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ROOM ACOUSTICS MODELING WITH THE DIGITAL WAVEGUIDE MESH — BOUNDARY STRUCTURES AND APPROXIMATION METHODS

Antti Kelloniemi

Dissertation for the degree of Doctor of Science in Technology to be presented with due permission for public examination and debate in Auditorium S4, Department of Electrical and Communications Engineering, Helsinki University of Technology, Espoo, Finland, on the 24th of November, at 12 o'clock noon.

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Author Antti Kelloniemi

Name of the dissertation

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Opponent	Dr. Rudolf Rabenstein
Supervisor	Prof. Vesa Välimäki
Instructor	Prof. Lauri Savioja

Abstract

The goal of this research was to develop realistic and efficient simulation models and approximation methods for sound wave propagation in rooms and other closed spaces. The focus was on optimization of boundary structures and on the use of models with high or low dimensionality in the simulation of three-dimensional wave propagation.

Modeling of wave phenomena such as diffusion and interference is needed for precise physical simulations of architectural acoustics. Approximation of sound propagation by geometrical methods is not sufficient in spaces with small dimensions or complicated shapes, and at low frequencies. The digital waveguide mesh method studied in this thesis research includes these phenomena automatically.

In this work, improvement of boundary model accuracy was pursued. Reflection coefficient values were modeled using novel equations relating the input and output signals at the mesh junctions. First, constant real-valued reflection coefficients were implemented with special attention to the absorbing boundary condition. Then, a more flexible solution was introduced, offering the possibility of directional dependency of the reflection coefficient. The last boundary structure proposed in this work makes it possible to define the boundary characteristics in a frequency-dependent manner.

In addition to the boundary methods, new ways of approximating the reverberation characteristics of a space have been addressed. As a full three-dimensional mesh is too demanding of computer resources and an unnecessarily exact simulation method for many uses, a computationally lighter solution using multiple two-dimensional meshes was introduced. Its benefits in spatial wave propagation modeling and visualization are discussed. Also, the concept of hyperdimensional meshes is exploited to show that small sized “hypermeshes” can be used as efficient models of high-frequency reverberation.

The novel techniques proposed in this research improve the accuracy and efficiency of the digital waveguide mesh models. These techniques are expected to be used in simulations of room acoustics and musical instruments, as well as in visualizations of wave propagation and in the creation of artificial reverberation effects.

Keywords acoustic signal processing, architectural acoustics, FDTD methods, modeling, visualization

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VÄITÖSKIRJAN TIIVISTELMÄ

Tekijä Antti Kelloniemi

Väitöskirjan nimi
Huoneakustiikan mallintaminen aaltojohtoverkkoa käyttäen — rajapintoja ja approksimaatiomenetelmiä

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Tiivistelmä

Tutkimuksen tavoitteena oli suljetussa tilassa tapahtuvan äänen aaltoliikkeen tehokas ja realistinen mallintaminen sekä tila-akustiikan approksimointi aaltojohtoverkko-tekniikkaa käyttäen. Työssä kehitettiin erityisesti rajapintojen mallinnusta sekä kolmiulotteisen aaltoliikkeen approksimointimenetelmiä.

Aaltoliikkeeseen liittyvien ilmiöiden kuten diffraktion ja interferenssin mallintaminen on tarpeen tarkkojen fysikaalisten tila-akustiikan simulaatioiden luomiseksi. Geometriset menetelmät ovat riittämättömiä äänen etenemisen mallintamisessa erityisesti matalilla taajuuksilla ja pienissä tai monimutkaisissa tiloissa. Aaltojohtoverkko-menetelmässä nämä ilmiöt tulevat automaattisesti huomioitua.

Tässä työssä keskityttiin reunojen mallintamisen tarkkuuden parantamiseen. Heijastuskertoimet mallinnettiin uusia verkon solmujen arvojen päivitysmenetelmiä käyttäen. Ensin mallinnettiin kertoimia, joilla on reaalinen vakioarvo. Erityistä huomiota kiinnitettiin täysin heijastamattomaan reunaehtoon. Seuraavaksi kehitettiin joustavampi ratkaisu, jota käyttäen suuntariippuvien kertoimien määrittäminen on mahdollista. Viimeisenä esitettiin reunaehtojen taajuusriippuvuuden mahdollistava menetelmä.

Rajapintojen toteutusmenetelmien lisäksi työssä käsiteltiin tila-akustiikan approksimointimenetelmiä. Täydellinen kolmiulotteinen aaltojohtoverkko on moneen käyttöön laskennallisesti liian raskas ja epätarkempikin ratkaisu olisi riittävä. Työssä esiteltiin vaihtoehtona useita kaksiulotteisia verkkoja käyttävä menetelmä, joka on merkittävästi kevyempi laskea. Kaksiulotteinen verkko on käytännöllinen myös aaltoliikkeen visualisoinnissa. Tutkimus ulotettiin kattamaan myös yli kolmiulotteiset aaltojohtoverkot. Pienten hyperulotteisten verkkojen todettiin toimivan tehokkaina kaiunnan malleina korkeilla taajuuksilla.

Työssä kehitetyt uudet menetelmät parantavat aaltojohtoverkko-mallinnuksen tarkkuutta ja tehokkuutta. Niiden uskotaan tulevan käyttöön tilojen ja soitinten akustisissa simulaatioissa, sekä aaltoliikkeen visualisoinneissa ja keinotekoisten kaiuntaefektien kehityksessä.

Asiasanat Akustinen signaalinkäsittely, tila-akustiikka, FDTD-menetelmät, mallintaminen, visualisointi

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Preface

This research was primarily carried out at the Telecommunications Software and Multimedia Laboratory of the Helsinki University of Technology in Espoo, Finland, from year 2003 to year 2006.

I would like to express my gratitude to Prof. Vesa Välimäki, my thesis supervisor, and to Prof. Lauri Savioja, project manager of the Experimental Virtual Environment (EVE) research group and the head of the Academy of Finland research project "Spatial Audio and Room Acoustics" (SARA, no. 201050), which was the main source of funding for this work. They were both encouraging and enthusiastic in introducing me to the world of scientific research.

I would like to thank my co-authors, Patty Huang, Jack Mullen, Dr. Damian T. Murphy, and Simon Shelley, and my colleagues, Toni Hirvonen, Perttu Hämäläinen, Prof. Matti Karjalainen, Dr. Tapio Lokki, Teemu Mäki-Patola, Jyri Pakarinen, Dr. Ville Pulkki, and Sampo Vesa, for the collaboration, the fruitful conversations, and all the help I have received from them. In addition, I am grateful for the comments of my thesis pre-examiners, Dr. Federico Fontana and Dr. Aki Härmä.

Special thanks should be given to Dr. Damian T. Murphy at the University of York in the United Kingdom for having me as a visiting researcher during spring 2006. Also, all the personnel of my current employer, Panphonics Oy, should be thanked for their positive attitudes during the finalization of my thesis.

I would like to extend my thanks to the organizations that have supported this work financially: the Academy of Finland, the Department of Computer Science and the Department of Electrical and Communications Engineering at Helsinki University of Technology, Tekniikan Edistämissäätiö (The Ministry of Trade and Industry), and the Nokia Foundation.

For their love and all the support, I wish to thank my mother Iiris Kelloniemi and father Paavo Kelloniemi. Finally, I am very grateful for the love and encouragement Aija Kiesi has given me during the years I have pursued this research.

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List of Publications

P1 A. Kelloniemi, D.T. Murphy, L. Savioja, and V. Välimäki. Boundary Conditions in a Multi-Dimensional Digital Waveguide Mesh. In *Proceedings of the 29th IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vol. 4, pp. 25-28, Montreal, Canada, May 2004.

P2 A. Kelloniemi, L. Savioja, and V. Välimäki. Spatial Filter Based Absorbing Boundary for the 2-D Digital Waveguide Mesh. *IEEE Signal Processing Letters*, Vol. 12, Issue 2, pp. 126-129, February 2005.

P3 A. Kelloniemi. Improved Adjustable Boundary Condition for the 2-D Digital Waveguide Mesh. In *Proceedings of the 8th International Conference on Digital Audio Effects (DAFx-05)*, pp. 237-242, Madrid, Spain, September 2005.

P4 A. Kelloniemi. Improved Adjustable Boundary Condition for the 3-D Digital Waveguide Mesh. In *Proceedings of the IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA'05)*, pp. 191-194, New Paltz, NY, USA, October 2005.

P5 A. Kelloniemi, V. Välimäki, and L. Savioja. Simulation of Room Acoustics Using 2-D Digital Waveguide Meshes. In *Proceedings of the 31st IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Vol. 5, pp. 313-316, Toulouse, France, May 2006.

P6 A. Kelloniemi. Frequency-Dependent Boundary Condition For the 3-D Digital Waveguide Mesh. In *Proceedings of the 9th International Conference on Digital Audio Effects (DAFx-06)*, Montreal, Canada, September 2006.

P7 D. T. Murphy, A. Kelloniemi, J. Mullen, and S. Shelley. Acoustic Modeling using the Digital Waveguide Mesh. Accepted for publication in *IEEE Signal Processing Magazine*, March 2007.

P8 A. Kelloniemi, P. Huang, V. Välimäki, and L. Savioja. Spatial Audio and Reverberation Modeling using Hyperdimensional Digital Waveguide Meshes.

Helsinki University of Technology, Laboratory of Acoustics and Audio Signal Processing, Report 80, Espoo, Finland, November 2006. *EURASIP Journal on Applied Signal Processing*, submitted for publication on September 19th, 2006.

Summary of Publications and Contribution

Publication 1: "Boundary Conditions in a Multi-Dimensional Digital Waveguide Mesh"

The predominant method for defining an adjustable boundary condition for a two-dimensional digital waveguide mesh was evaluated. It performs poorly, especially at low reflection coefficient values. A novel method for computing the boundary junction value was introduced, which was created as a linear combination of earlier solutions for reflection coefficient values $R = 0$ and $R = 1$. With the new method, significantly more accurate simulation results are obtained with values $0 < R \leq 1$. The idea of this reflection function was created and tested by the present author. 70% of the paper was written by him.

Publication 2: "Spatial Filter Based Absorbing Boundary for the 2-D Digital Waveguide Mesh"

It was reported in Publication 1 that the error in reflection magnitude at two-dimensional mesh boundaries is most erroneous when highly absorbing boundaries are desired. Thus, a new computational method for boundaries with minimal reflection was created. It relies on computing the boundary junction value by filtering the values of junctions in the neighborhood of each boundary junction. Filter coefficient values are numerically optimized to maximally absorb signals reaching the boundary at incident angles $\theta \leq 80^o$. Remarkable improvement in absorption was achieved.

The new method was invented, optimized, and tested by the present author, and 80% of the text was written by him. The optimization goals and methods were discussed among all the authors.

Publication 3: "Improved Adjustable Boundary Condition for the 2-D Digital Waveguide Mesh"

A novel adjustable boundary condition was created based on the absorbing boundary condition introduced in Publication 2 and on the use of admittance coefficients of interconnections in a mesh. A portion of the signal is reflected at the admittance boundary, and the rest is passed through. The excess signal is then absorbed at the edges of the mesh.

Benefits include more accurate results on all real-valued reflection coefficients with values ranging from -1 to 1, and the possibility to manage the admittance coefficients of interconnections separately for direction-dependent reflection characteristics. The research and writing for this publication were the work solely of the present author.

Publication 4: "Improved Adjustable Boundary Condition for the 3-D Digital Waveguide Mesh"

In this publication, the new method introduced in Publication 3 was extended to cover three dimensions. Like in two dimensions, an adjustable boundary was created using admittance coefficients and an absorbing mesh boundary. However, existing solutions for nonreflective boundaries in three dimensions are not as effective as the solution introduced in Publication 2 for the two-dimensional mesh. Therefore, the obtained results were not as good at low reflection coefficient values as in Publication 3. Still, the new solution outperformed the previous three-dimensional boundary method both in accuracy and in flexibility. The present author was the sole author of this publication.

Publication 5: "Simulation of Room Acoustics Using 2-D Digital Waveguide Meshes"

As the three-dimensional digital waveguide mesh requires heavy computation, ways of approximating room impulse responses with lighter means were studied. In this publication it was shown that a sum of responses of multiple correctly sized two-dimensional meshes provides the most significant low-frequency modes with a small fraction of the computational cost of a full three-dimensional model, though the Q-values and relative amplitudes of the mode peaks are erroneous.

As an acoustic visualization tool, the two-dimensional meshes proved to be superior over a three-dimensional simulation in that a certain layer of the acoustic wave field can be isolated and waves that propagate in other directions are automatically hidden, which makes the visualizations easier to follow.

The idea and simulations for this paper were created by and 70% of the text was written by the present author.

Publication 6: "Frequency-Dependent Boundary Condition for the 3-D Digital Waveguide Mesh"

As the earlier publications deal with only reflection coefficients that are defined with one real valued constant, the aim of this publication was to find a way to model more realistic boundaries with frequency-dependent characteristics. Use of boundary filters has been reported earlier for the wave variable mesh, but only the recent introduction of the Kirchhoff-wave (KW) converter made it possible to connect such filters also to the boundaries of a Kirchhoff variable mesh.

A simplified and more effective structure for KW conversion is introduced in this paper. It was also shown how the new structure is used at the boundaries of a three-dimensional mesh for room acoustics modeling. The present author was alone responsible for the contents of this publication.

Publication 7: "Acoustic Modeling using the Digital Waveguide Mesh"

In this publication, ways of using the digital waveguide mesh method for physical modeling of sound propagation are discussed. The paper includes both a review of previous evolution of the method, and new results on two-dimensional, three-dimensional, and hyperdimensional modeling with application examples.

The present author did the research for and writing of Section D, which discusses the hyperdimensional mesh structure and its uses. This section comprises 20% of the total work for this publication. The idea of a hyperdimensional mesh goes beyond modeling the physical behavior of sound waves. Extending the number of physical dimensions to four or above makes it possible to use small meshes to produce dense and irregular modal structures at high frequencies without low-frequency content.

Publication 8: “Spatial Audio and Reverberation Modeling using Hyperdimensional Digital Waveguide Meshes”

The last publication included in this thesis consistrs of a discussion of hyperdimensional mesh structures and their possible uses that is more thorough than was possible in Publication 7. The use of the structure as a multidimensional reverb is presented, for which the low correlation between different output signals from a hyperdimensional mesh are beneficial. Additionally, synthesis of the reverberations in a lecture hall and in a clavichord soundbox is discussed.

The present author created the hyperdimensional meshes and optimized their uses in the example cases in Section 4. The writing of the text was done collaboratively with all authors, with the present author contributing most in Sections 2, 3, 4.1, and 4.2. The present author’s portion of the total work was 50%.

List of Abbreviations

FDTD	Finite Difference Time Domain
FIR	Finite Impulse Response
K	Kirchhoff (variable)
W	Wave (variable)
1D, 1-D	One dimension, one dimensional
2D, 2-D	Two dimensions, two dimensional
3D, 3-D	Three dimensions, three dimensional
4D, 4-D	Four dimensions, four dimensional

List of Symbols

c	sound velocity
f	frequency
f_{s}	sampling frequency
$h_{\mathrm{Ba1}}, h_{\mathrm{Ba2}}, h_{\mathrm{Ba3}}$	weighting coefficients of the axial junction values
$h_{\mathrm{Bd1}}, h_{\mathrm{Bd2}}$	weighting coefficients of the diagonal junction values
l	index number
L	number of interconnections
$L_{\mathrm{x}}, L_{\mathrm{y}}, L_{\mathrm{z}}$	room dimensions
n	index number
N	number of dimensions
m	index number
p	junction value, sound pressure, displacement
p_{B}	boundary junction value
p_{l}	left-going wave variable value
p_{r}	right-going wave variable value
p_1, p_2, p_3, p_4	values of junctions perpendicular to the boundary, $1, 2, 3$, and 4 steps away from it
R	reflection coefficient
R_{obs}	observed reflection coefficient
T	sampling time interval
Δx	spatial sampling interval
Y	admittance
∂	partial derivate
θ	incident angle

Chapter 1

Introduction

This thesis and the work presented therein relates to the virtual modeling of architectural acoustics. Architectural acoustics includes all phenomena concerning the propagation of sound in rooms [1]. Many approaches are available for simulating sound propagation. There are ray tracing [2], image [3], wave digital filter [4], finite difference [5], and functional transformation methods [6, 7, 8], as well as digital waveguide networks [9], and digital waveguide mesh [10, 11, 12], to mention some, and each leads to different results [13, 14, 15, 16, 17, 18].

The best choice of method for any case of virtual modeling of acoustics can be found, when the benefits and limitations of each method are understood. The geometrical methods, namely ray tracing and image method, are very effective for wavelengths much shorter than the dimensions of the modeled space. The basic solutions of these are limited to cases where such phenomena as diffusion, diffraction, or interference do not occur.

If an approximate solution of the wave equation is searched at a steady state to solve the eigenvalues and eigenfunctions of an enclosure or resonating surface, for example, finite element method [19] or boundary element method [20] may be the most effective choises. Handling of complex geometries is easier with them than with finite difference methods. Finite element method has also been extended to time domain modeling [21], but there the finite difference and functional transformation methods are more used.

In this research, modeling of the actual sound waves is carried out using the digital waveguide mesh technique [10, 22, 23, 11]. The digital waveguide mesh belongs to the group of differential time-domain numerical modeling methods. It can be seen as a special case of much older techniques known as finite difference time domain methods [24, 25, 26], or as a subset of digital waveguide networks.

The digital waveguide mesh models multidimensional wave propagation in discrete time and space. The sound field in the complete computational domain is calculated as it evolves in time, which is especially useful for providing animated displays, producing realistic reverberation models, or for creating dynamic models with moving source or receiver locations or other changing parameters.

As a result of the discretization, the modeling is constrained by an upper frequency limit and finite spatial accuracy [27]. Both of these can be raised by making the mesh more dense, but this causes the computation time to grow. Discrete modeling also causes numerical dispersion error. Directional differences in the wave propagation speed can be minimized by the choice of mesh topology and by the use of interpolated meshes [28, 29, 9, 30]. The remaining frequency-dependent dispersion error can be minimized with frequency warping [31, 32].

Reflection and absorption occur when the sound waves incide against walls, furniture, or other boundaries. The complex changes caused to the phase and magnitude of the sound waves at the boundary depend on the boundary material and structure, as well as on the level, frequency, and propagation direction of the sound.

The digital waveguide mesh method accommodates primarily only one type of boundary. This boundary causes perfect phase-inversing reflections, which are not realistic in architectural acoustics. Approximate solutions of reflection coefficients between 0 and 1 have been found using one-dimensional boundary methods. Erroneous reflections caused by the dimensionality mismatch between the inner mesh structure and the boundary junctions deteriorate the accuracy of these solutions, especially under absorbing conditions [27, 33]. Currently research on methods for modeling boundaries with adjustable shape, diffusion, and reflection characteristics is carried out in many research centers around the world [34, 35, 36, 9, 37, 38, 17].

1.1 Scope of the Thesis

The primary topic of this thesis is the improvement of the modeling of boundaries in digital waveguide meshes. Earlier methods for approximating real boundaries were adopted from 1-D digital waveguides and do not give appropriate results in multidimensional modeling.

Good modeling of the reflection coefficient values was chosen as the primary target for the work presented here. Another important issue, diffusiveness, was left out of the scope of this thesis. Also, it was presumed from the similarity of the mesh structures in 2D and 3D that results obtained in 2D could be easily extended to 3D. Thus, the research was first carried out in 2D.

The scope of this thesis extends to research on the use of meshes with lower or higher dimensionality in approximations of 3-D wave propagation. As the full 3-D model is computationally too heavy and unnecessarily exact for some uses, lighter solutions were pursued for artificial reverberation and visualization use.

1.2 Real-Valued Reflection Coefficients

As a first approximation, reflection coefficients with constant values over the full ranges of simulation frequencies and incident angles were pursued. With the real-valued coefficients set at $-1 \leq R < 0$, the reflected signal should have exactly the opposite phase, and with values $0 < R \leq 1$ it should have exactly the same phase as the signal traveling toward the boundary. Only the magnitude of the reflected signal should change.

New reflection equations were defined for boundary junctions located at the edges of a 2-D mesh to better model values of R in the range $0 \leq R \leq 1$. With earlier techniques, in this range the relative error of the reflection coefficient was below 15% only with values of $R > 0.5$, while equally good results can be obtained with the new method with $R > 0.1$ in a limited frequency band. Still, only values at mesh junctions perpendicular to the boundary are taken into account, so unwanted reflections occur at high angles of incidence. Thus, even though the novel method extends the range of good approximation toward low reflection coefficient values, the residual reflections destroy the model's accuracy when maximal absorption is sought.

The problem was next addressed by defining an absorbing boundary condition for the 2-D mesh that is based on spatial filtering of the junction values close to the boundary. To provide better absorption, the scope of the boundary equation was spread to encompass other incident angles besides the perpendicular. The approach proved to be highly beneficial. With the earlier proposed Taylor series absorbing boundary, sufficient attenuation of 25 dB is obtained only at incident angles below 68.0^{o}, whereas the new method extends the usable range up to 79.5^{o}.

A new way of defining the amount of reflection at a junction was then developed using admittance coefficients of connections between junctions to implement changes in wave propagation media. Outside the simulation area, the mesh is truncated with an absorbing boundary. Reflection coefficient values of $-1 \leq R \leq 1$ can now be implemented with remarkably better accuracy both in 2-D and 3-D simulations. In 2D, error of the reflection coefficient value remains below 0.1 at angles of incidence less than 60^o at frequencies between 0.004 and 0.222 relative to the sampling frequency, and at $60^o \leq \theta < 80^o$ the same accuracy is reached at $0.005 < f/f_s < 0.114$. In 3D, the absolute error value is less than 0.1 at $\theta < 80^o$ within the frequency band $0.01 f_s$ to $0.18 f_s$ at $R > 0.3$ and within $0.01 < f/f_s < 0.23$ at $R < -0.6$. Between these reflection coefficient value ranges the performance of the boundaries is acceptable over narrower frequency bands. In addition, as the admittance values are set separately for each interconnection, direction-dependent characteristics can now be modeled.

1.3 Frequency-Dependent Reflection Coefficients

After reaching a reasonably accurate manner of defining real-valued reflection coefficients for the boundaries, frequency dependent solutions were pursued. The novel structure is based on implementing digital filters at the mesh edges.

Digital filters at the boundaries have been earlier studied for 2-D meshes using traveling wave (W) variables [39]. However, as the wave variable mesh is computationally wieldy when determined for multiple dimensions, the idea was reconsidered only after a structure for online conversion between Kirchhoff (K) variables and W variables was introduced [40]. In the work introduced here, the conversion is further simplified to provide an effective way to connect filters to the boundaries of a K mesh. If a junction were to have m Kirchhoff variable ports, m summations and $m - 1$ unit delays would be saved in comparison to the earlier converter structure.

1.4 Approximation of Room Acoustics Using 2-D Digital Waveguide Meshes

As the computation of acoustics of large halls using dense 3-D mesh structures has been noticed to be a demanding task for currently available personal computers, lighter approximation methods were studied. It was noticed that the most important modes inside cavities with simple geometries can be modeled by setting up three 2-D meshes to match the characteristic lengths of the space. The input and output points are projected on the chosen layers, the 2-D simulations are run separately, and the output signals are summed together.

As this method simulates only 1-D and 2-D standing waves, all the oblique modes are left out of the model. To add those, smaller meshes may be added. Adding one such mesh already raises the quality of the approximation remarkably.

Due to the use of fewer dimensions, the computational cost is reduced by up to more than 90%, depending on the size of the model. Some modes will be missing, but the frequencies of the lowest modes are modeled correctly. As some of the modes are created in multiple meshes and others occur only in one mesh, the frequency peaks of the summed output signal are erroneous in width and amplitude. Despite this, there are situations where this kind of computationally light approximation would be valuable.

Another use for 2-D meshes in modeling of spatial acoustics is found in visualization. In a 2-D mesh, only the waves propagating on the chosen plane will be seen, so the resulting visualizations are much easier to follow than full 3-D visualizations. This way the effects of changing the locations or physical characteristics of sound sources, listeners, or absorbing or diffusing surfaces can be easily investigated.

1.5 Artificial Reverberation With Hyperdimensional Digital Waveguide Meshes

In the practice of artificial reverberation, it is not important to create an exact physical model of some space over the full sound spectrum. Even if the recreation of the acoustics of a real hall or a musical instrument is the goal, it is enough to match some of the lowest modes precisely and to create enough modes at high frequencies to provide a random and dense distribution. Perceptually more important than getting the exact modal frequencies right is to correctly simulate the decay times at each frequency band [41].

The fact that the scattering function of the digital waveguide mesh does not restrict the dimensionality of the model to the limits of our physical world has been noted in earlier work [42, 43]. In the thesis research, hyperdimensional meshes are constructed to create high frequency reverberation. Adding a fourth dimension caused the modal frequencies to be less correlated. The modal density is high over the whole frequency band of the output signal, as the mesh dimensions are short relative to the number of junctions. The computational cost of a small hyperdimensional structure is relatively low, so in addition to being a mathematical curiosity, it could have value as an artificial reverberator.

1.6 Organization of the Thesis

This compendium summarizes the results achieved in the eight attached publications. The results and the author's contributions are summarized on pages 15 - 18. An overview of the work is presented in Chapters 1 - 5, on pages 23 - 54.

After the introduction, an overview of earlier research on digital waveguides is provided in Chapter 2, along with necessary background information on digital waveguide mesh methods. In Chapter 3 the methods used in the current research are described. New results obtained are introduced in Chapter 4, and conclusions are presented in Chapter 5.

Chapter 2

The Digital Waveguide Mesh — An Overview of Earlier Work

2.1 The Digital Waveguide

The digital waveguide method is popular in physically-based sound synthesis [17]. Digital waveguides are mostly used for modeling vibrations of different kinds of strings and tubes [44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56]. They have also been applied in efficient simulations of vibrating membranes [57, 58] and reverberant rooms [10, 59].

Derivation of the digital waveguide method can be initiated from the 1-D wave equation [60, 1]

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}, \tag{2.1}$$

where c denotes the wave speed and p is the displacement at location x.

In the 18th century, d'Alembert published the traveling wave solution to the 1-D wave equation [61]:

$$p(x,t) = p_{\mathrm{l}}(x - ct) + p_{\mathrm{r}}(x + ct), \tag{2.2}$$

where the displacement is now solved as a superposition of two waves, p_{l} and p_{r}, propagating to opposite directions. The concept is illustrated in Fig. 2.1.

In a digital waveguide, the wave values of Eq. 2.2 are sampled at discrete time and space intervals. This is the same approach as was first used in the Kelly-Lochbaum speech synthesis model [62]. Other modeling methods descending from the same basic idea are, for example, the transmission line matrix, the wave digital filter,

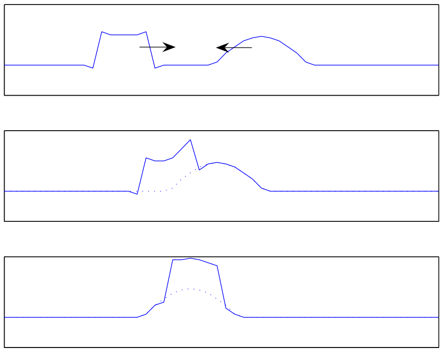


Figure 2.1: The amplitudes of two waves traveling in opposite directions are summed together.

and the finite-difference time-domain methods [24, 63, 64, 65, 66, 67, 5, 17]. The sampling theorem, also known as Nyquist (or Shannon) theorem [68, 69], shows that if the signals p_1 and p_2 are bandlimited to one half of the sampling frequency, f_s, they may be sampled without losing any information. In such a case, Eq. 2.2 can be expressed using discrete valued variables n for time-step and m for location,

$$p(m,n) = p_r[(n-m)T] + p_l[(n+m)T], \tag{2.3}$$

where T is the time interval between samples, defined as $T = \Delta x/c$, where Δx is the spatial interval between sampling points.

The digital waveguide models spatial sampling points located at equal distances that are covered in unit delay time. The traveling waves are passed through two parallel delay lines as depicted in Fig. 1 of Publication 7 [23]. Each discrete location in the delay lines can be seen as a scattering junction, its value $p(m,n)$ being a sum of input signals, which then are passed to the outputs.

For clarity, in the notation used from now on, T is suppressed, location index m is marked as a subscript and waves traveling toward a junction are marked with $^+$ and those traveling away from it with $^-$. The junction ports, each consisting of an input and an output, are indicated with integer indexes. Using this notation, Eq. 2.3 is written as

$$p_m(n) = \sum_{l=1}^{2} p_{m,l}^{+}(n), \tag{2.4}$$

and the output values at each port are solved as

$$p_{m,l}^{-}(n) = p_m(n) - p_{m,l}^{+}(n). \tag{2.5}$$

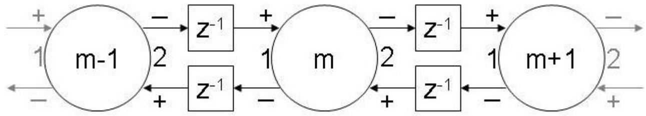


Figure 2.2: Three junctions of a digital waveguide, to illustrate the notation used in Eq. 2.6.

As shown in Fig. 2.2, the outgoing values from one junction are transformed into ingoing values of a neighboring junction by passing them trough the unit delays in the interconnections. Using this with Eqs. 2.4 and 2.5, another way of formulating the digital waveguide can be derived. The values of traveling waves are substituted with a sum of junction values one and two time-steps ago [70]:

$$\begin{aligned}
p_m(n) &= p^+_{m,1}(n) + p^+_{m,2}(n) \\
&= p^-_{m-1,2}(n-1) + p^-_{m+1,1}(n-1) \\
&= p_{m-1}(n-1) - p^+_{m-1,2}(n-1) + p_{m+1}(n-1) - p^+_{m+1,1}(n-1) \\
&= p_{m-1}(n-1) + p_{m+1}(n-1) - [p^+_{m-1,2}(n-1) + p^+_{m+1,1}(n-1)] \\
&= p_{m-1}(n-1) + p_{m+1}(n-1) - [p^-_{m,1}(n-2) + p^-_{m,2}(n-2)] \\
&= p_{m-1}(n-1) + p_{m+1}(n-1) \\
&\qquad - 2p_m(n-2) + [p^+_{m,1}(n-2) + p^+_{m,2}(n-2)] \\
&= p_{m-1}(n-1) + p_{m+1}(n-1) - p_m(n-2). \qquad (2.6)
\end{aligned}$$

The two ways of formulating the digital waveguide update function presented in Eqs. 2.4 and 2.6 are called respectively the *wave* and *Kirchhoff* formulations, or *W* and *K* formulations. In the W formulation, the values passed along the delay lines represent the traveling wave decomposition of the physical variables. Instead, in the K formulation, variables obey conservation laws equivalent to those known as Kirchhoff's laws when applied to energy and charge in electrical circuits [71].

In some references, the K formulation is also called the *FDTD formulation*. This is due to the fact that it is equivalent to the standard finite-difference time-domain method called the *leapfrog recursion* [26]. Different FDTD methods have been used for the modeling of wave propagation for a long time [5]. While the traveling wave formulation provides an exact model at frequencies below half of the sampling frequency, the group of FDTD methods includes many approximate methods. In FDTD simulations, two physical variables such as sound pressure and particle velocity are often used. In contrast, in the digital waveguide scheme only one variable is explicitly needed, although also a multivariable generalization of it has been presented [72].

In this thesis, the starting point is the digital waveguide method with its two equivalent formulations. The terminology used here is in accordance with this.

A structure to convert between the two variable types, depicted in Fig. 3 of Publication 7, was introduced in 2004 by Karjalainen and Erkut [40]. With it, the benefits of both formulations could be joined. Mixed modeling has been discussed also in other publications [26, 9, 73, 74]. With W variables, the structure is numerically more stable [75] and, due to its flexibility, also very usable in models with adjustable, frequency-dependent, or non-linear parameters [76, 77, 78, 79, 80, 81, 82]. In contrast, constructing waveguides of noninteger or time-varying lengths and simulating frequency-dependent effects using digital filters is much harder, or even impossible, with K variables [83]. On the other hand, use of the K formulation offers savings in computational and memory costs, which is a valuable fact when the method is extended to multiple dimensions.

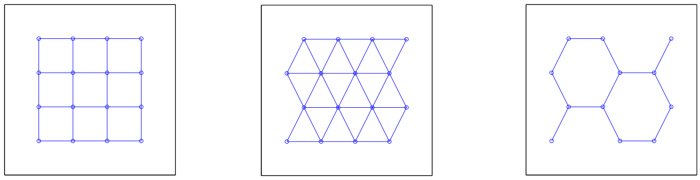
2.2 The Digital Waveguide Mesh

In the same way that digital waveguides provide a simulation of wave propagation in one dimension, a digital waveguide mesh can be constructed to model waves in multiple dimensions [11, 84]. Two-dimensional models are used for simulating the vibrations of membranes and plates [85, 86, 87, 88, 89, 90], but are also used in architectural acoustics simulations [91, 92, 93], and have recently been used in modeling of the vocal tract [94, 95, 96, 97]. The 3-D mesh topologies are used for simulation and auralization of room acoustics and of the reverberations of musical instrument bodies [42, 98, 28, 99, 100, 101], sometimes in combination with other digital waveguide structures of lower dimensionality [102, 103, 104].

A digital waveguide mesh is constructed by adding more ports to each scattering junction and thereby organizing a regular network of junctions interconnected by bidirectional unit delays. The general update equation of a digital waveguide mesh is written with K variables as

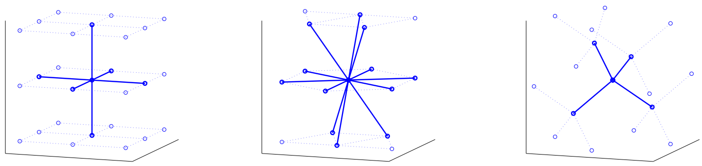
$$p_{\mathrm{c}}(n) = \frac{2\sum_{l=1}^{L} Y_l p_l(n-1)}{\sum_{l=1}^{L} Y_l} - p_{\mathrm{c}}(n-2), \tag{2.7}$$

which is a straightforward extension of Eq. 2.6 for a junction having L ports. The real-valued admittance coefficients, the Y_l of each interconnection, are now introduced. With them, changes in propagation media can be modeled. In a homogeneous mesh they are all identical and are normalized to $Y = 1$. This is why they were neglected earlier in the 1-D equations, even though they are equally useful in 1-D simulations [52, 17]. It should be noted that if the number of connections



(a) Rectilinear (b) Triangular (c) Hexagonal

Figure 2.3: Three ways of organizing a regular mesh in 2D.



(a) Rectilinear (b) Triangular (c) Tetrahedral

Figure 2.4: Three 3-D mesh topologies.

to a junction in a homogeneous mesh is an integer power of 2, the division can be implemented as a bit shift operation and the structure is thus multiply-free.

The sampling frequency, f_s, of a mesh and the nominal wave propagation speed, c, are related to each other as

$$f_\mathrm{s} = \frac{c\sqrt{N}}{\Delta x}, \tag{2.8}$$

where N is the number of dimensions in the mesh and Δx is the spatial sampling interval, or the length of the interconnections [27]. Because of the dispersion error discussed in more detail in Section 2.2.1, the relation is exact only at $f = 0$ and its accuracy depends on the used mesh topology [11, 84, 30].

2.2.1 Multidimensional Topologies

A 2-D mesh having junctions separated with unit delays can be constructed in various ways, as shown in Fig. 2.3 [84, 105]. In 3D, the number of possible choices grows still. Three topologies are chosen as examples in Fig. 2.4. In fact, the theory does not restrict the number of spatial dimensions [43, 27, 106]. However, the looks of hyperdimensional topologies are left for the reader to imagine.

Some of the structures are more often used than the others, which can be justified by examining for each topology the ratio of benefits over computational cost.

The easiest to handle are the rectilinear topologies. The regular matrices match well with structures offered by computer programming languages and the general shapes of simple membrane and room models.

In a rectilinear topology, the unit length that the signal optimally travels per time-step is the distance from a junction to its diagonal neighbor, as the nominal wave propagation speed in the mesh is relative to $1/\sqrt{N}$ where N is the number of dimensions [27]. Whatever the input/output junctions respectively chosen as excitation/measurement point, the impulse response contains zeros interleaved to non-zero sample values. This is because the signal runs from any input junction to any output junction through routes that always have either only odd or even numbers of unit delays [32]. Hence, the frequency response mirrors at one quarter of the sampling frequency, f_s.

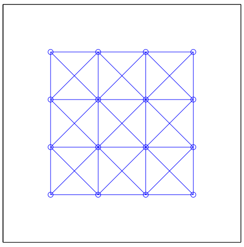
A computational error that occurs in multidimensional digital waveguide structures is known as numerical dispersion. For example, in a 2-D rectilinear mesh, the spatial distance from a junction to its first diagonal neighbor is $\sqrt{2}$ times the distance to an axial neighbor. However, the signal travels along the orhogonal steps from a junction to its axial neighbors in one time-step and to a diagonal neighbor, as well as to a second axial neighbor, in two time-steps. Thus, the signal reaches locations at physical distances of $\sqrt{2}$ spatial steps and 2 spatial steps away from the source at the same time instant.

The difference in the physical distances and signal path lengths causes a numerical error that is dependent on both the direction of wave propagation and the frequency of the signal [11, 84, 30]. The error is largest at high frequencies and in the axial directions of the mesh. The error can be made equal in all directions by using an interpolated mesh scheme, where the values at unit delay distance from a junction are treated as weighted sums of the values of the neighboring junctions and the center junction itself, as depicted in Fig. 2.5(a) [107, 108]. After this, the remaining frequency-dependent error can be rectified using the frequency warping technique [31, 109, 110, 111, 112, 113].

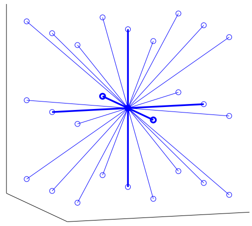
In other than rectilinear topologies, the dispersion errors are less dependent on the direction, as the physical distances of junctions better match the signal path lengths [114, 30]. However, these errors can still be reduced by frequency warping [115]

The triangular mesh is much used in 2-D modeling, as it generates the least directional errors [35, 116]. Still, because of its efficiency also the more sparse hexagonal mesh is used, and the rectilinear topology is used for its intuitivity [28].

In 3D, the most efficient structure is the tetrahedral one [30]. Again, in some cases a denser topology, such as hexagonal close packed structure, may be more favorable. Due to their simple geometry, both interpolated and noninterpolated versions of the



(a) 2-D



(b) 3-D

Figure 2.5: In addition to the axial connections, in the interpolated rectilinear topologies the junctions are connected with their diagonal neighbors.

rectilinear topology are also widely used.

2.3 Boundary Conditions

The effects that walls, furniture, and other boundaries have on the sound propagation are manifold. Changes in the magnitude and phase of the signal cause total or partial reflection and absorption, diffraction, or diffusion. The boundaries may be locally or nonlocally reactive, or an incident sound wave may cause vibrations or dislocations of the boundary, causing a dynamic change in the wave field.

In 1-D digital waveguides the boundaries are relatively easy to define. At any given time instant, there is only one signal sample coming to the end of a delay line. That signal is convolved with an appropriate filter function and fed to the other parallel delay line passing the signal in the opposite direction.

Unfortunately, in multidimensional meshes the case is not that simple. The equations of the digital waveguide mesh offer only one exact solution at the boundary: the phase-inversing total reflection. All other reflection characteristics require construction of special structures or require implementing particular boundary equations at the edges of meshes.

After the numerical errors inherent in the digital waveguide mesh method were extensively studied and it was seen to be a promising tool for sound propagation modeling in multiple dimensions, the boundary conditions became the interest of many researchers. In this dissertation work, reflection and absorption characteristics of the mesh boundaries were studied, whereas phase phenomena are discussed only very briefly.

As a basic solution for manipulating the reflection characteristics, 1-D boundary

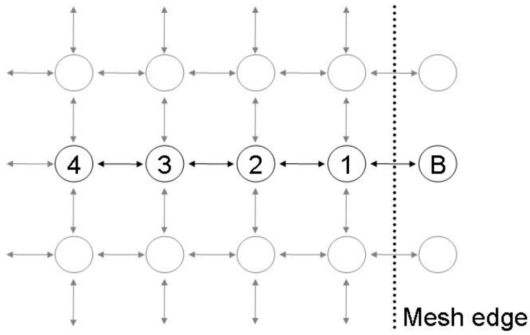


Figure 2.6: Junctions in a line perpendicular to the boundary are used in a Taylor-series absorbing boundary condition.

junctions can be implemented at the edges of a multidimensional mesh. The boundary junction value is computed as

$$p_{\mathrm{B}}(n) = (1+R)p_1(n-1) - Rp_{\mathrm{B}}(n-2), \tag{2.9}$$

where R is the desired reflection coefficient value $-1 \leq R \leq 1$, and indices B and 1 denote the boundary junction and its perpendicular neighbor, respectively [27]. The performance of this approach depends on the mesh topology. It implements the only exact solution, as the junction accepts no inputs with $R = -1$. However, it has been noted to cause unwanted reflections at low absolute values of $R \approx 0$.

The absorbing boundary condition was improved by Murphy and Mullen [33]. They suggested expanding the rule obtained from Eq. 2.9 with $R = 0$,

$$p_{\mathrm{B}}(n) = p_1(n-1), \tag{2.10}$$

into a Taylor series of the values of the junctions perpendicular to the boundary, as depicted in Fig. 2.6. For example, the third-degree Taylor series expansion of the right hand side of Eq. 2.10 would result in

$$p_{\mathrm{B}}(n) = \frac{8}{3}p_1(n-1) - \frac{5}{2}p_2(n-2) + p_3(n-3) - \frac{1}{6}p_4(n-4). \tag{2.11}$$

Boundaries constructed this way produce significantly better absorption than the original solution. However, it is still only a 1-D solution. Thus, signals encountering the boundary at large incident angles are not properly absorbed.

The so-called *perfectly matched layer* is a known effective absorbing boundary structure for the FDTD schemes [117, 118]. As it uses two physical propagation variables and requires large boundary layers around the simulation area, it was judged to be an unsuitable starting point for the development of absorbing boundaries for the digital waveguide mesh.

Another area of research is to look for a solution to the simulation of real boundaries by defining frequency-dependent boundary conditions. Setting up digital filters for the boundary has been suggested for the W mesh [39]. These filters have only 1-D connectivity to the mesh, so their performance is significantly deterioated at non-perpendicular angles of incidence [89]. The study of filters at the boundaries of multidimensional meshes started to gain interest again only after a conversion method between traveling wave variables and K variables was found [40, 26]. Wave variable junctions with connections to filters could now be implemented at the boundaries of a computationally more affordable K mesh [73].

In addition to reflection magnitude control, structures have been created for managing the exact boundary location and the diffusiveness of the reflected signal. In fine tuning the boundary location, fractional delays [87] are used in place of unit delays to break the regular grid. Diffuse boundaries have been constructed by mimicking a real-world diffuser construction with different lengths of delay lines connected to the mesh boundary [37] or with variable rotation of the output values of a junction [38].

Chapter 3

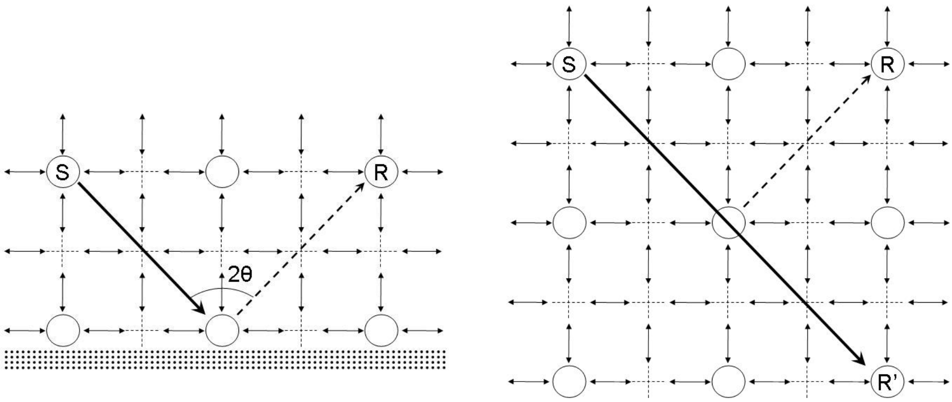
Methods for Testing New Structures

The boundary methods developed during this thesis work were mostly tested against theoretical goals. When measured values were used as the reference, the reverberation time in a hall was chosen instead of the direct reflection characteristics of a wall structure. Two reasons led to this approach: Firstly, there are no true real-world references for the simplest boundary solutions with reflection coefficients having constant values over all simulation frequencies. Secondly, exact simulation of a real-world boundary over wide ranges of frequencies and angles of incidence would have required extensive tuning of filter parameters. This work would have been invaluable for the main purpose, namely the investigation of the novel boundary structures.

The computation of the models used in this thesis was performed using MathWorks Matlab on a PC computer running on a Linux operating system. The digital waveguide meshes were implemented as matrices with dimensionality $N + 1$, where N is the number of spatial dimensions in the mesh and one dimension was added to record the state of the mesh at the necessary number of successive time instants.

3.1 Computing the Magnitude of a Reflected Signal

Most often, the magnitude of a reflected signal was needed to test the reflection coefficient value of the boundary structure currently under investigation. Aim of the research was to minimize the recorded error caused by deviation in the reflection magnitude at the point of specular reflection or by diffuse reflections occurring at



(a) (b)

Figure 3.1: The test setup: a) The mesh was initialized at source point S and the output was recorded at receiver points at the same distance from the tested boundary. b) For reference, a perfect reflection was simulated by receiving the signal at mirror locations R' when the mesh was made large enough to prevent any unwanted reflections.

any boundary junction.

The input was fed into a junction located close to the tested mesh edge and far from any other boundaries. The output signal was received at junctions located at equal distance from the tested boundary as shown in Fig. 3.1.

The direct signal from the input to the outputs was computed in a similar setup, but in a larger mesh having no boundaries close to the line of output points. Also, the signal was received in a line of mirror output points, R', two times further away from the line of output points than the tested boundary in the smaller mesh.

The direct signal was subtracted from the output signals received at points R as indicated in Fig. 3.1. For absorbing boundary conditions, the remaining reflected signal magnitude was recorded as error. For reflective boundaries, the result was compared to the respective output signals at points R', which represented the perfect specular reflection. The reference signal value was multiplied with the goal value of the reflection coefficient before comparing the two signal magnitudes and thus determining the error.

Mostly, the comparison was executed in the frequency domain, so the resulting reflection coefficient value was studied as a function of frequency and angle of incidence, but the signal phase was not recorded. When needed, the delay caused by the boundary was computed separately, again by comparing the reflected signals to the signals recorded at mirror image locations.

3.2 Comparing Simulated and Measured Signals

As the work continued from studies of separate boundary structures to the approximation of impulse responses of an actual room, the simulation results had to be compared with measurement data.

The data from one measurement series performed at lecture hall T3 at the Helsinki University of Technology was used as a reference. The room dimensions are marked in Fig. 4 of Publication 6. The impulse response of the room had been measured earlier by Tapio Lokki [119]. Both the frequency responses and decay times per octave were computed from the measurement data, and the simulation results were compared to these.

Chapter 4

Results

As a result of the work for this thesis, new boundary structures have been introduced for the digital waveguide mesh method. Accuracy of the boundary conditions with real-valued reflection coefficients $-1 \leq R \leq 1$ was increased in the work in Publications 1, 3 and 4. Special attention was given to absorbing boundaries with $R = 0$ in Publication 2. Also, instead of concentrating only on accuracy, the flexibility of the structures in modeling direction-dependent boundary conditions was addressed. Next, more realistic boundary conditions were pursued in Publication 6 by developing a structure that enables implementation of digital filters at the mesh boundaries.

In addition to the boundary methods, computationally lighter structures for approximating the room impulse response were studied. In Publication 5, multiple 2-D meshes were used for the purpose. Finally, the mesh was extended to hyper-dimensions, and possible uses of such nonphysical structure were investigated in Publication 7.

4.1 Absorbing Boundary Conditions

As mentioned in the review in Section 2.3, 1-D boundary conditions were earlier used also in multidimensional digital waveguide meshes. It was noted that the most error is created with the smallest absolute values of reflection coefficients. The unwanted reflections are caused by the change in dimensionality when going from the multidimensional equations of the inner mesh to the 1-D equations at the boundary junctions.

A 2-D absorbing boundary condition has been created to solve this problem, as

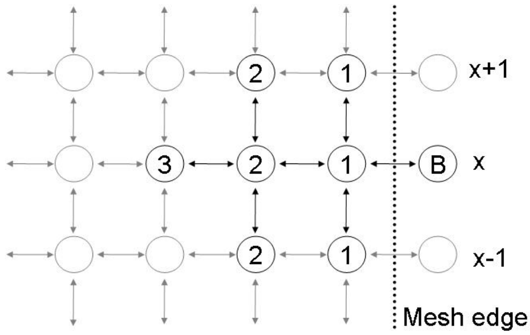


Figure 4.1: In the spatial-filter based absorbing boundary, junctions in a line perpendicular to the boundary and one step to the sides of these junctions are taken into account when updating the boundary junction value.

described in Publication 2. It is based on the best 1-D solution – the Taylor series boundary condition [33] – where the values of multiple junctions in a line perpendicular to the boundary are used when computing the value of each boundary junction as shown in Eq. 2.11 and Fig. 2.6.

In the new solution, the weights of the junction values are spread so that also the values of junctions one step lateral to the perpendicular line are taken into account as shown in Fig. 4.1. This is done to predict the waves coming toward the boundary from directions other than the perpendicular one. The ratios of spatial spreading of the weights have been optimized numerically using a Nelder-Mead algorithm, as discussed in detail in Publication 2, to reach the minimal amount of reflected signal. The resulting equation

$$\begin{aligned} p_{\mathrm{B}}(n) = {} & h_{\mathrm{Ba1}} p_{1,x}(n-1) + \frac{h_{\mathrm{Bd1}}}{2}[p_{1,x+1}(n-1) + p_{1,x-1}(n-1)] \\ & + h_{\mathrm{Ba2}} p_{2,x}(n-2) + \frac{h_{\mathrm{Bd2}}}{2}[p_{2,x+1}(n-2) + p_{2,x-1}(n-2)] \\ & + h_{\mathrm{Ba3}} p_{3,x}(n-3), \end{aligned} \tag{4.1}$$

where h_{Ba} and h_{Bd} are the weights of the axial and of the diagonal junctions, respectively, with values listed in Table 4.1.

The resulting reflection magnitude is diminished remarkably, as can be seen by comparing Figs. 3 and 4 in Publication 2. Good attenuation of at least -25 dB is reached at angles of incidence below 79.53°. In comparison, for the second-order Taylor series boundary the limit is 68°.

Coefficient	Value
h_{Ba1}	2.420878
h_{Ba2}	-2.338081
h_{Ba3}	0.9080989
h_{Bd1}	0.4859106
h_{Bd2}	-0.4768362

Table 4.1: The coefficient values for the absorbing boundary condition described in Eq. 4.1.

4.2 Adjustable Boundary Conditions

Adjustable boundary conditions have played a major role in digital waveguide mesh research after the numerical errors in the modeling of wave propagation in homogeneous media were systemically studied [115, 31, 113]. For acoustics modeling, it would be most convenient to have a general boundary structure that could be used as a model for many different kinds of surfaces by changing of intuitively understandable coefficient values. The pursuit of such a boundary structure was undertaken in this research work, as described in the following.

4.2.1 Real-Valued Reflection Coefficients in 2D

At the start of the work for this dissertation, the existing boundary solutions used in multidimensional meshes were evaluated. Soon it was noticed that the equation of the 1-D solution for the perfectly reflecting case, $R = 1$,

$$p_{\mathrm{B}}(n) = 2p_1(n-1) - p_{\mathrm{B}}(n-2), \tag{4.2}$$

is very similar in form to the first-degree Taylor series solution for the non-reflecting case, $R = 0$ [33],

$$p_{\mathrm{B}}(n) = 2p_1(n-1) - p_2(n-2). \tag{4.3}$$

An assumption was made that reflection coefficient values between 0 and 1 could be modeled by R-weighted linear combination of the absorbing and reflecting solutions. The resulting new rule for adjusting R is introduced in Publication 1 by combining Eq. 4.2 with Eq. 4.3:

$$\begin{aligned} p_{\mathrm{B}}(n) &= R(2p_1(n-1) - p_{\mathrm{B}}(n-2)) + (1-R)(2p_1(n-1) - p_2(n-2)) \\ &= 2p_1(n-1) - Rp_{\mathrm{B}}(n-2) - (1-R)p_2(n-2). \end{aligned} \tag{4.4}$$

The observed reflection coefficients, R_{obs} are closer to the desired value of R than those obtained with the earlier solution (Eq. 2.9) especially at low values of R, which can be seen by examining Figs. 3 and 4 in Publication 1. Only at low frequencies does the error still reach unacceptably high values. As a solution, at frequencies below $0.09f_{\mathrm{s}}$ Eq. 4.4 is replaced with

$$\begin{aligned} p_{\mathrm{B}}(n) =& 2p_1(n-1) - \frac{R}{2}(p_{\mathrm{B}}(n-1) + p_{\mathrm{B}}(n-3)) \\ & - \frac{1-R}{2}(p_1(n-1) + p_3(n-3)), \end{aligned} \quad (4.5)$$

where the junction values in the last two terms of Eq. 4.4 have been replaced with mean values of two junction values. The improvement at low frequencies can be seen by comparing Figs. 4 and 5 in Publication 1. The simulation has to be limited to either frequencies below or above 0.09 times the sampling frequency, and the more appropriate of the boundary equations (Eq. 4.4 or Eq. 4.5) has to be used depending on the frequency band.

Publication 3 provides the second solution for adjustable boundary conditions in 2D. It has been created by using the admittance coefficients introduced in Eq. 2.7 to construct reflective junctions, and by using the absorbing boundary condition (Eq. 4.1) to truncate the mesh.

The admittance coefficients can be defined separately for each interconnection of the junctions using the desired reflection coefficient values:

$$Y = \frac{1-R}{1+R}. \quad (4.6)$$

For example, at a wall location, a junction in an interpolated 2-D mesh could have the reflection coefficient value R at three junctions pointing inward in the mesh, and the other five junctions would have $Y = 1$, causing no reflection. The equation would then be written as

$$p_{\mathrm{c}}(n) = 2\frac{\sum_{l=1}^{3} Y p_l(n-1) + \sum_{l=4}^{8} p_l(n-1)}{3Y+5} - p_{\mathrm{c}}(n-2). \quad (4.7)$$

Part of the signal will then be reflected at the junction and part of it will pass through. When walls of a room are considered, the transmitted part of the signal is of no interest and will be absorbed at the edges of the mesh as depicted in Fig. 4.2. A homogeneous boundary layer of at least four junctions has to be left between the admittance change and the mesh edge to provide a stable absorbing boundary. The accuracy of the method is better than that of the earlier method as shown in Figs. 2, 3, and 4 of Publication 3.

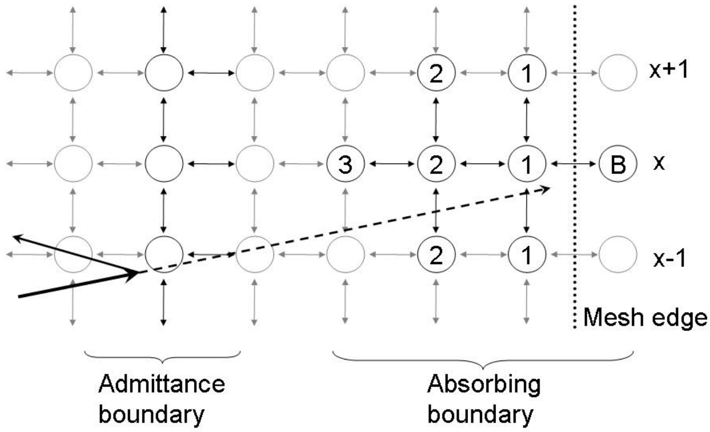


Figure 4.2: The admittance boundary reflects part of the signal, and the rest is absorbed with a spatial-filter based absorbing boundary at the mesh edge.

4.2.2 Real-Valued Reflection Coefficients in 3D

The reflection coefficients in the 3-D meshes were earlier produced just as for 2-D meshes, using 1-D boundary structures. The study in Publication 4 revealed the significant error caused by the structural change at the boundary. Thus, the solution described in Publication 3 for the 2-D mesh was adopted for the 3-D mesh.

The reflection coefficients of each port of a junction at a boundary location are set by admittance coefficients. Depending of the coefficient values, part of the signal is reflected back and part of it passes through the junction. The residue signal is then absorbed outside the simulation area limited by the admittance boundary. Unfortunately, as yet there is no absorbing boundary for the 3-D mesh that is as good as the one developed for 2-D meshes in Publication 2. Therefore, the 1-D solution has to be used at the mesh edges. The obtained results are nevertheless much better than those derived with the earlier boundary structure, as seen in Fig. 2 of Publication 4.

4.2.3 KW Converter and Digital Filters at the Boundary

In addition to real-valued reflection coefficients, frequency dependent boundaries are needed for realistic reflection modeling. There are known ways of manipulating the traveling wave variables with digital filters to obtain the wanted magnitude and phase characteristics. Unfortunately, implementing a whole multidimensional mesh using W variables consumes too many computational and memory resources to be realizable with current computers.

Implementation of the inner mesh using the computationally lighter K variables and implementation of the boundaries using W variables became possible after the introduction of KW conversion [40]. In the novel work discussed in Publication 6, the conversion is further simplified. The converter block is merged into a scattering junction as illustrated in Fig. 1 of Publication 6. The novel solution results in computations that are more straightforward.

The novel boundary solution involves matching the coefficients of a boundary filter to the wanted reflection characteristics. In this case filter coefficients were numerically optimized to match the reverberation times of the studied model to the measured values for lecture hall T3 at Helsinki University of Technology. In this work, low-order FIR filters were constructed as examples. Thus, exact locations and frequency-dependent reflection coefficient values were not pursued. With high-order filters the magnitude and phase of the reflected signal could be manipulated to simulate the effects of real materials.

4.3 2-D Meshes in Spatial Sound Propagation Modeling

The simulation of a large space using a full 3-D mesh on a wide frequency band was noticed to be too demanding a task for current personal computers. Researchers have earlier on proposed the use of digital waveguide models with fewer dimensions as a computationally lighter tool for visualization and low-precision modeling [93, 59, 57]. The novelty suggested in Publication 5 is to combine the outputs of multiple 2-D meshes as a simulation of wave propagation in 3D.

A rectangular room is discussed as a basic example. In such a space, standing waves occur at frequencies

$$f = \frac{c}{2}\sqrt{\left(\frac{n_{\mathrm{x}}}{L_{\mathrm{x}}}\right)^2 + \left(\frac{n_{\mathrm{y}}}{L_{\mathrm{y}}}\right)^2 + \left(\frac{n_{\mathrm{z}}}{L_{\mathrm{z}}}\right)^2}, \tag{4.8}$$

where L_{x}, L_{y}, and L_{z} are the room dimensions and n_{x}, n_{y}, and n_{z} are integer indexes of the modal frequencies along each dimension [1]. For any standing wave occurring at a surface perpendicular to two of the six walls, at least one of the indices equals zero. These are the lowest and often most important modes. Because the modal density grows at the high frequencies, human perception is not able to separate the modes from each other, so determinating the exact modal frequencies is not that important [120, 41].

Two-dimensional meshes can be designed to match the dimensions of cross-sections of a space so that all the 1-D and 2-D standing waves are simulated as sketched in Fig. 4.3. When input and output points are projected onto 2-D planes, simulations

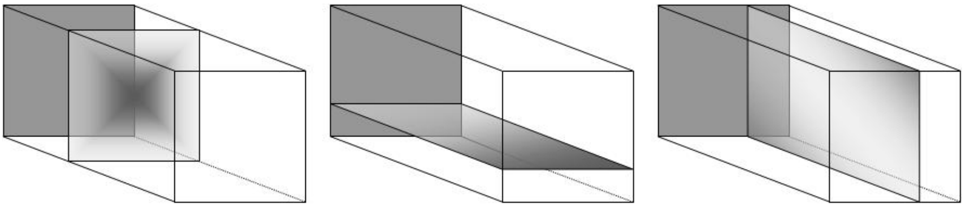


Figure 4.3: Two-dimensional meshes at the cross-sections of a 3-D space for modeling low frequency standing waves.

are run separately and output signals are summed together, and an approximation of the modal structure is created. As discussed in Publication 5, the relative heights and widths of the modal peaks are erroneous and all the 3-D modes are neglected in such a model.

To extend the number of modeled modes, smaller 2-D meshes can be added to the simulation. To match with the physical dimensions of the modeled space, one of the edges of such an additional mesh could have a length equal to L_{x} and another edge length could be a combination of the two other dimensions, L_{y} and L_{z}:

$$L_{\mathrm{yz}} = \left(\frac{1}{L_{\mathrm{y}}^2} + \frac{1}{L_{\mathrm{z}}^2} \right)^{-\frac{1}{2}} . \tag{4.9}$$

This way two of the indices in Eq. 4.8, namely n_{y} and n_{z}, are joined to get equal values. Already the addition of one smaller mesh of this kind enhances the simulation accuracy, whereas the computational requirements of the model are still only a fraction of the demands of a full 3-D model.

Another use for the 2-D meshes in spatial sound propagation modeling is found in visualization. As only the waves propagating on the chosen plane are seen at any given time, aspects hidden from the eye in a 3-D visualization become visible [121, 122, 123, 124]. As 3-D standing waves are typically weak and occur mostly at high frequencies, 2-D models are good for investigating the behavior of the most prominent modes, as shown in Figs. 3 and 4 of Publication 5. Seeing the wave propagation paths in 2-D may reveal sources of disturbing reflections, for example, and the locations of sound sources, listeners, or wall constructions may be optimized accordingly.

4.4 Hyperdimensional Mesh

The main idea behind hyperdimensional mesh topologies is that both Eqs. 2.7 and 4.8 can be extended to any number of spatial dimensions when the limits of our physical world are ignored. The procedure for constructing a hypermesh is equal to that of extending from 1-D to 2-D and 3-D meshes: More connections are added to each junction.

The research presented here focuses on the usage of the hypermesh as a high-frequency reverberation algorithm. At high frequencies, the most important feature is the frequency-dependent reverberation time. Modes are not perceived separately due to the high density and irregularity of the modal frequencies, so exact modeling is not needed [125, 126, 43, 127, 128, 120, 41]. This allows the use of non-physical structures as efficient reverberation algorithms. Widely used methods with good quality are constructed using comb and allpass filters, and feedback delay networks [129, 18]. Since Schroeder's pioneering publications, many efficient methods have emerged [125, 130, 126, 131, 132, 133, 134].

It is not the aim of this research to present a hypermesh reverberator that would be beneficial relative to the aforementioned methods in the view of computational cost. Hypermeshes are investigated relative to meshes of lower dimensionality. The benefits of extending the dimensionality as proposed in earlier work [42, 43] are studied in practice.

Every added dimension comes with a separate set of modal frequencies. This causes more irregularity to the modal structure if mesh dimensions are chosen so that the multiples of the base frequencies do not coincide. Maximal irregularity is achieved with dimensions chosen close together from a prime number series, as described in Publication 7. Because the number of junctions is kept constant while adding dimensions, the path lengths along each dimension diminish accordingly. This causes the modes to pack closer around $f_s/4$. As only short propagation paths are implemented, no sparse low-frequency modes are present in a typical hypermesh response, as seen in Fig. 10 of Publication 7.

The high number of uncorrelated signal paths in a mesh cause the outputs recorded at two locations to be incoherent, which is an important factor when regarding the perceived spaciousness of a sound [1]. Thus, hyperdimensional meshes could find use as a multichannel reverb algorithm as discussed in Section 4.1 of Publication 8. As exact modes are not needed, the user is not restricted to use the response up to the mirroring frequency. The efficiency of the model grows as the output can be used up to half of the sampling frequency.

Low-frequency modes are not created in small hypermeshes. This enables their use without highpass filtering. The low frequencies can then be modeled using some exact method. This approach is taken in the application examples of modeling the reverberation in a lecture hall and in a clavichord soundbox, which are respectively discussed in Sections 4.2 and 4.3 of Publication 8.

In the first example, a 3-D mesh is used for the low-frequency modes and two 4-D meshes are used for the frequencies above 1 kHz. The reverberation times of each mesh are set using FIR filters at the boundaries and the responses of the three parts of the multirate system are combined.

In the second case, the most prominent modes of the recorded soundbox response are isolated at frequencies below 500 Hz and a similar multirate system is created as in the previous case. This time the low-frequency modes are recreated using separate filters instead of by implementing a physical model of the structure.

Chapter 5

Conclusions and Future Directions

Methods to define boundary conditions in 2-D and 3-D digital waveguide meshes were developed in the research described in this thesis. The new methods enhanced the models by offering more accurate and easier ways to define the reflection coefficient values at the boundaries. First, boundary structures with unique real-valued reflection coefficients were developed. Second, direction-dependent and frequency-dependent methods were introduced.

As full digital waveguide mesh models of large spaces over wide frequency bands are computationally wieldy, approximation methods were discussed in the latter part of the thesis. Use of 2-D and hyperdimensional meshes was seen to be beneficial in low-accuracy physical modeling, visualization of the wave propagation, and high-frequency artificial reverberation.

This thesis summarizes work undertaken over the course of three years. The methods can be seen to develop from a simple solution for reflection coefficient values ranging from 0 to 1, to frequency-dependent boundaries and acoustical approximations. Still, all the solutions introduced in the publications are valid in their respective usages.

The digital waveguide mesh method is a topic of active research in universities and commercial companies around the world. Physical modeling of musical instruments and architectural acoustics benefits from the wave propagation model. The boundary methods introduced in this thesis are important steps toward the simulation of real boundaries. The focus here was on the magnitude of the signal, while researchers at other universities have concentrated on other phenomena such as diffusion, for example.

There is still work to do in model accuracy, efficiency, and in coherently linking together the results of many researchers. Besides optimizing the model, work should be done on measuring the reflection characteristics of real-world objects and boundaries, as reliable references are needed when developing a physical model of acoustical wave propagation.

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