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Acoustic Modeling using the Digital Waveguide Mesh

Damian Murphy, Antti Kelloniemi, Jack Mullen and Simon Shelley

I. INTRODUCTION

The digital waveguide mesh (DWM) is a numerical simulation technique based on the definition of a regular spatial sampling grid for a particular problem domain, in this specific case, a vibrating object capable of supporting acoustic wave propagation resulting in sound output. It is based on a simple and intuitive premise – the latter often considered important by the computer musicians who are the primary users of a sound synthesis algorithm – yet the emergent behavior is complex, natural and capable of high quality sound generation. Hence the DWM has been applied in many areas of computer music research since it was first introduced by Van Duyne and Smith in 1993 [1] and this paper is the first to attempt to consolidate and summarize this work. The interested reader is also directed to [2] where DWM modeling is considered in the more general context of discrete time physics based modeling for sound synthesis, and [3] where the DWM is examined within a rigorous theoretical and comparative framework for more established yet related wave scattering numerical simulation techniques.

II. THE 1-D DIGITAL WAVEGUIDE

The 1-D digital waveguide is based on a time and space discretization of the d'Alembert solution to the one-dimensional wave equation. This approach to sound synthesis was first used in the Kelly-Lochbaum model of the human vocal tract for speech synthesis [4] and has parallels with other, more generally applied wave variable scattering modeling paradigms such as the transmission line matrix (TLM) method [5] and wave digital filters (WDF) [6]. However it was Julius O. Smith III who first proposed the term *digital waveguide* and used these techniques initially for artificial reverberation [7] and later for sound synthesis [8], [9]. Digital waveguides have remained the most popular and successful physical modeling based sound synthesis technique to date, due to the realistic, high quality sounds that can be generated, often in real-time and so therefore also facilitating effective user

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interaction. This research has also been made more widely accessible through a range of commercially available physical modeling hardware synthesizers developed by Yamaha in the early 90s based on digital waveguide techniques [10]. The reader is referred to [9] and [11] for a thorough treatment and discussion of this area and a full derivation of some of the equations that are introduced in what follows. Consider the 1-D wave equation for transverse motion with speed c on an ideal, infinitely long, vibrating string:

$$\frac{\partial^2 y(t, x)}{\partial t^2} = c^2 \frac{\partial^2 y(t, x)}{\partial x^2} \quad (1)$$

The d'Alembert or traveling wave solution to (1) is defined as:

$$y(t, x) = y^+(t - x/c) + y^-(t + x/c) \quad (2)$$

where y^+ and y^- are arbitrary twice-differentiable functions denoting wave movement to the left and right respectively. Assuming that y^+ and y^- are bandlimited to half the sampling rate of the system allows the discrete time version of (2) to be defined for spatial sampling points mX and sampling interval nT such that $X = cT$:

$$y(nT, mX) = y^+(n - m) + y^-(n + m) \quad (3)$$

This solution can be implemented in an efficient and straightforward manner using two parallel digital delay lines to represent the left-going and right-going traveling wave components. Fig. 1 shows a digital waveguide implementation of an ideal string, rigidly terminated at either end of the M -sample delay lines, corresponding to the nut and bridge of a typical instrument. The system is excited with an appropriate input 'loaded' into the upper and lower delay lines at position $x_{in} = m_{in}cT$ and a physical output is obtained at $x_{out} = m_{out}cT$ by summing the upper and lower values according to (3), being exact at the sampling points of the system.

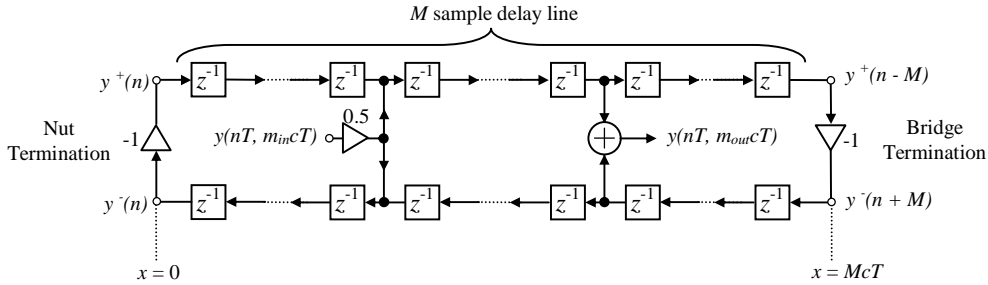


Fig. 1. The ideal lossless 1-D digital waveguide string model, M -samples long and rigidly terminated at either end.

A. The Scattering Junction

The terminations introduced in the 1-D string shown in Figure 2 are a special case of signal scattering. An input signal will propagate without loss until it is incident upon a change in system impedance, resulting in transmission and/or reflection of the incident signal. This example leads to the formal definition for a lossless scattering junction, now given without loss of generality in terms of acoustic pressure rather than string displacement. At such a junction, system continuity must be preserved in terms of pressures and volume velocities analogous to Kirchhoff's Laws for parallel connection of electrical circuit elements. Assuming N connected waveguide elements with the pressure in each defined as p_i and volume velocities as u_i then for lossless scattering the following must hold:

$$p_1 = p_2 = \dots = p_i = \dots = p_N = p_J \quad (4)$$

$$u_1 = u_2 = \dots = u_i = \dots = u_N = 0 \quad (5)$$

Note that p_J is defined as the actual pressure value at the point of connection for these N waveguide elements, referred to as the pressure value at scattering junction J . Scattering junctions, together with the 1-D waveguide elements described above, provide the basic building blocks for a digital waveguide physical model of a vibrating system. For instance, six 1-D strings could be coupled together via a scattering junction to simulate the bridge of a guitar, facilitating sympathetic resonances where excitation on one string causes low amplitude oscillation on one or more of the others due to energy transmitted through the bridge. Similarly this scattering junction could also allow coupling to a filter to simulate the effects of body resonances. Hence scattering junctions also act as system sampling points where physical variables may be tapped off for coupling with other aspects of the model or with the outside world. Similarly they can also be used to allow energy to be input to a system. In modeling a wind instrument such as a clarinet the bore can be implemented as a 1-D lossless waveguide coupled with the more complex, non-linear breath pressure/reed input function via an appropriate scattering junction implementation [9].

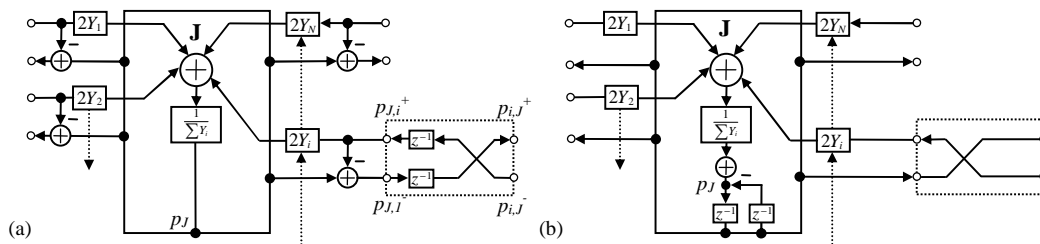


Fig. 2. Functional block diagrams for the general lossless scattering junction J with N neighbors. (a) The W-model case; (b) the K-model case. Note that in each example a single connecting waveguide element has been connected to terminal Y_i .

Fig. 2(a) shows the functional block diagram for a general lossless scattering junction J with N neighbors, with each connected unit waveguide element having an associated admittance Y_i . The impedance of a waveguide is given by $Z_i = p_i/u_i$ and hence the admittance $Y_i = 1/Z_i$. The signal $p_{J,i}^+$ represents the *incoming* signal to junction J along the waveguide from the opposite junction i . Similarly, the signal $p_{J,i}^-$ represents the *outgoing* signal from junction J along the waveguide to the opposite junction i . Connecting delay lines together at scattering junctions in a more general sense allows spatial and temporal sampling grids to be defined and gives rise to families of models that are more generally known as digital waveguide networks (DWN). The Kelly-Lochbaum vocal tract model and the simply terminated 1-D string as shown in Fig. 1 are both examples of specific DWNs. A DWN with a more complex arrangement of multi-port interconnections can be used to simulate reverberation, as in the first application of digital waveguides [7] and more recently explored in [12]. However a DWN consisting of (typically) unit delay waveguide elements and N -port lossless scattering junctions conforming to a regularly arranged and spaced grid structure gives rise to a particular family of 2-D or 3-D structures. These are called *digital waveguide meshes* and are more directly analogous in construction to the physical objects they are attempting to simulate.

III. THE DIGITAL WAVEGUIDE MESH

The digital waveguide mesh (DWM) was first proposed by Van Duyne and Smith [1] as an extension to 1-D digital waveguide sound synthesis appropriate for modeling plates and membranes, potentially leading to full 3-D object modeling. Acoustic wave propagation through a DWM is determined according to the scattering equations and associated mesh topology. For a lossless junction J according to conditions (4) and (5) or directly from Fig. 2(a) the sound pressure p_J at junction J for N connected waveguides can be expressed as:

$$p_J = \frac{2 \sum_{i=1}^N Y_i \cdot p_{J,i}^+}{\sum_{i=1}^N Y_i} \quad (6)$$

Noting from (3) that the total sound pressure p_J in a waveguide element connected to junction J can also be defined as the sum of the traveling waves in this element, or alternatively as the sum of the input and output gives:

$$p_J = p_{J,i}^+ + p_{J,i}^- \quad (7)$$

And finally, as the waveguide elements in a DWM are equivalent to bi-directional unit-delay lines, the input to scattering junction J at time index n , $p_{J,i}^+(n)$, is equal to the output from neighboring junction i into the connecting waveguide at the previous time step, $p_{i,J}^-(n-1)$. Expressing this relationship in the z -domain gives:

$$P_{J,i}^+ = z^{-1} \cdot P_{i,J}^- \quad (8)$$

Hence from (6) junction pressure values are calculated according to input values from immediate neighbors, output values are calculated using (7) and then propagated to neighbors via the bi-directional waveguide elements, becoming inputs at the next iteration according to (8). From (6), (7) and (8) via an appropriate linear transformation it is possible to derive an equivalent formulation in terms of junction pressure values only:

$$p_J = \frac{2 \sum_{i=1}^N Y_i \cdot p_i \cdot z^{-1}}{\sum_{i=1}^N Y_i} - p_J \cdot z^{-2} \quad (9)$$

Expression (9) can also be derived directly from a finite difference time domain (FDTD) formulation of the 2-D case of the wave equation in (1). The functional block diagram for the scattering junction implementation described by (9), equivalent to Fig. 2(a), is shown in Fig. 2(b). Digital waveguide models represent signal propagation via two directional wave components and schemes implemented in this way, according to (6), (7) and (8), are termed *W-models* or *W-DWMs* [2], [13], [14]. A linear transformation of a W-DWM leads to this alternative implementation as a Kirchhoff variable DWM (K-DWM) [2], [13], [14], as given in (9), and depending on physical quantities only rather than sampled traveling-wave components. In this form, and under certain conditions, a K-DWM can be computationally equivalent to an FDTD simulation.

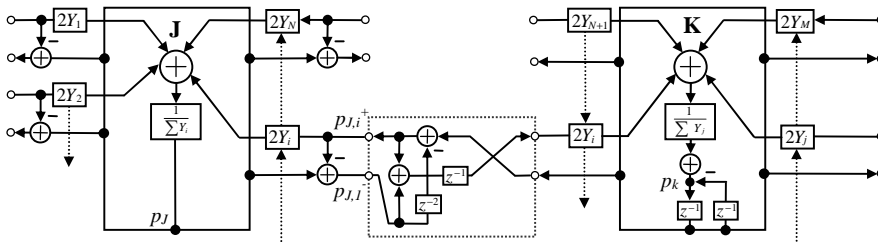


Fig.3. Functional block diagram for a W-DWM scattering junction J with N neighbors connected to a K-DWM scattering junction K via a KW-pipe connecting waveguide element.

Mixed modeling scenarios where K-DWM and W-DWM approaches have been interfaced in 1-D via a *KW-pipe* have been proposed in [14], [15], leading to the formulation of a 2-D hybrid DWM [13], [16], [17]. The 2-D hybrid

mesh combines the computational efficiency of the K-DWM approach in terms of computation time and memory use, with the flexibility of scattering-based boundary termination options for complex geometries through the use of KW-pipes. Typically KW-hybrid DWMs demonstrate a speed up in processing time of the order of 34% with a 50% decrease in use of main system memory [16]. A K-DWM scattering junction connected to a W-DWM scattering junction via a KW-pipe is shown in Fig. 3.

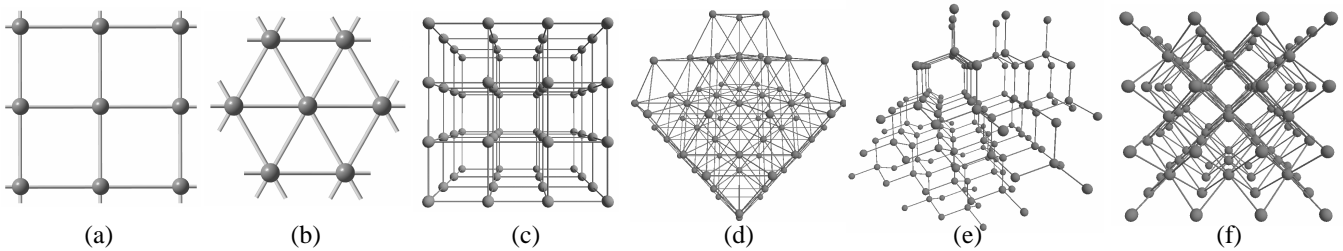


Fig. 4. DWM topologies. (a) 4-port 2-D rectilinear; (b) 6-port 2-D triangular; (c) 6-port 3-D rectilinear; (d) 12-port 3-D dodecahedral (CCP); (e) 4-port 3-D tetrahedral; (f) 8-port 3-D octahedral.

The W-DWM or K-DWM scattering equations can be used to implement a range of topologies/structures. In 2-D the most commonly implemented topologies are the 4-port rectilinear and 6-port triangular mesh structures shown in Fig. 4(a)-(b). A thorough comparison of their relative characteristics, together with those of the 3-port hexagonal mesh is presented in [18]. 2-D DWM models based on the rectilinear or triangular topology have been most commonly used for synthesis of percussion instruments such as plates, membranes and gongs [19], [20], [21], as well as for 2-D reverberation modeling [22]. 3-D topologies as shown in Fig 4(c)-(f) include the rectilinear [23], tetrahedral [24], [25] dodecahedral (also known as cubic close packed - CCP) [26] and octahedral structures, and a similar analysis of their characteristics is presented in [27].

3-D DWM structures are applied to a range of sound synthesis applications. The work of [28] combines a 2-D triangular mesh model of a drum membrane coupled to a 3-D rectilinear model of a drum-shell to give a more complete model of a percussion instrument. DWM models have been applied to simulate 3-D resonant objects [26], [29], [30], sometimes in combination or parallel with other digital waveguide models, for instance to provide synthesis of complex instrument resonances [31], [32], or to simulate a 3-D acoustic space with multiple 2-D cross-sectional simulations [33]. However most current research activity in 3-D DWM modeling is in their application to the accurate synthesis of acoustic spaces and this will be discussed in IV.C.

An additional subset of K-DWMs have also been subject to much investigation and these are based on an

interpolated rectilinear mesh structure in either 2-D [34] or 3-D [35]. Interpolated DWMs demonstrate wave propagation characteristics approaching that of triangular/dodecahedral topologies but without the additional overheads of a denser and more complex topological structure.

A. DWM Limitations

There are a number of important factors that impose limitations on DWM models as an optimal solution for all sound synthesis applications. One of the most significant advantages of the 1-D digital waveguide that originally made it a realistic proposition for applications in sound synthesis is the computational efficiency of the approach when compared with a brute force numerical solution to the system wave equation. This is further improved through the ability to commute losses to specific lumped points in the system, significantly reducing the number of calculations required per time-step iteration. Unfortunately the elegance of this approach is lost when moving to higher dimensions. With a DWM based system, acoustic wave propagation is determined by signal interaction at the scattering junctions and hence a calculation must take place at every junction for every time-step. Reducing the number of scattering junctions reduces the sample rate of the DWM and hence the effective bandwidth of the system. The advantage gained with the DWM approach however is in the structural immediacy of the simulation, allowing objects to be defined based only on physical and geometrical definitions, and the ability to observe and interact with the system at physically relevant or meaningful points.

A more specific DWM limitation is *dispersion error*, where the velocity of a propagating wave is dependent upon both its frequency and direction of travel, leading to wave propagation errors and a mistuning of the expected resonant modes. The degree of dispersion error is highly dependent upon mesh topology and has been investigated in, for example, [3], [18], [24], [27]. In 2-D both the interpolated and triangular DWMs demonstrate dispersion characteristics that are substantially reduced to a function of frequency only. In 3-D, minimization of dispersion can be similarly achieved through the use of interpolated or dodecahedral topologies. Appropriate pre- and post-processing of results from these mesh structures allows offline frequency warping techniques to be used to correct mis-tuned resonances [34], [35]. Alternatively frequency warping can be incorporated directly as part of a DWM scattering junction [36], [37]. However, although accurate synthesis of resonant modes is required for the dominant low frequency properties of a vibrating system, dispersion error is considered less important with increasing

frequency as modal density increases, and human perception of such variations becomes less critical. Oversampling a DWM can also offer improvements such that the required bandwidth lies within accepted limits, typically $0.25 \times f_{update}$ [1], where f_{update} for a DWM of dimension, D , and spatial sampling distance d is generally given by:

$$f_{update} = \frac{c\sqrt{D}}{d} \quad (10)$$

where c is the speed of sound. Ultimately f_{update} dictates the quality of audio signal output from a DWM with large sample rates requiring denser meshes, more computer memory and hence taking longer to run, limiting even the most efficient large-scale K-DWMs to offline generation only.

B. DWM Boundary Termination

There exist a number of possibilities for terminating a DWM at a system boundary. In [21] a 10×10 node 2-D rectilinear DWM is terminated with single one-pole allpass filters, which may be interpreted as a 1-D termination connected to an ideal spring, allowing modal frequencies in the DWM to be re-tuned or corrected appropriately. For curved boundaries, where the perimeter of the structure being modeled is not normal/parallel to the axes of the mesh, non-integer length waveguide elements called *rimguides* can be used [20] and have been demonstrated as appropriate for accurate low frequency modeling of circular membranes using a 2-D triangular mesh.

A commonly applied solution is to passively terminate a DWM using a simple 1-D connection that implements a change in admittance such that there is no signal return from the connected boundary over a finite time duration. Hence the associated input value for such a connection in Fig. 2, or using (6) or (9), is set to zero. This termination acts to reflect an incident signal according to the change in admittance of the connected waveguide elements. In the simplest case for a 1-port boundary-node p_B connected to a single N -port scattering junction p_1 with a change in waveguide admittance from Y to Y_B , a reflection coefficient $-1 \leq r \leq 1$ is determined such that:

$$r = \frac{Y - Y_B}{Y + Y_B} \quad (11)$$

p_B can therefore be calculated as a function of the sound pressure of the incident traveling wave variable from p_1 :

$$p_B = (1 + r) \cdot p_{B,1}^+ \quad (12)$$

In the equivalent K-DWM case, a passive termination is equivalently implemented as a feedback loop between waveguide element terminals in Fig. 2 with unit delay as derived in [13], [38] and given by:

$$p_B = (1+r)p_1 \cdot z^{-1} - r \cdot p_B \cdot z^{-2} \quad (13)$$

Note that $r = 1$ or $r = -1$ gives total reflection and $r = 0$ approximates anechoic conditions. Full derivations of boundary conditions for the general N -port boundary termination for K-, W-, and KW-hybrid cases is offered in [16] and a similar boundary implementation for a triangular DWM using multi-port reflection factors is presented in [29].

IV. APPLICATIONS OF THE DWM

The digital waveguide mesh in 2-D and 3-D has been applied to a diverse range of applications where simulation of acoustic wave propagation within an enclosed system is required. What follows is a summary of recent results and research in this area, namely for vocal tract synthesis, object modeling, synthesis of room impulse responses and how this method can be extended to abstract higher dimensions.

A. 2-D Vocal Tract Modeling for Speech Synthesis

The well established 1-D Kelly-Lochbaum vocal tract model [4] is based on a linear series of concatenated cylindrical acoustic tubes, each of different cross-sectional area, and each tube section implemented as a 1-D digital waveguide element. A number of developments on this basic model include nasal tract, lip radiation and wall losses to synthesize the singing voice [39] and the use of fractional waveguides to make lengthwise changes to tract shape [40], [41]. Standard waveguide elements have also been substituted for conical equivalents using scattering methods derived from the spherical wave equation. This increases model accuracy, giving higher-order area function approximation, but adds to the computational load and introduces possible stability problems [40], [42]. More recent work has explored the possibility of replacing the basic 1-D digital waveguide implementation with a 2-D DWM model that simulates the variation in cross-sectional area along the vocal tract directly through an appropriately shaped mesh geometry [43]. Formant patterns produced using the 2-D DWM implementation are equivalent to those produced by a very high-resolution 1-D digital waveguide acoustic tube based simulation. The 2-D model also offers simulation of cross-tract modes due to the additional dimension of freedom for acoustic oscillation and propagation and approximately linear control over formant bandwidths via the additional reflection parameter at the side walls of the vocal tract. Hence the 2-D DWM vocal tract offers improvements similar to other developments based on enhanced order acoustic tube area function approximation, together with additional model flexibility such as the ability to simulate a split in the air channel used in the creation of sounds such as /l/.

The disadvantages of this proposed voice synthesis mechanism rests in its inability to simulate smooth, continuous dynamic changes to the tract area functions to facilitate articulated voice synthesis, and the high mesh sample rate, f_{update} required to ensure accurate tracking and mapping of vocal tract shape resulting in an implementation that can only work offline. Both of these problems have been analysed and a solution proposed in a new implementation of the 2-D DWM [44]. In this new method, rather than mapping acoustic tube area function directly to the 2-D DWM geometry, a constant width 2-D rectangular DWM, 17.5 cm long with $f_{update} = 44.1\text{kHz}$, is used. The waveguide element impedance across the width of this rectangular geometry is then varied according to the area function information. A minimum impedance channel Z_{min} is defined as the lowest value across the range of vowels to be simulated, corresponding directly to the largest cross-sectional area A_{max} and from this a maximum tract width opening can be defined. An *impedance map* is constructed for a particular vowel tract shape such that each area function value $A(x)$ along the length of the tract walls corresponds to a maximum impedance value Z_x . An impedance curve varying from Z_x to Z_{min} and back to Z_x at the opposite wall is then defined across the tract according to a raised cosine function, with the minimum impedance channel equidistant between the tract walls.

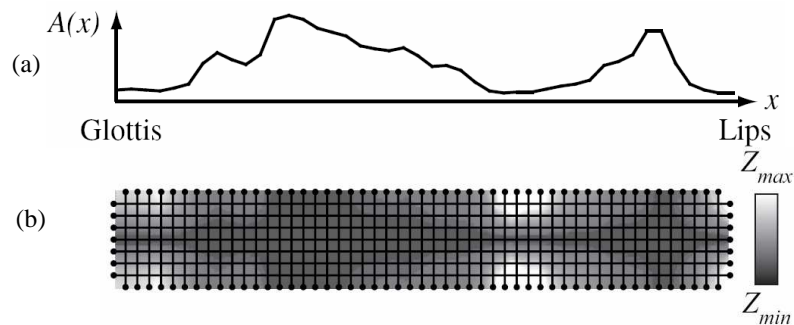


Fig. 5. Forming the impedance mapped /u/ vowel DWM. (a) Cross-sectional area function; (b) rectilinear mesh with raised cosine impedance map.

Fig. 5(a) shows the cross-sectional area function information $A(x)$ taken from MRI scans [45] as it varies along the length of the vocal tract from glottis to lips. Fig. 5(b) is the corresponding impedance map imposed across and along the underlying rectangular 2-D DWM based on a 4-port rectilinear topology. Areas of higher impedance are represented by a lighter shading and the minimum impedance channel can be observed as the darker area along the center of the map. Fig. 6(a) shows the resulting formant pattern for this vocal tract shape, when excited by a noise source at the glottis and measured at the lip end. The dotted lines have been generated from a high-resolution 1-D

waveguide model, using the same area functions for comparison purposes and measured average formant values are also shown.

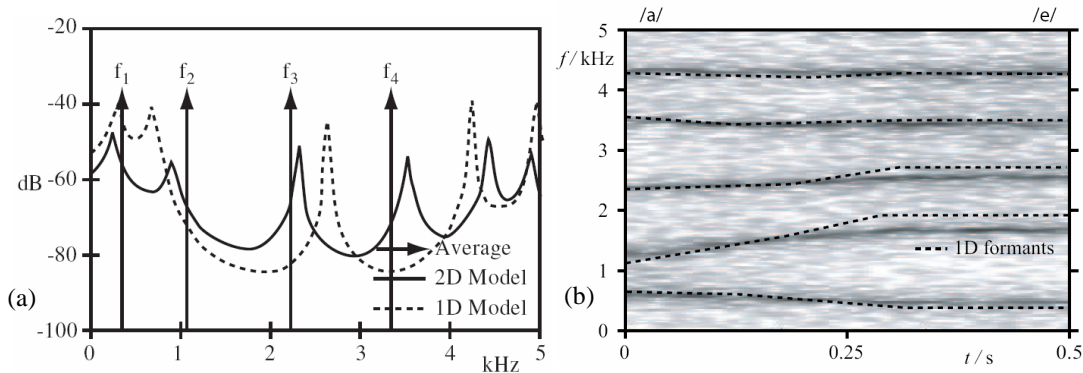


Fig. 6. Formant patterns from the impedance mapped DWM under noise excitation. (a) /u/ vowel compared with a high resolution 1-D model and average measured values; (b) /a/ to /e/ diphthong compared with same 1-D model.

Software developed to test the real-time dynamic behaviour of this 2-D DWM vocal tract model is available for download and use at [47] and initial results based on this system were first presented in [44]. This application also facilitates real-time dynamic articulation. An example is presented in Fig. 6(b) demonstrating a smooth linear interpolation between area function data for the /a/ - ‘bard’, and /e/ - ‘bed’, vowels, under noise source excitation to highlight the resulting change in formant patterns.

Fig. 6 shows that this new 2-D dynamically varying DWM demonstrates results in terms of simulated formant frequencies that are in good agreement with both a high resolution 1-D model and real world values. Fig. 6(a) seems to indicate that the 2-D model is closer to real world formant values than the high resolution 1-D case, although this accuracy actually varies with target vowel/tract-shape and the real-world values used. In general, the two modelling methods are in good agreement with one another. Also, Fig. 6(b) demonstrates a smooth transition between vocal tract shapes without any discontinuity for the 1-D and 2-D cases, being of particular importance in the latter example. Hence from these generally comparable results the 2-D dynamically varying DWM can be seen to offer an alternative to current 1-D dynamic vocal tract models, while also offering additional advantages over these 1-D implementations as discussed above and presented in [43] for static simulations of the vocal tract. The reader is invited to test the software presented in [47] and compare the audio output from both 1-D and 2-D models under LF glottal source excitation. Informal perceptual testing has demonstrated that users consider the 2-D example to be more ‘natural-sounding’ than the similar 1-D case.

This work is the first demonstration of a dynamically varying DWM model, in this case operating in real-time. Most prior DWM work has been based on a static representation of the acoustic system under study, partly due to the computational resources required for real-time implementation and user interaction, and partly due to possible discontinuities in the output from the resulting model. Hence this work potentially opens new areas of research and application areas for DWM modeling, possibly moving to direct user-input and feedback that has currently only been possible in 1-D digital waveguide synthesis. Further work in this area will concentrate on developing appropriate tract wall boundary filters and facilitate lengthwise shape changes for modeling lip protrusion required for accurate synthesis of the /u/ vowel. This work also demonstrates the potential of moving towards a full 3-D DWM model using 3-D MRI scans of the vocal tract shape incorporating complex-shape cross-sectional area data.

B. 2-D Object Modeling

The DWM is often used to synthesize the acoustic properties of a 2-D or 3-D resonant body as these objects are a fundamental component of most musical instruments, serving to both amplify and modify the characteristics of a source excitation. Given that the resonating aspects of most instrument bodies are relatively small implies that a high resolution DWM implementation is feasible – in real-time in the case of the vocal tract model above - with modern computing facilities. Consider the classic example of a 2-D ideal stretched circular membrane. The resonant frequencies, f_{mn} , can be defined according to the nature of their *nodal regions* where m represents the number of nodal lines positioned along the *diameter* of the membrane and n represents the number of circular nodal lines, including the boundary. The fundamental frequency of an ideal membrane, f_{01} , can be calculated according to its physical properties (for example as presented in [19]). Subsequent modes are fixed relative to f_{01} .

Fig. 7(a) shows the nodal regions of an ideal stretched circular membrane with diameter 0.5 m, implemented using a highly oversampled 2-D triangular DWM, with $f_{update} = 192$ kHz, resulting in a spatial sampling distance of 0.00253m and a total of 35742 junctions. The membrane is excited near the boundary with a low-pass filtered impulse and an output is obtained at a junction near the opposite boundary. To model an ideal membrane with clamped edges, the reflection coefficients, r , at the boundary of the mesh are set to -1. The modes (0,2), (1,1), (2,1) and (3,1), are shown, with associated frequencies given relative to f_{01} . Fig. 7(b) plots the spectrum of the output against the theoretical predicted frequencies for the fundamental and first nine modes.

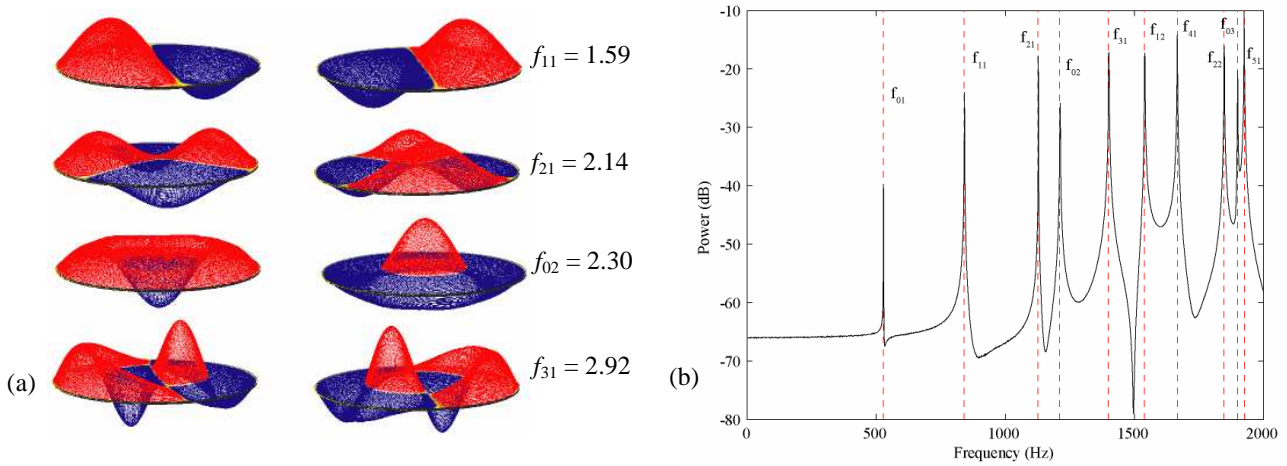


Fig. 7. 2-D triangular DWM model of a membrane with a diameter of 0.5m. (a) Animation captures from the resulting simulation demonstrating resonance at modes (0,2), (1,1), (2,1) and (3,1); (b) actual output spectrum compared with predicted modal frequencies.

Note that from Fig. 7(b) there is an exact correlation between the predicted modal frequencies and those obtained via simulation and this is due to the high mesh sample rate used, minimizing dispersion error effects for the bandwidth studied, and ensuring a smooth mesh fit to the circular boundary of the membrane without using ringguides.

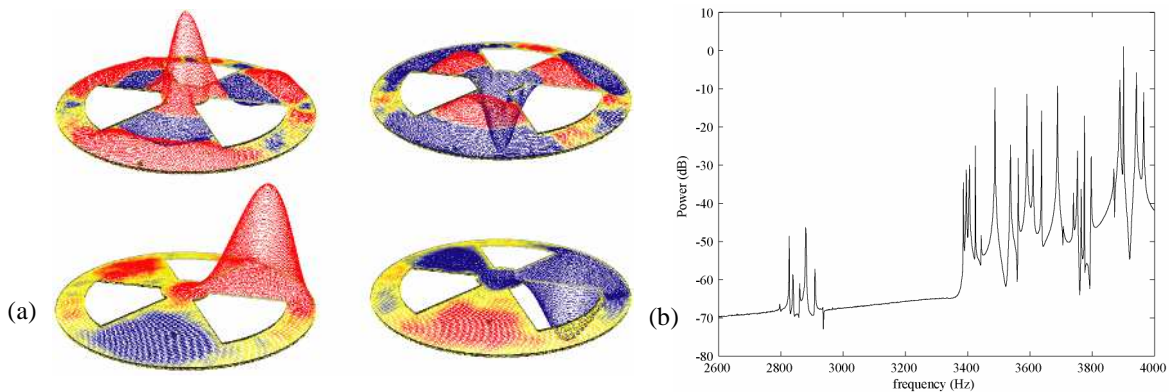


Fig. 8. 2-D triangular DWM model of a physically impossible system – a tri-foil radiation symbol membrane with a diameter of 0.5m. (a) Animation captures from the resulting simulation demonstrating modal resonances; (b) output spectrum.

An exciting possibility with physical modeling synthesis is that, with clear defined rules governing system behavior, it becomes relatively straightforward to extend these rules to situations that could not exist, or are difficult to control, in the real world. Fig. 8 presents one such example as an extension of the 2-D circular membrane, and shows the two lowest modes of resonance from a DWM membrane simulation of a tri-foil radiation symbol with diameter and f_{update} as before, this time resulting in a model consisting of 25549 junctions. Sound examples for these simple 2-D objects are available at [48]. In isolation the sounds produced from such basic 2-D membranes, although

percussive in nature, are somewhat uninspiring and require a more complete model for accurate and interesting object synthesis and hence these examples should be considered as a starting point only. Further research in improved modeling of resonant objects has considered DWNs for more complex theoretical multidimensional systems [3], specific aspects such as coupling a 2-D membrane to a 3-D resonator [28], improved non-linear excitation [49] and using simple DWM resonators to model the high frequency characteristics of complex instrument bodies [31]. Also of note is the Sounding Object Project that has explored physical modeling, including 2-D and 3-D DWMs of resonating objects, with a view to matching the perception of synthesized sounds to the modeled objects that created them [26], [29], [50].

C. *Synthesis of Room Impulse Responses*

The first application of DWMs in the field of room acoustics simulation was by Savioja et al. in 1994 [23]. Fundamentally, synthesizing the characteristics of a bounded space using a DWM is exactly the same as synthesizing the sound of a vibrating physical object. However in the latter example, sound output is generated directly from the modeled object by reading sample values at a scattering junction, whereas with room acoustics modeling it is a Room Impulse Response (RIR) that is synthesized rather than the actual sound source. The RIR is generally of little interest in terms of its direct audio quality, however when convolved with an arbitrary anechoic audio input the result is to perceive the audio source as if placed within the modeled space. Also the relative size of the DWMs used as 3-D acoustic spaces are many times larger than, for example, the 2-D vocal tract presented in IV.A, and hence take considerably longer to execute implying offline RIR synthesis only.

It has been shown that DWMs offer accurate RIR synthesis at low frequencies [51], and demonstrate natural wave phenomena such as interference and diffraction [52], with high frequency accuracy being limited by f_{update} and the dispersion error of the selected topology. This contrasts with other RIR synthesis methods based on geometric acoustic techniques [53] that are typically valid for high frequencies only. Other research has explored how 3-D spaces, or their reverberant characteristics, might be simulated using 2-D models, significantly reducing computational resources [22], [54]. However the accurate simulation of DWM boundaries is still a key research area with a view to how the physical properties of real materials might be modeled. This has included how 1-D boundary termination might be optimized for anechoic conditions [55], leading to a new spatially averaged approach for the

$0 \leq r \leq 1$ case [56]. Optimized absorption/reflection across a wide range of angles of incidence for $-1 \leq r \leq 1$ has been facilitated using the *admittance boundary* method [57], where a DWM is terminated with additional layers of boundary-nodes behind the actual boundary location, in turn terminated using an optimal anechoic solution.

For accurate simulation of real acoustic boundaries frequency dependent reflection/absorption must be implemented. In [58], a boundary-node is replaced with a boundary filter defined to optimally match given frequency dependent material reflection coefficients and implemented using a first-order IIR filter for a 2-D rectilinear K-DWM. This results in a good approximation to the target response, but is subject to the directional dependent characteristics of the mesh topology, being less accurate for certain angles of incidence.

The other important characteristic of a real-world acoustic boundary is whether a reflection is *specular*, where the angle of reflection is equal to the angle of incidence, or *diffuse* such that the incident energy is redistributed over a range of angles. Previous diffuse boundary implementations for a DWM are effective but limited, either in terms of accuracy [20], or by sacrificing user control for an optimal solution [59]. A new technique based on [20] simulates accurate diffusion with a high degree of control and consistency by rotating incoming junction signals via a circulant matrix at a *diffusing layer* of standard N -port W-DWM air-nodes adjacent to the boundary [60]. The model is lossless, and allows other boundary conditions, such as frequency dependent absorption, to be easily incorporated.

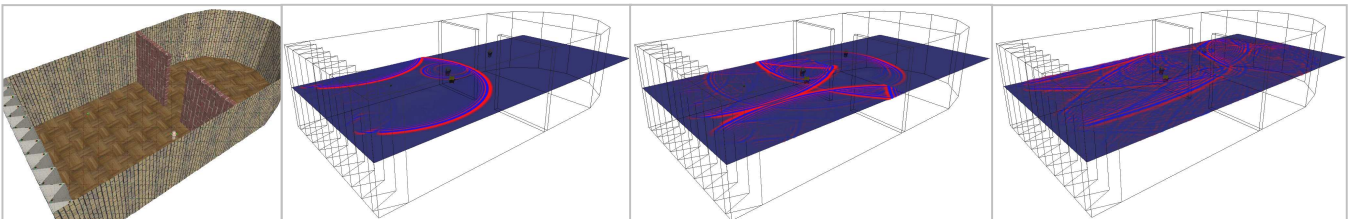


Fig. 9. DWM room acoustics modeling in *RoomWeaver* incorporating the latest implementations of frequency dependent and diffusive boundaries. From left to right, a completely defined acoustic space followed by wave propagation snapshots through a 2-D horizontal plane of the same space viewed in wireframe mode.

Much of this recent application focused research has been incorporated as part of the *RoomWeaver* DWM based room acoustics research tool first presented in [17] and shown in Fig. 9. The purpose of the system is to allow the user to intuitively set up enclosed space geometry, boundary surface, and source/receiver parameters required to generate a RIR by means of a simple scripting language and Graphical User Interface. High quality reverberation and auralization for a wide range of spaces/applications are possible using high resolution 2-D triangular, and 3-D mesh topologies both based on a KW-hybrid implementation. A range of RIRs synthesized according to varied

initial conditions and associated audio examples are available for download [48]. For complete synthesis of a sound event it would be desirable to incorporate a 3-D dynamically variable instrument model within a larger 3-D DWM of a performance space, requiring appropriate interfacing across DWM types according to the space and instrument models used. Although non-trivial this has been considered in the case of a drum model using a 2-D triangular DWM membrane with a 3-D rectilinear shell [28], and in the more general case [3]. However such complete synthesis could only be offered via the offline RIR generation/real-time convolution processing paradigm due to the computational expense of full 3-D space modelling. Such examples are presented in [48] for a 2-D DWM vocal tract processed with DWM synthesized RIRs.

D. The Hyper-Dimensional Digital Waveguide Mesh

From (6) it is clear that dimensionality is not inherent in the scattering equations. For example the 4-port lossless scattering junction is the main algorithmic building block of both the 2-D rectilinear and 3-D tetrahedral mesh. The spatial arrangement of the surrounding scattering junctions – the mesh topology – is the determining factor and hence it is possible to extend the scattering junction concept beyond three spatial dimensions to *hyper-dimensional DWMs* [23], [61], that have been shown as appropriate for simulating artificial reverberation [62].

At low frequencies, the acoustic characteristics of a room can be modeled with sufficient accuracy by approximating the main dimensions of a basic cuboid model and simulating the corresponding resonant modes. In real rooms there are typically additional architectural features that lead to frequency-dependent irregularity in these predicted trajectory lengths. Hence, at higher frequencies, a typical RIR will demonstrate a large number of densely and irregularly distributed modal peaks that are not determined by the basic geometry of the space alone. However for high quality artificial reverberation not all of these modes need to be simulated directly, with approximately 1500 modes distributed evenly or along a logarithmic scale between 80 Hz - 10 kHz being sufficient for diffuse and natural sounding artificial reverberation [63]. In a DWM, the number of primary resonant modes is equal to the number of dimensions, with higher dimensions leading to a more irregular arrangement of modal peaks. It is possible to extend these equations describing the resultant wave propagation to the hyper-dimensional case, where for each mesh dimension x_i , the primary mode has a frequency corresponding to $(c/2L_i)$ with L_i defined as the trajectory length. In an N -dimensional space standing waves occur at the following frequencies:

$$f_{n_1 n_2 n_3 \dots n_N} = \frac{c}{2} \sqrt{\left[\sum_{i=1}^N \left(\frac{n_i}{L_i} \right)^2 \right]} \quad (14)$$

where n_i is the integer index of the current mode for each dimension and c is the speed of sound. Furthermore, at a specific modal frequency, the sound pressure value at a point $(x_1, x_2, x_3, \dots, x_N)$ inside such is space is determined by:

$$p_{n_1 n_2 n_3 \dots n_N}(x_1, x_2, x_3, \dots, x_N) = A \sum_{i=1}^N \cos\left(\frac{n_i x_i}{L_i}\right) \quad (15)$$

where A is an arbitrary amplitude coefficient.

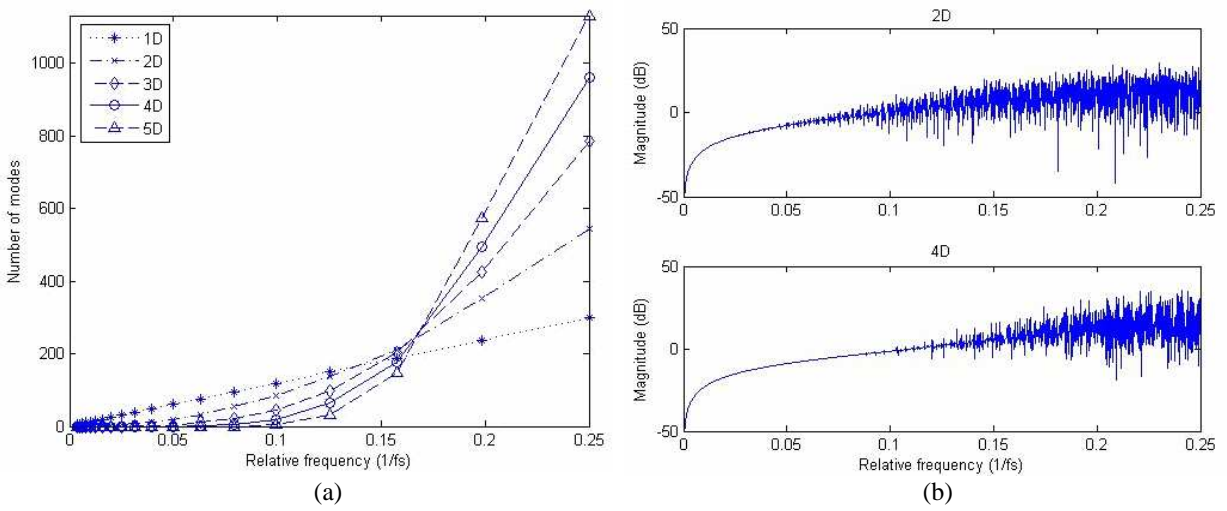


Fig.10. Frequency response information for 2310 node DWMs of varying dimension. (a) Theoretical modal distribution varying with increasing dimensionality; (b) frequency response of a 2-D and 4-D DWM.

Examples of modal distributions calculated using (15) are shown in Fig. 10(a). Note that the additional advantage of a hyper-dimensional DWM reverb is the resultant high density distribution of high frequency modes while simultaneously avoiding potentially problematic (in terms of perceived sound quality) low frequency resonances. This is due to the trajectories being kept shorter compared to a similar model with the same number of junctions but lower dimensionality and is demonstrated in Fig. 10(b) comparing 2-D and 4-D DWMs. Hence hyper-dimensional DWM reverb satisfies the requirement for a densely distributed high frequency modal response while giving the freedom for the more precise and sparsely arranged low frequency modes to be modeled with any other appropriate technique without frequency overlap. Further work for a more natural reverberant effect requires the implementation of frequency dependent losses to simulate air and boundary absorption as used in RIR synthesis with standard 2-D and 3-D DWM models.

V. CONCLUSIONS

The digital waveguide mesh has been an active area of music acoustics research for over ten years. Although founded in 1-D digital waveguide modeling, the principles on which it is based are not new to researchers grounded in numerical simulation, FDTD methods, electromagnetic simulation et al. This paper has attempted to provide a considered review of how the DWM has been applied to acoustic modeling and sound synthesis problems, including new 2-D object synthesis and an overview of recent research activities in articulatory vocal tract modeling, room impulse response synthesis and reverberation simulation. The extensive, although not by any means exhaustive, list of references indicates that though the DWM may have parallels in other disciplines, it still offers something new in the field of acoustic simulation and sound synthesis. Perhaps one reason for the continued interest in this area is the natural and intuitive complex emergent behavior that results from such simple, locally defined scattering equations. However despite this perceived simplicity, it is also clear that there are still many non-trivial problems to be solved. There are few current examples of useful and playable virtual instruments using DWM based sound synthesis (although a virtual drum in the London Science Museum that can be played in real-time and is a realization of the work presented in [28] is a notable exception) and this is mainly due to the computational resources required for such a real-time model. Non-real time operation is not a problem when simulating static, linear time invariant systems such as a representation of an acoustic space, and hence most recent DWM work has focused in this area. The real-time convolution of an audio input signal with the impulse response generated from such a model is trivial to implement on a modern computing platform and is now a commonly used sound processing operation. Offline only DWM based virtual instruments for sound synthesis prohibit user interaction and severely limit playability. However, some of the recent developments presented in this paper, particularly those relating to dynamic, real-time vocal tract simulation, are beginning to make significant inroads in this area and will hopefully lead to new DWM implementations that can be applied more generally - and more successfully - to the diverse range of possible sound synthesis applications.

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