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A time-domain approach to estimating the plucking point of guitar tones obtained with an under-saddle pickup

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Abstract

A method for estimating the plucking point of guitar tones is proposed. The algorithm is based on investigating the time lag between two consecutive pulses arriving at the bridge of the guitar. The signal is detected with an under-saddle pickup attached to the bridge. The method determines the minimum of the autocorrelation function for one period of the signal. The time lag of the minimum can be converted into the distance from the bridge where the string was plucked. The results obtained with the method are good, the error remains smaller than one centimetre, except for a few outliers. The algorithm is easy to implement and can be used to analyse playing styles. The efficiency of the method gives the potential to also use it in real-time computer music applications.

Keywords: Acoustic signal processing; Autocorrelation; Guitar; Musical acoustics; Plucked string instruments

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1. Introduction

The effect of the plucking point on the spectrum of a guitar sound is well known [1,2]. The harmonics that have a node at the plucking point are not excited and ideally have zero amplitude. This causes a comb-like characteristic in the magnitude spectrum of the string vibration as a function of the plucking point. Hence, by changing the plucking position, the timbre – most notably the brightness – of the played tone can be controlled. This phenomenon is often used as an expressive tool in music.

To understand the effect of the plucking point, let us examine the ratio d_R between the string length L_s and the plucking point d, the distance from the bridge. When ratio d_R has an integer value n, every nth harmonic will be missing or attenuated. Fig. 1 shows the magnitude response of a recorded guitar tone when the string has been plucked at one-eighth of its length from the bridge. In this case it is easy to observe the comb filtering effect of the plucking position: every eighth harmonic is missing or strongly attenuated. When d_R has a non-integer value the comb filtering effect is not as easy to observe. Fig. 2 displays a counterpart for Fig. 1 when the plucking position is 14 cm from the bridge on a 65.2 cm long string (i.e., about 1/4.66 of its length). The comb filtering effect is present but it is not trivial to determine the plucking point based on this.

Based on Fig. 2 one can understand the problems confronted when determining the plucking point from the observed spectral characteristics of a tone. Despite this,

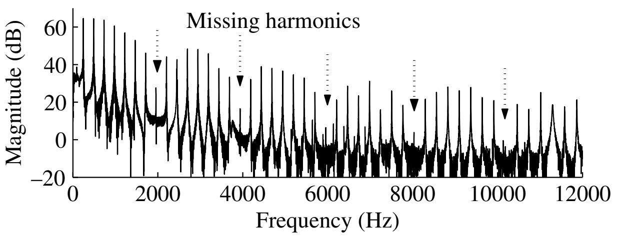

Fig. 1. Magnitude spectrum of an open B string (f_0 = 245 Hz) plucked at one-eighth of the string length from the bridge.

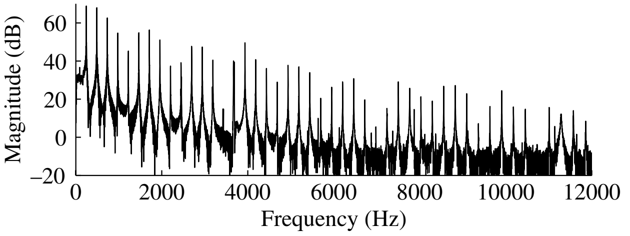

Fig. 2. Magnitude spectrum of an open B string when the plucking point is 1/4.66 of the string length from the bridge.

a frequency-domain based technique has been successfully used to determine the plucking point in guitar tones [3–5]. In this technique, an estimation of the plucking point is determined by finding the minimum error between the absolute value of an ideal string magnitude spectrum and a sampled-data spectrum.

In contrast, the algorithm proposed here takes a time-domain approach by observing the short-time autocorrelation function as suggested in [6]. In [5] the autocorrelation function is employed to obtain an initial estimate for an iterative frequency-domain method. We propose an algorithm to determine the plucking point of a guitar tone based on the autocorrelation function of one period of a guitar tone. By examining only one period, problems related to nonlinear coupling of harmonics [7] are avoided. In addition, a key feature here is that the signal is obtained from a guitar pickup placed under the saddle of the instrument. The pickup used in this study was a B-Band EMFi (electromechanical film) pickup [8]. The pickup responds to pressure, and, taking the area into consideration, one can say that the voltage of the pickup is proportional to force. For more information on the pickup see [9], where the same pickup was used.

With the pickup placed under the saddle a clean signal of the string vibrations is obtained. This is because the direct radiation of the strings and the filtering effect of the body are practically missing in the pickup signal [9]. This is illustrated in Fig. 3, which shows the time responses of a guitar pluck recorded simultaneously in anechoic conditions with a microphone placed 1 m in front of the guitar and with an under-saddle pickup. There is a small delay between the signals since it takes some time for the vibrations to travel to the microphone. The clean pickup signal will provide a robust possibility to analyse the plucking point. Moreover, the time-domain approach proposed in this article is straightforward and computationally efficient, giving the possibility of using it in real-time applications.

The structure of the article is as follows. In Section 2 we discuss the time-domain effects in string vibrations when the plucking position is varied. In Section 3 the

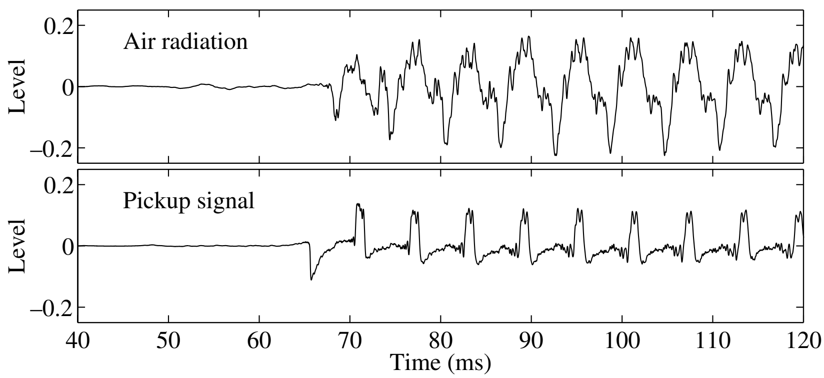

Fig. 3. Plucked guitar tone played on the second fret of the D string (f_0 = 166 Hz) as a function of time: (top pane) air radiation and (bottom pane) the corresponding signal from an under-saddle pickup.

plucking point estimation algorithm is introduced and Section 4 discusses the results obtained with the proposed method. Discussion and conclusions are presented in Sections 5 and 6, respectively.

2. Time-domain effects of varying the plucking point

Next, we discuss the behaviour of the string in the time-domain from the travelling wave theory viewpoint and, based on this, how the plucking point can be observed. As a string is plucked or struck, two transverse waves will travel in opposite directions reflecting back and forth between the boundaries. In the case of the guitar, the end where the saddle is attached is called the bridge and the other the nut. The resulting vibration is a combination of the normal modes of vibration [2]. The shape of the string can be obtained by adding up the modes. The reflecting waves or pulses form a standing wave on the string. Internal and external losses of the string cause the vibrations to decay until finally the string settles in its rest position.

Ideally, the travelling waves, resulting from a pluck, can be assumed as moving impulses. In the case of a real pluck the plectrum or nail exciting the string has a non-zero touching width, which adds a lowpass characteristic to the impulses. However, this lowpass effect is not crucial in this plucking point estimation method, therefore, in this discussion, the waves are treated as impulses. In addition, to simplify the analysis the reflection at the end points is assumed to be ideal with a reflection coefficient equal to -1.

Now we examine the impulses travelling in opposite directions with the help of Fig. 4. It shows the cross-sectional view along the string at three time instances, t_0, t_1, and t_2, and the input at the bridge (under-saddle pickup) as a function of time. When $t = t_0$, the string has been deflected from its rest position, but has not been released yet. When $t = t_1$, the impulse first travelling to the right, i_1, has reached the bridge. When $t = t_2$, the impulse first travelling to the left, i_2, has reached the bridge. Since the terminations are rigid the phase of each impulse is inverted at the boundaries.

An initial step in obtaining an approximation for the plucking point is to evaluate the time difference between the first two incoming impulses i_1 and i_2 in the following manner. The time for an impulse to travel from one end of the string to the other is

$$T_{1/2} = \frac{1}{f_0}\frac{1}{2}, \tag{1}$$

where f_0 is the fundamental frequency of the vibrating string and $T_{1/2}$ is half of the fundamental period. In this work f_0 is estimated from the examined signal as will be discussed in Section 3. The time it takes for impulses i_1 and i_2 to arrive at the bridge can be expressed, respectively, as

$$\tau_1 = \frac{d}{c} = \frac{d}{f_0\lambda} = \frac{d}{2f_0L_s} \tag{2}$$

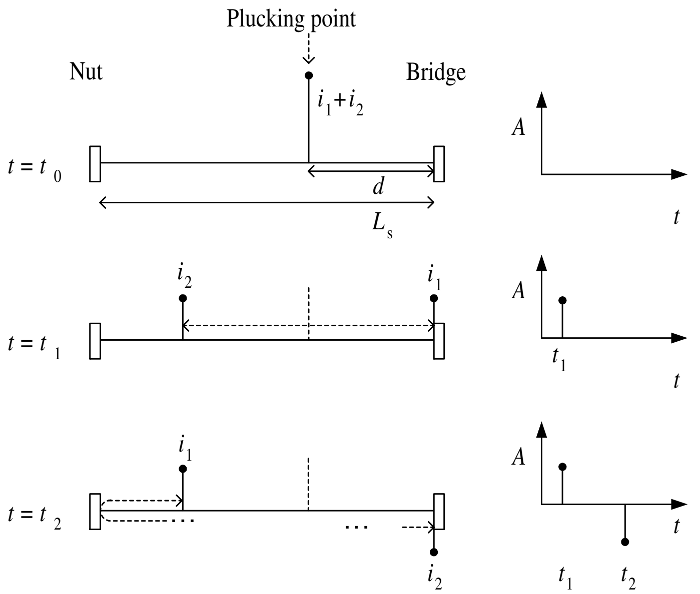

Fig. 4. On the left-hand side is the cross-sectional view along the string at three time instances, t_0, t_1, and t_2. The right-hand side shows the input at the bridge (under-saddle pickup) as a function of time.

and

$$\tau_2 = 2T_{1/2} - \tau_1 = \frac{1}{f_0} - \frac{d}{2f_0L_s} = \frac{2L_s - d}{2f_0L_s}, \tag{3}$$

where

$$c = f_0\lambda \tag{4}$$

and

$$\lambda = 2L_s. \tag{5}$$

L_s is the length of the string, d is the plucking point as the distance from the bridge, λ is the fundamental's (f_0's) wavelength on the string, and c is the velocity of the transverse wave. By determining the time difference $\Delta\tau = \tau_2 - \tau_1$, the plucking point can be solved. Furthermore, as the domain of interest is the digital one, the time difference between the first two arriving impulses is expressed as $\Delta T = \Delta\tau f_s$, where f_s is the sampling frequency. Hence, the plucking point, as a distance from the bridge, is expressed in the discrete domain as

$$d = \frac{L_s(f_s - \Delta T f_0)}{f_s}. \tag{6}$$

From the consideration above it can be understood that it is sufficient to examine only one period of the string vibration to determine the plucking point.

2.1. Comb filtering effect related to the plucking point

The comb filter characteristics produced by the first two impulses i_1 and i_2 are usually not equivalent to the comb characteristics seen in the magnitude spectrum of the complete tone of the vibrating string as missing harmonics. The first two pulses can be interpreted as the impulse response of a linear system that produces a comb filtering effect with notches much denser than is typically observed in the overall spectrum. The frequencies of the densely located notches are located at the following frequencies

$$f_{\text{notch},n} = n\frac{f_s}{\Delta T}, \quad (7)$$

where $n = 0, 1, 2, \ldots$ is the index of the notch frequency and ΔT is the time difference between the first two pulses, as defined previously. As is well known, the missing harmonics are located at frequencies which can be expressed as

$$f_{\text{miss-harm},n} = n\frac{f_s}{f_s/f_0 - \Delta T}. \quad (8)$$

Fig. 5 exemplifies the relation between the dense comb filtering effect caused by the first two pulses and the missing harmonic phenomenon. The figure shows the magnitude spectrum of a guitar tone when the plucking point is one-eighth of the string length, already shown in Fig. 1, together with the ideal comb filtering effect produced by the first two impulses (dashed line in Fig. 5). From Fig. 5 it can be observed that the first partial that coincides with a notch in the dense comb structure is the eighth one. All the other partials – except every eighth – are preserved because their frequencies do not coincide with the frequencies of the notches. This gives an insight to the comb filtering effect in a plucked or struck string sound, which is due to the plucking point, showing that the two are caused by the same physical phenomenon.

3. Plucking point estimation algorithm

In this section we describe the steps to obtain an estimate for the plucking point.

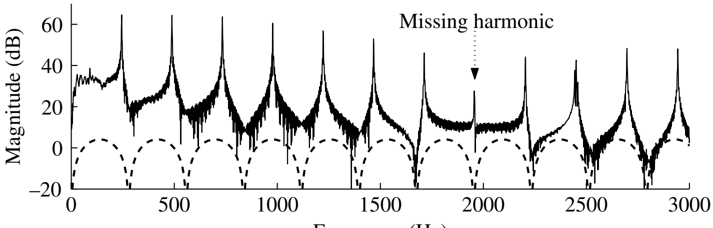

Fig. 5. The magnitude spectrum of a string plucked at one-eighth of its length (solid line) and the ideal comb filter created by the first two impulses (dashed line).

3.1. Onset detection

As a first step, the onset of a plucking event is determined. The onset of a pluck is typically well defined, in contrast to the onset of a bowed violin tone, for instance. The energy at high frequencies is notable during onsets of a plucking event. Initially, an onset event can be roughly determined by examining the energy of the highpass filtered version of the incoming signal in 20 m long frames with a 50% overlap. The highpass filter used was an elliptic second-order IIR digital filter with a −3 dB point at 6 kHz. Then the exact instant of the onset was determined by locating the maximum absolute signal value from five successive frames, namely the frame with the maximal energy and two frames before and after it. More elaborate onset detection schemes could be used, see, e.g., [10,11], but the one described above was found to be appropriate in this case.

3.2. Estimation of the fundamental frequency and extracting one period

After onset detection the fundamental frequency, f_0, of the analysed signal should be measured. For this any appropriate method can be used, such as the widely used autocorrelation based method discussed, see e.g. [12], or the YIN method [13], which is based on the autocorrelation method with a number of modifications. The autocorrelation method needs at least two to three pitch periods of the signal to analyse it properly. In the case of the guitar, where the lowest E string is tuned to around 82 Hz, this means that the analysis window should be at least 5.5 m long. Both the mentioned methods were tested, and the autocorrelation method [12] was selected due to its simplicity and computational efficiency.

With the knowledge of an onset occurrence and the fundamental frequency, one period from the beginning of a plucking event is extracted. For tones played close to the bridge it is crucial that the period extracted for analysis is the first one. This is because after the first pulse has arrived the first of the following pulse pairs is cancelled out strongly, so that the autocorrelation function method does not work. The cancellation takes place since the consecutive pulses are of finite width and occur close to each other in time. Fig. 6(a) illustrates the waveform of a tone played 2 cm from the bridge on the open G string. Fortunately, the used onset detection method does not fail to locate the first peak.

In contrast, for tones plucked far away from the bridge (over a third of the string length), in the very beginning of the tone the consecutive pulses look alike. Therefore, it is not crucial that the analysed period is the first one. However, it is essential that the build-up of the maximum peak is included. The build-up is included by searching the previous zero crossing of the signal, starting from the maximal value located as explained in Section 3.1. Fig. 6(b) shows the waveform of a tone played 32 cm from the bridge, close to the midpoint, on the open G string.

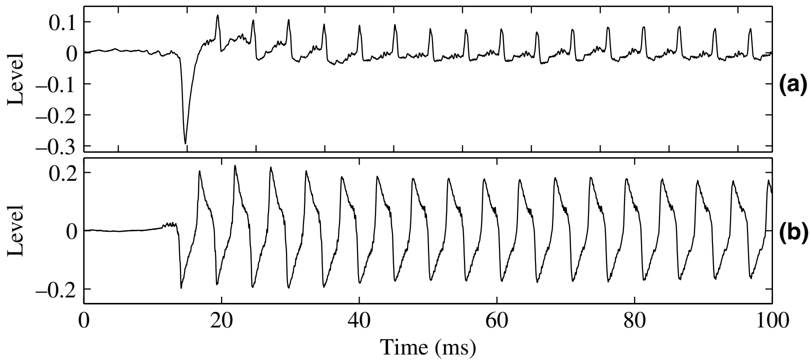

Fig. 6. The beginning of the waveforms of notes played on an open G string (f_0 = 194.0 Hz). The plucking point is: (a) 2 cm and (b) 32 cm from the bridge.

3.3. Determining minimum of autocorrelation function

After extracting the appropriate single period of the signal, its autocorrelation function (ACF) is calculated. The ACF of a discrete signal $x_1(n)$ may be defined as [12]

$$r_{1n}(k) = \frac{1}{N}\sum_{n=0}^{N-k} x_1(n)x_1(n+k), \tag{9}$$

where N is the length of the signal, and $r_{1n}(k)$ is the autocorrelation function of lag k. The autocorrelation function compares the signal to its shifted copy, and for a periodic signal it shows positive peaks at multiples of the period. In this application, the exact opposite quality is used in the following manner. The first two pulses arriving at the guitar pickup are antisymmetric, i.e., negative to one another. This will cause a strong negative correlation at a time lag corresponding to the plucking point, d, and will be seen as a negative peak in the ACF. Fig. 7 displays the extracted single periods and corresponding ACFs for the signals shown in Fig. 6. The locations of the ACF minima are marked with a circle at 4.51 ms (Fig. 7(b)) and 2.65 ms (Fig. 7(d)).

To improve the accuracy of the estimation, parabolic interpolation is used around the minimum of the ACF [13]. This is also how fractional values for the time lag can be obtained and not only integers. The fractional valued minimum index of the ACF is found by fitting a second-order polynomial to three values of the ACF, the previous sample from the minimum, the minimum value itself, and the next one, in the following manner:

$$\Delta i = \frac{1}{2}\frac{\alpha - \gamma}{\alpha - 2\beta + \gamma}, \tag{10}$$

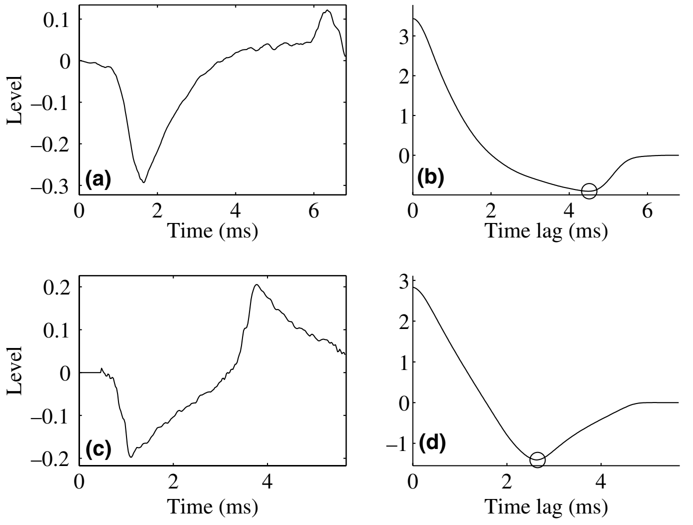

Fig. 7. (a) Waveform of the pickup signal when the plucking point is $d = 2$ cm, (b) its autocorrelation function, (c) the waveform when $d = 32$ cm, and (d) its autocorrelation function.

where Δi stands for the fractional valued offset ($-1\ldots1$), and α, β, and γ are the values of the three consecutive points, respectively.

3.4. Plucking point estimate

Now that the minimum index of the ACF has been determined, it can be used as ΔT in Eq. (6) and the estimate of the plucking point can be calculated. Rather than giving absolute values for the plucking point, we first define the relative plucking point, d_{rpp}, derived from Eq. (6) as follows:

$$d_{\mathrm{rpp}} = 1 - \frac{\Delta T f_0}{f_s}. \tag{11}$$

In this formulation the length of the string has been normalised to unity, and the actual plucking point measured from the bridge is calculated with Eq. (6), i.e., by multiplying d_{rpp} with the string length, L_s. When the length of the string is known, the fractional valued minimum index obtained from the ACF is used in Eq. (6).

For example, when the minimum of the ACF shown in Fig. 7(d) is applied to Eq. (6), the estimation for the plucking point is 31.98 cm from the bridge, indicating an error of 0.02 cm. In this case the values for the parameters are $L_s = 65.25$ cm, $f_0 = 193.8$ Hz, $T = 117.0$ samples, and $f_s = 44.1$ kHz. Next, the results obtained with the method described above will be presented more extensively.

4. Results

For testing the proposed algorithm a small database consisting of 374 recorded plucked guitar tones was created. The guitar used was a steel strung Landola, model D-805E, with DR strings, model PML-11. The recorded tones were plucked with a Jim Dunlop Jazz III plectrum. The guitar had the B-Band EMFi pickup attached under the saddle of the instrument. To create a logical database, the plucking point was first held constant while playing a chromatic scale from an open string up to the 12th fret. Secondly, the plucking point on an open string was varied in steps of 2 cm from 2 to 32 cm from the bridge. (A 2 cm change corresponds to a minor perceivable change in timbre). Both of these variations were recorded on all strings. The location of the targeted plucking point was marked on the string with a felt pen. An error margin of about ±2 mm is expected due to inaccuracies in string length measurements and the actual location of the plucking event.

In addition, for statistical analysis and to test the reliability of the algorithm further, the plucking event was repeated 20 times on selected strings and plucking points. A high (E_6) and low (A_4) frequency string was selected, and the plucking point was varied from near the bridge to beyond a typical plucking position (2, 4, 10, 12, and 20 cm). The set includes 200 plucks. In eight cases an error in onset detection was corrected manually.

Results for the plucking point estimation on fretted strings with a constant plucking point at 12 cm from the bridge are shown in Fig. 8. The results are grouped stringwise, such that each string number corresponds to the note of the open string as follows: #1 – E_6, #2 – B_5, #3 – G_5, #4 – D_5, #5 – A_4, #6 – E_4. The estimation error and fret number are shown on the y and the x axis,

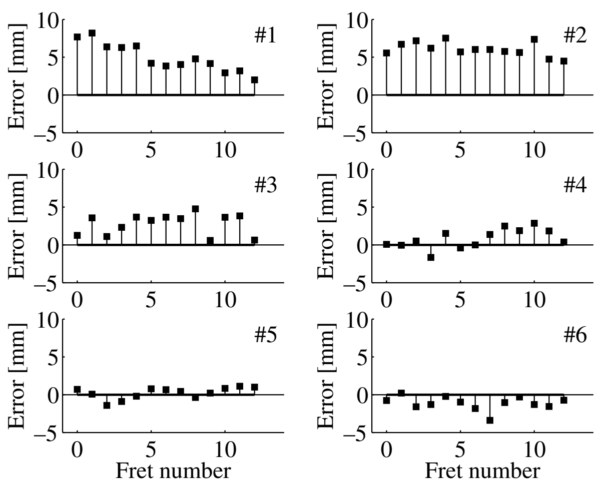

Fig. 8. The plucking point estimation error as a function of fret number for all six strings.

respectively. The error stays less than 1 cm for all the strings. Strings from 4 to 6 have a smaller error than the higher strings (1–3). This can be understood from the fact that as the fundamental frequency, f_0, increases the velocity of the pulses also increases. Therefore, small errors in the estimate will have a larger effect than at lower frequencies where the pulses travel slower. Fig. 8 also shows that the estimation algorithm works well for tones played on fretted strings. These are problematic cases for the frequency-domain method applied to microphone signals, as has been reported earlier [4].

Results for the algorithm when the plucking point has been varied in steps of 2 cm for all strings are shown in Fig. 9. The x axis indicates the actual plucking point and the y axis the estimated plucking point. Furthermore, the 'x's mark the estimated plucking point while the solid line indicates the actual plucking position. To a large degree the estimation stays reliable. The estimation error seems to increase as the plucking point approaches the bridge, but, as the following statistical analysis shows, the mean estimation error still stays smaller than a centimetre.

Fig. 10 displays the statistical results obtained for the plucking points repeated 20 times, so that the x axis indicates the plucking point (cm) and the y axis the estimation error (mm). In the figure, each box has lines at the upper (75%) and lower (25%) quartile values. The median value is indicated between these values with a line at the centre of the hour-glass shaped part of each box. The whiskers (|---|) show the extent of the rest of the data. Outliers, displayed with the star symbol (*), are data points with values beyond the end of the whiskers. Fig. 10(a) shows the estimation errors for the open E_6 string and (b) for the open A_4 string when the plucking points are 2, 4, 10, 12, and 20 cm. For plucks near the bridge on the A_4 string the error values of the outliers (4 data points) are

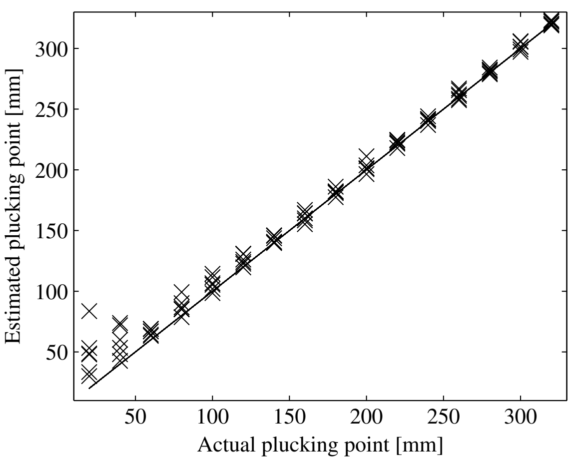

Fig. 9. Plucking point estimation results for all open strings when the plucking point is varied.

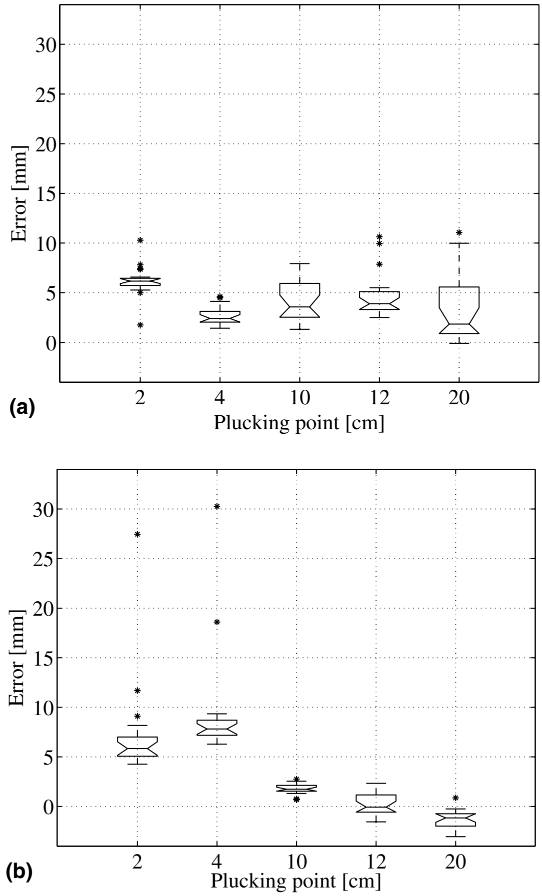

Fig. 10. Statistical plucking point estimation errors for selected plucking points for: (a) the E_6 string and (b) the A_4 string.

slightly larger than for the other cases. However, for all cases the median error values are less than a centimetre.

5. Discussion

The proposed plucking point estimation procedure leads to several potential applications. These include analysis of the plucking, estimation of plucking parameters for sound synthesis, and real-time control for computer music. We briefly discuss each of these in the following.

A standard microphone recording gives a blurred temporal response for a plucking excitation with finger or with a pick. This upsets any attempts to further analyse the excitation signal for guitar tones. The under-saddle pickup signal is cleaner than a standard microphone recording, and hence, this enables the analysis of the plucking point. Also, it gives the possibility to further analyse the excitation. One way is to cancel one of the two pulses related to the excitation and analyse and parameterise the remaining single pulse. This will open prospects in looking at different plucking styles or differences among plectra, for example.

In model-based sound synthesis, the difficulty in estimating the plucking point in a reliable way has prevented further parameterisation of sounds. In [3], [4], and [5], this possibility has been mentioned, because it is of interest to store only one excitation pulse for each tone and leave the plucking point as a free parameter which can be varied during synthesis. If cancellation of the plucking point is not done properly, synthesis of a different plucking point will cause the timbral effect to occur twice. This is undesirable. With the under-saddle pickup and the proposed estimation method, this further parameterisation becomes feasible. However, it also necessarily causes certain modifications to the synthesis model, because the modes of the instrument body are also lacking from the excitation pulse.

MIDI guitars have become useful controllers for those musicians who want to use synthesisers and other MIDI equipment, but who are not comfortable with a standard musical keyboard or a wind controller. The proposed method offers a new parameter to the arsenal of MIDI guitars, instead of detecting the onset time and fundamental frequency of each tone, it now becomes possible to estimate and transmit the plucking point. This parameter can be mapped to any parameter in the synthesiser, such as a parameter of an effect box or linear interpolation between two different synthesiser timbres. For example, timbre number 1 may be dominant when plucking close to the bridge and timbre number 2 may become louder when plucking closer to the sound hole. This additional parameter will be helpful to MIDI guitarists, who desperately need more ways to simultaneously control several parameters and nowadays also need to use pedals.

6. Conclusions

The estimation of the plucking point of guitar tones is made easy by using an under-saddle pickup. The pickup signal is digitised and the plucking point can be estimated with appropriate signal processing techniques. It turns out, as shown in this paper, that the time delay related to the plucking point can be reliably estimated based on the autocorrelation analysis of one period of the recorded signal. Previous attempts to measure the plucking point from an acoustic signal have been based on frequency-domain methods, which are more elaborate. The proposed time-domain method is easier to implement and yields accurate results. The errors in the estimated plucking point are usually less than 1 cm.

The estimation technique opens new possibilities, such as further analysis of playing styles, improved parameter estimation for the synthesis of plucked string instrument sounds, and the use of the plucking point as an additional parameter in real-time computer music applications.

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References

[1] Rossing TD. The science of sound. Reading, MA: Addison-Wesley; 1990.
[2] Fletcher NH, Rossing TD. The physics of musical instruments. Berlin: Springer; 1991.
[3] Bradley K, Cheng M, Stonick VL. Automated analysis and computationally efficient synthesis of acoustic guitar strings and body. In: Proceedings of IEEE ASSP workshop on applications of signal processing to audio and acoustics, New Paltz, New York; 1995. p. 238–41..
[4] Traube C, Smith JO. Estimating the plucking point on a guitar string. In: Proceedings of COST-G6 conference on digital audio effects, Verona, Italy; 2000. p. 153–58. Available online at http://profs.sci.univr.it/~dafx/DAFx-final-papers.html.
[5] Traube C, Depalle P. Extraction of the excitation point location on a string using weighted least-square estimation of a comb filter delay. In: Proceedings of international conference on digital audio effects, London, UK; 2003. p. 188–91. Available online at http://www.elec.qmul.ac.uk/dafx03/proceedings/.
[6] Välimäki V, Huopaniemi J, Karjalainen M, Jánosy Z. Physical modeling of plucked string instruments with application to real-time sound synthesis. J Audio Eng Soc 1996;44(5):331–353.
[7] Legge KA, Fletcher NH. Nonlinear generation of missing modes on a vibrating string. J Acoust Soc Am 1984;76(1):5–12.
[8] Backman J. Audio applications of electrothermomechanical film. J Audio Eng Soc 1990;38(5):364–371.
[9] Karjalainen M, Välimäki V, Penttinen H, Saastamoinen H. DSP equalization of electret film pickup for the acoustic guitar. J Audio Eng Soc 2000;48(12):1183–1193.
[10] Duxbury C, Sandler M, Davies M. A hybrid approach to musical note onset detection. In: Proceedings of COST-G6 conference on digital audio effects, Hamburg, Germany; 2002. p. 33–8. Available online at http://www.dafx.de/.
[11] Masri P. Computer modelling of sound for transformation and synthesis of musical signals. Ph.D. Thesis. University of Bristol, 1996. Available online at http://www.een.bris.ac.uk/Research/CCR/dmr/index.html.
[12] Rabiner LR. On the use of autocorrelation analysis for pitch detection. IEEE Trans Acoust Speech Signal Process 1977;25(1):24–33.
[13] de Cheveigné A, Kawahara H. YIN, a fundamental frequency estimator for speech and music. J Acoust Soc Am 2002;111(4):1917–1930.