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# BEAMFORMING WITH WIDE NULL SECTORS FOR REALISTIC ARRAYS USING DIRECTIONAL WEIGHTING

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## Abstract

Mutual coupling and other non-idealities are a typical source of problems in small arrays. The element patterns are perturbed, which causes difficulties in standard array pattern synthesis. One common way to avoid the problems is to correct the element patterns computationally before array pattern generation. However, a complex-valued array pattern can be generated also directly without the use of a set of corrected complex-valued element patterns. When the desired array pattern is an amplitude pattern with an arbitrary phase pattern, an iterative process is needed to find the input coefficients. Here it is demonstrated how to generate simple box-type amplitude patterns for a six-element microstrip array. The main focus is in wide null generation and in the comparison of the null region weighting in pattern generation with the standard procedure without weighting. The results show, that the used direction-dependent weighting in array amplitude pattern generation gives generally better agreement of the resulting amplitude pattern with the desired pattern in the dB scale.

## Contents

# 1 Introduction

Inside an antenna array, mutual coupling and other non-idealities cause element pattern distortion which is problematic in small arrays [1], [2]. The element patterns can be corrected partially with a matrix operation after which the array can be used more realistically as an ideal array. The matrix correction of element patterns is defined for complex-valued element pattern vectors, and the corrected inputs (or outputs) are defined multiplying the uncorrected inputs with the correction matrix. The linear least square error (LSE) solution for array correction can be found using the matrix pseudoinverse, which is widely used in practical array problems [2]–[5]. However, for any physically realized array, the feed coefficients that give the best fit with a wanted complex array pattern can also be found without a preceding element pattern correction to a desired array [6]. In this work the weighting function depends on the wanted array pattern and the resulting correction matrix is different for each case of wanted array patterns. Thus the correction matrix itself is not in the focus of interest. In the cases considered in this paper, only the wanted array amplitude/power pattern is predetermined when the array phase pattern is arbitrary, and an iterative process needs to be performed to find the optimal array input/feed coefficients [7], [8].

In mobile communications the generation of nulls in the array pattern is important [9]–[13], [14]. The nulls can be narrow or wide nulls. The directions of desired nulls can be emphasized, which is the proposed method in this work. It causes the array pattern outside the emphasized region to agree less with the wanted pattern. In null-pronounced weighting the pattern amplitudes of the wanted null directions are multiplied with a pronouncing weighting factor or cost function. In the robust weighting presented here, all the amplitude values of the measured array element pattern vectors and also the desired array pattern vector itself are multiplied component-wise with the inverse values of the desired array amplitude pattern before the array input vectors are calculated. This weighting used with iteration gives a criterion similar to LSE on the dB scale. The effect of weighting is examined in this paper from a practical point of view, by characterizing the fitting accuracy of wanted and generated array amplitude patterns in the dB scale.

When the number of users exceeds the element number in a base station array, simplified array patterns become more useful. The classic method to optimize signal to noise and interference ratio [15] needs updated information on noise and interference and can be not optimal when extended to downlink direction and take too match resources to be used in mobile communications. In mobile communications the smart sectorization is a challenge for future networks [16]. Wide angular regions with lowered radiation level (nulls) are generated for directions, where interferers can be located. The reason of uncertainty of the interferer localization can be the rapid movement [17]–[20]. Another reason for wide nulling can be the nulling on the frequency band [21]. The simple patterns having regions with different constant gains can be useful also when there are groups of users at different distances from the base station or with different service levels. In this paper, the pattern generation is tested using box-type array patterns for a six-element array. Mainly the difference between pattern generation with and without weighting is compared. However, the presented data can be used at the same time to characterize what is the practical accuracy for box-type array pattern generation with a real six-element array. There is a growing interest to small adaptive arrays in mobile radio communications but the literature concerning pattern synthesis is still mainly available for large arrays used in static radio links, where the accuracy requirements and resources to reach them are much higher than in mobile communications.

## 2 Theory

When the array element patterns and the complex-valued desired array pattern are known, the input coefficients can be calculated using matrix inversion [22]. When the number of elements is less than the number of pattern measurement points, the array patterns can be generated to match the desired ones using the matrix pseudoinverse, which is widely used due to simplicity and because it gives the best agreement in terms of the LSE criterion [2], [3], [5]. The array element weights or input coefficients $\vec{\mathbf{a}}_{opt}$ can be calculated accordingly [6] with

$$\mathbf{a}_{opt}^{T} = \vec{\mathbf{\Psi}}_{desired} \left\{ \mathbf{F}_{meas}^{H} \left[ \mathbf{F}_{meas} \mathbf{F}_{meas}^{H} \right]^{-1} \right\}, \qquad (1)$$

where the complex vector of wanted array field pattern $\vec{\mathbf{\Psi}}_{desired}$ is multiplied by the pseudoinverse $\mathbf{F}_{meas}^{H} \left[ \mathbf{F}_{meas} \mathbf{F}_{meas}^{H} \right]^{-1}$ of the matrix $\mathbf{F}_{meas}$ containing all the measured or computed element patterns of the array. In this work matrix $\mathbf{F}_{meas}$ is a complex value $N \times M$ matrix of $N = 6$ element patterns and with $M = 359$ observation directions. When weighting is used in the input coefficient calculations, all the single patterns in the matrices or vectors are multiplied component-wise with a weighting vector $\vec{\mathbf{\Psi}}_{w}$ having the length equal to that of the element pattern, yielding

$$\mathbf{a}_{opt}^{T} = \left( \vec{\mathbf{\Psi}}_{w} \otimes \vec{\mathbf{\Psi}}_{desired} \right) \left\{ \left( \mathbf{F}_{w} \otimes \mathbf{F}_{meas} \right)^{H} \left[ \left( \mathbf{F}_{w} \otimes \mathbf{F}_{meas} \right) \left( \mathbf{F}_{w} \otimes \mathbf{F}_{meas} \right)^{H} \right]^{-1} \right\}, \qquad (2)$$

where the rows of the weighting matrix $\mathbf{F}_{w}$ are all equal to the pattern weighting vector $\vec{\mathbf{\Psi}}_{w}$ and $\otimes$ denotes component-wise multiplication. With this kind of weighting when all the responses in a given direction are multiplied with the same factor, the exact solution, if it exists, does not change. Usually, the solution of (2) is a LSE solution and when it is done with weighting for $\mathbf{F}_{meas}$ and $\vec{\mathbf{\Psi}}_{w}$, the final error in each direction is a statistical LSE error divided by the corresponding weighting factor. This means, that in some directions the accuracy is increased or lowered with weighting.

In many cases a solution with a small relative error is wanted. If the amplitudes in the matrix $\vec{\mathbf{\Psi}}_{desired}$ in (1) are equal, then the complex LSE error is the relative error for a complex valued $\vec{\mathbf{\Psi}}_{desired}$. With weighting vector inverted to amplitudes of $\vec{\mathbf{\Psi}}_{desired}$ in (2) we can have equal amplitudes. In this work the nulls of the wanted array pattern are pronounced with the weighting vector

$$\vec{\mathbf{\Psi}}_w(\theta_i) = \frac{1}{\Delta + \left|\vec{\mathbf{\Psi}}_{desired}(\theta_i)\right|^{N_W}}, \tag{3}$$

where $\vec{\mathbf{\Psi}}_{desired}(\theta)$ is the wanted array pattern, $N_w$ is a weighting exponent, and $\Delta$ is a positive number, which defines the upper limit of weighting. If $N_w = 0$, then there is no weighting and the pattern matching is the usual LSE presented in (1). If $N_w = 1$, then the LSE correction done for weighted patterns decreases the linear error in the low level region and minimizes the mean relative error over the whole calculation region. Mainly the values $N_w = 0$ and $N_w = 1$ are used here in the comparison, but also the cases $N_w < 0$, $0 < N_w < 1$ and $N_w > 1$ are presented briefly. When the desired array pattern contains zero values the parameter $\Delta$ is necessary to avoid division by zero. The value of $\Delta$ should not be chosen below the noise/uncertainty level to avoid the sensitivity of the pattern generation on pattern measurement errors or noise. The desired array patterns used here have "zeros" with a given dB value of radiation intensity; they are not exactly zeros and thus the parameter $\Delta$ is chosen $\Delta = 0$ if not mentioned otherwise. Here is demonstrated the array pattern generation without any noise added, which also allows this parameter to equal zero.

In (1) the pseudoinverse matrix is the same for different desired array patterns and can be stored. In (2) the calculation should be repeated for each desired array pattern and thus the element patterns should be stored. Equation (2) differs mainly by notation from the one presented in [7], where a diagonal matrix for the power weighting is used instead of component-wise multiplication, and where the real and imaginary components of patterns are separated and weighted independently. When the general reference phase for an array is arbitrary, there is no need for different weighting vectors for real and imaginary parts. The phase information of the weighting vector in (2) has no effect and the weighting vector can be given as an amplitude vector. The form in (2) helps to see, that the solution for the case with weighting remains a basic LSE solution like (1), but for modified pattern vectors, which is not seen as well in the reduced form used in [23].

The presented Equations (1) - (3) are for complex-valued patterns. In the case of a desired amplitude-only array pattern, the phase is simply allowed to vary freely during the iteration to ensure best matching of desired and generated array amplitude patterns. Here, in the iteration process the first desired complex array pattern is the desired box-type amplitude pattern combined with a phase pattern of zeros. The use of simplified desired array pattern of this kind leads into computational efficiency [24]. In the following iteration the desired array phase pattern at each iteration step is set to be the obtained array phase pattern of the previous iteration cycle

$$\arg(\vec{\mathbf{\Psi}}_{des,K}) = \arg(\vec{\mathbf{a}}^{\mathrm{T}}_{opt,K-1}\mathbf{F}_{meas}), \tag{4}$$

where $K$ is the number of the current iteration cycle. The idea to accept the reached phase pattern as the next desired phase pattern in iteration process was used earlier in [8], where also the practical question of the possible convergence to a local error minimum, which can lead to a poor iteration result, is mentioned.

The criterion used to characterize the validity of the generated array amplitude pattern could be simply the mean square error, *mse*, between the wanted and generated array amplitude patterns, because it is in accordance with the used pseudoinverse LSE method. The preliminary results, however, showed that this criterion is not always suitable. The wanted array amplitude patterns are in this work box-type patterns with fixed high and low radiation intensity levels at certain angular regions. Therefore, it was decided to use the quadratic sum of *mse*'s from the corresponding amplitude levels as the validity criterion. This joint error (*JE*) is calculated by

$$JE = \sqrt{mse_{high}^2 + mse_{low}^2} = \sqrt{\left( \frac{1}{M_{high}} \sum_{m_h=1}^{M_{high}} \left| e_{high,m_h}^2 \right| + \frac{1}{M_{low}} \sum_{m_l=1}^{M_{low}} \left| e_{low,m_l}^2 \right| \right)} , \qquad (5)$$
$$e_{high\ (or\ low)}(\theta_m) = \left\{ \vec{\mathbf{a}}^{\mathrm{T}} \mathbf{F}_{meas} \right\}_m - \vec{\mathbf{\Psi}}_{desired}(\theta_m), \quad m \in \{\text{"high" (or "low")}\}$$

where $M_{high}$ and $M_{low}$ are the number of measurement points in high and low radiation intensity levels. When the error is defined to characterize the dB-scale fitting of curves, the corresponding error $e_{high\ (or\ low)}(\theta_m)$ is the difference between the dB-scale values for azimuth direction $\theta_m$. When the obtained and desired amplitude patterns are compared with (5) they need to have the same mean value. On linear and dB scales we get different equations defining the rescaling factors $r_{lin}$ and $r_{dB}$

$$\text{linear scale:} \quad r_{lin} \sum_{m=1}^{M} \left| \left\{ \mathbf{a}_{opt}^{T} \mathbf{F}_{meas} \right\}_m \right| = \sum_{m=1}^{M} \left| \vec{\mathbf{\Psi}}_{desired}(\theta_m) \right| \qquad (6)$$

$$\text{dB scale:} \quad r_{dB} M / 20 + \sum_{m=1}^{M} \log_{10} \left| \left\{ \mathbf{a}_{opt}^{T} \mathbf{F}_{meas} \right\}_m \right| = \sum_{m=1}^{M} \log_{10} \left| \vec{\mathbf{\Psi}}_{desired}(\theta_m) \right| , \qquad (7)$$

where index $m$ is the order number of the direction angle and $\mathbf{a}_{opt}$ is the iterated input vector which should be rescaled for *mse* calculation. On the linear scale in the case without weighting the resulting array pattern and the wanted array pattern have automatically the same mean value as a property of the pseudoinverse but in other cases (weighting or logarithmic scale) the rescaling is needed before error calculation. For linear scale comparison the scaling factor $r_{lin}$ is a multiplication factor. When the patterns are compared with (5) on the scale of relative shift, i.e. on the dB scale the resulting input vector is normalized with a shift $r_{dB}$ in order to have the same mean dB-value for the desired and obtained array patterns. In the following the joint errors calculated by (5) are denoted by *JE* and *JEdB* for linear scale and dB-scale errors, respectively. The pattern correlation without mean extraction used in [4], [5] could be used also here to characterize the amplitude pattern in the case of linear scale, but not in the case of dB-scale, where power scaling is defined as a shift of the curve. For simplicity the MSE criterion is used for both, linear and dB-scale, when the resulting array pattern is validated. Also the sum of correlations calculated independently for different levels would not be as meaningful as the final fitting criterion in (5) calculated as joint two-level *mse*.

The final joint two-level MSE validity criterion in (5) is not to be confused with the LSE criterion for complex-valued vectors included in the pseudoinverse method. When the pseudoinverse is combined with (4) in the iteration process we can expect this to lead to the LSE criterion for amplitude vectors. The joint two-level MSE is used after the pattern synthesis to validate the resulting pattern. It is a counterpart for sharp beam or null region underweighting in the calculations; the result pattern can be optimal with respect to the mean squared error calculated for the whole amplitude pattern, but this optimum does not always have practical meaning and therefore some additional characterization as (5) is needed. Equation (5) does not basically depend on the number of sampling points at the high and low radiation levels and thus lowers the underweighting of a narrow sector in calculation. Because the final joint two-level MSE validity criterion is not the criterion included in the step of the iteration completed by (2) and (3), it is not obvious, that the result is good enough in the terms of this final validity criterion calculated by (5). Whether weighting is the better method or not with the chosen validity criterion is to be clarified within this work.

The pure statistical LSE criterion as used in the array pattern synthesis in [25] is a general criterion useful for array patterns that contain many details. The minimax criterion used in [26] is closer to the traditional standard pattern synthesis with minimized sidelobe level for a given half power beam width, where the radiation intensity values between high and low intensity levels should be mainly “forbidden”. The final two-level criterion in (5) is a compromise between the pure LSE criterion and the minimax criterion and it is practical also when an angular sector with lowered radiation power is defined for other reasons than just to obtain as low level as possible. It should be noted for completeness, that instead of the square sum of deviations in (5) also the stronger criterion of absolute sums of deviations could be used as well as a measure of level separations.

## 3 Results

The array used in this study is a small linear 5.2 GHz microstrip array with six elements described in more details in [5]. The element spacing is $0.5\lambda$. The ground plane dimensions are $0.9\lambda \times 3.3\lambda$. The array pattern optimization is done for pattern values at azimuth angles from –90° to 90° at 0° elevation. Array patterns in the horizontal plane were measured with an increment of one degree in the main polarization.

### 3.1 Basic example of array pattern generation and pattern weighting

Box-type array patterns were generated both for wide nulls and for wide beams. In Fig. 1(a) an example for a wide null in forward direction with a null depth *ND* = –40dB and a null width *NW* = 60° is presented. In Fig. 1(b) the generation of a beam in the forward direction with a beam width *BW* = 60° and a side lobe level *SLL* = –40dB is presented. In subfigures (a) and (b) of Fig. 1 the patterns presented on dB scale are normalized to

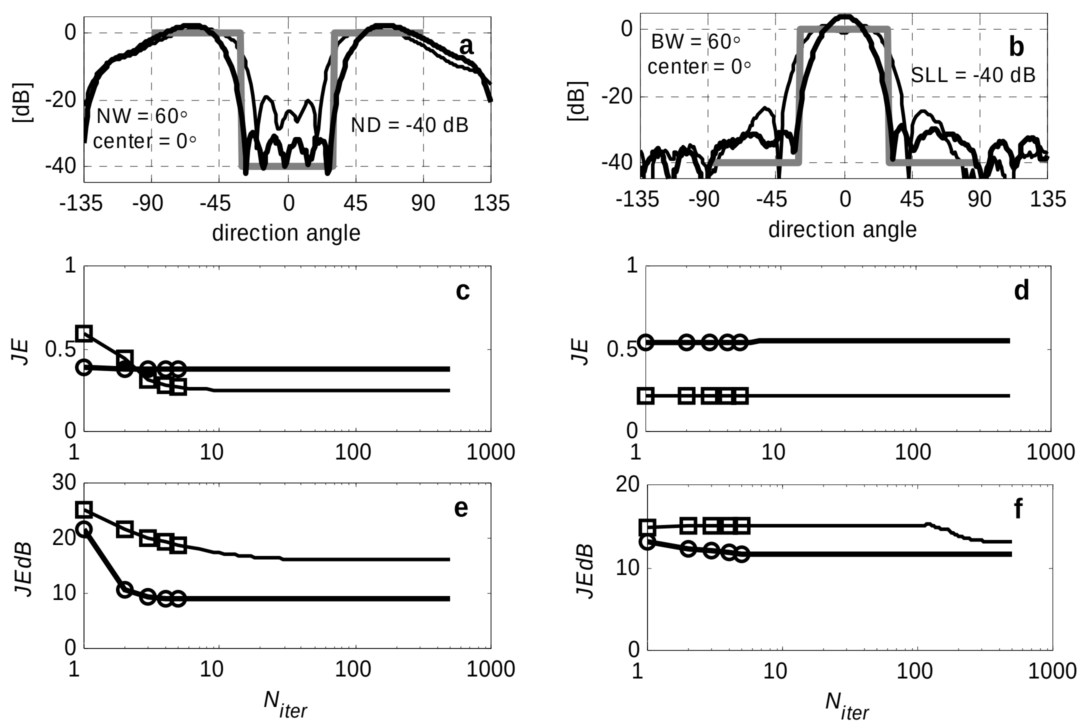



Figure 1. *Generated box-type array patterns a) with a wide null and b) with a beam. The desired array patterns (thick gray solid line) and the array patterns generated using iteration are without weighting (solid line) and with weighting of the null regions (thick solid line). The evolution of iteration is presented in figures c) - f) below the corresponding array patterns. In c) and d) is the evolution of the linear scale joint two-level mse, JE and in e) and f) is the evolution of the joint dB-scale two-level mse, JEdB, where the thin line is for the non-weighted and the thick line for the weighted case. The first 5 cycles of iteration are marked separately.*

have the same linear scale amplitude mean. The presented final array patterns have been obtained after 500 iteration cycles. We can see that the null-pronouncing weighting causes stronger defined nulls and sidelobes compared to the case without weighting. The cost for better curve matching on null/sidelobe levels is worse matching on the high levels. Both the joint two-level mean square errors, in linear (*JE*) and in dB (*JEdB*) scale, are presented as a function of iteration cycles to indicate the curve fitting and thus the convergence of the iteration process. With one exception, the final matching is obtained with only a few iteration cycles. In the pattern generation example of Fig. 1 we get after 500 iteration cycles in the case without weighting *JE* = 0.24 and in the case with weighting *JE* = 0.38. The joint two-level *mse* of dB scale values is in the case without weighting *JEdB* = 16.2 dB-units and with weighting *JEdB* = 8.9 dB-units. The corresponding values for the beam generation are, *JE* = 0.22, *JE* = 0.54, *JEdB* = 13.1 dB-units and *JEdB* = 11.6 dB-units, respectively. The limiting linear scale *JE* is lower (i.e. better) for the case without weighting than for the case with weighting. On the dB scale the limiting *JEdB* behaves the opposite way: the limiting *JEdB*'s are lower in the case with weighting than without weighting. For wide null generation the joint two-level *mse* for the case with weighting is, however, at the beginning of iteration lower on both linear and dB scales. In Fig. 1 the iteration converges in a few cycles, which is a characteristic detected also in [7] for iterative pattern synthesis using weighting.

Mutual coupling affects the element patterns and thus the input vector generated using idealized model element patterns gives in reality only approximately the wanted array pattern. For more exact array pattern generation the real element patterns are needed, unless the element patterns are corrected beforehand using the matrix method [1], [2], [5]. The effect of using an inadaptable set of element patterns, e.g. ideal patterns instead of real or corrected element patterns, can be demonstrated, if the weights found for real element patterns are used for ideal element patterns. Using the same input vector as in Fig. 1(a) the array pattern is now generated using ideal element patterns. Here, the ideal element pattern is the mean complex-valued element pattern, where the phase shift due to

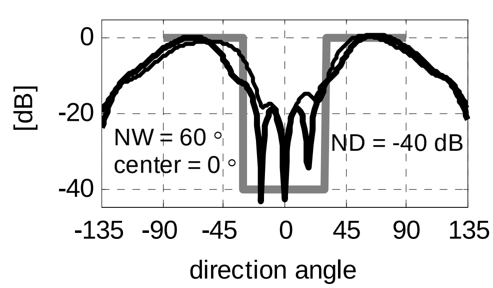



Figure 2. *Array patterns with a wide null for an array with ideal element patterns. The array patterns are generated using port weights found for measured element patterns. The desired array pattern is denoted by a thick gray solid line, the array pattern without weighting by a thin black solid line and for the case using weighting by a thick black solid line.*

element position is extracted before computing the mean element pattern. In Fig. 2 we see the array patterns both with and without weighting. The increase of the null level and loss of accuracy in the wide null region is seen in both cases. The null depth increase is of the same magnitude as if noise at a level 25 dB below the wanted signal level is added to the array output.

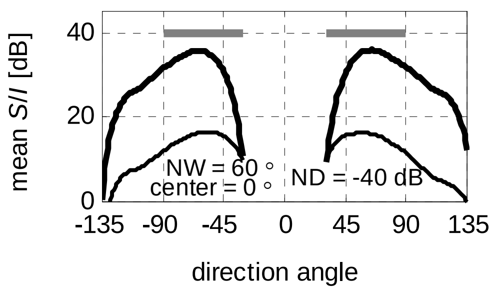
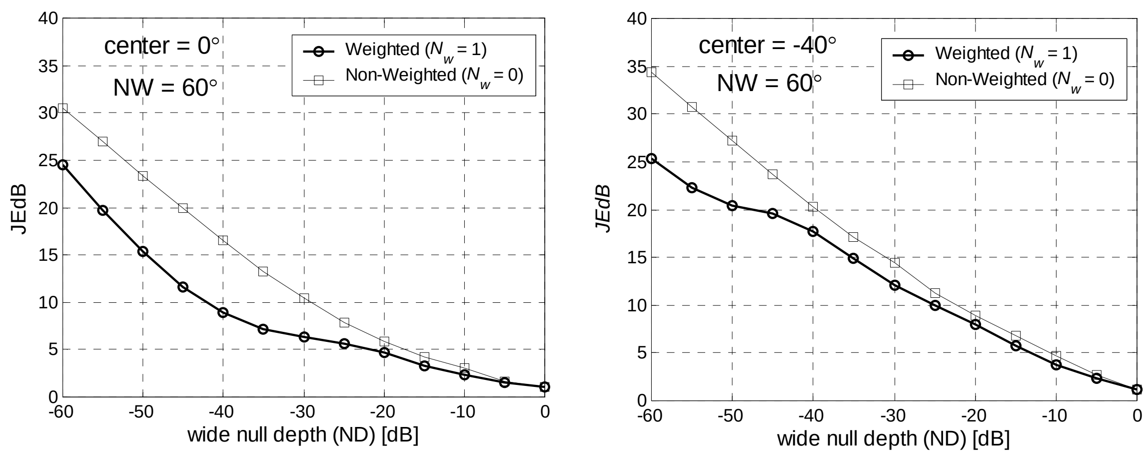



Figure 3. *The mean signal-to-interference ratio S/I as a function of signal arrival direction, when the signal and interferer sources are of equal strength. The interferer is located in the sector –30° to +30° and the signal source outside this sector. The array patterns with a wide null are the same as in Fig. 1(a), the mean S/I is denoted by a thick gray solid line for the desired array pattern, by thin black solid line for the array pattern generated without weighting and by a thick black solid line when using weighting.*

Sectors with a decreased received power level are useful for interference cancellation, for which the signal-to-interference ratio *S/I* is of interest. As an application example we consider the case when an interferer has a broad angular-domain distribution and the same power level as the wanted signal. The interferer is canceled with spatial filtering using the array pattern with wide null in the same sector. The signal to interference ratio *S/I* is calculated and averaged for the same array pattern as in Fig. 1(a). The mean *S/I* is presented in Fig. 3 as a function of the angle of arrival. Outside the wide null sector that is cancelled, the shape of the directional dependency is the same as for the corresponding array pattern in Fig. 1(a). We can see that using directional weighting in array pattern calculation, the mean of the signal-to-interference ratio *S/I* is in the presented case on a wide angular region about 20 dB higher than in the case without weighting. The maximum of the mean *S/I* is 36 dB when the 40 dB level was set as the goal. It is preferable to use the weighting for the spatial filtering of the interferer as long as it can be ensured, that the noise level is at the array ports low enough e.g. in this example it needs to be less than the maximum of *S/I* level without weighting (about −18 dB of the signal level).

## 3.2 Comparison of array pattern generation with and without robust weighting for different array patterns

As was mentioned in Chapter 2, the robust weighting is not always the optimal method in array pattern generation, especially when the obtained array pattern is characterized on the dB scale. In this part of the work it is examined, which robust method, with weighting or not, is the better one for different null widths NW (or BW), null depths ND (or SLL) and null (or beam) center positions. When the null depth *ND* is altered while the null center is in forward direction and *NW* = 60°, the result in Fig. 4(a) shows, that the weighting is always better compared to the case without weighting. However, the difference between these methods is not as significant when *ND* > −20 dB. This means, that with signal-to-noise ratio *SNR* < 20 dB there is no practical difference between the methods for the examined array. In the case when *ND* = −50 to −30 dB the result for the case with weighting is about 5 to 8 dB-units better than without it.

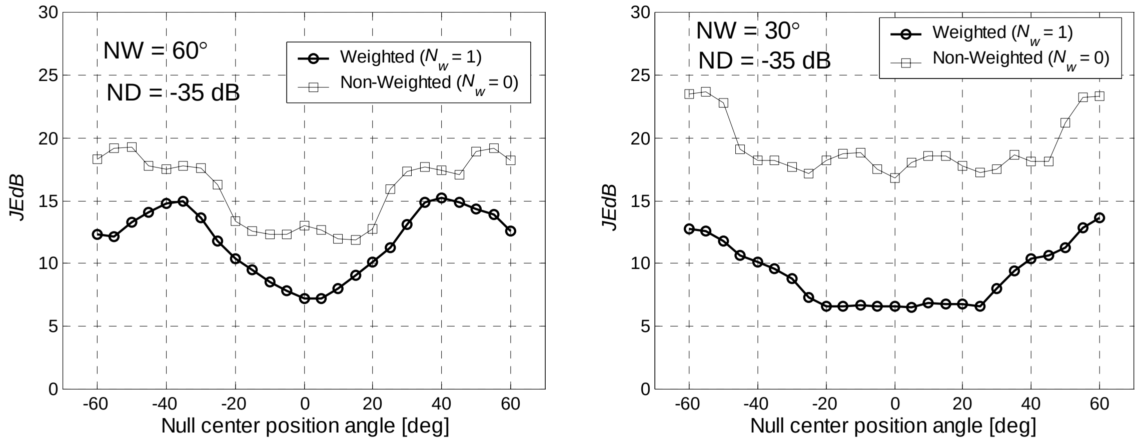



(a) (b)

Figure 4. *The dependency of the joint two-level dB-scale mse (JEdB) on the null dept for 60° null width (NW) and a) with center at 0° and b) with center at –40°. The number of iterations is 200.*

When the null center position is shifted −40° from the forward direction the error in the patterns increases. The result presented in Fig. 4(b) shows that the weighting is also in this case better compared to the case without it. As in previous cases there is almost no difference between these methods, when *ND* > −20 dB. When *ND* = −40 to −30 dB the difference is from 2 to 3 dB-units.

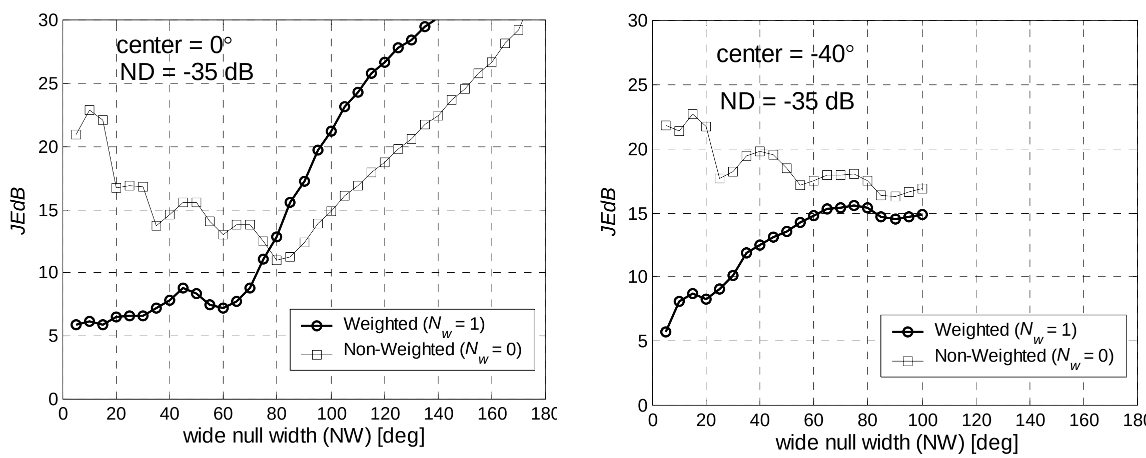
The dependency of the *JEdB* on the wide null center position is presented in Fig. 5(a) and in Fig. 5(b) for null widths of 60° and 30°, respectively. We see that the fitting errors depend strongly on the null width and position, and that the direction for which the wide null results in the best fitting is the forward direction. In the case of *NW* = 30° the weighting has good characteristics over a wider region than in the previous case. In these cases weighting is the better method for any position of the null center. As can be seen in Fig. 5(a) the difference between the methods is not as significant for wider null widths.




(a) (b)

Figure 5. *The dependency of the JEdB on the null position for null depth ND = –35 dB and with null width a) NW = 60° and b) NW = 30°. The number of iterations is 200.*

In Fig. 6(a) the effect of the null width (NW) is presented for a wide null in the forward direction. We can see that weighting gives better results when *NW* < 70°, but the method without weighting is better when *NW* > 80°. However, for *NW* > 100° in the case without weighting the realized gap between the points of the two levels (absolute value of the desired *ND* plus joint two-level dB-*mse*) is decreased to values < 20dB, which is far from the absolute value of the wanted *ND* of 35 dB-units. When the null center is shifted




(a) (b)

Figure 6. *The dependency of the joint two-level dB-scale mse (JEdB) on the null width (NW) for null depth (ND) of –35 dB and a) with center position in the forward direction, and b) in the direction of –40°. The number of iterations is 200.*

to –40° in Fig. 6(b) the result becomes worse for both methods, but weighting remains better.

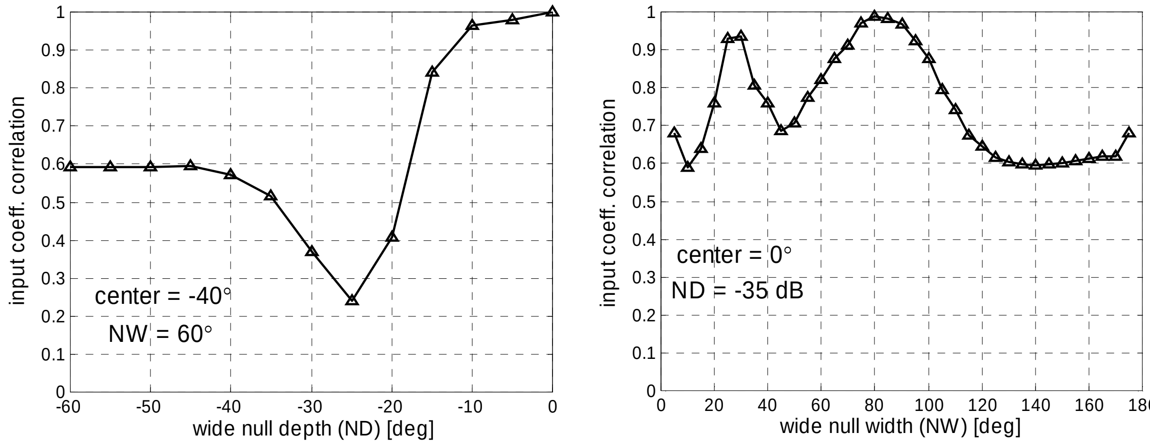
The considerations that were done for nulls, are also applied to beam generation by simply inverting the corresponding desired array amplitude pattern. This leads to beam patterns with beam direction, beam width and SLL that are the same as the corresponding wide null direction, NW and ND as in Fig. 1. The results show, that weighting is optimal when generating wider beams ($BW > 50°$) in the forward direction −40° to +40° and is also somewhat better for beams with lower sidelobes ($SLL < -35$ dB). However, the beam generation without weighting is better for sharp beams ($BW < 30°$) and for directions close to the endfire. The beam generation is basically complementary to null generation: good parameters for null generation without weighting are good parameters for beam generation with weighting, and vice versa.

In realistic arrays the effects of mutual coupling are always present. In the array under test the scattering matrix is measured and the eigenvalues and -vectors of the matrix $\mathbf{S}^H\mathbf{S}$ are computed. The eigenvalues are the same as the relative reflected power with the corresponding eigenvectors as voltage wave inputs to the array. At 5.2 GHz the mean eigenvalue of $\mathbf{S}^H\mathbf{S}$ is 3.1%, when the minimum and maximum eigenvalues are 0.2% and 6.7%. The array patterns generated with the pure eigenvectors are somewhat exotic and not similar to the box-type patterns considered here. The radiated power stability is proven by generating array patterns with the computed best input coefficients concerning the cases presented in Fig. 4, Fig. 5 and Fig. 6. The radiated power to the pattern azimuth regions of −180° to 180° and −90° to 90° are compared with the corresponding radiated power with input vector [1 1 1 1 1 1]. The comparison shows, that the radiated power to the given regions is 95% to 120% as compared with the reference and does not have any abrupt change when the wanted array pattern is changing. This means that the instability of the radiated power is not a problem in broad null generation for array with spacing $d = 0.5\lambda$, as it is e. g. in superdirective beam generation for an array with spacing $0.5\lambda$ or less [27].

The cases with and without weighting can be examined also by the closeness of the final iterated input vectors. The input vectors $\vec{\mathbf{a}}_w$ and $\vec{\mathbf{a}}_n$ generated for a wanted array pattern with and without weighting, are compared with help of the correlation function

$$c\left(\vec{\mathbf{a}}_w,\vec{\mathbf{a}}_n\right)=\left|\vec{\mathbf{a}}_w^H\vec{\mathbf{a}}_n \Big/ \sqrt{\left(\vec{\mathbf{a}}_w^H\vec{\mathbf{a}}_w\right)\left(\vec{\mathbf{a}}_n^H\vec{\mathbf{a}}_n\right)}\right|. \tag{8}$$

The comparison of the input coefficients with the correlation function in (8) is presented in Fig. 7. The presented cases of desired patterns are the same as in Fig. 4(b) and 6(a). These experiments show that when the final error for the used criterion is low enough, the




(a) (b)

Figure 7. *The correlation between the input coefficients iterated with and without weighting. The dependency on a) the depth and b) the width of the wide null. Number of iterations is 200.*

iterated input coefficients of different cases are close to each other. When the error is not close to zero, the solutions of the input coefficients can be close or far from each other. This shows that the weighting method does not in general mean only fine tuning of the coefficients found without weighting.

### 3.3 Advanced pattern weighting

Different values of the pattern weighting vector exponent $N_w$ in (3) and also some additional pattern weighting vectors are analyzed next. The basic weighting vector $\vec{\mathbf{\Psi}}_w$ in (3) is produced by components of the additional weighting vector $\vec{\mathbf{\Psi}}_a$ to obtain the modified weighting vector

$$\vec{\mathbf{\Psi}}_m = \vec{\mathbf{\Psi}}_a \otimes \vec{\mathbf{\Psi}}_w . \tag{9}$$

The corresponding modified weighting matrix contains in each row the modified weighting vector. To avoid sharp beam or null underweighting, the high level and low level pattern regions can be weighted additionally with the inverses of their sampling point number $M_{low}$ and $M_{high}$

$$\vec{\mathbf{\Psi}}_a(\theta_{i\,\in\{level\}}) = \left(1/M_{level}\right)^{N_p} , \tag{10}$$

where *level* = “low” or “high” and $N_p$ is the exponent of the additional weighting. The case $N_p = 1$ is similar to the equalization of the effective number of calculation/sampling points in array pattern high and low level regions. To pronounce steep change from low to high radiation intensity level, the points at the boundary region can also be additionally weighted. Also when a sharp null/beam is close to the calculation area limit, this additional weighting can be needed.

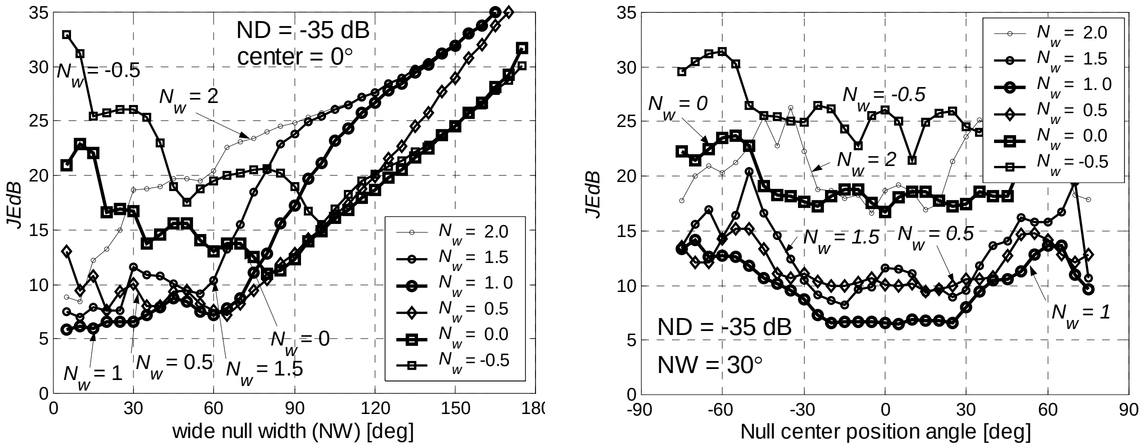
The influence of the phase pattern changes on the iteration convergence was also analyzed. In iterations with (4) the desired pattern phase change is the shift from the previous to the latest reached phase pattern. This shift can be included also with other ratios than one. It was noticed for examples with untypical pattern parameters, that in order to improve final results the last reached phases should be accepted only partly as the new desired phases with a phase shift ratio < 1. Additionally, defining a too low desired null/sidelobe dB-level causes the final array pattern to lose validity when using the robust weighting method with $N_w = 1$. This is due to the fact that the dB-scale has no lower limit, and taking the low level closer to zero leads to poorer relative fitting at the high intensity level. The parameter $\Delta$ in (3) can be used also to define the lowest level, which should be taken into account in calculations, to stabilize the algorithm if too low null levels are chosen. Here any practical need to use $\Delta \neq 0$ for stabilizing was not detected for nulls with $ND > -80$ dB for $N_w = 1$, which is practically a very low level. It was noticed, that for stabilizing a low level parameter $\Delta$, which takes into account the mean element pattern, give better results in the case of very deep wide zeros or sidelobes, namely

$$\Delta_{corr} = \frac{L_{high}}{L_{low} / \mathrm{ND}} \Delta \, , \tag{11}$$

where $L_{high}$ is the mean value of element pattern amplitude in array pattern high level region and $L_{low}$ the corresponding low level region mean value, when element pattern amplitude maximum is normalized to 1. With a correction (9) a constant $\Delta$ can be used for different array patterns with different high and low level radiations in the case of very deep nulls.

In Fig. 8 the *JEdB* is shown as a function of null width and null center position, when the weighting exponent $N_w$ in (3) is varied from –0.5 to 2 in 0.5 steps. The basic cases without weighting ($N_w = 0$) and with weighting ($N_w = 1$) are the same as in Fig. 6(a) and 3(b), respectively. We see, that using quadratic joint two-level MSE-criterion defined in (5) $N_w = 1$ results in the best pattern. In general, the values $N_w \approx 0.5$ to 1 seem to be the best for wide null/beam generation, when the wanted array pattern is varied within the most realistic ranges, but also minima for *JEdB* with $N_w = -1.5$ to 1.5 are detected. The negative values are better in the case of very sharp beams (which on the other hand can generally not be satisfactorily generated with few array elements).

In Section 3.2 the two regions (low and high) were weighted independently. In principle, there are three separated radiation intensity regions: left high/low radiation intensity sector, null/beam sector and right high/low radiation intensity sector and they can be weighted individually. All the presented box-type array patterns belong to amplitude patterns containing three sectors with different levels of radiation intensity. Sectorization




(a) (b)

Figure 8. *The effect of the weighting exponent $N_w$. In a) the wide null width is altered and in b) the wide null position is altered. Null depth is ND=–35dB and null width NW=30°. Number of iterations is 200.*

has advantages in mobile communications [11], [28], [29]. A weighting example with 3 sectors with different desired radiation intensity levels is presented in Fig. 9. A narrow sector or narrow sectors close to the calculation region limit are typically difficult to match. Also it is not easy to find the optimal matching criterion to reach satisfactorily the desired pattern from the practical point of view. The criterion for the matching in the three-level case is taken as

$$JEdB_{3L} = \sqrt{2/3\left(mse_{dB,1}^2 + mse_{dB,2}^2 + mse_{dB,3}^2\right)}, \qquad (12)$$

where the square root of 2/3 of the sum of level errors is used to have a criterion with values comparable to the two-level matching criterion.

In Fig. 9(a) the basic weighting exponent $N_w$ is altered. For the case without weighting $JEdB_{3L}$ = 13.0 dB-units ($N_w = 0$). The lowest error without additional weighting is $JEdB_{3L}$ = 11.3 dB-units with $N_w$ = 0.5. For $N_w$ = 1 $JEdB_{3L}$ = 12.7 dB-units. In Fig. 9(b) the exponent $N_p$ of additive weighting is altered, when the basic weighting exponent $N_w$ = 1. The sectors are weighted additionally multiplying by $(1/M_i)^{N_p}$, where $M_i$ are the numbers of sampling points for sectors with $i$ = 1,2,3. The joint tree-level dB-*mse*'s, $JEdB_{3L}$'s, are 11.9, 11.0 and 11.8 dB-units for additional exponent values $N_p$ = 1, $N_p$ = 2 and $N_p$ = 4, respectively.

We see in the example of Fig. 9 that the weighting with $N_w$ = 1 can be used also well in this problematic case without additional weighting with $N_p$ = 0, because the differences in fitting errors are not significant, varying less than 10% in the dB-scale. We can also see that for this case none of the presented solutions is very good. The region close to the calculation area limit could be weighted even more. In general a better weighting could be obtained by taking into account the evolution of error during the iteration as was proposed in [7] and done in [8], forcing the maximum errors closer to zero. As in [23] the weighting could be more sensitive to direction and also take into account the error sign, as proposed in [30] to obtain an array pattern with fewer points between the two levels. This is the usual goal of pattern synthesis for link arrays. The increasing weighting for problematic directions during the iteration [31] is also an alternative which can to be added to the basic algorithm.

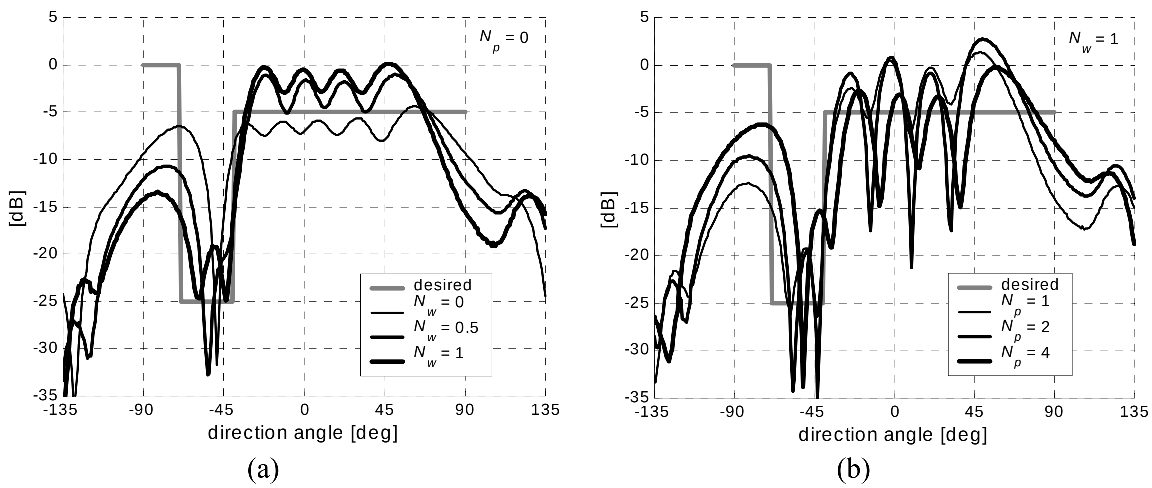



Figure 9. *The matching of generated array patterns with the desired 3-level pattern, when different weighting exponents are used. The weighting is realized a) varying the weighting exponent $N_w$ and b) varying additional weighting exponent $N_p$ related to effective sampling point number on a sector.*

# 4 Conclusions

The generation of box-type array patterns with wide nulls using an amplitude only criterion has been verified. The basic two cases, the one without and the other with weighting, are compared. Which one is better in the sense of LSE depends on the choice of the amplitude scaling, i.e. in this work the logarithmic, relative shift scale fitting has been preferred over the linear scale. It has been shown, that the robust null-pronouncing weighting in wide null generation results in the best fitting on the dB-scale for a wide range of pattern parameters. The results also show, that the basic LSE criterion on linear or on logarithmic scale is not always enough for evaluating the pattern generation. Additional case-sensitive criteria, such as the joint two-level *mse* (*JE* or *JEdB*), presented here as a final validity criterion, could be used to improve the iteration results, as well as the different additional weightings described in Section 3.3. However, the presented iteration with the robust weighting is easier to implement and could be compared in terms of computational complexity with the iteration using relative-error sensitive weighting presented in [8].

In this approach, all the adaptivity is used to optimize the wide null generation. Usually the wide sector nulling problem is a part of a more complicated adaptive array pattern generation [32], [33] and in order to have adaptivity left for other purposes the wide null matching with 6-element arrays cannot be in general as good as presented here. The used method with and without weighting can be used to generate beams and is a simple alternative for standard pattern synthesis particularly for small arrays in which the element patterns can differ a lot from each other and the standard synthesis with idealized patterns gives poor fitting for wide zeros. The need to store the measured patterns for each element as well as somewhat time-consuming additional calculations in the iteration process are disadvantages of the presented pattern weighting method, however, the advantage of improved dB scale fitting can at least compensate the disadvantages.

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