Antenna array pattern correlation and input type

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When mutual coupling is present in an antenna array, the radiation patterns of the active elements as well as the pattern correlations depend on the representation of the input signal in terms of the input type. The pattern correlation is presented for different input types, i.e. for voltages, currents and voltage waves. It is shown for a microstrip array that a suitable choice of the reference plane of input voltages and currents can lead to a lower pattern correlation than for voltage waves.

Introduction: Element pattern correlation in an array is a basic characteristic that explains the existence of correlation between the signals received or transmitted by different elements in an antenna array. The connection between antenna array element patterns and mutual coupling can be used to estimate the correlation between the RF signals in the array branches, when signals are arriving from several directions [1]. The RF signals at the array input, the input type, can be voltages, currents or voltage waves, for which mutual coupling is presented with admittance matrix, impedance matrix and scattering matrix, respectively. The pattern correlation has been considered in past years using only one of the representations for the mutual coupling, i.e. the impedance or the scattering matrix presentation [2–4].

Theory: For the case of a lossless array the dependency of the pattern correlation matrix $\boldsymbol{R}_{pat}$ on the scattering matrix is presented in [2] as

$$\begin{aligned}\boldsymbol{R}_{pat} &= \boldsymbol{D}^{-1}(\boldsymbol{I}-\boldsymbol{S}^H\boldsymbol{S})\boldsymbol{D}^{-1} = \boldsymbol{D}^{-1}(\boldsymbol{I}-\boldsymbol{P}_{refl})\boldsymbol{D}^{-1}\\ &= \boldsymbol{D}^{-1}\boldsymbol{P}_{rad}\boldsymbol{D}^{-1} = \boldsymbol{D}^{-1}\boldsymbol{F}^H\boldsymbol{F}\boldsymbol{D}^{-1} = \boldsymbol{F}_0^H\boldsymbol{F}_0\end{aligned} \tag{1}$$

where $\boldsymbol{S}$ is the scattering matrix, $\boldsymbol{I}$ is the unity matrix, $\boldsymbol{F}$ is the array pattern matrix, $\boldsymbol{D}$ is a diagonal matrix containing the square roots of the diagonals of the radiated power rate matrix $\boldsymbol{P}_{rad}$, and H denotes a Hermitian transpose. For a non-dissipative array the connection between the reflected power rate matrix $\boldsymbol{P}_{refl}$ and radiated power rate matrix $\boldsymbol{P}_{rad}$ is also given in (1). The array pattern matrix $\boldsymbol{F}$ is defined for voltage wave inputs and it contains the active element patterns of the array elements when placed in an array. Matrix $\boldsymbol{F}_0$ consists of complex-valued element patterns normalised to unity. Let the voltage wave inputs $\vec{V}^+$ be known. Then the corresponding voltage inputs can be found by

$$\vec{V} = (\boldsymbol{I}+\boldsymbol{S})\vec{V}^+ \tag{2}$$

and the corresponding current inputs $\vec{I}$ can be found by

$$\vec{I} = (\boldsymbol{I}-\boldsymbol{S})\vec{V}^+ \tag{3}$$

The active element patterns are obtained with a nonzero input only at a single array element. If $\boldsymbol{S}$ contains nonzero off-diagonal values, the active element patterns are different for each type of input. The (active) element pattern matrix in the case of voltage inputs is

$$\boldsymbol{F}_V = (\boldsymbol{I}+\boldsymbol{S})^{-1}\boldsymbol{F}_s \tag{4}$$

where $\boldsymbol{F}_s$ is the element pattern matrix in the case of voltage wave inputs (scattering matrix presentation). The corresponding element pattern matrix in the case of current inputs is

$$\boldsymbol{F}_I = (\boldsymbol{I}-\boldsymbol{S})^{-1}\boldsymbol{F}_s \tag{5}$$

The radiated power P_{rad} for a lossless array can be defined by

$$\begin{aligned}P_{rad} &= \mathrm{Re}\{\vec{V}^H\vec{I}\} = \mathrm{Re}\{\vec{I}^H\vec{V}\} = \frac{1}{2}(\vec{V}^H\vec{I}+\vec{I}^H\vec{V})\\ &= \frac{1}{2}(\vec{I}^H\boldsymbol{Z}^H\vec{I}+\vec{I}^H\boldsymbol{Z}\vec{I}) = \frac{1}{2}(\vec{V}^H\boldsymbol{Y}\vec{V}+\vec{V}^H\boldsymbol{Y}^H\vec{V})\end{aligned} \tag{6}$$

where $\boldsymbol{Z}$ and $\boldsymbol{Y}$ are the unnormalised impedance and admittance matrices. Furthermore, we can use the dependency between the scattering and normalised impedance matrix $\boldsymbol{z}$ [5]

$$\boldsymbol{z} = \boldsymbol{Z}_0^{-1}\boldsymbol{Z} = (\boldsymbol{I}+\boldsymbol{S})(\boldsymbol{I}-\boldsymbol{S})^{-1} \tag{7}$$

where $\boldsymbol{I}$ is the unity matrix and $\boldsymbol{Z}_0$ is a diagonal matrix with load/generator impedances of array elements, which are usually 50 Ω, and in the following they are assumed to be real-valued (jX_g can be always compensated for conjugationally by $-jX_g$) and equal to each other.

For power-normalised voltage inputs $\vec{v} = \vec{V}/\sqrt{Z_0}$, we get

$$\begin{aligned}P_{rad} &= \frac{1}{2}(\vec{V}^H\boldsymbol{Y}^H\vec{V}+\vec{V}^H\boldsymbol{Y}\vec{V}) = \frac{1}{2}\vec{V}^H(\boldsymbol{Y}^H+\boldsymbol{Y})\vec{V}\\ &= \frac{1}{2}\vec{V}^H\boldsymbol{Z}_0^{-1/2}(\boldsymbol{y}+\boldsymbol{y}^H)\boldsymbol{Z}_0^{-1/2}\vec{V} = \frac{1}{2}\vec{v}^H(\boldsymbol{y}+\boldsymbol{y}^H)\vec{v}\end{aligned} \tag{8}$$

and for power-normalised current inputs $\vec{i} = \vec{I}\sqrt{Z_0}$, we get

$$\begin{aligned}P_{rad} &= \frac{1}{2}(\vec{I}^H\boldsymbol{Z}\vec{I}+\vec{I}^H\boldsymbol{Z}^H\vec{I}) = \frac{1}{2}\vec{I}^H(\boldsymbol{Z}+\boldsymbol{Z}^H)\vec{I}\\ &= \frac{1}{2}\vec{I}^H\boldsymbol{Z}_0^{1/2}(\boldsymbol{z}+\boldsymbol{z}^H)\boldsymbol{Z}_0^{1/2}\vec{I} = \frac{1}{2}\vec{i}^H(\boldsymbol{z}+\boldsymbol{z}^H)\vec{i}\end{aligned} \tag{9}$$

The scaling power $|\vec{v}|^2 = |\vec{V}|^2/Z_0$ is the power radiated by an array, whose elements are not coupled and have a radiation impedance equal to the system impedance Z_0, when the input voltage $\vec{v} = \vec{V}/\sqrt{Z_0}$. Accordingly, $|\vec{i}|^2 = Z_0|\vec{I}|^2$ is the power radiated by the same array when input current is $\vec{i} = \vec{I}/\sqrt{Z_0}$. In the general case with mutual coupling present, these powers are not identical, and they do not represent the actual radiated powers in either half of the allowed generator powers. But with help of the scaling powers, we can now write the radiated power matrices for power-normalised current and voltage inputs, respectively:

$$\boldsymbol{P}_{rad,I} = \boldsymbol{F}_I^H\boldsymbol{F}_I = \frac{1}{2}(\boldsymbol{z}+\boldsymbol{z}^H) = \mathrm{Re}\{\boldsymbol{z}\} \tag{10}$$

$$\boldsymbol{P}_{rad,V} = \boldsymbol{F}_V^H\boldsymbol{F}_V = \frac{1}{2}(\boldsymbol{y}+\boldsymbol{y}^H) = \mathrm{Re}\{\boldsymbol{y}\} \tag{11}$$

If the diagonal elements of the impedance or the admittance matrix are close to unity, then (10) and (11) present the pattern correlation for current-driven or voltage-driven arrays, respectively. When the mutual coupling is strong, we can hardly expect diagonal values close to unity, and then the patterns should be rescaled using diagonals as in (1) to obtain the corresponding presentation for the correlation matrix [1]. Finally we can formulate for the pattern correlation for a lossless array two different input presentations:

$$\begin{aligned}\boldsymbol{R}_{pat,I} &= \boldsymbol{R}_{pat,Z} = \boldsymbol{F}_{I,0}^H\boldsymbol{F}_{I,0} = \boldsymbol{D}_I^{-1}\boldsymbol{F}_I^H\boldsymbol{F}_I\boldsymbol{D}_I^{-1}\\ &= \frac{1}{2}\boldsymbol{D}_I^{-1}(\boldsymbol{z}+\boldsymbol{z}^H)\boldsymbol{D}_I^{-1} = \boldsymbol{D}_I^{-1}\mathrm{Re}\{\boldsymbol{z}\}\boldsymbol{D}_I^{-1}\end{aligned} \tag{12}$$

$$\begin{aligned}\boldsymbol{R}_{pat,V} &= \boldsymbol{R}_{pat,Y} = \boldsymbol{F}_{V,0}^H\boldsymbol{F}_{V,0} = \boldsymbol{D}_V^{-1}\boldsymbol{F}_V^H\boldsymbol{F}_V\boldsymbol{D}_V^{-1}\\ &= \frac{1}{2}\boldsymbol{D}_V^{-1}(\boldsymbol{y}+\boldsymbol{y}^H)\boldsymbol{D}_V^{-1} = \boldsymbol{D}_V^{-1}\mathrm{Re}\{\boldsymbol{y}\}\boldsymbol{D}_V^{-1}\end{aligned} \tag{13}$$

Measurements and results: In this work six-element microstrip arrays are examined. They were introduced in [6]. Fig. 1 shows the equivalent circuit for a voltage source connected with a cable to the array antenna port. The generator impedance is $R_g = 50\ \Omega$ and $X_g = 0$, and the reference plane is placed at the beginning of the coaxial probe of the SMA connector. For different positions of the reference plane along a 50 Ω line we have computed the mean cross-correlation of element patterns for a microstrip antenna array by

$$\langle r_{ij,i\neq j}\rangle(\Delta\phi_{ref}) = \frac{\left(-N_{el}+\sum_{i,j=1}^{N_{el}}|\boldsymbol{R}_{pat}(i,j,\Delta\phi_{ref})|\right)}{(N_{el}^2-N_{el})} \tag{14}$$

where the number of elements $N_{el} = 6$ and $\Delta\phi_{ref}$ is the change in the reference phase angle. Only the measured scattering matrix and its derivatives $\boldsymbol{z}$ and $\boldsymbol{y}$ are used in the calculations. Fig. 2 shows resulting dependencies for an array with element spacing $d = 0.5\lambda$ for different input types at 5.25 GHz. The reference phase angle of the scattering matrix has no effect on pattern correlation. For impedance and admittance matrix presentations there is a dependency on the reference plane, and the corresponding pattern correlations are the same with a 90° shift, when they are both periodic with a 180° period. In Fig. 2 the pattern correlation for the voltage wave input is always lower than the others. For arrays with stronger coupling (e.g. $d = 0.3$ and 0.4λ) the pattern correlation for voltage/current input can also be lower than for voltage wave input.

When the generator impedance is Z_0 ($R_g = Z_0$, $X_g = 0$), the amplitude of the array element input voltage wave at an antenna input is half the generator voltage and their phases differ only by a constant shift, which is convenient in practice. The generator voltages or currents define the corresponding vectors or currents at antenna input ports or at some other reference plane with a non-diagonal matrix. This means that, using an array with voltage or current feed, the RF signal inputs in generators or outputs in loads should be multiplied by the inverse of the same matrix, which is less convenient than for voltage wave feed, but can give better signals/patterns [6].

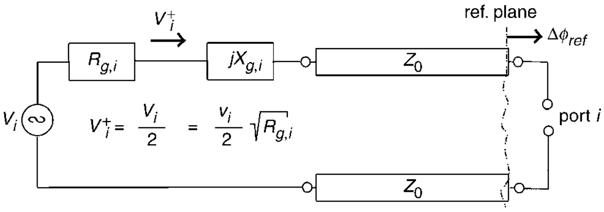

Fig. 1 *Input circuit model for antenna element in array connected with cable to voltage generator*

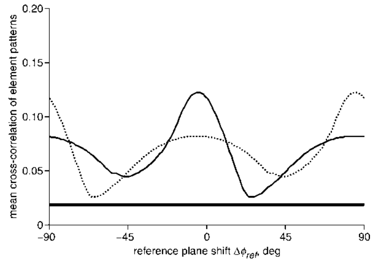

Fig. 2 *Mean pattern correlation for different input types in microstrip array*

—— $\langle|\boldsymbol{R}_{pat,S}(i,j)|\rangle$ —— $\langle|\boldsymbol{R}_{pat,y}(i,j)|\rangle$ -------- $\langle|\boldsymbol{R}_{pat,z}(i,j)|\rangle$

The pattern correlation in impedance or admittance matrix presentations does not depend on the generator side impedances seen from array ports. This means that, if we find a reference plane with minimum correlation, we can change generator/load impedances, cable lengths or matching circuit in the generator side of the reference plane to find better matching of the array. However, the minimum pattern correlation is not necessarily the most important criterion. When there is no mutual coupling in an array, the pattern correlation is always zero. The pattern correlation, however, can still be zero in the presence of mutual coupling. By (1) it is zero, when the scattering matrix columns are orthogonal, and by (12) and (13) it is zero, when the off-diagonal elements of the impedance or admittance matrix are imaginary.

In antenna simulations the system impedance and input circuits are typically not considered, whereas in measurements they are present. Usually the cable lengths are fixed, and cables and generators/loads are designed for 50 Ω. If the patterns are defined with generator/load voltages (scattering matrix presentation), they, and as following, their correlation depend also on the load/generator impedance [4]. The hardware solution for array matching is to use a multiport, which diagonalises the impedance matrix. Because the ports of the matching multiport should be coupled there is practically no hope of matching the array perfectly [7, 8]. When a matching circuit is added to the array, the resulting element patterns and their correlation are also changed.

Conclusions: An exact matrix expression for the correlation of active element patterns in an array is presented for different input types. Dependencies are derived for a non-dissipative array and they are represented by the scattering matrix, by the impedance matrix and by the admittance matrix. These matrices are easier to obtain than measured 3D pattern matrices with two polarisations and thus they are preferable when characterising the array performance for general adaptive use. Experiments with microstrip arrays show that, for any impedance or admittance matrix, an optimal reference plane can be found, which results in minimum pattern correlation. In the case of close element spacing the minimum correlation level for the impedance or admittance matrix can be lower than for the scattering matrix. The use of array input voltages or currents instead of voltage waves requires matrix computing for the generator/load signals.

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