Helsinki University of Technology Radio Laboratory Publications

Teknillisen korkeakoulun Radiolaboratorion julkaisuja

Espoo, July 2006 REPORT S 278

STATISTICAL ANALYSIS OF THE WIRELESS PROPAGATION CHANNEL AND ITS MUTUAL INFORMATION

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Dissertation for the degree of Doctor of Science in Technology to be presented with due permission for public examination and debate in Auditorium S5 at Helsinki University of Technology (Espoo, Finland) on the 19th of July 2006 at 12 o'clock noon.

Helsinki University of Technology

Department of Electrical and Communications Engineering

Radio Laboratory

Teknillinen korkeakoulu

Sähkö- ja tietoliikennetekniikan osasto

Radiolaboratorio

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FI-02015 TKK
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ISBN 951-22-8246-1 (printed)
ISBN 951-22-8247-X (electronic)

http://lib.tkk.fi/Diss/

ISSN 1456-3835

Otamedia Oy
Espoo 2006

Preface

The research work that lead to this thesis was carried out in the Radio Laboratory of Helsinki University of Technology (TKK) during 2003-2006. A substantial part of the work was completed in the inspiring atmosphere of the Institute of Communications and Radio-Frequency Engineering of Vienna University of Technology between October 2004 and July 2005. The work was funded by National Technology Agency of Finland (TEKES) and Academy of Finland via its GETA postgraduate school and Centre of Excellence program. The additional financial support from Nokia Foundation, Foundation of Commercial and Technical Sciences and Research Foundation of Helsinki University of Technology is gratefully acknowledged.

I thank my thesis supervisors Dr. Hassan El-Sallabi and Prof, Pertti Vainikainen for their numerous helpful comments, guidance, and continuous support. The effort and encouraging feedback of the two thesis pre-examiners, Prof. Jørgen Bach Andersen and Prof. Alister Burr, is also greatly appreciated.

As co-authors, Pasi Suvikunnas, Lasse Vuokko and Filip Mikas have significantly contributed to the thesis. Our collaboration has been very fruitful and smooth, many thanks for that.

Sorting out all the colleagues and friends whom to thank is a task more formidable than the thesis itself, so I'll just skip the list here and thank you in person.

The warmest of thanks go to my parents, grandparents, brother and sister, who have been for years wondering what I'm doing at the university. Partial answer may be found from this thesis.

Finally, Mei Yen has been there to support me regardless of whether it was shining, raining or snowing. Thank you so much.

Espoo, June 18, 2006

Jari Salo

Abstract

This thesis considers the statistical analysis of the mobile radio propagation channel and certain functions of it. First, mutual information of the fading multiple-input multiple-output (MIMO) wireless channel is analyzed. Specifically, an approximate distribution of mutual information for Rayleigh fading MIMO channels as well as an upper bound for the ergodic mutual information for Rician fading channels are derived. A novel decomposition of mutual information at high SNR is also considered. The decomposition also leads to an appealing measure for spatial multiplexing efficiency, or multipath richness, of a MIMO radio channel. As the second topic, the amplitude distribution function of the general multiple scattering radio channel is derived. Rayleigh, Rice, and double-Rayleigh distributions are special cases of the general result. In the third topic, an additive model is proposed as a physical basis for shadow fading – as an alternative to the conventional multiplicative one. A simple analytical justification for the log-normality of small area shadow fading is provided. The additive model is also shown to be supported by radio channel measurements.

Tiivistelmä

Työ käsittelee siirtyvän tietoliikenteen radiokanavan ja tiettyjen siihen liittyvien funktioiden tilastollista analyysiä. Ensimmäisenä aiheena analysoidaan häipyvän ns. moniantenniradiokanavan keskinäisinformaatiota. Tarkemmin eriteltynä työssä johdetaan likimääräinen lauseke Rayleigh-häipyvän moniantennikanavan keskinäisinformaation todennäköisyysjakaumalle. Lisäksi johdetaan yläraja Rice-häipyvän kanavan ergodiselle keskinäisinformaatiolle, sekä uudentyyppinen keskinäisinformaatiohajotelma, joka pätee korkeilla signaalikohinasuhteilla. Hajotelman perusteella ehdotetaan myös tunnuslukua moniantennikanavan ns. multipath richnessille. Toisena aiheena työssä johdetaan yleistetyn monisirontakanavan amplitudin todennäköisyysjakauma. Rayleigh-, Rice- ja kaksois-Rayleigh-jakaumat ovat yleisen tuloksen erikoistapauksia. Työn kolmannessa aiheessa osoitetaan että myös summamalli – verrattuna perinteiseen tulomalliin – voi olla selityksenä ns. hitaan häipymän lognormaaliuteen. Työssä esitetään mittaustuloksia, jotka tukevat summamallia.

Contents

List of Original Publications

[P1] J. Salo, F. Mikas and P. Vainikainen, "An Upper Bound on the Ergodic Mutual Information in Rician Fading MIMO Channels," *IEEE Transactions on Wireless Communications,* vol. 5, no. 6, pp. 1415–1421, Jun. 2006.

[P2] J. Salo, P. Suvikunnas, H. M. El-Sallabi and P. Vainikainen, "Approximate Distribution of Capacity of Rayleigh Fading MIMO Channels," *Electronics Letters,* vol. 40, no. 12, pp. 741–742, Jun. 2004.

[P3] J. Salo, P. Suvikunnas, H. M. El-Sallabi and P. Vainikainen, "Some Insights into MIMO Mutual Information: the High SNR Case," to appear in *IEEE Transactions on Wireless Communications.*

[P4] J. Salo, P. Suvikunnas, H. M. El-Sallabi and P. Vainikainen, "Ellipticity Statistic as a Measure of MIMO Multipath Richness," *Electronics Letters,* vol. 42, no. 3, pp. 45–46, Feb. 2006.

[P5] J. Salo, H. M. El-Sallabi, and P. Vainikainen, "Distribution of the Product of Independent Rayleigh Random Variables," *IEEE Transactions in Antennas and Propagation*, vol 54, no. 2, pp. 639 - 643, Feb. 2006.

[P6] J. Salo, H. M. El-Sallabi, and P. Vainikainen, "Statistical Analysis of the Multiple Scattering Radio Channel," to appear in *IEEE Transactions on Antennas and Propagation.*

[P7] J. Salo, H. M. El-Sallabi and P. Vainikainen, "Impact of Double-Rayleigh Fading on System Performance," *in Proc. IEEE International Symposium on Wireless Pervasive Computing,* Phuket, Thailand, Jan. 16-18, 2006, pp. 1-5.

[P8] J. Salo, L. Vuokko and P. Vainikainen, "Why is Shadow Fading Lognormal?" *Proc. International Symposium on Wireless Personal Multimedia Communications,* Aalborg, Denmark, Sept. 18-22, 2005, pp. 522-526.

[P9] J. Salo, L. Vuokko, H. M. El-Sallabi, and P. Vainikainen, "An Additive Model as a Physical Basis for Shadow Fading," to appear in *IEEE Transactions on Vehicular Technology.*

List of Abbreviations

The following abbreviations are used in the summary part of this thesis.

AWGN	Additive White Gaussian Noise
cdf	cumulative distribution function
CSI	Channel State Information
CSI-RX	Channel State Information at the receiver
MI	Mutual Information
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
NLOS	Non-Line-Of-Sight
LOS	Line-Of-Sight
pdf	probability density function
RF	Radio Frequency
RX	Receiver
SIMO	Single-Input Multiple-Output
SNR	Signal-to-Noise-Ratio
TDMA	Time Domain Multiple Access
TX	Transmitter

List of Symbols

The following symbols are used in the summary part of this thesis.

$\sim$	equivalence in distribution, 'distributed as'
$\otimes$	Kronecker product
$\{\mathbf{x}_t\}_{t=1}^{T}$	a set $\{\mathbf{x}_1, \ldots, \mathbf{x}_T\}$, $\{\mathbf{x}_t\} \equiv \{\mathbf{x}\}_{t=1}^{\infty}$
$\mathbf{A}^T$	transpose of $\mathbf{A}$
$\mathbf{A}^H$	conjugate transpose of $\mathbf{A}$
$\lvert\mathbf{A}\rvert$	determinant of $\mathbf{A}$
$\lVert\mathbf{A}\rVert_{\mathrm{F}}$	Frobenius norm of $\mathbf{A}$
$\mathrm{cov}[\mathbf{x}]$	covariance matrix of $\mathbf{x}$
$\mathrm{diag}(\mathbf{a})$	diagonal matrix with elements of $\mathbf{a}$ on the diagonal
$\mathrm{etr}(\mathbf{A})$	$\exp[\mathrm{tr}(\mathbf{A})]$
$\mathrm{E}_X[g(X)]$	expected value of $g(X)$ with respect to random variable X
$\mathbf{I}_n$	$n \times n$ identity matrix
$\lg(x)$	base-10 logarithm of x
$\ln(x)$	natural logarithm of x
$\log_2(x)$	base-2 logarithm of x
$\Pr(A)$	probability of event A
$\mathrm{rank}(\mathbf{A})$	rank of $\mathbf{A}$
$\mathrm{tr}(\mathbf{A})$	trace of $\mathbf{A}$
$\mathrm{var}[X]$	variance of X
$\mathrm{vec}(\mathbf{A})$	columnwise vectorization of $\mathbf{A}$
$\mathbf{A}$	matrix (boldface upper case)
$\mathbf{a}$	column vector (boldface lower case)
$C(\Theta)$	ergodic capacity for deterministic channel parameters Θ
$C_p(\Theta)$	outage capacity for deterministic channel parameters Θ and outage probability p

$\mathbb{C}$	field of complex numbers
$\mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Theta})$	matrix variate complex Gaussian distribution with $n \times m$ mean matrix $\mathbf{M}$, $n \times n$ column vector covariance matrix $\mathbf{\Sigma}$ and $m \times m$ row vector covariance matrix $\mathbf{\Theta}$
$\mathcal{CN}(\mathbf{\Sigma})$	n-variate zero-mean complex Gaussian distribution with $n \times n$ covariance matrix $\mathbf{\Sigma}$, i.e., $\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}, 1)$
$f_X(x)$	probability density function (pdf) of X
$F_X(t)$	cumulative distribution function (cdf) of X
$I(\Theta)$	ergodic mutual information for deterministic channel parameters Θ
$I_p(\Theta)$	outage mutual information for deterministic channel parameters Θ and outage probability p
j	imaginary unit, $j = \sqrt{-1}$
K	$\min(n_\mathrm{r}, n_\mathrm{t})$
L	$\max(n_\mathrm{r}, n_\mathrm{t})$
n_r	number of receive antennas
n_t	number of transmit antennas
P	total power of the transmitted signal
ρ	signal-to-noise-ratio, $\rho = \frac{P}{\sigma^2}$
σ^2	noise variance at the output of receiver antenna
t	discrete time index

Chapter 1

Introduction

The archetypal task of a communications engineer is to design a transmitter and a receiver that are able to realize a desired quality of service of communication. To accomplish this task the engineer has to take into account the properties of the communications channel linking the transmitter and the receiver. In radio communications the data rate currently achievable with practical technology is in the order of few dozens of megabits per second. The main obstacle in increasing the data rate is the radio channel that gives rise to a variety of design challenges not easily overcome. One of the greatest discoveries in communications theory is that the fundamental limit, i.e., channel capacity, is not dependent on the actual transmitter or receiver technology used; it depends solely on the properties of the communications channel itself [1]. It is, therefore, of great interest to research the radio communications channel in order to determine these limits, and to answer many other questions of practical importance.

In many radio systems the channel varies in time and can typically only be modelled as a random entity. It then follows that the analysis of the channel properties, such as capacity and bit error probability, has to be carried out using statistical tools. In this thesis, a small subset of such channel functions of the random radio channel is analyzed. The focus is on the following three topics.

Topic 1: Statistical analysis of the mutual information of a radio channel with multiple antennas both at the transmitter and receiver link ends. Mutual information is a quantity that essentially defines the greatest rate of information transmission a communications system can achieve. Formulas that allow a simple estimation of mutual information for a common type of fading radio channel are derived. Some insight into the mathematical structure of mutual information is provided.

Topic 2: Statistical analysis of the multiple scattering radio channel. It has been theorized that under certain radio propagation conditions the radio waves must propagate to the receiver through a "keyhole". This leads to a multiplicative process called double scattering, where two sums of incoherent sinusoidal waveforms multiply each other. In this thesis, probability distribution functions for the amplitude of a radio signal received over a more generalized multiple scattering channel are derived. This function is a necessary prerequisite for many types of theoretical and measurement analysis one might wish to perform, such as evaluation of the performance of communications systems.

Topic 3: Consider a mobile terminal moving in an urban environment. Suppose that the received signal power is averaged in logarithmic scale over a distance of about $5-10$ meters and denote this average with P_1. Collecting a set of such averages, say $P_1, \ldots, P_{1000}$, and plotting its histogram, it has been observed in numerous measurements conducted worldwide that the histogram practically always resembles the Gaussian distribution. Yet, no widely accepted theory for exactly why this happens has been so far presented. This variation of the average power is called shadow fading. This thesis proposes a theory stating that the Gaussian nature of shadow fading may be caused by the summation of plane waves whose powers are change slowly as the receiver moves. This hypothesis is of interest because it is based on an additive signal model, whereas most of the earlier proposed theories are based on a multiplicative one.

The thesis is presented in the compendium format commonly used at the Helsinki University of Technology. Chapters 2–5 give an overview and literature survey of the research topics considered in this thesis. More specifically, in Chapter 2 the mutual information of MIMO communications channels is discussed (Topic 1). Chapter 3 introduces the essentials of multiple scattering radio channels (Topic 2). Chapter 4 gives an overview of known results concerning shadow fading (Topic 3). In addition, at the end of each of these chapters the related main research results are summarized. The details can be found in the original papers found at the end of this thesis. Chapter 5 concludes the summary part of the thesis. Some definitions are collected in the appendices for easy reference.

Contributions of the author

In [P1]–[P9], the first author mentioned wrote the manuscripts and derived the original results, except as noted in the following. In [P1], Filip Mikas assisted in the derivation of the main theorem. In [P3]–[P4], the idea of separating the effects of MIMO "eigenvalue dispersion" and SNR on mutual information was conceived in discussions with Pasi Suvikunnas. In [P4], Pasi Suvikunnas pre-processed the measured channel matrices. In [P8]–[P9], Lasse Vuokko pre-processed and clustered the channel measurement data. In [P1]–[P9], all co-authors commented on the manuscripts. In [P9], comments from Hassan El-Sallabi considerably improved the presentation of the manuscript. Pertti Vainikainen and Hassan El-Sallabi supervised the work.

Chapter 2

Mutual Information of Wireless MIMO Channels

This chapter pertains to publications [P1]–[P4]. First, in Section 2.1 the definitions of mutual information and capacity in multiple-input multiple-output (MIMO) channels are given. The focus is on the topics surrounding those studied in this thesis and thus omit, for example, multiuser systems and frequency-selective MIMO channels. Section 2.2 summarizes the papers [P1]–[P4].

The probability distributions used in this thesis have been summarized in Appendix A. A brief recapitulation is in order: $\mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Theta})$ denotes a matrix variate complex Gaussian distribution with $n_\mathrm{r} \times n_\mathrm{t}$ mean matrix $\mathbf{M}$, $n_\mathrm{r} \times n_\mathrm{r}$ column covariance matrix $\mathbf{\Sigma}$ and $n_\mathrm{t} \times n_\mathrm{t}$ row covariance matrix $\mathbf{\Theta}$, i.e., $\mathbf{H} \sim \mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Theta})$ has $\mathrm{cov}[\mathrm{vec}(\mathbf{H})] = \mathbf{\Sigma} \otimes \mathbf{\Theta}$, and $\mathrm{E}[\mathbf{H}] = \mathbf{M}$. Notation $\mathbf{x} \sim \mathcal{CN}(\mathbf{\Sigma})$ means that $\mathbf{x}$ has an n-variate zero-mean complex Gaussian distribution with $n \times n$ covariance matrix $\mathrm{cov}[\mathbf{x}] = \mathbf{\Sigma}$.

2.1 Problem Formulation and Relevant Literature

Consider the classic model for a communication system shown in Figure 2.1. From an information theoretic perspective the transmitter is just a random number generator that produces a channel input vector $\mathbf{x}_t \in \mathbb{C}^{n_\mathrm{r} \times 1}$ at a discrete time instant t from a probability law defined by $f_\mathbf{x}(\mathbf{x})$. The channel output at time t is given by the relation

$$\mathbf{y}_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{n}_t \,, \quad t = 1, 2, \ldots \tag{2.1}$$

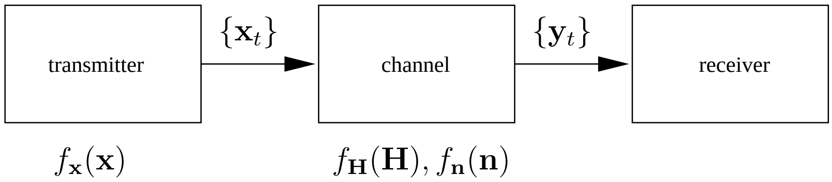

Figure 2.1: A MIMO communication system.

Here $\mathbf{H}_t \in \mathbb{C}^{n_\mathrm{r}\times n_\mathrm{t}}$ is a channel matrix and $\mathbf{n}_t \in \mathbb{C}^{n_\mathrm{t}\times 1}$ is a noise vector. The channel properties are defined by the distribution of the channel matrix, denoted by $f_{\mathbf{H}}(\mathbf{H})$, and the distribution of the additive noise, denoted by $f_{\mathbf{n}}(\mathbf{n})$. The following assumptions are retained throughout:

A1 Channel input $\mathbf{x}_t$ satisfies $\mathrm{E}[\mathbf{x}^H\mathbf{x}] = P$. It is also independent of channel variables $\mathbf{H}_t$ and $\mathbf{n}_t$ for all values of t;

A2 Channel noise $\mathbf{n}_t$ is independent between time instants and distributed according to $\mathcal{CN}(\sigma^2\mathbf{I}_{n_\mathrm{r}})$. It is also independent of $\mathbf{x}_t$ and $\mathbf{H}_t$ for all values of t;

A3 Channel distribution satisfies $\mathrm{E}[\|\mathbf{H}\|_\mathrm{F}^2] = n_\mathrm{r}n_\mathrm{t}$.

It is assumed that the length of the transmitted code word is infinite. With assumptions A1–A3, the maximum achievable information rate over the channel depends on $f_{\mathbf{x}}(\mathbf{x})$, $f_{\mathbf{H}}(\mathbf{H})$, σ^2, and the available knowledge of $\mathbf{H}_t$ at the receiver. A special case is the deterministic channel, or AWGN channel, for which $\mathbf{H} \sim \mathcal{CN}(\mathbf{H}, \mathbf{0}, \mathbf{0})$.

Definition 1 (Deterministic channel) *The channel matrix $\mathbf{H}_t$ is a deterministic constant over all values of t. This is called the AWGN channel.*

A MIMO AWGN channel may be a good model, for example, in satellite links that transmit information using orthogonal polarizations.

Assuming that $\mathbf{H}$ is random, the two extreme cases of channel coherence are defined in Definition 2 and Definition 3. Note that the term “fast fading” has a slightly different meaning than in communications where “fast” refers to a case where the channel varies within a symbol interval. Here it is assumed that the channel does not change during a symbol interval.

Definition 2 (Fast fading channel) *An independent realization of $\mathbf{H}_t$ is drawn from $f_{\mathbf{H}}(\mathbf{H})$ for all values of t. The transmitted code word spans over an infinite number of channel realizations.*

The fast fading channel is a reasonable model in systems where the transmitted code words are very long with respect to the channel coherence time.

Definition 3 (Slow fading channel) *An independent realization of* $\mathbf{H}_t$ *is drawn from* $f_{\mathbf{H}}(\mathbf{H})$ *once only and then remains constant for all values of* t*. The transmitted code word spans over one random channel realization only.*

The slow fading channel is a good model when the channel can be assumed to be constant during the transmission of a single code word.

Mutual information (MI) forms the basis for evaluating the upper performance limits for all communication systems. The following definition, quoted here for completeness, applies to arbitrary continuous channel input and output distributions [2].

Definition 4 (Mutual information) *The mutual information between two random variables* $\mathbf{x}$ *and* $\mathbf{y}$ *with joint density* $f_{\mathbf{xy}}(\mathbf{x}, \mathbf{y})$ *is defined as*

$$I = \int f_{\mathbf{xy}}(\mathbf{x}, \mathbf{y}) \log \frac{f_{\mathbf{xy}}(\mathbf{x}, \mathbf{y})}{f_{\mathbf{x}}(\mathbf{x}) f_{\mathbf{y}}(\mathbf{y})} \, \mathrm{d}\mathbf{x} \, \mathrm{d}\mathbf{y} \, . \tag{2.2}$$

By convention, the logarithm is usually taken to base 2. As the focus of this thesis is mostly on the analysis of fading MIMO channels, mutual information in (2.2) will be a function of the distribution of the channel matrix.

Suppose that the channel input distribution is fixed. For a random channel two notions of mutual information arise. For the fast fading channel the maximum achievable information rate is given by the *ergodic mutual information*, while for the slow fading channel the maximum achievable information rate can be characterized by means of the *outage mutual information* [3]. In both cases the available channel knowledge at the receiver plays an important role. Note that channel knowledge potentially available at the transmitter is irrelevant, as it is not used since $f_{\mathbf{x}}(\mathbf{x})$ is assumed to be fixed.

The simplest case arises when the receiver knows $\mathbf{H}_t$ perfectly; this is condition is called full receiver channel state information, or full RX-CSI, for short. For the AWGN channel from Definition 1, the ergodic MI is denoted by $I(\rho, \mathbf{H})$. For example, assuming that A1–A3 hold and $\mathbf{x} \sim \mathcal{CN}(\frac{P}{n_{\mathrm{t}}}\mathbf{I}_{n_{\mathrm{t}}})$ it is given by [4, 5]

$$\begin{aligned} I(\rho, \mathbf{H}) &= \log_2 \left(\frac{|\operatorname{cov}[\mathbf{x}]| |\operatorname{cov}[\mathbf{y}]|}{\left| \operatorname{cov}\left[[\mathbf{x}^T \ \mathbf{y}^T]^T \right] \right|} \right) & (2.3) \\ &= \log_2 \left| \mathbf{I}_{n_{\mathrm{r}}} + \frac{\rho}{n_{\mathrm{t}}} \mathbf{H}\mathbf{H}^H \right| . & (2.4) \end{aligned}$$

Ergodic mutual information is defined by letting $\mathbf{H}$ be random, and considering the average of $I(\rho, \mathbf{H})$ over $\mathbf{H}$. The outage MI, in turn, is defined as a percentile of $I(\rho, \mathbf{H})$.

Definition 5 (Ergodic MI, full RX-CSI) *Fix the channel input distribution as $f_{\mathbf{x}}(\mathbf{x})$. Assume a fast fading channel that is perfectly known at the receiver for all values of t. Denote $\rho = \frac{P}{\sigma^2}$. The ergodic mutual information is defined as*

$$I(\rho) = \mathrm{E}_{\mathbf{H}}\left[I(\rho, \mathbf{H})\right] . \tag{2.5}$$

Definition 6 (Outage MI, full RX-CSI) *Fix the channel input distribution as $f_{\mathbf{x}}(\mathbf{x})$. Assume a slow fading channel that is perfectly known at the receiver. For an outage probability p, the outage mutual information $I_p(\rho)$ is defined by*

$$\Pr\left[I(\rho, \mathbf{H}) \leq I_p(\rho)\right] = p\,, \tag{2.6}$$

where $I(\rho, \mathbf{H})$ was given in (2.4).

From the above definitions it is clear that in order to evaluate $I(\rho)$ or $I_p(\rho)$ the channel distribution must be known, either explicitly or in the form of its characteristic or moment generating function. For Gaussian channel input where $\mathbf{x} \sim \mathcal{CN}\left(\frac{P}{n_{\mathrm{t}}}\mathbf{I}_{n_{\mathrm{t}}}\right)$ and $\mathbf{H} \sim \mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Theta})$ the results available on ergodic MI at the time of writing are summarized in Table 2.1. Only known exact results (or bounds if exact results are not available) and certain approximations related to this work are given in the table. The approximations are often simpler to compute numerically and often provide insight into the behavior of MI. The proof technique in [6] and [7] (see also [8]) is based on the Gross-Richards representation of the hypergeometric function of two matrix arguments [9]. Interestingly enough, although it is stated in [7] that the entries of $\mathbf{H}$ are assumed independent, it appears that the results therein hold also for one-sided spatial correlation, i.e., $\mathbf{H} \sim \mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{I}_L)$, .

A number of other asymptotic and non-asymptotic approximations can also be found in the literature [10–15].

In Table 2.2 the key results for outage mutual information for the Gaussian-input channel where $\mathbf{H} \sim \mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Theta})$ are listed. Various other approximations based on random matrix theory have also been derived [16]. These approximations hold asymptotically for a large SNR or large number of antennas and, in some cases, for almost arbitrary channel distributions [16]. Gaussian approximations are also available [7, 17, 18].

Channel capacity is defined as the supremum of mutual information over all channel input distributions.

Table 2.1: Summary of known results for ergodic MI, $\mathbf{x} \sim \mathcal{CN}(\frac{P}{n_t}\mathbf{I}_{n_t})$, full CSI at the receiver.

$f_{\mathbf{H}}(\mathbf{H})$	Results†
$\mathcal{CN}(\mathbf{0}, \mathbf{I}_{n_r}, \mathbf{I}_{n_t})$	Coincides with ergodic capacity [19]. $I(\rho)$ given in integral form in [19] and as a finite series involving exponential integrals in [14].
$\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}, \mathbf{\Theta})$	Exact $I(\rho)$ is given in [6, Th. 4] for the case where both $\mathbf{\Sigma}$ and $\mathbf{\Theta}$ have distinct eigenvalues. Upper bounds are given in [14] and [20].
$\mathcal{CN}(\mathbf{M}, \mathbf{I}_{n_r}, \mathbf{I}_{n_t})$	Exact result is given in [7]. A simple lower bound for $\text{rank}(\mathbf{M}) = 1$ appears in [21].
$\mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{I}_L)$	Upper bound is given in [P1]. The proof is based on the generalization of [22] to complex matrices. Exact ergodic MI is an open problem (but see [7]).
$\mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Theta})$	Exact solution is an open problem. Upper and lower bounds can be found in [23, 24].

† $K = \min(n_r, n_t)$, $L = \max(n_r, n_t)$. For a given transmit power P, $I(\rho)$ for $\mathbf{H} \sim \mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Theta})$ is the same as that of $\mathbf{H} \sim \mathcal{CN}(\mathbf{M}^H, \mathbf{\Theta}, \mathbf{\Sigma})$.

Definition 7 (Ergodic capacity) *Assume a fast fading channel that is perfectly known at the receiver for all values of t. The ergodic capacity is defined as*

$$C(\rho) = \sup_{f_{\mathbf{x}}} \mathrm{E}_{\mathbf{H}} \left[I(\rho, \mathbf{H}) \right] . \tag{2.7}$$

Given perfect CSI at the receiver and the average transmit power constraint (A1) the capacity-achieving channel input is $\mathbf{x} \sim \mathcal{CN}(\mathbf{Q})$. This is because Gaussian distribution has maximum entropy over all distributions with a given covariance matrix. Therefore, finding the capacity-achieving distribution in (2.7) is reduced to the problem of finding the optimal covariance matrix $\mathbf{Q}$, such that $\text{tr}(\mathbf{Q}) = P$. It should be noted, however, that for other channel input constraints, the optimal channel input is not, in general, Gaussian.

In the important special case of $\mathbf{H} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{n_r}, \mathbf{I}_{n_t})$, i.e., "Rayleigh iid channel", the capacity-achieving channel input is distributed as $\mathcal{CN}(\frac{P}{n_t}\mathbf{I}_{n_t})$, and the corresponding ergodic MI in Table 2.1 coincides with the channel capacity [19]. For the AWGN (deterministic) channel the optimum channel input is, in turn, $\mathcal{CN}(\mathbf{\Sigma})$, where the eigenvalues of $\mathbf{\Sigma}$ are determined based

Table 2.2: Summary of known results for the distribution of MI, $\mathbf{x} \sim \mathcal{CN}(\frac{P}{n_\mathrm{t}}\mathbf{I}_{n_\mathrm{t}})$, full CSI at the receiver.

$f_{\mathbf{H}}(\mathbf{H})$	Results[†]
$\mathcal{CN}(\mathbf{0}, \mathbf{I}_{n_\mathrm{r}}, \mathbf{I}_{n_\mathrm{t}})$	Gaussian approximations given in [17] and [18]. A high-SNR approximation in terms of the Meijer G-function appears in [P2].
$\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}, \mathbf{I}_L)$	Exact characteristic function of MI is given in [25] and [26]. Mean and variance for parametric density approximation are given in [26].
$\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}, \mathbf{\Theta})$	Exact distribution functions (pdf and cdf) for $K = 2, 3$ are given in single integral form in [27]. Moment generating function is derived in [6].
$\mathcal{CN}(\mathbf{M}, \mathbf{I}_{n_\mathrm{r}}, \mathbf{I}_{n_\mathrm{t}})$	Exact distribution functions for $K = 2$ and $\mathrm{rank}(\mathbf{M}) = 1$ are given in [28] in single integral form. Moment generating function, as well as mean and variance for parametric density approximation, can be found in [7].
$\mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Theta})$	Exact distribution is an open problem. High-SNR expressions for the mean and variance are derived in [23] for parametric density approximation.

[†] $K = \min(n_\mathrm{r}, n_\mathrm{t})$, $L = \max(n_\mathrm{r}, n_\mathrm{t})$. For a given transmit power P, the distribution of MI is the same for $\mathbf{H} \sim \mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Theta})$ and $\mathbf{H} \sim \mathcal{CN}(\mathbf{M}^H, \mathbf{\Theta}, \mathbf{\Sigma})$.

on the water-filling principle [19]. Key results on the ergodic capacity are collected in Table 2.3.

Definition 8 (Outage capacity) *Assume a slow fading channel that is perfectly known at the receiver. The outage capacity $C_p(\rho)$ for outage probability p is defined as*

$$C_p(\rho) = \sup_{f_{\mathbf{x}}} \left[I_p(\rho, \mathbf{H}) \right] . \tag{2.8}$$

As with the ergodic capacity, from the assumptions of perfect CSI at the receiver and the average power constraint A1, it follows that the channel input that achieves the outage capacity is $\mathbf{x} \sim \mathcal{CN}(\mathbf{Q})$. For the "Rayleigh iid" channel it is conjectured in [19] that $\mathbf{Q}$ is diagonal, possibly with some zero entries on the diagonal and the non-zero entries being equal. The optimal channel input is, in general, different in the ergodic and outage scenarios. In the general case of $\mathbf{H} \sim \mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Theta})$, the outage capacity is an open

Table 2.3: Summary of known results for the ergodic capacity.

$f_{\mathbf{H}}(\mathbf{H})$	CSI-RX	Results
$\mathcal{CN}(\mathbf{H}, \mathbf{0}, \mathbf{0})$ (AWGN)	full	Optimum is $\mathbf{x} \sim \mathcal{CN}(\mathbf{Q})$. The eigenvectors of $\mathbf{Q}$ equal the eigenvectors of $\mathbf{H}^H\mathbf{H}$. The eigenvalues of $\mathbf{Q}$ are given by the water-filling solution [19].
$\mathcal{CN}(\mathbf{0}, \mathbf{I}_{n_\mathrm{r}}, \mathbf{I}_{n_\mathrm{t}})$	full	Optimum is $\mathbf{x} \sim \mathcal{CN}(\frac{P}{n_\mathrm{t}}\mathbf{I}_{n_\mathrm{t}})$ [19]. The capacity is given by the corresponding ergodic MI, see Table 2.1.
$\mathcal{CN}(\mathbf{0}, \mathbf{I}_{n_\mathrm{r}}, \mathbf{I}_{n_\mathrm{t}})$	none	Optimum channel input is a product of isotropically distributed unitary matrix and a nonnegative diagonal matrix [29]. (Block fading channel)
$\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}, \mathbf{\Theta})$	full	Optimum is $\mathbf{x} \sim \mathcal{CN}(\mathbf{Q})$. The eigenvectors of $\mathbf{Q}$ equal the eigenvectors of $\mathbf{\Sigma}$. Eigenvalues of $\mathbf{Q}$ are the solution of an optimization problem given in [30].
$\mathcal{CN}(\mathbf{M}, \mathbf{I}_{n_\mathrm{r}}, \mathbf{I}_{n_\mathrm{t}})$	full	Optimum is $\mathbf{x} \sim \mathcal{CN}(\mathbf{Q})$. The eigenvectors of $\mathbf{Q}$ equal the eigenvectors of $\mathbf{M}^H\mathbf{M}$. The eigenvalues of $\mathbf{Q}$ are the solution of an optimization problem given in [31].
$\mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Theta})$		Open problem

problem. In the special case $\mathbf{H} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{n_\mathrm{r}}, \mathbf{\Theta})$ the properties of the optimal input are examined in [32], where it was shown that optimal $\mathbf{Q}$ has the same eigenvectors as $\mathbf{\Theta}$. The eigenvalues of the optimal $\mathbf{Q}$ depend on the outage probability and can be solved from the distribution of the mutual information, which should be available as a function of $\mathbf{Q}$. For the special case $n_\mathrm{t} \leq 2$, the distribution is known in integral form [32]. Other than these special cases, the outage capacity remains largely an open problem.

2.2 Contributions of this thesis

In this thesis the statistics of the mutual information are analyzed in the case of isotropic Gaussian input $\mathbf{x} \sim \mathcal{CN}(\frac{P}{n_\mathrm{t}}\mathbf{I}_{n_\mathrm{t}})$. An overview of the main results in this thesis are given in the following sections, and details can be found in [P1]–[P4].

2.2.1 Ergodic Mutual Information of the Rician Fading MIMO Channel [P1]

In [P1], an upper bound for the ergodic mutual information for Gaussian channel with one-sided spatial correlation and a nonzero mean is derived. The derivation is based on Jensen's inequality and the expected values of elementary symmetric functions of a complex non-central Wishart matrix. For real non-central Wishart, these expected values are derived in [22]. The main contribution of this thesis is the extension of these results to the complex case, and then applying the result to upper bound the ergodic MI. The resulting bounds are also shown to be tight in the sense that at a high SNR, the relative error of the upper bound approaches zero, and at a low SNR the bound becomes exact. At a high SNR, the bounding error itself is upper bounded, and shown to be very small, particularly if $K < L$.

Other recent papers addressing the ergodic mutual information for the same or closely related channel statistics have appeared in literature include [7, 23, 24, 33, 34].

2.2.2 Approximate Distribution of the Mutual Information of the Rayleigh Fading MIMO Channel [P2]

The exact distribution function of mutual information is unknown even in the simplest case of $\mathbf{H} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{n_\mathrm{r}}, \mathbf{I}_{n_\mathrm{t}})$. As noted in Table 2.2, the exact characteristic function is known, however, and can be numerically Fourier transformed to give the outage mutual information [25,26]. Gaussian approximations for the MI are also available [17,18]. Nevertheless, it is of academic interest to seek exact or approximate distribution functions by more direct methods. In [P2], a high-SNR approximation for the mutual information of the Rayleigh iid channel, i.e., $\mathbf{H} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{n_\mathrm{r}}, \mathbf{I}_{n_\mathrm{t}})$, is derived. The basic idea is to use the Minkowski determinant inequality [35] (used earlier in [15]) to lower bound the log determinant, and then solve the exact density of $|\mathbf{H}\mathbf{H}^H|$ using an inverse Mellin transform. After a transformation of random variable, this gives an approximate density of the MI. The approximation becomes sharper for a high SNR, or when $K \ll L$. The distribution function is obtained by integrating the density, and provides an upper bound on the outage probability.

2.2.3 Decomposition of the Mutual Information at High SNR [P3]

It is shown in [P3] that at a high SNR mutual information can be presented as a sum of terms depending on average SNR, SNR fading, and eigenvalue dispersion characterized by the so-called ellipticity statistic [36]. The result gives some insight into the structure of MI, and suggests that spatial multiplexing capability of the MIMO channel can be characterized by the ellipticity statistic, which is merely the ratio of geometric and arithmetic means of the channel eigenvalues. The MI decomposition was originally conceived for the study of the effect of MIMO antennas on the spatial multiplexing properties of the system. In [P3], the distribution functions of the individual terms in the decomposition were also derived under the assumption that $\mathbf{H} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{n_\mathrm{r}}, \mathbf{I}_{n_\mathrm{t}})$. The results complement the earlier statistical analysis in [15], and differ from those results in that [P3] analyzes the individual components of the mutual information.

2.2.4 Quantifying MIMO Multipath Richness [P4]

In [P4] it is argued that ellipticity statistic, defined as the ratio of geometric and arithmetic means of channel eigenvalues, can be used for characterizing the multipath richness of MIMO channels. This observation is motivated by the theoretical findings in [P3], and illustrated here by means of a real-world measurement example.

Chapter 3

Statistical Analysis of the Multiple Scattering Channel

This chapter pertains to publications [P5]–[P7]. In Section 3.1 the multiple scattering signal model is defined, and its relation to Rice, Rayleigh and double-Rayleigh distributions is shown. Examples of propagation scenarios where the model may be applicable are given, and key references to relevant literature are listed. Section 3.2 overviews the related contributions in this thesis.

For definitions of the probability distributions the reader is referred to Appendix A.

3.1 Problem Formulation and Relevant Literature

The multiple scattering scattering channel model is a generalization of the classical single scattering (Rayleigh) model. Mathematically, the narrow-band channel impulse response can be written as [37, 38]

$$H = w_0 H_0 + w_1 H_1 + w_2 H_2 H_3 + w_3 H_4 H_5 H_6 + \dots , \tag{3.1}$$

where $H_0 = e^{j\theta}$ with θ uniformly distributed over $[0, 2\pi)$. Throughout this thesis it is assumed that $\{H_i\}$ are isotropic Gaussian random variables with $\mathrm{E}[|H_i|^2] = 1$ and that the mixture weights $\{w_n\}$ are deterministic constants. The following amplitude distributions emerge as special instances of (3.1).

- The amplitude $R = |H|$ is *Rayleigh* fading if $w_n = 0$ for all $n \neq 1$.

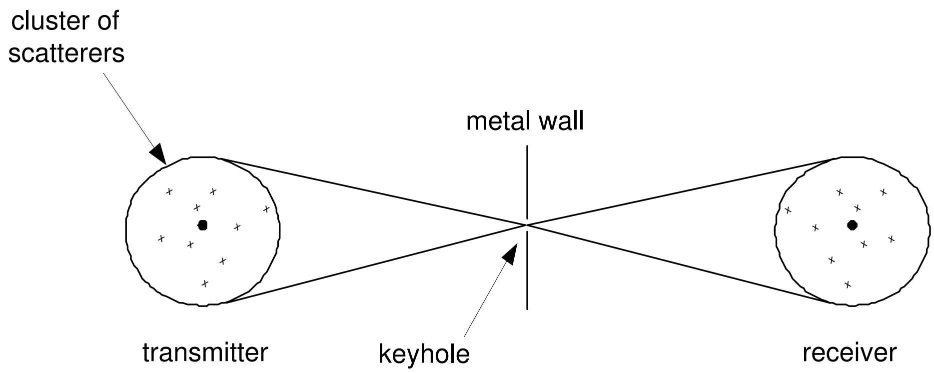

Figure 3.1: Conceptual illustration of keyhole and double scattering.

- The amplitude R is *Rician* fading with a K factor of $\frac{w_0^2}{w_1^2}$ if $w_n = 0$, $n > 1$.
- *Double-Rayleigh* distributed amplitude results as the special case where $w_n = 0$ for all $n \neq 2$.
- Setting $w_n = 0$ for all $n \neq \{1, 2\}$ results in the *leaky keyhole* channel, that can be used for modelling keyhole propagation where a complex Gaussian component leaks past the keyhole [39].

The question arises of the type of physical propagation scenario that gives rise to the general model in (3.1). To explain this, one can start with the concept of keyhole shown in Fig. 3.1, and first introduced in the context of multiple-input multiple-output (MIMO) channels in [40]. In Fig. 3.1, the signal can propagate from the transmitter to the receiver only through a small aperture in the metal wall. Assuming that the number of scatterers on the transmitter side of the metal wall is large, the signal at the keyhole will have Rayleigh-distributed amplitude. It should be pointed out that to observe the amplitude variation the phases of the plane waves should be random. This can occur if either the transmitter, the cloud of scatterers, or the keyhole is moving. On the receiver side, similar reasoning convinces one that from the keyhole to the receiver the signal experiences another Rayleigh-distributed fading process. Mathematically, the keyhole functions as a multiplier between the two fading processes, hence resulting in a received amplitude that is a product of the Rayleigh random variables, i.e., double-Rayleigh fading. The process where the received signal can be modelled as a product of two scattering processes is called *double scattering.*

At this point, a short remark on terminology is in order. As in [37, 38], multiple scattering means here a radio signal that can be modelled as a linear combination of signal components with constant, Rayleigh, double-Rayleigh etc distributed amplitude corresponding to the mathematical signal model defined in (3.1). In radio channel modelling the terms single and double scattering can also be used to refer to single-*bounce* and double-*bounce* interactions, especially in the context of geometric channel models. However, it should be noted that a double-bounce (multiple-bounce) signal does not in general have double-Rayleigh (multiple-Rayleigh) amplitude distribution as this requires the existence of a keyhole in the signal path (but see Fig. 3.2a for an example how this could occur).

The model in Fig. 3.1 with the metal wall and small aperture is somewhat contrived and is used here only for illustrating the concept of double scattering. There exist, however, systems that can be mathematically modelled as keyholes. These real-world "keyholes" can occur in the following forms:

- In the case of outdoor-to-indoor propagation, where the signal travels through a narrow hole or a slit into the building and experiences scattering both inside and outside the building. Note that if the transmitter is fixed, the occurrence of double-Rayleigh amplitude would require the outdoor environment and the receiver to be moving.

- *Fig. 3.2a:* When two rings of scatterers are separated by a large distance, all propagation paths travel via the same narrow pipe that can be thought of as a keyhole if $r \ll R$ [37]. The MIMO case has been considered in [41].

- *Fig. 3.2b:* Propagation in amplify-and-forward wireless relay networks. A simple configuration with one repeater is shown in Fig. 3.2b. The A-F node amplifies the received signal and functions essentially as a keyhole. Here it is assumed that the A-F node is a noiseless analogue repeater with fixed gain, or slow power control. The wireless peer-to-peer concept has recently received a great deal of attention in the communications community; see e.g. [42–45]. Amplify-and-forward is one example of a relay functionality proposed for use in a relay network. In a relay network with n moving A-F nodes, the amplitude of the received signal is a product of n Rayleigh random variables. It should be noted that the noise model of an A-F node influences the noise seen by the receiver, and consequently in bit error analysis one needs to take into account dependency of the receiver noise and channel gain [46].

- *Fig. 3.2c:* Propagation via diffracting wedges, such as street corners or rooftops. The wedge functions as a multiplier for the two Rayleigh processes [47].

Strictly speaking, the examples above assume single-polarized transmission and reception and ideal polarization discrimination of the propagation environment.

Although the examples above represent real-world keyholes, the assumption of “pure” keyhole propagation is somewhat unrealistic, as evidenced by the difficulties experienced in reproducing the keyhole effect in controlled laboratory measurements [48]. It is more plausible that, in addition to the double scattering keyhole signal, a single scattering leakage signal also occurs as shown in Fig. 3.3. The resulting “leaky keyhole” channel can then be modelled as a mixture of single and double scattering components having their own mixture weights:

$$H = w_1 H_1 + w_2 H_2 H_3. \tag{3.2}$$

More generally, the inclusion of the line-of-sight and higher order scattering components results in the general multiple scattering model given in (3.1). This model is quite general and, with its various special cases, provides a versatile tool for the analysis of channel measurements and communication systems.

This section concludes with a summary of known analytical and experimental results regarding the general multiple scattering model. The double-Rayleigh distribution was apparently first applied in radio propagation context in [47]. Its derivation can be found in [49–51]. As mentioned, the model (3.1) was first proposed in the landmark papers [37, 38] which are also responsible for inspiring much of the work in this thesis. The notion of keyhole was conceived for the study of rank-reduced MIMO channels [40] and further elaborated in [41, 52]. As the present focus is in the amplitude distribution, the spatial dimension, i.e., MIMO channels, are not considered in this part of the thesis.

Multiple scattering distribution has been found to provide a good explanation for channel amplitude measurements reported in [37, 38, 51, 53, 54]. Double scattering was predicted in simulations in [47] and verified experimentally in [48].

Compared with the number of published channel measurements there are not many results available that consider the multiple scattering model. In some sense this is not very surprising considering its relative unfamiliarity to researchers. More importantly, the measurement of the phenomenon – and the associated data analysis – appears to be far from straightforward: this raises the question of whether one should indeed be concerned with the multiple scattering phenomenon in the first place. After all, if it is as difficult to detect in measurements as it seems, it is unlikely to affect communication

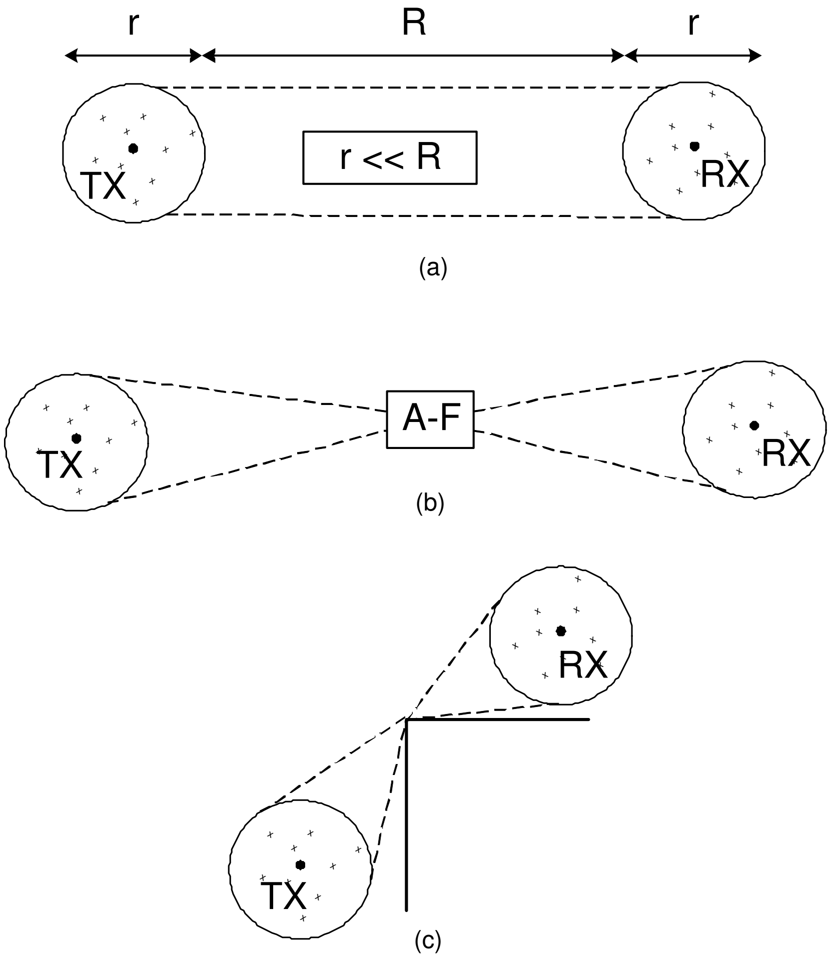

Figure 3.2: Three propagation scenarios with a keyhole: (a) keyhole created by two rings of scatterers separated by large distance [41]; (b) amplify-and-forward relay; (c) propagation via diffracting street corner [47].

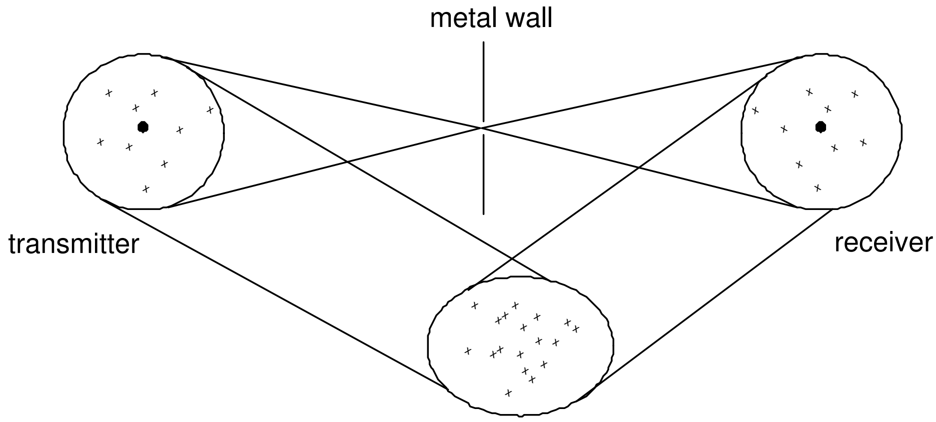

Figure 3.3: "Leaky keyhole" channel: a more realistic propagation model. Leakage through the keyhole is modelled as a Rayleigh fading signal.

system performance. Two justifications for studying multiple scattering are pointed out: first, the lack of analytical tools may have hindered earlier studies and resulted in false interpretations of the measurement analysis; second, the emergence of new radio concepts such as wireless relay networks give rise to new types of propagation scenarios where both ends of the radio link are moving, thus creating a physical setting where multiple scattering can take place.

3.2 Contributions of this thesis

3.2.1 Distribution of the Product of Independent Rayleigh Random Variables [P5]

In this paper the distribution functions (pdf and cdf) of the product of n independent Rayleigh random variables are derived. The result generalizes the double-Rayleigh distribution to arbitrary $n > 2$. The distribution functions are derived using an inverse Mellin transform technique as in [P2], and again given in terms of the Meijer G-function. Series forms are also provided for $n = \{3, 4, 5\}$. A method-of-moments based estimator for the parameter of the distribution is also derived and its variance is evaluated in closed form.

Comment: After publication of [P5] it was discovered that the probability density function in [P5, Eq. (8)] is a special case of the more general density of a product of generalized gamma variates, which can be stated in terms of the Fox's H-function [55], which is the most general function known. In fact, even more general characterization is possible by considering products and ratios of the so-called H-function variates [56] of which the generalized gamma (and a number of other) random variables are special cases. However, it should be noted that apart from [P5, Eq. (8)] the remaining results in [P5] seem not to have appeared elsewhere[1].

3.2.2 Statistical Analysis of the Multiple Scattering Radio Channel [P6]

In this paper the results of [P5] are generalized, and the amplitude distribution functions for the general multiple scattering model in (3.1) are derived. The functions are given in integral and series forms. The case of second-order scattering where $w_n = 0$ for all $n > 2$ is considered in detail. A method-of-moments based estimator for the distribution parameters is also derived. An upper bound for the mean square error of the estimator is derived for the leaky keyhole channel.

3.2.3 Impact of Double Scattering on System Performance [P7]

In this paper it is shown that the double-Rayleigh fading induces a hefty degradation on system performance hence lending motivation for the study of multiple scattering channels. An analytical expression for the bit error rate of some common modulation schemes is derived. From the result it can also be seen that the diversity order is not a useful channel measure for the double-Rayleigh channel. This behavior is in sharp contrast to other amplitude models, such as Rayleigh and Nakagami-m.

[1] In addition to [P5, Eq. (8)], other recent special cases of [55, 56] have been reported in [57, 58], including the distribution of the product of independent Nakagami-m variables.

Chapter 4

An Additive Model for Shadow Fading

This chapter pertains to publications [P8]–[P9]. In Section 4.1 the components of the classic three-stage product model for the received signal power in mobile communications are described. Special emphasis is given to modelling and theory of shadow fading, and important literature references are given. In Section 4.2 the contributions of the thesis are given.

4.1 Problem Formulation and Relevant Literature

In mobile communications, signal power variations are usually modelled as a product of large-scale fading and small-scale fading [59]. Large-scale fading can be further subdivided into path loss and shadow fading. Here "small scale" means a distance of around a single wavelength. Large-scale fading, on the other hand, refers to signal power variation over distances above ten wavelengths or so.

In Table 4.1 some common models for predicting path loss over irregular terrain are listed. A number of site-specific models that take into account the geometry of the propagation environment have also been proposed. For extensive overviews, the reader is referred to [60–63]. Path loss models fall into three groups: empirical, analytical, and hybrids of the two [63]. Empirical models are fully measurement-based, while the analytical ones are constructed from a simplified electro-magnetic representation of the propagation environment.

In contrast to path loss, small-scale fading is usually modelled stochastically. Numerous probability laws for the amplitude of small-scale fading have

Table 4.1: Summary of common path loss models.

model	environ.	notes
Okumura	macro	$150-1920$ MHz, transmitter height $30-1000$ m, distances $1-100$ km. Measurement results given as a collection of curves [64].
Hata	macro	Okumura's results given as formulas for $100-1500$ MHz, transmitter height $30-200$ m, distance $1-20$ km [65].
COST-231-Hata	macro	Extension of Hata's formulas to $1500-2000$ MHz range [66].
Lee	macro	"Above" 30 MHz, mobile height 3 m, three cities given as reference [67].
Walfisch-Bertoni	urban macro	Gives the excess path loss due to over-the-rooftop propagation and diffraction to street level [68]. Transmitter above rooftop level. Deterministic model.
Maciel-Bertoni-Xia	urban macro/ micro	Generalizes Walfisch-Bertoni over-the-rooftop propagation model to transmitters below rooftop height [69]. Deterministic model.

been proposed. The standard one is the Rayleigh distribution, which arises from central limit theorem arguments [70]. If one strong ray dominates the received signal, the amplitude model generalizes to the Rice distribution [71]. Further generalization is the case with two strong rays in random noise, called the two-wave-diffuse-power distribution [72]. Nakagami-m is an approximate distribution for the amplitude of a sum of random complex vectors and also includes the Rayleigh distribution as a special case [49]. Weibull distribution is another generalization of the Rayleigh distribution proposed as an amplitude model, although it lacks a physical basis [73]. The multiple scattering distribution, of which the double-Rayleigh distribution is a special case, has also been investigated [37, 38, 47].

It is evident that there are a number of approaches – many of which are motivated by the physics of radio propagation – for modelling both path loss and small-scale fading. The same cannot be stated about shadow fading, for which little physical insight has accumulated so far. For shadow fading, or slow fading, the prevailing approach has been black box modelling. Since the phenomenon was first observed [74], a large number of measurement results have been reported in literature, e.g. [64, 74–81] [82]. Comparison between the numerous measurements is sometimes difficult due to differences in frequency, measurement environment, measurement route length, and data processing methods. Textbooks giving a readable synthesis of the key conclusions from the measurement results include [59, 83–86]. Some important points are reviewed in the following paragraphs.

In measurement literature the empirical distribution of shadow fading is almost without exception found approximately log-normal, probably partly due to lack of a physical basis indicating that a more exotic model should be used. In some cases, the log-normal distribution fit does not fit the tails very well [64, 80, 81].

The shadow fading standard deviation has been reported to be between 2 and 11.4 dB in suburban macro cells [87] [88], between 3.7 and 10 dB in urban macro cells [76] [89], between 4 and 10 dB for rural macro cells [88], between 3.5 and 7.7 dB for NLOS urban micro cells [77] [90]. The results can be seen to vary considerably. In addition to environmental differences and data processing, it was noted in [64, 75–77, 91] that the size of the measurement area affects the standard deviation of shadow fading: measurements over small areas result in a lower value.

The standard deviation has been observed to be mostly independent of the distance between the transmitter and the receiver when the propagation area is homogeneous [85]. On the other hand, the standard deviation increases with carrier frequency [84].

Spatial 50% autocorrelation distance of the shadow fading process is reported as less than 10 meters for urban micro cells [77,92,93], $50-120$ meters for urban macro cells [76, 81, 89], and $50 - 400$ meters for suburban macro cells [76,88,89,92]. A widely accepted model for the autocorrelation function is the exponential model [92].

Angular correlation of shadow fading between signals received from two base stations has also been studied. In [81, 88, 94] low (less than 0.5) or no correlation was found. On the other hand, in [78, 93, 95] a moderate or large correlation was found.

One should probably differentiate between two types of shadow fading: global and local shadow fading. *Global shadow fading* means the residual error in the prediction of a path loss model. It is mainly used for coverage prediction in network planning. Measurement analysis of global shadow fading is typically analyzed by fitting a path loss model to a distance versus received-signal-power graph in a log-log scale. The shadow fading distribution parameters are determined from the residual fitting error. Most of the measurement results mentioned above are the result of this type of analysis.

Local shadow fading means a variation in the mean power in a smaller area, where path loss can be assumed constant. The received spatially average signal power also exhibits log-normal variation also in this scale [61], albeit with a much lower standard deviation. Standard deviations of $2.7-5.6$ dB have been reported for measurement distances of $200-800$ meters in both urban and rural environments [75–77, 91]. These values are clearly at the lower part of the spectrum of the usual reported range of deviations. Ideally, the measurement analysis of local shadow fading is performed by dividing the measurement route into segments where path loss can be assumed constant. Its main usage is the continuous simulation of the radio channel of a moving receiver.

Some theories to explain the shadow fading mechanism have appeared in literature. They are summarized below.

T1 The classical theory is the multiplicative model that is based on the central limit theorem, from which it follows that a logarithm of a large number of random multiplicative attenuations approaches a Gaussian distributed random variable [84, 96].

T2 In [97, 98] it was reported that randomizing the building heights and rooftop construction in the Walfisch-Bertoni type model [68] results in approximately log-normal variation in the predicted signal level. Randomizing rooftop heights alone was not sufficient to reproduce the log-normal distribution [97].

T3 In [99] it was shown that by randomizing building heights in a multiple knife-edge diffraction model the variation in the local mean is approximately log-normal. Note that this result contrasts to certain extent with those above, and also illustrates how using a different path loss model and simulation assumptions can lead to slightly different conclusions.

T4 In a line-of-sight (LOS) microcell, the sum of LOS, ground-reflected and wall-reflected phasors can result in slow, large-scale variations in the received signal power; these variations have approximately Nakagami-m or log-normal distribution [100].

T5 Multiple scattering in the radio channel, discussed in Chapter 3, can result in an amplitude distribution that closely emulates the Suzuki distribution [37] which is a mixture of Rayleigh and log-normal distributions [101]. This way the multiple scattering phenomenon can explain the signal level variations usually perceived as log-normal without a need for an artificial factor multiplying the small-scale fading signal.

Theory T1 has some shortcomings that have inspired researchers to examine alternatives. In [102] it was shown that a large number of random multiplicative attenuations is needed in order for their log-sum to approach a Gaussian random variable. This result applied to many distributions of interest. Furthermore, if the number of multiplicative attenuations were large, then the path loss should increase exponentially with distance. The variance of path loss should probably also increase with distance [37]. These claims contradict measurement observations. In T2 the number of absorbing screens modelling a building varied between 55 and 105, which corresponds to a quite large homogeneous propagation area.

Theories T1–T3 focus on explaining the global shadow fading in urban street canyons, based on an underlying path loss model. Theories T4 and T5 do not assume any path loss model and could provide an explanation for local shadow fading, but are also somewhat restricted in their application area: namely, LOS microcells and strong multiple scattering propagation conditions. The multiple scattering theory was supported by measurement results [37,38] which indicate that a good fit to measurement can be obtained using the general model (3.1). It should be noted, however, that a good model fit alone does not guarantee that the underlying propagation phenomenon is based on multiple scattering.

4.2 Contributions of this thesis [P8]-[P9]

An additive model as a physical basis for local shadow fading is proposed. The narrowband received signal is modelled as a sum of sinusoids [83] whose amplitudes vary slowly with respect to the phasor summation, giving rise to small-scale fading. It is shown in [P9] that if the sum of powers of the sinusoids satisfy the central limit theorem [103], the resulting signal will be approximately Gaussian in log domain. The simple proof is based on a theorem stating that a logarithm of a positive approximately Gaussian random variable is also (approximately) Gaussian if its variance is small compared to its mean [104]. This will be the case if the number of sinusoids is sufficiently large; in practice $4-10$ can be enough depending on the distributions of the sinusoid powers.

A special case of the model in [P9] was proposed first in [P8], where it was assumed that the sinusoids (i.e., plane waves) are grouped in clusters. A slightly simpler and less general proof was given in [P8]. The advantage of the cluster-based additive shadow fading model is that it can be verified by measurements, and it also offers insight into how to model cluster and composite shadow fading with geometric channel models, e.g. [105, 106].

Chapter 5

Summary

Three topics were studied in this thesis:

Topic 1: the statistical analysis of mutual information of fading MIMO channels;

Topic 2: the statistical analysis of the multiple scattering radio channel;

Topic 3: an additive model as a physical basis for shadow fading.

The main contributions to these topics are as follows.

- A closed-form upper bound for ergodic mutual information of Rician fading MIMO channels with one-sided correlation was derived. An approximate distribution of mutual information of Rayleigh fading MIMO channels was derived. A mutual information decomposition separating the effects of the average SNR, the instantaneous SNR due to channel fading, and channel eigenvalue dispersion was proposed. As an application of the latter result, a measure for quantifying MIMO multipath richness was also proposed.

- Amplitude distribution functions (pdf and cdf) of the general multiple scattering radio channel were derived. A building block of the general result is the n-Rayleigh distribution functions; these were also derived. The impact of double-Rayleigh fading on symbol error probability of some common modulation schemes was evaluated.

- An additive model as a physical basis for local shadow fading was proposed. It was shown that the addition of time-variant plane wave powers can result in approximately log-normally distributed received power (when averaged over small-scale fading). Theoretical results were verified with simulations and, for a simplified cluster-based additive model, with measurements.

Some open questions and potential topics for future research include:

- measurement analysis to determine the dependence of multipath richness on propagation environment, carrier frequency, antenna beamwidth, transmitter–receiver distance, etc;
- derivation of numerically less cumbersome forms for the multiple scattering distribution or its special cases; methods and bounds for the estimation of mixture weights of the multiple scattering distribution from measurement data; measurement analysis of multiple scattering channels;
- extended measurement analysis of cluster shadow fading; construction of cluster-based channel model with cluster shadow fading, generalizing the theory of local (sum model) and global (product model) shadow fading to a more general sum-product model that could explain the phenomenon in all distance scales.

Appendix A

Summary of probability distributions

In this appendix the probability distributions used in the thesis are summarized.

Isotropic random variable

In this thesis all isotropic variables are complex-valued. A random vector $\mathbf{x}$ is an isotropic random variable if both $\mathbf{x}$ and $\mathbf{Qx}$ have the same distribution for a unitary matrix $\mathbf{Q}$. In the scalar case this specializes to $\mathbf{Q} = e^{j\theta}$, with θ real. If $\mathbf{x}$ has Gaussian distribution, $\mathbf{x}$ is called an isotropic Gaussian random variable. Another term for isotropic is circular.

Rayleigh distribution

Let $R_1 = |w_1 H_1|$, where w_1 is a positive constant and H_1 is a complex isotropic Gaussian random variable with unit variance. Then R_1 has the Rayleigh distribution and its probability density function is

$$f_{R_1}(r) = \frac{2r}{w_1^2} \exp\left(-\frac{r}{w_1^2}\right) . \tag{A.1}$$

Double-Rayleigh distribution

Let $R_2 = |w_2 H_2 H_3|$, where w_2 is a positive constant and H_2 and H_3 are complex isotropic Gaussian random variables with unit variance. Then R_2 has the double-Rayleigh distribution and its probability density function is [47]

$$f_{R_2}(r) = \frac{4r}{w_2^2} K_0\left(\frac{2r}{w_2}\right), \tag{A.2}$$

where $K_0(x)$ is the modified Bessel function of the second kind.

n-Rayleigh distribution

Let $R_n = w_n \prod_{i=1}^{n} |H_i|$, where w_n is a positive constant and $\{H_i\}_{i=1}^{n}$ are independent identically distributed complex isotropic Gaussian random variables with unit variance. Then R_n has the n-Rayleigh distribution and its probability density function is [P5]

$$f_{R_n}(r) = \frac{2}{w_n} G_{0,n}^{n,0}\left(\frac{r^2}{w_n^2}\Big|_{\{\frac{1}{2}\}_n}^{-}\right), \tag{A.3}$$

where $G(\cdot)$ is the Meijer G-function defined in Appendix B. The following notation has been used:

$$\{a\}_n = \{\underbrace{a, a, \ldots, a}_{n \text{ times}}\}. \tag{A.4}$$

Multiple scattering distribution

Define

$$C_0 = w_0 e^{j\theta}, \tag{A.5}$$

$$C_n = w_n \prod_{i=1}^{n} H_i, \quad n > 0. \tag{A.6}$$

It is assumed that $\{w_i\}$ are positive constants, θ has uniform distribution over $(0, 2\pi]$, and $\{H_i\}$ are independent identically distributed complex isotropic Gaussian random variables with unit variance. The probability density function of the random variable

$$R_N^{\star} = \left|\sum_{n=0}^{N} C_n\right| \tag{A.7}$$

is then given in integral form as [P6]

$$f_{R_N^\star}(r) = r \int_0^\infty \omega \prod_{n=0}^{N} \Phi_{C_n}(\omega) J_0(r\omega)\ \mathrm{d}\omega\,, \tag{A.8}$$

where

$$\Phi_{C_0}(\omega) = J_0(w_0\omega)\,, \tag{A.9}$$

$$\Phi_{C_1}(\omega) = \exp\left(-\frac{w_1^2\omega^2}{4}\right)\,, \tag{A.10}$$

$$\Phi_{C_2}(\omega) = \frac{4}{4 + w_2^2\omega^2}\,, \tag{A.11}$$

$$\Phi_{C_3}(\omega) = \left(\frac{2}{w_3\omega}\right)^2 e^{(\frac{2}{w_3\omega})^2}\,\mathrm{E}_1\left((\frac{2}{w_3\omega})^2\right)\,, \tag{A.12}$$

$$\Phi_{C_n}(\omega) = \frac{2}{w_n\omega} G^{n-1,1}_{1,n-1}\left(\left(\frac{2}{w_n\omega}\right)^2\Big|^{\frac{1}{2}}_{\{\frac{1}{2}\}_{n-1}}\right)\,, \quad n > 0\,, \tag{A.13}$$

where and $J_0(x)$ is the Bessel function of the first kind and $\mathrm{E}_1(x)$ is the exponential integral. The case $n = 1$ is the characteristic function of an isotropic complex Gaussian random variable, and hence well-known from literature [72]. Interestingly, $\Phi_{C_2}(\omega)$ is the characteristic function of the Laplace distribution, which implies that the real and imaginary parts of double-Rayleigh fading channel are Laplace distributed; this has been noted also in [107].

Matrix variate complex Gaussian distribution

The random matrix $\mathbf{X} \in \mathbb{C}^{n_\mathrm{r}\times n_\mathrm{t}}$ has complex matrix variate Gaussian distribution with mean matrix $\mathbf{M} \in \mathbb{C}^{n_\mathrm{r}\times n_\mathrm{t}}$ and positive-definite covariance parameters $\mathbf{\Sigma} \in \mathbb{C}^{n_\mathrm{r}\times n_\mathrm{r}}$ and $\mathbf{\Theta} \in \mathbb{C}^{n_\mathrm{t}\times n_\mathrm{t}}$ if its probability density function is given by [23]

$$f_{\mathbf{X}}(\mathbf{X}) = \frac{1}{\pi^{n_\mathrm{r}n_\mathrm{t}}|\mathbf{\Sigma}|^{n_\mathrm{t}}|\mathbf{\Theta}|^{n_\mathrm{r}}}\,\mathrm{etr}\left[-\mathbf{\Sigma}^{-1}(\mathbf{X}-\mathbf{M})\mathbf{\Theta}^{-1}(\mathbf{X}-\mathbf{M})^H\right]\,. \tag{A.14}$$

Note that $\mathrm{E}[\mathbf{X}] = \mathbf{M}$ and $\mathrm{cov}[\mathrm{vec}(\mathbf{X}^H)] = \mathbf{\Sigma}\otimes\mathbf{\Theta}\,.$

Multivariate complex Gaussian distribution

The random vector $\mathbf{x} \in \mathbb{C}^{n\times 1}$ has complex multivariate Gaussian distribution with mean vector $\mathbf{m} \in \mathbb{C}^{n\times 1}$ and positive-definite covariance parameters

$\mathbf{\Sigma} \in \mathbb{C}^{n\times n}$ if its probability density function is given by

$$f_{\mathbf{x}}(\mathbf{x}) = \frac{1}{\pi^n|\mathbf{\Sigma}|} \operatorname{etr}\left[-\mathbf{\Sigma}^{-1}(\mathbf{x}-\mathbf{m})(\mathbf{x}-\mathbf{m})^H\right] . \tag{A.15}$$

An $\mathbf{x}$ drawn from a zero-mean distribution, i.e., $\mathbf{m} = \mathbf{0}$, is denoted with $\mathbf{x} \sim \mathcal{CN}(\mathbf{\Sigma})$. Note that $\mathrm{E}[\mathbf{x}] = \mathbf{0}$ and $\mathrm{cov}[\mathbf{x}] = \mathbf{\Sigma}$.

Complex central Wishart distribution

Let $\mathbf{X} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}, \mathbf{I}_L)$ be a $K \times L$ matrix with $K \leq L$. Then the $K \times K$ matrix $\mathbf{W} = \mathbf{X}\mathbf{X}^H$ has the complex central Wishart distribution, and its pdf is [108]

$$f_{\mathbf{W}}(\mathbf{W}) = \frac{|\mathbf{W}|^{L-K}}{\Gamma_K(L)|\mathbf{\Sigma}|^L} \operatorname{etr}\left[-\mathbf{\Sigma}^{-1}\mathbf{W}\right] , \tag{A.16}$$

where $\Gamma_n(m) = \pi^{n(n-1)/2}\prod_{k=1}^{n}\Gamma(m-k+1)$ and $\Gamma(x)$ is the gamma function.

Complex non-central Wishart distribution

Let $\mathbf{X} \sim \mathcal{CN}(\mathbf{M}, \mathbf{\Sigma}, \mathbf{I}_L)$ be a $K \times L$ matrix with $K \leq L$. Then the $K \times K$ matrix $\mathbf{S} = \mathbf{X}\mathbf{X}^H$ has the complex non-central Wishart distribution, and its pdf is [109]

$$f_{\mathbf{S}}(\mathbf{S}) = \frac{|\mathbf{S}|^{L-K}}{\Gamma_K(L)|\mathbf{\Sigma}|^L} \operatorname{etr}\left[-\mathbf{\Sigma}^{-1}\mathbf{S}\right] \operatorname{etr}\left[-\mathbf{\Theta}\right] {}_0F_1\left(L; \mathbf{\Theta}\mathbf{\Sigma}^{-1}\mathbf{S}\right) \tag{A.17}$$

where $\mathbf{\Theta} = \mathbf{\Sigma}^{-1}\mathbf{M}\mathbf{M}^H$ is the non-centrality parameter and ${}_0F_1(b; \mathbf{A})$ is a hypergeometric function of complex matrix argument [110].

Appendix B

Meijer G-function

In this appendix the Meijer G-function is summarized for the convenience of the reader. Further information including properties and tables of Meijer G-function identities can be found in [111–115].

The Meijer G-function is a generalization of the hypergeometric function and can be defined using the contour integral representation

$$G_{p,q}^{m,n}\left(z\Big|_{b_1,\dots,b_q}^{a_1,\dots,a_p}\right) = \frac{1}{j2\pi}\int_{\mathcal{L}} \frac{\prod_{i=1}^{m}\Gamma(b_i+s)\prod_{i=1}^{n}\Gamma(1-a_i-s)}{\prod_{i=n+1}^{p}\Gamma(a_i+s)\prod_{i=m+1}^{q}\Gamma(1-b_i-s)} z^{-s}\,\mathrm{d}s, \tag{B.1}$$

where z, $\{a_i\}_{i=1}^{p}$, and $\{b_i\}_{i=1}^{q}$ can be complex, although this thesis deals only with real-valued parameters. The integration contour $\mathcal{L}$ must be selected so that it separates the poles of $\{\Gamma(b_i+s)\}_{i=1}^{m}$ and $\{\Gamma(1-a_i-s)\}_{i=1}^{n}$. Depending on the parameters, there exist up to three such contours. The value of the G-function is the same regardless of which contour is used.

The existence of (B.1) depends on the parameters and have been summarized in Table B.1. In Table B.1 the latter two cases follow from the first two by using the analytical continuation property:

$$G_{p,q}^{m,n}\left(z\,\Big|_{b_1,\dots,b_q}^{a_1,\dots,a_p}\right) = G_{q,p}^{n,m}\left(\frac{1}{z}\,\Big|_{1-a_1,\dots,1-a_q}^{1-b_1,\dots,1-b_p}\right). \tag{B.2}$$

The Meijer G-function includes for specialized values of its parameters a large number of special functions of engineering importance, including the hypergeometric function and all types of Bessel functions. Very general integral identities have been derived for it, thus making it a useful tool for integration. Another application of the Meijer G-function is the derivation of product distributions.

Numerical values for the Meijer G-function can be computed with certain

Table B.1: Existence of (B.1).

case	condition	existence of $G(z)$
I	$q > p\,,\, q \geq 1$	exists for all z, $z \neq 0$
II	$q = p\,,\, q \geq 1$	exists for $\lvert z\rvert < 1$
III	$p > q\,,\, p \geq 1$	exists for all z, $z \neq 0$
IV	$p = q\,,\, p \geq 1$	exists for $\lvert z\rvert > 1$

widely-available mathematics software packages, including Mathematica and Maple.

The Meijer G-function is a special case of the Fox's H-function [113]. Unfortunately, currently there exist no mathematical software to compute numerical values for the H-function.

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