# PATTERN CORRELATION AND MISMATCH IN TWO-ELEMENT ANTENNA ARRAYS

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**ABSTRACT**: *In an antenna array the element pattern correlation and the array matching depend on each other. For lossless two-element arrays the general formulation of the dependency of array matching on pattern correlation is given and solved numerically for some basic cases. Array matching is given as a characteristic mismatch coefficient and as the mean relative reflected power.*

## 1. INTRODUCTION

In an antenna array, mutual coupling can be used to estimate the correlation between the RF signals in the array branches [1]. This problem has been considered recently in [2–4]. In this work we use the voltage wave and scattering matrix presentations for mutual coupling and demonstrate how to estimate the pattern correlation when the array matching is known, and vice versa.

## 2. THEORY

The pattern correlation matrix $\mathbf{R}_{pat}$ for a two-element array is defined as

$$\mathbf{R}_{pat} = \begin{bmatrix} 1 & \mathbf{f}_{01}^H \mathbf{f}_{02} \\ \mathbf{f}_{02}^H \mathbf{f}_{01} & 1 \end{bmatrix} = \begin{bmatrix} 1 & R_{12} \\ R_{12}^* & 1 \end{bmatrix} \quad \text{with} \quad \sqrt{\mathbf{f}_{01}^H \mathbf{f}_{01}} = \sqrt{\mathbf{f}_{02}^H \mathbf{f}_{02}} = 1, \tag{1}$$

where $H$ denotes a Hermitian transpose, and vectors $\mathbf{f}_{01}$ and $\mathbf{f}_{02}$ give with a certain angular grid the normalized complex 3D field patterns of antenna elements 1 and 2. The eigenstructure of the correlation matrix is

$$\vec{\mathbf{X}}_1 = \frac{\begin{bmatrix} e^{j\varphi/2} & e^{-j\varphi/2} \end{bmatrix}}{\sqrt{2}}, \lambda_{R,1} = 1 + |R_{12}| \text{ and } \vec{\mathbf{X}}_2 = \frac{\begin{bmatrix} e^{j\varphi/2} & -e^{-j\varphi/2} \end{bmatrix}}{\sqrt{2}}, \ \lambda_{R,2} = 1 - |R_{12}|, \tag{2}$$

where $\vec{\mathbf{X}}_i$ is an eigenvector and $\lambda_{R,i}$ is the corresponding eigenvalue and $\varphi$ denotes phase delay. If the antenna pair is symmetric, the eigenvectors are [1 1] and [1 –1] and the off-diagonal elements $R_{12}$ and $R_{21}$ are real-valued and equal to each other. For a lossless antenna pair the connection between the pattern correlation and scattering parameters $S_{ij}$ is according to [2, 3]

$$R_{12} = -\frac{S_{11}^* S_{12} + S_{12}^* S_{22}}{\sqrt{1 - |S_{11}|^2 - |S_{12}|^2} \cdot \sqrt{1 - |S_{22}|^2 - |S_{12}|^2}} = R_{21}^*. \tag{3}$$

As can be seen from (3) there is a set of possible scattering parameters for each given value of pattern correlation. Thus, the minimum of the antenna pair mismatch would be useful to estimate for each level of correlation. It can be shown with (3) that when the antenna pair is symmetric (i.e. $S_{11} = S_{22}$), $R_{12}$ is real-valued. The same can be proven also with element patterns: if the pattern correlation $\mathbf{f}_{01}^{H}\mathbf{f}_{02}$ has complex conjugate pairs in symmetric directions the summation results in a real value. With non-zero mutual coupling coefficients the pattern correlation is still zero if scattering matrix columns are orthogonal. This requires matching symmetry $|S_{11}| = |S_{22}|$ and some phase relation between the scattering parameters [2]. When only the absolute values of the scattering parameters are given we can find from (3) the lowest and highest pattern correlation levels

$$\frac{|S_{12}|^2\left(|S_{11}|-|S_{22}|\right)^2}{\left(1-|S_{11}|^2-|S_{12}|^2\right)\left(1-|S_{12}|^2-|S_{22}|^2\right)} \le |R_{12}|^2 \le \frac{|S_{12}|^2\left(|S_{11}|+|S_{22}|\right)^2}{\left(1-|S_{11}|^2-|S_{12}|^2\right)\left(1-|S_{12}|^2-|S_{22}|^2\right)}, \qquad (4)$$

which shows also that in the case without matching symmetry the correlation cannot be zero with non-zero scattering parameters.

When the array is in general adaptive use without any regular relation between the signals in the branches, then the mean relative reflected power, the reflected power rate, can be calculated as

$$\left\langle P_{refl}/P_{in}\right\rangle = \frac{1}{N_{el}}\sum_{i,j=1}^{N_{el}}\left|S_{ij}^2\right|, \qquad (5)$$

where $N_{el} = 2$ is the number of the elements in the array. It can be shown with help of (3) and (5) that with fixed pattern correlation the mean reflected power rate has a minimum in the case of a symmetric array with matching balance $S_{11} = \pm S_{12} = S_{22}$ [2,

5]. The corresponding minimum value of the mean reflected power rate is then $2|S_{11}|^2$. As a lower limit for the backward coupling with a given pattern correlation we can define the critical mismatch coefficient $t_{cr}$

$$t_{cr} = \min\left\{\left|t_{ch,b}\right|\right\}, \text{ when } t_{ch,b} = S_{11} = S_{12} = S_{22}\,, \tag{6}$$

where $t_{ch,b}$ is the characteristic, realized, mismatch coefficient for an antenna pair with matching balance $S_{11} = S_{12} = S_{22}$. We see the dependency of the limiting case in Fig. 1, where the correlation $|R_{12}|$ is presented as a function of $t_{cr}$ calculated by (3). The area below the curve is the allowed area for $t_{ch,b}$. Apart from the special case of matching balance we can find some other characteristic mismatch coefficients in the allowed area. If only absolute value of the scattering parameters is considered, then for a given $R_{12}$ the characteristic matching coefficient $t_{ch,n}$ can be defined

$$t_{ch,n} = \left|S_{11}\right| = \left|S_{12}\right| = \left|S_{22}\right| \geq t_{cr}\,. \tag{7}$$

The norm-balanced case in (7) is more general and includes the balanced case (6), but is still unusual. In the general case the characteristic matching coefficient belonging to the allowed area and limited by $t_{cr}$ can be presented using (5) and (6) as

$$t_{ch,g} = \sqrt{\tfrac{1}{2}\left\langle P_{refl} / P_{in}\right\rangle} = \sqrt{\tfrac{1}{2}\left|S_{12}\right|^2 + \tfrac{1}{4}\left(\left|S_{11}\right|^2 + \left|S_{22}\right|^2\right)} \geq t_{cr}\,. \tag{8}$$

For a realized array with known element pattern correlations and with known scattering parameters we can find with (3) and Fig. 1, how close the matching is to the optimal value $t_{cr}$. When only absolute values of scattering parameters are given, we can find with (8) and Fig. 1 the maximum value of correlation.

As we have seen, the pattern correlation can be characterized with scattering parameters. For each level of correlation the best possible matching is presented as

$t_{cr}$. Another informative case to analyze the relation between matching characteristics and pattern correlation is to find the minimum level of mismatch, when the complex-valued element patterns are known with a certain power scaling. The basic relation is

$$\mathbf{P}_{rad} = \mathbf{I} - \mathbf{P}_{refl} = \mathbf{I} - \mathbf{S}^H\mathbf{S} = \begin{bmatrix} 1-P_{11} & -P_{12} \\ -P_{12}^* & 1-P_{22} \end{bmatrix} = \begin{bmatrix} g_1 & R_{12}\sqrt{g_1 g_2} \\ R_{12}^*\sqrt{g_1 g_2} & g_2 \end{bmatrix}, \qquad (9)$$

where $\mathbf{P}_{rad}$ is the relative radiated power matrix, $P_{ij}$ are the components of reflected power rate matrix $\mathbf{P}_{refl}$ and $g_1$ and $g_2$ are real-valued scaling factors (relative gains), with $0 \le g_i \le 1$. Assuming the same scenario of general adaptive use of the array as in (5) the mean reflected power rate is

$$\left\langle P_{refl} / P_{in} \right\rangle = \left\langle \lambda_p \right\rangle = \frac{P_{11} + P_{22}}{2} = 1 - \frac{g_1 + g_2}{2}, \qquad (10)$$

where $\left\langle \lambda_p \right\rangle$ is the mean eigenvalue of $\mathbf{P}_{refl}$. We can find the eigenvalues of $\mathbf{P}_{refl}$ in (9) and rewrite those using (10)

$$\lambda_P = \tfrac{1}{2}\left[ P_{11} + P_{22} \pm \sqrt{\left(P_{11}+P_{22}\right)^2 - 4\left(P_{11}P_{22} - \left|P_{12}\right|^2\right)} \right] = \left\langle \lambda_P \right\rangle \pm \sqrt{\left(1-\left\langle \lambda_P \right\rangle\right)^2 \left|R_{12}\right|^2 + \left(\frac{g_1 - g_2}{2}\right)^2 \left(1 - \left|R_{12}\right|^2\right)} \qquad (11)$$

The eigenvalues in (11) must be in the region from zero to unity [6], and setting the smaller eigenvalue to be not less than zero we can find

$$\left\langle \lambda_P \right\rangle \ge \frac{1}{1-\left|R_{12}\right|^2}\left[ -\left|R_{12}\right|^2 + \sqrt{\left|R_{12}\right|^2 + \left(\frac{g_1 - g_2}{2}\right)^2 \left(1 - \left|R_{12}\right|^2\right)^2} \right]. \qquad (12)$$

In (12) the minimum of the mean reflected power rate depends only on the pattern correlation and on the scaling asymmetry $|g_1 - g_2| = |P_{11} - P_{22}|$. We see from (12) that the best matching for a given $|R_{12}|$ can be reached with $g_1 = g_2 = g = 1 - P_{ii} = 1 - |S_{ii}|^2 - |S_{12}|^2$, $i = 1,2$, which requires matching symmetry $|S_{11}| = |S_{22}|$. Matching

symmetry does not necessarily require element symmetry in the antenna pair. In the case of matching symmetry we get

$$\langle \lambda_P \rangle \geq \frac{R_{12}}{1+R_{12}}, \text{and } \lambda_P = P_{ii} \pm P_{12} = 1 - g \pm gR_{12} = \langle \lambda_P \rangle \pm \left(1 - \langle \lambda_P \rangle\right) R_{12}, i = 1,2. \quad (13)$$

Instead of the reflected power rate matrix we can also use the radiated power rate matrix. Their eigenvalues are related

$$\lambda_{rad} = 1 - \lambda_P \text{ and } \langle \lambda_{rad} \rangle = 1 - \langle \lambda_P \rangle. \quad (14)$$

It is possible to prove with (2), that in the case of a symmetric antenna pair the eigenstructure of $\mathbf{P}_{rad}$ and $\mathbf{P}_{refl}$ consists of the same eigenvectors [1 1] and [1 –1] as the eigenstructure of $\mathbf{R}_{pat}$. According to (13) and (14) the eigenvalues $\lambda_{rad,1}$ and $\lambda_{rad,2}$ of $\mathbf{P}_{rad}$ are the same as $\lambda_{R,1}$ and $\lambda_{R,2}$ of $\mathbf{R}_{pat}$ in (2) but rescaled (divided) at least by a factor $1+|R_{12}|$.

## 3. SIMULATIONS

Using (13) we have calculated the minimum of the mean reflected power rate as a function of the element distance in three cases of identical element patterns, namely for a pair of isotropic elements, for a pair with planar (in horizontal plane) omnidirectional patterns (flat isotropic) and for a pair of beams with planar Gaussian (flat Gaussian) element patterns

$$f_i = e^{\pm \frac{kd}{2} \sin\varphi} e^{-\frac{1}{2}\left(\varphi / 35^\circ\right)^2}, \quad (15)$$

where $d$ is the element spacing, $k$ is the wave vector, and $\varphi$ is the azimuth angle. The pattern correlations in the first two cases are known: $R_{12}$ = sinc($kd$) for isotropic and $R_{12} = J_o(kd)$ for the planar omnidirectional patterns [7]. In the fourth mixed case one of the antennas is the planar omnidirectional and the second has the Gaussian

pattern presented in (15). This kind of antenna pair can be useful when there is a preferred direction. The effect of matching asymmetry is presented by (12) with $|g_1 - g_2|$ = 0.2 for isotropic patterns. The matching asymmetry is realistic also for an antenna pair with slightly different resonant frequencies.

From the results presented in Fig. 2 we see that at small inter-element distances the mismatch cannot be ignored. With equal patterns the limit for the minimum of the mean reflected power rate is 0.5. When the elements are in the same point the correlation is unity and we can find by (13) that one of the radiating modes (mode [1, –1]) does not radiate at all. In the mixed case both radiating modes radiate also when the elements are in the same point. The effect of matching asymmetry is that the matching gets worse; the mean reflected power rate increases smoothly (see Fig. 2) so that the minimum value is shifted up from zero by $|g_1 - g_2|/2$ = 0.1 whereas the maximum value of 0.5 is the same as in the case with matching symmetry.

High element pattern orthogonality for tightly spaced elements can be obtained simply with orthogonal polarizations or with weakly overlapping element patterns, but the basic relations are the same as presented here. For real antennas the element patterns often become rippled when the elements are close to each other and thus (12) and (13) give only an estimation of the matching when used with isolated element patterns. The tendency of overestimating the correlation in antenna arrays has been noticed in [2, 4], and thus the presented method for estimating the mismatch can be only a preliminary test for an antenna array with close spacing. If

the losses in the antenna system can not be ignored, then the estimation of matching or pattern correlation becomes more difficult [8].

## 4. CONCLUSION

For two-element arrays the exact formulation for the relation between the element pattern correlation and the array matching is given. The best possible matching level found for each correlation level is presented as the critical mismatch coefficient and as the minimum of the mean reflected power rate. The formulation is given for arrays in general adaptive use. The presented formulation covers all cases of non-dissipative antenna pairs and is tested for some basic cases of two-element antenna arrays.

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**Figure captions:**

Fig. 1.
Antenna array pattern correlation vs. critical mismatch parameter

Fig. 2.
The dependency of the minimum mismatch on element separation for an antenna pair with different basic element patterns.

Figure 1

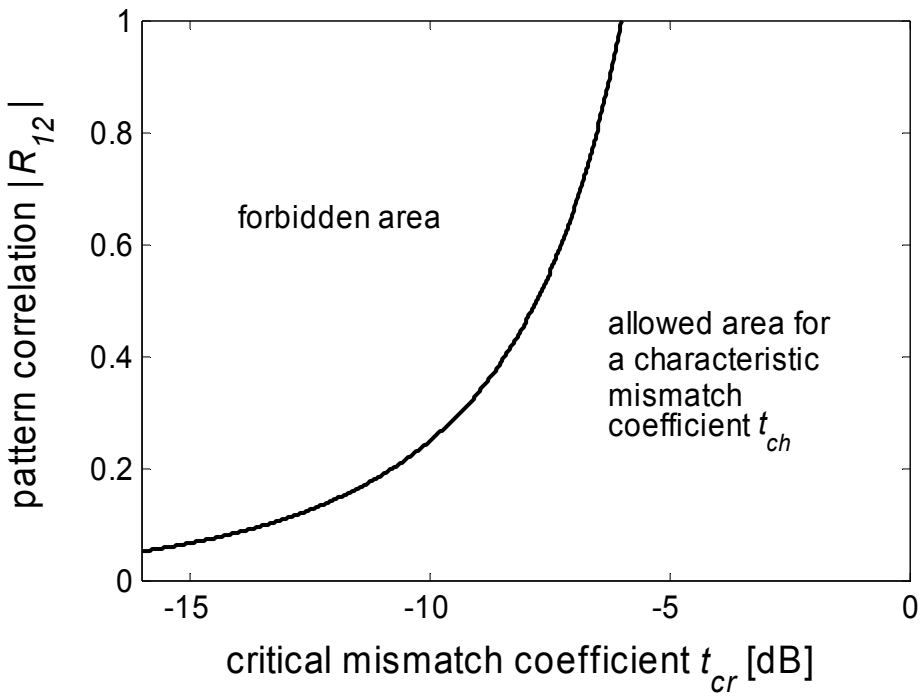


Fig. 1. Antenna array pattern correlation vs. critical mismatch parameter.

Figure 2

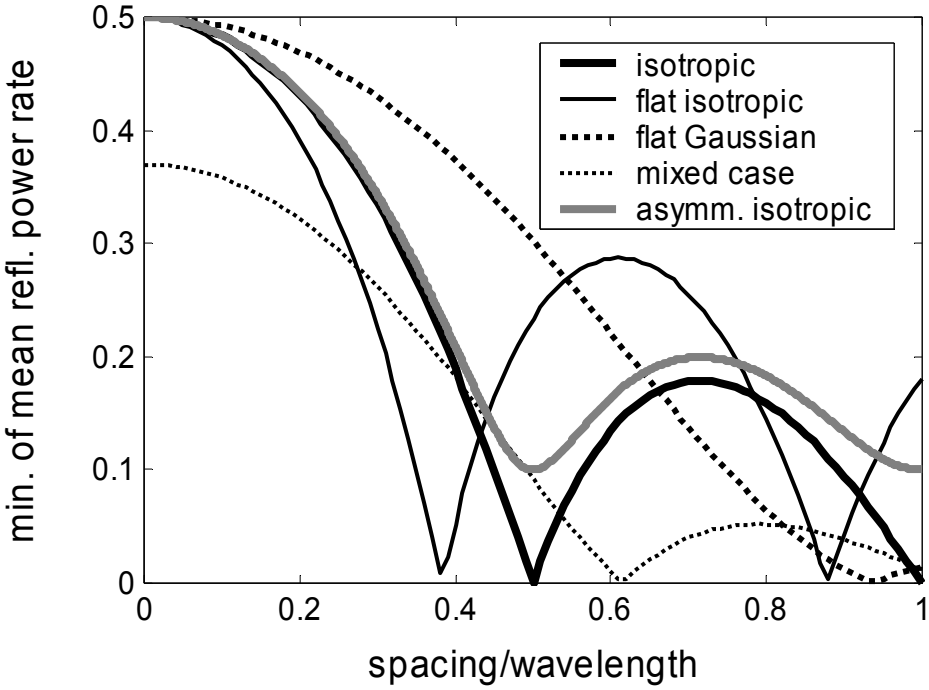


Fig. 2. The dependency of the minimum mismatch on element separation for an antenna pair with different basic element patterns.