Determination of the diffuser reference plane for accurate illuminance responsivity calibrations

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It is difficult to predict where the effective measurement plane is situated with dome-shaped diffusers often used in commercial photometers and radiometers. Insufficient knowledge of this plane could lead to large systematic errors in calibration of the illuminance responsivity of photometers. We propose a method that can be used to determine this reference plane accurately, based on the inverse-square law between the measured signal and the distance from the source. The method is demonstrated with three commercial photometers with dome-shaped diffusers of different geometries. By taking into account the measured shifts of the reference planes ($5.0 \pm 0.5 \text{ mm}$, $7.8 \pm 0.3 \text{ mm}$, and $8.5 \pm 0.7 \text{ mm}$), we reduced the systematic measurement errors up to 2% to statistical uncertainty components at the level of 0.2%. © 2005 Optical Society of America

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1. Introduction

Commercially available luxmeters are used for many purposes including photography, occupational health, and lighting design. Luxmeters with good cosine correction for angular responsivity are available at a reasonable price. However, diffusers that improve the cosine response also change the effective plane of illuminance measurement, introducing a potential source of measurement error when the luxmeter is calibrated for illuminance responsivity. It is generally known that large measurement errors can occur if the luxmeter is calibrated for a high illuminance level close to the light source and the true position of the reference plane is not taken into account.

To perform accurate illuminance measurements, the reference plane of a photometer must be known precisely.¹ This can be achieved with reference photometers that maintain the national illuminance responsivity scales, which usually have open apertures or thin, flat diffusers.^{2–4} Hand-held, commercial

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luxmeters are often equipped with dome-shaped or cylindrical diffusers and the reference plane of these luxmeters is, for simplicity, usually chosen to be at the top of the diffuser.

Luxmeters with diffusers can be calibrated by use of only one illuminance level far away from the light source, where the effect that is due to the error in the reference plane position is small and the obtained correction factor applies reasonably well. However, when the luxmeter is used in measurements at shorter distances from the source, the correction factor might no longer be valid. Furthermore, calibrations at high illuminance levels need to be performed close to the source.

An additional problem in determining the reference plane of the luxmeter is related to the position of the light source. The proper distance between the photometer and the lamp filament cannot be measured directly because of the glass envelope of the lamp that refracts light and could also act as a lens.⁵ A telescope could in principle be used to determine the location of the filament visually, but, when viewing from the side direction, the effect of the lamp envelope might not be evaluated correctly in the direction of the calibrated luxmeter.

Here we propose a method for accurately determining the reference plane of a diffuser and test the method with careful measurements of three different types of luxmeter. First, we determine the effective filament position of the lamp from the distance dependence of the signal with a reference photometer having a known aperture plane. Second, with the

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known lamp filament position the reference plane of the luxmeter is determined from the distance dependence of the signal applying again the inverse-square law. Furthermore, we show that the measured distance offsets from the top of the diffuser cannot be simply derived by calculating the geometry of the diffuser. To the best of our knowledge, we believe that this type of systematic measurement and analysis has not been reported previously.

2. Materials and Methods

A. Inverse-Square-Law Method

The proposed inverse-square-law method is based on measurements of illuminance at varying distances from a photometric lamp. The measurements are performed with a reference photometer and the photometer (luxmeter) whose reference plane is to be determined. It is required that the reference photometer has a well-defined input aperture. The outermost component of the photometer must be a limiting aperture that defines the measurement plane. Dome diffusers are not suitable for reference photometers.

When illuminances are measured at different distances from a point source, the measured values obey the inverse-square law

$$E_v = I_v / (d + \Delta d_S + \Delta d_P)^2, \tag{1}$$

where E_v is the measured illuminance, I_v is the luminous intensity of the point source, d is the distance between the selected reference planes of the source and the photometer, Δd_S is the distance offset of the source, and Δd_P is the distance offset of the photometer used. When lamps are used as the light source instead of an ideal point source, the effect of transverse dimensions of the lamp filament can be taken into account by use of the modified inverse-square law,⁶ but for the purposes of this paper, Eq. (1) is sufficiently accurate. Distance d should be defined between such points that their separation is convenient to measure directly. These may be selected, for example, as the outermost surfaces of the photometer and the lamp holder. Obtained correction terms Δd_S and Δd_P will give positions of the reference planes that obey the inverse-square law with respect to the measured signal and the distance between the reference planes.

The lamp is first measured with the reference photometer to obtain the offset Δd_S for the lamp. The analysis takes place by fitting Eq. (1) to the measurement results by the least-squares method. Luminous intensity I_v and distance offset Δd_S are used as free parameters ($\Delta d_P = 0$ for the reference photometer). The measurement and the analysis are then repeated with the unknown photometer to obtain the offset $\Delta d_P (d + \Delta d_S \text{ is known from the previous measure$ ment). Measurement positions should be selected in such a way that the applicability of the inverse-square law is not limited because of the too-short lampphotometer distance and that the variation of the signal is sufficient between the nearest and the most



Fig. 1. Schematic drawings of the investigated diffusers. Dimensions of the diffusers in the drawings are as follows. (a) D = 24.3 mm, L = 8.0 mm, (b) D = 16.0 mm, L = 7.1 mm, (c) D = 30.0 mm, L = 15.0 mm, W = 26.1 mm.

distant measurement positions. The photometers must also have adequate resolution and linearity.

B. Reference Photometers

In this study we used three-commercial standard photometers as reference photometers, all of type PRC TH15 manufactured by PRC Krochmann GmbH, Germany. All these photometers have a welldefined reference plane, either a circular aperture with an 8 mm diameter (HUT-1 and HUT-2) or a planar Teflon diffuser of the same size (LM-1). The aperture planes and the front surface of the flat diffuser were used as the reference planes for distance measurements. The photocurrents were measured with a current-to-voltage converter and a digital voltmeter.

C. Tested Photometers

As test devices we used three photometer heads, schematic drawings of the diffusers are shown in Fig. 1. Distances d were measured from the top of the diffusers (the outermost point). Minolta T-1 had a dome diffuser [Fig. 1(a)] and Hagner E2 had two measuring heads with a dome-shaped and a cylindrical diffuser [Figs. 1(b) and 1(c)].

D. Lamp and Optical Rail

Measurements were conducted at the facility for luminous intensity and illuminance measurements⁴ utilizing a 4.5-m optical rail with an accurate length scale. All the detectors were mounted, one at a time, on the same rail carrier by use of magnetic baseplates. Lamp and detectors were aligned to the same optical axis by use of a two-beam alignment laser.

The photometric lamp used was an Osram Wi41/G, provided by PRC Krochmann GmbH. This lamp has a magnetic holder for the alignment mirror in front of the lamp. When in place, the front surface of the mirror is a good and reliable reference plane for distance measurements.

3. Measurement Results

A. Distance Offset of the Light Source

The lamp was first measured with the reference photometers HUT-1, HUT-2, and LM-1. Illuminance values were measured at several distances d between 500 and 2500 mm. The distance offset Δd_s was cal-



Fig. 2. Results of repeated measurements to determine the distance offset Δd_s of the light source with three different reference photometers. Vertical bars indicate the standard deviation of the results for each photometer.

Table 1. Measured Distance Offsets Δd_P of the Reference Planes of the Diffusers^{*a*}

		Offset $\Delta d_{S} + \Delta d_{P} (\text{mm})$		
Measurement Set	HUT-2	Diffuser (a)	Diffuser (b)	Diffuser (c)
1	81.4	86.2		
2	81.5	87.5		
3	81.4	85.7	89.2	
4	81.7		90.0	
5	81.5		88.6	88.6
6	81.8		90.0	91.3
7	82.1			91.0
Offset $\Delta d_{P}~(\mathrm{mm})$		5.0 ± 0.5	7.8 ± 0.3	8.5 ± 0.7

^aDiffusers are denoted as in Fig 1. Uncertainties are calculated as the standard deviations of the mean for each diffuser.

culated separately for each measurement sequence with $\Delta d_P = 0$. At d = 500 mm, the transverse dimensions of the lamp filament cause a deviation of less than 0.1 mm for the offset that would be observed for an ideal point source.⁶

The results from 17 measurements (lamp operated several times) are presented in Fig. 2. It can be seen that the filament plane of the lamp was on average (79.6 ± 0.1) mm behind the alignment mirror. Values measured with different photometers are in good agreement. This indicates that the thin, flat diffuser in LM-1 does not influence the results within the uncertainties.

The standard deviation of the results was 0.4 mm and the standard deviation of the mean was 0.1 mm, but differences up to 1.4 mm were measured with the same photometer between different burns of the lamp. The exact reason for the variation of the distance offset is unknown, but it is possible that the effective position of the lamp filament deviates slightly in different burns. We conclude that, if the desired measurement accuracy is of the order of 1 mm, it is sufficient to calibrate the lamp filament position once. If higher accuracy is desired, the effective filament position must be determined separately for each lamp burn.

B. Reference Planes of Dome-Shaped Diffusers

Reference planes of the tested photometers were determined by use of standard photometer HUT-2 as the reference. The lamp and the luxmeters on magnetic baseplates were mounted on the optical bench and aligned. Illuminance values were measured at six distances (500–1500 mm) from the alignment mirror. Illuminances were first measured at all distances by use of HUT-2, and then the signals were determined by placing the top of the diffuser of the tested photometers at the same positions. Magnetic baseplates were used to interchange the prealigned photometers, so that the lamp did not have to be switched off during the change. This was to ensure that the observed variation of the filament position between burns did not influence the results.

Data analysis was carried out for each photometer by use of Eq. (1). We performed the least-squares fit by varying parameter $\Delta d_S + \Delta d_P$, which includes offsets for both the photometer and the lamp. These results are presented in Table 1. Distance offset Δd_P for HUT-2 is known to be zero. Therefore the lamp offset Δd_S can be accounted for by subtracting the results for HUT-2 from the results for test photometers. Note that different lamps are used in the measurements of Fig. 2 and Table 1.

In all three photometer heads, the reference plane according to which the inverse-square law holds is several millimeters behind the top of the diffuser. If the reference plane is known before the calibration, distances for illuminance calibrations should be measured with respect to this plane.

C. Correction of Erroneous Illuminance Responsivity Calibrations

Calibration of illuminance responsivity at various distances with both the reference photometer and the tested photometer allows simple correction for the distance offset. The offset Δd_S required to define the filament position of the lamp is obtained from the measurement with the reference photometer. The least-squares fit process also gives the luminous intensity of the lamp. Measurements at various distances with the test photometers give their offsets Δd_P . A knowledge of the distances allows for correct illuminance levels at each measurement position to be calculated from the luminous intensity value. If the luminous intensity value is not needed, the correct illuminance values $E_{v,\text{test}}$ for the test photometer can also be calculated as

$$E_{v,\text{test}} = E_{v,\text{ref}} [(d + \Delta d_S) / (d + \Delta d_S + \Delta d_P)]^2, \quad (2)$$

where $E_{v,ref}$ is the illuminance measured by the reference photometer.

The original illuminance correction factors obtained from the measurement of the photometer with diffuser (b) are presented in Fig. 3 as crosses. For an ideal case, the correction factors would not depend on distance. Since the reference plane is behind the top of the diffuser, the correction factors decrease when distance is increased. Without any correction that is



Fig. 3. Original (crosses) and new (circles) correction factors for the photometer with diffuser (b) in Fig. 1.

due to the reference plane position, errors of the order of 2% in the correction factors are obtained. The data were corrected using Eq. (2). These new correction factors are shown in Fig. 3 as circles. It can be seen that the correction factors are nearly constant (1.137 with a standard deviation of 0.001), indicating that the distance dependence has disappeared.

4. Geometric Calculation of the Distance Offset

A straightforward model to calculate the photometer distance offset from the shape of the dome diffuser is based on assuming that each infinitesimal surface element of the diffuser contributes to the photometer signal an amount proportional to the illuminance and cross-sectional area of that element. Figure 4 shows the geometry for a spherical diffuser for which the radius of the dome is r_0 and the diffuser surface extends up to the cone angle θ_0 relative to the optical axis. As calculated in Appendix A, the reference plane of the photometer is then shifted by distance

$$\Delta d_{C} = r_{0} \bigg[1 - \frac{2}{3} (1 - \cos^{3} \theta_{0}) / \sin^{2} \theta_{0} \bigg]$$
(3)

behind the top of the diffuser. A similar calculation for a photometer with a cylindrical dome diffuser shows that the offset is

$$\Delta d_C = R_0 \left[1 - \frac{1}{4} (2\Theta_0 + \sin 2\Theta_0) / \sin \Theta_0 \right], \qquad (4)$$



Fig. 4. Coordinate system for the calculation of the distance offset from the geometry.

where R_0 is the radius of the cylinder and the diffuser surface is limited by angles $-\Theta_0$ and $+\Theta_0$ relative to the optical axis. In the case of a hemisphere or a half of a cylinder ($\theta_0 = \Theta_0 = \pi/2$), the offsets are Δd_C = 0.333 r_0 or $\Delta d_C = 0.215R_0$, respectively.

For the diffuser of Fig. 1(a), the geometric parameters are given by $r_0 = L/2 + D^2/(8L) = 13.2 \text{ mm}$ and $\theta_0 = \arcsin[D/(2r_0)] = 66.7^\circ$, leading to the distance offset $\Delta d_C = 3.4$ mm calculated with Eq. (3). This number can be compared with the measured value $\Delta d_P = (5.0 \pm 0.5)$ mm from Table 1. For the photometer heads of Figs. 1(b) and 1(c), the calculated offsets are 2.6 and 3.2 mm with Eqs. (3) and (4), whereas the measured values Δd_P are (7.8) \pm 0.3) and (8.5 \pm 0.7) mm, respectively. The calculated offsets are much smaller than the measured values even if the indicated standard uncertainties are taken into account. It is concluded that the position of the reference plane of the photometer head with the dome diffuser cannot be calculated from the shape of the diffuser. The measured distance offset could be affected by diffuser material, together with both the internal and the external structure of the photometer head.

5. Concluding Remarks

The method described improves the accuracy of illuminance measurements and greatly reduces the effect of reference plane displacement. It can be concluded that the deeper the diffuser, the larger the measurement error. The problem does not concern only luxmeters but all devices that measure optical radiation and have diffusers. Such devices include, e.g., spectroradiometers and optical powermeters. The problem can be emphasized with large dome diffusers used in solar UV measurements. Spectroradiometers are usually calibrated at a distance of 500 mm from a spectral irradiance standard lamp, whereas the actual measurement distance to the Sun is practically infinite. Therefore, further study of the distance offsets of spectroradiometers and UV radiometers in general can be considered necessary.

To determine the reference plane of a diffuser experimentally is not difficult, but it is time-consuming. Naturally, the institutes that perform many radiometric and photometric calibrations are not willing to pay much attention to this matter without compensation. Therefore, it would be most convenient if the manufacturers of the luxmeters would make such measurements in advance and include the results in technical specifications before the devices are brought to the market.

Appendix A

Here we describe the details of deriving Eq. (3) in the text. The illuminance at the perpendicular projection of the narrow circular stripe of width $r_0 d\theta$ in Fig. 4 is $I_v/r^2(\theta)$, where

$$r(\theta) = d + \Delta d_S + r_0 - r_0 \cos \theta \tag{A1}$$

is the distance of the stripe from the source. Since the distance is much larger than the transverse dimensions of the diffuser (or of the light source), $d + \Delta d_S \gg r_0$; terms of the order of $[r_0/(d + \Delta d_S + r_0)]^2$ are neglected in this calculation. As seen from the source, the perpendicular projected area of the infinitesimal stripe is

$$\mathrm{d}A_{\perp} = 2\pi r_0^2 \sin\theta\cos\theta\,\mathrm{d}\theta. \tag{A2}$$

Integration over the surface of an ideal diffuser gives the calculated illuminance signal of the photometer:

$$E_{C} = \frac{\int \left[I_{v}/r^{2}(\theta)\right] \mathrm{d}A_{\perp}}{\int \mathrm{d}A_{\perp}} = \frac{2I_{v}}{(d + \Delta d_{S} + r_{0})^{2} \sin^{2}\theta_{0}}$$
$$\times \int_{0}^{\theta_{0}} \frac{\sin\theta\cos\theta \mathrm{d}\theta}{\left[1 - r_{0}\cos\theta/(d + \Delta d_{S} + r_{0})\right]^{2}}, \qquad (A3)$$

where the cross-sectional area is used as a weighting factor in combining the signals from different stripes. The latter form of Eq. (A3) is used to derive a series expansion for $1/r^2(\theta)$, which is straightforward to integrate for the two lowest-order terms. After integration, the series expansion can be used in the reverse direction and the dependence of the illuminance sig-

nal on r_0 and θ_0 can be rewritten into the denominator with the following result:

$$E_{C} = \frac{I_{v}}{\left[d + \Delta d_{S} + r_{0} - 2/3r_{0}(1 - \cos^{3} \theta_{0})/\sin^{2} \theta_{0}\right]^{2}}.$$
(A4)

Comparison of Eqs. (1) and (A4) leads to Eq. (3) in the text.

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