laminate containing one layer for each orientation angle is analysed in the zero-curvature state. This yields an identical result with an analysis of a symmetric laminate having the same layer orientations and proportions of layer orientations. The FPF model is used in the strength analyses since the constraints define the required performance of an intact laminate. Failure is predicted with the failure criterion specified by the analysis option settings.

#### **Load-carrying capability**

The load-carrying capability of a laminate is evaluated with the tool *Laminate failure* of the analysis system. The FPF model is used in the analysis since laminates must withstand applied loads without any failure. A laminate is identified to withstand a load if the reserve factor provided by the analysis satisfies the condition  $RF \geq 1$ . When a load consists of a constant and variable load vector, a laminate is identified to withstand the load if the lowest of the reserve factors, as specified in Section 4.3.4, satisfies the condition.

An in-plane isotropic laminate is analysed using the failure criterion selected for the ply type in the problem specification. The reserve factors provided by the analysis for the single-layer laminate are identical with the reserve factors of the corresponding multi-layer laminates, since laminates formed from in-plane isotropic plies have constant in-plane properties through their thickness.

A simplified failure analysis approach is applied for homogenised laminates. The analysis is performed by applying a failure criterion in the laminate level, i.e. by treating the homogenised laminate as a ply. The in-plane strengths computed for the corresponding actual laminate in the principal loading conditions with the conventional ply-level failure analysis are used as reference stresses in the failure criterion function. To approximate the effects of internal stresses, these in-plane strengths are computed in the operating environment defined by the load. A more thorough description of the approach is given by Palanterä *et al.*<sup>62</sup>

It is known that the applied laminate-level failure analysis may provide unrealistic results.<sup>62</sup> Figure B1 of Appendix B demonstrates this by displaying combinations of (normalised) failure stresses  $\sigma_x$  and  $\sigma_y$  computed for a set of actual and homogenised laminates. The actual laminates have been analysed conventionally by applying the maximum stress criterion and the ply-level failure analysis. The homogenised laminates have been analysed by applying the same failure criterion in the laminate level. Since in-plane strengths computed for the corresponding actual laminates have been used as reference stresses in the laminate-level analyses, the results of the two approaches are equal in the principal loading conditions. However, they typically differ from each other when both  $\sigma_x$  and  $\sigma_y$  have non-zero values.

The laminate set analysed in Figure B1 consists of symmetrical  $[0^\circ/90^\circ/45^\circ/-45^\circ]$ ,  $[0^\circ/0^\circ/45^\circ/-45^\circ]$ ,  $[45^\circ/-45^\circ]$  and  $[0^\circ/90^\circ]$  laminates, which represent relatively well the different types of laminates considered in the ply evaluation phase. Poor correlation of the failure envelopes is evident. Figure B2 of Appendix B points out that the correlation is also poor when the interactive Tsai-Hill criterion is used in ply- and laminate-level failure analyses. The other interactive criteria give similar results. The correlation is typically better with all criteria when a normal and shear load, i.e.  $\sigma_x$  and  $\tau_{xy}$  or  $\sigma_y$  and  $\tau_{xy}$ , are applied simultaneously. However, it may get worse when all in-plane stress components have non-zero values.

According to Figure B3 of Appendix B, a reasonable correlation of failure envelopes is achieved when the maximum strain criterion is applied in the ply- and laminate-level analyses. The failure envelopes of actual laminates computed with the criterion also agree relatively well with the envelopes based on other criteria, except for the third quadrant where the Hoffman and Tsai-Wu criteria give considerably different results (Fig. B4 of Appendix B).

Based on this short evaluation, the maximum strain criterion is specified to be used in all laminate-level failure analyses. Another criterion selected by the user with analysis option settings is thus ignored in the ply evaluation phase. The selected approach can be considered a simple compromise that is used to speed-up the ply evaluation phase. The applicability of the approach is further discussed in Chapter 9.

### **Deformations owing to in-plane forces and moments**

According to Chapter 3, deformation attributes specify, with absolute strain values, maximum in-plane strains due to effective in-plane forces and moments applied to a laminate. Deformations owing to a load specified by in-plane forces and moments are sought amongst the surface strains that are known to represent the highest and lowest strain levels in a laminate. In the most general case, when the load consists of a constant and variable load

vector and both vectors specify an external load, the values are computed for a homogenised laminate as follows:

1. Since deformation attributes specify maximum in-plane strains owing to effective in-plane forces and moments, the effective load vectors defined by the nominal load vectors  $\{F\}$  and by the factors of safety  $FoS$  assigned to the load vectors are computed with the equations

$$\begin{aligned}\{F\}^c_{effective} &= FoS^c \cdot \{F\}^c \\ \{F\}^v_{effective} &= FoS^v \cdot \{F\}^v \\ \{F\}^r_{effective} &= FoS^c \cdot \{F\}^c + FoS^v \cdot \{F\}^v\end{aligned}\quad (9a-c)$$

where the superscripts  $c$ ,  $v$  and  $r$  refer, respectively, to the constant, variable and resultant load vectors.

2. Surface strains owing to each effective load vector are computed with closed form solutions available:

$$\begin{aligned}\epsilon_x^{t,NM} &= \frac{1}{h} \left( \frac{1}{E_x} N_x - \frac{\nu_{yx}}{E_y} N_y \right) - \frac{6}{h^2} \left( \frac{1}{E_x} M_x - \frac{\nu_{yx}}{E_y} M_y \right) \\ \epsilon_x^{b,NM} &= \frac{1}{h} \left( \frac{1}{E_x} N_x - \frac{\nu_{yx}}{E_y} N_y \right) + \frac{6}{h^2} \left( \frac{1}{E_x} M_x - \frac{\nu_{yx}}{E_y} M_y \right) \\ \epsilon_y^{t,NM} &= \frac{1}{h} \left( -\frac{\nu_{xy}}{E_x} N_x + \frac{1}{E_y} N_y \right) - \frac{6}{h^2} \left( -\frac{\nu_{xy}}{E_x} M_x + \frac{1}{E_y} M_y \right) \\ \epsilon_y^{b,NM} &= \frac{1}{h} \left( -\frac{\nu_{xy}}{E_x} N_x + \frac{1}{E_y} N_y \right) + \frac{6}{h^2} \left( -\frac{\nu_{xy}}{E_x} M_x + \frac{1}{E_y} M_y \right) \\ \gamma_{xy}^{t,NM} &= \frac{1}{h} \frac{1}{G_{xy}} N_{xy} - \frac{6}{h^2} \frac{1}{G_{xy}} M_{xy} \\ \gamma_{xy}^{b,NM} &= \frac{1}{h} \frac{1}{G_{xy}} N_{xy} + \frac{6}{h^2} \frac{1}{G_{xy}} M_{xy}\end{aligned}\quad (10a-f)$$

where  $N_x$ ,  $N_y$ ,  $N_{xy}$ ,  $M_x$ ,  $M_y$ ,  $M_{xy}$  are the effective in-plane forces and moments specified by the load vector,  $E_x$ ,  $E_y$ ,  $G_{xy}$ ,  $\nu_{xy}$  and  $\nu_{yx}$  are the in-plane engineering constants of the laminate, and  $h$  is the thickness of the laminate. The superscripts  $t$  and  $b$  on the left-

hand side of the equations refer, respectively, to the top and bottom surfaces. The superscript  $NM$  refers to strains owing to a load given in the form of in-plane forces and moments.

3. The maximum tensile, compressive and shear strains owing to each effective load vector are computed with the equations

$$\begin{aligned} (\epsilon_t^{NM})_{c,v,r} &= \left( \max \left[ \epsilon_x^{t,NM}; \epsilon_x^{b,NM}; \epsilon_y^{t,NM}; \epsilon_y^{b,NM}; 0 \right] \right)_{c,v,r} \\ (\epsilon_c^{NM})_{c,v,r} &= \left( \left| \min \left[ \epsilon_x^{t,NM}; \epsilon_x^{b,NM}; \epsilon_y^{t,NM}; \epsilon_y^{b,NM}; 0 \right] \right| \right)_{c,v,r} \\ (\gamma^{NM})_{c,v,r} &= \left( \max \left[ \left| \gamma_{xy}^{t,NM} \right|; \left| \gamma_{xy}^{b,NM} \right| \right] \right)_{c,v,r} \end{aligned} \quad (11a-c)$$

Equation (11a) is formulated so that the tensile strain attribute gets the value zero when both surfaces are compressed in the  $x$ - and  $y$ -directions. Analogously, Eq. (11b) sets the compressive strain value to zero when both surfaces are tensionally strained in the  $x$ - and  $y$ -directions.

4. The values of the deformation attributes are the maximum values given by Eqs. (11) for the three load vectors:

$$\begin{aligned} \epsilon_t^{NM} &= \max \left[ (\epsilon_t^{NM})_c; (\epsilon_t^{NM})_v; (\epsilon_t^{NM})_r \right] \\ \epsilon_c^{NM} &= \max \left[ (\epsilon_c^{NM})_c; (\epsilon_c^{NM})_v; (\epsilon_c^{NM})_r \right] \\ \gamma^{NM} &= \max \left[ (\gamma^{NM})_c; (\gamma^{NM})_v; (\gamma^{NM})_r \right] \end{aligned} \quad (12a-c)$$

The process also provides deformation attribute values for in-plane isotropic laminates when it is noted that for such laminates  $E_x = E_y = E$ ,  $\nu_{xy} = \nu_{yx} = \nu$ , and  $G_{xy} = G = E/[2(1+\nu)]$ .

### Deformations owing to midplane strains and curvatures

The search procedure applied requires that laminate deformations owing to a load specified by midplane strains and curvatures are computed twice: first by ignoring, then by taking into account the curvatures. The techniques used in the computation are described below for homogenised laminates. The same techniques are applied for in-plane isotropic laminates by accounting for simplifications owing to in-plane isotropy.

When the curvatures are ignored, the deformation attribute values are derived from the laminate midplane strains that are known to represent the highest and lowest strain levels of the laminates with no curvatures. The values are computed as follows:

1. The effective constant, variable and resultant load vectors are computed with Eqs. (9) by ignoring the curvatures, if such exist in the load vectors.

2. The midplane strains owing to applied effective in-plane forces are computed for each load vector by subtracting, as needed, free hygrothermal strains from actual strains specified by the load vector:

$$\begin{aligned}\varepsilon_x^{o,\varepsilon\kappa} &= \varepsilon_x^o - \Delta T^o \alpha_x - \Delta m^o \beta_x \\ \varepsilon_y^{o,\varepsilon\kappa} &= \varepsilon_y^o - \Delta T^o \alpha_y - \Delta m^o \beta_y \\ \gamma_{xy}^{o,\varepsilon\kappa} &= \gamma_{xy}^o\end{aligned}\tag{13a-c}$$

The terms  $\varepsilon_x^o$ ,  $\varepsilon_y^o$  and  $\gamma_{xy}^o$  on the right-hand side of the equations are the effective actual midplane strains specified by the load vector. The terms  $\Delta T^o$  and  $\Delta m^o$  are, respectively, the effective temperature and moisture content differences in between the operating temperature and the stress-free temperature in the midplane of the laminate. The terms  $\alpha_x$  and  $\alpha_y$  are the thermal expansion coefficients of the laminate in the  $x$ - and  $y$ -directions. The terms  $\beta_x$  and  $\beta_y$  are the corresponding moisture expansion coefficients. The superscript  $\varepsilon\kappa$  on the left-hand side of the equations refers to strains owing to a load specified by the midplane strains and curvatures.

3. The maximum tensile, compressive and shear strains owing to each effective load vector are computed with the equations

$$\begin{aligned}(\varepsilon_t^{\varepsilon\kappa})^{c,v,r} &= \left( \max \left[ \varepsilon_x^{o,\varepsilon\kappa}; \varepsilon_y^{o,\varepsilon\kappa}; 0 \right] \right)^{c,v,r} \\ (\varepsilon_c^{\varepsilon\kappa})^{c,v,r} &= \left( \left| \min \left[ \varepsilon_x^{o,\varepsilon\kappa}; \varepsilon_y^{o,\varepsilon\kappa}; 0 \right] \right| \right)^{c,v,r} \\ (\gamma^{\varepsilon\kappa})^{c,v,r} &= \left( \left| \gamma_{xy}^{o,\varepsilon\kappa} \right| \right)^{c,v,r}\end{aligned}\tag{14a-c}$$

These equations are formulated analogously with Eqs. (11) to set the maximum tensile (compressive) strain to zero when the midplane is in compression (in tension) in the  $x$ - and  $y$ -directions.

4. Equations analogous to Eqs. (12) are used to compute the values of the deformation attributes.

When curvatures  $\kappa_x$ ,  $\kappa_y$  and  $\kappa_{xy}$  are accounted for, the deformation attribute values are derived from laminate surface strains which are known to represent the highest and lowest strain levels of curved laminates. The values are computed as follows:

1. The effective constant, variable and resultant load vectors are computed with Eqs. (9).
2. The surface strains owing to effective in-plane forces and moments are computed with the equations

$$\begin{aligned}
\varepsilon_x^{t,\varepsilon\kappa} &= \varepsilon_x^o - \frac{h}{2}\kappa_x - \Delta T^t \alpha_x - \Delta m^t \beta_x \\
\varepsilon_x^{b,\varepsilon\kappa} &= \varepsilon_x^o + \frac{h}{2}\kappa_x - \Delta T^b \alpha_x - \Delta m^b \beta_x \\
\varepsilon_y^{t,\varepsilon\kappa} &= \varepsilon_y^o - \frac{h}{2}\kappa_y - \Delta T^t \alpha_y - \Delta m^t \beta_y \\
\varepsilon_y^{b,\varepsilon\kappa} &= \varepsilon_y^o + \frac{h}{2}\kappa_y - \Delta T^b \alpha_y - \Delta m^b \beta_y \\
\gamma_{xy}^{t,\varepsilon\kappa} &= \gamma_{xy}^o - \frac{h}{2}\kappa_{xy} \\
\gamma_{xy}^{b,\varepsilon\kappa} &= \gamma_{xy}^o + \frac{h}{2}\kappa_{xy}
\end{aligned} \tag{15a-f}$$

3. Equations analogous to Eqs. (11) are used to compute the maximum strains.
4. Equations analogous to Eqs. (12) are used to compute the values of the deformation attributes.

### 5.3 EVALUATION PROCEDURE

Ply evaluation is performed in three phases. In the first phase, the feasibility of each ply is evaluated as follows:

1. Ply feasibility is studied against constraints set for the ply-specific attributes. If the ply is feasible with respect to these constraints, the evaluation is continued. If one or more constraints are not satisfied, the ply is identified as infeasible and the evaluation is interrupted.
2. A set of homogenised laminates representing ply performance is created from the ply. A constant value is given at this stage for laminate thickness.
3. Ply performance is evaluated against possible constraints set for the lay-up-dependent attributes. The evaluation is performed by computing attribute values for the created set of laminates and by comparing them against the constraints. The ply is identified to be feasible with respect to the constraints, if one or several laminates satisfy all constraints. The laminates that do not satisfy all constraints are rejected from the set of laminates representing ply performance.
4. Ply performance is evaluated against possible constraints set for the thickness-dependent attributes. The evaluation is performed by searching for each remaining laminate a thickness range in which it simultaneously satisfies all constraints. If such a range exists at least for one laminate, the ply is identified to be feasible with respect to

all constraints. The laminates with no feasible thickness range are rejected from the set of laminates representing ply performance.

In the second phase, the laminate that best meets the design target is sought for a feasible ply amongst the remaining laminates representing its performance:

5. A feasible thickness that maximises the preference function is sought for each laminate.
6. The laminates with their best possible thickness values are ranked with respect to each other with the laminate evaluation tool.
7. The laminate with the highest value of preference function is selected to represent ply performance. Such a laminate is called the *representative laminate* of the ply.

Finally, when each candidate ply has been evaluated, the mutual quality of feasible plies is determined. This phase is performed by ranking representative laminates of the plies with respect to each other with the laminate evaluation tool.

The ply evaluation procedure is illustrated in Figure 6. A detailed description of the sub-procedures is given below.

### 5.3.1 Feasibility of ply-specific attributes

The feasibility of ply-specific attributes is determined for each ply as follows:

- *Ply types*: The type of the ply is read from the ply specification data. The type is feasible, if it exists in the set of acceptable ply types specified by the constraint.
- *Maximum/minimum operating temperature/pressure*: A value for the attribute is read from the ply specification data. The value is feasible, if it is within the limits specified by the constraint.
- *Manufacturing technique*: A value for the attribute is the set of applicable techniques read from the ply specification data. The value is feasible, if the set contains at least one of the techniques specified by the constraint.

### 5.3.2 Feasibility of stiffness, hygrothermal expansion and strength

A laminate representing ply performance is evaluated against the constraints set for engineering constants, hygrothermal expansion coefficients and strengths by determining the attribute values for the laminate and by comparing them with the constraints.

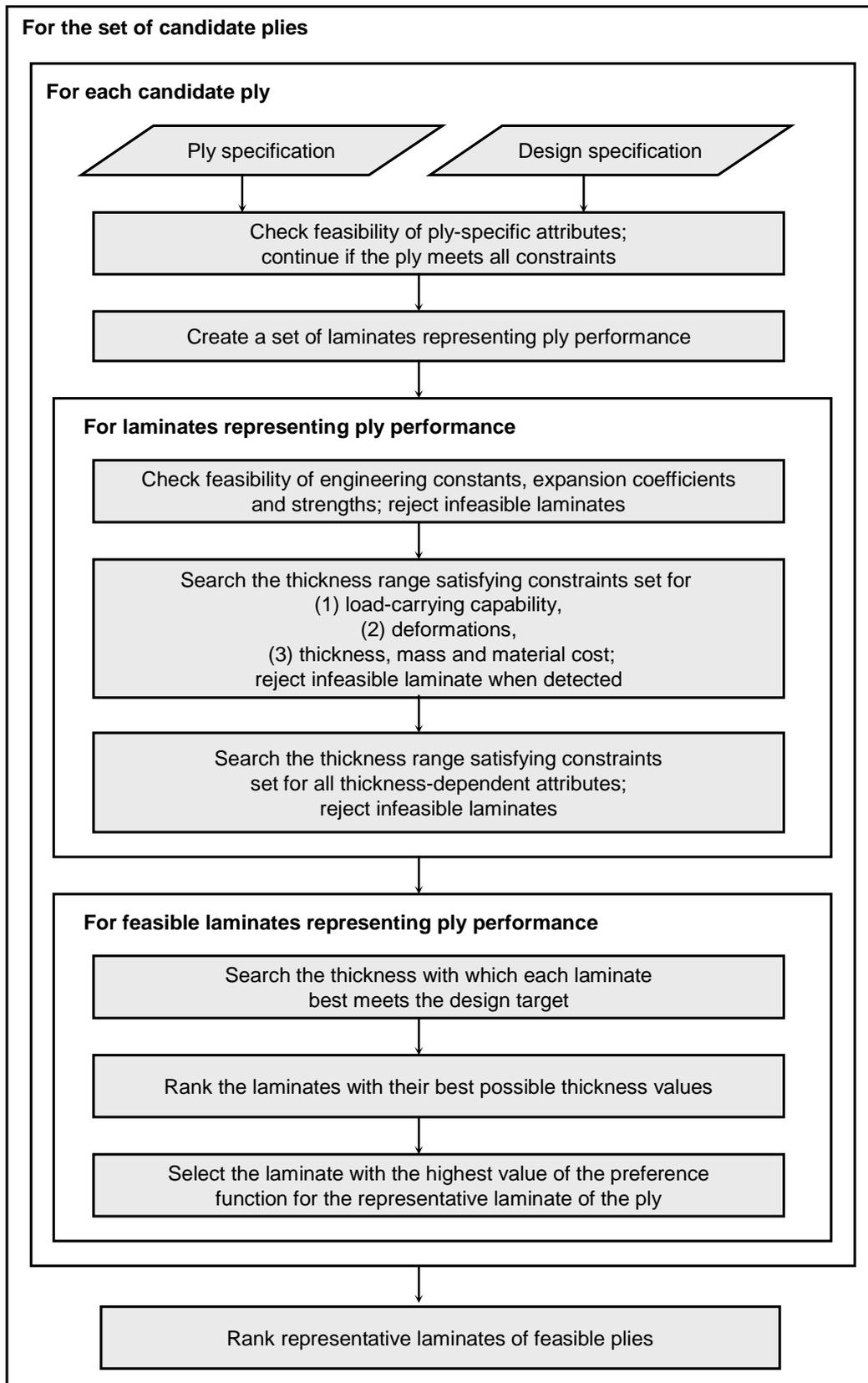


Figure 6 Ply evaluation procedure.

### 5.3.3 Feasibility of load-carrying capability

A laminate representing ply performance is evaluated against a constraint set for the attribute *Load-carrying capability* by searching for a feasible thickness range in which the laminate is capable of carrying the specified loads. Such a thickness range is first sought for each load by considering separately the loads specified by in-plane forces and moments and the loads specified by midplane strains and curvatures. This is followed by a search of the thickness range that provides a feasible load-carrying capability with all loads.

#### Load specified by in-plane forces and moments

Laminate stresses owing to applied in-plane forces and moments decrease with increasing laminate thickness. Thus, the thickness with which and above which a laminate is able to carry a load must be found. This lower bound of the feasible thickness range is sought iteratively:

1. The initial value of the thickness  $(h^{NM})_L$  is set to the maximum 250 mm thickness considered. The superscript *NM* refers here to a thickness specified by in-plane forces and moments and the subscript *L* to a thickness specified by a load.
2. The value of the reserve factor *RF* is computed for the laminate.
3. Conclusions are made and actions taken:
  - 3.1 If  $RF < 1$  and  $(h^{NM})_L = 250$  mm, it is concluded that the laminate is not able to carry the load with any thickness in the thickness range considered by the system. The search is stopped.
  - 3.2 If  $RF \geq 1$  and  $(h^{NM})_L < 0.1$  mm, it is concluded that the laminate is able to carry the load with any thickness in the thickness range considered by the system. The search is stopped.
  - 3.3 If  $|RF - 1| \leq 0.01$ ,  $(h^{NM})_L$  is selected to represent the lower bound of the thickness range with which the laminate is able to carry the load. The search is stopped.
  - 3.4 If the conditions above are not satisfied, the laminate thickness is redefined to the following value and the search is continued with Step 2:

$$(h^{NM})_L = \frac{(h^{NM})_L}{\sqrt[3]{RF}} \quad (16)$$

The cubic root of the reserve factor is used in the denominator of Eq. (16) since it was found to provide a smooth and relatively fast convergence towards the final solution.

#### Load specified by midplane strains and curvatures

The thickness range with which a laminate is able to carry a load specified by midplane strains and curvatures is sought as follows:

1. The initial value of the thickness  $(h^{EK})_L$  is set to the minimum thickness 0.1 mm

considered in the evaluation process. The superscript  $\varepsilon\kappa$  in the symbol refers to a thickness specified by midplane strains and curvatures.

2. The value of the reserve factor  $RF$  is computed for the laminate by ignoring curvatures if such exist in the load specification.
3. Conclusions are made and actions taken:
  - 3.1 If  $RF < 1$ , it is concluded that no feasible thickness exists for the laminate since the computed value of the reserve factor is independent of laminate thickness and, when possible curvatures are accounted for, the value decreases. The same conclusion is made when  $RF = 1$  and the load contains curvatures though in some special cases applied curvatures do not decrease the reserve factor. The search is stopped.
  - 3.2 If  $RF \geq 1$  and the load does not contain curvatures, it is concluded that the laminate is able to carry the load with any thickness. The search is stopped.
  - 3.3 If  $RF > 1$  and the load contains curvatures, it is concluded that there exists a thickness with which and below which the laminate is able to carry the load. The search is continued with Step 4.
4. The value of the reserve factor  $RF$  is computed for the laminate by accounting for the curvatures.
5. Further conclusions are made and actions taken as follows:
  - 5.1 If  $|RF - 1| \leq 0.01$  and  $0.1 \text{ mm} \leq (h^{\varepsilon\kappa})_L \leq 250 \text{ mm}$ ,  $(h^{\varepsilon\kappa})_L$  is selected for the upper bound of the thickness range with which the laminate is able to carry the load. The search is stopped.
  - 5.2 If  $|RF - 1| \leq 0.01$  and  $(h^{\varepsilon\kappa})_L < 0.1 \text{ mm}$ , it is concluded that the laminate is not able to carry the load in the thickness range  $0.1 \dots 250 \text{ mm}$  considered by the system. The search is stopped.
  - 5.3 If  $RF \geq 1$  and  $(h^{\varepsilon\kappa})_L \geq 250 \text{ mm}$ , it is concluded that the laminate is able to carry the load in the thickness range  $0.1 \dots 250 \text{ mm}$  considered by the system. The search is stopped.
  - 5.4 If the conditions above are not satisfied, the laminate thickness is redefined to the following value and the search is continued with Step 4:

$$(h^{\varepsilon\kappa})_L = RF \cdot (h^{\varepsilon\kappa})_L \quad (17)$$

### All loads

If a feasible thickness range does not exist for a laminate with a given load, it is naturally concluded that the laminate is not able to carry all loads in the thickness range considered by the system. If feasible thickness ranges exist separately for each load, the thickness range that provides a feasible load-carrying capability with all loads is sought:

1. The lower bounds of feasible thickness ranges sought for loads specified by in-plane forces and moments are compared with each other. The highest of these is selected to represent the lower bound of the feasible thickness range,  $(h_L)_{\min}$ , in which the laminate

is capable of carrying all  $m$  loads specified by in-plane forces and moments:

$$(h_L)_{\min} = \max \left[ (h^{NM})_{L,i} \right] \quad , \quad i = 1, 2, \dots, m \quad (18)$$

2. The upper bounds of feasible thickness ranges sought for loads specified by midplane strains and curvatures are compared with each other. The lowest of these is selected to represent the upper bound of the feasible thickness range,  $(h_L)_{\max}$ , in which the laminate is capable of carrying all  $n$  loads specified by midplane strains and curvatures:

$$(h_L)_{\max} = \min \left[ (h^{\epsilon\kappa})_{L,i} \right] \quad , \quad i = 1, 2, \dots, n \quad (19)$$

3. Conclusions on the load-carrying capability of the laminate are made:
  - if  $(h_L)_{\max} \geq (h_L)_{\min}$ , it is concluded that load-carrying capability of the laminate is feasible in the thickness range  $[(h_L)_{\min}, (h_L)_{\max}]$
  - if  $(h_L)_{\max} < (h_L)_{\min}$ , it is concluded that load-carrying capability of the laminate is infeasible.

### 5.3.4 Feasibility of deformations

A thickness range that provides feasible deformations for a laminate is first sought for each load by considering separately the loads specified by in-plane forces and moments, and the loads specified by midplane strains and curvatures. This is followed by a search of the thickness range that provides feasible deformations with all loads. The search procedure is detailed below.

#### Load specified by in-plane forces and moments

When possible laminate failure is not considered, there always exists a thickness with which and above which a laminate loaded by in-plane forces and moments satisfies a deformation constraint. This conclusion can be made since surface strains, representing the highest and lowest strain levels in a laminate, decrease with increasing thickness of the laminate. The lower bound of the feasible thickness range is determined from closed form solutions of surface strains.

The minimum thickness  $(h^{NM})_{\epsilon_t}$  with which a laminate satisfies a constraint set for the tensile strain  $\epsilon_t$  is solved by replacing surface strains on the left-hand side of Eqs. (10a-d) with the constraint. The sought thickness is the highest of the real and positive roots of the equations. If no positive real root exists, the surface strains are compressive. It is then concluded that any thickness satisfies the constraint.

The minimum thickness  $(h^{NM})_{\epsilon_c}$  with which a laminate satisfies a constraint set for the maximum compressive strain  $\epsilon_c$  is solved analogously by replacing surface strains on the left-hand side of Eqs. (10a-d) with the constraint that is expressed with a negative strain value.

The minimum thickness  $(h^{NM})_{\gamma}$  with which a laminate satisfies a constraint set for the shear strain  $\gamma$  is solved by replacing surface strains on the left hand side of Eqs. (10e,f) with the constraint that is expressed first with a positive strain value and then with a negative strain value. The sought thickness is the highest of the real and positive roots of the four equations.

If any of the thickness values is higher than 250 mm, it is concluded that the laminate does not meet the corresponding deformation constraint in the thickness range considered by the system.

### Load specified by midplane strains and curvatures

The thickness range in which a laminate loaded by midplane strains and curvatures is able to satisfy the deformation constraints is sought in two phases. In the first phase, the values of deformation attributes are computed with Eqs. (13) and (14) by ignoring curvatures. The values are further compared with the constraints and the following conclusions are made and actions taken:

- If any of the constraints is violated, it is concluded that no feasible thickness exists for the laminate in the default thickness range of the system. The search is stopped.
- If all constraints are satisfied and the load contains no curvatures, it is concluded that the laminate satisfies the deformation constraints with any thickness. The search is stopped.
- If all constraints are satisfied and the load contains curvatures, it is concluded that there exists a thickness with which and below which the laminate satisfies a deformation constraint.

With the third conclusion, the upper bound of the feasible thickness range is further determined corresponding to each constraint:

- The maximum thickness  $(h^{EK})_{\epsilon_t}$  satisfying the constraint set for the maximum tensile strain is solved by replacing the surface strains on the left-hand side of Eqs. (15a-d) with the constraint. The sought thickness is the lowest of the positive values given by the equations. If all the values are negative, the surface strains are compressive. It is then concluded that any thickness satisfies the constraint.
- The maximum thickness  $(h^{EK})_{\epsilon_c}$  satisfying the constraint set for the maximum compressive strain is solved analogously by replacing surface strains on the left-hand side of Eqs. (15a-d) with the constraint that is expressed with a negative strain value.
- The maximum thickness  $(h^{EK})_{\gamma}$  satisfying the constraint set for the shear strain is solved by replacing surface strains on the left hand side of Eqs. (15e,f) with the constraint that is expressed first with a positive strain value and then with a negative strain value. The sought thickness is the lowest of the positive values given by the equations.

If any of the thickness values is lower than 0.1 mm, it is concluded that the laminate does not meet the corresponding deformation constraint in the default thickness range of the system.

### All loads

The thickness range providing feasible deformations with all loads is finally sought for a laminate. Corresponding to each constraint, the search is started only when a feasible thickness range exists with each load. If not, it is concluded that the laminate is not able to satisfy the

constraint with any thickness. Since deformations of a failed laminate are meaningless, the search is limited to the range  $[(h_L)_{\min}, (h_L)_{\max}]$ , i.e. to the range that provides a feasible load-carrying capability for the laminate.

The following procedure is used in the search:

1. The lower bounds of feasible thickness ranges are computed corresponding to each deformation constraint:

$$\begin{aligned} (h_{\epsilon})_{\min} &= \max \left[ (h^{NM})_{\epsilon,i} ; (h_L)_{\min} \right] \\ (h_{\epsilon c})_{\min} &= \max \left[ (h^{NM})_{\epsilon c,i} ; (h_L)_{\min} \right] , \quad i = 1, 2, \dots, m \\ (h_{\gamma})_{\min} &= \max \left[ (h^{NM})_{\gamma,i} ; (h_L)_{\min} \right] \end{aligned} \quad (20a-c)$$

2. The upper bounds of feasible thickness ranges are computed corresponding to each deformation constraint:

$$\begin{aligned} (h_{\epsilon})_{\max} &= \min \left[ (h^{\epsilon K})_{\epsilon,i} ; (h_L)_{\max} \right] \\ (h_{\epsilon c})_{\max} &= \min \left[ (h^{\epsilon K})_{\epsilon c,i} ; (h_L)_{\max} \right] , \quad i = 1, 2, \dots, n \\ (h_{\gamma})_{\max} &= \min \left[ (h^{\epsilon K})_{\gamma,i} ; (h_L)_{\max} \right] \end{aligned} \quad (21a-c)$$

3. The lower and upper bounds of the thickness range, in which all deformation constraints are satisfied, are computed:

$$\begin{aligned} (h_{\epsilon})_{\min} &= \max \left[ (h_{\epsilon})_{\min} ; (h_{\epsilon c})_{\min} ; (h_{\gamma})_{\min} \right] \\ (h_{\epsilon})_{\max} &= \min \left[ (h_{\epsilon})_{\max} ; (h_{\epsilon c})_{\max} ; (h_{\gamma})_{\max} \right] \end{aligned} \quad (22a,b)$$

4. Conclusions on the feasibility of the laminate with respect to deformation constraints are made:

- If  $(h_{\epsilon})_{\max} \geq (h_{\epsilon})_{\min}$  ( $(h_{\epsilon c})_{\max} \geq (h_{\epsilon c})_{\min}$ ,  $(h_{\gamma})_{\max} \geq (h_{\gamma})_{\min}$ ), it is concluded that the maximum tensile (compressive, shear) strain of the laminate is feasible in the thickness range  $[(h_{\epsilon})_{\min}, (h_{\epsilon})_{\max}]$  ( $[(h_{\epsilon c})_{\min}, (h_{\epsilon c})_{\max}]$ ,  $[(h_{\gamma})_{\min}, (h_{\gamma})_{\max}]$ )
- If  $(h_{\epsilon})_{\max} < (h_{\epsilon})_{\min}$  ( $(h_{\epsilon c})_{\max} < (h_{\epsilon c})_{\min}$ ,  $(h_{\gamma})_{\max} < (h_{\gamma})_{\min}$ ), it is concluded that the maximum tensile (compressive, shear) strain of the laminate is infeasible
- If  $(h_{\epsilon})_{\max} \geq (h_{\epsilon})_{\min}$ , it is concluded that all deformations of the laminate are feasible in the thickness range  $[(h_{\epsilon})_{\min}, (h_{\epsilon})_{\max}]$
- If  $(h_{\epsilon})_{\max} < (h_{\epsilon})_{\min}$ , it is concluded that the laminate is not able to satisfy all deformation constraints with any thickness.

### 5.3.5 Feasibility of thickness, mass and material cost

The feasibility of a laminate with respect to constraints set for thickness, mass and material cost is evaluated by searching for a thickness range in which the laminate satisfies the constraint and simultaneously provides a feasible load-carrying capability.

Since a failed laminate is not considered in the evaluation, the upper and lower bounds of the thickness range, in which a constraint set for the attribute *Thickness* is satisfied, are given by the equations

$$\begin{aligned}(h_h)_{\max} &= \min [ h_{\max} ; (h_L)_{\max} ] \\ (h_h)_{\min} &= \max [ h_{\min} ; (h_L)_{\min} ]\end{aligned}\tag{23a,b}$$

where  $h_{\max}$  and  $h_{\min}$  are, respectively, the highest and lowest acceptable thickness values specified by the constraint. If  $(h_h)_{\max} \geq (h_h)_{\min}$ , it is concluded that the laminate satisfies the constraint in the thickness range  $[(h_h)_{\min}, (h_h)_{\max}]$ . If  $(h_h)_{\max} < (h_h)_{\min}$ , it is concluded that the laminate is infeasible with respect to the constraint.

For defining the thickness ranges that satisfy constraints set for the attributes *Mass* and *Material cost*, the masses and material costs per unit area are first computed for the laminate with the equations

$$\begin{aligned}m_A &= h \cdot \rho \\ p_A &= h \cdot p_V\end{aligned}\tag{24a,b}$$

where  $\rho$  and  $p_V$  are, respectively, the ply density and ply material cost per unit volume. The values of  $\rho$  and  $p_V$  are read from the ply specification data.

According to Chapter 3, constraints specifying the highest acceptable values can only be set for the attributes *Mass* and *Material cost*. Thus, the upper and lower bounds of the thickness range, in which a constraint set for the attribute *Mass* is satisfied, are given by the equations

$$\begin{aligned}(h_m)_{\max} &= \min [ m_{\max} / \rho ; (h_L)_{\max} ] \\ (h_m)_{\min} &= (h_L)_{\min}\end{aligned}\tag{25a,b}$$

where  $m_{\max}$  is the highest applicable mass per unit area. Analogously, the upper and lower bounds of the thickness range, in which a constraint set for the attribute *Material cost* is satisfied, are given by the equations

$$\begin{aligned} (h_p)_{\max} &= \min [ p_{A\max} / p_V ; (h_L)_{\max} ] \\ (h_p)_{\min} &= (h_L)_{\min} \end{aligned} \quad (26a,b)$$

where  $p_{A\max}$  is the highest applicable material cost per unit area.

### 5.3.6 Overall feasibility

The overall feasibility of a laminate is evaluated by searching for a thickness range in which the laminate simultaneously satisfies all the constraints set for thickness-dependent attributes. The lower and upper bounds of the feasible thickness range are computed with equations

$$\begin{aligned} (h_{all})_{\max} &= \min [ (h_L)_{\max} ; (h_\varepsilon)_{\max} ; (h_h)_{\max} ; (h_m)_{\max} ; (h_p)_{\max} ] \\ (h_{all})_{\min} &= \max [ (h_L)_{\min} ; (h_\varepsilon)_{\min} ; (h_h)_{\min} ; (h_m)_{\min} ; (h_p)_{\min} ] \end{aligned} \quad (27a,b)$$

If  $(h_{all})_{\max} \geq (h_{all})_{\min}$ , it is concluded that the laminate satisfies all constraints in the thickness range  $[(h_{all})_{\min}, (h_{all})_{\max}]$ . If  $(h_{all})_{\max} < (h_{all})_{\min}$ , it is concluded that the laminate is not able to satisfy all constraints with any thickness.

### 5.3.7 Representative laminate of a ply

The representative laminate, i.e. the laminate with the best performance, is sought for a ply in two steps. In the first step, the optimum thickness is sought for each remaining laminate representing ply performance. In the second step, the laminates with their optimum thickness values are ranked with respect to each other.

#### Optimum thickness of a laminate

When the design specification contains no loads and constraints/objectives have not been set for thickness-dependent attributes (deformations, thickness, mass, material cost), the optimum thickness is set with an in-built system rule to 0.1mm, i.e. to the lowest value considered.

When the design specification contains loads and/or when objectives have been set for thickness-dependent attributes, the optimum thickness is determined for each laminate with a simple search procedure by applying the multiobjective design technique described in Chapter 4. The preference function is constructed by taking into account all objectives. The search proceeds as follows:

1. If the lowest feasible thickness  $(h_{all})_{\min}$  is not specified by constraints, it is set to the system default value 0.1 mm.
2. If the highest feasible thickness  $(h_{all})_{\max}$  is not specified by constraints, it is set to the system default value 250 mm.

3. If  $(h_{all})_{\min} = (h_{all})_{\max}$ , this thickness is defined to be the optimum thickness and the search is stopped.
4. Five laminates with the following thickness values are formed:

$$h_i = (h_{all})_{\min} + i \frac{\Delta h}{4} \quad , \quad i = 0,1,2,3,4 \quad (28)$$

$$\Delta h = (h_{all})_{\max} - (h_{all})_{\min}$$

The design attribute values are further computed for the laminates and multiobjective design is applied to rank the laminates with respect to each other. The lowest of the thickness values maximising the preference function is selected for a base thickness  $h_{base}$  of the following search.

5. The value of  $\Delta h$  is halved and feasible laminates with the following thickness values are ranked with respect to each other:

$$h_i = h_{base} + i \cdot \Delta h \quad , \quad i = -1,0,1 \quad (29)$$

$$(h_{all})_{\min} \leq h_i \leq (h_{all})_{\max}$$

6. The following conclusions are made and actions taken:
  - The lowest of the thickness values maximising the preference function is selected for a new base thickness  $h_{base}$
  - If  $\Delta h > 0.1$  mm, the process is continued with Step 5
  - If  $\Delta h \leq 0.1$  mm,  $h_{base}$  is defined to be the optimum thickness and the search is stopped.

### Laminate ranking

Laminates with their optimum thickness values are ranked with respect to each other with the laminate evaluation tool described in Chapter 4. If one laminate maximises the preference function, this laminate is defined to be the representative laminate. If several laminates with the same maximum value of the preference function exist, these laminates are ranked with in-built system rules. The rules are applied one by one and the laminates are rejected by the rules until one laminate, the representative laminate, is left:

- With the first rule, the thinnest laminate is selected because it is normally easiest to manufacture.
- The second rule is applied when the design specification contains loads. According to the rule, the laminate with the highest reserve factor is selected, i.e. the laminate with the best load-carrying capability is preferred.

- The third rule aims to find the laminate with the smallest variation in its strength values, i.e. the laminate having a satisfactory capability to carry all types of loads. This type of laminate is normally preferred when there is a possibility to select from amongst several laminates. The rule thus searches for the highest value of the strength ratio

$$\frac{\sigma_f}{\sigma_{f \max}} = \min \left[ \frac{X_t}{\sigma_{f \max}}; \frac{X_c}{\sigma_{f \max}}; \frac{Y_t}{\sigma_{f \max}}; \frac{Y_c}{\sigma_{f \max}}; \frac{S}{\sigma_{f \max}} \right] \quad (30)$$

$$\sigma_{f \max} = \max [X_t; X_c; Y_t; Y_c; S]$$

where  $X_t$ ,  $X_c$ ,  $Y_t$ ,  $Y_c$  and  $S$  are laminate in-plane strengths in the principal loading conditions.

- In the rare case that two or more laminates have the same value of the strength ratio defined by Eq. (30), the fourth rule is applied with an aim of finding the laminate that is easy to manufacture. The rule is formulated for the case  $\theta_{\min} = 0^\circ$  and  $\theta_{\max} = 90^\circ$ , but it is also applied when  $\theta_{\min} \neq 0^\circ$  and/or  $\theta_{\max} \neq 90^\circ$ . With this rule, the laminate with the smallest proportion of the orientation  $\pm\theta$  is selected.
- If more than one laminate still exists, the representative laminate is defined to be the one with the smallest proportion of  $90^\circ$  orientation ( $\pm\theta_{\max}$ ) since a  $0^\circ$  layer is normally easier to lay-up than a  $90^\circ$  layer.

### 5.3.8 Mutual quality of plies

In the last phase of the ply evaluation process, the feasible plies are ranked with respect to each other by ranking their representative laminates with the laminate evaluation tool. The following design attribute values are used for the representative laminates in ranking:

- Engineering constants, expansion coefficients and strengths computed for the representative laminate (in-plane orthotropic ply) or given for the ply in its specification data (in-plane isotropic ply)
- Deformations, thickness, mass and material cost computed for the representative laminate of the ply
- Maximum and minimum operating temperatures and pressures given for the ply in its specification data.

## 6 PLY EVALUATION - SANDWICH LAMINATES

Both core plies and reinforced/homogeneous plies need to be evaluated when sandwich laminates are being created. The specification of such a problem, the evaluation process and the evaluation techniques are described in this chapter.

### 6.1 PROBLEM SPECIFICATION

A laminate creation problem is restricted to sandwich laminates when the set of candidate plies contains core plies. In other aspects, the ply evaluation problem is specified as described in Section 5.1. The reference environment need not be defined for core plies since a core layer is assumed to have a negligible effect on the in-plane behaviour and in-plane strength of a sandwich laminate.

### 6.2 EVALUATION OF CORE PLYS

To simplify the evaluation process, core plies are evaluated by considering only those design attributes for which a value, or a representative value, can be derived from the ply specification data without laminate-level analyses. The evaluation techniques are described below. If all plies are found to be infeasible, the evaluation is stopped. If feasible core plies exist, the process continues with the evaluation of reinforced and homogeneous plies as described in Section 6.3.

#### 6.2.1 Constraints and objectives considered

The constraints and objectives considered in the evaluation of core plies are:

- Constraint set for the attribute *Ply types*
- Constraints and objectives set for operating temperatures and pressures
- Minimisation of the attributes *Mass* and *Material cost*.

The system default constraint set for the attribute *Stacking* is always identified to be satisfied since any type of stacking can be realised with any core ply in conventional sandwich laminates. A constraint set for the attribute *Layer angle* is not considered since, by default, orthotropic core layers are assumed to be oriented so that their principal axes coincide with the principal  $x$ - and  $y$ -axes of the laminate.

The following constraints and objectives are not considered since a value, or a representative value, cannot be derived for the attribute from the ply specification data:

- Constraints and objectives set for mechanical and hygrothermal properties
- Constraint set for load-carrying capability
- Constraints and objectives set for deformations
- Constraints and objectives other than the minimisation set for the attributes *Mass* and *Material cost*
- Constraint and objective set for the attribute *Thickness*.

## 6.2.2 Design attribute values

### Ply types

A value for the attribute *Ply types* is always included in the ply specification. Thus, the attribute value, either *honeycomb core* or *homogeneous core*, is read for a core ply from the specification data.

### Mass and material cost

When no constraint has been set for the attribute *Load-carrying capability*, representative values for the attributes *Mass* and *Material cost* are read for a core ply from the ply specification data: the value of ply density is given to the attribute *Mass* and the value of price per unit volume to the attribute *Material cost*.

When a constraint has been set for the attribute *Load-carrying capability*, the attributes *Mass* and *Material cost* get, respectively, the values

$$m_r = \frac{\rho_c}{E_3^{1/3} \left( \frac{G_{23} G_{31}}{2} \right)^{1/3}} \quad (31a,b)$$

$$p_r = \frac{p_c}{E_3^{1/3} \left( \frac{G_{23} G_{31}}{2} \right)^{1/3}}$$

where  $m_r$  is the representative mass and  $p_r$  the representative material cost. The term  $E_3$  is the out-of-plane Young's modulus,  $G_{23}$  and  $G_{31}$  are the out-of-plane shear moduli,  $\rho_c$  is the material density, and  $p_c$  is the material price per unit volume of the ply.

The denominator in Eqs. (31) describes how well an isotropic core layer is able to resist wrinkling failure of the face sheet.<sup>61</sup> It is used as a measure for core material efficiency since local instability is normally the only failure mode affected by the core layer properties. For simplicity, Eqs. (31) are used also for honeycombs, though local instability modes in sandwich laminates with honeycomb cores may be different. Worth of noting is that the expressions of the representative values are of similar form with the expressions of other representative values that are commonly used in material ranking.

### Operating temperatures and pressures

The values for the maximum and minimum operating temperatures and pressures are read for a core ply from the ply specification data.

### 6.2.3 Feasibility of a ply

The feasibility of a core ply is evaluated by comparing its specification data with the constraints. A value of the attribute *Ply types* is feasible if it is included in the set of acceptable plies. An operating temperature or pressure is feasible if it is within the limits specified by the constraint set for the attribute.

### 6.2.4 Mutual quality of plies

The mutual quality of core plies is evaluated with the multiobjective design technique described in Chapter 4. The evaluation may result in a situation where two or more plies with the same maximum value of the preference function exist. In this case, the final ranking of the core plies is performed with in-built system rules. The rules compare core ply properties for which objectives cannot be set. They are applied one by one and the core plies are rejected by the rules until only one ply is left. Before applying a rule, the system checks that each ply specification contains the data necessary for the evaluation. Where data is missing, the rule is passed. The following rules are applied:

- The aim of the first rule is to find a ply with the best out-of-plane stiffness properties that are typically the most important properties of a core material. Since it is not known whether the Young's modulus or shear moduli are more important, the highest value is sought for the product of  $E_3$  and  $(G_{23} + G_{31})/2$ , i.e. for the product of the out-of-plane Young's modulus and the average out-of-plane shear modulus.
- The second rule aims to find a ply with the best out-of-plane strength properties. The highest value is sought for the product of the first failure compression strength  $Z_c$  and the average first failure out-of-plane shear strength  $(R + Q)/2$ .
- The third rule compares the mechanical behaviour of plies: if the selection is to be made between an isotropic and orthotropic core ply, the former is selected since sandwich laminates with isotropic core plies are normally easier to design and manufacture.
- In a rare situation that the best ply is not found with the three rules, the fourth rule simply selects the newest ply based on the input/modification times of plies.

## 6.3 EVALUATION OF REINFORCED AND HOMOGENEOUS PLYS

Reinforced and homogeneous plies are evaluated using the procedure developed for solid laminates (Fig. 6). However, the ply performance is now studied by evaluating sandwich laminates, the face sheets of which are formed from reinforced and homogeneous candidate plies. The best ranked core ply is used as a core layer. The evaluation techniques are described below. As in the creation of solid laminates, the evaluation process provides an initial solution for the laminate creation problem for each feasible reinforced and homogeneous ply.

### 6.3.1 Laminates representing ply performance

The evaluation of reinforced and homogeneous plies is simplified by setting the following system constraints for sandwich laminates representing ply performance:

- A laminate has a symmetric and balanced lay-up and it contains one core layer that is formed from the best core ply.
- An orthotropic core layer is oriented so that its principal axis 1 coincides with the  $x$ -axis of the laminate.
- A core layer is assumed to have a negligible effect on the in-plane behaviour and on the in-plane strength of a sandwich laminate. This is normally a satisfactory assumption since the in-plane moduli of commonly used core plies are very low compared with the moduli of plies from which face sheets are formed.
- The face sheets of a laminate are formed independently from each reinforced/homogeneous candidate ply. This corresponds to the practice used in the evaluation of solid laminates.
- The minimum face sheet thickness considered is 0.1 mm. The maximum thickness of the laminate is limited to 250 mm.
- Face sheets formed from orthotropic and 23 transversely isotropic plies have a homogenised structure, as described in Chapter 5. The layer orientations and proportions of layer orientations specified in Section 5.2 are considered.
- Two ratios of the core thickness  $c$  and the total laminate thickness  $h$  are considered. The two ratios are  $c/h = 0.8$  and  $c/h = 0.5$ . They are selected to represent, respectively, typical thin-faced and thick-faced sandwich laminates.

When compared to the creation of solid laminates, the number of laminates representing ply performance is doubled since sandwich laminates with two thickness ratios represent the performance of each ply.

### 6.3.2 Laminate analyses

The feasibility of sandwich laminates with in-plane isotropic face sheets and with face sheets formed from homogenised laminates need to be analysed in the evaluation of homogeneous and reinforced plies. The applied analysis techniques are described below.

#### **Mechanical and hygrothermal behaviour and strength**

According to Chapter 3, the effect of the core layer is ignored when the in-plane engineering constants and strengths are computed for a sandwich laminate. The values of these attributes are thus computed as in the creation of solid laminates.

The effect of the core layer is accounted for in the computation of hygrothermal expansion coefficients. The values of these attributes are computed by analysing a corresponding actual sandwich laminate with the tool *Laminate 2.5D behaviour*.

### Load-carrying capability

The tool *Laminate failure* is used in the failure analyses. Face sheet failure is predicted as described in Subsection 5.2.3. The analysis tool accounts for the wrinkling and core failures if defined so with analysis option settings. Core failure is predicted with the selected failure criterion.

### Deformations

The technique described in Subsection 5.2.3 is used in the deformation analyses of sandwich laminates. To take into account the multi-layered sandwich structure, Eqs. (10) are rewritten to the form:

$$\begin{aligned}
 \varepsilon_x^{t,NM} &= \frac{1}{h} \left( \frac{1}{E_x} N_x - \frac{\nu_{yx}}{E_y} N_y \right) - \frac{6}{h^2} \left( \frac{1}{E_x^f} M_x - \frac{\nu_{yx}^f}{E_y^f} M_y \right) \\
 \varepsilon_x^{b,NM} &= \frac{1}{h} \left( \frac{1}{E_x} N_x - \frac{\nu_{yx}}{E_y} N_y \right) + \frac{6}{h^2} \left( \frac{1}{E_x^f} M_x - \frac{\nu_{yx}^f}{E_y^f} M_y \right) \\
 \varepsilon_y^{t,NM} &= \frac{1}{h} \left( -\frac{\nu_{xy}}{E_x} N_x + \frac{1}{E_y} N_y \right) - \frac{6}{h^2} \left( -\frac{\nu_{xy}^f}{E_x^f} M_x + \frac{1}{E_y^f} M_y \right) \\
 \varepsilon_y^{b,NM} &= \frac{1}{h} \left( -\frac{\nu_{xy}}{E_x} N_x + \frac{1}{E_y} N_y \right) + \frac{6}{h^2} \left( -\frac{\nu_{xy}^f}{E_x^f} M_x + \frac{1}{E_y^f} M_y \right) \\
 \gamma_{xy}^{t,NM} &= \frac{1}{h} \frac{1}{G_{xy}} N_{xy} - \frac{6}{h^2} \frac{1}{G_{xy}^f} M_{xy} \\
 \gamma_{xy}^{b,NM} &= \frac{1}{h} \frac{1}{G_{xy}} N_{xy} + \frac{6}{h^2} \frac{1}{G_{xy}^f} M_{xy}
 \end{aligned} \tag{32a-f}$$

where  $E_x, E_y, G_{xy}, \nu_{xy}, \nu_{yx}$  are the in-plane engineering constants and  $E_x^f, E_y^f, G_{xy}^f, \nu_{xy}^f, \nu_{yx}^f$  are the flexural engineering constants of the sandwich laminate. For simplicity, these are computed by ignoring the core layer stiffness, i.e. with the equations:

$$\begin{aligned}
E_x &= (1 - c/h) E_{xf} \\
E_y &= (1 - c/h) E_{yf} \\
G_{xy} &= (1 - c/h) G_{xyf} \\
\nu_{xy} &= \nu_{xy}^f = \nu_{xyf} \\
\nu_{yx} &= \nu_{yx}^f = \nu_{yxf} \\
E_x^f &= [1 - (c/h)^3] E_{xf} \\
E_y^f &= [1 - (c/h)^3] E_{yf} \\
G_{xy}^f &= [1 - (c/h)^3] G_{xyf}
\end{aligned} \tag{33a-h}$$

where the subscript  $f$  refers to the face sheets.

### Mass and material cost

Masses and material costs per unit area are computed for sandwich laminates with equations

$$\begin{aligned}
m_A &= c \rho_c + (h - c) \rho_f \\
p_A &= c p_{vc} + (h - c) p_{vf}
\end{aligned} \tag{34a,b}$$

where  $\rho$  and  $p_v$  are, respectively, the ply density and material cost per unit volume. The subscripts  $c$  and  $f$  refer, respectively, to the core layer and to the face sheets.

### 6.3.3 Feasible thickness range of a laminate

The feasible thickness range of a sandwich laminate is computed as defined in Section 5.3. The equations are modified, as needed, to take into account the differences of solid and sandwich laminates:

- Equations (32) are used instead of Eqs. (10)
- Equations (34) are used instead of Eqs. (24)
- Equation (25a), used in the computation of a thickness range that satisfies a constraint set for the mass, is replaced by the equation:

$$(h_m)_{\max} = \min \left[ \frac{m_{\max}}{(c/h)\rho_c + (1-c/h)\rho_f}; (h_L)_{\max} \right] \tag{35}$$

- Equation (26a), used in the computation of a thickness range that satisfies a constraint set for the material cost, is replaced by the equation:

$$(h_p)_{\max} = \min \left[ \frac{P_{A \max}}{(c/h)p_{Vc} + (1-c/h)p_{Vf}} ; (h_L)_{\max} \right] \quad (36)$$

## 7 LAMINATE SEARCH

The ply evaluation phase provides a representative laminate for each feasible reinforced and homogeneous ply. The representative laminate formed from an in-plane isotropic ply is also an optimal solution, since no simplifications are made in the evaluation of such plies. However, owing to the simplifications described in the previous chapters, representative laminates formed from other types of plies are normally not optimal and may even be infeasible. The laminate creation process can then be completed with the laminate search phase. Section 7.1 below describes how a laminate search problem is specified. The search techniques and the search procedure are described in the following sections.

### 7.1 PROBLEM SPECIFICATION

The following data specify a laminate search problem:

- Constraints and objectives set for the laminate being sought
- Representative laminate formed in the ply evaluation phase
- Specification of the ply forming the representative solid laminate or specifications of the reinforced/homogeneous and core plies forming the representative sandwich laminate
- Reference temperature and/or moisture content for the in-plane orthotropic ply when the design specification contains thermal and/or moisture loads
- Analysis option settings, as needed.

In practice, laminate search in the ESAComp system is a direct continuation of the ply evaluation phase. It is specified with a selection of a reinforced/homogeneous ply that has been identified as feasible in the ply evaluation phase.

### 7.2 SEARCH TECHNIQUES

#### 7.2.1 Design space – solid laminates

The design space of the ply evaluation phase, specified in Section 5.2, is modified as follows in the search for an optimal solid laminate:

- The set of possible values of layer orientation angles is extended. When the layer orientation constraint is specified with discrete angles, all acceptable angles are accounted for. When a range of acceptable values has been specified, the values considered are the boundary values  $\theta_{min}$  and  $\theta_{max}$ , and the values:

$$\theta = i \cdot 5^\circ \quad , \quad i = 1, 2, \dots, 17 \quad (37)$$

$$\theta_{min} < \theta < \theta_{max}$$

In addition, the angles minimising the longitudinal and transverse expansion

coefficients of  $\pm\theta$  laminates are accounted for when a constraint or objective has been set for an expansion coefficient. The argumentation for the selected step of  $5^\circ$  is that better accuracy can seldom be achieved with existing manufacturing techniques. Additionally, the versatile tools of the analysis system allow the design to be fine tuned as needed.

- The set of possible values of  $p_{\theta_{min}}$ ,  $p_\theta$  and  $p_{\theta_{max}}$ , defining the proportions of layers with the orientations  $\pm\theta_{min}$ ,  $\pm\theta$  and  $\pm\theta_{max}$ , is extended. The proportions may get the values:

$$p_{\theta_{min}, \theta, \theta_{max}} = i \cdot 0.05 \quad , \quad i = 0, 1, 2, \dots, 20 \quad (38)$$

$$p_{\theta_{min}} + p_\theta + p_{\theta_{max}} = 1$$

A smaller step size is not seen necessary since the proportions cannot be tailored very accurately in practical applications.

The design variables in the search of solid laminates are thus laminate thickness  $h$ , layer orientation angle  $\theta$  and layer proportions  $p_{\theta_{min}}$ ,  $p_\theta$  and  $p_{\theta_{max}}$ .

### 7.2.2 Design space - sandwich laminates

The design space of the ply evaluation phase, specified in Section 6.3, is modified as follows in the search for an optimal sandwich laminate:

- The layer orientation angle  $\theta$  and layer proportions in face sheets may get values as described above for solid laminates
- Any value in the range of  $0 < c/h < 1$  is possible for the ratio of the core and laminate thickness.

The design variables in the search of sandwich laminates are thus:

- Face sheet thickness  $t$  and core thickness  $c$  for laminates with in-plane isotropic face sheets
- Face sheet thickness  $t$  and core thickness  $c$ , layer orientation  $\theta$ , and layer proportions  $p_{\theta_{min}}$ ,  $p_\theta$  and  $p_{\theta_{max}}$  for laminates with in-plane orthotropic face sheets.

### 7.2.3 Laminate analyses

The laminate analyses are performed in the laminate search phase with the analysis tools of the program. To obtain realistic results also in failure analyses, homogenised laminates are modified for the analyses by constructing their surfaces from actual layers. The internal stresses and stress interactions, as specified by the user selected failure criterion, are then taken into account.

The modification of a solid homogenised laminate is performed as follows:

- The surfaces, each with the thickness of 1 % from the total laminate thickness, are formed from actual layers so that they contain a layer with each orientation angle

included in the laminate. Values of the design variables  $p_{\theta_{min}}$ ,  $p_{\theta}$  and  $p_{\theta_{max}}$  define the layer thickness values. The layers are arranged to the order  $+\theta_{min}/-\theta_{min}/+\theta/-\theta/+\theta_{max}/-\theta_{max}$ , the first layer being the surface layer.

- In between the surfaces, with the thickness of 98 % of the total thickness, is a layer formed from the homogenised laminate with the orientation of  $0^\circ$ . High enough strength values are set to this layer so that it will not be critical in any failure analysis.

In the most general case, i.e. with three different layer orientation angles, the stacking of a modified laminate is thus  $[+\theta_{min}/-\theta_{min}/+\theta/-\theta/+\theta_{max}/-\theta_{max}/0^\circ_{hom}]_{SO}$ . The subscript *hom* in the code refers to the homogenised laminate structure and the subscript SO to a symmetric laminate with an odd number of layers.

Homogenised laminates forming face sheets of a sandwich laminate are modified analogously: in both face sheets the surface layer, with the thickness of 1 % of the total face sheet thickness, is formed from actual layers. Sandwich laminates are thus of the type  $[+\theta_{min}/-\theta_{min}/+\theta/-\theta/+\theta_{max}/-\theta_{max}/0^\circ_{hom}/0^\circ_c]_{SO}$ , where the subscript *c* refers to the core layer.

With the specified modification, failure is predicted for laminate surfaces which are known to be critical in all applicable load cases. Further, since the surface thickness is only 1 % of the total laminate thickness, failure analysis results represent well the laminate performance in bending as well.

## 7.3 SEARCH PROCEDURE

### 7.3.1 Initial solution

The representative laminate may be infeasible because the laminate-level failure analysis technique is used in ply evaluation. Therefore, the feasibility of the laminate is checked in the beginning of the laminate search phase if the design specification contains loads.

Comparative analyses of actual and homogenised laminates indicate that a feasible actual laminate, if such exists, can normally be found in the thickness range  $0.5 h_r \dots 2.0 h_r$ , where  $h_r$  is the thickness of the representative laminate. Based on this, a feasible initial solution is sought by evaluating a set of laminates formed from the representative laminate by varying its thickness:

$$h = (0.5 + 0.25 i) \cdot h_r \quad , \quad i = 0, 1, \dots, 6 \quad (39)$$

$$0.1 \text{ mm} \leq h \leq 250 \text{ mm}$$

In the search of sandwich laminates, the set is formed with the thickness ratio  $c/h$  of the representative laminate.

The laminates are evaluated with the laminate evaluation tool. It automatically takes into account all constraints, which must be done since another thickness may provide a feasible

load-carrying capability but result in a violation of some other constraint. If feasible solutions exist amongst the laminates, the one that maximises the preference function is selected for the initial solution of the search. If two or more feasible laminates maximise the preference function, the thinnest of these is used as an initial solution. If no feasible solution exists, the search process is interrupted.

It should be noted that the specified search of an initial solution is simple and may fail even when a laminate with a feasible thickness exists. In most cases, however, a feasible initial solution, if such exists, can be found with the technique.

### 7.3.2 Orthotropic sandwich laminates

Simple methods are used in the laminate search since the optimum solution is normally close to the initial solution. In the most general case, when a sandwich laminate with orthotropic face sheets is being created, the search proceeds as follows:

1. The initial solution, defined in Subsection 7.3.1, is identified to be the first *reference solution* and the first *base solution*.
2. The values of the design variables other than the face sheet thickness  $t$  and the core thickness  $c$  are fixed to their base solution values. The best possible values for the variables  $t$  and  $c$  are further searched for:

- 2.1 An initial step is specified for the face sheet and core thickness:

$$\begin{aligned}\Delta t &= 0.05 \cdot t_{ref} \\ \Delta c &= 0.05 \cdot c_{ref}\end{aligned}\tag{40}$$

The subscript *ref* refers here to the reference solution.

- 2.2 A set of laminates containing the base solution and maximum of eight laminates around the base solution is formed:

$$\begin{aligned}t &= [t_{base} - \Delta t ; t_{base} ; t_{base} + \Delta t] \quad , \quad 0.1 \text{ mm} \leq t \\ c &= [c_{base} - \Delta c ; c_{base} ; c_{base} + \Delta c] \quad , \quad 0.2 \text{ mm} < 2t + c \leq 250 \text{ mm}\end{aligned}\tag{41}$$

The subscript *base* refers here to the base solution.

- 2.3 The laminate evaluation tool is used to find feasible laminates in the set and to compute the values of the preference function for the feasible laminates.
- 2.4 Feasible laminates in the set are evaluated and actions are taken:

- If one of the laminates around the base solution maximises the preference function, the laminate is identified to be a new base solution. The process is then continued with Sub-step 2.
  - If at least two laminates around the base solution maximise the preference function, a new base solution is searched for with the in-built system rules described in Subsection 5.3.7. The process is then continued with Sub-step 2.
  - If the base solution only maximises the preference function, the process is continued with Sub-step 5.
  - If the base solution and at least one laminate around it maximise the preference function, the best of these laminates is searched for with the in-built system rules described in Subsection 5.3.7. The laminate is identified to be a new base solution. The process is then continued either with Sub-step 5 (no change in base solution) or with Sub-step 2 (base solution changed).
- 2.5 The value of  $\Delta t$  is checked. If  $\Delta t \geq 0.01 t_{ref}$ , the values of  $\Delta t$  and  $\Delta c$  are halved and the process is continued with Sub-step 2. If  $\Delta t < 0.01 t_{ref}$ , the process is continued with Step 3.
- 3 The values of the design variables other than the layer orientation angle  $\theta$  are fixed to their base solution values. The best possible value for the layer orientation angle is further searched for:
- 3.1 The series of acceptable values of the layer orientation angle  $\theta$  is formed:
- $$\theta = [\theta_i] \quad , \quad \theta_{i-1} < \theta_i \quad (42)$$
- 3.2 A set of laminates with the following values of the layer orientation angle is formed:
- $$\theta = [\theta_{base-1} ; \theta_{base} ; \theta_{base+1}] \quad , \quad \theta_{min} < \theta < \theta_{max} \quad (43)$$
- The angles  $\theta_{base-1}$  and  $\theta_{base+1}$  are, respectively, the angles preceding and following the angle  $\theta_{base}$  in the series of acceptable angles.
- 3.3 The laminate evaluation tool is used to find feasible laminates in the set and to compute the values of the preference function for the feasible laminates.
- 3.4 Conclusions are made and actions taken as specified in Sub-step 2.4.
- 3.5 The process is continued with Step 4.
- 4 The values of the design variables other than the proportions of layer orientations  $p_{\theta_{min}}$ ,  $p_{\theta}$  and  $p_{\theta_{max}}$  are fixed to their base solution values. The best possible values for the proportions are further searched for:

- 4.1 A step of  $\Delta p = 0.05$  is specified.
- 4.2 A set of laminates containing the base solution and maximum of six laminates around the base solution is formed:

$$\begin{aligned}
 p_{\theta \min} &= [(p_{\theta \min})_{base} - \Delta p ; (p_{\theta \min})_{base} ; (p_{\theta \min})_{base} + \Delta p] \\
 p_{\theta} &= [(p_{\theta})_{base} - \Delta p ; (p_{\theta})_{base} ; (p_{\theta})_{base} + \Delta p] \\
 p_{\theta \max} &= [(p_{\theta \max})_{base} - \Delta p ; (p_{\theta \max})_{base} ; (p_{\theta \max})_{base} + \Delta p] \quad (44) \\
 0 \leq p_{\theta \min} \leq 1 \quad , \quad 0 \leq p_{\theta} \leq 1 \quad , \quad 0 \leq p_{\theta \max} \leq 1
 \end{aligned}$$

$$p_{\theta \min} + p_{\theta} + p_{\theta \max} = 1$$

- 4.3 The laminate evaluation tool is used to find feasible laminates in the set and to compute the values of the preference function for the feasible laminates.
- 4.4 Conclusions are made and actions taken as specified in Sub-step 2.4.
- 4.5 The process is continued with Step 5.
- 5 The base solution and the reference solution are compared with each other. The base solution is defined to be the best solution if the following conditions are satisfied:

$$\frac{|t_{base} - t_{ref}|}{t_{ref}} \leq 0.01$$

$$\frac{|c_{base} - c_{ref}|}{c_{ref}} \leq 0.01$$

$$\theta_{base} = \theta_{ref} \quad (45a-e)$$

$$(p_{\theta \max})_{base} = (p_{\theta \max})_{ref}$$

$$(p_{\theta \min})_{base} = (p_{\theta \min})_{ref}$$

The process is then continued with Step 6. If the conditions are not satisfied, the base solution is selected for a new reference solution and the process is continued with Step 2.

- 6 As a final step in the search process, the following laminates are evaluated with the laminate evaluation tool:
  - Representative laminate
  - Initial solution if it is not the representative laminate
  - Best solution.

### 7.3.3 Other laminates

The search procedure specified in the previous section is simplified, as follows, in the creation of other types of laminates:

- In the creation of solid laminates, Step 2 is performed by ignoring the core thickness  $c$  and by replacing the face sheet thickness  $t$  with the laminate thickness  $h$  that may get values  $0.1 \text{ mm} \leq h \leq 250 \text{ mm}$ . Equation (45b) is further ignored when the base solution and the reference solution are compared with each other.
- Steps 3 and 4 and the conditions specified by Eqs. (45c-e) are ignored in the creation of sandwich laminates with in-plane isotropic face sheets.

## 8 SYSTEM PERFORMANCE

The performance of the design system is demonstrated with test problems. The aim is to show that the design procedures provide the best solution within their operational limits. Laminate and ply evaluations are performed with the ESAComp program containing the developed design tools. Laminate search is performed with the prototype tool developed for the purpose.<sup>59</sup>

### 8.1 PLY SPECIFICATIONS IN TEST PROBLEMS

Table 3 lists the plies used in the test problems. Table 4 provides the numeric specification data for reinforced and homogeneous plies. The core ply specifications are given in Table 5. The plies are later referred to with their identification codes.

The following should be noted concerning the specification data:

- Out-of-plane properties are not specified for any reinforced or homogeneous ply since they are not needed in the test problems.
- The ply T300/5208 is specified with the data given by Massard & Paterson<sup>63</sup> to be able to compare results provided by the laminate creation tool with the results given in the reference.
- Typical data is used in the specifications of other reinforced and homogeneous plies.
- Typical data is mostly used in the specifications of core plies. The in-plane moduli and in-plane failure strains of honeycomb plies are fictitious but result in realistic behaviour of a sandwich laminate: Firstly, owing to the specified low moduli, the core layers practically have no effect on the in-plane stiffness. Secondly, the specified high in-plane failure strains guarantee that the primary failure mode in in-plane loading is a face sheet or wrinkling failure.

### 8.2 LAMINATE EVALUATION

#### 8.2.1 Test problems

The performance of the laminate evaluation tool is demonstrated with two laminate sets defined in Table 6. The evaluation of the first set aims to show how the best solution is found amongst laminates with different layer orientations. The evaluation of the second set has a similar aim but, instead of the layer orientations, the proportions of layer orientations are varied.

**Table 3** Plies in the test problems.

Identification	Physical nature	Mech. behaviour	Type
T300/5208	Reinforced	23 transv. isotropic	Unidirectional carbon/epoxy
Aramid/EP	Reinforced	23 transv. isotropic	Unidirectional aramid/epoxy
E-glass/EP	Reinforced	23 transv. isotropic	Unidirectional glass/epoxy
Al 2024	Homogeneous	Isotropic	Aluminium sheet
PVC-60	Homogeneous core	Isotropic	PVC foam
PVC-100	Homogeneous core	Isotropic	PVC foam
Nomex-50	Honeycomb core	Orthotropic	Impregnated aramid, hexagonal cell
Al-54	Honeycomb core	Orthotropic	Aluminium, hexagonal cell

**Table 4** Specification data for reinforced and homogeneous plies in the test problems.

Property	T300/5208	Aramid/EP	E-glass/EP	Al 2024
$t$ (mm)	0.125	0.200	0.190	-
$\rho$ (kg/m <sup>3</sup> )	1600	1350	2000	2800
$m_A$ (g/m <sup>2</sup> )	200	270	380	-
$T_{sf}$ (°C)	100	25	25	25
$E_1$ (GPa)	181	75	45	72
$E_2$ (GPa)	10.3	5.5	10	72
$G_{12}$ (GPa)	7.2	2.0	5.0	27.7
$\nu_{12}$	0.28	0.34	0.30	0.30
$\alpha_1$ (e-6/°C)	0.02	- 4.0	5.5	23
$\alpha_2$ (e-6/°C)	22.5	100	25	23
$X_t$ (MPa)	1500	1400	1100	300
$X_c$ (MPa)	1500	250	675	300
$Y_t$ (MPa)	40	25	35	300
$Y_c$ (MPa)	246	100	120	300
$S$ (MPa)	68	40	80	150

**Table 5** Specification data for core plies in the test problems.

Property	PVC-60	PVC-100	Nomex-50	Al-54
$\rho$ (kg/m <sup>3</sup> )	60	100	50	54
$E_1$ (GPa)	0.03	0.11	1e-009	1e-009
$E_2$ (GPa)	0.03	0.11	1e-009	1e-009
$G_{12}$ (GPa)	0.012	0.038	1e-009	1e-009
$\nu_{12}$	0.25	0.45	0.5	0.5
$E_3$ (GPa)	0.03	0.11	0.145	0.793
$G_{23}$ (GPa)	0.012	0.038	0.021	0.152
$G_{31}$ (GPa)	0.012	0.038	0.045	0.345
$X_t$ (MPa)	1.1	2.6	1e-006	1e-006
$X_c$ (MPa)	0.4	1.8	1e-006	1e-006
$Y_t$ (MPa)	1.1	2.6	1e-006	1e-006
$Y_c$ (MPa)	0.4	1.8	1e-006	1e-006
$Z_t$ (MPa)	1.1	2.6	2.14	2.90
$Z_c$ (MPa)	0.4	1.8	2.14	2.90
$S$ (MPa)	0.5	1.6	1e-006	1e-006
$R$ (MPa)	0.5	1.6	1.28	2.00
$Q$ (MPa)	0.5	1.6	0.62	1.21

**Table 6** Laminate sets in laminate evaluation problems.

Set n:o	Ply	Lay-ups	Values of parameters
1	T300/5208	$[+\theta/-\theta]_4SE$	$\theta = 0, 5, 10, \dots, 90$
2	T300/5208	$[(0)n1/(90)n2/(45)n3/(-45)n4]SE$	$n1 = 0, 2, 4, 6, 8$ $n2 = 0, 2, 4, 6, 8$ $n3 = n4 = 0, 1, 2, 3, 4$ $n = \sum ni = 16$

The design specifications used in the laminate evaluation are given in Table 7. The specifications are relatively simple but typical in composite design:

1. The engineering constants  $E_x$ ,  $E_y$  and  $G_{xy}$  are constrained to provide the required in-plane stiffness for the laminate. In addition, thermal expansion in the longitudinal direction is constrained to obtain the required dimensional stability. An objective specifying maximisation of the engineering constant  $E_x$  is set to obtain a laminate that is as stiff as possible in the longitudinal direction.
2. Another objective specifying maximisation of the shear modulus  $G_{xy}$  is set. The weighting factors of the two objectives are set to one, which indicates that the objectives are equally important.
3. An additional constraint is set: the required load-carrying capability is specified with an in-plane load that the laminate must withstand without any failure.
4. The load constraint is modified: the operating temperature where the load is applied is given in the form of a constant load. Table 4 defines the stress-free temperatures of the laminates formed from the ply T300/5208.
5. Additional deformation constraints are set to obtain a satisfactory performance in long-term loading.

**Table 7** Design specifications in laminate evaluation problems.

Spec. n:o	Constraints	Objectives	Applied loads
1	$E_x \geq 12 \text{ GPa}$ ; $E_y \geq 12 \text{ GPa}$ ; $G_{xy} \geq 10 \text{ GPa}$ $-3.5\text{e-}6/^{\circ}\text{C} \leq \alpha_x \leq 3.5\text{e-}6/^{\circ}\text{C}$	$\max E_x$	
2	$E_x \geq 12 \text{ GPa}$ ; $E_y \geq 12 \text{ GPa}$ ; $G_{xy} \geq 10 \text{ GPa}$ $-3.5\text{e-}6/^{\circ}\text{C} \leq \alpha_x \leq 3.5\text{e-}6/^{\circ}\text{C}$	$\max E_x$ ; $w = 1$ $\max G_{xy}$ ; $w = 1$	
3	$E_x \geq 12 \text{ GPa}$ ; $E_y \geq 12 \text{ GPa}$ ; $G_{xy} \geq 10 \text{ GPa}$ $-3.5\text{e-}6/^{\circ}\text{C} \leq \alpha_x \leq 3.5\text{e-}6/^{\circ}\text{C}$ $RF \geq 1$	$\max E_x$ ; $w = 1$ $\max G_{xy}$ ; $w = 1$	$\{F\}_1$
4	$E_x \geq 12 \text{ GPa}$ ; $E_y \geq 12 \text{ GPa}$ ; $G_{xy} \geq 10 \text{ GPa}$ $-3.5\text{e-}6/^{\circ}\text{C} \leq \alpha_x \leq 3.5\text{e-}6/^{\circ}\text{C}$ $RF \geq 1$	$\max E_x$ ; $w = 1$ $\max G_{xy}$ ; $w = 1$	$\{F\}_2$
5	$E_x \geq 12 \text{ GPa}$ ; $E_y \geq 12 \text{ GPa}$ ; $G_{xy} \geq 10 \text{ GPa}$ $-3.5\text{e-}6/^{\circ}\text{C} \leq \alpha_x \leq 3.5\text{e-}6/^{\circ}\text{C}$ $RF \geq 1$ $\varepsilon_l \leq 0.3 \%$ ; $\varepsilon_c \leq 0.3 \%$ ; $\gamma \leq 0.3 \%$	$\max E_x$ ; $w = 1$ $\max G_{xy}$ ; $w = 1$	$\{F\}_2$
$\{F\}_1 = \{F\}^v = \{N_x, N_y, N_{xy}\}^T = \{-500, -200, 200\}^T \text{ kN/m}$ $\{F\}_2 = \{F\}^c + \{F\}^v$ ; $\{F\}^c = \{T/N_x, N_y, N_{xy}\}^T = \{-80 \mid 0, 0, 0\}^T \text{ } ^{\circ}\text{C} \mid \text{kN/m}$ $\{F\}^v = \{T/N_x, N_y, N_{xy}\}^T = \{0 \mid -500, -200, 200\}^T \text{ } ^{\circ}\text{C} \mid \text{kN/m}$			

All laminates in the first and second set are evaluated against the first design specification. For convenience, the laminates that do not satisfy the constraints of the specification are rejected from the sets that are evaluated against the design specifications 2 ... 5.

System default settings are used for relevant analysis options with design specifications 3, 4 and 5 containing a load. These are:

- Failure criterion: Tsai-Hill
- Factors of safety:  $FoS^c = FoS^v = 1$
- Stress/strain recovery plane: top/bottom.

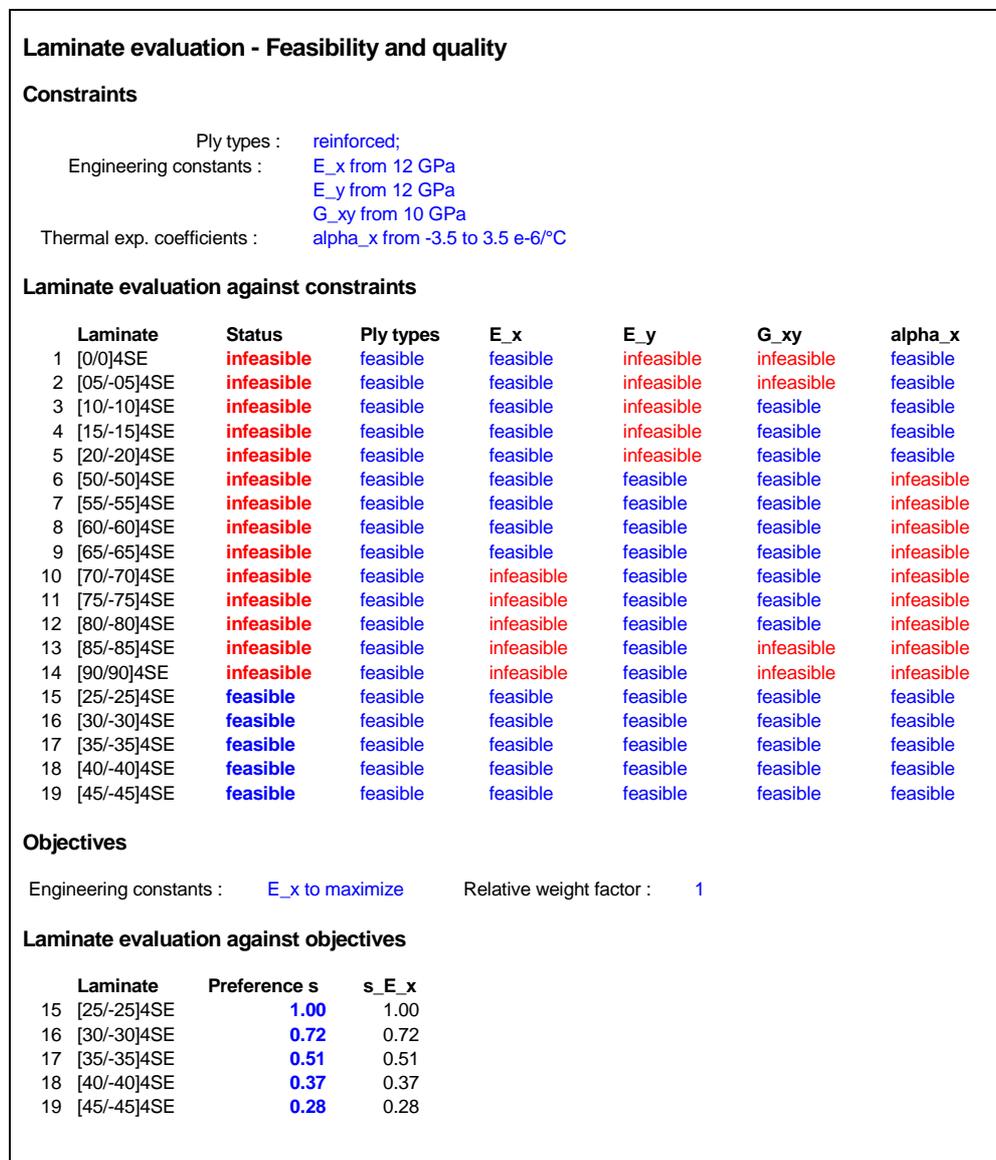
The last setting indicates that the lower of the reserve factors or margins of safety, computed for each layer at its top and bottom surfaces, represents the failure margin of the layer. Since symmetrical laminates are evaluated and only in-plane loads are applied in the test problems, the results are the same as with the other possible option setting defining the layer midplane to be the stress/strain recovery plane.

### 8.2.2 Results - first laminate set

The evaluation of the first laminate set against the first design specification shows that laminates with layer orientation angles close to  $0^\circ$  have an infeasible shear modulus and/or Young's modulus  $E_y$  (Fig. 7). Analogously, laminates with layer orientation angles close to  $90^\circ$  are infeasible because of their low shear modulus and/or low Young's modulus  $E_x$ . The laminates with layer orientation angles in between  $50^\circ$  and  $90^\circ$  are infeasible because thermal expansion coefficients  $\alpha_x$  of the laminates are out of bounds.

The laminates with layer angles from  $25^\circ$  to  $45^\circ$  satisfy all the constraints. Thus, their overall status is feasible. The lower portion of the result display in Figure 7 shows how these laminates meet the objective set for the engineering constant  $E_x$ . The laminates are listed in their ranking order with values of the preference function and the component objective function. The two values are naturally identical for each laminate since only one objective is set. The preference order of the laminates is obvious since it is well known that the modulus  $E_x$  of angle-ply laminates, formed from unidirectional plies, decreases with an increasing value of the layer orientation angle  $\theta$  in the range  $25^\circ \leq \theta \leq 45^\circ$ .

Figure 8 shows how the feasible laminates rank with the objectives of the second design specification. The new objective, maximisation of the shear modulus, does not change the preference order of the laminates compared with their order in the first test problem. The result is obvious since the layer orientation angle in these angle-ply laminates has a more radical effect on the Young's modulus than on the shear modulus. However, the values of the preference function indicate that the laminates are now much closer to each other in their overall quality because the Young's modulus  $E_x$  decreases and the shear modulus  $G_{xy}$  increases with an increasing layer orientation angle.



**Figure 7** Laminate evaluation: laminate set n:o 1, design specification n:o 1.



The evaluation of the first laminate set against the third design specification provides the results shown in Figure 9. The laminate with layer orientations  $\pm 45^\circ$  is now infeasible since it is not capable of carrying the specified load. The other four laminates are feasible and their ranking order is the same as in the previous test problem. However, the values of the shear modulus objective function and the preference function have slightly changed since the  $[\pm 45^\circ]$ -laminate that provided maximum shear stiffness in the previous test problem is not feasible.

Laminate evaluation - Feasibility and quality							
Factor of safety :		FoS <sup>v</sup> = 1					
Failure criterion :		Tsai-Hill					
Stress/strain recovery :		layer top/bottom					
Constraints							
Ply types :		reinforced;					
Engineering constants :		E <sub>x</sub> from 12 GPa					
		E <sub>y</sub> from 12 GPa					
		G <sub>xy</sub> from 10 GPa					
Thermal exp. coefficients :		alpha <sub>x</sub> from -3.5 to 3.5 e-6/°C					
Load 1 :		{-500, -200, 200} kN/m					
Laminate evaluation against constraints							
Laminate	Status	Ply types	E <sub>x</sub>	E <sub>y</sub>	G <sub>xy</sub>	alpha <sub>x</sub>	Load carrying capability
1 [45/-45]4SE	<b>infeasible</b>	feasible	feasible	feasible	feasible	feasible	<b>infeasible</b>
2 [25/-25]4SE	<b>feasible</b>	feasible	feasible	feasible	feasible	feasible	feasible
3 [30/-30]4SE	<b>feasible</b>	feasible	feasible	feasible	feasible	feasible	feasible
4 [35/-35]4SE	<b>feasible</b>	feasible	feasible	feasible	feasible	feasible	feasible
5 [40/-40]4SE	<b>feasible</b>	feasible	feasible	feasible	feasible	feasible	feasible
Objectives							
Engineering constants :		E <sub>x</sub> to maximize		Relative weight factor :		0.5	
		G <sub>xy</sub> to maximize				0.5	
Laminate evaluation against objectives							
Laminate	Preference s	s <sub>E<sub>x</sub></sub>	s <sub>G<sub>xy</sub></sub>				
2 [25/-25]4SE	<b>0.83</b>	1.00	0.67				
3 [30/-30]4SE	<b>0.76</b>	0.72	0.81				
4 [35/-35]4SE	<b>0.72</b>	0.51	0.92				
5 [40/-40]4SE	<b>0.69</b>	0.37	1.00				

**Figure 9** Laminate evaluation: laminate set n:o 1, design specification n:o 3.



Laminate evaluation - Attribute feasibility				
Laminate		Lay-up		
1	[25/-25]4SE Thu Aug 13 10:59:59 1998	((+25a/-25a)4)SE a T300/5208		
2	[30/-30]4SE Thu Aug 13 11:00:20 1998	((+30a/-30a)4)SE a T300/5208		
3	[35/-35]4SE Thu Aug 13 11:00:40 1998	((+35a/-35a)4)SE a T300/5208		
Load 1 : {-500, -200, 200} kN/m + {-80} C				
Factors of safety : FoS^c = 1, FoS^v = 1				
Failure criterion : Tsai-Hill				
Stress/strain recovery : layer top/bottom				
Laminate	Status	[25/-25]4SE	[30/-30]4SE	[35/-35]4SE
		infeasible	feasible	feasible
Lay-up				
Ply types	reinforced;	Solid;Reinf.	Solid;Reinf.	Solid;Reinf.
Mechanical and hygrothermal behavior				
E_x	from 12 GPa	90.07	64.81	46.09
E_y	from 12 GPa	12.55	14.01	16.25
G_xy	from 10 GPa	30.30	36.74	41.98
alpha_x	from -3.5 to 3.5 e-6/°C	-2.21	-2.37	-1.92
Deformations				
eps_t	to 0.3 %	0.0000	0.0000	0.0250
eps_c	to 0.3 %	-0.3920	-0.1841	-0.2864
eps_s	to 0.3 %	0.3300	0.2722	0.2382
Load carrying capability				
		feasible	feasible	feasible
Load carrying capability				
				Load 1
[25/-25]4SE	MoS_FPF	(%)	38	
	MoS^c_FPF	(%)	81	
	MoS^v_FPF	(%)	73	
	MoS^v+c_FPF	(%)	88	
	MoS^r_FPF	(%)	27	
[30/-30]4SE	MoS_FPF	(%)	125	
	MoS^c_FPF	(%)	42	
	MoS^v_FPF	(%)	157	
	MoS^v+c_FPF	(%)	69	
	MoS^r_FPF	(%)	64	
[35/-35]4SE	MoS_FPF	(%)	110	
	MoS^c_FPF	(%)	20	
	MoS^v_FPF	(%)	138	
	MoS^v+c_FPF	(%)	28	
	MoS^r_FPF	(%)	26	

Figure 11 Laminate evaluation: feasibility of laminates, laminate set n:o 1, design specification n:o 5.

### 8.2.3 Results - second laminate set

The evaluation results of the second laminate set resemble those obtained for the first set:

1. According to Figure 12, seven candidate laminates satisfy all the constraints of the first design specification. The preference order of the laminates is obvious since it is well known that the modulus  $E_x$  of  $[0^\circ/90^\circ/\pm 45^\circ]$ -laminates, formed from unidirectional plies, decreases when the number of  $0^\circ$  layers decreases.
2. An additional objective set for the shear modulus  $G_{xy}$  changes the values of the preference function (Fig. 13). The ranking order of the laminates is also changed compared with the previous test problem, though the best laminate is still the same.
3. Six candidate laminates are able to carry the in-plane load of the third design specification (Fig. 14). The values of the preference function are changed compared with the second test problem since the  $[\pm 45^\circ]$ -laminate that provided the best shear stiffness in the previous test problem is infeasible. Three laminates possess the same maximum value of the preference function.
4. Three candidate laminates satisfy the load constraint of the fourth design specification that defines the temperature where the external load is applied (Fig. 15). The laminate with 50 % of layers in the  $0^\circ$  direction and 50 % in the  $\pm 45^\circ$  directions now performs best.
5. Figure 16 shows that only one candidate laminate is able to satisfy the constraints of the fifth design specification. The other two laminates that satisfied the fourth design specification are ruled out because maximum compressive and/or shear strains owing to the applied load exceed the allowed values.

### 8.2.4 Verification of results

The laminate evaluation results were verified as follows:

- The values of all quantitative attributes were computed with analysis tools of the system to confirm that the evaluation tool uses these tools properly.
- The design attribute values were compared with the constraints to confirm that all conclusions on the feasibility/infeasibility of attribute values are correct.
- The values of the component objective functions were computed manually with the equations of Chapter 4 to confirm that the equations are properly used.
- The values of the preference function were computed manually to confirm that the values are properly derived.

Laminate evaluation - Feasibility and quality						
<b>Constraints</b>						
Ply types :	reinforced;					
Engineering constants :	E <sub>x</sub> from 12 GPa					
	E <sub>y</sub> from 12 GPa					
	G <sub>xy</sub> from 10 GPa					
Thermal exp. coefficients :	alpha <sub>x</sub> from -3.5 to 3.5 e-6/°C					
<b>Laminate evaluation against constraints</b>						
Laminate	Status	Ply types	E <sub>x</sub>	E <sub>y</sub>	G <sub>xy</sub>	alpha <sub>x</sub>
1 [0_0/90_2/45_3/-45_3]SE	infeasible	feasible	feasible	feasible	feasible	infeasible
2 [0_0/90_4/45_2/-45_2]SE	infeasible	feasible	feasible	feasible	feasible	infeasible
3 [0_0/90_6/45_1/-45_1]SE	infeasible	feasible	feasible	feasible	feasible	infeasible
4 [0_0/90_8/45_0/-45_0]SE	infeasible	feasible	infeasible	feasible	infeasible	infeasible
5 [0_2/90_6/45_0/-45_0]SE	infeasible	feasible	feasible	feasible	infeasible	infeasible
6 [0_4/90_4/45_0/-45_0]SE	infeasible	feasible	feasible	feasible	infeasible	feasible
7 [0_6/90_2/45_0/-45_0]SE	infeasible	feasible	feasible	feasible	infeasible	feasible
8 [0_8/90_0/45_0/-45_0]SE	infeasible	feasible	feasible	infeasible	infeasible	feasible
9 [0_6/90_0/45_1/-45_1]SE	feasible	feasible	feasible	feasible	feasible	feasible
10 [0_4/90_2/45_1/-45_1]SE	feasible	feasible	feasible	feasible	feasible	feasible
11 [0_4/90_0/45_2/-45_2]SE	feasible	feasible	feasible	feasible	feasible	feasible
12 [0_2/90_2/45_2/-45_2]SE	feasible	feasible	feasible	feasible	feasible	feasible
13 [0_2/90_0/45_3/-45_3]SE	feasible	feasible	feasible	feasible	feasible	feasible
14 [0_2/90_4/45_1/-45_1]SE	feasible	feasible	feasible	feasible	feasible	feasible
15 [0_0/90_0/45_4/-45_4]SE	feasible	feasible	feasible	feasible	feasible	feasible
<b>Objectives</b>						
Engineering constants :	E <sub>x</sub> to maximize	Relative weight factor :	1			
<b>Laminate evaluation against objectives</b>						
Laminate	Preference s	s_E <sub>x</sub>				
9 [0_6/90_0/45_1/-45_1]SE	1.00	1.00				
10 [0_4/90_2/45_1/-45_1]SE	0.73	0.73				
11 [0_4/90_0/45_2/-45_2]SE	0.73	0.73				
12 [0_2/90_2/45_2/-45_2]SE	0.49	0.49				
13 [0_2/90_0/45_3/-45_3]SE	0.45	0.45				
14 [0_2/90_4/45_1/-45_1]SE	0.44	0.44				
15 [0_0/90_0/45_4/-45_4]SE	0.18	0.18				

Figure 12 Laminate evaluation: laminate set n:o 2, design specification n:o 1.





## 8.3 PLY EVALUATION

### 8.3.1 Test problems

The performance of the ply evaluation process is demonstrated by solving the nine test problems specified in Table 8.

**Table 8** Candidate plies and design specifications in laminate creation problems.

Test problem	Candidate plies	Constraints	Objectives	Applied loads
1	Set 1	$E_x \geq 12 \text{ GPa} ; E_y \geq 12 \text{ GPa} ; G_{xy} \geq 10 \text{ GPa}$	$\max E_x$	
2	Set 1	$E_x \geq 12 \text{ GPa} ; E_y \geq 12 \text{ GPa} ; G_{xy} \geq 10 \text{ GPa}$ $-3.5\text{e-}6/^{\circ}\text{C} \leq \alpha_x \leq 3.5\text{e-}6/^{\circ}\text{C}$	$\max E_x$	
3	Set 1	$RF \geq 1$	$\min h$	$\{F\}_1$
4	Set 1	$RF \geq 1$	$\min h$	$\{F\}_1 ; \{F\}_2$
5	Set 1	$RF \geq 1$	$\min h$	$\{F\}_3 ; \{F\}_4$
6	Set 1	$RF \geq 1$	$\min h$	$\{F\}_5$
7	Set 1	$RF \geq 1$	$\min h$	$\{F\}_6$
8	Set 1	$RF \geq 1$	$\max h$	$\{F\}_7$
9	Set 2	$RF \geq 1$	$\min m_A$	$\{F\}_6$

Set 1: Al 2024 + Aramid/EP + E-glass/EP + T300/5208  
Set 2: Set 1 + Al-54 + Nomex-50 + PVC-60 + PVC-100

$\{F\}_1 = \{F\}^v = \{N_x, N_y, N_{xy}\}^T = \{0, 0, 2\}^T \text{ MN/m}$   
 $\{F\}_2 = \{F\}^v = \{N_x, N_y, N_{xy}\}^T = \{4, 2, 0\}^T \text{ MN/m}$   
 $\{F\}_3 = \{F\}^c + \{F\}^v ; \{F\}^c = \{T / N_x, N_y, N_{xy}\}^T = \{20 | 0, 0, 0\}^T \text{ } ^{\circ}\text{C} | \text{ MN/m}$   
 $\{F\}^v = \{T | N_x, N_y, N_{xy}\}^T = \{0 | 0, 0, 2\}^T \text{ } ^{\circ}\text{C} | \text{ MN/m}$   
 $\{F\}_4 = \{F\}^c + \{F\}^v ; \{F\}^c = \{T | N_x, N_y, N_{xy}\}^T = \{0 | 4, 2, 0\}^T \text{ } ^{\circ}\text{C} | \text{ MN/m}$   
 $\{F\}^v = \{T | N_x, N_y, N_{xy}\}^T = \{20 | 0, 0, 0\}^T \text{ } ^{\circ}\text{C} | \text{ MN/m}$   
 $\{F\}_5 = \{F\}^v = \{M_x, M_y, M_{xy}\}^T = \{2, -1, 0.5\}^T \text{ kNm/m}$   
 $\{F\}_6 = \{F\}^v = \{N_x, N_y, N_{xy} / M_x, M_y, M_{xy}\}^T = \{1, 1, 0.25 | 1, 2, 0\}^T \text{ MN/m} | \text{ kNm/m}$   
 $\{F\}_7 = \{F\}^v = \{\kappa_x, \kappa_y, \kappa_{xy}\}^T = \{1, 0, 0\}^T \text{ 1/m}$

Three orthotropic plies and one isotropic ply are evaluated in Problems 1...8. In Problems 1 and 2, constraints and objectives are set for the engineering constants and expansion coefficients only. Different types of loads are applied to the laminate in Problems 3...7, the objective being minimisation of thickness. The thickest laminate capable of withstanding a specified curvature is searched for in Problem 8.

Four core plies are added to the set of candidate plies in Problem 9, indicating that a sandwich laminate is being searched for. A load containing in-plane forces and moments is applied to the laminate, the objective being the minimisation of mass.

Problems 3...9 are solved by applying the maximum strain criterion and the Tsai-Hill criterion for orthotropic plies. The results then indicate how the simplified laminate-level failure analysis performs with different failure criteria. The von Mises failure criterion is applied for “laminates” formed from the homogeneous Al 2024 ply.

### 8.3.2 Results

The ply evaluation results are summarised in Tables 9, 10 and 11. Table 9 gives results for Problems 1 and 2, in which constraints are set for the engineering constants and expansion coefficients. Tables 10 and 11 give results for Problems 3...9, in which a constraint is set for load-carrying capability. The applied failure criterion for in-plane orthotropic plies is the maximum strain criterion in Table 10 and the Tsai-Hill criterion in Table 11. The tables are constructed, column-by-column, as follows:

1. The number of the test problem is identified.
2. The candidate plies are listed in their preference order.
3. The layer orientations and proportions of layer orientations in representative solid laminates, and in the face sheets of representative sandwich laminates, are given in the form  $\pm\theta_{min}/p_{\theta min}$  ;  $\pm\theta/p_{\theta}$  ;  $\pm\theta_{max}/p_{\theta max}$ . Alternatively, the infeasibility of a ply is identified.
4. The thickness values of representative laminates are given, as well as the thickness ratios of representative sandwich laminates when applicable (Problem 9).
5. The value of the preference function is given for each feasible candidate ply.
6. The Margins of Safety (*MoS*) are given in Tables 10 and 11 for laminates that are created from representative laminates by modelling their surfaces with actual layers as in the laminate search phase (Chapter 7). When two loads are applied, the lower of the values is shown. The values indicate how the simplified laminate-level failure analysis performs compared with the normally used ply-level failure analysis.

**Table 9** Ply evaluation results, Problems 1 and 2.

Test problem	Candidate ply	Feasibility / Representative laminate	<i>h</i> (mm)	<i>s</i>
1	T300/5208	0/ 75% ; ±45/ 25% ; 90/ 0%	0.10	1.00
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	0.10	0.50
	Aramid/EP	0/ 50% ; ±45/ 50% ; 90/ 0%	0.10	0.29
	E-glass/EP	0/ 25% ; ±45/ 75% ; 90/ 0%	0.10	0.16
2	T300/5208	0/ 75% ; ±29/ 25% ; 90/ 0%	0.10	1.00
	Aramid/EP	0/ 0% ; ±32/ 75% ; 90/ 25%	0.10	0.53
	E-glass/EP	Infeasible		
	Al 2024	Infeasible		

**Table 10** Ply evaluation results, Problems 3...9, maximum strain / von Mises criterion.

Test problem	Candidate ply	Feasibility / Representative laminate	$h / c/h$ (mm / -)	$s$	$MoS$ <sup>1</sup> (%)
3	T300/5208	0/ 0% ; ±45/ 100% ; 90/ 0%	5.53	1.00	0
	Aramid/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	15.51	0.67	0
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	11.56	0.36	0
	E-glass/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	22.87	0.00	0
4	T300/5208	0/ 25% ; ±45/ 75% ; 90/ 0%	7.20	1.00	1
	Aramid/EP	0/ 25% ; ±45/ 75% ; 90/ 0%	19.99	0.55	0
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	11.56	0.39	0
	E-glass/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	22.87	0.00	-17
5	T300/5208	0/ 25% ; ±45/ 75% ; 90/ 0%	12.18	1.00	-2
	Aramid/EP	0/ 25% ; ±45/ 75% ; 90/ 0%	20.35	0.71	0
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	11.56	0.53	0
	E-glass/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	23.39	0.00	-17
6	T300/5208	0/ 50% ; ±45/ 25% ; 90/ 25%	5.68	1.00	0...1
	Aramid/EP	0/ 75% ; ±45/ 0% ; 90/ 25%	9.13	0.73	0...2
	E-glass/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	9.71	0.12	-16
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	7.46	0.00	0
7	T300/5208	0/ 25% ; ±45/ 25% ; 90/ 50%	6.47	1.00	-21...-20
	Aramid/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	10.52	0.66	-5...-4
	E-glass/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	9.14	0.29	-57...-56
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	7.71	0.00	0
8	E-glass/EP	0/ 100% ; ±45/ 0% ; 90/ 0%	23.33	1.00	29
	Aramid/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	21.88	0.94	-39
	T300/5208	0/ 100% ; ±45/ 0% ; 90/ 0%	16.57	0.71	0
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	8.53	0.37	0
9 <sup>2</sup>	T300/5208	0/ 0% ; ±45/ 100% ; 90/ 0%	16.27 / 0.80	1.00	-16
	Aramid/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	19.84 / 0.80	0.95	-48
	E-glass/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	20.72 / 0.80	0.51	-70
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	20.96 / 0.80	0.00	0

<sup>1</sup>  $MoS$  values computed for laminates that are created from representative laminates by modelling their surfaces with actual layers as described in Chapter 7. As applicable, the range of  $MoS$  values achieved with different stacking sequences of surface layers is given.

<sup>2</sup> The best core ply selected by the tool for the representative laminates is Al-54.

**Table 11** Ply evaluation results, Problems 3...9, Tsai-Hill / von Mises criterion.

Test problem	Candidate ply	Feasibility / Representative laminate	$h / c/h$ (mm / -)	$s$	$MoS$ <sup>1</sup> (%)
3	T300/5208	0/ 0% ; ±45/ 100% ; 90/ 0%	4.78	1.00	0
	Aramid/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	17.37	0.41	0
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	11.56	0.08	0
	E-glass/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	17.24	0.00	0
4	T300/5208	0/ 25% ; ±45/ 75% ; 90/ 0%	7.46	1.00	-26
	Aramid/EP	0/ 25% ; ±45/ 75% ; 90/ 0%	22.38	0.36	0
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	11.56	0.29	0
	E-glass/EP	0/ 25% ; ±45/ 75% ; 90/ 0%	20.28	0.00	-42
5	T300/5208	0/ 25% ; ±45/ 75% ; 90/ 0%	9.80	1.00	-41
	Aramid/EP	0/ 25% ; ±45/ 75% ; 90/ 0%	23.89	0.35	0
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	11.56	0.35	0
	E-glass/EP	0/ 25% ; ±45/ 75% ; 90/ 0%	20.68	0.00	-42
6	T300/5208	0/ 50% ; ±45/ 25% ; 90/ 25%	5.55	1.00	-8...-7
	Aramid/EP	0/ 75% ; ±45/ 0% ; 90/ 25%	9.12	0.71	-28...-27
	E-glass/EP	0/ 75% ; ±45/ 0% ; 90/ 25%	10.27	0.03	18...19
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	7.46	0.00	0
7	T300/5208	0/ 0% ; ±45/ 75% ; 90/ 25%	6.29	1.00	-32
	Aramid/EP	0/ 25% ; ±45/ 25% ; 90/ 50%	10.38	0.68	-36
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	7.71	0.07	0
	E-glass/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	11.22	0.00	-54
8	E-glass/EP	0/ 100% ; ±45/ 0% ; 90/ 0%	30.00	1.00	-31
	Aramid/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	21.04	0.70	-41
	T300/5208	0/ 100% ; ±45/ 0% ; 90/ 0%	16.57	0.55	-14
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	8.53	0.28	0
9 <sup>2</sup>	T300/5208	0/ 25% ; ±45/ 50% ; 90/ 25%	16.51 / 0.80	1.00	-35
	Aramid/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	20.38 / 0.80	0.94	-57
	E-glass/EP	0/ 0% ; ±45/ 100% ; 90/ 0%	27.17 / 0.80	0.09	-66
	Al 2024	0/ 100% ; ±45/ 0% ; 90/ 0%	20.96 / 0.80	0.00	0

<sup>1</sup>  $MoS$  values computed for laminates that are created from representative laminates by modelling their surfaces with actual layers as described in Chapter 7. As applicable, the range of  $MoS$  values achieved with different stacking sequences of surface layers is given.

<sup>2</sup> The best core ply selected by the tool for the representative laminates is Al-54.

The results of Problem 1 (Table 9) show that a laminate satisfying the specified stiffness constraints exists for all plies, though the representative laminates formed from the Aramid/EP and E-glass/EP plies are not very efficient with respect to the objective. Since constraints or objectives have not been set for the thickness-dependent attributes, the in-built system rules specify the thickness values of the laminates as 0.1 mm, i.e. at the lowest possible value in the thickness range considered by the tool. The results provided by the ply evaluation tool are shown in Figure 17.

In Problem 2, the same set of plies is evaluated with an additional constraint set for the thermal expansion coefficient  $\alpha_x$ . The results in Table 9 indicate that the E-glass/EP or Al 2024 plies are infeasible owing to the new constraint. The representative laminates of the other two plies differ from those of Problem 1, containing layers with the orientations that minimise the longitudinal expansion coefficient of  $\pm\theta$ -laminates. The in-built system rules again specify thickness values of the representative laminates to the lowest value considered by the tool.

Laminate creation - Ply feasibility and quality					
<b>Evaluated plies</b>					
Ply 1 :	T300/5208				
	Thu Jun 11 10:49:37 1998				
Ply 2 :	Al 2024				
	Thu Jun 11 10:47:35 1998				
Ply 3 :	Aramid/EP				
	Wed Mar 21 11:44:20 2001				
Ply 4 :	E-glass/EP				
	Thu Jun 11 10:50:27 1998				
<b>Constraints</b>					
<b>Ply feasibility</b>		<b>Ply 1</b>	<b>Ply 2</b>	<b>Ply 3</b>	<b>Ply 4</b>
<b>Status</b>		<b>feasible</b>	<b>feasible</b>	<b>feasible</b>	<b>feasible</b>
<b>Mech. and hygroth. constr.</b>		<b>feasible</b>	<b>feasible</b>	<b>feasible</b>	<b>feasible</b>
Engineering constants		<b>feasible</b>	<b>feasible</b>	<b>feasible</b>	<b>feasible</b>
E_x from 12 GPa		143.11	72.00	41.69	22.70
E_y from 12 GPa		20.84	72.00	12.14	15.17
G_xy from 10 GPa		17.03	27.70	10.68	10.62
<b>Objectives</b>					
Engineering constants :	E_x to maximize	Relative weight factor :		1	
<b>Ply quality</b>					
<b>Ply</b>	<b>Preference</b>	<b>E_x</b>			
1 T300/5208	1.00	1.00			
2 Al 2024	0.50	0.50			
3 Aramid/EP	0.29	0.29			
4 E-glass/EP	0.16	0.16			
<b>Representative laminates</b>					
<b>Ply</b>	<b>theta_1 / p_1</b>	<b>theta_2 / p_2</b>	<b>theta_3 / p_3</b>	<b>h_opt</b>	
				mm	
1 T300/5208	0° / 75%	± 45° / 25%	90° / 0%	0.10	
2 Al 2024	0° / 100%	± 45° / 0%	90° / 0%	0.10	
3 Aramid/EP	0° / 50%	± 45° / 50%	90° / 0%	0.10	
4 E-glass/EP	0° / 25%	± 45° / 75%	90° / 0%	0.10	

Figure 17 Ply evaluation results, Problem 1.

In Problem 3, the minimum thickness is sought for a laminate subjected to a shear load. Tables 10 and 11 indicate that all plies are feasible. The representative laminates formed from in-plane orthotropic plies are of the type  $\pm 45^\circ$ , which is an obvious solution for laminates loaded in shear. The values of the preference function clearly indicate the efficiency of the T300/5208 ply compared with other plies. The *MoS* values computed for laminates with actual surface layers are all zero, i.e. in this special case the simplified laminate-level failure analysis provides the same results as the normal ply-level failure analysis.

The thickness of the representative laminate of T300/5208 is 4.78 mm with the Tsai-Hill criterion. This is in line with the reference solution given by Massard and Paterson:<sup>63</sup> by applying the Tsai-Hill criterion they end up with a  $\pm 45^\circ$ -laminate formed from 42 layers of 0.125 mm thick. The layer orientations are thus the same but the thickness of the laminate, 5.25 mm, is higher, representing the best feasible actual laminate that can be formed from the ply with a symmetrical and balanced lay-up.

In Problem 4, the laminate being sought must withstand a shear load and a biaxial load. All plies are again feasible, but representative laminates of the in-plane orthotropic plies now differ in laminate structure (Tables 10 and 11): the  $\pm 45^\circ$  E-glass/EP laminate performs best when the maximum strain criterion is applied, while the other representative laminates have 25% of layers in the  $0^\circ$  direction and 75 % of layers in the  $\pm 45^\circ$  directions. The preference order of the plies is the same as in Problem 3. The lowest *MoS* value computed for laminates with actual surface layers is -42 % for the E-glass/EP laminate analysed with the Tsai-Hill criterion. The performance of the simplified laminate-level failure analysis is thus relatively poor for these laminate/load combinations.

Massard and Paterson again give a reference solution to Problem 4 in the form of a symmetrical and balanced actual laminate made of a 0.125 mm thick T300/5208 ply.<sup>63</sup> With the Tsai-Hill criterion they end up with a laminate with 78 layers, 22 (28.2%) of them in the  $0^\circ$  direction and 56 (71.8%) in the  $\pm 45^\circ$  directions. The proportions of the layer orientations are close to the values of the representative laminate but the reference laminate is again considerably thicker. This is partly explained by the homogenised nature of the representative laminate, partly by the error owing to the simplified failure analysis approach (according to Table 11, the *MoS* value of the representative laminate with actual surface layers is -26%).

Problem 5 is a modification of Problem 4: the operating temperature (20 °C) is added to the design specification in the form of a constant load vector. According to Table 3, the reference (stress-free) temperature 100 °C is specified for laminates formed from the T300/5208 ply and 25 °C for laminates formed from other candidate plies. Tables 10 and 11 show that representative laminates are the same as in Problem 4. The preference order of the plies is also the same. The thickness values of the representative laminates formed from the T300/5208 ply clearly increase due to the internal thermal stresses. The highest absolute *MoS* values computed for representative laminates with actual surface layers are of the same magnitude as in Problem 4.

In problem 6, the laminate being searched for must withstand a moment load. Tables 10 and 11 show that the selected failure criterion affects the representative laminate of the E-glass/EP ply but not the representative laminates of other plies. The mutual preference order of the E-

glass/EP and Al 2024 plies is changed compared with the results of the earlier problems. The performance of the simplified laminate-level failure analysis is only satisfactory: the lowest  $MoS$  values computed for representative laminates with actual surface layers are -27...-28 %.

In Problem 7, with the load containing in-plane forces and moments, four types of laminates exist amongst the representative laminates of the in-plane orthotropic plies. The mutual preference order of the E-glass/EP and Al 2024 plies depends on the failure criterion applied. The performance of the simplified laminate-level failure analysis is poor: the lowest  $MoS$  values computed for representative laminates with actual surface layers are -56...-57 %.

In Problem 8, the thickest laminate capable of withstanding the specified curvature is searched for. Tables 10 and 11 indicate that the preference order of the in-plane orthotropic plies changes radically compared to other test problems. The low modulus E-glass/EP and Aramid/EP plies now perform better than the high modulus T300/5208 ply. The representative laminates of the E-glass/EP and T300/5208 plies are unidirectional (100 % of layers in the  $0^\circ$  direction), but of the type  $\pm 45^\circ$  for the Aramid/EP ply. The lowest  $MoS$  value computed for representative laminates with actual surface layers is -41 %. The performance of the simplified laminate-level failure analysis is thus relatively poor also in this case.

In Problem 9, four core plies are added to the set of candidate plies, indicating that a sandwich laminate is being searched for. The design specification is the same as in Problem 7. The best performing core ply is the Al-54 ply which is also used in representative laminates. Face sheets of the representative laminates are of the type  $\pm 45^\circ$  with one exception: a quasi-isotropic face sheet made of the T300/5208 ply performs best when the Tsai-Hill criterion is applied (Table 11). The thickness ratio  $c/h$  is 0.80 for all representative laminates. The performance of the simplified laminate-level failure analysis is again poor: the  $MoS$  value -70 %, computed for the representative laminate of the E-glass/EP-laminate with actual surface layers, is the lowest  $MoS$  value when all test problems are accounted for.

It should further be noted that layers in the representative laminates of the Al 2024 ply are always in the  $0^\circ$  direction since the layer orientation has no effect on laminate properties. Also, the reference  $MoS$  values are zero for laminates formed from the Al 2024 ply since no approximations are made in failure analyses of laminates formed from in-plane isotropic plies.

### 8.3.3 Verification of results

The results of Problems 1 and 2 were verified by utilising the laminate evaluation tool: all homogenised laminates considered in ply evaluation were specified and evaluated to confirm that the representative laminates are feasible and maximise the preference function. The ESAComp analysis tool *Laminate 2.5D behaviour* was further used to check that values of the angles minimising the expansion coefficients are correct in Problem 2.

Results of Problems 3...9 were verified by utilising the ESAComp analysis tool *Laminate failure*: all the homogenised laminates considered by the tool were specified using the thickness of the representative laminate for each laminate. The analysis confirmed that the representative laminate is the only feasible laminate in the set.

## 8.4 LAMINATE SEARCH

### 8.4.1 Test problems

Problems 1...9 specified in Table 9 are further used to demonstrate the performance of the laminate search process. For each test problem, the optimum laminate that can be formed from the T300/5208 ply is searched for. Problems 3...9 are solved by applying one (maximum strain) criterion only since the normal ply level failure analysis approach is used in laminate search.

The test problems are solved with and without layer orientation constraints to demonstrate that the optimum solution can be found in both cases. In the former case, the allowed layer orientation angles are  $0^\circ$ ,  $\pm 45^\circ$  and  $90^\circ$ . The results then also indicate how these normally applied layer orientation constraints penalise the design.

### 8.4.2 Results

Table 12 gives the results obtained by the prototype tool for Problems 1 and 2. The table is constructed, column-by-column, as follows:

1. The number of the test problem is identified
2. The layer orientations and proportions of the layer orientations are listed for the representative and best laminates in the form  $\pm\theta_{min}/p_{\theta min}; \pm\theta/p_{\theta}; \pm\theta_{max}/p_{\theta max}$
3. The values of the attribute for which an objective has been set are given.

**Table 12** Laminate search results for the test problems 1 and 2.

Test problem	Representative laminate Best laminate	Attribute values
1(a)	0/ 75% ; $\pm 45$ / 25% ; 90/ 0% 0/ 90% ; $\pm 45$ / 10% ; 90/ 0%	$E_x = 143.1$ GPa $E_x = 166.2$ GPa
1(b)	0/ 75% ; $\pm 45$ / 25% ; 90/ 0% 0/ 85% ; $\pm 30$ / 15% ; 90/ 0%	$E_x = 143.1$ GPa $E_x = 166.6$ GPa
2(a)	0/ 75% ; $\pm 45$ / 25% ; 90/ 0% 0/ 90% ; $\pm 45$ / 10% ; 90/ 0%	$E_x = 143.1$ GPa $E_x = 166.2$ GPa
2(b)	0/ 75% ; $\pm 29$ / 25% ; 90/ 0% 0/ 80% ; $\pm 29$ / 20% ; 90/ 0%	$E_x = 157.2$ GPa $E_x = 162.3$ GPa

(a) Layer orientations constrained to  $0^\circ$ ,  $\pm 45^\circ$  and  $90^\circ$

(b) No layer orientation constraints

The results of Problem 1 indicate that the search phase considerably improves the solution found in the ply evaluation phase. The layer orientation constraints in this case have only a minor effect on the optimum value of the modulus  $E_x$ .

In Problem 2, the representative laminates with and without the layer orientation constraints differ considerably owing to the fact that the angle minimising the thermal expansion coefficient ( $29^\circ$ ) is also considered when the layer orientations are not constrained. The representative laminate achieved without layer orientation constraints is much more efficient than the one based on the constrained layer orientations. The search phase improves the solution in both cases. However, without layer orientation constraints the end result is a local optimum with a slightly lower  $E_x$  value compared to the solution computed with constrained layer orientations.

Table 13 gives results provided by the laminate search phase for Problems 3...9. Again, the number of the test problem is identified in the first column. The second column lists the laminates in the sequence (1) representative laminate, (2) initial solution, (3) best laminate with constrained layer orientations and (4) best laminate without layer orientation constraints. This compressed notation can be used since representative laminates and initial solutions for each problem are the same with and without layer orientation constraints (no constraints or objectives have been set for expansion coefficients). The third column gives the thickness values of the laminates. The fourth column lists the thickness ratios  $c/h$  for sandwich laminates of Problem 9.

According to Table 10, the representative laminates in Problems 3 and 4 are feasible also when their surfaces are modelled with actual layers. Thus, the representative laminates and the initial solutions have the same thickness (Table 13). In Problem 3, the solution does not improve in the laminate search phase. This is natural since the laminate-level failure analysis used in ply evaluation gives for this laminate and load case the same result as the ply-level failure analysis. Also, the  $\pm 45^\circ$ -laminate is known to be the best laminate for shear loading. In Problem 4, the search phase with layer orientation constraints slightly improves the design for the reason that the set of possible proportions of layer orientations is extended. A considerable improvement is further achieved when the layer orientation  $\theta$  is not constrained.

In Problem 5, the representative laminate is infeasible when ply level failure analysis is applied ( $MoS = -2\%$ ). The system therefore searches a feasible initial solution in the beginning of the search phase. The best solution is close to the representative laminate when the layer orientation  $\theta$  is constrained to  $45^\circ$ . With no layer orientation constraint, a considerably thinner feasible laminate is found.

In Problem 6, the representative laminate is feasible when ply level failure analysis is applied. Search practically does not improve the design even when the layer orientation  $\theta$  is not constrained.

In Problem 7, the representative laminate is infeasible when ply level failure analysis is applied ( $MoS = -20\%$ ). The solutions obtained in the search phase with and without the layer orientation constraints are close to each other in quality.

In Problem 8, the representative laminate and the best solutions are practically the same because the structure of the representative laminate is already optimal for the load case and the laminate-level failure analysis used in ply evaluation phase gives for this laminate and load case the same result as the ply-level failure analysis.

In Problem 9, the representative sandwich laminate is infeasible. The proportions of layer orientations do not change in search. The original value  $45^\circ$  of  $\theta$  also appears to be best even when layer orientations are not constrained. Thus, the best solutions are the same with and without layer orientation constraints. The thickness ratio  $c/h$  slightly decreases in search.

**Table 13** Laminate search results for the test problems 3...9.

Test problem	Representative laminate Initial solution Best laminate (a) Best laminate (b)	$h$ (mm)	$c/h$
3	0/ 0% ; $\pm 45$ / 100% ; 90/ 0%	5.53	
	0/ 0% ; $\pm 45$ / 100% ; 90/ 0%	5.53	
	0/ 0% ; $\pm 45$ / 100% ; 90/ 0%	5.53	
	0/ 0% ; $\pm 45$ / 100% ; 90/ 0%	5.53	
4	0/ 25% ; $\pm 45$ / 75% ; 90/ 0%	7.20	
	0/ 25% ; $\pm 45$ / 75% ; 90/ 0%	7.20	
	0/ 15% ; $\pm 45$ / 85% ; 90/ 0%	6.98	
	0/ 10% ; $\pm 40$ / 90% ; 90/ 0%	6.10	
5	0/ 25% ; $\pm 45$ / 75% ; 90/ 0%	12.18	
	0/ 25% ; $\pm 45$ / 75% ; 90/ 0%	15.23	
	0/ 15% ; $\pm 45$ / 85% ; 90/ 0%	12.23	
	0/ 0% ; $\pm 40$ / 100% ; 90/ 0%	9.56	
6	0/ 50% ; $\pm 45$ / 25% ; 90/ 25%	5.68	
	0/ 50% ; $\pm 45$ / 25% ; 90/ 25%	5.68	
	0/ 55% ; $\pm 45$ / 20% ; 90/ 25%	5.50	
	0/ 50% ; $\pm 40$ / 20% ; 90/ 30%	5.54	
7	0/ 25% ; $\pm 45$ / 25% ; 90/ 50%	6.47	
	0/ 25% ; $\pm 45$ / 25% ; 90/ 50%	8.09	
	0/ 10% ; $\pm 45$ / 65% ; 90/ 25%	6.87	
	0/ 0% ; $\pm 40$ / 70% ; 90/ 30%	6.92	
8	0/ 100% ; $\pm 45$ / 0% ; 90/ 0%	16.57	
	0/ 100% ; $\pm 45$ / 0% ; 90/ 0%	16.57	
	0/ 100% ; $\pm 45$ / 0% ; 90/ 0%	16.57	
	0/ 100% ; $\pm 45$ / 0% ; 90/ 0%	16.57	
9	0/ 0% ; $\pm 45$ / 100% ; 90/ 0%	16.27	0.80
	0/ 0% ; $\pm 45$ / 100% ; 90/ 0%	20.34	0.80
	0/ 0% ; $\pm 45$ / 100% ; 90/ 0%	17.29	0.78
	0/ 0% ; $\pm 45$ / 100% ; 90/ 0%	17.29	0.78

(a) Layer orientations constrained to  $0^\circ$ ,  $\pm 45^\circ$  and  $90^\circ$

(b) No layer orientation constraints

### **8.4.3 Verification of results**

The prototype tool used in problem solving provided as a result the best solution and the laminates in the design space around the best solution. This set of laminates was evaluated with the laminate evaluation tool to confirm that the best solution is feasible and maximises the preference function, i.e. is at least a local optimum in the design space and with the search method applied.

## **8.5 OPERATING SPEED**

The operating speed of the system naturally depends on the hardware and on the operating system. The solving times for the laminate and ply evaluation problems were measured using a personal computer (IBM Think Pad A22p, Intel Pentium III processor, 128 MB RAM) with the Windows XP 2002 operating system. The measured times for the laminate evaluation problems were 3...15 seconds and for the ply evaluation problems 5...55 seconds. The longest solving time was measured in ply evaluation for Problem 9, in which the set of candidate plies contained 4 core plies and 4 reinforced/homogeneous plies.

## 9 DISCUSSION

The key features of the system developed, as well as possible system enhancements and extensions are discussed in this chapter.

### 9.1 SYSTEM DEVELOPED

The important topics specific to the developed design system are multiobjective design, the simplifications related to the design space in laminate creation, the approximate failure analysis techniques used in ply evaluation, the applied laminate search method and, naturally, the overall performance of the system.

The system utilises ESAComp analysis tools when computing the design attribute values of laminates. The tools are based on widely accepted theories, i.e. on the Classical Lamination Theory and on commonly used failure criteria. A discussion on the applicability of these theories is beyond the scope of this thesis.

#### 9.1.1 Multiobjective design approach

Different forms of component objective functions can be used in multiobjective design. The linear functions, described in Chapter 4, were selected partly because of their simplicity, partly because no significant benefits were seen in the use of more complex functions.

The reference values of design attributes also play an important role in the ranking of feasible laminates. The argumentation for the reference values used in the current system is given in Chapter 3. It is worth noting that the design attribute values are always ranked with respect to each other. In other words, values of the component objective functions and the preference function may change when the set of feasible objects changes. This was demonstrated with the test problems in Chapter 8.

#### 9.1.2 Design space in laminate creation

The number of plies, layer orientation angles and mutual proportions of layer orientation angles are restricted in the laminate creation process. The argumentation for the restrictions is given in Chapters 5 and 7.

Another simplification of the design system is the concept of homogenised laminates applied for in-plane orthotropic plies. The concept was developed to simplify the laminate creation process. In general, a homogenised laminate provided by the design system as a result of the laminate creation process is useful for the designer as far as it represents well an actual laminate that can be formed from a ply. This is the situation when:

- Few layer orientations are in use
- Layers with different orientations are evenly dispersed through the thickness of the laminate, and

- Laminate thickness is considerably higher than the thickness of the ply forming the laminate.

Homogenised laminates created by the design system automatically satisfy the first condition since the number of layer orientation angles is restricted. The dispersion of layers with different orientations is also possible and, in practice, should always be done since thick stacks of unidirectional layers are susceptible to matrix cracking. A thick homogenised laminate thus closely represents an actual laminate created by respecting widely accepted design practices.

On the other hand, an actual laminate close to a thin homogenised laminate cannot normally be formed. This is a limitation of the approach. However, even in this case a homogenised laminate gives a good start for the design of an actual relatively simple laminate. It also shows what could be achieved if the ply thickness could be tailored for the purpose.

Finally, it should be noted that with the adopted approach one design variable of laminates, the stacking sequence, is not available for obtaining the best performance e.g. under bending loads.

### **9.1.3 Failure analyses in ply evaluation**

An obvious limitation of the existing system is the inaccuracy of the simplified failure analysis approach used in the ply evaluation phase. This limitation is not critical since the laminate search phase reveals the possible infeasibility of the representative laminate and is normally capable of finding a feasible and efficient laminate, if such exists.

### **9.1.4 Laminate search method**

The applied laminate search method in laminate creation is simple but satisfactory when the optimum solution is relatively close to the initial solution. This is normally the situation after the ply evaluation phase. The search may end up with a local optimum, which is a common problem of inverse problem solving.

### **9.1.5 System performance**

The capabilities of the tools to find solutions within their operational limits were demonstrated with the test problems. The operating speed of the system was noted to be satisfactory. No systematic approach has been taken to get feedback on the usefulness of the design tools. It is though clear that laminate evaluation and, especially, laminate creation are commonly faced design problems that are laborious to perform without any design aids.

## **9.2 POSSIBLE ENHANCEMENTS**

As was noted in the previous section, the inaccuracy of the simplified failure analysis approach used in ply evaluation is a clear limitation of the existing system. Two different approaches can be taken to improve the system: One solution is to develop for each failure criterion a laminate-level failure analysis that approximates well the ply-level failure analysis results. Another possibility is to use the approach applied in the laminate search phase, i.e. to model

the laminate surfaces with actual layers already in the ply evaluation phase. The latter approach would probably be a better choice owing to its simplicity. Its drawback is increased solution time in ply evaluation. However, this is not seen to be a major problem since, as was reported in Chapter 8, the solution times are very reasonable with the current system. Moreover, the performance of computers is constantly improving.

### 9.3 POSSIBLE EXTENSIONS

Possible extensions of the system are briefly described below. An obvious extension, implementation of the laminate search module to the ESAComp system, is not discussed since this feature was originally planned to be included in the existing system but, as was noted in Chapter 1, is not yet functional.

#### 9.3.1 Accounting for laminate degradation

According to Chapter 3, all the constraints and objectives in the current system are interpreted to describe the required and desired performance of an intact laminate. Since it is well known that many laminates are capable of carrying considerable excess loads after the first ply failure, it would be beneficial to extend the design system so that the intactness of the laminate is not the default assumption.

In the design specification, the extension can be arranged by modifying the constraint that can be set for the attribute *Load carrying capability*. In the modified form, it would be possible to specify that the laminate must stay intact or that ply failures are acceptable. This type of constraint could be given even when no loads are included in the design specification. In problem solving, a degraded laminate would be analysed with ESAComp analysis tools provided for such laminates. The tools assume that each layer is degraded and carries loads with a reduced performance, possessing linear-elastic behaviour even when degraded. These are common assumptions in simple analyses of degraded laminates.

In case the laminate being sought is allowed to degrade, a laminate evaluation problem should be interpreted as follows:

- If the design specification does not contain loads, or if the loads in the design specification do not result in ply failure, the laminate is assumed to stay intact. Thus, laminate performance is represented by the engineering constants, expansion coefficients and deformations of the intact laminate. However, since laminates are allowed to degrade, the laminate strength is represented by the higher of the strengths computed for the intact and degraded laminates.
- If any of the loads included in the design specification results in ply failure, the engineering constants, expansion coefficients and strengths computed for the degraded laminate represent its performance. The load-carrying capability is studied by applying loads, one by one, to the degraded laminate.

In laminate creation, the new option would result in a need to create both an intact laminate and a degraded laminate from a ply since it is impossible to predict which of the laminates has

a better overall performance. In other words, even when ply failures are acceptable, the performance of an intact laminate may be better than the performance of a degraded laminate.

### **9.3.2 Laminate search**

A natural objective in further development of the laminate creation tool is to introduce a module capable of creating actual laminates. This module could replace or, preferably, complement the existing search tool.

The new module could be based on two design approaches: When the initial solution is thick, the module could create an actual laminate that resembles the optimal homogenised laminate as closely as possible. An evaluation of the laminate and fine-tuning of the lay-up would complete the creation process. When the initial solution is thin, an actual laminate close to the optimal homogenised laminate cannot normally be formed. In this case the module could search for an actual laminate by adding layers to the laminate one by one. The optimal homogenised laminate would serve as a reference in the selection of layer orientation angles.

### **9.3.3 Additional design tools**

According to Chapter 1, the current analysis system provides a possibility to specify and analyse structural elements such as bars, beams and plates. The design system can also be extended to enable the evaluation and creation of laminates for these elements. The user would be able to prepare a design specification for an element, to evaluate feasibility and quality of laminates for the element, and to create feasible and effective laminates for the element.

Many of the design attributes introduced for continuous laminates are also important in the design of bars, beams and plates. Thus, the design specification and design tools of the current system form a good basis for the extensions. New design attributes can be introduced as needed.

The development of plate design tools has already been started. In the first phase, a laminate evaluation tool has been developed for transversely loaded plates. The new design attribute introduced is the maximum deflection of the plate.

## 10 SUMMARY

The overall objective of this work was to develop a laminate design system that can be implemented into a laminate analysis program. The objective was detailed by defining that the system should be capable of finding feasible and efficient plies and laminates for an application. The first version of the system was restricted to the design of continuous laminates.

The system developed allows specifying a design target with constraints and objectives that can be set for important design attributes of a laminate. Two design tools are available. One tool provided for laminate evaluation searches, amongst a set of candidate laminates, feasible laminates that satisfy all constraints. The tool further ranks feasible laminates by evaluating how closely they meet the objectives. The second tool creates feasible laminates from a set of candidate plies and searches for the laminate that best meets the objectives. The laminate evaluation tool is introduced since the designer may face a problem where he or she has a specified set of laminates to choose from. The tool is also a valuable aid in the laminate creation process where the feasibility and quality of candidate solutions must be studied. The usefulness of the laminate creation tool is obvious.

The set of design attributes provided for a laminate contains both quantitative and qualitative attributes. Qualitative attributes are included since all the important properties of a laminate cannot be expressed quantitatively. Constraints can be set for all the attributes, and objectives for most of the quantitative attributes. An objective is always accompanied by a weighting factor that indicates the importance of the objective.

The laminate evaluation tool computes the design attribute values of candidate laminates by using the analysis tools of the system. It further compares the values with the constraints. If feasible laminates satisfying all constraints exist, the tool finally ranks the feasible laminates with the multiobjective design technique.

The laminate creation problem is simplified such that so-called homogenised laminates are created instead of actual laminates. These are laminates in which layers with different orientations are merged in specified proportions of the orientations to obtain a laminate that is homogeneous through its thickness.

The laminate creation tool performs its task in two phases. In the first phase it evaluates the feasibility and quality of candidate plies. To speed up the design process, the design space is suppressed and approximate failure analysis techniques are used. In the second phase, the tool attempts to find, in an extended design space, an optimum laminate that can be formed from a selected feasible ply. The solution of the first phase is used as a starting point in the search.

Since the laminate evaluation tool uses analysis tools of the system, the design attribute values computed for the candidate laminates are as good as the theories on which the analysis tools are based. The Classical Lamination Theory and failure criteria used by the tools are commonly accepted. The multiobjective design technique used in the laminate ranking could be applied in many ways. The selected approach is based on linear component objective

functions. The approach is simple but ranks laminates realistically when the functions are formed using applicable reference values for the design attributes.

The simplifications introduced in the laminate creation are necessary in order to obtain a solution within a decent time span, even with a large number of candidate plies. Many of the simplifications are in accordance with commonly accepted design practices. Thus, they very seldom restrict the use of the tool. For example, unsymmetrical and/or unbalanced laminates, which are not considered in the creation process, are seldom used in practical applications. The creation of homogenised laminates instead of actual laminates is only a partial solution for a design problem. Therefore, future work should be focused on the development of a module with which actual laminates can be created. The inaccurate failure analysis technique used in the ply evaluation phase is another topic that could and should be improved.

Despite the limitations, the design system should already be a practical aid for the designer of laminate structures. No major problems are foreseen in the extension of the system as planned, i.e. for the design of other structural elements such as bars, beams and plates. Another planned enhancement is to extend the design system so that the intactness of the laminate is not the default assumption.

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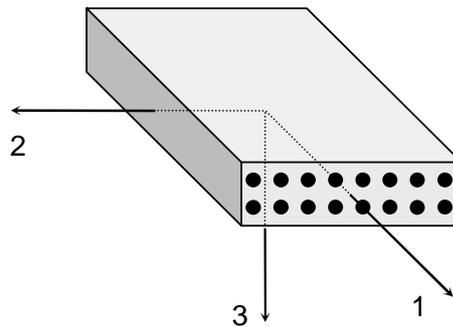


## Appendix A Notation and Conventions

The notation and conventions used in laminate analysis and design have not become established. The important notation and conventions used in the ESAComp software and throughout in this thesis are therefore summarised below.

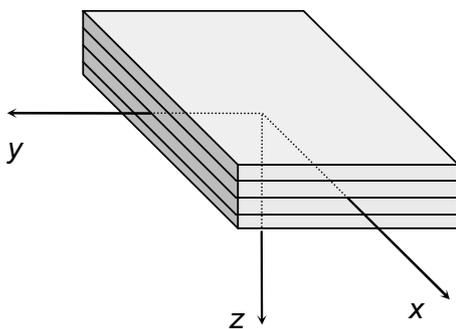
### Coordinate systems

Plies are specified in the *principal coordinate system* 123. According to Figure A1, the axes 1 and 2 define the ply plane.

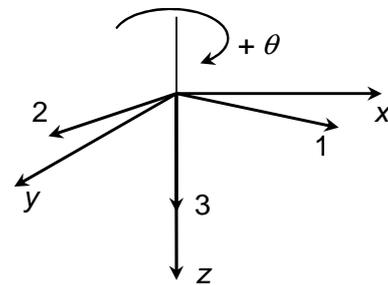


**Figure A1** Principal coordinate system 123 for plies.

Laminates are specified in the *laminate coordinate system*  $xyz$ . The  $x$ - and  $y$ -axes define the laminate plane. The  $xy$ -plane is thus parallel to the 12 planes of all layers and the  $z$ -axis coincides with the 3-axes of the layers (Fig. A2). The origin of the system is always fixed to the laminate midplane. The positive rotation of the layer axes 1 and 2 with respect to the laminate axes  $x$  and  $y$  is defined in Figure A3.



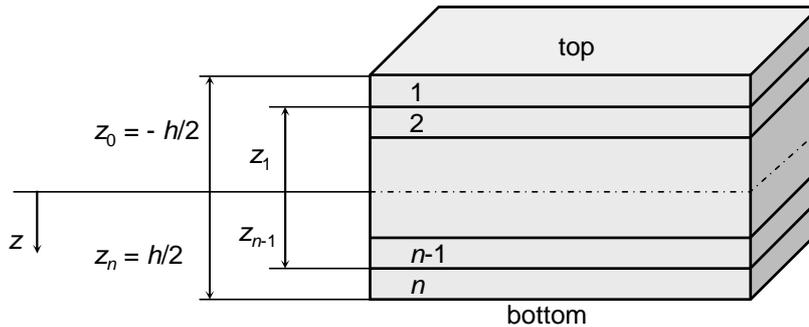
**Figure A2** Laminate coordinate system  $xyz$ .



**Figure A3** Positive rotation of axes 1 and 2 with respect to axes  $x$  and  $y$ .

## Laminate lay-up

The laminate lay-up is specified layer by layer from the *top surface* to the *bottom surface*, the former being the surface on the negative side of the  $z$ -coordinate. The thickness of the resulting laminate is denoted by  $h$ . Thus, values of the  $z$ -coordinates of the top and bottom surfaces are  $z = -h/2$  and  $z = h/2$ , respectively. The layer numbering convention is illustrated in Figure A4.



**Figure A4** Layer numbering convention for laminates.

## Engineering constants

The so-called *row normalised notation* is used for engineering constants of plies and laminates. The Poisson's ratio with the subscript 12 ( $\nu_{12}$ ), for example, defines the ratio of the compressive strain in the direction 2 ( $y$ ) to the tensile strain in the direction 1 ( $x$ ) when the ply is loaded in tension in the direction 1 ( $x$ ).

## Stresses and strains

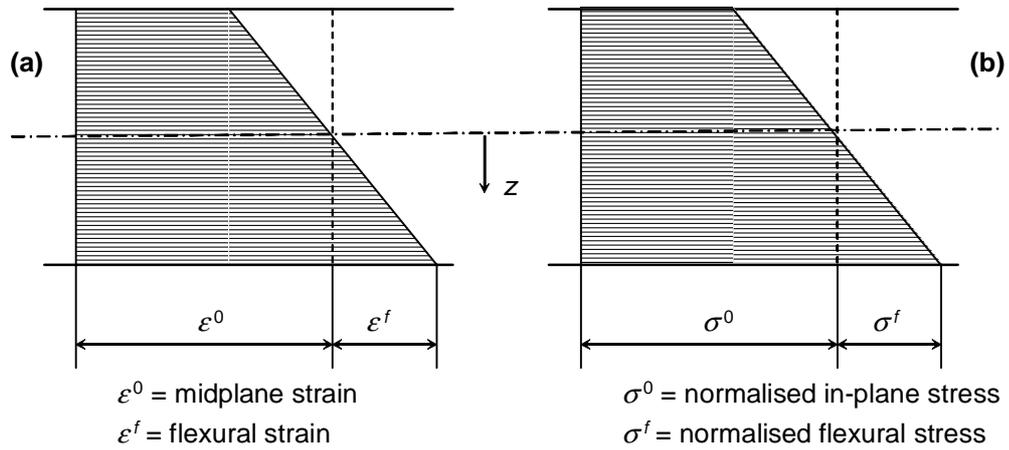
The so-called *engineering shear strains* are used. These are twice the magnitudes of the corresponding tensor strains.

The normal stresses and strains in the principal coordinate system 123 of a ply (Fig. A1) are referred to with the subscripts 1, 2 and 3, and the shear stresses and strains with the subscripts 23, 31 and 12. In the coordinate system  $xyz$  (Fig. A2), the normal stresses and strains are referred to with the subscripts  $x$ ,  $y$  and  $z$ , and the shear stresses and strains with the subscripts  $yz$ ,  $zx$  and  $xy$ .

The strain state of a laminate is described either by midplane strains and curvatures or by midplane and flexural strains (Fig. A5a). The stress state is described either with resultant in-plane forces and resultant moments or with normalised in-plane and flexural stresses:

- *Resultant in-plane forces* are the forces per unit width corresponding to the stress state of the laminate. They are thus integrals of the corresponding stress components over the laminate thickness.
- *Resultant in-plane moments* are the moments per unit width corresponding to the stress state of the laminate.

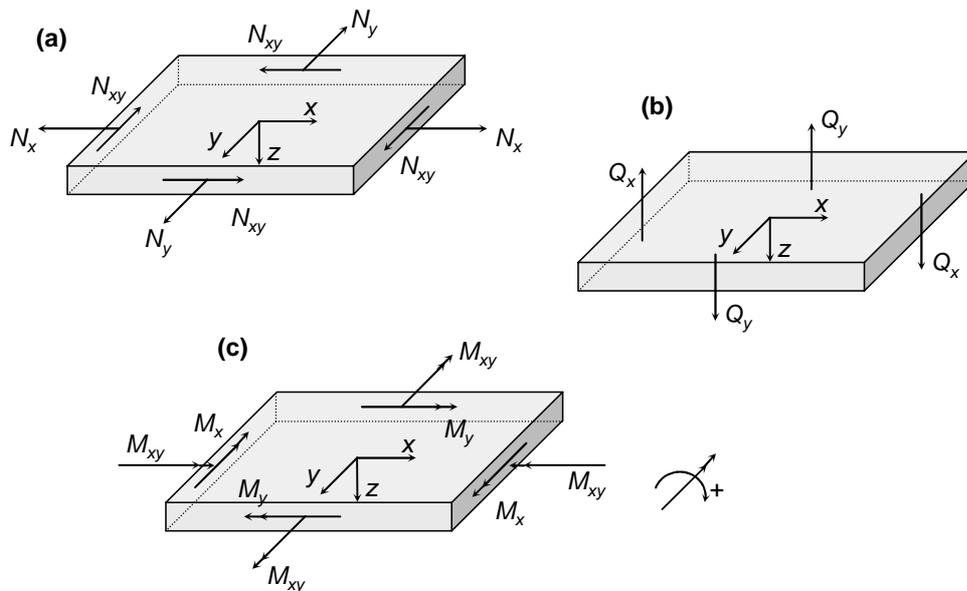
- *Normalised in-plane stresses* are the average in-plane stresses achieved by dividing the corresponding in-plane resultant forces by the laminate thickness (Fig. A5b).
- *Normalised flexural stresses* are the laminate surface stresses that would, in the case of a linearly varying stress distribution through the thickness of the laminate, and with the change of sign at the laminate midplane, give the same moment effect as the actual stress distribution (Fig. A5b).



**Figure A5** The (a) strain and (b) normalised stress distributions through the thickness of a laminate.

## Loads

The positive directions of in-plane forces and moments applied to a laminate are defined in Figure A6. The forces and moments are applied in the laminate midplane.



**Figure A6** Positive directions of (a) in-plane and (b) out-of-plane forces and (c) moments per unit width applied to a laminate.

## Factors of safety

Factors of safety are applied to loads. Since a load may consist of two load vectors, it is possible to define two factors of safety, one for the constant load vector and another for the variable load vector. It is further possible to define a coefficient by which these factors are multiplied to achieve factors of safety for wrinkling analyses of sandwich laminates. This coefficient is called the *stability factor*.

## Reserve factors and margins of safety

The results of the failure analyses are expressed in the form of reserve factors and margins of safety against ply failure. For sandwich laminates, the reserve factor and margin of safety against wrinkling failure are also provided.

The *reserve factor (RF)* is specified to be a factor which, when multiplying the applied effective load, results in the failure load. A reserve factor that is smaller than one indicates that the load is not acceptable. According to this definition, the ESAComp practice is to consider the limit value  $RF = 1$  acceptable.

The *margin of safety (MoS)* is an alternative for the reserve factor. It describes the relative margin between the applied effective load and the failure load. A reserve factor of 1.25, for example, corresponds to a 0.25 or 25 % margin of safety.

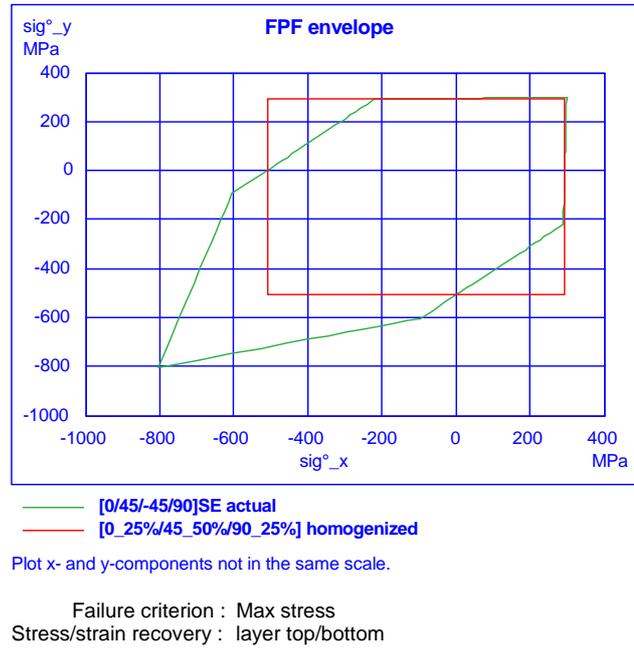
## Appendix B Laminate Failure Envelopes

Figures B1...B3 display the failure envelopes of actual and homogenised laminates in biaxial in-plane loading in the normalised stress space. The failure criteria applied are the maximum stress criterion (Fig. B1), the Tsai-Hill criterion (Fig. B2) and the maximum strain criterion (Fig. B3). The envelopes are given for the following actual laminates and for the corresponding homogenised laminates:

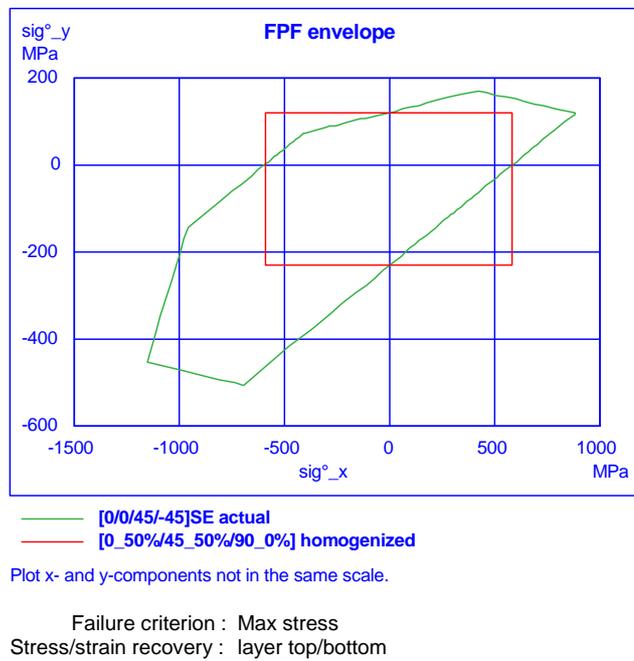
- $[0^\circ/45^\circ/-45^\circ/90^\circ]_{SE}$
- $[0^\circ/0^\circ/45^\circ/-45^\circ]_{SE}$
- $[45^\circ/-45^\circ]_{SE}$
- $[0^\circ/90^\circ]_{SE}$

Figure B4 displays the failure envelopes of the actual laminates with different failure criteria.

The laminates are formed from the ply T300/5208. The ply properties are defined in Table 4 (Chapter 8).

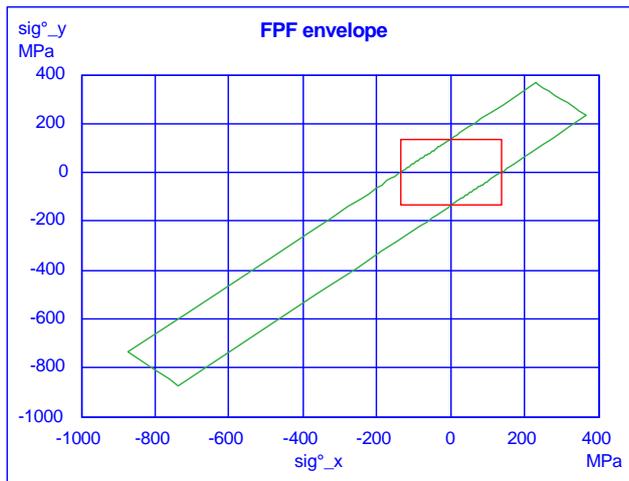


(a) [0/45/-45/90]-laminates



(b) [0/0/45/-45]-laminates

**Figure B1** Failure envelopes in the (normalised) stress space computed with the maximum stress criterion for a set of actual and homogenised laminates.

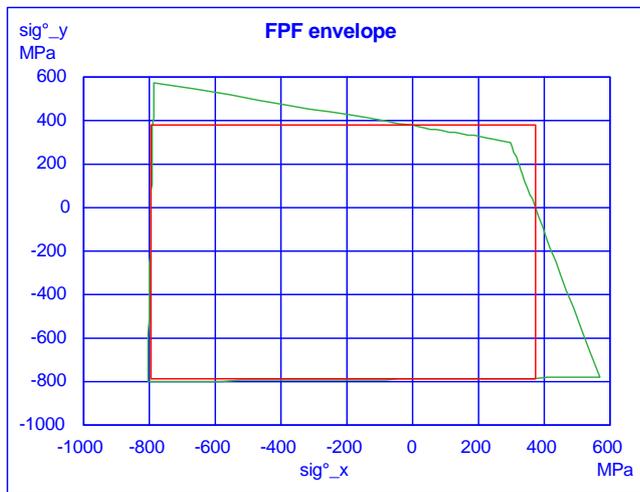


— [45/-45]SE actual  
 — [0\_0%/45\_100%/90\_0%] homogenized

Plot x- and y-components not in the same scale.

Failure criterion : Max stress  
 Stress/strain recovery : layer top/bottom

(c) [45/-45]-laminates



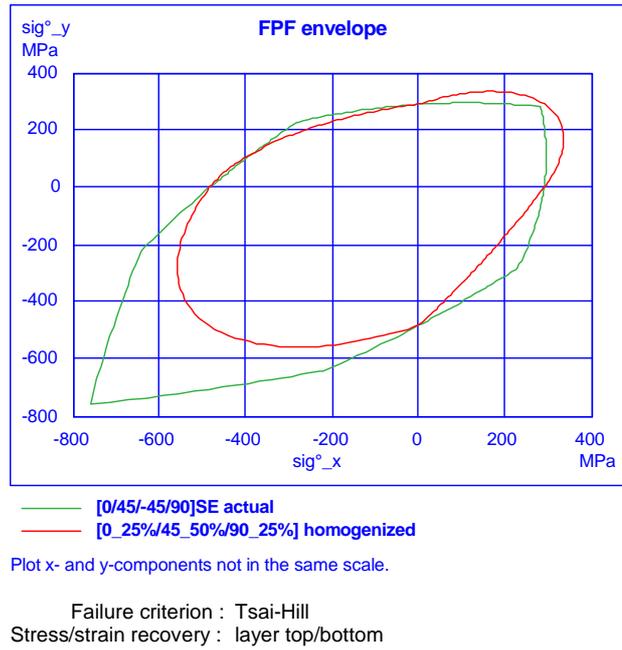
— [0/90]SE actual  
 — [0\_50%/45\_0%/90\_50%] homogenized

Plot x- and y-components not in the same scale.

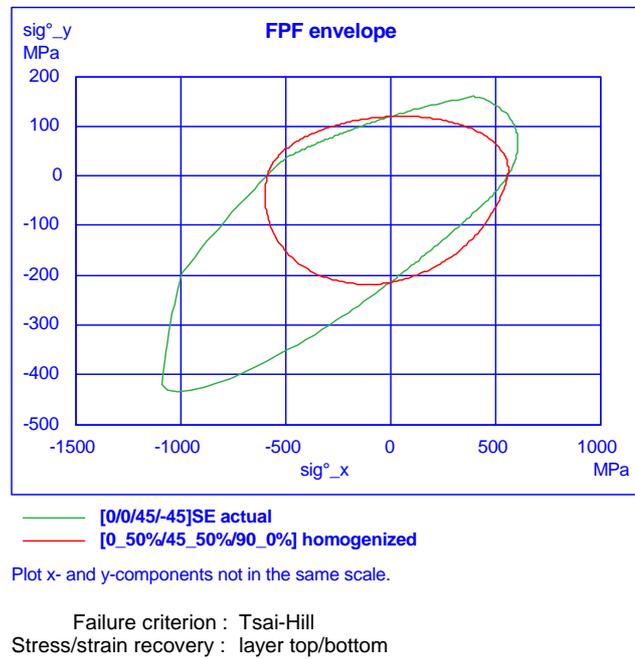
Failure criterion : Max stress  
 Stress/strain recovery : layer top/bottom

(d) [0/90]-laminates

**Figure B1** (continued) Failure envelopes in the (normalised) stress space computed with the maximum stress criterion for a set of actual and homogenised laminates.

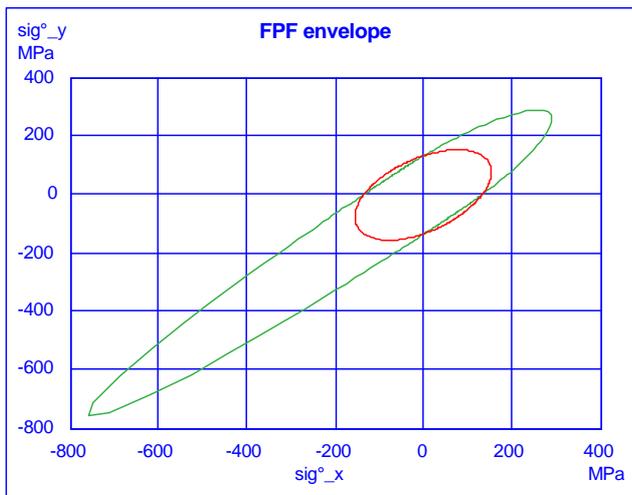


(a) [0/45/-45/90]-laminates



(b) [0/0/45/-45]-laminates

**Figure B2** Failure envelopes in the (normalised) stress space computed with the Tsai-Hill criterion for a set of actual and homogenised laminates.

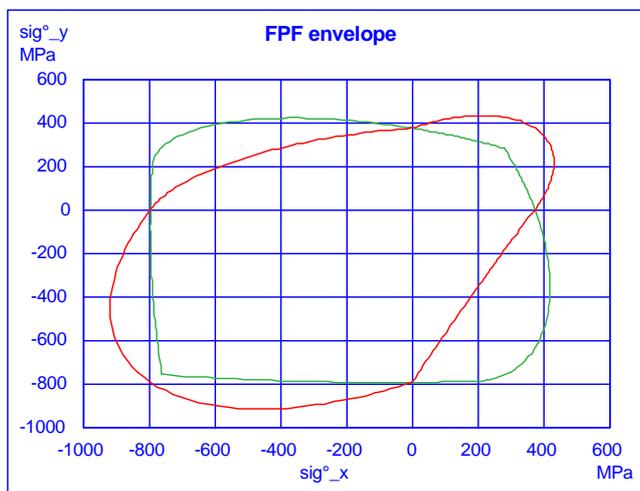


— [45/-45]SE actual  
 — [0\_0%/45\_100%/90\_0%] homogenized

Plot x- and y-components not in the same scale.

Failure criterion : Tsai-Hill  
 Stress/strain recovery : layer top/bottom

(c) [45/-45]-laminates



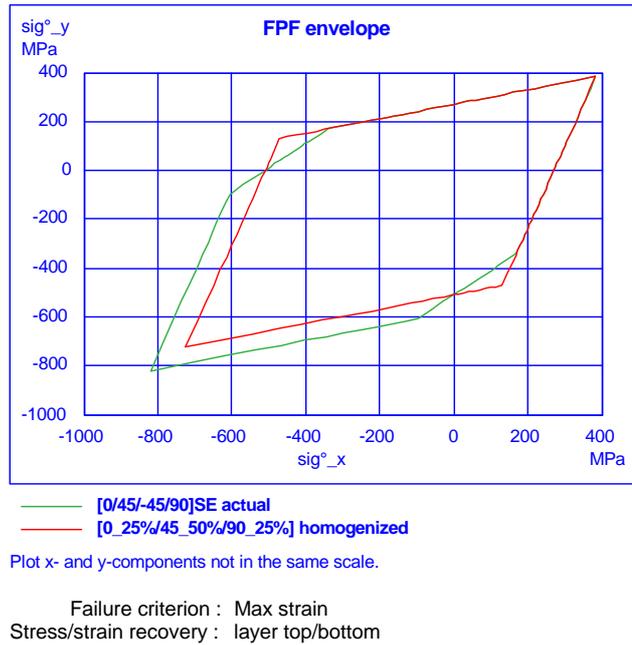
— [0/90]SE actual  
 — [0\_50%/45\_0%/90\_50%] homogenized

Plot x- and y-components not in the same scale.

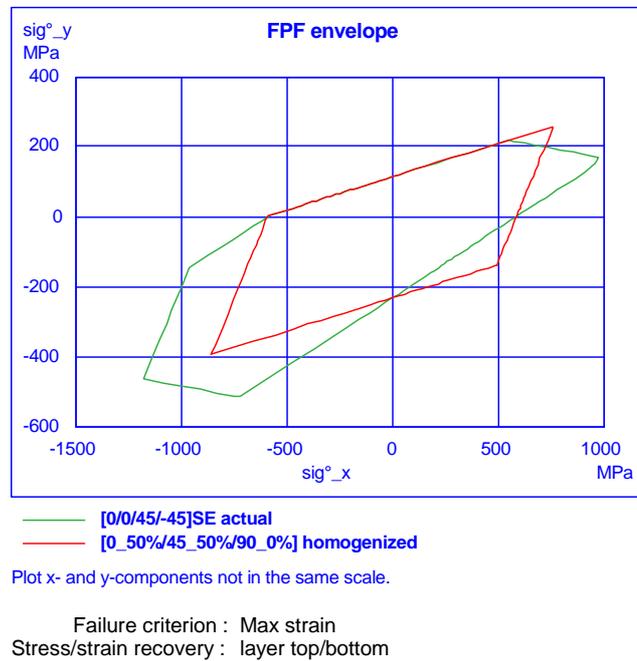
Failure criterion : Tsai-Hill  
 Stress/strain recovery : layer top/bottom

(d) [0/90]-laminates.

**Figure B2** (continued) Failure envelopes in the (normalised) stress space computed with the Tsai-Hill criterion for a set of actual and homogenised laminates.

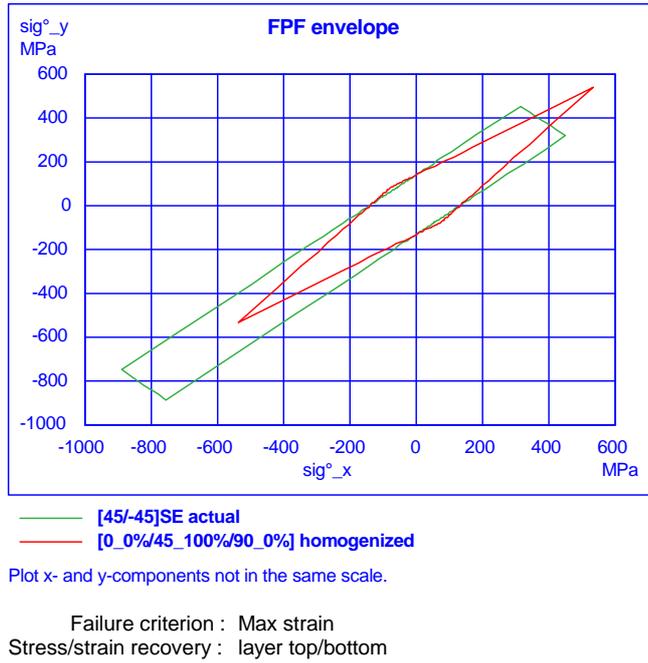


(a) [0/45/-45/90]-laminates

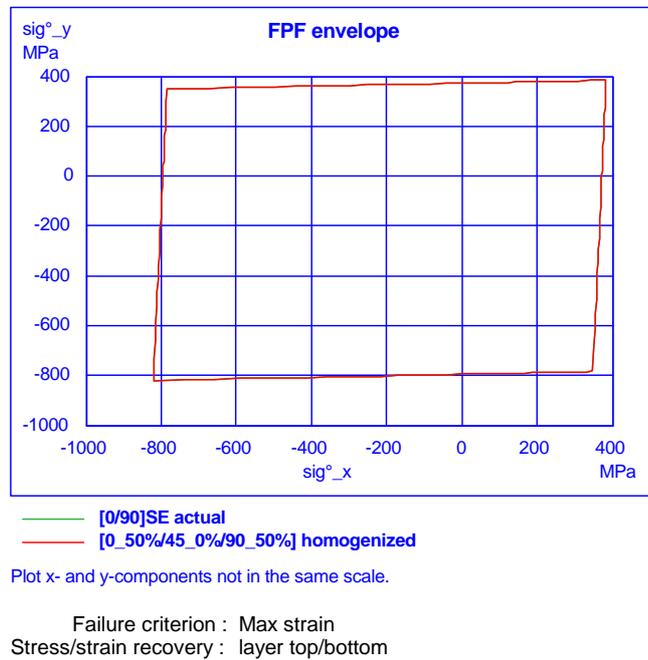


(b) [0/0/45/-45]-laminates

**Figure B3** Failure envelopes in the (normalised) stress space computed with the maximum strain criterion for a set of actual and homogenised laminates.

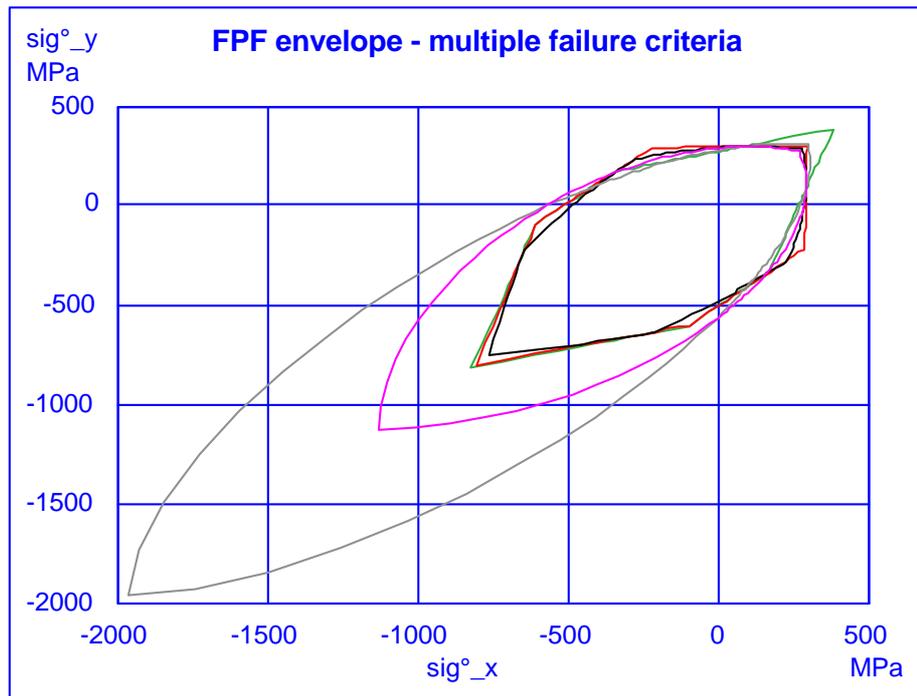


(c) [45/-45]-laminates



(d) [0/90]-laminates

**Figure B3** (continued) Failure envelopes in the (normalised) stress space computed with the maximum strain criterion for a set of actual and homogenised laminates.



- Max strain
- Max stress
- Tsai-Hill
- Tsai-Wu,  $F_{12}^* = -0.5$
- Hoffman

Plot x- and y-components not in the same scale.

Stress/strain recovery : layer top/bottom

Laminate : **[0/45/-45/90]SE actual**

Modified : Sat Oct 20 16:52:19 2001

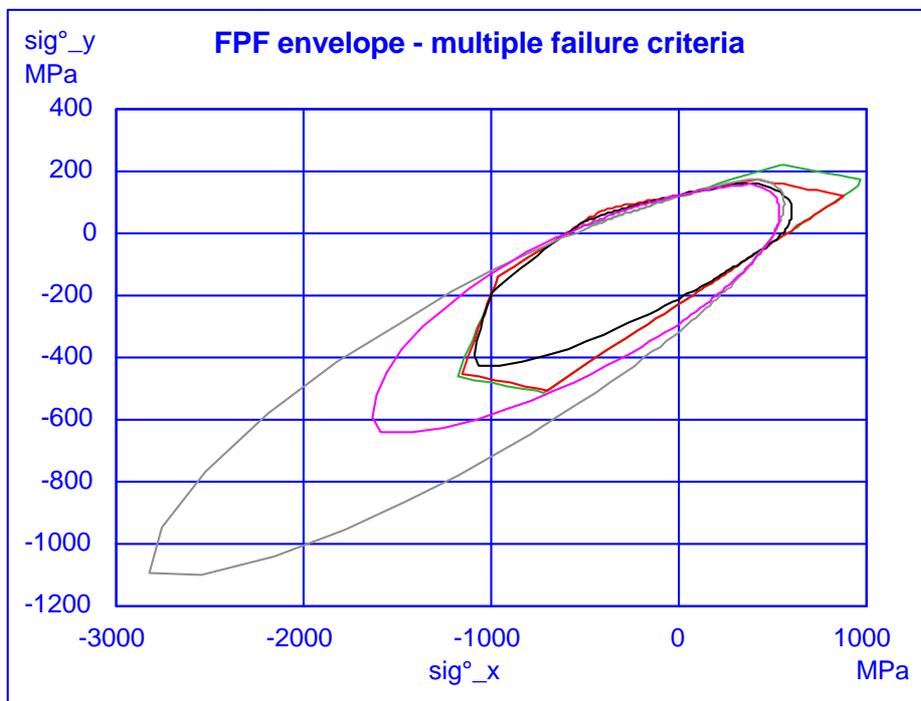
Lay-up : (0a/+45a/-45a/90a)SE h = 1 mm

Ply

a T300/5208

(a) [0/45/-45/90]-laminate

**Figure B4** Failure envelopes in the normalised stress space computed with different failure criteria for a set of actual laminates.



- Max strain
- Max stress
- Tsai-Hill
- Tsai-Wu,  $F_{12}^*=-0.5$
- Hoffman

Plot x- and y-components not in the same scale.

Stress/strain recovery : layer top/bottom

Laminate : **[0/0/45/-45]SE actual**

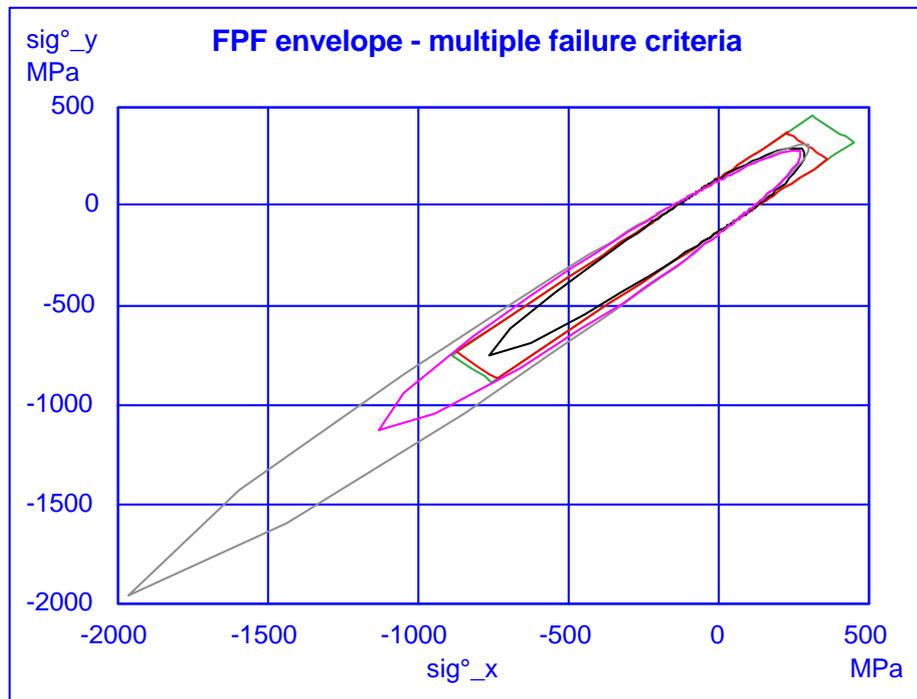
Modified : Sat Oct 20 16:52:28 2001

Lay-up : (0a/0a/+45a/-45a)SE h = 1 mm

Ply  
a T300/5208

(b) [0/0/45/-45]-laminate

**Figure B4** (continued) Failure envelopes in the normalised stress space computed with different failure criteria for a set of actual laminates.



- Max strain
- Max stress
- Tsai-Hill
- Tsai-Wu,  $F_{12}^* = -0.5$
- Hoffman

Plot x- and y-components not in the same scale.

Stress/strain recovery : layer top/bottom

Laminate : **[45/-45]SE actual**

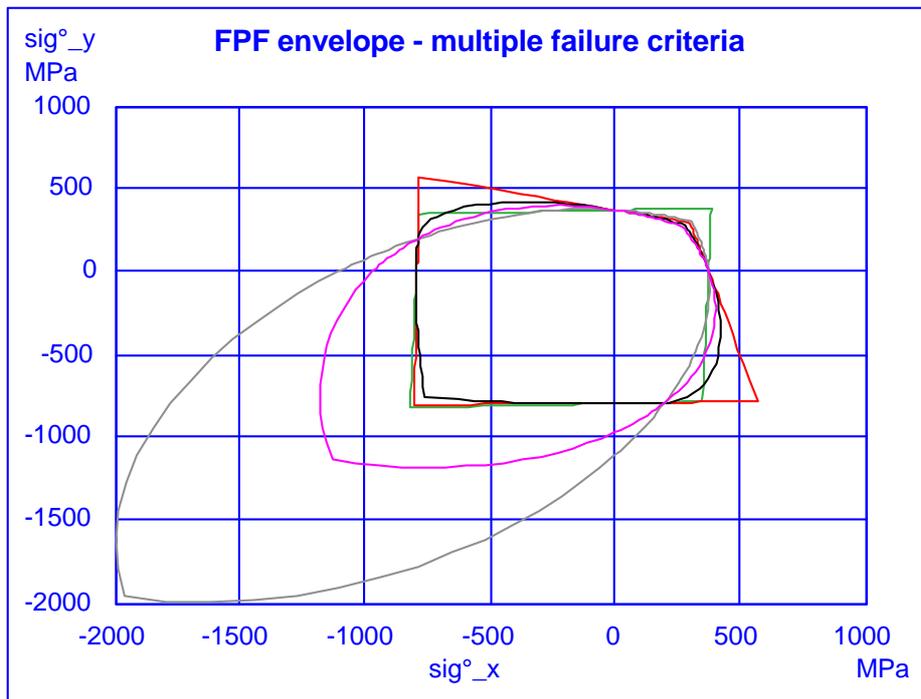
Modified : Sat Oct 20 16:52:13 2001

Lay-up : (+45a/-45a)SE h = 0.5 mm

Ply  
a T300/5208

(c) [45/-45]-laminate

**Figure B4** (continued) Failure envelopes in the normalised stress space computed with different failure criteria for a set of actual laminates.



- Max strain
- Max stress
- Tsai-Hill
- Tsai-Wu,  $F_{12}^* = -0.5$
- Hoffman

Plot x- and y-components not in the same scale.

Stress/strain recovery : layer top/bottom

Laminate : **[0/90]SE actual**  
 Modified : Sat Oct 20 16:52:36 2001

Lay-up : (0a/90a)SE h = 0.5 mm

Ply  
 a T300/5208

(d) [0/90]-laminate

**Figure B4** (continued) Failure envelopes in the normalised stress space computed with different failure criteria for a set of actual laminates.