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JOINT TIME-DOMAIN TRACKING OF CHANNEL AND FREQUENCY OFFSET FOR OFDM SYSTEMS

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ABSTRACT

This paper addresses the problem of joint channel and frequency offset estimation and tracking for OFDM systems. The proposed method is based on Extended Kalman filtering. The channel taps and the frequency offset are estimated in time-domain while equalization is performed in frequency domain. Simulation results demonstrate accurate tracking capability of the algorithm in scenarios when the channels are time-frequency selective. The frequency offset may be slowly or abruptly changing in time.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is an emerging transmission technique which has been applied in high speed modems, wireless LAN's and digital audio and video broadcasting. This multicarrier technique is very appealing in wireless communications due to the fact that it has the ability to turn a frequency selective channel into a set of parallel narrowband channels. This leads to very simple equalization since the transmission becomes free of Intersymbol Interference (ISI). Hence, OFDM is a viable candidate for future beyond 3G wireless communications standards.

In order to enjoy all the benefits of OFDM transmission, two key tasks must be successfully accomplished: frequency offset and channel estimation. One of the main drawbacks of OFDM is its high sensitivity to frequency offsets caused by the oscillator inaccuracies and the Doppler shift due to mobility. These factors give rise to inter-carrier interference (ICI). This issue becomes critical when higher data rates are required, and an increased number of subcarriers and consequently a reduced inter-carrier spacing is needed. Frequency and time selectivity of broadband mobile channels should also be compensated for, in order to fully achieve the benefits of OFDM transmissions.

In OFDM, regular Kalman Filter has been employed for estimating time-frequency selective channels assuming perfect carrier synchronization [1]. On the other hand, Extended Kalman Filter (EKF) has been used for frequency offset estimation in the case of AWGN channels [2].

In this paper a joint channel and frequency offset time-domain tracking algorithm based on EKF is proposed.

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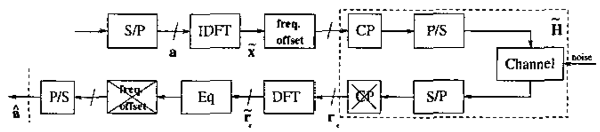

Figure 1. OFDM transmission chain.

Channel and frequency offset estimation is followed by equalization in frequency domain.

By building a state-space model which includes both the channel and the frequency offset as the state, we are able to apply EKF in time-domain in order to estimate the state. Though estimation and tracking are possible in the frequency plane, time-domain processing allows significant complexity reduction. EKF is used due to the nonlinearity in the measurement equation introduced by the frequency offset. When dealing with non-linear state-space models in communications, Extended Kalman Filter proves to be a valuable tool [3].

The rest of the paper is organized as follows. A brief presentation of the system model is given next. In Section 3, we present the state-space model which allows the implementation of EKF for channel and frequency offset tracking. We also present how equalization is performed. In Section 4, we report simulation results using time-frequency selective channels and time varying frequency offset. Finally, Section 5 concludes the paper.

2. SYSTEM MODEL

The OFDM transmission model used in this paper is presented in Figure 1. In the following, the transmission process is represented in a formal way, using the notation introduced in [4]. Let us look at the transmission procedure for the k^{th} block. As the cyclic prefix provides us with inter-block interference free transmission, we can then process each OFDM block independently. The k^{th} modulated OFDM block is written as $\tilde{\mathbf{x}}(k) = \mathbf{F}_N\mathbf{a}(k)$, where $\mathbf{F}_N$ is the $N \times N$ inverse discrete Fourier transform (IDFT) matrix, N being the total number of subcarriers and $\mathbf{a}(k)$ is the $N \times 1$ complex symbol vector. Let us consider the frequency offset at the transmitter side. A discussion of the system model with offset at the receiver side is presented in section 3.2.. The modulated block with frequency offset at the transmitter is:

$$\tilde{\mathbf{x}}_\epsilon(k) = \mathbf{C}_\epsilon\, \tilde{\mathbf{x}}(k), \tag{1}$$

where $\mathbf{C}_\epsilon$ is the $N \times N$ frequency offset matrix having the form $\mathbf{C}_\epsilon = \text{diag}\left\{\exp\left(j\frac{2\pi n\epsilon}{N}\right)\right\}$ with $n = 0, \ldots, N-1$. The quantity ϵ, chosen as $0 \le \epsilon < 1$, is referred to as *normalized frequency offset*.

The received $N \times 1$ signal block after cyclic prefix insertion, followed by transmission on the wireless channel and cyclic prefix removal (Figure 1) is expressed as:

$$\mathbf{r}_\epsilon(k) = \widetilde{\mathbf{H}}(k)\mathbf{C}_\epsilon\, \tilde{\mathbf{x}}(k) + \mathbf{w}(k). \tag{2}$$

Due to cyclic prefix insertion and removal operations, $\widetilde{\mathbf{H}}(k)$ is a $N \times N$ circulant matrix, with the (i,l)th entry given by $h_{(i-l)modN}$. The channel taps $\{h_l\}_{l=0,\ldots,L_h-1}$ are assumed to be constant within the duration of one ODFM block, and are also supposed to vary independently in time. The channel is considered to have a maximum of L_h taps and is no longer than L_{CP}, the length of the cyclic prefix, in order to avoid inter-block interference. The noise term $\mathbf{w}$ is assumed to be circular white Gaussian.

Since circulant matrices implement circular convolutions, they are diagonalized by DFT and IDFT operations and the following equality holds:

$$\mathbf{F}_N^H \widetilde{\mathbf{H}}(k)\mathbf{F}_N = \mathbf{D}(k), \tag{3}$$

where

$$\mathbf{D}(k) = \text{diag}\left\{\sum_{l=0}^{N-1} h_l(k)\exp\left(-j\frac{2\pi nl}{N}\right)\right\}_{n=0,\ldots,N-1} \tag{4}$$

contains the frequency response of the channel, evaluated at the subcarrier frequencies.

After having performed the Fourier transform of (2) we obtain:

$$\tilde{\mathbf{r}}_\epsilon(k) = \mathbf{F}_N^H \widetilde{\mathbf{H}}(k)\mathbf{C}_\epsilon \mathbf{F}_N\, \mathbf{a}(k) + \tilde{\mathbf{w}}(k), \tag{5}$$

where $\tilde{\mathbf{w}}(k) = \mathbf{F}_N^H\, \mathbf{w}(k)$. Right multiplication by $\mathbf{F}_N^H$ of equation (3) leads to $\mathbf{F}_N^H \widetilde{\mathbf{H}}(k) = \mathbf{D}(k)\mathbf{F}_N^H$, and using this fact, (5) becomes:

$$\tilde{\mathbf{r}}_\epsilon(k) = \mathbf{D}(k)\left(\mathbf{F}_N^H \mathbf{C}_\epsilon \mathbf{F}_N\right)\mathbf{a}(k) + \tilde{\mathbf{w}}(k). \tag{6}$$

Hence, the following observations can be made:

- If $\epsilon = 0$: then $\mathbf{C}_\epsilon \equiv \mathbf{I}$ and (6) becomes: $\tilde{\mathbf{r}}_\epsilon(k) = \tilde{\mathbf{r}}_0(k) = \mathbf{D}(k)\,\mathbf{a}(k) + \tilde{\mathbf{w}}(k) \equiv \tilde{\mathbf{r}}(k)$.
- If $0 < \epsilon < 1$: then $\mathbf{F}_N^H \mathbf{C}_\epsilon \mathbf{F}_N \neq \mathbf{I}$ and hence the matrix $\mathbf{D}(k)\left(\mathbf{F}_N^H \mathbf{C}_\epsilon \mathbf{F}_N\right)$ is not diagonal, which leads to intercarrier interference, breaking the orthogonality property of the OFDM transmission.

Using a zero forcing type of receiver, the channel needs to be equalized first and then offset is compensated for in the following way:

$$\hat{\mathbf{a}}(k) = \left(\mathbf{F}_N^H \mathbf{C}_\epsilon \mathbf{F}_N\right)^{-1} \mathbf{D}(k)^{-1}\, \tilde{\mathbf{r}}_\epsilon(k). \tag{7}$$

Finally a decision is made on $\hat{\mathbf{a}}(k)$, in order to estimate the originally transmitted symbols. Equation (7) requires the knowledge of both the offset value and the channel coefficients at the receiver side.

3. CHANNEL AND OFFSET ESTIMATION AND TRACKING

In this section, we propose a method stemming from Extended Kalman Filtering (EKF) to estimate and track channel coefficients and frequency offset over time. As stated in the previous section, the offset can be modeled at the transmitter or at the receiver. In the following we investigate both cases and we emphasize their differences in EKF equations and in the equalization stage.

3.1. Frequency offset at transmitter side

In this subsection we consider that the offset is induced at the transmitter side. The OFDM transmission model is formulated as a state-space model. We start by defining the $[(L_h + 1) \times 1]$ state vector as follows: $\mathbf{s}(k) = [h_0(k), h_1(k) \cdots h_{L_h-1}(k), \epsilon(k)]^T = \left[\mathbf{h}^T(k), \epsilon(k)\right]^T$. Then, the state equation is written as:

$$\mathbf{s}(k) = \mathbf{A}\,\mathbf{s}(k-1) + \mathbf{v}(k), \tag{8}$$

where $\mathbf{A}$ and $\mathbf{v}$ are the state transition matrix and the state noise, respectively. The $[(L_h+1) \times (L_h+1)]$ state transition matrix $\mathbf{A}$ is chosen to be close to the identity matrix. An important observation is the fact that the state equation is linear. Let us first recall the received time-domain signal, including the frequency offset at the transmitter side (2), and give two equivalent formulations:

$$\begin{aligned} \mathbf{r}_\epsilon(k) &= \widetilde{\mathbf{H}}(k)\, \tilde{\mathbf{x}}_\epsilon(k) + \mathbf{w}(k) & (9) \\ &= \widetilde{\mathbf{X}}_\epsilon(k)\, \mathbf{h}(k) + \mathbf{w}(k), & (10) \end{aligned}$$

where $\widetilde{\mathbf{X}}_\epsilon(k)$ is the circulant matrix of size $N \times L_h$ made from vector $\tilde{\mathbf{x}}_\epsilon(k)$ defined in (1). We can also summarize equations (9) and (10) as:

$$\mathbf{r}_\epsilon(k) = \mathcal{G}\left(\mathbf{s}(k)\right) + \mathbf{w}(k), \tag{11}$$

where $\mathcal{G}$ is a non-linear function of the state vector $\mathbf{s}(k)$.

Equations (8) and (9) form the state-space equations. Considering that the noise term $\mathbf{w}$ is Gaussian and the measurement equation is nonlinear, Extended Kalman Filtering [5] can be applied to estimate the state which contains the channel and the frequency offset. By performing a linearization of the nonlinear state-space equations, regular Kalman filter can be applied to estimate the state. Our state equation is already linear, hence no action needs to be taken. In the case of the measurement equation we have to take the derivative of $\mathcal{G}$ with respect to $\mathbf{s}$. Due to the structure of $\mathbf{s}(k) = \left[\mathbf{h}^T(k), \epsilon(k)\right]^T$, at any time index k the following holds:

$$\left[\frac{\partial \mathcal{G}}{\partial \mathbf{s}}\right] = \left[\frac{\partial \mathcal{G}}{\partial \mathbf{h}}, \frac{\partial \mathcal{G}}{\partial \epsilon}\right]. \tag{12}$$

By using both formulas (9) and (10) we obtain:

$$\mathbf{G}_{N\times(L_h+1)}(k) = \left[\frac{\partial \mathcal{G}}{\partial \mathbf{s}}\right] = \left[\widetilde{\mathbf{X}}_\epsilon, \widetilde{\mathbf{H}}\,\frac{\partial\left(\tilde{\mathbf{x}}_\epsilon\right)}{\partial \epsilon}\right], \tag{13}$$

where derivatives are evaluated at $\hat{\mathbf{s}}(k-1|k-1)$, and $\partial\left(\tilde{\mathbf{x}}_\epsilon\right)/\partial\epsilon = \mathbf{C}'_\epsilon\, \tilde{\mathbf{x}}$, with $\mathbf{C}'_\epsilon$ defined as:

$$\mathbf{C}'_\epsilon = \text{diag}\left\{(j\,2\pi n/N)\exp\left(j\,2\pi n\epsilon/N\right)\right\}_{n=0,\ldots,N-1}. \tag{14}$$

The estimated state at time index k is given by:

$$\hat{\mathbf{s}}(k|k) = \hat{\mathbf{s}}(k|k-1) + \mathbf{K}(k)\left[\mathbf{r}_\epsilon(k) - \mathcal{G}\left(\hat{\mathbf{s}}(k|k-1)\right)\right]. \quad (15)$$

where $\hat{\mathbf{s}}(k|k-1) = \mathbf{A}\,\hat{\mathbf{s}}(k-1|k-1)$. The Kalman gain $\mathbf{K}(k)$ is updated according to:

$$\mathbf{K}(k) = \mathbf{P}(k|k-1)\mathbf{G}^H(k)\left[\mathbf{G}(k)\mathbf{P}(k|k-1)\mathbf{G}^H(k) + \mathbf{R}_\mathbf{w}\right]^{-1}, \quad (16)$$

where $\mathbf{R}_\mathbf{w} = \sigma^2\mathbf{I}$ is the $N \times N$ covariance matrix of the measurement noise. The computation of the prediction error covariance matrix is done according to:

$$\mathbf{P}(k|k-1) = \mathbf{A}\mathbf{P}(k-1|k-1)\mathbf{A}^T + \mathbf{Q}, \quad (17)$$

with $\mathbf{Q}$ having the structure $\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_h & \mathbf{0}_{L_h \times 1} \\ \mathbf{0}_{1 \times L_h} & \sigma_\epsilon^2 \end{bmatrix}$ where $\mathbf{Q}_h = \sigma_h^2\mathbf{I}$, σ_h^2 being the variance of the channel's state noise and σ_ϵ^2 is the offset's state noise variance. The filtering error covariance matrix is given by:

$$\mathbf{P}(k|k) = \left[\mathbf{I} - \mathbf{K}(k)\mathbf{G}(k)\right]\mathbf{P}(k|k-1). \quad (18)$$

The overall structure of channel tracking algorithm is illustrated in Figure 2. The algorithm works as follows. For OFDM symbol at time k:

1. Decode the received vector $\mathbf{r}_\epsilon(k)$ and obtain the symbols estimate $\hat{\mathbf{a}}(k)$, using $\hat{\mathbf{h}}(k-1|k-1)$ and $\hat{\epsilon}(k-1|k-1)$, i.e. the filtered estimate of the channel at symbol time $k-1$ and the estimated frequency offset at time $k-1$.
2. Re-modulate $\hat{\mathbf{a}}(k)$: $\hat{\tilde{\mathbf{x}}}(k) = \mathbf{F}_N\,\hat{\mathbf{a}}(k)$.
3. Introduce the estimated frequency offset:
$$\hat{\tilde{\mathbf{x}}}_\epsilon(k) = \mathbf{C}_{\hat{\epsilon}}(k-1)\,\hat{\tilde{\mathbf{x}}}(k).$$
4. Build matrices $\hat{\widetilde{\mathbf{X}}}_\epsilon(\mathbf{k})$ and $\hat{\widetilde{\mathbf{H}}}$ in order to obtain $\hat{\mathbf{G}}(k)$.
5. Run EKF to finally obtain $\hat{\mathbf{h}}(k|k)$ and $\hat{\epsilon}(k)$.

Except for the first OFDM symbol, the algorithm works in a decision directed mode. For a better initialization one can consider few (two, three) OFDM symbols are known at the beginning of the transmission. Some pilot symbols more may be needed to deal with fast varying channels and divergence of EKF.

The major computational cost lies in the calculation of the matrix inversion in the Kalman gain expression (16). By applying the matrix inversion lemma, the number of operations can be reduced to $O\left((L_h+1)\,N^2\right)$ when tracking is done in time-domain. The complexity is of order $O\left(N^3\right)$ when the processing takes place in the frequency domain. In practice $L_h \ll N$, hence significantly lower complexity is achieved when working in time-domain.

3.2. Frequency offset at receiver side

Previous derivations assumed frequency offset was present at transmitter side. Equivalently it can be modeled at the receiver also. In this section, we briefly rewrite the transmission equations as well as the derivatives needed for EKF, in the case where the receiver is not perfectly matched in frequency to the transmitter.

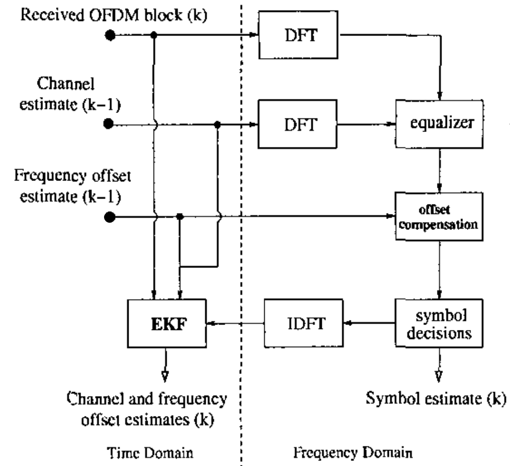

Figure 2. Time-domain channel and offset estimation and tracking, frequency domain equalization.

First, the $N \times 1$ received time-domain signal block is written as:

$$\begin{aligned} \mathbf{r}_\epsilon(k) &= \mathbf{C}_\epsilon\widetilde{\mathbf{H}}(k)\,\tilde{\mathbf{x}}(k) + \mathbf{w}(k) && (19)\\ &= \mathbf{C}_\epsilon\widetilde{\mathbf{X}}(k)\,\mathbf{h}(k) + \mathbf{w}(k) && (20)\\ &= \mathcal{G}\left(\mathbf{s}(k)\right) + \mathbf{w}(k), && (21) \end{aligned}$$

where $\widetilde{\mathbf{X}}(k)$ is a circulant matrix of size $N \times L_h$ made from vector $\tilde{\mathbf{x}}(k)$. Performing the DFT, we obtain:

$$\begin{aligned} \tilde{\mathbf{r}}_\epsilon(k) &= \mathbf{F}_N^H\mathbf{C}_\epsilon\widetilde{\mathbf{H}}(k)\mathbf{F}_N\,\mathbf{a}(k) + \tilde{\mathbf{w}}(k) && (22)\\ &= \left(\mathbf{F}_N^H\mathbf{C}_\epsilon\mathbf{F}_N\right)\mathbf{D}(k)\,\mathbf{a}(k) + \tilde{\mathbf{w}}(k), && (23) \end{aligned}$$

Zero-forcing equalization leads to:

$$\hat{\mathbf{a}}(k) = \mathbf{D}(k)^{-1}\left(\mathbf{F}_N^H\mathbf{C}_\epsilon\mathbf{F}_N\right)^{-1}\tilde{\mathbf{r}}_\epsilon(k). \quad (24)$$

State equation (8) remains unchanged, whereas measurement equation is now given by (19) or equivalently by (20). Then, at time index k the matrix $\mathbf{G}$ of derivatives is expressed as:

$$\mathbf{G}_{N\times(L_h+1)}(k) = \left[\frac{\partial\mathcal{G}}{\partial\mathbf{s}}\right] = \left[\mathbf{C}_\epsilon\widetilde{\mathbf{X}}, \mathbf{C}'_\epsilon\widetilde{\mathbf{H}}\,\tilde{\mathbf{x}}\right], \quad (25)$$

where derivatives are evaluated at $\hat{\mathbf{s}}(k-1|k-1)$, and $\mathbf{C}'_\epsilon$ is defined as in (14). Finally, by using these modifications, the EKF can be run in the same way as described in Section 3.1.

4. SIMULATIONS

In our simulation studies, results are presented in the case where frequency offset is introduced at the transmitter side. However, placing the offset at the receiver front-end exhibits similar tracking and bit error rate performance.

In our setup the carrier frequency is $f_0 = 2.4$ GHz and the number of subcarriers is set to $N = 128$. The available bandwidth is chosen equal to $B = 1$ MHz leading to a subcarrier symbol rate of 7.8 KHz. BPSK symbol modulation is employed. The wireless channel is Rayleigh fading with independent propagation paths. The power delay profile is $[0, -1, -3, -9]$ [dB] and $[0, 1, 2, 3]$ [μs] and the Doppler spectrum is Jakes's. This type of scenario corresponds to a Typical Urban environment [6]. The receiver speed is 30 km/h.

In our first example, 250 OFDM blocks are transmitted. The wireless channel is considered to be remain stationary during the OFDM block time. The frequency offset is considered to change abruptly after each 50 blocks. In order to emphasize the capability of the EKF to track the frequency offset we have slowly modified the frequency offset value for 50 blocks between the values 0.4 and 0.25. As a reminder, ϵ was defined as the normalized frequency offset ($0 \leq \epsilon < 1$), that is effective frequency deviation lies in the interval $[0, B/N[$, B being the total bandwidth allocated to the system. The first three OFDM blocks are known to the receiver, in order to acquire the track of both the channel taps and the frequency offset. Re-training blocks are then sent periodically (every 50 blocks in our simulations), in order to avoid possible divergence of the Kalman filter. Otherwise the transmission algorithm works in a decision directed mode.

The performance of the proposed method in tracking the channel and frequency offset over time is studied in simulation. Real and imaginary parts of **h** are depicted in Figure 3, in the case of 30 km/h velocity of the mobile terminal and at 15 dB SNR. The time variations of the four channel taps are accurately tracked, even for those with low average power. Since the equalization stage operates in the frequency domain, accuracy in estimating frequency responses of the channels at the subcarrier frequencies needs to be investigated. Figure 5 and 6 respectively show amplitude and phase responses, for the true and the estimated channel, at a given OFDM block time. Since time-domain estimation performs well, channel transfer functions are consequently also modeled accurately, in both amplitude and phase.

The frequency offset tracking result is depicted in Figure 4. We observe that the algorithm is able to track both the slow and abrupt changes of the frequency offset.

The final performance criterion is the bit error rate as a function of SNR, presented in Figure 7 for a terminal velocity equal to 30 km/h. A lower bound for the performance of the tracking algorithm is given by using the ideal channel state information (CSI) i.e. perfectly known channel, and perfect knowledge of the frequency offset at the receiver side. As shown by simulation plots, tracking both channel and frequency offset in time-domain provides us with results close to those obtained with known channel and offset, for a wide range of SNR (0 to 20 dB).

For completeness, results for various configurations (estimated or true channel, estimated or known offset value) are also included. Those demonstrate the accuracy of time-domain offset estimation, since bit error rate curves in the case of true and estimated offset values almost overlap. Finally the curve with channel equalization only, without any offset compensation, points out the importance of frequency offset estimation in OFDM systems, which otherwise suffer considerably from inter-carrier interference. Investigations for lower terminal velocities show an even better performance, close to ideal case.

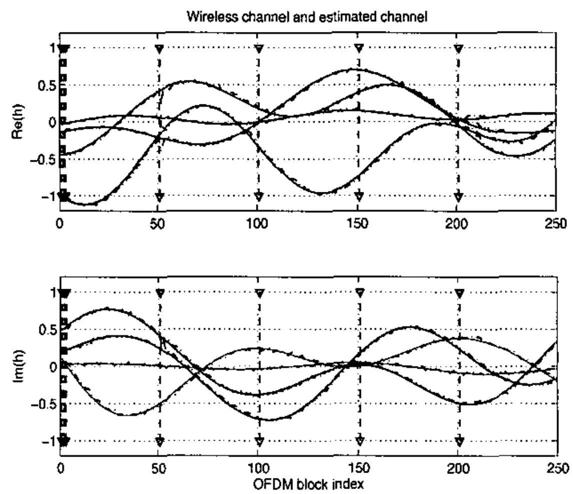

Figure 3. Time-domain tracking (SNR=15 dB, $v =$ 30 km/h). Continuous line is the true channel, dash line is the estimated channel; vertical lines indicate re-training time instants.

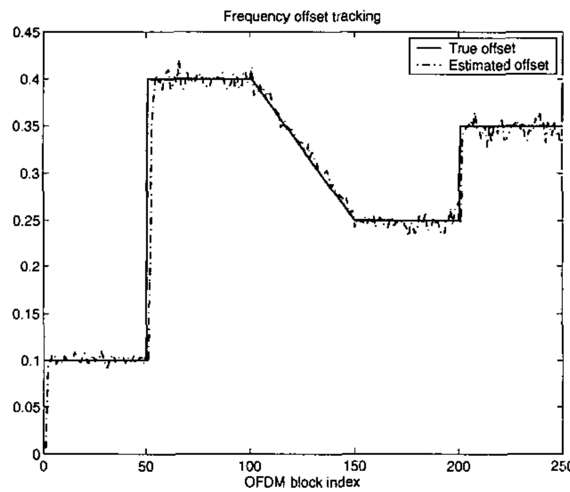

Figure 4. Frequency offset tracking (SNR=15dB, v = 30 km/h). Continuous line is the true offset, dash line is the estimated offset.

5. CONCLUSIONS

In this paper, a channel estimation and frequency offset tracking method stemming from Extended Kalman Filter is proposed. The tracking stage runs in time-domain and is the core of the presented algorithm. Reliable offset tracking capability is also shown in cases when the frequency offset varies abruptly in time. However, in difficult environments characterized by low SNR and fast varying channels, additional pilot symbol structure may be needed to avoid the divergence of the algorithm. Even though additional DFT's are needed, complexity requirements are also lowered. The

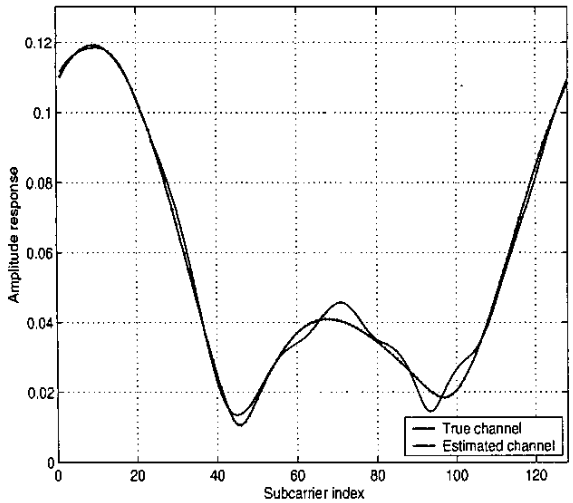

Figure 5. Amplitude response (SNR=15 dB, $v = 30$ km/h).

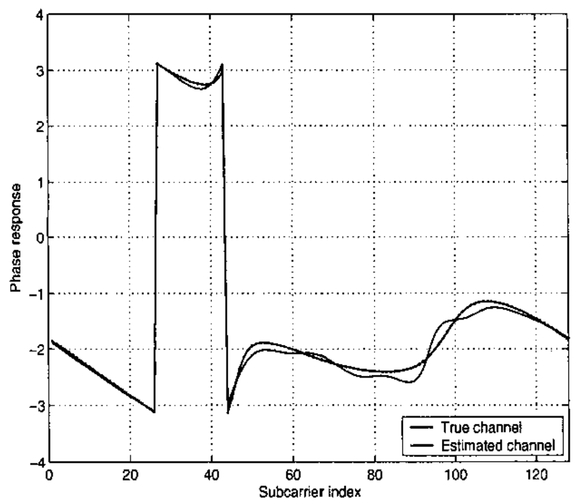

Figure 6. Phase response (SNR $= 15$ dB, $v = 30$ km/h).

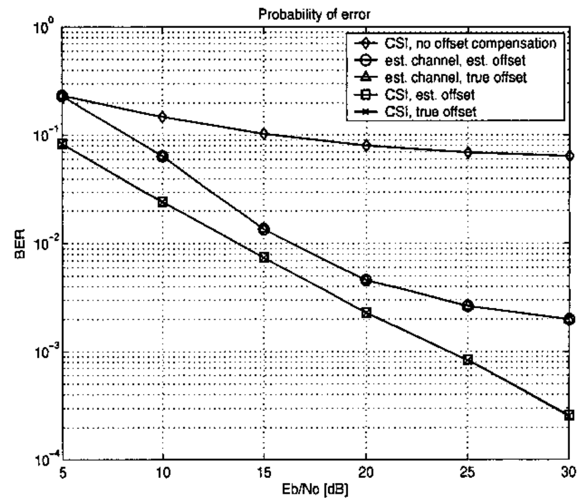

Figure 7. Bit error rate performance (over 10000 blocks, $v = 30$ km/h).

reliable performance of the method is demonstrated for mobile user in a typical urban scenario.

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