Optimal Virtual Element Patterns for Adaptive Arrays

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***Abstract*—A major problem in the use of small arrays is the perturbation in the element patterns due to mutual coupling and diffraction effects. The element patterns differ from each other and need to be corrected in order to obtain an array pattern that is close to the desired one. A matrix method can be used to correct the element patterns by modifying the original input/output weights to corresponding weights for corrected elements and vice versa. In this paper the main goal of the array correction is the maximal identity of the element patterns. In addition to identical element patterns the corrected array is characterized by uniform element spacing, which can be chosen to differ significantly from the element spacing in the real array making the corrected array more like a pure virtual array. For the array correction the linear least square error method has been used. To show the applicability of the virtual array, this method is applied to a typical beam scan case, and also over a frequency band.**

***Index Terms*—Adaptive arrays, microstrip antennas, pattern correction, virtual arrays.**

I. Introduction

Mutual coupling and other nonidealities are a challenge in array pattern generation with small arrays. The distortion of element patterns in a small array due to mutual coupling can be partially corrected by tuning the feed voltages [1]. This linear correction is achieved by multiplying the desired feed vector with a correction matrix. A linear least square error (LSE) feed voltage correction method established in [2] was used here. We examine a small linear microstrip antenna array as a potential candidate for an adaptive antenna in mobile communications. A more detailed description of the LSE method in pattern correction is in [3], where the pattern correction for a small microstrip array was examined using different ideal desired element patterns such as those of isotropic elements or an isolated microstrip antenna element. The selection of the desired patterns can take lots of analyzing time. In contrast, in this study the desired element pattern is not fixed before the correction procedure. Instead, the goal is to find the desired element pattern, which gives the best correction results; in the optimal case the final corrected element patterns are as identical as possible to each other. Then the standard pattern synthesis can be used with better accuracy [4]. The spacing of the desired elements is fixed, but not necessarily the same as in the real array. Also the resulted element pattern can differ drastically from the real element pattern. Therefore, the term virtual array is used. The input/output coefficients between virtual and real array are defined with matrix operation. In this work the virtual array generation is tested for different microstrip arrays with different real element spacing. The virtual array generation is applied also over a frequency band and for a beam scan.

This paper is organized as follows: Section II contains the basic theory of pattern correction to the virtual array with iteration. The used arrays and patterns are described shortly in Section III. Section IV concentrates on array pattern correction at certain frequency: first the results are compared with the results of the older method, which uses desired patterns when virtual spacing is the same as the real spacing. Then the effect of the change of virtual spacing in element pattern, fitting accuracy and scan properties is analyzed. In Section V the behavior of the correction is described on a wider frequency band.

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II. Theory

The pattern correction can be presented as

$$\mathbf{F}_{\mathrm{rep}} \approx \mathbf{K} \cdot \mathbf{F}_{\mathrm{meas}} \tag{1}$$

where $\mathbf{F}_{\mathrm{meas}}$ is a matrix of array manifold containing the measured element patterns of an array with N_{el} elements, $\mathbf{K}$ is $N_{\mathrm{el}} \times N_{\mathrm{el}}$ correction matrix and matrix $\mathbf{F}_{\mathrm{rep}}$ contains the corrected/repaired element patterns. As in [3] the pseudoinverse is used as the LSE method for solving the pattern correction. In this work the correction is used without any predetermined information on the wanted array element patterns. Instead, the correction matrix is obtained, which gives element patterns most similar to one another. The resulting optimal element and array patterns can be verified with the absolute value of the pattern correlation defined in [3], [5]. This is closest to unity in the final corrected case.

The following iteration procedure is used for the pattern correction. In the process the desired pattern is a variable and the matrix correction from measured to new desired pattern matrix is defined as

$$\begin{aligned}\mathbf{F}_{\mathrm{rep},K+1} &= \mathbf{K}_{K+1}\mathbf{F}_{\mathrm{meas}} \\ &= \left\{\mathbf{F}_{\mathrm{des},K+1}\mathbf{F}_{\mathrm{meas}}^{H}\left(\mathbf{F}_{\mathrm{meas}}\mathbf{F}_{\mathrm{meas}}^{H}\right)^{-1}\right\}\mathbf{F}_{\mathrm{meas}}\end{aligned} \tag{2}$$

where K is the iteration index and $\mathbf{F}_{\mathrm{des},K+1}$ is the latest desired matrix of element patterns. To define the new desired array pattern matrix $\mathbf{F}_{\mathrm{des},K+1}$ the reference points of the element patterns in the old repaired array pattern matrix $\mathbf{F}_{\mathrm{rep},K}$ are shifted to the array center as if the elements were relocated in the array center

$$G_{\mathrm{rep},K}(d,n,m) = e^{j((2n-1)/2)kd\sin\theta_m}F_{\mathrm{rep},K}(n,m) \tag{3}$$

where n is the element index, m is the index of azimuth angle θ defined with a step of 1°, d is the element spacing, and k is the wave number. Then the mean pattern of these centralized element patterns is calculated over the $N_{\rm el}$ elements in the array

$$\langle G_{{\rm rep},K}\rangle = \frac{1}{N_{\rm el}}\sum_{n=1}^{N_{\rm el}} e^{j((2n-1)/2)kd\sin\theta_m} F_{{\rm rep},K}(n,m) \quad (4)$$

and a new desired array pattern matrix is defined as

$$F_{{\rm des},K+1}(d,n,m) = e^{-j((2n-1)/2)kd\sin\theta_m}\langle G_{{\rm rep},K}\rangle \quad (5)$$

where the same element spacing d as in (3) and (4) is used. In the more general case of a nonuniform linear array d can be a function of the element index n and the correction can be used easily also for planar arrays. The change of the number of elements $N_{\rm el}$ is not allowed as is allowed in [6]. The first desired array pattern matrix can be defined by (5) using the mean of the centralized element patterns of the measured array element patterns, which are first centralized by (3). In this iteration the element spacing in the wanted array should be given and it is not necessarily exactly that of the real array. Later in the paper we use for the computational element spacing d in (3)–(5) the term virtual element spacing with notation $d_{\rm virt}$ and for the real element spacing we use the notation d. One usual case of array calibration is to correct the element positions [7]–[10] and the basic case to use the correction method to a virtual array is to find a simplified linear array with constant spacing, which is not far from the real spacing. The final (latest) correction matrix is used to correct the measured element patterns according to (1).

The pattern correction enhances direction finding and interference canceling, due to more identical element patterns and exact placement of zeros [11], [12]. For an adaptive array the main advantage of pattern correction to simplified and identical patterns is the economy of resources compared with the use of measured patterns with more complicated algorithms and avoidance of errors caused by element pattern perturbation when simple algorithms are used. When using the corrected array as a virtual array in a subsequent adaptive algorithm, only the correction matrix, the virtual element spacing and the latest iterated mean of centralized element patterns need to be stored. And if this mean pattern is smooth enough, it can be approximated with only a few parameters.

III. Used Arrays and Patterns

The examined arrays are the same 6-element microstrip arrays with spacings 0.3λ to 0.93λ that were used in [3]. They have a small-size ground plane, which means that in addition to mutual coupling, the element patterns are perturbed also by the edge diffraction. A microstrip antenna array has no front-back symmetry and thus the region of azimuth angle for calculations using (3)–(5) should not be beyond -90° to 90°, which is the region used in this work. The number of iterations is 50. The arrays are used mainly at 5.2 GHz, when the spacings given for arrays are for 5.3 GHz in this work. The reason is that the patch antennas in the array are equal, not tuned individually, and the mutual coupling mainly decreases the resonant frequencies, which are different in different arrays and different for different element positions. The used patterns are for main polarization and measured in an anechoic chamber with 1° increment.

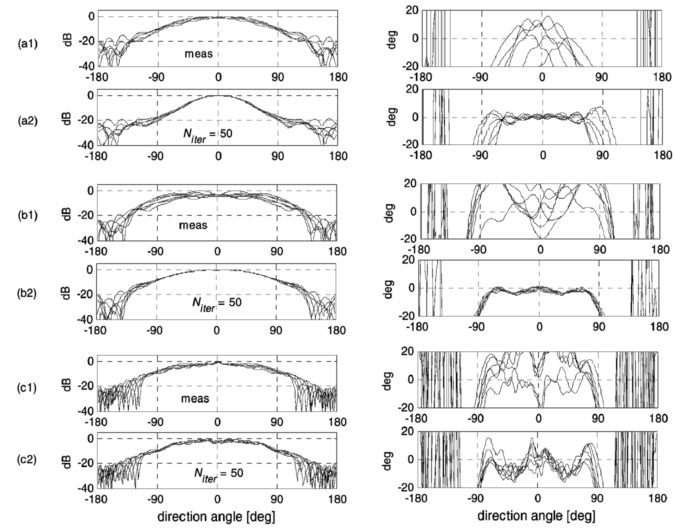

Fig. 1. Different array element patterns at 5.2 GHz. In the left column are element amplitude patterns and in the right column the corresponding phase patterns. In row (a1) are measured and (a2) corrected patterns for the array with element spacing $d = 0.48\lambda$ at 5.3 GHz. In row (b1) are measured and (b2) corrected patterns for the array with element spacing $d = 0.3\lambda$ and in row (c1) are measured and (c2) corrected patterns for the array with element spacing $d = 0.93\lambda$. The correction matrix is defined after 50 iterations.

IV. Virtual Array at a Certain Frequency

A. *General Element Pattern Generation for a Virtual Array With the Same Element Spacing as in the Real Array*

For all the examined arrays the results at 5.2 GHz show that a correction matrix, which gives nearly identical element patterns, can be found. In Fig. 1 we see the measured and the corrected element patterns for arrays with element spacings of about 0.3λ, 0.5λ and 0.93λ. The generated corrected prototype element patterns are different for different arrays. This means that the presented method is not a method to find a general nonperturbed pattern for all arrays with radiating elements of given type and with different element spacings in the real array. For the array with element spacing of 0.5λ the virtual element pattern relative gain in the endfire direction is decreased causing decreased element beam width. This demonstrates that the ideal element pattern is not well predicted with this method, which is a cost of higher similarity of element patterns.

We see in Fig. 1 also that the element patterns are random when the direction angle (azimuth) is more than 140° for the arrays with element spacing of 0.3λ and 0.5λ, and when the direction angle is more than 120° for the array with element spacing 0.93λ. This means that the effect of correction calculated in region from -90° to 90° affects also beyond this region. When the first desired element patterns were varied (instead of the mean pattern an isotropic, microstrip model, or sinusoidal pattern was used at first), this change in the initial pattern had no effect on the final iterated correction matrix, which indicates that the iteration procedure is stable for simple element patterns.

The correlation of one element pattern with the mean element pattern is presented in Table I for all elements in examined arrays. The element pattern is for an element relocated with the corresponding pattern phase changes in the array center. In this case the real element spacing d is used as virtual spacing and the

TABLE I
ABSOLUTE VALUE OF THE ELEMENT PATTERN CORRELATIONS WITH THE MEAN ELEMENT PATTERN IN THE EXAMINED ARRAYS BEFORE AND AFTER CORRECTION

d/λ at 5.3 GHz	Structure	Before/ After (b/a)	el. 1	el. 2	el. 3	el. 4	el. 5	el. 6	Mean of Corre-lations
0.30	1B), 2A)	b	0.9573	0.9391	0.9630	0.9637	0.9305	0.9520	0.9509
		a	0.9997	0.9998	0.9998	0.9998	0.9998	0.9997	0.9998
0.40	1B), 2A)	b	0.9741	0.9603	0.9541	0.9578	0.9649	0.9831	0.9657
		a	0.9989	0.9991	0.9996	0.9996	0.9991	0.9989	0.9992
0.48	1B), 2A)	b	0.9268	0.9276	0.9326	0.9376	0.9332	0.9352	0.9322
		a	0.9984	0.9991	0.9992	0.9992	0.9992	0.9984	0.9989
0.48	1A), 2A)	b	0.9812	0.9782	0.9777	0.9791	0.9822	0.9849	0.9806
		a	0.9924	0.9963	0.9979	0.9980	0.9960	0.9909	0.9953
0.50	1A), 2B)	b	0.9869	0.9759	0.9753	0.9758	0.9749	0.9836	0.9787
		a	0.9985	0.9996	0.9996	0.9996	0.9996	0.9985	0.9992
0.70	1B), 2A), 3)	b	0.9812	0.9782	0.9777	0.9791	0.9822	0.9849	0.9806
		a	0.9924	0.9963	0.9979	0.9980	0.9960	0.9909	0.9953
0.93	1B), 2A), 4)	b	0.9809	0.9757	0.9715	0.9741	0.9795	0.9811	0.9771
		a	0.9947	0.9957	0.9960	0.9968	0.9956	0.9901	0.9948

[1A)] usual periodical array, [1B)] antenna array with symmetrized input ports
[2A)] vertically stacked elements, [2B)] horizontally stacked elements
[3)], [4)] the substrate plate is different (larger) than for other arrays

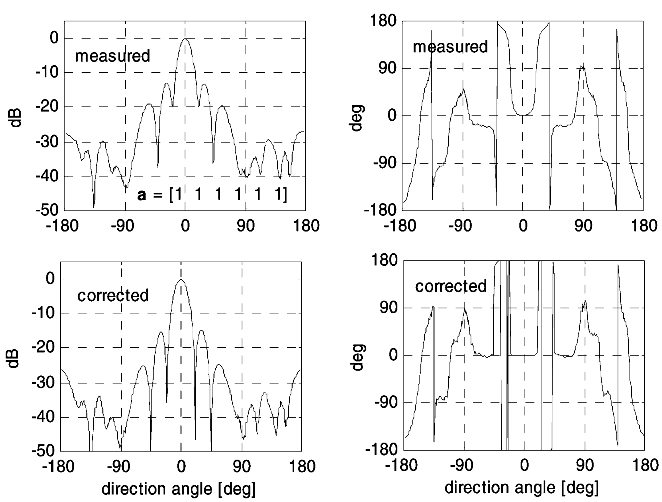

Fig. 2. Array pattern for array with element spacing about $d = 0.48\lambda$ presented for measured and corrected element patterns at 5.2 GHz. On the left are amplitude patterns and on the right the corresponding phase patterns. The used input vector is $\mathbf{a} = [1\ 1\ 1\ 1\ 1\ 1]$.

reconstructed element location is in the center of the real array. In Table I the element for which the pattern differs most from the mean pattern can have any position. In many cases the center element differs most from the mean element before correction, while after the correction the pattern of the edge element has the lowest correlation with the mean element pattern.

The effectiveness of the obtained correction can be characterized also for array patterns as was done in [3]. In Fig. 2 we see the array amplitude and phase patterns for the corrected and uncorrected array with element spacing $d = 0.48\lambda$ in the case when the input vector is $\mathbf{a} = [1\ 1\ 1\ 1\ 1\ 1]$. In the uncorrected case the element patterns are the measured ones and they are used without any input vector correction as if they were identical in amplitude and phase. The phase reference point for each antenna element is at its geometrical position. The result shows, that the correction works well: the array pattern is more regular and has pronounced deep zeros after the correction. The correlation of the resulted array pattern with the final ideal desired array pattern with identical element patterns is 0.9998 in region $-90°$ to $90°$ and 0.9978 in region $-180°$ to $180°$. These correction results are better than those obtained for this array in [3], where the desired element patterns were fixed before the correction and where the corresponding correlations with the desired array pattern were 0.9822 and 0.9794, for the angular regions from $-90°$ to $90°$ and from $-180°$ to $180°$, respectively.

B. Effect of Virtual Array Spacing

A virtual array with lower element spacing can be utilized, if in (3)–(5) the element spacing is less than the real spacing. In Fig. 3 we see correction examples for an array with real element spacing $d = 0.48\lambda$ with different virtual spacing of 100%, 60%, and 0% of the real spacing. We see, that the element pattern becomes more directed when the spacing in the virtual array decreases. When the elements are virtually in the same point, i.e., $d_{\mathrm{virt}} = 0$, the virtual element patterns are exactly identical combinations of the real element patterns with weights $[1\ 1\ 1\ 1\ 1\ 1]$. In this trivial case the adaptivity of the virtual array is totally lost and one array pattern is generated and it cannot be scanned. This extreme case demonstrates well, that the general adaptivity and resolution of the virtual array decreases with decreasing virtual spacing. At the same time the array correction becomes better in the forward direction. The complex correlations of patterns for elements moved to the center of the virtual array are before and after correction 0.9322 and 0.9989 for $d_{\mathrm{virt}} = 0.48\lambda$, 0.588 and 0.9999 for array with $d_{\mathrm{virt}} = 0.29\lambda$, and 0.404 and 1.0000 for $d_{\mathrm{virt}} = 0.00\lambda$.

In some directions the differences between element patterns are increased compared to other directions, which results to inaccuracies in generated array patterns in these directions. From Fig. 3(b2) we can see, that array patterns generated for an array with $d_{\mathrm{virt}} = 0.3\lambda$ are well predicted in the directions $-45°$ to $45°$ but not well predicted in the directions of azimuth angles $|\theta| > 45°$ due to increased amplitude and phase randomness of the virtual element patterns.

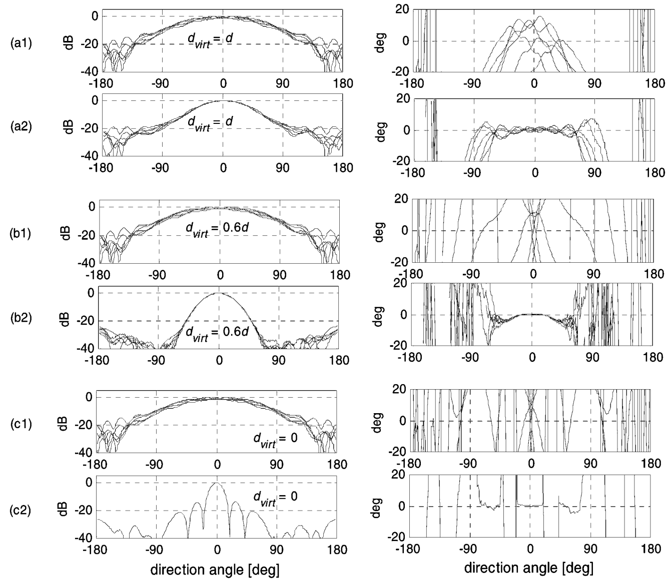

Fig. 3. Array element patterns of centralized elements generated using array with element spacing $d = 0.48\lambda$ at 5.3 GHz, corrected to different virtual spacing of about 0.5λ, 0.3λ and 0.0λ. In row (a1) are uncorrected and (a2) corrected array element patterns with $d_{\mathrm{virt}} = d$. In row (b1) are uncorrected and (b2) corrected array element patterns with $d_{\mathrm{virt}} = 0.6d$. In row (c1) are uncorrected and (c2) corrected array element patterns with $d_{\mathrm{virt}} = 0$. On the left are the element amplitude patterns and on the right the element phase patterns of the centralized elements.

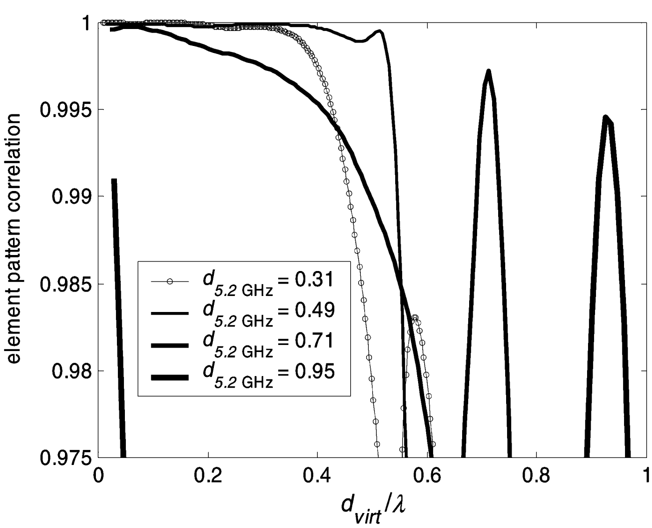

Fig. 4. Obtained mean of element pattern correlations for different arrays with different virtual element spacings. The real spacings at 5.2 GHz are given in the legend.

In Fig. 4 the dependence of the mean corrected element pattern correlation with the iterated element pattern on virtual array spacing is presented for some arrays with different real element spacing. For all arrays with the virtual spacing equal to the real spacing, the correction gives good correlation values. The arrays with real spacing of 0.3λ, 0.5λ and 0.7λ can be used for the virtual array for spacing from 0 to 0.5λ with correlation > 0.980 after correction. The array with $d = 0.9\lambda$ can be used only for virtual distances close to its real spacing (or close to zero). It was noticed in [3] that a desired array with slightly decreased element spacing results better correlation, when an isotropic array is used as a desired array. Also in algorithms a

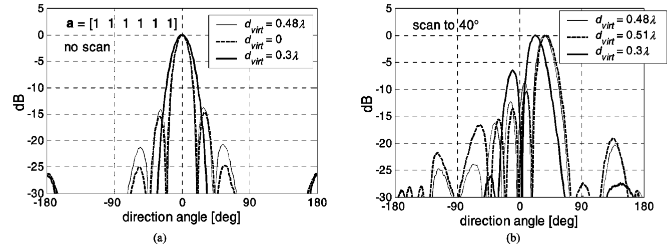

Fig. 5. Array patterns generated from array with spacing $d = 0.48\lambda$ with different virtual spacing d_{virt} in (a) without scan with virtual element weights [1 1 1 1 1 1] and in (b) where the beam with virtual element weights [1 1 1 1 1 1] for forward direction are scanned to 40° by direction angle of the array factor of the virtual array. Virtual spacing is given in the legends in each figure.

slightly increased spacing can give better results [13]. Here the best correction result for virtual array spacing close to the real spacing 0.48λ is obtained with slightly increased virtual array spacing 105% of real spacing and gives correlation 0.9996 vs. 0.9990 for the 100% spacing. The increase of correlation with slightly increased virtual spacing is detected in this work also for two other arrays with real spacing close to 0.5λ.

The beam scan was verified for different virtual spacings for an array with a real spacing of 0.48λ. In Fig. 5(a) the array patterns with a beam in the forward direction for virtual arrays with different spacings are presented. In Fig. 5(b) some beams with different virtual arrays are scanned over a range of 40°, when the scan angle 40° is defined as the array factor $e^{j((2N-1)/2)kd_{\mathrm{virt}}\sin\theta_m} + e^{j((2N-3)/2)kd_{\mathrm{virt}}\sin\theta_m} + \cdots$ direction angle shift θ_m. We see that for the array with $d = 0.5\lambda$ the beam maximum shift of 40° is not reached using the virtual array with $d_{\mathrm{virt}} = 0.3\lambda$. This is due to the fact, that the element pattern is more directive than in the array with $d = 0.5\lambda$, and thus the beam maximum is shifted to the array center. For algorithmic simplicity it would be better, that the virtual array beam maximum shifts by the scan angle defined with progressive input coefficients, which is a property of isotropic elements. However, the approximate direction of the scanned beam can be found simply knowing the angle respective to the progressive input. The corrected scan angle, which gives the same beam maximum shift as for the real array with true element spacing can be calculated approximately as

$$\theta_{\mathrm{corr}} = \arcsin\left(\frac{d\sin(\theta_{\mathrm{scan}})}{d_{\mathrm{virt}}}\right). \tag{6}$$

The effect of the element pattern on the array pattern is well known and is taken into account in some standard algorithms [14], [15]. The only element pattern without a question of element pattern form effect on beam position is the isotropic one. With a virtual spacing of $d_{\mathrm{virt}} = 107\%$ the element patterns agree well with each other and the peak shift agrees well with the scan angle. The best agreement between the beam maximum shift and the scan angle can be reached with a virtual spacing of 0.56λ (117%), but then the element pattern correlation after the correction is only 0.975, but is better than the value 0.932 for the uncorrected array. The fact that we can increase the virtual distance without loosing accuracy is due to the substrate, which affects the element radiation. The array with d of 0.3λ is on a substrate plate with the same dimensions as the array with spacing $d = 0.5\lambda$ and this allows increase the virtual spacing significantly. For an array with $d = 0.3\lambda$ the mean element pattern correlation of 0.982 for the corresponding virtual array with $d_{\rm virt} = 0.5\lambda$ (see Fig. 4) is higher than for noncorrected arrays with real spacing of about $d = 0.5\lambda$ in Table I. The corresponding virtual element pattern differs from that presented in Fig. 1(b2); it is directed more to the sides having a dual-peak shape (not shown), which might be useful for increasing the resolution of direction of arrival (DOA) algorithms in off-broadside directions.

For the real array with $d = 0.5\lambda$ the array patterns generated with input coefficients $[1\ 1\ 1\ 1\ 1\ 1]$ for different virtual arrays with virtual spacing $d_{\rm virt} = 0.5\lambda$ and $d_{\rm virt} = 0.3\lambda$ are less different than one would expect. This means, that the changes in the array factor of the virtual array are partly compensated with the change of the virtual array element pattern width.

The array mutual coupling compensation/array calibration has been noticed to increase the signal to noise ratio SNR and signal to interference ratio SINR [11], [12], [16], [17]. If the pattern correction matrix compensates also mutual coupling, then the SNR should increase with pattern correction. However, the correction to open/shorted condition of antenna elements, used in [16] and [18], is not a good method in pattern correction for microstrip arrays [3]. In the case of low virtual spacing the element patterns are close to each other. This causes in general increased signal and noise correlation and if a noise/signal covariance matrix is used to increase SNR or SINR [16], [19], it should compensate higher correlation and thus becomes more different from a diagonal matrix and might be not accurately inverted. On the other hand, the weight vectors of virtual array elements with close element spacing are all near to $[1\ 1\ 1\ 1\ 1\ 1]$, which means that if the noise is uncorrelated in the real array, then the SNR's in the virtual array branches are higher and can compensate the effect of the loss of adaptivity (more static beams, highly correlated signal and noise). Thus it is not obvious, what is the effect of low virtual spacing to the array performance, when a signal is arriving from the forward direction, where the virtual array element beams are more directed. All the elements of the real array are weighted similarly in the correction to the virtual array and thus there is no problem with unbalance between the real array elements, if the real elements are not very different. However, a general rule is that when the element spacing in virtual array is decreasing, the correction matrix differs more and more from a diagonal one and problems can arise for the adaptive use of the virtual array.

V. Using the Virtual Array for a Wider Frequency Band

In the previous chapters, single frequency was considered for the correction. However, the arrays are usually used on a wider frequency band. Thus the correction needs to be characterized for a frequency band as was done in [3]. In Fig. 6 we see the correction results for all array element patterns corrected separately at each frequency for an array with element spacing $d = 0.48\lambda$. The corresponding real element spacing is used for the virtual array spacing at each frequency. It can be seen, that the element pattern width is decreasing with increasing frequency, and that the corrected element pattern is narrower than the uncorrected element pattern.

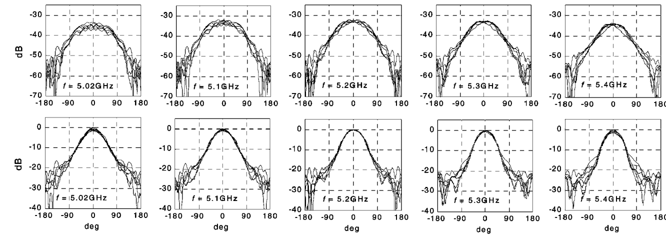

Fig. 6. Uncorrected and corrected array element patterns of the array with element spacing $d = 0.48\lambda$. The frequency is shown in each figure. The correction with 50 iterations is done individually on each frequency using the corresponding real spacing as the virtual spacing. On top are measured and at bottom corrected element patterns.

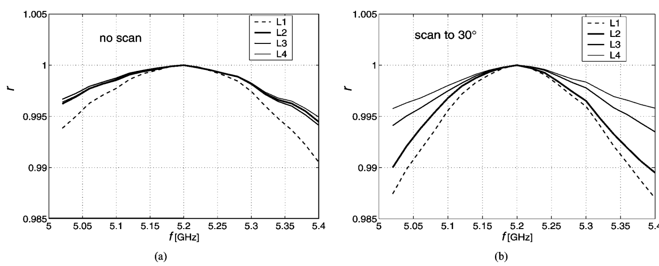

Fig. 7. Frequency dependency of the correlation of the array pattern with the array pattern of center-frequency 5.2 GHz is presented for different array patterns. The array patterns are generated using input weights $[1\ 1\ 1\ 1\ 1\ 1]$. In (a) the beam is in forward direction and in (b) the beam is scanned to 30 degrees. The dashed line (L1) is for the uncorrected, measured array. The thick solid line (L2) is for the correction using the correction matrix of the center frequency for all frequencies. For solid line (L3) the correction is done independently on each frequency using constant physical element spacing (constant spacing in millimeters) and in the case of thin solid line (L4) using frequency-independent phase distance (constant spacing in wavelengths).

According to [3] one important goal for array pattern correction is to obtain a frequency-independent array pattern, i.e., to obtain the same pattern with the same array input weights over the whole frequency range of interest. In a real application it would not be practical to do the correction at each frequency separately. Therefore, the correction using one correction matrix over the frequency band is compared with the correction done independently on each frequency. For different correction cases the correlation of the array patterns with the corresponding array pattern at the center frequency 5.2 GHz is presented in Fig. 7. The array patterns are obtained with the element weights $[1\ 1\ 1\ 1\ 1\ 1]$.

We see in Fig. 7(a) that with the correction of the array element patterns the array pattern correlation with corresponding center frequency array pattern increases. The correction using only the center frequency correction matrix gives as high correlation on the frequency bands as the individual correction at each frequency. The individual correction at each frequency is done in one case with a constant physical element spacing d and in other case with a constant phase distance d/λ. In all cases with correction the correlation is similar.

In Fig. 7(b) the array is scanned to 30° and we see, that also in this case the correction gives a set of array element patterns closer to each other than is in the case without correction. When using the same correction matrix for all frequencies the correlation with the center frequency array pattern is lower with a scan than in the case without a scan. With a scan it is also lower than in the cases of frequency-dependent correction matrix, for which the largest correlation with the center frequency array pattern is

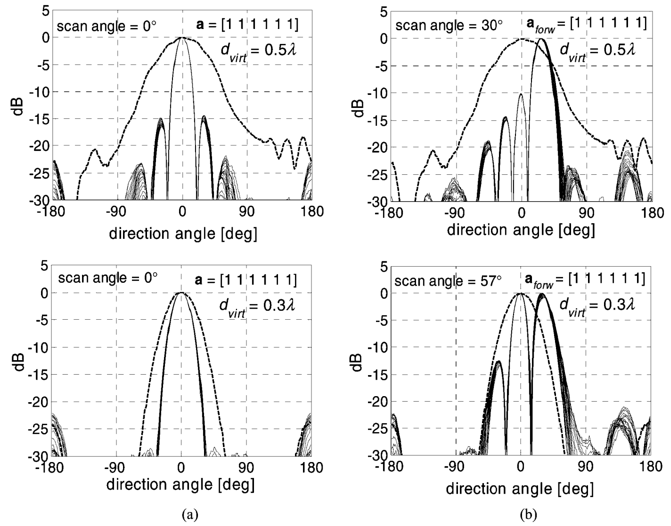

Fig. 8. Corrected array patterns with input [1 1 1 1 1 1] of forward beam in (a) for an array with virtual spacing of 100% (top) and 60% (down) without scan and in (b) for array with virtual spacing of 100% (top) and 60% (down) with scan of 30° and 57°, respectively. Scan angle is the array factor direction angle shift. The element distance in physical array is $d = 0.5\lambda$. Array patterns are presented at all examined frequencies. The correction is done using center-frequency correction matrix. The dashed line is the corresponding virtual element pattern.

obtained with constant d/λ and it is about the same as for case when the beam is in forward direction. The result means, that for the given array the corrected array patterns are more regular due to more identical element patterns and at the same time less dependent on the frequency compared with the uncorrected array.

Better frequency-independence than when using a constant $d_{\mathrm{virt}}/\lambda$ can be obtained if the array element patterns are corrected separately for each element to an iterated mean element pattern on the frequency band, but this leads to different element patterns, which is typically not desired in an array processing algorithm. The frequency dependence of the array pattern increases when the virtual spacing increases beyond the real spacing region.

In Fig. 8 the frequency dependence of the array pattern of the corrected array is presented for two virtual spacings ($d_{\mathrm{virt}} = 0.3\lambda$ and $d_{\mathrm{virt}} = 0.5\lambda$) of an array with real spacing $d = 0.5\lambda$ with and without a scan. The scan angle for the virtual array with virtual spacing $d_{\mathrm{virt}} = 0.3\lambda$ is determined with the help of (6) compensating the directivity growth/beam width increase effect, i.e we get about the same beam maximum shift as for $d_{\mathrm{virt}} = 0.5\lambda$. We can see that the frequency dependence increases with increasing shift from forward direction. When the main beam is shifted about 30° we see, that for the virtual array with $d_{\mathrm{virt}} = 0.3\lambda$ the second beam increases. The absence of the grating lobe is an advantage of lower element spacing, but here the other grown beams are problematic. This demonstrates a case of a wide-angle scan when there is no advantage in using of a virtual array.

VI. Conclusion

It was shown that the virtual array is a useful tool in the generation of a regular array manifold for a small linear array. In most of the presented cases the generated virtual element patterns are very similar to one another, which is a requirement for the standard pattern synthesis and for many DSP algorithms [1], [4]. As opposed to what was expected in the case of a wide spread of angles of arrival, a decrease in virtual element spacing did not improve the scanning properties of the array. When using a constant correction matrix over a frequency band, only a moderate array-pattern correction could be obtained.

From the presented real array with element spacing $d = 0.5\lambda$ we can easily generate a virtual array with element spacing $d_{\mathrm{virt}} = 0.3\lambda$ with good accuracy, if we allow the element pattern width to decrease. This means, that the clearly closer real spacing d is necessary only when real array with wider beam is needed. When a compact virtual array with narrower element beams can be used the presented method is attractive. One benefit of the virtual array with lowered element spacing is that the mutual coupling can be neglected, if the spacing of the corresponding real array is great enough. On the other hand, the examined array with spacing $d = 0.3\lambda$ can be used with good accuracy as a virtual array even with increased spacing of 0.5λ. Also the array with real spacing 0.5λ can be used with good accuracy with slightly increased spacing of the virtual array. This indicate, that the presented method of virtual array can be used for small arrays mounted on different devices, causing a greater effective ground plate, to increase the practical, computational resolution in addition to the good level of the pattern correction.

The linear correction method using input/output coefficient multiplication with a correction matrix does not change the manifold of different array patterns that can be generated with a real array. The advantage in the use of virtual arrays is algorithmic simplicity, predicted by the demonstrated results especially for virtual arrays with element spacing not far from the real spacing. The form and the accuracy of the generated virtual element pattern indicate well, for which different purposes it can be used. Any reached element pattern is always optimal for a certain use. So, for a real array different virtual arrays could be stored (storing corresponding correction matrix, element spacing, and simplified element pattern for each virtual array) and used at the same time for different purposes. As we have seen, in many cases the correction result is very good and thus the virtual array method presented here can be one alternative in linear pattern correction presented in [1]–[4] which is an alternative for more accurate and complicated array design done, for example, in [20]. In the future it will be of interest to prove this method for an array, e.g., conformal array, which is mounted on some device in such a way that the element patterns are more degenerated than for the small arrays examined so far.

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