

THE DEPENDENCY OF PATTERN CORRELATION ON MUTUAL COUPLING AND LOSSES IN ANTENNA ARRAYS

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ABSTRACT: *The element pattern correlation explains the existence of correlation between the signals received by different antenna elements in an antenna array. Usually the array losses are ignored. A more realistic model for pattern correlation in a two-element array including losses is presented and discussed.*

1. INTRODUCTION

The connection between element patterns and mutual coupling can be used to estimate the correlation between the RF signals in the array branches [1]. The pattern correlation matrix is a basic array characteristic, characterizing the correlation between the RF signals in different array branches when the signals arrive from many different directions. It does not depend on the propagation environment, and estimates the lowest limit of signal correlation for the given array. In past years there has been an increasing interest in this problem mainly due to the intense work on MIMO links [2-4]. Usually the losses are not taken into account. However, in practical use of mobile terminals different loss mechanisms are present and in more realistic models they need to be included. Furthermore, to obtain and use the set of element patterns, with phase and polarization information defined in all spherical directions, is not as convenient as to obtain and use the measured or calculated scattering matrix. Therefore, as in [2] and [3], the voltage wave and scattering matrix formulation is used to describe mutual coupling, and a more general case including the effect of power dissipation is introduced.

2. THEORY

The basic dependency of the pattern correlation matrix on the scattering matrix in the lossless case was presented in [2]. For a lossless array the radiated power P_{rad} can be calculated as

$$P_{rad} = \vec{\mathbf{a}}^H \mathbf{F}^H \mathbf{F} \vec{\mathbf{a}} = \vec{\mathbf{a}}^H (\mathbf{I} - \mathbf{S}^H \mathbf{S}) \vec{\mathbf{a}} = P_{in} - P_{refl}, \quad (1)$$

where $\vec{\mathbf{a}}$ is the input voltage wave vector, \mathbf{S} is the scattering matrix and \mathbf{F} is the array pattern matrix and H denotes a hermitian transpose. P_{in} is the input and P_{refl} the reflected power. The array pattern matrix \mathbf{F} is defined for voltage wave inputs and it contains the active element patterns of the elements in an array. The basic matrix expression for the power balance for a lossless array is [2]

$$\mathbf{I} - \mathbf{S}^H \mathbf{S} = \mathbf{I} - \mathbf{P}_{refl} = \mathbf{S}^H \mathbf{S} = \mathbf{F}^H \mathbf{F} = \mathbf{P}_{rad}. \quad (2)$$

Matrix $\mathbf{S}^H \mathbf{S}$ is the reflected power rate matrix, denoted \mathbf{P}_{refl} , and matrix $\mathbf{F}^H \mathbf{F}$ is the radiated power rate matrix, denoted \mathbf{P}_{rad} . They give the reflected and radiated powers relative to the input power with $P_{in} = \mathbf{a}^H \mathbf{a} = 1$. Further, the array element patterns in (2) can be normalized to unity norm ($\mathbf{F} \rightarrow \mathbf{F}_0$) and this leads to a correlation matrix equation

$$\mathbf{R}_{pat} = \mathbf{D}^{-1} (\mathbf{I} - \mathbf{S}^H \mathbf{S}) \mathbf{D}^{-1} = \mathbf{D}^{-1} \mathbf{F}^H \mathbf{F} \mathbf{D}^{-1} = \mathbf{F}_0^H \mathbf{F}_0, \quad (3)$$

where \mathbf{R}_{pat} is the pattern correlation matrix and \mathbf{D} is a diagonal matrix containing the square roots of the diagonal elements of $\mathbf{F}^H \mathbf{F}$.

The presented model in [2] does not take the losses into account. Because the system is linear, we can write similarly to (2) for the more general case with losses

$$\mathbf{I} - \mathbf{S}_L^H \mathbf{S}_L = \mathbf{L}^H \mathbf{L} + \mathbf{F}_L^H \mathbf{F}_L, \quad (4)$$

where subscript L denotes losses and $\mathbf{L}^H \mathbf{L}$ is a Hermitian matrix of dissipated power [5], which depend on the losses in the array structure (material losses) and in the near-field region (obstacles, human body, etc.). If we have the scattering matrix and the array element pattern matrices with true power scaling then we can find the matrix of dissipated power rate as

$$\mathbf{P}_{loss} = \mathbf{L}^H \mathbf{L} = \mathbf{I} - \mathbf{S}_L^H \mathbf{S}_L - \mathbf{F}_L^H \mathbf{F}_L. \quad (5)$$

Similarly to \mathbf{P}_{refl} and \mathbf{P}_{rad} the matrix of dissipated power $\mathbf{L}^H \mathbf{L}$ contains linear relations depending on port inputs. In the simplest case $\mathbf{L}^H \mathbf{L}$ is a diagonal matrix. In general \mathbf{L} cannot be solved if the array structure is not known in more detail, and thus we consider only $\mathbf{L}^H \mathbf{L}$. With non-zero losses the pattern correlation is

$$\mathbf{R}_{pat,L} = \mathbf{D}_L^{-1} (\mathbf{I} - \mathbf{S}_L^H \mathbf{S}_L - \mathbf{L}^H \mathbf{L}) \mathbf{D}_L^{-1} = \mathbf{D}_L^{-1} \mathbf{P}_{rad,L} \mathbf{D}_L^{-1} = \mathbf{D}_L^{-1} \mathbf{F}_L^H \mathbf{F}_L \mathbf{D}_L^{-1} = \mathbf{F}_{L,0}^H \mathbf{F}_{L,0}, \quad (6)$$

where \mathbf{D}_L contains the roots of the diagonals of $\mathbf{F}_L^H \mathbf{F}_L$. There is no principal difference between Equations (3) and (6), which is seen when we define a matrix of radiated power rate $\mathbf{P}_{rad} = \mathbf{I} - \mathbf{S}^H \mathbf{S}$ and place it in (3), or in the case with losses we define $\mathbf{P}_{rad,L} = \mathbf{I} - \mathbf{S}_L^H \mathbf{S}_L - \mathbf{L}^H \mathbf{L}$ and place it in (6). The result is always a Hermitian matrix and its effect on radiated power is defined in the same manner. However, the practical difference between (3) and (6) is significant. In practice only the scattering matrix can be measured in a convenient way, whereas the element pattern matrix \mathbf{F} would be difficult to measure for all directions with two polarizations and to calibrate it with the true power, and thus (5) is an unrealistic way to calculate $\mathbf{L}^H \mathbf{L}$.

The impedance matrix representation is an alternative to the scattering matrix formulation of the presented problem [4,6]. However, to describe the losses, instead of adding a term to the impedance matrix, the real part of the impedance matrix should be divided into two parts, separating array and radiation losses.

When we add lossy material to the antenna array structure, the scattering matrix, matrix of losses, and pattern matrices change, and as a result the patterns can become more or less correlated, and the matching can become better or worse. The diagonal elements of \mathbf{P}_{refl} define the mean value of the reflected power in the adaptive use of the array, when the signal strengths and delays in the array branches are changing. The mean radiation efficiency of the array in general can get only worse by adding lossy material, if the changes in scattering matrix are negligible, because the diagonals of a Hermitian matrix describing power loss are always real and positive. But, if the array is used with some preferred inputs, the radiation efficiency can also become greater if off-diagonal elements compensate each other. We see in (5) that the correlation matrix can be diagonal for an unmatched lossy array, if the matrices $\mathbf{S}^H \mathbf{S}$ and $\mathbf{L}^H \mathbf{L}$ have opposite off-diagonal elements. In a lossless array the cross-correlation is zero according to (3) if the array is fully matched (i.e. $\mathbf{S} = 0$), but according to (6) in a lossy array perfect matching ($\mathbf{S}_L = 0$) is not a criterion for zero cross-correlation, because even then $\mathbf{L}^H \mathbf{L}$ can have non-zero off-diagonal elements. If we have a two-element array of a symmetric antenna pair, then the off-diagonal values of \mathbf{P}_{rad} are real [2]. The same symmetry rule is obviously true also for the matrix of lost power $\mathbf{L}^H \mathbf{L}$.

3. SIMULATIONS

Let us consider the example of an asymmetric two-element array where the elements are as well matched. This means, that \mathbf{P}_{refl} can have complex-valued off-diagonal elements. Let the two diagonal elements of $\mathbf{P}_{\text{refl}} = \mathbf{S}^H \mathbf{S}$ be 0.2 and the mutual terms be $\{\mathbf{S}^H \mathbf{S}\}_{12} = \{\mathbf{S}^H \mathbf{S}\}_{21}^* = 0.2e^{j\pi(70^\circ/180^\circ)}$. Due to hermittivity $\mathbf{L}^H \mathbf{L}$ consists

of real-valued diagonal elements and transpose-conjugated off-diagonal elements. Let the matrix components of $\mathbf{L}^H \mathbf{L}$ have equal amplitudes and changing phases in the off-diagonal elements. Let us consider three different cases of amplitudes with 0.1, 0.2 or 0.4 for the components of $\mathbf{L}^H \mathbf{L}$. The case without losses is given as a reference. The pattern correlation value $|R(1,2)| = r_{12}$ is presented in Fig. 1 in each case depending on the phase difference between the mutual components of $\mathbf{L}^H \mathbf{L}$ and $\mathbf{S}^H \mathbf{S}$. We see, that depending on the phase relations of the matrices $\mathbf{S}^H \mathbf{S}$ and $\mathbf{L}^H \mathbf{L}$, the pattern correlation r_{12} can decrease as well as increase. When the amplitudes of the off-diagonal elements of $\mathbf{S}^H \mathbf{S}$ and $\mathbf{L}^H \mathbf{L}$ are of equal strength, the radiated power rate and correlation matrices $\mathbf{P}_{rad,L}$ and \mathbf{R}_{pat} can be diagonalized due to compensation with opposite values (phase of $\{\mathbf{L}^H \mathbf{L}\}_{12}$ minus phase of $\{\mathbf{S}^H \mathbf{S}\}_{12} = 250^\circ - 70^\circ = 180^\circ$). With equal amplitudes of $\mathbf{S}^H \mathbf{S}$ and $\mathbf{L}^H \mathbf{L}$ the level of correlation can be twice the correlation caused only by $\mathbf{S}^H \mathbf{S}$. Because the system is passive and can only consume power, the off-diagonal values $\mathbf{P}_{rad,L} = \mathbf{I} - \mathbf{S}_L^H \mathbf{S}_L - \mathbf{L}^H \mathbf{L}$ for two-element arrays with equal self-matching and self losses are limited by

$$|\mathbf{P}_{rad}(1,2)| = |\mathbf{P}_{rad}(2,1)| \leq 1 - |\mathbf{P}_{rad}(1,1)| = 1 - |\mathbf{P}_{rad}(2,2)|. \quad (7)$$

Figure 2 shows the eigenvalues of \mathbf{P}_{rad} in each case of the loss level presented in Fig. 1. These eigenvalues give the radiated power rates when the corresponding eigenvectors are the array inputs. The case when the pattern correlation is unity in Fig. 1 is the case when there is only one radiating mode with radiation eigenvalue 1 in Fig. 2 and when the other eigenvalue is zero. Let us look further the case, where the minimum correlation is zero, which occurs when scattering and loss levels are equal and the phase differences between the phases of off-diagonal

elements of \mathbf{P}_{refl} and $\mathbf{L}^H \mathbf{L}$ are 180° (then the phase shift of $\{\mathbf{L}^H \mathbf{L}\}_{12}$ is 250°). The eigenvalues of the radiated power rate matrix \mathbf{P}_{rad} are both smaller than one. This demonstrates, that the matching can be far from perfect, even when the pattern correlation is zero. The mean of eigenvalues of \mathbf{P}_{rad} is the mean gain of the array in adaptive use without any preferred input/output, and according to Fig. 2 it does not depend on the phase of the off-diagonal element of \mathbf{P}_{rad} , and it is always decreasing with increasing losses. The means of the eigenvalue pairs presented in Fig. 2 are 0.8, 0.7, 0.6 and 0.5 in order of increasing losses. Fig. 1 shows that it is possible that the correlation decreases with increasing losses, but it also shows the general tendency of pattern correlation to increase. If the antenna pair is physically symmetric and the added losses are asymmetric (i.e. the dissipating material is not very close to the array), then Fig. 1 and Fig. 2 are realistic. But if the pair and the added losses are symmetric, then only the phase differences of 0° or 180° are possible, with lowest or highest level of pattern correlation and with maximum or minimum difference between the reflected power eigenvalues.

4. CONCLUSION

An exact matrix expression is presented for the dependency of the pattern correlation on the reflected power and dissipated power rates. In the lossless case the scattering matrix defines the pattern correlation matrix exactly. However, in the more realistic case of lossy arrays, i. e. for handheld devices, the losses need to be taken into account. Since in the general case the losses cannot be determined exactly, the pattern correlation can only be estimated.

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Figure Captions:

Fig. 1. Pattern correlation in some asymmetric and lossy two-element array.

Fig. 2. Eigenvalues λ_1 and λ_2 of radiated power in the lossy two-element array.

Figure 1

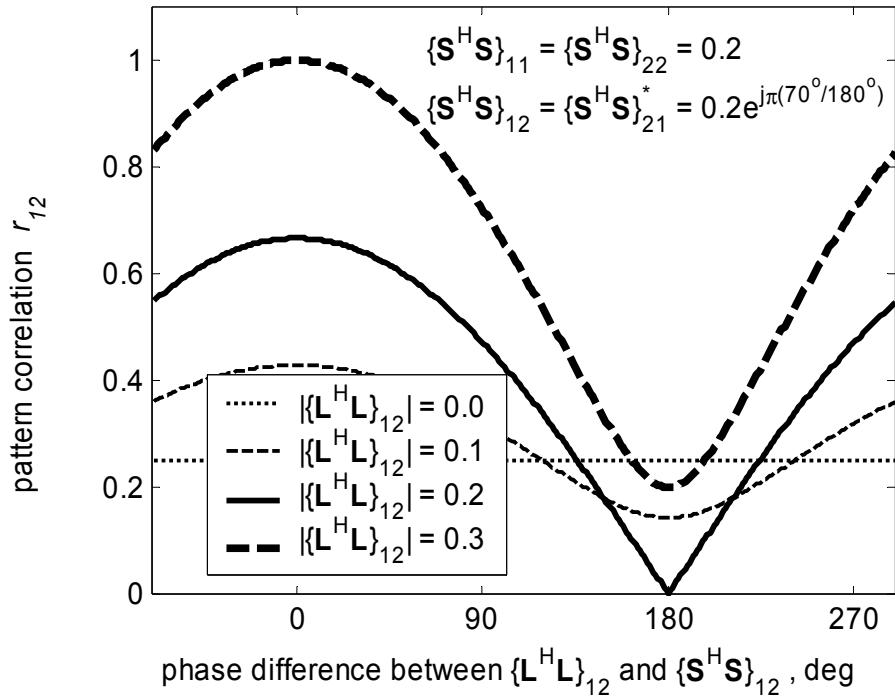
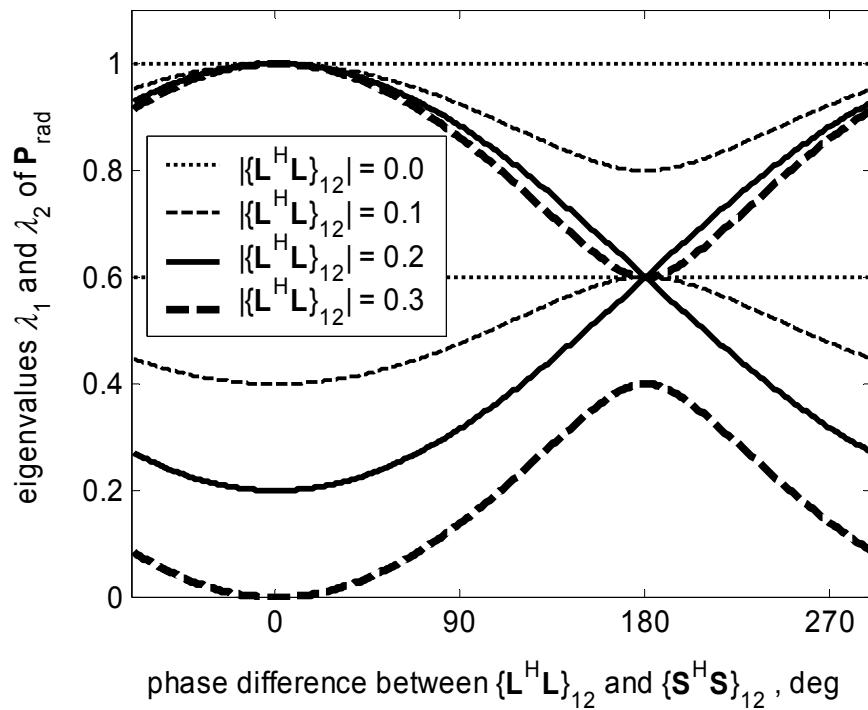


Fig. 1. Pattern correlation in some asymmetric and lossy two-element array.

Figure 2

Fig. 2. Eigenvalues λ_1 and λ_2 of radiated power in the lossy two-element array.