Mesoscopic Josephson Junction as a Noise Detector

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ABSTRACT

Small Josephson junctions are known to be very susceptible to noise. We have utilized this property in developing methods to measure noise as well as environmental resonance modes in mesoscopic systems. We review recent results on tunnel junction systems and show also that higher order moments of shot noise can be addressed with the present method based on the noise-induced modification of incoherent tunneling of Cooper pairs.

Keywords: Noise spectroscopy, shot noise, higher moments, Josephson junction

1. INTRODUCTION

At present there is a strong effort to measure noise and its higher moments¹ in mesoscopic solid state devices as well as to understand the role of quantum noise in dephasing of qubits.² These phenomena are quite intriguing: contrary to the traditional views, the noise spectrum tends to enter in an unsymmetrized form.³ Novel detectors, based on a variety of different physical principles, have recently been suggested in order to access these phenomena.^{4–8} Our experimental approach is to use incoherent Cooper pair tunneling in a mesoscopic Josephson junction as a probe.

In a single Josephson junction (JJ) with weak dissipation, *i.e.* with a strongly resistive environment, Coulomb blockade of Cooper pair current takes place.⁹ The Coulomb blockade of Cooper pairs is very sensitive to fluctuations. Inherently, it is influenced by Johnson-Nyquist noise, which is predicted to result in a power-law-like increase of conductance both as a function of temperature and voltage. The exponent of the power law, $2\rho - 2$, is governed by the parameter $\rho = R/R_Q$ where R describes the dissipative ohmic environment and $R_Q = h/4e^2 = 6.5 \text{ k}\Omega$. Hence, in the case of large exponents $2\rho - 2 \gg 1$, there is a strong influence of tiny changes in temperature on resistance, or alternatively, a high sensitivity to external noise sources with similar spectrum as the Johnson-Nyquist noise.

The situation is slightly different with shot noise sources, for which a theory has recently been developed.¹⁰ It has been shown that extremely good sensitivity can be obtained also in this case.¹¹ Therefore, a JJ detector is a good candidate, in certain systems, for high-resolution noise measurements as will be demonstrated by the experimental examples discussed in this paper. Moreover, a Josephson junction detector is inherently very sensitive to odd moments of shot noise and, thus, it can be employed to study the non-Gaussian nature of such noise sources.

In this paper we will review both the theoretical background for the noise spectroscopy using a JJ as well as its first applications to experimental investigations.

2. THEORETICAL BACKGROUND

2.1. Mesoscopic Josephson Junction

In a superconducting tunnel junction, the coupling of the order parameter across an insulating barrier leads to coherent tunneling of Cooper pairs.¹² This supercurrent is described by a sinusoidal current-phase relationship:

$$I = I_c \sin \varphi, \tag{1}$$

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where I_c is the maximum value of the current, and the phase

$$\varphi(t) = \int_{-\infty}^{t} \frac{2e}{\hbar} V(t') dt' \tag{2}$$

is defined as an integral of the voltage V across the tunnel barrier. The Josephson coupling energy $E = -E_J \cos(\varphi)$ is related to the critical current of the junction by the identity $E_J = \frac{\hbar I_c}{2e}$. From Eqs. 1 and 2, one finds that, at small phase deviations, a classical Josephson junction is equivalent to an inductance given by

$$L = \frac{\Phi_0}{2\pi I_c},\tag{3}$$

where $\Phi_0 = h/2e$ is the flux quantum.

In mesoscopic tunnel junctions, the discreteness of charge starts to play a role via the Coulomb energy $\mathcal{E}_c = \frac{Q^2}{2C}$, where C is the capacitance of the junction and Q is the charge on the capacitor plates. In quantum theory, charge is described by the operator $\hat{Q} = -i2e\frac{\partial}{\partial\varphi}$. This operator is canonically conjugate to $\hat{\varphi}$, i.e. $[\hat{Q},\hat{\varphi}] = i2e$. Hence, there is a Heisenberg uncertainty relation, $\Delta Q \Delta \varphi \sim 2e$, which implies that the charge and the phase of the superconducting junction cannot be defined simultaneously. This leads to a delocalization of the phase and to Coulomb blockade of the supercurrent, as experimentally shown by Haviland $et\ al.^{13}$ in the case when Josephson energy is on the order of the single-electron Coulomb energy, i.e. $E_J/E_c \sim 1$ where $E_c = \frac{e^2}{2C}$. The same conclusion of delocalization applies even for large values of the ratio E_J/E_c .

2.2. Phase Fluctuation theory (P(E)-theory)

When strong phase fluctuations eliminate the supercurrent, the current is carried by incoherent tunnelling of Cooper pairs at subgap voltages ($V < 2\Delta$). This is controlled by the probability P(E) of exchanging energy E with the environment. According to the perturbation theory for $E_J \ll E_C$ the current is given by $^{14-16}$

$$I(V) = \frac{\pi e E_J^2}{\hbar} \left[P(2eV) - P(-2eV) \right]. \tag{4}$$

The P(E) function,

$$P(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} dt \exp\left[J(t) + \frac{iEt}{\hbar}\right], \qquad (5)$$

is a Fourier transform of the correlation function

$$\exp|J(t)| = \langle \exp(i\varphi(t)) \exp(-i\varphi(0)) \rangle, \tag{6}$$

which takes into account the phase fluctuations of the phase $\varphi(t)$ over the junction. For Gaussian noise, this simplifies so that $J(t) = \langle |\varphi(t) - \varphi(0)|\varphi(0)\rangle$.

For a linear environment, J(t) is given by

$$J(t) = 2 \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \frac{\text{Re}Z(\omega)}{R_Q} \frac{e^{-i\omega t} - 1}{1 - e^{-\beta\hbar\omega}}.$$
 (7)

For ohmic environment, $Z(\omega) = (1/R + i\omega C_T)^{-1}$ where C_T is the junction capacitance. Thus, by denoting $\tau = RC_T$, the real part becomes

$$Re\{Z(\omega)\} = \frac{R}{1 + (\omega\tau)^2}.$$
 (8)

A few P(E) functions calculated for resistive environments with $\rho = R/R_Q = 2...10$ using the method of Ref. 17 are displayed in Fig. 1.

The P(E) function is of the form $P(E) \propto E^{2\rho-1}$ at small energies. Since $P(-2eV) \ll P(+2eV)$ due to detailed balance at low temperatures, one may neglect the backward tunneling rate and Eq. 4 yields a pure

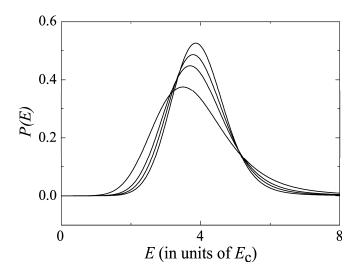


Figure 1. P(E) functions calculated for $\rho = 2, 4, 6$, and 10. These curves can be used to obtain IV curves for mesoscopic Josephson junctions using Eq. 4.

power law for the IV curve. Similarly, the temperature dependence of zero bias conductance takes a power-law form

$$G_0 = \frac{4\pi\rho\tau^2 e^2 E_J^2}{\hbar^3} \left(\frac{2\pi\rho\tau k_B}{\hbar}\right)^{2\rho-2} T^{2\rho-2},\tag{9}$$

as derived e.g. in Ref. 16.

At energies around E_C , the P(E) function is strongly peaked for a high-resistance environment ($\rho \gg 1$). It may be approximated by a Gaussian function

$$P(E) = \frac{1}{\sqrt{4\pi E_C k_B T}} \exp\left[-\frac{(E - E_C)^2}{4E_C k_B T}\right],\tag{10}$$

where the width is governed by thermal fluctuations in the resistance R. Consequently, the IV-curve has a rather broad peak centered around $V = 2E_C/e$ due to the 2e charge of Cooper pairs.

Experimentally, the applicability of P(E) theory has been confirmed quite well.^{18–20} Thus, extra peak structure or change in effective temperature may be taken as signs of additional imposed noise. When the resistance of the environment is large, even a tiny change in effective temperature, $T \to T + \delta T$, results in a large modification of $G_0 \propto (T + \delta T)^{2\rho-2}$, which means a high sensitivity against added Johnson-Nyquist type of noise.

3. ENVIRONMENTAL RESONANCES

Incoherent Cooper pair tunneling in a mesoscopic Josephson junction has been employed for studies of resonances in short transmission lines²¹ and in small mesoscopic SQUID loops.²² Small SQUID (Superconducting QUantum Interference Device) loops (see Fig. 2) are quite useful in such studies since they behave as tunable single Josephson junctions, which are equivalent to tunable inductors. To first approximation, these loops act as single mode resonators with a single peak in the $ReZ(\omega)$ function (one peak at both positive and negative frequencies). Combined with the capacitance of its environment C_{env} , the inductance of a Josephson junction will form an LC-oscillator with a characteristic plasma frequency of

$$f_p = \frac{1}{2\pi\sqrt{LC}} = \frac{\sqrt{8E_J E_c}}{h},\tag{11}$$

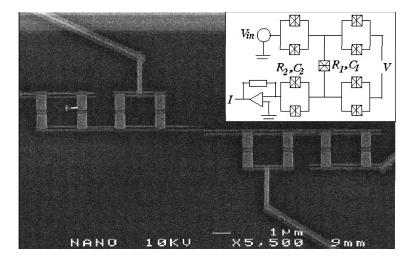


Figure 2. SEM micrograph of a circuit with a detector junction having an environment of four SQUID loops. The probe junction has an area of $100 \times 100 \text{ nm}^2$ and the SQUID junctions $150 \times 550 \text{ nm}^2$. The inset shows the schematic of the circuit for a 4-lead measurement configuration. Capacitances $C_1 = 0.7 \text{ fF}$ and $C_2 = 5.2 \text{ fF}$ for the small and large junctions, respectively. The ratio E_J/E_C for the environment junctions could be tuned over 0.1-8 which means that the plasma frequency could be tuned over $f_p = 0.2-10 \text{ GHz}$.

where the Coulomb energy is calculated for the total capacitance $C = C_T + C_{env}$. Consequently, an underdamped Josephson junction behaves as a harmonic oscillator with a level spacing of hf_p .^{23, 24}

Due to multiphoton processes, the importance of which depends on the quality factor of the mode, the P(E) function displays a characteristic sequence of equally spaced peaks.^{15,16} This behavior is seen in the measured IV curve of Fig. 3, where the fundamental spacing equals to $hf_p/2e$ where the factor of two is due to Cooper pair tunneling. In fact, the spacing of peaks in Fig. 3 is not fully equal, and clear deviations from the harmonic oscillator behavior is observed.

Lorentzian lines have been fitted to the measured IV-curve in order to determine the actual eigenfrequencies. The width of the peaks is rather broad as seen in Fig. 3. This is a consequence of several reasons. It depends foremost on the intrinsic Q-value of the resonances, but also it depends on the amount how much the detector disturbs the studied SQUID loops, i.e on the detector current. In order to keep the line width small, the current $(\propto E_J^2)$ through the detector junction should to be kept low enough so that the investigated junctions, i.e. the SQUID loops, have time to relax to the ground state before the next tunneling event. In other words, the rate f = I/2e should be smaller than the inverse of the intrinsic life time of the SQUIDs' energy levels.

Locations of the overlapping peaks were found to depend only on the magnetic flux, *i.e.* on the tuning of the plasma frequency. The identification of the peaks is difficult, and for this purpose the a large set of data, illustrated in Fig. 4, has to be analyzed as a whole. Fig. 4 is given in terms of reduced magnetic flux Φ/Φ_0 threading the SQUID loops. The flux can be converted into inductance using the formula $L=\Phi_0/(2\pi I_C\sin(\Phi/\Phi_0))$ which means that the plasma frequency $f_p\propto \sqrt{|\sin(\Phi/\Phi_0)|}$. The main resonance peak at V_1 in Fig. 3 is identified as the basic plasma frequency but the other peaks, not exactly multiphoton peaks, are related to the actual shape of the nonlinear potential.²²

4. SHOT NOISE

In previous sections it was demonstrated that a mesoscopic Josephson junction is a sensitive tool for measuring Gaussian (thermal) noise and fluctuations coming from environmental resonances. More importantly, such a detector can also be employed to measure shot noise. This has been shown in our measurements on a three terminal device displayed in Fig. 5. The circuit contains three elements: a SQUID-loop Josephson junction (JJ1+JJ2), a superconducting-normal tunnel junction (SIN), and a thin film Cr resistor ($R \sim 100 \text{ k}\Omega$), located

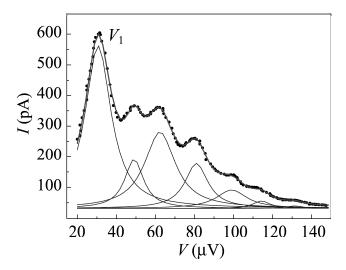


Figure 3. Probe junction IV-curve measured in the SQUID-loop resonator environment of Fig. 2 with plasma frequency $f_p = 12$ GHz (at flux $\Phi/\Phi_0 = -0.33$, i.e., $E_J = 70\mu\text{eV}$). The curves are Lorentzian lines fitted simultaneously to the data.

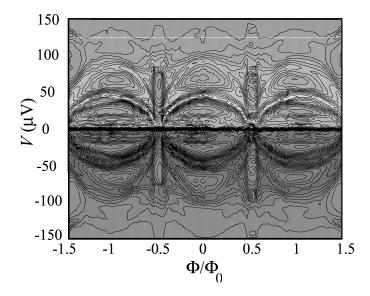


Figure 4. Contour plot of the measured current in the probe junction of Fig. 2 in the flux-voltage plane; flux penetrating the SQUID loops is given in reduced units Φ/Φ_0 where $\Phi_0 = h/2e$ is the flux quantum. The plasma frequency $(f_p \propto \sqrt{|\sin(\Phi/\Phi_0)|})$ results in the three almost circular ridges seen in the figure.

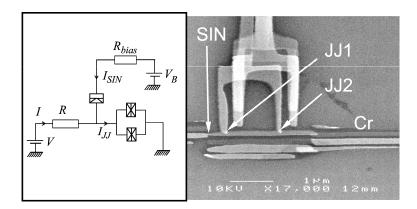


Figure 5. A scanning electron microscope picture of our three terminal device and a schematic view of the circuit (left). The chrome resistor is denoted by Cr, the superconductor-normal junction by SIN, and the Josephson junction in a SQUID-loop configuration by JJ1 and JJ2.

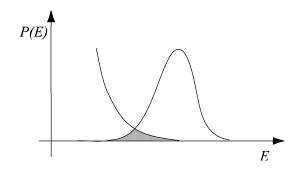


Figure 6. Graphical illustration of the convolution process used to estimate the total $P(E)_{|E=0}$ function of the shot noise source + resistor combination. For details, see text.

within a few μ m from the junctions. This kind of configuration results in a well-developed Coulomb blockade in the Josephson junction.

The simplest way to account for shot noise due to current I_{SIN} is to equate the available noise power with a noise temperature T_N : $F2eI_{SIN} = k_BT_N$ where F is a Fano-factor. Consequently, the zero-bias conductance G_0 under the influence of a quasiparticle current I_{SIN} corresponds to the conductance at an elevated effective temperature of $T + T_N$ (see Eq. 9). Hence, a comparison of the current-induced conductance change with the temperature dependence allows one to determine T_N and thereby the Fano-factor. Even though qualitative agreement has been obtained with the experimental results, this kind of approach is rigorous only at $\rho < 1$.

The problem for the theory arises from the fact that short times $t \sim \tau/\sqrt{\rho}$ become important when $\rho > 1.^{10}$ In this case, the modification due to shot noise turns from a nonanalytic power law into a linear dependence on I_{SIN} . This may be understood in simple terms on the basis of Fig. 6 which illustrates graphically the convolution of two uncorrelated noise sources with $P_1(E)$ and $P_2(E)$

$$P(E)_{TOT} = P_1(E) * P_2(E) = \int_{-\infty}^{\infty} P_1(\widetilde{E}) P_2(E - \widetilde{E}) d\widetilde{E}, \tag{12}$$

which yields the P(E) function of the combined noise. Note that since we are interested in the zero bias conductance, we need evaluate only $P(E)_{TOT}$ at E=0. In the regime $t \leq \sqrt{\tau/\rho}$, the correlator $\exp\left[J(t)\right]$ may be approximated by 1+J(t). Assuming Gaussian noise and disregarding the delta-function contributions, its Fourier transform, equal to P(E), becomes the power spectrum of phase fluctuations S_{φ} . This can be obtained from the voltage fluctuations $S_V(\omega) = S_I R^2/(1+\omega^2 R^2 C^2)$ induced by the current noise S_I :

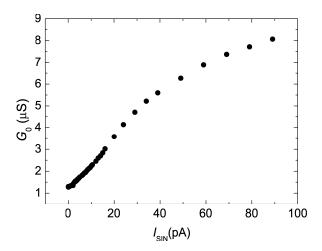


Figure 7. Zero bias conductance $G_0 = \frac{dI_{JJ}}{dV}|_{I_{JJ}=0}$ vs. I_{SIN} for the JJ + R_C section of the circuit illustrated in Fig. 5. Sample parameters: $R_T^{JJ} = 3k\Omega$, $R_T^{SIN} = 4.3 \ k\Omega$, $R_C = 67 \ k\Omega$, $R_B = 53k\Omega$, $E_C = 35 \ \mu eV$ $E_J = 14 \ \mu eV$.

$$S_{\varphi}(\omega) = \frac{4e^2}{\hbar^2 \omega^2} \frac{S_I R^2}{1 + \omega^2 R^2 C^2}.$$
 (13)

Using Eq. 13 for the shot noise term and a peaked P(E) function (see Fig. 1) for the resistive environment, the convolution governing the zero bias conductance (at E=0) becomes as shown in Fig. 6. The linear dependence on I_{SIN} becomes thus apparent. A proper calculation yields¹⁰

$$G_S = \frac{\pi^{5/2}}{32\sqrt{2\ln\rho}} \left(\frac{E_J}{E_C}\right)^2 \frac{1}{V_C} \rho^{3/2} |I_{SIN}|,$$
(14)

which is obtained in the limit of zero temperature and zero current. Here V_C denotes the Coulomb voltage E_C/e .

Measured data on the shot noise dependence of G_0 is illustrated in Fig. 7. The linear dependence on I_{SIN} is clearly visible in the data at small currents. The numerical value obtained using Eq. 14 falls short by a factor of four from the measured points. We have also performed measurements varying the Josephson coupling energy and verified the E_J^2 dependence.¹¹ In a suitably biased regime (around 10 pA), a change in the noise due to a change in current by 0.1 pA can clearly be observed.

The above results were obtained within the assumption that the noise is Gaussian. However, there are features in the experimental data that cannot be explained within such an approximation. One illustration of such data is given in Fig. 8 which displays the measured $\frac{dI_{JJ}}{dV}$ at zero quasiparticle current as well as at a few values of I_{SIN} ranging from 0.01 nA to 0.1 nA; the data was taken at the minimum value of $E_J = 14~\mu\text{eV}$. The bias voltage V to the JJ was applied via the chrome resistor while the SIN-junction was current biased through $R_{bias} = 100~\text{M}\Omega$. The location of the minimum amplitude of the Coulomb blockade dip $G_{min} = \left[\frac{dI_{JJ}}{dV}\right]_{min}$ is seen to shift monotonically with I_{SIN} . This shift amounts to $\Delta I_{JJ} \sim 0.20 \cdot I_{SIN}$. A detailed analysis $^{10,\,11}$ connects this shift to a ratchet effect, caused by odd moments of the shot noise.

5. DISCUSSION

Altogether, a Josephson junction noise detector provides a novel alternative to be considered for high-resolution noise measurements. Its main virtue, the high sensitivity, comes from the large detector band width: αE_C , where $\alpha \sim 1$ is a constant depending on the impedance of the environment and the operating temperature. Junction detectors will surpass the sensitivity of regular high-resolution noise measurements²⁵ when the selection of the

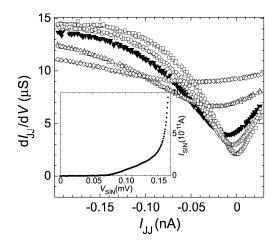


Figure 8. Differential conductance $\frac{dI_{JJ}}{dV}$ vs. current I_{JJ} for the JJ + R_C section of the circuit illustrated in Fig. 5 with a small current bias (via 100 MΩ resistor) in the SIN junction: $I_{SIN} = 0$ (□), 0.01 (⋄), 0.02 (▼), 0.05 (△), 0.1 nA (⋄). The inset shows the IV-curve for the SIN junction. T = 82 mK.

parameter values and the operating conditions are properly made. In the case of shot noise, however, the response is sensitive to both the second moment as well as to higher moments simultaneously. A JJ detector has frequency selectivity as was illustrated in the studies of environmental modes, although the resolution is somewhat limited and the role of multiphonon processes has to be understood. In the best case we have been able to resolve peaks of width of $0.5 - 1~\mathrm{GHz}.^{22}$

The main limitations of our method are set by the difficulties in determining the absolute magnitude of noise, and the requirement that the studied object has to be in the immediate vicinity of the detector. The former calls for a reliable calibration scheme, perhaps based on a well known normal tunnel junction, while the latter is a feature that is not a problem when studying mesoscopic samples.

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